Trigonometric functions

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \tfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha) = 2\sin\alpha\cos\alpha;\ \tan(2\alpha) = \tfrac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha =$$
$$= 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$
$$\sin\alpha + \sin\beta = 2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2} = \pm\sqrt{\tfrac{1-\cos\alpha}{2}} \qquad \cos\tfrac{\alpha}{2} = \pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$
$$\tan\tfrac{\alpha}{2} = \tfrac{\sin\alpha}{1+\cos\alpha} = \tfrac{1-\cos\alpha}{\sin\alpha} = \pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha = \tfrac{\tan^2\alpha}{1+\tan^2\alpha} \qquad \sin\alpha = \tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha = \tfrac{1}{1+\tan^2\alpha} \qquad \cos\alpha = \tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x + b\cos x =$$
$$= \operatorname{sgn}(a)\sqrt{a^2+b^2}\sin\left(x + \operatorname{atan}\tfrac{b}{a}\right)$$
$$= \operatorname{sgn}(b)\sqrt{a^2+b^2}\cos\left(x - \operatorname{atan}\tfrac{a}{b}\right)$$
$$\operatorname{acos} x + \operatorname{asin} x = \tfrac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$$
$$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$$
$$\tanh(x+y) = \tfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\left(\begin{smallmatrix}\sinh x\\ \cosh x\end{smallmatrix}\right) = \tfrac{1}{2}\left(\begin{smallmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{smallmatrix}\right)$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\cosh^2 x = \tfrac{1}{1-\tanh^2 x}$$
$$\sinh(ix) = i\sin x;\ \cosh(ix) = \cos x$$
$$\left(\begin{smallmatrix}\operatorname{asinh} x\\ \operatorname{acosh} x\end{smallmatrix}\right) = \log\left(x + \sqrt{x^2 + \left(\begin{smallmatrix}1\\ -1\end{smallmatrix}\right)}\right)$$
$$\operatorname{atanh} x = \tfrac{1}{2}\log\tfrac{1+x}{1-x}$$

Areas

$$\text{triangle: } \sqrt{p(p-a)(p-b)(p-c)}$$
$$\text{quad: } \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd\cos^2\tfrac{\alpha+\gamma}{2}}$$
$$\text{Pick: } A = \left(I + \tfrac{B}{2} - 1\right)A_{\text{check}}$$

Combinatorics

$$P_n^{(m_1,m_2,\dots)} = \tfrac{n!}{m_1!m_2!\dots} \qquad C_{n,k} = \binom{n}{k} = \tfrac{n!}{k!(n-k)!} \qquad C'_{n,k} = \binom{n+k-1}{k}$$
$$D_{n,k} = \tfrac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C} = \tfrac{ABC-AB}{9\times C \quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}} = \sqrt{\tfrac{a+\sqrt{a^2-b}}{2}} \pm \sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{k=0}^n a^k = \tfrac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{k=0}^n ka^k = \tfrac{a}{(1-a)^2}\left(1 + na^{n+1} - (n+1)a^n\right)$$
$$\textstyle\sum_{x=1}^n x^3 = \left(\sum_{x=1}^n x\right)^2 = \tfrac{1}{4}n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2 = \tfrac{1}{6}n(n+1)(2n+1)$$
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$
$$(a+b)^n = \textstyle\sum_{k=0}^n \binom{n}{k} a^{n-k}b^k$$
$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\Gamma(1+z) = \textstyle\int_0^\infty t^z e^{-t}dt = z!$$
$$n! \approx \left(\tfrac{n}{e}\right)^n\sqrt{2\pi n};\ \log n! \approx n(\log n - 1)$$
$$\tfrac{\mathrm{d}}{\mathrm{d}x}\textstyle\int_0^x g(x,y)\mathrm{d}y = \int_0^x \frac{\partial g}{\partial x}(x,y)\mathrm{d}y + g(x,x)$$
$$\sqrt{z} = \sqrt{\tfrac{|z|+\operatorname{Re} z}{2}} \pm i\sqrt{\tfrac{|z|-\operatorname{Re} z}{2}}$$
$$\langle \operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle = \tfrac{1}{2}\operatorname{Re}(ab^*)$$
$$\operatorname{erf}(x) = \tfrac{2}{\sqrt{\pi}}\textstyle\int_0^x e^{-t^2}\mathrm{d}t$$
$$f^{(n)}(z_0) = \tfrac{n!}{2\pi i}\textstyle\oint \frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$
$$f(z) = \textstyle\sum_{k=-\infty}^\infty \left(\frac{1}{2\pi i}\oint \frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\right)(z-z_0)^k$$
$$\operatorname{sinc} x = \tfrac{\sin x}{x}$$
$$\zeta(s) = \textstyle\sum_{n=1}^\infty \frac{1}{n^s};\ \operatorname{Li}_s(z) = \sum_{n=1}^\infty \frac{z^n}{n^s}$$

Derivatives

$$\tan' x = 1 + \tan^2 x$$
$$\cot' x = -1 - \cot^2 x$$
$$\operatorname{atan}' x = -\operatorname{acot}' x = \tfrac{1}{1+x^2}$$
$$\operatorname{asin}' x = -\operatorname{acos}' x = \tfrac{1}{\sqrt{1-x^2}}$$
$$(a^x)' = a^x\ln a$$
$$\log_a' x = \tfrac{1}{x\ln a}$$
$$\cosh' x = \sinh x$$
$$\tanh' x = 1 - \tanh^2 x$$
$$\operatorname{atanh}' x = \operatorname{acoth}' x = \tfrac{1}{1-x^2}$$
$$\operatorname{asinh}' x = \tfrac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}' x = \tfrac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})' = \tfrac{1}{f'(f^{-1})}$$
$$\left(\tfrac{1}{x}\right)' = -\tfrac{\dot{x}}{x^2}$$
$$\left(\tfrac{x}{y}\right)' = \tfrac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)' = x^y\left(\dot{y}\ln x + y\tfrac{\dot{x}}{x}\right)$$
$$\tfrac{\partial(x,y)}{\partial(u,v)} := \tfrac{\partial x}{\partial u}\tfrac{\partial y}{\partial v} - \tfrac{\partial x}{\partial v}\tfrac{\partial y}{\partial u}$$
$$\tfrac{\partial(x,y)}{\partial(u,y)} = \tfrac{\partial x}{\partial u}\Big|_y = -\tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_x$$
$$\tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_x\tfrac{\partial u}{\partial x}\Big|_y = -1$$
$$\tfrac{\partial x}{\partial u}\Big|_y = \tfrac{\partial x}{\partial u}\Big|_v - \tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_v$$
$$\tfrac{\partial x}{\partial u}\Big|_v = \tfrac{\partial x}{\partial y}\Big|_v\tfrac{\partial y}{\partial u}\Big|_v$$

Integrals

$$\textstyle\int x^a = \frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x = \frac{a^x}{\ln a}$$
$$\textstyle\int \frac{1}{x} = \ln|x|$$
$$\textstyle\int \tan x = -\ln|\cos x|$$
$$\textstyle\int \cot x = \ln|\sin x|$$
$$\textstyle\int \frac{1}{\sin x} = \ln\left|\tan\frac{x}{2}\right|$$
$$\textstyle\int \frac{1}{\cos x} = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$
$$\textstyle\int \ln x = x(\ln x - 1)$$
$$\textstyle\int \tanh x = \ln\cosh x$$
$$\textstyle\int \coth x = \ln|\sinh x|$$
$$\textstyle\int \frac{1}{\sqrt{a^2-x^2}} = \operatorname{asin}\frac{x}{a}$$
$$\textstyle\int \frac{1}{a^2+x^2} = \frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy = x\int y - \int(\dot{x}\int y)$$
$$\textstyle\int_{-\infty}^\infty e^{-x^2} = \sqrt{\pi}$$
$$\textstyle\int \mathrm{d}x e^{-\frac{1}{2}x^\top Ax + b^\top x} =$$
$$= \tfrac{1}{\sqrt{\det\frac{A}{2\pi}}}e^{\frac{1}{2}b^\top A^{-1}b}$$
$$\textstyle\int_{-\infty}^\infty e^{i\omega t}\mathrm{d}t = 2\pi\delta(\omega)$$
$$\textstyle\int \frac{\mathrm{d}u}{(1+u^2)^{3/2}} = \frac{u}{\sqrt{1+u^2}}$$
$$\textstyle\int \mathrm{d}\Omega_d = \frac{d\pi^{d/2}}{\Gamma(\frac{d}{2}+1)}$$
$$\textstyle\int_0^\infty \frac{x^3}{e^x-1} = \frac{\pi^4}{15}$$
$$\textstyle\int_0^\infty \frac{x^{n-1}}{e^x-1} = \Gamma(n)\zeta(n)$$

Differential equations

$$\dot{x} + \dot{a}x = b : x = e^{-a}\left(\textstyle\int be^a + c_1\right)$$
$$x\ddot{x} = k\dot{x}^2 : x = c_2\sqrt[1-k]{(1-k)t + c_1}$$
$$\dot{x} + ax^2 = b : x = \sqrt{\tfrac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$
$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = fe^{-i\omega t} : x = \tfrac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x = x - \tfrac{x^3}{3!} + \tfrac{x^5}{5!} - \tfrac{x^7}{7!} + \cdots$$
$$\cos x = 1 - \tfrac{x^2}{2} + \tfrac{x^4}{4!} - \tfrac{x^6}{6!} + \cdots$$
$$\tan x = x + \tfrac{x^3}{3} + \tfrac{2}{15}x^5 + \tfrac{17}{315}x^7 + \mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x} = \tfrac{1}{x} + \tfrac{x}{6} + \tfrac{7}{360}x^3 + \tfrac{31}{15120}x^5 + \mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x} = 1 + \tfrac{x^2}{2} + \tfrac{5}{24}x^4 + \tfrac{61}{720}x^6 + \tfrac{277}{8064}x^8 + \mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x} = \tfrac{1}{x} - \tfrac{x}{3} - \tfrac{x^3}{45} - \tfrac{2}{945}x^5 + \mathrm{O}(x^7)$$
$$\operatorname{asin} x = x + \tfrac{x^3}{6} + \tfrac{3}{40}x^5 + \tfrac{5}{112}x^7 + \mathrm{O}(x^9)$$
$$\operatorname{atan} x = x - \tfrac{x^3}{3} + \tfrac{x^5}{5} - \tfrac{x^7}{7} + \cdots$$
$$\tfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$
$$e^x = 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{3!} + \cdots$$
$$\ln(1+x) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \tfrac{x^4}{4} + \cdots$$
$$\sinh x = x + \tfrac{x^3}{3!} + \tfrac{x^5}{5!} + \tfrac{x^7}{7!} + \cdots$$
$$\cosh x = 1 + \tfrac{x^2}{2} + \tfrac{x^4}{4!} + \tfrac{x^6}{6!} + \cdots$$
$$\tanh x = x - \tfrac{x^3}{3} + \tfrac{2}{15}x^5 - \tfrac{17}{315}x^7 + \mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x} = \tfrac{1}{x} - \tfrac{x}{6} + \tfrac{7}{360}x^3 - \tfrac{31}{15120}x^5 + \mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x} = 1 - \tfrac{x^2}{2} + \tfrac{5}{24}x^4 - \tfrac{61}{720}x^6 + \tfrac{277}{8064}x^8 + \mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x} = \tfrac{1}{x} + \tfrac{x}{3} - \tfrac{x^3}{45} + \tfrac{2}{945}x^5 + \mathrm{O}(x^7)$$
$$\operatorname{asinh} x = x - \tfrac{x^3}{6} + \tfrac{3}{40}x^5 - \tfrac{5}{112}x^7 + \mathrm{O}(x^9)$$
$$(1+x)^n = 1 + nx + \tfrac{n(n-1)}{2}x^2 + \mathrm{O}(x^3)$$
$$(1+x)^x = 1 + x^2 - \tfrac{x^3}{2} + \tfrac{5}{6}x^4 - \tfrac{3}{4}x^5 + \mathrm{O}(x^6)$$
$$x! = 1 - \gamma x + \left(\tfrac{\gamma^2}{2} + \tfrac{\pi^2}{12}\right)x^2 + \mathrm{O}(x^3)$$
$$J_0(x) = 1 - \tfrac{x^2}{4} + \tfrac{x^4}{64} - \tfrac{x^6}{2304} + \tfrac{x^8}{147456} + \mathrm{O}(x^{10})$$
$$J_1(x) = \tfrac{x}{2} - \tfrac{x^3}{16} + \tfrac{x^5}{384} - \tfrac{x^7}{18432} + \mathrm{O}(x^9)$$

Fourier

Fourier: $c_n = \frac{2}{T}\int_0^T f(t)\cos\left(n\frac{t}{T}\right)\mathrm{d}t$

$\mathcal{F}[f](\omega) = \hat{f}(\omega) = \int \mathrm{d}t e^{i\omega t} f(t)$

$f, g \in L^2 : (\hat{f}, \hat{g}) = 2\pi(f, g)$

$\mathcal{F}\left[\frac{\sin t}{t}\right] = \sqrt{\frac{\pi}{2}}\chi_{[-1;1]}(\omega)$

$t^{k\leq n} f(t) \in L^1 : \mathcal{F}[t^n f(t)] = (-i)^n \hat{f}^{(n)}$

$f^{(k\leq n)} \in L^1 : \mathcal{F}[f^{(n)}] = (-i\omega)^n \hat{f}$

$\mathcal{F}^2 f = 2\pi f(-t);\ (\omega\hat{f})' = -\mathcal{F}[tf']$

$f \star g = g \star f;\ \hat{f}\star\hat{g} = 2\pi\mathcal{F}[fg]$

$f \in L^1,\ g \in L^p : \mathcal{F}[f\star g] = \hat{f}\hat{g}$

$f \star g(x) = \int f(x-y)g(y)\mathrm{d}y$

$(f\star g)' = f'\star g = f\star g'$

$f(x+\Delta)\star g = f\star g(x+\Delta)$

$f \in L^1,\ g \in L^p \ \Rightarrow\ f\star g \in L^p$

$f, g \in L^2 : f\star g = \frac{1}{2\pi}\int \hat{f}\hat{g}e^{-i\omega t}\mathrm{d}\omega$

