Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$

$$\binom{\sinh x}{\cosh x}=\frac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\frac{1}{1-\tanh^2x}$$

$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\operatorname{atanh}x=\frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$

Combinatorics

$$D_{n,k}=\frac{n!}{(n-k)!}\qquad P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\int_0^\infty t^ze^{-t}dt$$
$$\tilde f(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ikx}f(x)$$

Derivatives

$$\tan'x=1+\tan^2x$$
$$\cot'x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\frac{1}{1+x^2}$$
$$\operatorname{asin}'x=-\operatorname{acos}'x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log'_a x=\frac{1}{x\ln a}$$
$$\cosh'x=\sinh x$$
$$\tanh'x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\frac{1}{1-x^2}$$
$$\operatorname{asinh}'x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}'x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$
$$\left(\frac{1}{x}\right)'=-\frac{\dot x}{x^2}$$
$$\left(\frac{x}{y}\right)'=\frac{\dot xy-x\dot y}{y^2}$$
$$(x^y)'=x^y\left(\dot y\ln x+y\frac{\dot x}{x}\right)$$

Integrals

$$\int x^a=\frac{x^{a+1}}{a+1}\qquad\int a^x=\frac{a^x}{\ln a}$$
$$\int\frac{1}{x}=\ln|x|\qquad\int\tan x=-\ln|\cos x|\qquad\int\cot x=\ln|\sin x|$$
$$\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|\qquad\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\qquad\int\ln x=x(\ln x-1)$$
$$\int\tanh x=\ln\cosh x\qquad\int\coth x=\ln|\sinh x|\qquad\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}\qquad\int xy=x\int y-\int\left(\dot x\int y\right)\qquad\int e^{yx}x=e^{yx}\left(\frac{y}{x}-\frac{1}{y^2}\right)$$

Differential equations

$$\dot x+\dot ax=b:x=e^{-a}\left(\int be^a+c_1\right)$$
$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot x=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$

Taylor

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$
$$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$
$$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+O(x^9)$$
$$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+O(x^7)$$
$$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+O(x^{10})$$
$$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+O(x^7)$$
$$\operatorname{asin}x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+O(x^9)$$

$$\operatorname{acos}x=\frac{\pi}{2}-\operatorname{asin}x$$
$$\operatorname{atan}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$
$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$
$$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+O(x^9)$$
$$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+O(x^7)$$
$$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+O(x^{10})$$
$$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+O(x^7)$$
$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+O(x^3)$$
$$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+O(x^6)$$
$$x!=1-\gamma x+\left(\frac{\gamma^2}{2}+\frac{\pi^2}{12}\right)x^2+O(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\lor j=k\lor k=i\\ 1 & i+1\equiv j\land j+1\equiv k\\ -1 & i\equiv j+1\land j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$
$$\vec a\times\vec b=\varepsilon_{ijk}a_jb_k\hat e_i$$
$$(\vec a\times\vec b)\vec c=(\vec c\times\vec a)\vec b$$
$$(\vec a\times\vec b)\times\vec c=-(\vec b\vec c)\vec a+(\vec a\vec c)\vec b$$
$$(\vec a\times\vec b)(\vec c\times\vec d)=(\vec a\vec c)(\vec b\vec d)-(\vec a\vec d)(\vec b\vec c)$$

$$|\vec u\times\vec v|^2=u^2v^2-(\vec u\vec v)^2$$
$$\vec\nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right);\ \Box=\frac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec\nabla V=\frac{\partial V}{\partial\rho}\hat\rho+\frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat\phi+\frac{\partial V}{\partial z}\hat z$$
$$\vec\nabla\vec v=\frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$
$$\vec\nabla\times\vec v=\left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat\rho+$$
$$+\left(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\right)\hat\phi+\frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\right)$$
$$\nabla^2V=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2}+\frac{\partial^2V}{\partial z^2}$$
$$\vec\nabla V=\frac{\partial V}{\partial r}\hat r+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v=\frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec\nabla\times\vec v=\frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta}-\frac{\partial v_\theta}{\partial\varphi}\right)\hat r+$$
$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(rv_\varphi)}{\partial r}\right)\hat\theta+\frac{1}{r}\left(\frac{\partial(rv_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}\right)\hat\varphi$$
$$\nabla^2V=\frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2}+\frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta}+\frac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\vec\nabla(f\vec v)=(\vec\nabla f)\vec v+f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v)=\vec\nabla f\times\vec v+f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v)=-\nabla^2\vec v+\vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w)=\vec w(\vec\nabla\times\vec v)-\vec v(\vec\nabla\times\vec w)$$