$\|f\| = 1 : \Delta\omega\Delta t \geq \frac{1}{2}$

$\Delta\omega\Delta t = \frac{1}{2} : f(t) = g(t;\bar{t},\Delta t)e^{-i\bar{\omega}t}$

Distributions

$\mathcal{D} := \{f \in C^\infty \mid \exists K \text{ compact} : f(\mathscr{C} K) = 0\}$

$\mathcal{S} := \{f \in C^\infty \mid |x^n f^{(k)}| \leq A_{nk}\} \supset \mathcal{D}$

$\langle 1, f\rangle := \int f;\ \langle gT, f\rangle := \langle T, gf\rangle$

$T \in \mathcal{S}' : \langle\mathcal{F}T, f\rangle := \langle T, \mathcal{F}f\rangle$

$\langle T', f\rangle := -\langle T, f'\rangle;\ \langle\delta, f\rangle := f(0)$

$\langle T\otimes S, \phi\rangle := \langle T(x), \langle S(y), \phi(x+y)\rangle\rangle$

$\langle T\star S, \phi\rangle := \langle T\otimes S, \phi(x+y)\rangle$

$\mathcal{F}1 = 2\pi\delta(\omega);\ \mathcal{F}\,\mathrm{sgn} = 2i\mathcal{P}\frac{1}{\omega}$

$\mathcal{F}\theta = i\mathcal{P}\frac{1}{\omega} + \pi\delta(\omega)$

$\lim_{\varepsilon\to 0}\frac{1}{x+i\varepsilon} = \mathcal{P}\frac{1}{x} - i\pi\delta(x)$

$x^n T = 0 \ \Rightarrow\ T = \sum_{k=0}^{n-1} a_k\delta^{(k)}$

$xT = S \ \Rightarrow\ T = S/x + k\delta$

$T, S \in \mathcal{D}' : T\otimes S = S\otimes T$

$\sum_{n=0}^\infty e^{inx} = \mathcal{P}\frac{1}{1-e^{ix}} + \pi\sum_{n=-\infty}^\infty \delta(x - 2n\pi)$

$\delta^{(n)}\star f = f^{(n)}$

$\delta(g(x)) = \sum_{x_i\in g^{-1}(0)}\frac{\delta(x-x_i)}{|g'(x_i)|}$

Bessel functions

sol. of $x^2 f'' + xf' + (x^2-\alpha^2)f = 0$

$\alpha =$ "order"

$J_\alpha =$ "first kind, normal"

$\alpha \in \mathbb{Z}_0 \vee \alpha > 0 : J_\alpha(0) = 0$

$J_0(0) = 1;$ otherwise $|J_\alpha(0)| = \infty$

$J_0(z) = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\, e^{z\cos\theta};\ J_1(z) = J_0'(z)$

$\alpha \notin \mathbb{Z} : J_\alpha, J_{-\alpha}$ indep.

$\alpha \in \mathbb{Z} : J_{-\alpha} = (-1)^\alpha J_\alpha$

$Y_\alpha =$ "second kind, normal" (also N_α)

$\alpha \notin \mathbb{Z} : Y_\alpha = \frac{\cos(\alpha\pi)J_\alpha - J_{-\alpha}}{\sin(\alpha\pi)}$

$\alpha \in \mathbb{Z} : Y_\alpha = \lim_{\alpha'\to\alpha} Y_{\alpha'}$

$\alpha \in \mathbb{Z} : Y_\alpha, J_\alpha$ indep.

$\alpha \in \mathbb{Z} : Y_{-\alpha} = (-1)^\alpha Y_\alpha$

$\frac{2\alpha}{x}J_\alpha(x) = J_{\alpha-1}(x) + J_{\alpha+1}(x)$

$2J_\alpha'(x) = J_{\alpha-1}(x) - J_{\alpha+1}(x)$

$\int_0^1 xJ_\alpha(xu_{\alpha,m})J_\alpha(xu_{\alpha,n}) = \frac{\delta_{mn}}{2}J_{\alpha+1}^2(u_{\alpha,m})$

$u_{\alpha,n} = n$th. zero of J_α

Cylindrical harmonics

$V(\rho,\phi,z) = \sum_{n=0}^\infty\int \mathrm{d}k A_{nk}P_{nk}(\rho)\Phi_n(\phi)Z_k(z)$

$P_{nk}(\rho) =$ comb. of $J_n(k\rho), Y_n(k\rho)$

$\Phi_n(\phi) =$ comb. of $e^{\pm in\phi}$

$Z_k(z) =$ comb. of $e^{\pm kz}$

Spherical harmonics

$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^\infty\frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right);\ \sum_m |Y_{lm}|^2 = \frac{2l+1}{4\pi}$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2-1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$

$P_l(1) = 1;\ (P_n, P_m) = \frac{2\delta_{nm}}{2n+1};\ (Y_{lm}, Y_{l'm'}) = \delta_{ll'}\delta_{mm'}$

$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2-1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2-1\right)^l,\ 0 \leq m \leq l$

$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m Y_{lm}^*$

$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l Y_{lm}^*(\theta',\varphi')Y_{lm}(\theta,\varphi)$

Inequalities

$|a| - |b| \leq |a+b| \leq |a| + |b|$

$x > -1 : 1 + nx \leq (1+x)^n$

$\frac{|a^n-b^n|}{|a-b|<1} \leq n(1+|b|)^{n-1}$

$\sqrt[p]{\sum(a_i+b_i)^p} \leq \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$

$\sum a_i b_i \leq \left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^p y^q \leq \left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac{1}{n}\sum a_i^{p\leq q}} \leq \sqrt[q]{\frac{1}{n}\sum a_i^q}$

$\sum\left(\frac{a_1+\ldots a_i}{i}\right)^p \leq \left(\frac{p}{p-1}\right)^p\sum a_i^p$

$x \geq 0, |\ddot{x}| \leq M : |\dot{x}| \leq \sqrt{2Mx}$

$\frac{1}{1+x} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$

$\dim V = \dim\ell(V) + \dim(V\cap\ker\ell)$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772\cdot 10^{-1}$

$G = 6.674\cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206\cdot 10^{-2}\,\frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022\cdot 10^{23}\,\frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381\cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617\cdot 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602\cdot 10^{-19}\,\mathrm{A\,s}$

$m_\mathrm{e} = 9.109\cdot 10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673\cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675\cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{e} = 5.110\cdot 10^{-1}\,\mathrm{MeV}$

$m_\mathrm{p} = 9.383\cdot 10^2\,\mathrm{MeV}$

$m_\mathrm{n} = 9.396\cdot 10^2\,\mathrm{MeV}$

$m_\mathrm{n} - m_\mathrm{p} = 1.293\,\mathrm{MeV}$

$\mathrm{amu} = 1.661\cdot 10^{-27}\,\mathrm{kg}$

$h = 6.626\cdot 10^{-34}\,\mathrm{J\,s}$

$h = 4.136\cdot 10^{-15}\,\mathrm{eV\,s}$

$\varepsilon_0 = 8.854\cdot 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988\cdot 10^9\,\frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257\cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274\cdot 10^{-24}\,\mathrm{A\,m}^2$

$\mu_\mathrm{B} = 5.788\cdot 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{T}}$

$\alpha = 7.297\cdot 10^{-3}$

$\mathrm{barn} = 1\cdot 10^{-28}\,\mathrm{m}^2$

$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464\cdot 10^{-3}\,\frac{\mathrm{W}}{\mathrm{sr}}$

$r_B = 5.292\cdot 10^{-11}\,\mathrm{m}$

$\mathrm{Rydberg} = 1.361\cdot 10^1\,\mathrm{eV}$

$r_e = 2.818\cdot 10^{-15}\,\mathrm{m}$

$\mathrm{Debye} = 3.336\cdot 10^{-30}\,\mathrm{C\,m}$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i = j \vee j = k \vee k = i \\ 1 & i+1 \equiv j \wedge j+1 \equiv k \\ -1 & i \equiv j+1 \wedge j \equiv k+1 \end{cases}$$

$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

$\vec{a}\times\vec{b} = \varepsilon_{ijk}a_j b_k\hat{e}_i;\ (\vec{a}\otimes\vec{b})_{ij} = a_i b_j$

$(\vec{a}\times\vec{b})\vec{c} = (\vec{c}\times\vec{a})\vec{b}$

$(\vec{a}\times\vec{b})\times\vec{c} = -(\vec{b}\vec{c})\vec{a} + (\vec{a}\vec{c})\vec{b}$

$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d}) = (\vec{a}\vec{c})(\vec{b}\vec{d}) - (\vec{a}\vec{d})(\vec{b}\vec{c})$

$|\vec{u}\times\vec{v}|^2 = u^2v^2 - (\vec{u}\vec{v})^2$

$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right);\ \Box = \frac{\partial^2}{\partial t^2} - \nabla^2$

$\vec{\nabla}V = \frac{\partial V}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$

$\vec{\nabla}\vec{v} = \frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

$\vec{\nabla}\times\vec{v} = \left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\rho} +$

$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\phi}\right)\hat{z}$

$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$

$$\vec\nabla V = \tfrac{\partial V}{\partial r}\hat r + \tfrac{1}{r}\tfrac{\partial V}{\partial\theta}\hat\theta + \tfrac{1}{r\sin\theta}\tfrac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v = \tfrac{1}{r^2}\tfrac{\partial(r^2 v_r)}{\partial r} + \tfrac{1}{r\sin\theta}\tfrac{\partial(v_\theta\sin\theta)}{\partial\theta} + \tfrac{1}{r\sin\theta}\tfrac{\partial v_\varphi}{\partial\varphi}$$

$$\vec\nabla\times\vec v = \tfrac{1}{r\sin\theta}\big(\tfrac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \tfrac{\partial v_\theta}{\partial\varphi}\big)\hat r +$$

$$+\tfrac{1}{r}\big(\tfrac{1}{\sin\theta}\tfrac{\partial v_r}{\partial\varphi} - \tfrac{\partial(rv_\varphi)}{\partial r}\big)\hat\theta + \tfrac{1}{r}\big(\tfrac{\partial(rv_\theta)}{\partial r} - \tfrac{\partial v_r}{\partial\theta}\big)\hat\varphi$$

$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\tfrac{1}{r^2}\tfrac{\partial}{\partial r}\left(r^2\tfrac{\partial V}{\partial r}\right) = \tfrac{1}{r}\tfrac{\partial^2}{\partial r^2}(rV) = \tfrac{2}{r}\tfrac{\partial V}{\partial r} + \tfrac{\partial^2 V}{\partial r^2}$$

$$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$$

$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$

$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$

$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$

$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$

$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\,\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\,\vec\nabla)\vec w$$

$$\tfrac12\vec\nabla v^2 = (\vec v\,\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$

$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\tfrac{\partial g}{\partial n} - g\tfrac{\partial f}{\partial n}\right)\mathrm{d}S$$

$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$

$$\delta(\vec r-\vec r_0) = \tfrac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r_0^2\sin\theta_0}$$

$$\nabla^2\tfrac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$

$$\Delta x_{\mathrm{hist}} \approx \tfrac{x_{\max}-x_{\min}}{\sqrt N}$$

$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$

$$\text{median} = F^{-1}(\tfrac12)$$

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$

$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$

$$\alpha_n = E[x^n]$$

$$M_n = E[(x-\mu)^n]$$

$$\sigma^2 = M_2 = E[x^2] - \mu^2$$

$$\mathrm{FWHM} \approx 2\sigma$$

$$\gamma_1 = \tfrac{M_3}{\sigma^3},\ \gamma_2 = \tfrac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$

$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$

$$\alpha_n = i^{-n}\tfrac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$

$$h\ge 0 : P(h\ge k) \le \tfrac{E[h]}{k}$$

$$P(|x-\mu| > k\sigma) \le \tfrac{1}{k^2}$$

$$B(k;n,p) = \tbinom{n}{k}p^k(1-p)^{n-k}$$

$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$

$$P(k;\mu) = \tfrac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$

$$u(x;a,b) = \tfrac{1}{b-a},\ x\in[a;b]$$

$$\mu_u = \tfrac{b+a}{2},\ \sigma_u^2 = \tfrac{(b-a)^2}{12}$$

$$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \tfrac{1}{\lambda},\ \sigma_\varepsilon^2 = \tfrac{1}{\lambda^2}$$

$$g(x;\mu,\sigma) = \tfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac12(\frac{x-\mu}{\sigma})^2}$$

$$g(\vec x;\vec\mu,V) = \tfrac{e^{-\frac12(\vec x-\vec\mu)^\top V^{-1}(\vec x-\vec\mu)}}{\sqrt{\det(2\pi V)}}$$

$$\mathrm{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$

$$z = \tfrac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$

$$\chi^2 = \sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$$

$$\wp(x;n) = \tfrac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$$

$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$

$$n\ge 30 : \wp(x;n) \approx g(x;n,\sqrt{2n})$$

$$n\ge 8 : p[\sqrt{2\chi^2}] \approx g(;\sqrt{2n-1},1)$$

$$S(x;n) = \tfrac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac n2)}\left(1+\tfrac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

$$\mu_S = 0,\ \sigma_S^2 = \tfrac{n}{n-2}$$

$$p\left[z\sqrt{\tfrac{n}{\chi^2}}\right] = S(,n)$$

$$n\ge 35 : S(x;n) \approx g(x;0,1)$$

$$c(x;a) = \tfrac{a}{\pi}\tfrac{1}{a^2+x^2}$$

$$\sigma_{xy} = E[xy] - \mu_x\mu_y \le \sigma_x\sigma_y$$

$$\rho_{xy} = \tfrac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}| \le 1$$

$$\mu_{f(x)} \approx f(\mu_x)$$

$$\sigma_{fg} \approx \sigma_{x_ix_j}\tfrac{\partial f}{\partial x_i}\Big|_{\mu_{x_i}}\tfrac{\partial g}{\partial x_j}\Big|_{\mu_{x_j}}$$

$$\mu \approx m = \tfrac1n\sum_{i=1}^n x_i$$

$$\sigma^2 \approx s^2 = \tfrac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$

$$s_m^2 = \tfrac{s^2}{n}$$

$$p\left[\tfrac{m-\mu}{s_m}\right] = S(;n)$$

$$H_{0\ \text{sign.lev.}} = \int_{\text{reject}} p(S|H_0)\,\mathrm{d}S$$

$$H_{0\ \text{pow.vs.}\ H_1} = \int_{\text{reject}} p(S|H_1)\,\mathrm{d}S$$

Least squares

$$f(x) = mx+q,\quad f(x) = a,$$

$$f(x) = bx,\quad f(x;\boldsymbol\theta) = \theta_i h_i(x)$$

$$\hat m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\sigma_{\hat m}^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\hat q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\sigma_{\hat q}^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\sigma_{\hat m,\hat q} = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\hat a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \sigma_{\hat a}^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$

$$\hat{\mathbf a} = (\sum V_{\mathbf y}^{-1})^{-1}(\sum V_{\mathbf y}^{-1}\mathbf y)$$

$$V_{\hat{\mathbf a}} = (\sum V_{\mathbf y}^{-1})^{-1}$$

$$\hat b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \sigma_{\hat b}^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

$$H_{ij} := h_j(x_i)$$

$$\hat{\boldsymbol\theta} = (H^\top V_{\mathbf y}^{-1}H)^{-1}H^\top V_{\mathbf y}^{-1}\mathbf y$$

$$V_{\hat{\boldsymbol\theta}} = (H^\top V_{\mathbf y}^{-1}H)^{-1}$$

Kinematics

$$\tfrac{1}{R} = \left|\tfrac{v_xa_y - v_ya_x}{v^3}\right|$$

$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\dot\theta\hat\theta + \dot\theta\hat\varphi$$

$$\dot{\vec w} = \tfrac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \tfrac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \tfrac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$$

$$\theta\equiv\tfrac{\pi}{2} \to \dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$

$$\theta\equiv\tfrac{\pi}{2} \to \ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi$$

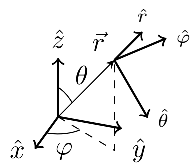
$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$

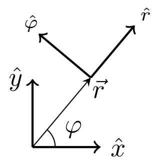
$$\ddot{\vec r}\hat r = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$

$$\ddot{\vec r}\hat\theta = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$

$$\ddot{\vec r}\hat\varphi = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$

$$\vec A = \ddot{\vec r} + \vec A_{\mathrm T} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \tfrac{\mathrm d}{\mathrm dt}\alpha(\beta,t) = \tfrac{\partial\alpha}{\partial\beta}\dot\beta + \tfrac{\partial\alpha}{\partial t}$$

$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \tfrac{\mathrm d(mT)}{\mathrm dt} = \vec F\vec p$$

$$M := \sum_i m_i;\ \vec R := \tfrac{m_i\vec r_i}{M}$$

$$T = \tfrac12 M\dot{\vec R}^2 + \tfrac12 m_i(\dot{\vec r}_i - \dot{\vec R})^2$$

$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$$

$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$

$$\tau_1 = I_1\dot\omega_1 + (I_3 - I_2)\omega_3\omega_2$$

$$\mathcal L(q,\dot q,t) = T - V + \tfrac{\mathrm d}{\mathrm dt}f(q,t)$$

$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$$

$$\tfrac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$

$$p := \tfrac{\partial\mathcal L}{\partial\dot q};\ \dot p = \tfrac{\partial\mathcal L}{\partial q}$$

$$\mathcal H(q,p,t) = \dot qp - \mathcal L$$

$$\dot q = \tfrac{\partial\mathcal H}{\partial p};\ \dot p = -\tfrac{\partial\mathcal H}{\partial q}$$

$$\tfrac{\mathrm d\mathcal H}{\mathrm dt} = \tfrac{\partial\mathcal H}{\partial t} = -\tfrac{\partial\mathcal L}{\partial t}$$

$$\{u,v\} = \tfrac{\partial u}{\partial q}\tfrac{\partial v}{\partial p} - \tfrac{\partial u}{\partial p}\tfrac{\partial v}{\partial q}$$

$$\tfrac{\mathrm du}{\mathrm dt} = \{u,\mathcal H\} + \tfrac{\partial u}{\partial t}$$

$$\eta = (q,p);\ \Gamma = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$\dot\eta = \Gamma\tfrac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \tfrac{\partial u}{\partial\eta}\Gamma\tfrac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac12 mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac23 mr^2$

ball: $\frac25 mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2 + \frac34 r^2\right)$

ellipsoid: $I_a = \frac15 m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle = -2\langle T\rangle$$

$$U_{\mathrm{eff}} = U + \tfrac{L^2}{2mr^2}$$

$$\tfrac{1}{\mu} = \tfrac{1}{m_1} + \tfrac{1}{m_2}$$

$$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$$

$$T = \tfrac12 M\dot{\vec R}^2 + \tfrac12\mu\dot{\vec r}^2$$

$$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$$

$$k = \tfrac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \tfrac{2EL^2}{\mu\alpha^2}}$$

$$r = \tfrac{k}{1+\varepsilon\cos\theta}$$

$$a = \tfrac{k}{|1-\varepsilon^2|} = \tfrac{\alpha}{2|E|}$$

$$a^3\omega^2 = G(m_1+m_2) = \tfrac{\alpha}{\mu}$$

$$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$$

Relativity

$$\beta = \tfrac{v}{c} = \tanh\chi$$

$$\gamma = \tfrac{1}{\sqrt{1-\beta^2}} = \cosh\chi$$

$$\vec p = \gamma m\vec v;\ \mathcal E = \gamma mc^2$$

$$\text{free particle: } \mathcal L = \tfrac{mc^2}{\gamma}$$

$$\tfrac{\mathrm d\mathcal E}{\mathrm dt} = \vec v\tfrac{\mathrm d\vec p}{\mathrm dt};\ \tfrac{\mathrm dp}{\mathrm dt} = \tfrac{\mathrm d\mathcal E}{\mathrm dx}$$

$$\begin{pmatrix}ct'\\ x'\end{pmatrix} = \gamma\begin{pmatrix}1 & -\beta\\ -\beta & 1\end{pmatrix}\begin{pmatrix}ct\\ x\end{pmatrix}$$

$$\chi'' = \chi' + \chi$$

$$V'_\parallel = \tfrac{V_\parallel - v}{1-\frac{vV_\parallel}{c^2}}$$

$$V'_\perp = \tfrac{1}{\gamma}\tfrac{V_\perp}{1-\frac{vV_\parallel}{c^2}}$$

$$\tfrac{V'}{c} = 1 - \tfrac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$$

$$\mathrm d\tau = \tfrac{1}{\gamma}\mathrm dt$$

$$x^\mu = (ct,\vec x)$$

$$v^\mu = \tfrac{\mathrm dx^\mu}{\mathrm d\tau} = \gamma(c,\vec v)$$

$$a^\mu = \tfrac{\mathrm dv^\mu}{\mathrm d\tau} = \gamma\left(\tfrac{\mathrm d\gamma}{\mathrm dt}c, \tfrac{\mathrm d(\gamma\vec v)}{\mathrm dt}\right)$$

$$p^\mu = mv^\mu = \left(\tfrac{\mathcal E}{c},\vec p\right)$$

$$\tfrac{\mathrm dp^\mu}{\mathrm d\tau} = \gamma\left(\tfrac{\vec v}{c}\tfrac{\mathrm d\vec p}{\mathrm dt}, \tfrac{\mathrm d\vec p}{\mathrm dt}\right)$$

$$\partial_\mu = \tfrac{\partial}{\partial x^\mu} = \left(\tfrac1c\tfrac{\partial}{\partial t},\vec\nabla\right)$$

$$g_{\mu\nu} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

$$x_\mu = g_{\mu\nu}x^\nu$$

$$\partial_\mu\partial^\mu = \Box$$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1+\beta$

$\mathrm{SO}_{1,3} = \left\{ \Lambda \;\middle|\; \begin{smallmatrix} \Lambda^\top g \Lambda = g \\ \det \Lambda \geq 0 \end{smallmatrix} \right\}$

$(\Lambda^0{}_0)^2 \geq 1$

$\Lambda = \begin{pmatrix} \gamma & -\gamma\vec{\beta} \\ -\gamma\vec{\beta} & I + \frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta} \end{pmatrix}$

$M \to \sum_i m_i$

$E_1^{\max} = \frac{M^2+m_1^2-\sum_{i\neq 1} m_i^2}{2M}$

$A + B_{\text{still}} \to \sum_i m_i$

$E_A^{\min} = \frac{(\sum_i m_i)^2 - m_A^2 - m_B^2}{2m_B}$

m, M_{still} 1D coll.

$E'_m = \frac{(M+m)^2 E_m + 2Mm^2}{M^2+m^2+2ME_m}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu\mathrm{d}N$

$C_V := T\frac{\partial S}{\partial T}\big|_V = \frac{\partial U}{\partial T}\big|_V$

$C_p := T\frac{\partial S}{\partial T}\big|_p = \frac{\partial U}{\partial T}\big|_p + p\frac{\partial V}{\partial T}\big|_p = \frac{\partial H}{\partial T}\big|_p$

$\mu_J := \frac{\partial T}{\partial V}\big|_{U,N};\ \alpha := \frac{1}{V}\frac{\partial V}{\partial T}\big|_p$

$\kappa_T := -\frac{1}{V}\frac{\partial V}{\partial p}\big|_T;\ \kappa_S := -\frac{1}{V}\frac{\partial V}{\partial p}\big|_S$

$\gamma := \frac{C_p}{C_V} = \frac{\kappa_T}{\kappa_S};\ C_p - C_V = \frac{\alpha^2 VT}{\kappa_T}$

$\kappa, C \geq 0$

Euler: $U = ST - pV + \mu N$

Gibbs-Duhem: $S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix S, V, N : $\min U$ at equilibrium

Fix T, V, N : $\min F := U - TS$

Fix T, p, N : $\min G := F + pV = \mu N$

Fix S, p, N : $\min H := U + pV$

$\Omega(T,V,\mu) := U - TS - \mu N = -pV$

$$\begin{array}{ccc} V & F & T \\ U & & G \\ S & H & p \end{array}$$

$\frac{\partial}{\partial T}\frac{G}{T}\big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\big|_V = -\frac{U}{T^2}$

$\delta E \leq T\delta S - p\delta V + H\delta\int M\mathrm{d}V + \mu\delta N$

$\mathrm{d}U = T\mathrm{d}S + H\mathrm{d}\langle M\rangle_V;\ \chi := \frac{\partial M}{\partial H}$

Clausius-Clapeyron: $\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{s_2-s_1}{v_2-v_1}$

TM examples

IDEAL GAS: $pV = Nk_{\mathrm{B}}T$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R,\ \gamma = 1 + \frac{2}{\mathrm{dof}}$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

V.D.WAALS: $\left(p - \frac{N^2a^2}{V^2}\right)(V - Nb) = Nk_{\mathrm{B}}T$

Statistical mechanics

MICROCANONICAL

$\rho := \frac{1}{N!}\begin{cases} \text{const.} & E < \mathcal{H} < E + \Delta \\ 0 & \text{otherwise} \end{cases}$

$\Gamma(E) := \frac{1}{N!}\int_{E<\mathcal{H}<E+\Delta} \mathrm{d}p\mathrm{d}q$

$S := k_{\mathrm{B}}\log\Gamma(E)$

$T := \frac{\partial E}{\partial S}\big|_V;\ p := -\frac{\partial E}{\partial V}\big|_S$

$S = -k_{\mathrm{B}}\langle\log\rho\rangle$

$\left\langle x_i\frac{\partial\mathcal{H}}{\partial x_j}\right\rangle = \delta_{ij}kT$

CANONICAL

$\rho = \frac{1}{h^{3N}N!}e^{-\beta\mathcal{H}};\ \beta := \frac{1}{k_{\mathrm{B}}T}$

$Z := \frac{1}{h^{3N}N!}\int \mathrm{d}p\mathrm{d}q e^{-\beta\mathcal{H}}$

$F = -\frac{\log Z}{\beta};\ U = -\frac{\partial\log Z}{\partial\beta}$

GRAND CANONICAL

$\rho = \frac{1}{h^{3N}N!}e^{-\beta(\mathcal{H}-\mu N+pV)}$

$\mathcal{Z} := \sum_{N=0}^\infty z^N Z;\ z := e^{\beta\mu}$

$N = z\frac{\partial}{\partial z}\log\mathcal{Z}\big|_{\beta,V}$

$U = -\frac{\partial}{\partial\beta}\log\mathcal{Z}\big|_{z,V};\ \Omega = -\frac{\log\mathcal{Z}}{\beta}$

$\lambda_T^2 := \frac{h^2\beta}{2\pi m};\ z \ll 1 \to z \approx \frac{N\lambda_T^3}{V}$

VIRIAL EXPANSION

$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v(|\vec{r}_i - \vec{r}_j|)$

$\frac{pV}{Nk_{\mathrm{B}}T} = \sum_{l=1}^\infty B_l(T)\left(\frac{N}{V}\right)^{l-1}$

$\Omega = -Vk_{\mathrm{B}}T\sum_{l=1}^\infty\left(\frac{z}{\lambda_T^3}\right)^l b_l(T)$