$\vec\nabla\times(\vec v\times\vec w)=(\vec\nabla\vec w+\vec w\,\vec\nabla)\vec v-(\vec\nabla\vec v+\vec v\,\vec\nabla)\vec w$

$\frac12\vec\nabla v^2=(\vec v\,\vec\nabla)\vec v+\vec v\times(\vec\nabla\times\vec v)$

$\int\vec\nabla\vec v\mathrm{d}^3x=\oint\vec v\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S=\oint\vec v\mathrm{d}\vec l$

$\int(f\nabla^2g-g\nabla^2f)\,\mathrm{d}^3x=\oint_S\left(f\frac{\partial g}{\partial n}-g\frac{\partial f}{\partial n}\right)\mathrm{d}S$

$\oint\vec v\times\mathrm{d}\vec S=-\int(\vec\nabla\times\vec v)\mathrm{d}^3x$

$\delta(\vec r-\vec r_0)=\frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$

$\nabla^2\frac{1}{|\vec r-\vec r_0|}=-4\pi\delta(\vec r-\vec r_0)$

$\delta(g(x))=\frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$

$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle=\frac12\,\mathrm{Re}(a\bar b)$

Statistics

$P(E\cap E_1)=P(E_1)\cdot P(E|E_1)$

$\Delta x_{\mathrm{hist}}\approx\frac{x_{\max}-x_{\min}}{\sqrt N}$

$P(x\le k)=F(k)=\int_{-\infty}^k p(x)$

$\mathrm{median}=F^{-1}(\frac12)$

$E[f(x)]=\int_{-\infty}^\infty f(x)p(x)$

$\mu=E[x]=\int_{-\infty}^\infty xp(x)$

$\alpha_n=E[x^n]$

$M_n=E[(x-\mu)^n]$

$\sigma^2=M_2=E[x^2]-\mu^2$

$\mathrm{FWHM}\approx2\sigma$

$\gamma_1=\frac{M_3}{\sigma^3},\ \gamma_2=\frac{M_4}{\sigma^4}$

$\phi[y](t)=E[e^{ity}]$

$\phi[y_1+\lambda y_2]=\phi[y_1]\phi[\lambda y_2]$

$\alpha_n=i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$

$h\ge0: P(h\ge k)\le\frac{E[h]}{k}$

$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$

$B(n,p,k)=\binom{n}{k}p^k(1-p)^{n-k}$

$\mu_B=np,\ \sigma_B^2=np(1-p)$

$P(\mu,k)=\frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2=\mu$

$u(x,a,b)=\frac{1}{b-a},\ x\in[a;b]$

$\mu_u=\frac{b+a}{2},\ \sigma_u^2=\frac{(b-a)^2}{12}$

$\varepsilon(x,\lambda)=\lambda e^{-\lambda x},\ x\ge0$

$\mu_\varepsilon=\frac1\lambda,\ \sigma_\varepsilon^2=\frac{1}{\lambda^2}$

$g(x,\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac12(\frac{x-\mu}{\sigma})^2}$

$\mathrm{FWHM}_g=2\sigma\sqrt{2\ln2}$

$z=\frac{x-\mu}{\sigma};\ \mu,\sigma[z]=0,1$

$\chi^2=\sum_{i=1}^n z_i^2$

$\wp_n(x)=\frac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$

$\mu_\wp=n,\ \sigma_\wp^2=2n$

$n\ge30:\wp_n(x)\approx g(x,n,\sqrt{2n})$

$n\ge8: p[\sqrt{2\chi^2}]\approx g(,\sqrt{2n-1},1)$

$S(x,n)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac n2)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$

$\mu_S=0,\ \sigma_S^2=\frac{n}{n-2}$

$p\left[z\sqrt{\frac{n}{\chi^2}}\right]=S(,n)$

$n\ge35: S(x,n)\approx g(x,0,1)$

$c(x,a)=\frac a\pi\frac{1}{a^2+x^2}$

$\sigma_{xy}=E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$

$\rho=\frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le1$

$\mu[f(x_1,...)]\approx f(\mu_1,...)$

$\sigma^2[f(x_1,...)]\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$

$\mu\approx m=\frac1n\sum_{i=1}^n x_i$

$\sigma^2\approx s^2=\frac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$

$s_m^2=\frac{s^2}{n}$

$p\left[\frac{m-\mu}{s_m}\right]=S(,n)$

Fit

$f(x)=mx+q,\quad f(x)=a$

$f(x)=bx$

$m=\frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$

$\Delta m^2=\frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$

$q=\frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$

$\Delta q^2=\frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$

$a=\frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2=\frac{1}{\sum\frac{1}{\Delta y^2}}$