$B_1 = 1,\ B_2 = -b_2,\ B_3 = 4b_2^2 - 2b_3$

$f_{ij} := e^{-\beta v(|\vec{r}_i-\vec{r}_j|)} - 1$

$\mathcal{U}_l := \sum_{\substack{\text{connected} \\ l\text{ vertices} \\ \text{graphs}}} \prod_{\substack{\text{connections} \\ \langle i,j\rangle}} f_{ij}$

$b_l = \frac{1}{l!}\frac{1}{V}\int \mathrm{d}\vec{r}_1\cdots\mathrm{d}\vec{r}_l\,\mathcal{U}_l$

$\mathcal{W}_l := \sum_{\substack{\text{conn. } l \text{ vert. graphs} \\ \text{that remain conn.} \\ \text{removing a point}}} \prod_{\substack{\text{conn.} \\ \langle i,j\rangle}} f_{ij}$

$B_l = -\frac{1}{l(l-2)!}\frac{1}{V}\int \mathrm{d}\vec{r}_1\cdots\mathrm{d}\vec{r}_l\,\mathcal{W}_l$

Statistical QM

FERMIONS

$\Omega(T,V,\mu) = -\frac{1}{\beta}\sum_\alpha\log\left(1 + e^{-\beta(\varepsilon_\alpha-\mu)}\right)$

$f(\epsilon) := \frac{1}{1+e^{\beta\epsilon}}$

$n(\varepsilon, T, \mu) = f(\varepsilon - \mu)$

$\int_{-\infty}^\infty \mathrm{d}\epsilon\, f(\epsilon-\mu)\phi(\epsilon) = \frac{\pi D}{\sin(\pi D)}\Phi(\mu)$

$\Phi(\mu) := \int_{-\infty}^\mu \mathrm{d}\epsilon\,\phi(\epsilon);\ D := \frac{1}{\beta}\frac{\mathrm{d}}{\mathrm{d}\mu}$

$\frac{pV}{NkT} = 1 + 2^{-5/2}\frac{N\lambda_T^3}{V} + O((N\lambda_T^3/V)^2)$

$\epsilon = \frac{p^2}{2m} \to \epsilon_F = \frac{h^2}{2m}\left(\frac{N}{V}\right)^{2/3}\left(\frac{3}{4\pi g}\right)^{2/3}$

BOSONS

$\Omega(T,V,\mu) = \frac{1}{\beta}\sum_\alpha\log\left(1 - e^{-\beta(\varepsilon_\alpha-\mu)}\right)$

$\epsilon \propto p^2 : n(T,z) = n_0 + g\lambda_T^{-d}\,\mathrm{Li}_{\frac{d}{2}}(z)$

$\epsilon \propto p^2 : p(T,z) = gk_{\mathrm{B}}T\lambda_T^{-d}\,\mathrm{Li}_{\frac{d}{2}+1}(z)$

$\frac{pV}{NkT} = 1 - 2^{-5/2}\frac{N\lambda_T^3}{V} + O((N\lambda_T^3/V)^2)$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t},\ Z = \frac{V}{I}$

$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$

$Z_{\text{series}} = \sum_k Z_k,\ \frac{1}{Z_{\text{parallel}}} = \sum_k\frac{1}{Z_k}$

$\sum_{\text{loop}} V_k = 0,\ \sum_{\text{node}} I_k = 0$

$\mathcal{E} = -L\dot{I},\ L = \frac{\Phi_B}{I}$

$I_{A\to C} = I_0\left(e^{\frac{V_{AC}}{V_T}} - 1\right),\ V_T = \eta\frac{k_{\mathrm{B}}T}{q_{\mathrm{e}}}$

$I_{E,\text{out}} = I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right) - \alpha_R I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$

$I_{C,\text{in}} = -I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right) + \alpha_F I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H =$ heat transfer

$G = H - TS$

$a_i\mathrm{A}_i \to b_j\mathrm{B}_j$

$\Delta H_{\mathrm{r}}^{\mathrm{o}} = b_j\Delta H_{\mathrm{f}}^{\mathrm{o}}(\mathrm{B}_j) - a_i\Delta H_{\mathrm{f}}^{\mathrm{o}}(\mathrm{A}_i)$

$\forall\, i, j : v_{\mathrm{r}} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_{\mathrm{r}} = k[\mathrm{A}_i]^{m_i}$

$k = Ae^{-\frac{E_{\mathrm{a}}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod[\mathrm{B}_j]^{b_j}}{\prod[\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod\chi_{\mathrm{B}_j}^{b_j}}{\prod\chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_{\text{tot}}}$

$K_c = K_p(RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_{\text{tot}}^{\sum a_i - \sum b_j}$

$\Delta G_{\mathrm{r}}^{\mathrm{o}} = -RT\ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT\ln\frac{Q}{K}$

$\ln\frac{K_2}{K_1} = -\frac{\Delta H^{\mathrm{o}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_{\mathrm{w}} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^{\mathrm{o}} - \frac{RT}{n_{\mathrm{e}}N_Aq_{\mathrm{e}}}\ln Q$ (Nerst)

(std) $\Delta E = \Delta E^{\mathrm{o}} - \frac{0.059}{n_{\mathrm{e}}}\log_{10}Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

CGS→MKS Substitutions: $\vec{E}, V \times\sqrt{4\pi\varepsilon_0}$ $\quad \rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$ $\quad \vec{H}\times\sqrt{4\pi\mu_0}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \mu/\mu_0$ $\quad L\times 4\pi\varepsilon_0$

$c \mapsto \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ $\quad \vec{D}\times\sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \vec{B}, \vec{A}\times\sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{M}\times\sqrt{\frac{\mu_0}{4\pi}}$ $\quad \varepsilon/\varepsilon_0$ $\quad R, Z\times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

MKS→CGS $\mu_0 \mapsto \frac{4\pi}{c^2}$ $\quad \vec{D}/4\pi$ $\quad \vec{B}, \vec{A}/c$ $\quad \vec{M}\times c$ $\quad \varepsilon/4\pi$ $\quad R, Z\times 1$ $\quad C\times 1$

$\varepsilon_0 \mapsto \frac{1}{4\pi}$ $\quad \vec{E}, V\times 1$ $\quad \rho, \vec{J}, I, \vec{P}\times 1$ $\quad \vec{H}\times\frac{c}{4\pi}$ $\quad \sigma$ (cond.) $\times 1$ $\quad \mu\times\frac{4\pi}{c^2}$ $\quad L\times 1$

Electrostatics (CGS)

$\vec{F}_{12} = q_1q_2\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_1-\vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int\mathrm{d}^3r'\,\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|};\ \rho_q = \delta(\vec{r}-\vec{r}_q)$

$\oint\vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\times\vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde{U} = \frac{1}{2}\frac{q_iq_j}{|\vec{r}_i-\vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int\vec{E}_i\vec{E}_j\,\mathrm{d}^3x$

$V(\vec{r}) = \int\rho G_{\mathrm{D}}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_{\mathrm{D}}}{\partial n}\,\mathrm{d}S$

$V(\vec{r}) = \langle V\rangle_S + \int\rho G_{\mathrm{N}}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S\frac{\partial V}{\partial n}G_{\mathrm{N}}(\vec{r})\,\mathrm{d}S$

$$\nabla^2_y G(\vec x,\vec y) = -4\pi\delta(\vec x-\vec y);\ G_{\mathrm D}(\vec x,\vec y)\big|_{\vec y\in S}=0;\ \frac{\partial G_{\mathrm N}}{\partial n}\Big|_{\vec y\in S} = -\frac{4\pi}{S}$$

$$U_{\mathrm{sphere}} = \frac35 \frac{Q^2}{R};\ \vec p = \int \mathrm d^3r\, \rho\vec r;\ \vec E_{\mathrm{dip}} = \frac{3(\vec p\hat r)\hat r-\vec p}{r^3};\ V_{\mathrm{dip}}=\frac{\vec p\hat r}{r^2}$$

force on a dipole: $\vec F_{\mathrm{dip}} = (\vec p\vec\nabla)\vec E$

$$Q_{ij}=\int \mathrm d^3r\,\rho(\vec r)(3r_ir_j-\delta_{ij}r^2);\ V_{\mathrm{quad}}=\frac{1}{6r^5}Q_{ij}(3r_ir_j-\delta_{ij}r^2)$$

$$V(r,\theta)=\sum_{l=0}^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$V(r,\theta,\varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l \left(A_{lm}r^l+\frac{B_{lm}}{r^{l+1}}\right)Y_{lm}(\theta,\varphi)$$

$$V(r>\operatorname{diam\,supp}\rho,\theta,\varphi) = \sum_{l=0}^\infty \frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^\infty r^2\mathrm dr\int_0^{2\pi}\mathrm d\varphi\int_0^\pi \sin\theta\, \mathrm d\theta\, r^l\rho(r,\theta,\varphi)Y^*_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec\nabla\vec J = -\frac{\partial\rho}{\partial t}=0;\ I=\int\vec J \mathrm d\vec S$$

solenoid: $B=4\pi\frac{j_{\mathrm s}}{c}$; wires: $\frac{\mathrm dF}{\mathrm dl}=\frac{2}{c^2}\frac{I_1I_2}{d}$

$$\mathrm d\vec F = \frac{I\mathrm d\vec l}{c}\times\vec B = \mathrm d^3x\frac{\vec J}{c}\times\vec B;\ \vec F_q = q\frac{\dot{\vec r}}{c}\times\vec B$$

$$\mathrm d\vec B = \frac{I\mathrm d\vec l}{c}\times\frac{\vec r}{r^3};\ \vec B_q = q\frac{\dot{\vec r}}{c}\times\frac{\vec r}{r^3}$$

$$\vec B=\vec\nabla\times\vec A;\ \vec A = \int \mathrm d^3r'\frac{\vec J'}{c}\frac{1}{|\vec r-\vec r'|}+\vec\nabla\Lambda$$

$$\vec B = \int \mathrm d^3r'\frac{\vec J'}{c}\times\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$$

$$\varphi = \frac Ic\Omega,\ \vec B=-\vec\nabla\varphi$$

$$\vec\nabla\vec A=0\to\nabla^2\vec A=-4\pi\frac{\vec J}{c}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec B=4\pi\frac{\vec J}{c};\ \oint\vec B \mathrm d\vec l = 4\pi\frac Ic$$

$$\vec m = \frac12\int \mathrm d^3r'\left(\vec r'\times\frac{\vec J'}{c}\right) = \frac{1}{2c}\frac qm\vec L = \frac{\vec SI}{c}$$

$$\vec A_{\mathrm{dm}}=\frac{\vec m\times\vec r}{r^3};\ \vec\tau=\vec m\times\vec B$$

$$\vec F_{\mathrm{dmdm}} = -\vec\nabla_R\frac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$$

loop axis: $\vec B=\hat z\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac Ic$

Electromagnetism (CGS)

Faraday: $\mathcal E = -\frac1c\frac{\mathrm d\Phi_B}{\mathrm dt}$; $\int \mathrm d^3x\vec J=\dot{\vec p}$

$$\vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t};\ \vec\nabla\vec E=4\pi\rho;\ \vec\nabla\vec J=-\frac{\partial\rho}{\partial t}$$

$$\vec\nabla\times\vec B=4\pi\frac{\vec J}{c}+\frac1c\frac{\partial\vec E}{\partial t};\ \vec\nabla\vec B=0$$

$$\mathrm d\vec F=\mathrm d^3x\left(\rho\vec E+\frac{\vec J}{c}\times\vec B\right);\ \vec F_q=q\left(\vec E+\frac{\dot{\vec r}}{c}\times\vec B\right)$$

$$u=\frac{E^2+B^2}{8\pi};\ \vec S=\frac{c}{4\pi}\vec E\times\vec B;\ \vec g=\frac{\vec S}{c^2}$$

$$\mathbf T^E = \frac1{4\pi}\left(\vec E\otimes\vec E-\frac12E^2\right);\ \mathbf T=\mathbf T^E+\mathbf T^B$$

$$-\frac{\partial u}{\partial t}=\vec J\vec E+\vec\nabla\vec S;\ -\frac{\partial\vec g}{\partial t}=\rho\vec E+\frac{\vec J}{c}\times\vec B-\vec\nabla\mathbf T$$

$$\vec B=\vec\nabla\times\vec A;\ \vec E=-\vec\nabla\phi-\frac1c\frac{\partial\vec A}{\partial t}$$

$$-\nabla^2\phi-\frac1c\frac{\partial}{\partial t}\vec\nabla\vec A=4\pi\rho$$

$$\vec\nabla\left(\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}\right)-\nabla^2\vec A+\frac1c\frac{\partial^2\vec A}{\partial t^2}=4\pi\frac{\vec J}{c}$$

$$(\phi,\vec A)\cong\left(\phi-\frac1c\frac{\partial\Lambda}{\partial t},\vec A+\vec\nabla\Lambda\right)$$

$$(\phi,\vec A)=\int \mathrm d^3r'\frac{\left(\rho,\frac{\vec J}{c}\right)\left(\vec r',t-\frac1c|\vec r-\vec r'|\right)}{|\vec r-\vec r'|}$$

$$\vec\nabla\vec A=0\to\Box\vec A=\frac{4\pi}{c}(\vec J-\vec J_L)=:\frac{4\pi}{c}\vec J_T$$

$$\vec J_L=\frac1{4\pi}\vec\nabla\frac{\partial\phi}{\partial t}=-\frac1{4\pi}\vec\nabla\int\frac{\vec\nabla'\vec J'}{|\vec r-\vec r'|}\mathrm d^3r'$$

$$\vec E'_\parallel=\vec E_\parallel;\ \vec B'_\parallel=\vec B_\parallel$$

$$\vec E'_\perp=\gamma\left(\vec E_\perp+\frac{\vec v}{c}\times\vec B\right)$$

$$\vec B'_\perp=\gamma\left(\vec B_\perp-\frac{\vec v}{c}\times\vec E\right)$$

$$\vec\omega_{\mathrm{Larmor}}=-\frac{q}{2mc}\vec B$$

plane wave: $\begin{cases}\vec E=\vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B=\hat k\times\vec E\\ \omega=ck\end{cases}$

$$\vec B_{\mathrm{diprad}}=\frac1{c^2}\frac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}};\ \vec E_{\mathrm{diprad}}=\vec B_{\mathrm{diprad}}\times\hat r$$

Larmor: $P=\frac{2}{3c^3}|\ddot{\vec p}|^2$

Rel. Larmor: $P=\frac{2}{3c^3}q^2\gamma^6(a^2-(\vec a\times\vec\beta)^2)$

$$\vec A_{\mathrm{dm}}=\frac1c\frac{\dot{\vec m}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}}$$

L.W.: $(\phi,\vec A)=\frac{q(1,\frac{\vec v}{c})}{[r-\frac{\vec v\vec r}{c}]_{t_{\mathrm{rit}}}}$; $t_{\mathrm{rit}}=t-\frac rc\big|_{t_{\mathrm{rit}}}$

$$A^\mu=(\phi,\vec A);\ J^\mu=(c\rho,\vec J)$$

Lorenz gauge: $\partial_\alpha A^\alpha=0$

Temporal gauge: $\phi=0$

Axial gauge: $A_3=0$

Coulomb gauge: $\vec\nabla\vec A=0$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu;\ \mathscr F^{\mu\nu}=\frac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu}=\begin{pmatrix}0&-E_x&-E_y&-E_z\\ E_x&0&-B_z&B_y\\ E_y&B_z&0&-B_x\\ E_z&-B_y&B_x&0\end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu}=4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr F^{\alpha\nu}=0;\ \frac{\mathrm dp^\mu}{\mathrm d\tau}=qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma}+\partial_\nu F_{\sigma\mu}+\partial_\sigma F_{\mu\nu}=0;\ \det F=(\vec E\vec B)^2$$

$$F^{\alpha\beta}F_{\alpha\beta}=2(B^2-E^2);\ F^{\alpha\beta}\mathscr F_{\alpha\beta}=4\vec E\vec B$$

$$\Theta^{\mu\nu}=\frac1{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu}+\frac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu}=\begin{pmatrix}u & c\vec g\\ c\vec g & -\mathbf T\end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu}=\frac{J_\alpha}{c}F^{\alpha\nu}=-G^\nu$$

$$\mathcal L=\frac{mc^2}{\gamma}-q\vec A\frac{\vec v}{c}+q\phi;\ \mathcal H=\frac1{2m}\left(\vec p-\frac{q\vec A}{c}\right)^2+q\phi$$

plane wave: $\mathbf T=-u\hat k\otimes\hat k$; $\Theta^{\mu\nu}=u\hat k^\mu\hat k^\nu$

E.M. in matter (CGS)

$$\vec\nabla\vec D=4\pi\rho_{\mathrm{ext}};\ \vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec H=4\pi\frac{\vec J_{\mathrm{ext}}}{c}+\frac1c\frac{\partial\vec D}{\partial t}$$

$$\vec P=\frac{\mathrm d\langle\vec p\rangle}{\mathrm dV};\ \vec M=\frac{\mathrm d\langle\vec m\rangle}{\mathrm dV}$$

$$\rho_{\mathrm{pol}}=-\vec\nabla\vec P;\ \sigma_{\mathrm{pol}}=\hat n\vec P;\ \frac{\vec J_{\mathrm{mag}}}{c}=\vec\nabla\times\vec M$$

$$\vec D_{\mathrm{pol}}=\vec E+4\pi\vec P;\ \vec H_{\mathrm{mag}}=\vec B-4\pi\vec M$$

static linear isotropic: $\vec P=\chi\vec E$

static linear: $P_i=\chi_{ij}E_j$

static linear: $\varepsilon=1+4\pi\chi$

static: $\Delta D_\perp=4\pi\sigma_{\mathrm{ext}}$; $\Delta E_\parallel=0$

static linear: $u=\frac1{8\pi}\vec E\vec D$

$$\Delta U_{\mathrm{dielectric}}=-\frac12\int \mathrm d^3r\,\vec P\vec E_0$$

plane capacitor: $C=\frac{\varepsilon}{4\pi}\frac Sd$

cilindric capacitor: $C=\frac{L}{2\log\frac Rr}$

atomic polarizability: $\vec p=\alpha\vec E_{\mathrm{loc}}$

non-interacting gas: $\vec p=\alpha\vec E_0$; $\chi=n\alpha$

hom. cubic isotropic: $\chi=\frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi}3n\alpha$

perm. dipole: $\chi=\frac13\frac{np_0^2}{kT}$

local field: $\vec E_{\mathrm{loc}}=\vec E+\frac{4\pi}3\vec P$

$$\vec J\vec E=-\vec\nabla\left(\frac{c}{4\pi}\vec E\times\vec H\right)-\frac1{4\pi}\left(\vec E\frac{\partial\vec D}{\partial t}+\vec H\frac{\partial\vec B}{\partial t}\right)$$

$$n=\sqrt{\varepsilon\mu};\ k=n\frac\omega c$$

plane wave: $B=nE$

$$\vec J_{\mathrm c}=\sigma\vec E;\ \varepsilon_\sigma=1+i\frac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm p}^2=4\pi\frac{n_{\mathrm{vol}}q^2}{m};\ \omega_{\mathrm{cyclo}}=\frac{qB}{mc}$$

I: $u=\frac1{8\pi}(\vec E\vec D+\vec H\vec B)$

I: $\langle S_z\rangle=\frac cn\langle u\rangle$

II: $u=\frac1{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\frac{\partial}{\partial\omega}(\mu\omega)H^2\right)$

II: $\langle S_z\rangle=v_{\mathrm g}\langle u\rangle$; $v_{\mathrm g}=\frac{\partial\omega}{\partial k}=\frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$

III: $\langle W\rangle=\frac{\omega}{4\pi}\left(\operatorname{Im}\varepsilon\langle E^2\rangle+\operatorname{Im}\mu\langle H^2\rangle\right)$

Fresnel TE (S): $\frac{E_{\mathrm t}}{E_{\mathrm i}}=\frac{2}{1+\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\mathrm r}}{E_{\mathrm i}}=\frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_{\mathrm t}}{E_{\mathrm i}}=\frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\mathrm r}}{E_{\mathrm i}}=\frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz}=\pm\sqrt{\varepsilon_2\left(\frac\omega c\right)^2-k_x^2}$, $\operatorname{Im}k_{tz}>0$

Drüde-Lorentz: $\varepsilon=1-\frac{\omega_{\mathrm p}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$$P(t)=\int_{-\infty}^\infty g(t-t')E(t')\mathrm dt'$$

$$P(\omega)=\chi(\omega)E(\omega)$$

$$\chi(\omega)=\int_{-\infty}^\infty e^{i\omega t}g(t)\mathrm dt;\ \chi(-\omega)=\chi^*(\omega)$$

$$g(t<0)=0\implies$$

$$\operatorname{Re}\varepsilon(\omega)=1+\frac2\pi\fint_0^\infty\frac{\omega'\left(\operatorname{Im}\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm d\omega'$$

$$\operatorname{Im}\varepsilon(\omega)=-\frac{2\omega}{\pi}\fint_0^\infty\frac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm d\omega'+\frac{4\pi\sigma_0}{\omega}$$

sum rule: $\frac\pi2\omega_{\mathrm p}^2=\int_0^\infty\omega\operatorname{Im}\varepsilon \mathrm d\omega$

sum rule: $2\pi^2\sigma_0=\int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm d\omega$

sum rule: $\int_0^\infty(\operatorname{Re}n-1)\mathrm d\omega=0$

Miller rule: $\chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$

Quantum mechanics (CGS)

$$r_{\mathrm e}=\frac{e^2}{m_{\mathrm e}c^2};\ \alpha=\frac{e^2}{\hbar c};\ \mu_{\mathrm B}=\frac{e\hbar}{2m_{\mathrm e}c}$$

$$\lambda_{\mathrm{Broglie}}=\frac hp$$

Planck: $\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm d\nu$

$$i\hbar\frac{\partial\mathcal U}{\partial t}=\mathcal H\mathcal U;\ \frac{\partial\mathcal H}{\partial t}=0\Rightarrow\mathcal U(t)=e^{-\frac{i\mathcal Ht}{\hbar}}$$

$$[\mathcal H(t),\mathcal H(t')]=0\Rightarrow\mathcal U(t)=e^{-\frac{i\int_0^t\mathrm dt\,\mathcal H(t)}{\hbar}}$$

$$\mathcal U=\left(\frac{-i}{\hbar}\right)^k\int_0^t\mathrm dt_1\cdots\int_0^{t_{k-1}}\mathrm dt_k\,\mathcal H(t_1)\cdots\mathcal H(t_k)$$

$$A_H(t)=\mathcal U(t)^\dagger A\,\mathcal U(t)$$

$$\frac{\partial\mathcal H}{\partial t}=0\Rightarrow\frac{\mathrm dA_H}{\mathrm dt}=\frac{[A_H,\mathcal H]}{i\hbar}$$

$$(A\otimes B)(|a\rangle\otimes|b\rangle)=A|a\rangle\otimes B|b\rangle$$

$$(\langle a|\otimes\langle b|)(|c\rangle\otimes|d\rangle)=\langle a|c\rangle\langle b|d\rangle$$

$$A^{(1)}+B^{(2)}=A^{(1)}\otimes\mathbb 1^{(2)}+\mathbb 1^{(1)}\otimes B^{(2)}$$

$$[A,BC]=[A,B]C+B[A,C]$$

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$$

$$[X,P]=i\hbar;\ \langle x|p\rangle=\frac1{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$$

$$\langle x|X|\psi\rangle = x\langle x|\psi\rangle;\ \langle x|P|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|\psi\rangle$$
$$\left\langle (A-\langle A\rangle)^2\right\rangle\left\langle (B-\langle B\rangle)^2\right\rangle \ge \tfrac{1}{4}|\langle [A,B]\rangle|^2$$
$$e^B A e^{-B} = A + [B,A] + \tfrac{1}{2!}[B,[B,A]] + \cdots$$
$$[A,B]\propto 1 \ \Rightarrow\ [A,f(B)] = [A,B]f'(B)$$
$$[A,B]\propto 1 \ \Rightarrow\ e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$$
$$e^{\frac{ip'X}{\hbar}}|p\rangle = |p+p'\rangle;\ e^{-\frac{iPx'}{\hbar}}|x\rangle = |x+x'\rangle$$
$$\psi(x) = \langle x|\psi\rangle;\ \rho = |\psi|^2;\ \psi = \sqrt{\rho}e^{\frac{iS}{\hbar}}$$
$$\mathcal{H} = \tfrac{\vec{P}^2}{2m} + V(\vec{X}) : \vec{j} = \tfrac{\hbar}{m}\mathrm{Im}(\psi^*\vec{\nabla}\psi) = \tfrac{\rho\vec{\nabla}S}{m}$$
$$\tfrac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j};\ \int \mathrm{d}^3x\,\vec{j} = \tfrac{\langle\vec{P}\rangle}{m}$$

$$\psi(x,t) = \int \mathrm{d}x' K(x,t;x')\psi(x',t=0)$$
$$K(x,t;x') = \sum_E \psi_E(x')^*\psi_E(x)e^{-\frac{iEt}{\hbar}} = \langle x|e^{-\frac{i\mathcal{H}t}{\hbar}}|x'\rangle$$
$$\left(\mathcal{H} - i\hbar\tfrac{\partial}{\partial t}\right)K(x,t;x') = -i\hbar\delta(x-x')\delta(t)$$
$$\rho[|\alpha_i\rangle, w_i] := \sum_i w_i|\alpha_i\rangle\langle\alpha_i|$$
$$\mathrm{tr}\,\rho = 1;\ [A] := \mathrm{tr}(\rho A)$$
$$\#\{w_i > 0\} = 1 \iff \mathrm{tr}(\rho^2) = 1$$
$$\#\{w_i > 0\} > 1 \iff 0 < \mathrm{tr}(\rho^2) < 1$$
$$\tfrac{\partial\rho}{\partial t} = -\tfrac{[\rho,\mathcal{H}]}{i\hbar}$$
$$W_\psi(x,p) = \int \tfrac{\mathrm{d}y}{2\pi\hbar}\left\langle x+\tfrac{y}{2}\middle|\psi\right\rangle\left\langle\psi\middle|x-\tfrac{y}{2}\right\rangle e^{-\frac{ipy}{2}}$$

$$V(x+a) = V(x) \ \Rightarrow\ \psi_{nk}(x) = e^{ikx}u_{nk}(x),$$
$$u_{nk}(x+a) = u_{nk}(x)$$
$$\text{trasm.} \approx e^{-\int_0^d \frac{\sqrt{2m(V(x)-E)}}{\hbar}\mathrm{d}x}$$
$$\mathcal{H}_{\mathrm{em}} = \tfrac{1}{2m}\left(\vec{p} - \tfrac{e}{c}\vec{A}\right)^2 + e\phi(\vec{x}) - \vec{\mu}\vec{B}$$
$$\vec{j}_{\mathrm{em}} = \tfrac{e\hbar}{m}\mathrm{Im}(\psi^*\vec{\nabla}\psi) - \tfrac{e^2}{mc}|\psi|^2\vec{A} + c\vec{\nabla}\times(\psi^*\vec{\mu}\psi) =$$
$$= \tfrac{e}{m}\mathrm{Im}\left(\psi^*\left(\vec{p} - \tfrac{e}{c}\vec{A}\right)\psi\right) + c\vec{\nabla}\times(\psi^*\vec{\mu}\psi)$$
$$D_\mu = \partial_\mu + i\tfrac{q}{\hbar c}A_\mu$$
$$\vec{A}\mapsto\vec{A} + \vec{\nabla}\Lambda \ \Rightarrow\ |\psi\rangle \mapsto e^{\frac{iq\Lambda}{\hbar c}}|\psi\rangle$$
$$\vec{\mu} = g\tfrac{e\hbar}{2mc}\vec{L}$$