$b=\frac{\sum xy}{\sum x^2},\ \Delta b^2=\frac{\Delta y^2}{\sum x^2}$

Kinematics

$\frac1R=\left|\frac{v_xa_y-v_ya_x}{v^3}\right|$

$\vec\omega=\dot\varphi\cos\theta\hat r-\dot\varphi\sin\theta\hat\theta+\dot\theta\hat\varphi$

$\dot{\vec w}=\frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r+\frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta+\frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi+\vec\omega\times\vec w$

$\theta\equiv\frac\pi2\to\dot{\vec r}=\dot r\hat r+r\dot\varphi\hat\varphi$

$\theta\equiv\frac\pi2\to\ddot{\vec r}=(\ddot r-r\dot\varphi^2)\hat r+(r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$

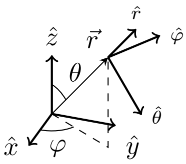
$\dot{\vec r}=\dot r\hat r+r\dot\theta\hat\theta+r\dot\varphi\sin\theta\hat\varphi$

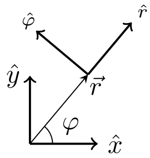
$\langle\ddot{\vec r},\hat r\rangle=\ddot r-r\dot\theta^2-r\dot\varphi^2\sin^2\theta$

$\langle\ddot{\vec r},\hat\theta\rangle=r\ddot\theta+2\dot r\dot\theta-r\dot\varphi^2\sin\theta\cos\theta$

$\langle\ddot{\vec r},\hat\varphi\rangle=r\ddot\varphi\sin\theta+2\dot r\dot\varphi\sin\theta+2r\dot\theta\dot\varphi\cos\theta$

$\vec A=\ddot{\vec r}+\vec A_{\mathrm T}+\vec\omega\times(\vec\omega\times\vec r)+\dot{\vec\omega}\times\vec r+2\vec\omega\times\dot{\vec r}$





Mechanics

$\dot\alpha=\frac{\mathrm d}{\mathrm dt}\alpha(\beta,t)=\frac{\partial\alpha}{\partial\beta}\dot\beta+\frac{\partial\alpha}{\partial t}$

$\vec p:=m\dot{\vec r};\ \vec F=\dot{\vec p};\ \frac{\mathrm d(mT)}{\mathrm dt}=\vec F\vec p$

$M:=\sum_i m_i;\ \vec R:=\frac{m_i\vec r_i}{M}$

$T=\frac12M\dot{\vec R}^2+\frac12m_i(\dot{\vec r}_i-\dot{\vec R})^2$

$\vec L=\vec R\times M\dot{\vec R}+(\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$

$\vec\tau_O=\dot{\vec L}_O+\vec v_O\times\vec p$

$\tau_1=I_1\dot\omega_1+(I_3-I_2)\omega_3\omega_2$

$\mathcal L(q,\dot q,t)=T-V+\frac{\mathrm d}{\mathrm dt}f(q,t)$

$S[q]=\int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$

$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\big|_{\epsilon(t_1)=\epsilon(t_2)=0}=0$

$p:=\frac{\partial\mathcal L}{\partial\dot q};\ \dot p=\frac{\partial\mathcal L}{\partial q}$

$\mathcal H(q,p,t)=\dot qp-\mathcal L$

$\dot q=\frac{\partial\mathcal H}{\partial p};\ \dot p=-\frac{\partial\mathcal H}{\partial q}$

$\frac{\mathrm d\mathcal H}{\mathrm dt}=\frac{\partial\mathcal H}{\partial t}=-\frac{\partial\mathcal L}{\partial t}$

$\{u,v\}=\frac{\partial u}{\partial q}\frac{\partial v}{\partial p}-\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$

$\frac{\mathrm du}{\mathrm dt}=\{u,\mathcal H\}+\frac{\partial u}{\partial t}$

$\eta=(q,p);\ \Gamma=\left(\begin{smallmatrix}0&1\\-1&0\end{smallmatrix}\right)$

$\dot\eta=\Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\}=\frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac12mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac23mr^2$

ball: $\frac25mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2+\frac34r^2\right)$

ellipsoid: $I_a=\frac15m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$\langle U\rangle\approx-2\langle T\rangle$

$U_{\mathrm{eff}}=U+\frac{L^2}{2mr^2}$

$\frac1\mu=\frac{1}{m_1}+\frac{1}{m_2}$

$\vec r=\vec r_1-\vec r_2,\ \alpha=Gm_1m_2$

$T=\frac12M\dot{\vec R}^2+\frac12\mu\dot{\vec r}^2$

$\vec L=\vec R\times M\dot{\vec R}+\vec r\times\mu\dot{\vec r}$

$k=\frac{L^2}{\mu\alpha},\ \varepsilon=\sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$