QM perturbative

$$\mathcal{H} = \mathcal{H}_0 + V$$
$$E = E_0 + \epsilon_1 + \epsilon_2 + \cdots$$
$$V \mapsto kV \ \Rightarrow\ \epsilon_n \mapsto k^n\epsilon_n$$
$$|\psi\rangle = |\psi_0\rangle + |\psi_1\rangle + \cdots;\ \langle\psi_0|\psi\rangle = 1$$
$$\epsilon_1 = \langle\psi_0|V|\psi_0\rangle$$
$$|\psi_1\rangle = \sum_{\alpha\neq\psi_0}|\alpha\rangle\tfrac{\langle\alpha|V|\psi_0\rangle}{E_0 - E_\alpha}$$
$$\epsilon_2 = \sum_{\alpha\neq\psi_0}\tfrac{|\langle\alpha|V|\psi_0\rangle|^2}{E_0 - E_\alpha} = \langle\psi_0|V|\psi_1\rangle$$
$$\epsilon_n = \langle\psi_0|V|\psi_{n-1}\rangle$$

$$Q := \sum_{\alpha\neq\psi_0}|\alpha\rangle\langle\alpha|;\ G_Q := Q\tfrac{1}{E_0 - \mathcal{H}_0}Q$$
$$|\psi_n\rangle = G_Q V|\psi_{n-1}\rangle - \sum_{s=1}^{n-1}\epsilon_s G_Q|\psi_{n-s}\rangle$$
$$\mathcal{H} = \mathcal{H}_0 + V(t)$$
$$|\psi(t)\rangle = \sum_k a_k(t)e^{-iE_kt/\hbar}|k\rangle$$
$$a_k = a_k^{(0)} + a_k^{(1)} + \cdots$$
$$\hbar\omega_{ab} := E_a - E_b;\ V_{ab} := \langle a|V|b\rangle$$
$$a_k^{(n+1)}(t) = \tfrac{i}{\hbar}\sum_s\int_0^t \mathrm{d}\tau\, e^{i\omega_{ks}\tau}V_{ks}(\tau)a_s^{(n)}(\tau)$$
$$\underset{f\neq i}{P_{i\to f}}(t) = \left|\tfrac{1}{\hbar}\int_0^t \mathrm{d}\tau\, e^{i\omega_{fi}\tau}V_{fi}(\tau)\right|^2$$

$$\underset{f\neq i}{P_{i\to f}} = \left|\tfrac{1}{\hbar\omega_{fi}}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega_{fi}t}V'_{fi}(t) + O(V^2)\right|^2$$
$$|\psi(t)\rangle =: e^{-i\mathcal{H}_0t/\hbar}|\tilde{\psi}(t)\rangle$$
$$|\tilde{\psi}(t)\rangle =: \tilde{U}(t)|\tilde{\psi}(0)\rangle$$
$$i\hbar\tfrac{\partial\tilde{U}}{\partial t} =: \tilde{\mathcal{H}}\tilde{U};\ \tilde{\mathcal{H}}(t) = e^{i\mathcal{H}_0t/\hbar}V(t)e^{-i\mathcal{H}_0t/\hbar}$$
$$R_{i\to f} \approx \tfrac{2\pi}{\hbar}\left|V_{fi} + \sum_{s\neq i}\tfrac{V_{fs}V_{si}}{E_i - E_s + i\varepsilon}\right|^2\delta(E_f - E_i)$$
$$V(t) = Fe^{-i\omega t} + F^\dagger e^{i\omega t} \to$$
$$\to R_{i\to f} \approx \tfrac{2\pi}{\hbar}|F_{fi}|^2\delta(E_f - E_i \pm \hbar\omega)$$

QM scattering

$$\tfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f|^2;\ \text{elastic: } f = \tfrac{k}{2\pi\hbar v}\left\langle\vec{k}_f\middle|T\middle|\vec{k}_i\right\rangle$$
$$\text{anelastic: } f = \sqrt{\tfrac{k'^2}{4\pi^2\hbar^2vv'}}\left\langle\vec{k}',b\middle|T\middle|\vec{k},a\right\rangle$$

$$\text{Born: } \langle|T|\rangle \approx \langle|V|\rangle$$
$$\vec{q} := \vec{k}' - \vec{k};\ \left\langle\vec{k}'\middle|V\middle|\vec{k}\right\rangle = \int \mathrm{d}\vec{x}\, V(\vec{x})e^{-i\vec{q}\vec{x}}$$
$$V = V(r) : \left\langle\vec{k}'\middle|V\middle|\vec{k}\right\rangle = 4\pi\int_0^\infty \mathrm{d}r\, rV\tfrac{\sin(qr)}{q}$$

$$\psi(\vec{x}) = e^{i\vec{k}\vec{x}} - \tfrac{m}{2\pi\hbar^2}\int \mathrm{d}\vec{y}\,\tfrac{e^{i\vec{k}|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|}V(\vec{y})\psi(\vec{y})$$
$$f = -\tfrac{m}{2\pi\hbar^2}\int \mathrm{d}\vec{y}\, e^{-i\vec{k}'\vec{y}}V(\vec{y})\psi(\vec{y})$$
$$\mathrm{d}\Omega = 2\pi\tfrac{q\mathrm{d}q}{k^2};\ \sigma_{\mathrm{tot}} = \tfrac{4\pi}{k}\mathrm{Im}(f(0))$$
$$\text{charge: } f = -\tfrac{2m}{\hbar^2}\tfrac{q_{\mathrm{part}}}{q^2}\left(Q_{\mathrm{tot}} - \int \mathrm{d}\vec{r}\,\rho(\vec{r})e^{-i\vec{q}\vec{r}}\right)$$

QM rotations

$$\sigma_1 = \left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right);\ \sigma_2 = \left(\begin{smallmatrix}0&-i\\i&0\end{smallmatrix}\right);\ \sigma_3 = \left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$$
$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k;\ \mathrm{tr}(\hat{n}\vec{\sigma}) = 0$$
$$[\sigma_i,\sigma_j] = 2i\varepsilon_{ijk}\sigma_k;\ \{\sigma_i,\sigma_j\} = 2\delta_{ij}$$
$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = \vec{a}\vec{b} + i\vec{\sigma}(\vec{a}\times\vec{b})$$
$$e^{i\vec{\sigma}\hat{n}\alpha} = \cos\alpha + i(\vec{\sigma}\hat{n})\sin\alpha$$
$$|\vec{\sigma}\hat{n},1\rangle = \cos\tfrac{\theta}{2}|\sigma_3,1\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|\sigma_3,-1\rangle$$
$$M = \vec{v}\vec{\sigma} \ \Rightarrow\ \lambda_M = \pm|\vec{v}|$$
$$U(R_{\hat{n},\phi}) = \exp\left(-i\vec{J}\hat{n}\phi\right)$$
$$[J_i,J_j] = i\varepsilon_{ijk}J_k;\ J_\pm := J_x \pm iJ_y$$
$$[J_+,J_-] = 2J_z;\ [J_z,J_\pm] = \pm J_\pm$$
$$[J^2,J_\pm] = [J^2,J_z] = 0$$
$$J^2|j,m\rangle = j(j+1)|j,m\rangle$$
$$J_z|j,m\rangle = m|j,m\rangle$$
$$J_\pm|j,m\rangle = \sqrt{j(j+1) - m(m\pm 1)}\,|j,m\pm 1\rangle$$
$$m = -j, j-1, \ldots, j;\ 2j \in \mathbb{N}$$

$$U \in \mathrm{SU}_2 : U = \left(\begin{smallmatrix}a&b\\-b^*&a^*\end{smallmatrix}\right),\ |a|^2 + |b|^2 = 1$$
$$U = e^{-\frac{i\sigma_z\alpha}{2}}e^{-\frac{i\sigma_y\beta}{2}}e^{-\frac{i\sigma_z\gamma}{2}}$$
$$a = \cos\tfrac{\phi}{2} - in_z\sin\tfrac{\phi}{2} = e^{-i\frac{\alpha+\gamma}{2}}\cos\tfrac{\beta}{2}$$
$$b = -\sin\tfrac{\phi}{2}(n_y + in_x) = -e^{-i\frac{\alpha-\gamma}{2}}\sin\tfrac{\beta}{2}$$
$$U(\vec{v}\vec{\sigma})U^\dagger = (R_{\hat{n},\phi}\vec{v})\vec{\sigma}$$
$$\vec{L} = \vec{X}\times\vec{P};\ L_z = \tfrac{\hbar}{i}\tfrac{\partial}{\partial\varphi}$$
$$L_\pm = \hbar e^{\pm i\varphi}\left(\pm\tfrac{\partial}{\partial\theta} + i\cot\theta\tfrac{\partial}{\partial\varphi}\right)$$
$$\vec{L}^2 = -\hbar^2\left(\tfrac{1}{\sin^2\theta}\tfrac{\partial^2}{\partial\varphi^2} + \tfrac{1}{\sin\theta}\tfrac{\partial}{\partial\theta}\left(\tfrac{1}{\sin\theta}\tfrac{\partial}{\partial\theta}\right)\right)$$
$$\vec{L}^2Y_{lm} = \hbar l(l+1)Y_{lm};\ L_zY_{lm} = \hbar mY_{lm}$$
$$A = \vec{A} :\leftrightarrow [J_i,A_j] = i\varepsilon_{ijk}A_k$$
$$T = \mathbf{T} :\leftrightarrow [J_z,T_q] = qT_q,$$
$$\left[J_\pm,T_q^{(k)}\right] = \sqrt{k(k+1) - q(q\pm 1)}\,T_{q\pm 1}^{(k)}$$
$$U(R)^\dagger\vec{A}U(R) = R\vec{A}$$
$$\left[\vec{\theta}\vec{J},\vec{A}\right] = -i\vec{\theta}\times\vec{A}$$

$$\left[\hat{n}\vec{J},T_q^{(k)}\right] = \sum_{q'}\langle kq'|\hat{n}\vec{J}|kq\rangle T_{q'}^{(k)}$$
$$\langle jm'|e^{-i\alpha J_z}e^{-i\beta J_y}e^{-i\gamma J_z}|jm\rangle =$$
$$= e^{-i\alpha m'}\langle jm'|e^{-i\beta J_y}|jm\rangle e^{-i\gamma m}$$
$$\langle jm'|e^{-i\beta J_y}|jm\rangle =$$
$$= (-1)^{m'-m}\sqrt{\tfrac{(j+m')!(j-m')!}{(j+m)!(j-m)!}}\left(\cos\tfrac{\beta}{2}\right)^{2j}\cdot$$
$$\cdot\sum_\mu(-1)^\mu\binom{j+m}{\mu}\binom{j-m}{j-m'-\mu}\left(\tan\tfrac{\beta}{2}\right)^{m'-m+2\mu}$$
$$Y_{lk}(\theta,\varphi) = \sqrt{\tfrac{2l+1}{4\pi}}\langle lk|e^{-i\theta J_y}|l0\rangle e^{ik\varphi}$$
$$C_{M;m_1m_2}^{J;j_1j_2} := \langle j_1m_1;j_2m_2|j_1j_2JM\rangle$$
$$C \neq 0 \ \Rightarrow\ M = m_1 + m_2;\ C \in \mathbb{R}$$
$$C \neq 0 \ \Rightarrow\ |j_1 - j_2| \le J \le j_1 + j_2$$
$$C_{M;m_1m_2}^{J;j_1j_2} = (-1)^{J-j_1-j_2}C_{M;m_2m_1}^{J;j_2j_1}$$
$$\langle\alpha jm|T_q^{(k)}|\beta j'm'\rangle = \langle\alpha j\|T^{(k)}\|\beta j'\rangle C_{m;m'q}^{j;j'k}$$
$$J_x^{(1)} = \tfrac{1}{\sqrt{2}}\left(\begin{smallmatrix}&1&\\1&&1\\&1&\end{smallmatrix}\right)$$
$$\langle l+1,m|\cos\theta|lm\rangle = \sqrt{\tfrac{(l+1)^2-m^2}{(2l+1)(2l+3)}}$$

QM solutions

$$\text{BOX: } \mathcal{H} = \tfrac{P^2}{2m} + \begin{cases}0 & 0 < x < L\\ \infty & \text{otherwise}\end{cases}$$
$$E_n = \tfrac{n^2\pi^2\hbar^2}{2mL^2},\ n \ge 1$$
$$\psi_n(x) = \sqrt{\tfrac{2}{L}}\sin\left(n\pi\tfrac{x}{L}\right) = \sqrt{\tfrac{2}{L}}\sin\left(\sqrt{\tfrac{2mE}{\hbar^2}}x\right)$$
$$\Delta x^2 = L^2\left(\tfrac{1}{12} - \tfrac{1}{2n^2\pi^2}\right);\ \Delta p = \tfrac{\hbar n\pi}{L}$$
$$\text{HARM.: } \mathcal{H} = \tfrac{P^2}{2m} + \tfrac{m\omega^2X^2}{2};\ x_0 := \sqrt{\tfrac{\hbar}{m\omega}}$$

$$A := \tfrac{1}{\sqrt{2}x_0}\left(X + \tfrac{iP}{m\omega}\right);\ A^\dagger = \tfrac{1}{\sqrt{2}x_0}\left(X - \tfrac{iP}{m\omega}\right)$$
$$N := A^\dagger A = \tfrac{\mathcal{H}}{\hbar\omega} - \tfrac{1}{2};\ \mathcal{H} = \hbar\omega\left(N + \tfrac{1}{2}\right)$$
$$[A,A^\dagger] = 1;\ [N,A] = -A;\ [N,A^\dagger] = A^\dagger$$
$$A^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle;\ A|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$|n\rangle = \tfrac{(A^\dagger)^n}{\sqrt{n!}}|0\rangle,\ n = 0,1,\ldots$$
$$\psi_n(x) = \tfrac{1}{\pi^{\frac{1}{4}}\sqrt{2^nn!x_0}}H_n\left(\tfrac{x}{x_0}\right)e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$

$$\sum_{n=0}^\infty H_n(x)\tfrac{t^n}{n!} = e^{-t^2+2tx}$$
$$H_n(x) = e^{\frac{x^2}{2}}\left(x - \tfrac{\mathrm{d}}{\mathrm{d}x}\right)^ne^{-\frac{x^2}{2}}$$
$$H_n(x) = (-1)^ne^{x^2}\tfrac{\mathrm{d}^n}{\mathrm{d}x^n}e^{-x^2}$$
$$H_n(-x) = (-1)^nH_n(x)$$
$$n \text{ even: } H_n(0) = (-1)^{\frac{n}{2}}\tfrac{n!}{(n/2)!}$$
$$H_n'(x) = 2nH_{n-1}(x);\ H_0 = 1$$
$$H_1 = 2x;\ H_2 = 4x^2 - 2;\ H_3 = 8x^3 - 12x$$

$$H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x)$$
$$H_n''(x)=2xH_n'(x)-2nH_n(x)$$
$$\int_{-\infty}^{\infty}\mathrm{d}x H_n(x)H_m(x)e^{-x^2}=\sqrt{\pi}2^n n!\delta_{nm}$$
$$A_H(t)=Ae^{-i\omega t}$$
$$A\left|\alpha\right\rangle=\alpha\left|\alpha\right\rangle,\ \left|\alpha\right\rangle=\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-\frac{1}{2}|\alpha|^2}\left|n\right\rangle$$
$$D(\alpha):=e^{\alpha A^\dagger-\alpha^*A};\ D(\alpha)\left|0\right\rangle=\left|\alpha\right\rangle$$
$$\text{DELTA: }\mathcal{H}=\frac{P^2}{2m}-\lambda\delta(x),\ \lambda>0$$
$$x_0:=\frac{\hbar^2}{\lambda m};\ \beta:=\frac{m\lambda}{\hbar^2};\ k^2:=\frac{2mE}{\hbar^2}$$
$$\psi_{\text{bounded}}(x)=\frac{1}{\sqrt{x_0}}e^{-\frac{|x|}{x_0}},\ E_{\text{bounded}}=-\frac{\lambda}{2x_0}$$
$$R=\frac{1}{-1+i\frac{k}{\beta}};\ T=\frac{1}{1+i\frac{\beta}{k}}=R+1$$
$$\text{STEP: }\mathcal{H}=\frac{P^2}{2m}+\begin{cases}0 & x<0\\ V_0>0 & x>0\end{cases}$$
$$k^2:=\frac{2mE}{\hbar^2},\ q^2:=\frac{2m(E-V_0)}{\hbar^2}$$
$$\psi_{\text{right}}(x)\propto\begin{cases}e^{ikx}+\frac{k-q}{k+q}e^{-ikx} & x<0\\ \frac{2k}{k+q}e^{iqx} & x>0\end{cases}$$

$$\text{HYDROGEN: }\mathcal{H}=\frac{\vec{P}^2}{2M}-\frac{e^2}{X}$$
$$a:=r_B:=\frac{\hbar^2}{Me^2};\ \text{Rydberg}:=\frac{e^2}{2a}$$
$$E_n=-\frac{1}{n^2}\frac{e^2}{2a};\ \text{degen.}=n^2$$
$$\psi_{nlm}=R_{nl}Y_{lm};\ \vec{j}=\frac{\hbar}{M}\hat{\varphi}\frac{m}{r\sin\theta}|\psi|^2$$
$$R_{nl}=2\sqrt{\frac{(n-l-1)!}{a^3n^4(n+l)!}}e^{-\frac{r}{na}}\left(\frac{2r}{na}\right)^l L_{n+l}^{2l+1}\left(\frac{2r}{na}\right)$$
$$L_n^{(j)}(x)=\sum_{m=0}^{n-j}(-1)^m\binom{n}{n-j-m}\frac{x^m}{m!}$$
$$L_k(x)=e^x\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(x^ke^{-x}\right)$$
$$L_k^{(j)}=(-1)^j\frac{\mathrm{d}^j}{\mathrm{d}x^j}L_k(x)$$
$$R_{1s}=2a^{-\frac{3}{2}}e^{-\frac{r}{a}}$$
$$R_{2s}=\frac{1}{\sqrt{2}}a^{-\frac{3}{2}}\left(1-\frac{r}{2a}\right)e^{-\frac{r}{2a}}$$
$$R_{2p}=\frac{1}{2\sqrt{6}}a^{-\frac{3}{2}}\frac{r}{a}e^{-\frac{r}{2a}}$$
$$\Delta\mathcal{H}_{\text{f.s.}}=-\frac{1}{8}\frac{P^4}{m^3c^2}+\pi K\delta(\vec{X})+(g-1)K\frac{\vec{L}\vec{S}}{X^3}$$
$$K:=\frac{Ze^2\hbar^2}{2m^2c^2};\ \vec{J}=\vec{L}+\vec{S}$$
$$\Delta E_{\text{f.s.}}(n,l,j)=-Z^4\frac{\alpha^2}{2n^3}\left(\frac{1}{j+\frac{1}{2}}-\frac{3}{4n}\right)$$

$$\Delta\mathcal{H}_{\text{h.s.}}=C\vec{I}\vec{J}$$
$$C_{nlj}=2g_N\mu_N\mu_B\begin{cases}\frac{8\pi}{3}|\psi(0)|^2 & l=0\\ \left\langle\frac{1}{R^3}\right\rangle\frac{l(l+1)}{j(j+1)} & l>0\end{cases}$$
$$\Delta E_{jm}^{\text{Zeeman}}=\mu_BBm\Big(1+(g-1)\cdot$$
$$\cdot\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}\Big)$$
$$\vec{\omega}_{\text{Thomas}}=\frac{\vec{a}\times\vec{v}}{2c^2}$$
$$\text{RUTHERFORD: }V(r)=\frac{zZe^2}{r}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left(\frac{zZe^2}{2mv^2}\right)^2\left(\sin^4(\theta/2)\right)^{-1}$$
$$\text{YUKAWA: }V(r)=\frac{\alpha}{r}e^{-\mu r}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left(\frac{2m\alpha}{\hbar^2}\right)^2\left(|\vec{k}'-\vec{k}|^2+\mu^2\right)^{-2}$$
$$q:=\vec{k}'-\vec{k};\ \frac{\mathrm{d}\sigma}{\mathrm{d}q}=\frac{2\pi q}{k^2}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$
$$\sigma=16\pi\left(\frac{m\alpha}{\hbar^2}\right)^2\frac{1}{\mu^2}\frac{1}{4k^2+\mu^2}$$
$$\text{CHARGED SHERE }e\rho(r)$$
$$\left\langle\vec{k}'\middle|V\middle|\vec{k}\right\rangle=\frac{4\pi}{q^2}ze^2\int\mathrm{d}\vec{r}\,\rho(r)e^{-i\vec{q}\vec{r}}$$

Particle physics

$$M(A,Z)=Zm_{\text{p}}+(A-Z)m_{\text{n}}-B(A,Z)$$
$$B(A,Z)=a_vA-a_sA^{2/3}-a_c\frac{Z(Z-1)}{A^{1/3}}-a_{\text{sym}}\frac{(A-2Z)^2}{A}+a_pA^{-3/4}\Delta$$
$$\Delta=\begin{cases}0 & & A\text{ odd}\\ \left.\begin{matrix}1 & Z\text{ even}\\ -1 & Z\text{ odd}\end{matrix}\right\} & & A\text{ even}\end{cases}$$
$$a_v=15.5;\ a_s=16.8;\ a_c=0.72;\ a_{\text{sym}}=23;\ a_p=34\ [\text{MeV}]$$
$$\frac{\partial M}{\partial Z}=0:Z=\frac{m_{\text{n}}-m_{\text{p}}+4a_{\text{sym}}}{\frac{2a_c}{A^{1/3}}+\frac{8a_{\text{sym}}}{A}};\ r_{\text{nuc}}\approx1.5\sqrt[3]{A}\,\text{fm}$$
$$s_{ab}:=(p_a+p_b)^2;\ M\to abc:(m_a+m_b)^2\le s_{ab}\le(M-m_c)^2$$

$$M\to abc:s_{ab}+s_{bc}+s_{ac}=M^2+m_a^2+m_b^2+m_c^2$$
$$a_iA_i\to b_jB_j:Q:=a_im_{A_i}-b_jm_{B_j}$$
$$p=qBR$$
$$\frac{\mathrm{d}^3\vec{p}}{2E}=\mathrm{d}^4p\delta(p^2-m^2)\theta(p_0)$$
$$\mathrm{d}L_p=\left(\prod_n\frac{\mathrm{d}^3\vec{p}_n}{2E_n}\right)\delta^4(p_{\text{in}}-\sum\nolimits_n p_n);\ \mathrm{d}\sigma=f_{\text{coll}}(p_1,\dots,p_n)\mathrm{d}L_p$$
$$\text{two body: }\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1}=f(\Omega_1)\frac{p_1}{4\sqrt{s}};\ \sqrt{s}=\text{c.m. energy}$$
$$\text{Rutherford: }\tan\frac{\theta}{2}=\frac{1}{4\pi\varepsilon_0}\frac{Qqm}{p^2b};\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|;\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{d_{\min}^2}{16}\frac{1}{\sin^4\frac{\theta}{2}}$$
$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{Mott}}=\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{Rutherford}}\cdot\cos^2\frac{\theta}{2}$$
$$\text{mass defect}:=M-A\cdot\text{amu}$$

QFT fields

$$\varphi(x)=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(a(\vec{p})e^{-ip\cdot x}+a^\dagger(\vec{p})e^{ip\cdot x}\right)$$
$$\phi(x)=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(a_+(\vec{p})e^{-ip\cdot x}+a_-^\dagger(\vec{p})e^{ip\cdot x}\right)$$
$$\phi^*(x)=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(a_+^\dagger(\vec{p})e^{ip\cdot x}+a_-(\vec{p})e^{-ip\cdot x}\right)$$

$$B_\mu(x)=\sum\nolimits_b\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(a_b(\vec{p})\varepsilon_\mu(\vec{p},b)e^{-ip\cdot x}+a_b^\dagger(\vec{p})\varepsilon_\mu^*(\vec{p},b)e^{ip\cdot x}\right)$$
$$A_\mu(x)=\sum\nolimits_\lambda\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2|\vec{p}|}}\left(a_\lambda(\vec{p})\varepsilon_\mu(\vec{p},\lambda)e^{-ip\cdot x}+a_\lambda^\dagger(\vec{p})\varepsilon_\mu^*(\vec{p},\lambda)e^{ip\cdot x}\right)$$
$$\psi(x)=\sum\nolimits_r\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(b_r(\vec{p})u_r(\vec{p})e^{-ip\cdot x}+d_r^\dagger(\vec{p})v_r(\vec{p})e^{ip\cdot x}\right)$$
$$\bar{\psi}(x)=\sum\nolimits_r\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(b_r^\dagger(\vec{p})\bar{u}_r(\vec{p})e^{ip\cdot x}+d_r(\vec{p})\bar{v}_r(\vec{p})e^{-ip\cdot x}\right)$$

QFT ($\hbar=c=1$)

$$-\sigma^2\vec{\sigma}\sigma^2=\vec{\sigma}^\top$$
$$-\sigma^2\vec{\sigma}^*\sigma^2=\vec{\sigma}$$
$$[J^{\mu\nu},J^{\rho\sigma}]=i(\eta^{\mu\sigma}J^{\nu\rho}+\eta^{\nu\rho}J^{\mu\sigma}-\eta^{\mu\rho}J^{\nu\sigma}-\eta^{\nu\sigma}J^{\mu\rho})$$
$$J^i:=\frac{1}{2}\varepsilon^{ijk}J^{jk}$$
$$K^i:=\frac{1}{2}J^{0i}$$
$$\vec{J}_\pm:=\frac{1}{2}(\vec{J}\pm i\vec{K})$$
$$\left[J^i,J^j\right]=i\varepsilon^{ijk}J^k$$
$$J^{ij}=\varepsilon^{ijk}J^k$$
$$\left[K^i,K^j\right]=-iJ^{ij}$$
$$\left[J^i,K^j\right]=i\varepsilon^{ijk}K^k$$
$$[J_+,J_-]=0$$
$$[J_\pm^i,J_\pm^j]=i\varepsilon^{ijk}J_\pm^k$$
$$\text{fermions: }\left\{a,a^\dagger\right\}=1;\ \{a,a\}=\left\{a^\dagger,a^\dagger\right\}=0$$
$$b_\alpha:=\eta_\alpha a_\alpha$$
$$\eta_\alpha:=\prod_{\beta=1}^{\alpha-1}\begin{pmatrix}-1&\\&1\end{pmatrix}_\beta$$