$r=\frac{k}{1+\varepsilon\cos\theta}$

$a=\frac{k}{|1-\varepsilon^2|}=\frac{\alpha}{2|E|}$

$a^3\omega^2=G(m_1+m_2)$

$\vec A=\mu\dot{\vec r}\times\vec L-\mu\alpha\hat r,\ \dot{\vec A}=0$

Inequalities

$|a|-|b|\le|a+b|\le|a|+|b|$

$x>-1: 1+nx\le(1+x)^n$

$\frac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$

$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$

$\sum a_ib_i\le\left(\sum a_i^p\right)^{\frac1p}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^py^q\le\left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac1n\sum a_i^{p\le q}}\le\sqrt[q]{\frac1n\sum a_i^q}$

$\sum\left(\frac{a_1+\dots a_i}{i}\right)^p\le\left(\frac{p}{p-1}\right)^p\sum a_i^p$

$x\ge0,|\ddot x|\le M: |\dot x|\le\sqrt{2Mx}$

$\frac{1}{1+x}<\ln\left(1+\frac1x\right)<\frac1x$

Vector spaces

$(V,\mathbb K,+,\cdot)$ vector space; $\quad\mathbb K$ field

$\exists\vec0\in V:\vec v+\vec0=\vec v$

$\cdot:\mathbb K\times V\to V;\quad\lambda\cdot(\vec v+\vec w)=\lambda\vec v+\lambda\vec w$

$0_{\mathbb K}\cdot\vec v=\vec0,\ 1_{\mathbb K}\cdot\vec v=\vec v$

$\lambda\in\mathbb K,\ \vec v,\vec w\in V\ \Rightarrow\ \vec v+\vec w\in V,\ \lambda\vec v\in V$

$\dim(U+V)=\dim U+\dim V-\dim(U\cap V)$

ℓ linear $:\ell(\vec v+\lambda\vec w)=\ell(\vec v)+\lambda\ell(\vec w)$

$\ker\ell=\{\vec v\in V\,|\,\ell(\vec v)=0\}$

$\dim V=\dim\ell(V)+\dim(V\cap\ker\ell)$

$\langle,\rangle:V\times V\to\mathbb K;\quad\langle\vec v,\vec w\rangle=\langle\vec w,\vec v\rangle$

$\langle\vec v+\lambda\vec w,\vec u\rangle=\langle\vec v,\vec u\rangle+\lambda\langle\vec w,\vec u\rangle$

$\|\|:V\to\mathbb K;\quad\|\vec v\|=0\to\vec v=\vec0$

$\|\lambda\vec v\|=|\lambda|\|\vec v\|;\quad\|\vec v+\vec w\|\le\|\vec v\|+\|\vec w\|$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M	N	Ξ	O	Π	P	Σ	T	Y	Φ	X	Ψ	Ω
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ	ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$; $e = 2.718$; $\gamma = 5.772 \cdot 10^{-1}$; $G = 6.674 \cdot 10^{-11}\,\frac{\mathrm{m^3}}{\mathrm{kg\,s^2}}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$; $R = 8.206 \cdot 10^{-2}\,\frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$; $N_\mathrm{A} = 6.022 \cdot 10^{23}\,\frac{1}{\mathrm{mol}}$; $k = 1.381 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$; $q_\mathrm{e} = 1.602 \cdot 10^{-19}\,\mathrm{A\,s}$; $m_\mathrm{e} = 9.109 \cdot 10^{-31}\,\mathrm{kg}$; $m_\mathrm{p} = 1.673 \cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675 \cdot 10^{-27}\,\mathrm{kg}$; $\mathrm{amu} = 1.661 \cdot 10^{-27}\,\mathrm{kg}$; $h = 6.626 \cdot 10^{-34}\,\mathrm{J\,s}$; $\varepsilon_0 = 8.854 \cdot 10^{-12}\,\frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}$

$\mu_0 = 1.257 \cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A^2}}$; $\mu_\mathrm{B} = 9.274 \cdot 10^{-24}\,\mathrm{A\,m^2}$; $\alpha = 7.297 \cdot 10^{-3}$; $\mathrm{eV} = 1 \cdot 10^{-12}\,\mathrm{erg}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i \mathrm{A}_i \;\to\; b_j \mathrm{B}_j$