$$\langle\vec{x}|\vec{p}\rangle:=\frac{1}{\sqrt{V}}e^{i\vec{p}\vec{x}}\ (\text{then }V\to\infty)$$
$$\langle\vec{p}_1|\vec{p}_2\rangle=\frac{(2\pi)^3}{V}\delta(\vec{p}_1-\vec{p}_2)$$
$$\tilde{U}(\infty,-\infty)=T_W\exp\left(i\int\mathrm{d}^4x\mathcal{L}_I[\Phi](x)\right)$$
$$S[\Phi]=:\int\mathrm{d}^4x\mathcal{L}[\Phi](x)=:\int\mathrm{d}^4x(\mathcal{L}_0[\Phi](x)+\mathcal{L}_I[\Phi](x))$$
$$T_W\Phi_1(x)\Phi_2(y):=\theta(x^0-y^0)\Phi_1(x)\Phi_2(y)\pm\theta(y^0-x^0)\Phi_2(y)\Phi_1(x),$$
$$-\text{ iff }\Phi_1\text{ and }\Phi_2\text{ fermions}$$
$$T_W\frac{\partial\Phi_1}{\partial x}\frac{\partial\Phi_2}{\partial y}:=\frac{\partial}{\partial x}\frac{\partial}{\partial y}T_W\Phi_1(x)\Phi_2(y)$$
$$S[\varphi]=S[x\mapsto\varphi(x+\lambda)]+\mathrm{O}(x^{()}\lambda^2)\to$$
$$\to T^{\mu\nu}=\partial^\mu\varphi\partial^\nu\varphi-\frac{1}{2}\eta^{\mu\nu}\mathcal{L}$$
$$\text{standard: }\gamma_S^0=\begin{pmatrix}\mathbb{1}&\\&-\mathbb{1}\end{pmatrix}$$
$$\text{standard: }\gamma_S^i=\begin{pmatrix}&\sigma^i\\-\sigma^i&\end{pmatrix}$$
$$\bar{u}:=u^\dagger\gamma^0\underset{(\text{standard})}{=}\begin{pmatrix}u_1^*&u_2^*&-u_3^*&-u_4^*\end{pmatrix}$$
$$\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3\underset{(\text{standard})}{=}\begin{pmatrix}&\mathbb{1}\\\mathbb{1}&\end{pmatrix}$$
$$\bar{a}b\text{ scalar}$$
$$\bar{a}\gamma^5b\text{ pseudoscalar}$$

$$\bar{a}\gamma^\mu b \text{ vector}$$
$$\bar{a}\gamma^\mu\gamma^5 b \text{ pseudovector}$$
$$\bar{a}\sigma^{\mu\nu} b \text{ tensor}$$
$$:a^\dagger\cdots a\cdots a^\dagger: := a^\dagger a^\dagger\cdots aa + ... \text{ (creation to the left)}$$
$$\text{real scalar } \mathcal{L}_0 = \tfrac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2),\ (\partial^\mu\partial_\mu + m^2)\varphi = 0$$
$$\text{complex scalar } \mathcal{L}_0 = \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi,\ (\partial_\mu\partial^\mu + m^2)\begin{pmatrix}\phi\\ \phi^*\end{pmatrix} = 0$$
$$\text{real vector } \mathcal{L}_0 = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{m^2}{2}B^\mu B_\mu,\ F^{\mu\nu} := \partial^\mu B^\nu - \partial^\nu B^\mu,$$
$$(\partial_\mu\partial^\mu + m^2)B^\nu = 0,\ \partial_\mu B^\mu = 0$$
$$\text{Dirac spinor } \mathcal{L}_0 = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi,\ (i\gamma^\mu\partial_\mu - m)\psi = 0$$
$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$
$$\Lambda(\omega) = e^{\frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}}$$
$$R(\omega) = e^{\frac{i}{4}\omega_{\mu\nu}\Sigma^{\mu\nu}}$$
$$R^{-1}(\omega)\gamma^\mu R(\omega) = \lambda^\mu_\nu(\omega)\gamma^\nu$$
$$R^{-1}(\omega) = \gamma^0 R^\dagger(\omega)\gamma^0$$
$$\bar{u}_r(\vec{p})u_s(\vec{p}) = -\bar{v}_r(\vec{p})v_s(\vec{p}) = 2m\delta_{rs}$$
$$u_r^\dagger(\vec{p})u_s(\vec{p}) = v_r^\dagger(\vec{p})v_s(\vec{p}) = 2E(\vec{p})\delta_{rs}$$
$$\bar{v}_r(\vec{p})u_s(\vec{p}) = \bar{u}_r(\vec{p})v_s(\vec{p}) = 0$$
$$v_r^\dagger(\vec{p})u_s(-\vec{p}) = u_r^\dagger(\vec{p})v_s(-\vec{p}) = 0$$
$$u_r^\alpha(\vec{p})\bar{u}_r^\beta(\vec{p}) - v_r^\alpha(\vec{p})\bar{v}_r^\beta(\vec{p}) = 2m\delta^{\alpha\beta}$$
$$u_r^\alpha(\vec{p})\bar{u}_r^\beta(\vec{p}) = (\not{p} + m)^{\alpha\beta}$$
$$v_r^\alpha(\vec{p})\bar{v}_r^\beta(\vec{p}) = (\not{p} - m)^{\alpha\beta}$$
$$\left(\gamma^0\right)^\dagger = \gamma^0$$
$$\left(\gamma^i\right)^\dagger = -\gamma^i$$
$$\gamma^0\left(\gamma^i\right)^\dagger\gamma^0 = \gamma^i$$
$$\left(\gamma^5\right)^\dagger = \gamma^5$$
$$\left\{\gamma^5, \gamma^\mu\right\} = 0$$
$$\left[\gamma^5, \Sigma^{\mu\nu}\right] = 0$$
$$\operatorname{tr}\gamma^5 = 0$$
$$\not{p} := \gamma^\mu p_\mu$$
$$\left\{\gamma^5, \gamma^\mu\right\} = 0$$
$$\not{p}\not{q} = pq - i\Sigma_{\mu\nu}p^\mu q^\nu$$
$$\gamma_\mu\not{p}\gamma^\mu = -2\not{p}$$
$$\gamma_\mu\not{p}\not{q}\not{k}\gamma^\mu = -2\not{k}\not{q}\not{p}$$
$$\gamma_\mu\not{p}\not{q}\gamma^\mu = 4pq$$
$$\operatorname{tr}(\underbrace{\gamma^\mu\gamma^\nu\cdots\gamma^\sigma}_{\text{dispari}}) = 0$$
$$\operatorname{tr}\left(\gamma^5\not{p}\not{q}\right) = 0$$
$$\operatorname{tr}\left(\not{p}\not{q}\right) = 4pq$$
$$\operatorname{tr}\left(\gamma^5\not{p}\not{q}\not{k}\not{l}\right) = 4i\varepsilon_{\mu\nu\rho\sigma}p^\mu q^\nu k^\rho l^\sigma$$
$$\operatorname{tr}\left(\not{p}\not{q}\not{k}\not{l}\right) = 4\big((pq)(kl) - (pk)(ql) + (pl)(qk)\big)$$
$$\text{base } \{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \Sigma^{\mu\nu}\}$$
$$E(\vec{p}) := \sqrt{m^2 + \vec{p}^2}$$
$$\{b_r(\vec{p}), b_s^\dagger(\vec{k})\} = \{d_r(\vec{p}), d_s^\dagger(\vec{k})\} = \tfrac{(2\pi)^3}{V}\delta_{rs}\delta^3(\vec{p} - \vec{k})$$
$$\{a(\vec{p}), a^\dagger(\vec{k})\} = \tfrac{(2\pi)^3}{V}\delta^3(\vec{p} - \vec{k})$$
$$\theta^{\mu\nu} := i\bar\psi\gamma^\mu\partial^\nu\psi$$
$$P^\mu := \int \mathrm{d}^3x\,\theta^{0\nu} = \int \mathrm{d}^3x\,\psi^\dagger i\partial^\nu\psi$$
$$\theta_a := \tfrac{1}{2}\varepsilon_{abc}\omega^{bc}$$
$$\eta_a := \omega^{0a}$$
$$\Lambda(\omega) = e^{i(\theta_a J^a + \eta_a K^a)}$$
$$J^a K^a = 0$$
$$J^a J^a - K^a K^a = 3$$
$$J_R = J_-$$
$$J_L = J_+$$
$$\psi_R = \tfrac{1+\gamma^5}{2}\psi \underset{\text{(standard)}}{=} \begin{pmatrix}\xi_R\\ \xi_R\end{pmatrix},\ \xi_R = \tfrac{1}{2}\begin{pmatrix}\psi_1+\psi_3\\ \psi_2+\psi_4\end{pmatrix}$$
$$\psi_L = \tfrac{1-\gamma^5}{2}\psi \underset{\text{(standard)}}{=} \begin{pmatrix}\xi_L\\ -\xi_L\end{pmatrix},\ \xi_L = \tfrac{1}{2}\begin{pmatrix}\psi_1-\psi_3\\ \psi_2-\psi_4\end{pmatrix}$$
$$\Sigma_S^{0a} = i\begin{pmatrix} & \sigma^a\\ \sigma^a & \end{pmatrix}$$
$$\Sigma_S^{ab} = \varepsilon^{abc}\begin{pmatrix}\sigma^c & \\ & \sigma^c\end{pmatrix}$$
$$\tfrac{i}{4}\omega_{\mu\nu}\Sigma^{\mu\nu} = \tfrac{1}{2}\begin{pmatrix} i\theta_a\sigma^a & -\eta_a\sigma^a\\ -\eta_a\sigma^a & i\theta_a\sigma^a\end{pmatrix}$$
$$\text{chiral: } \gamma_C^0 = \begin{pmatrix} & 1\\ 1 & \end{pmatrix}$$
$$\text{chiral: } \gamma_C^i = \begin{pmatrix} & -\sigma^i\\ \sigma^i & \end{pmatrix}$$
$$\gamma_C^5 = \begin{pmatrix}1 & \\ & -1\end{pmatrix}$$
$$\gamma_C^0\gamma_C^\mu = \begin{pmatrix}\sigma^\mu & \\ & \sigma_\mu\end{pmatrix}$$
$$\tfrac{1}{2}(\sigma_\mu\sigma^\nu + \sigma_\nu\sigma^\mu) = \tfrac{1}{2}(\sigma^\mu\sigma_\nu + \sigma^\nu\sigma_\mu) = \eta^{\mu\nu}$$
$$\sigma_\mu\sigma^\nu\sigma^\mu = -2\sigma_\nu$$
$$\sigma^\mu\sigma_\nu\sigma^\mu = -2\sigma^\nu$$
$$\sigma^{\mu\nu} := \tfrac{i}{2}(\sigma_\mu\sigma^\nu - \sigma_\nu\sigma^\mu)$$
$$\bar\sigma^{\mu\nu} := \tfrac{i}{2}(\sigma^\mu\sigma_\nu - \sigma^\nu\sigma_\mu)$$
$$\sigma^{0j} = i\sigma^j$$
$$\bar\sigma^{0j} = -i\sigma^j$$
$$\sigma^{ij} = \bar\sigma^{ij} = \varepsilon^{ijk}\sigma^k$$
$$\bar\sigma^{\mu\nu} = -\tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\sigma^{\rho\sigma}$$
$$\operatorname{tr}(\sigma_\mu\sigma^\nu) = 2\eta^{\mu\nu}$$
$$\operatorname{tr}(\sigma_\mu\sigma^\nu\sigma_\rho\sigma^\sigma) = 2(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - i\varepsilon^{\mu\nu\rho\sigma})$$
$$\sigma_C^{\mu\nu} = \begin{pmatrix}\sigma^{\mu\nu} & \\ & \bar\sigma^{\mu\nu}\end{pmatrix}$$
$$\gamma_C^\mu = W^{-1}\gamma_S^\mu W$$
$$W = \tfrac{1}{\sqrt{2}}\begin{pmatrix}\mathbb{1} & \mathbb{1}\\ \mathbb{1} & -\mathbb{1}\end{pmatrix}$$
$$\text{parity: } \psi(x) \xmapsto{P} \gamma^0\psi(Px)$$
$$\text{charge: } \psi \xmapsto{C} i\gamma^2\psi^\dagger$$
$$\bar\psi \xmapsto{C} -\psi(i\gamma^2)^{-1}$$
$$\text{time: } \psi(x) \xmapsto{T} i\gamma^1\gamma^3\psi(Tx)$$
$$\tilde{U}(\infty, -\infty) = T_W e^{iS_I} =: e^{i\theta_0}\mathbb{S} =: e^{i\theta_0}(1 + i\mathbb{T})$$
$$A(\{\vec{k}_{\text{out}}\}, \{\vec{k}_{\text{in}}\}) =: (2\pi)^4\delta^4(\textstyle\sum k_{\text{out}} - \sum k_{\text{in}})\cdot$$
$$\cdot\prod \tfrac{1}{\sqrt{2E(\vec{k}_{\text{out}})}}\prod \tfrac{1}{\sqrt{2E(\vec{k}_{\text{in}})}} V^{-(\#\text{in}+\#\text{out})/2}\mathcal{M}(\{\vec{k}_{\text{out}}\}, \{\vec{k}_{\text{in}}\})$$
$$\tfrac{\mathrm{d}P}{\mathrm{d}t} = \tfrac{(2\pi)^4\delta^4(\sum k_{\text{out}} - \sum k_{\text{in}})}{V^{\#\text{in}-1}\prod 2E(\vec{k}_{\text{in}})}|\mathcal{M}|^2\prod\tfrac{\mathrm{d}^3k_{\text{out}}}{(2\pi)^3 2E(\vec{k}_{\text{out}})}$$
$$\mathrm{d}\sigma_{12\to\text{out}} = \tfrac{1}{4\sqrt{(k_1k_2)^2 - m_1^2m_2^2}}\prod\tfrac{\mathrm{d}^3k_{\text{out}}}{(2\pi)^3 2E(\vec{k}_{\text{out}})}\cdot$$
$$\cdot|\mathcal{M}|^2(2\pi)^4\delta^4(k_1 + k_2 - \textstyle\sum k_{\text{out}})$$
$$\overline{\textstyle\prod_\mu\gamma^\mu} := \gamma^0\left(\textstyle\prod_\mu\gamma^\mu\right)^\dagger\gamma^0 = \textstyle\prod_{\text{rev. }\mu}\bar\gamma^\mu$$
$$\bar\gamma^\mu = \gamma^\mu$$
$$\bar\gamma^5 = -\gamma^5$$
$$\textstyle\sum_{rs}\left|\bar{u}_s(\vec{k})\prod_\mu\gamma^\mu v_s(\vec{p})\right|^2 = \operatorname{tr}\left((\not{k} + m)\prod_\mu\gamma^\mu(\not{p} - m)\overline{\prod_\mu\gamma^\mu}\right)$$