$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$

$\forall\, i, j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$

$k = A e^{-\frac{E_\mathrm{a}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0}$, $[\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma \frac{p}{p_0}$, $p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}}$, $K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}}$, $K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}}$, $\chi = \frac{n}{n_\mathrm{tot}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_n = K_\chi n_\mathrm{tot}^{\sum b_j - \sum a_i}$

$\Delta G^\mathrm{o}_\mathrm{r} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln \frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$; $\mathrm{d}L = p\mathrm{d}V$; $\mathrm{d}S = \frac{\mathrm{d}Q}{T}$; $C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V$; $C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p$; $\gamma = \frac{C_p}{C_V}$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n}$, $c_V = \frac{\mathrm{dof}}{2} R$, $c_p = c_V + R$

$c_V = \frac{R}{\gamma - 1}$, $c_p = \frac{\gamma}{\gamma - 1} R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Electronics (MKS)

$\begin{pmatrix} V \\ I \end{pmatrix} = \begin{pmatrix} V_0 \\ I_0 \end{pmatrix} e^{i\omega t}$

$Z = \frac{V}{I}$; $Z_R = R$; $Z_C = -i\frac{1}{\omega C}$; $Z_L = i\omega L$

$Z_\mathrm{series} = \sum_k Z_k$; $\frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$

$\sum_\mathrm{loop} V_k = 0$; $\sum_\mathrm{node} I_k = 0$

$\mathcal{E} = -L\dot{I}$; $L = \frac{\Phi_B}{I}$

Relativity

$\beta = \frac{v}{c}$; $\gamma = \frac{1}{\sqrt{1-\beta^2}}$; $\vec{p} = \gamma m \vec{v}$

$\mathcal{E} = \gamma m c^2$; $\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$

$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$; $x^\mu = (ct, \vec{x})$; $v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = m v^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$; $\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu} x^\nu$; $\partial_\mu \partial^\mu = \Box$; $p^\mu p_\mu = (mc)^2$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3}$; $\vec{E}_1 = \frac{\vec{F}_{12}}{q_2}$; $V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$; $\rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi \int \rho\, \mathrm{d}^3 x$; $-\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho$; $\vec{\nabla} \times \vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\, \mathrm{d}^3 x$; $\tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{ij}\int \vec{E}_i \vec{E}_j\, \mathrm{d}^3 x$

$V(\vec{r}) = \int \rho G_\mathrm{D}(\vec{r})\, \mathrm{d}^3 x - \frac{1}{4\pi}\oint_S V \frac{\partial G_\mathrm{D}}{\partial n}\, \mathrm{d}S$

$V(\vec{r}) = \langle V \rangle_S + \int \rho G_\mathrm{N}(\vec{r})\, \mathrm{d}^3 x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n} G_\mathrm{N}(\vec{r})\, \mathrm{d}S$

$\nabla^2_y G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y})$; $G_\mathrm{D}(\vec{x}, \vec{y})|_{\vec{y} \in S} = 0$; $\frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y} \in S} = -\frac{4\pi}{S}$

$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R}$; $\vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3}$; $V_\mathrm{dip} = \frac{\vec{p}\hat{r}}{r^2}$

$V(r, \theta) = \sum_{l=0}^\infty \left(A_l r^l - \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$

$V(r, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^{l} \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$

$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l$; $f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 f P_l$

$P_l(1) = 1$; $\langle P_n | P_m \rangle = \frac{2\delta_{nm}}{2n+1}$; $\langle Y_{lm} | Y_{l'm'} \rangle = \delta_{ll'}\delta_{mm'}$

$P_0 = 1$; $P_1 = x$; $P_2 = \frac{3x^2 - 1}{2}$; $Y_{00} = \frac{1}{\sqrt{4\pi}}$; $Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi}$; $Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi}$; $Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l$, $|m| \le l$

$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos\theta)$; $Y_{l,-m} = (-1)^m \overline{Y}_{lm}$

$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^{l} \overline{Y}_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi)$

$V(r > \operatorname{diam}\operatorname{supp}\rho, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^{l} \frac{4\pi}{2l+1} q_{lm}[\rho] \frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$

$q_{lm}[\rho] = \int_0^\infty r^2 \mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \int_0^\pi \sin\theta\, \mathrm{d}\theta\, r^l \rho(r, \theta, \varphi) \overline{Y}_{lm}(\theta, \varphi)$

Magnetostatics (CGS)

$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0$; $I = \int \vec{J}\mathrm{d}\vec{S}$

solenoid: $B = 4\pi\frac{j_\mathrm{s}}{c}$

$\mathrm{d}\vec{F} = \frac{I\mathrm{d}\vec{l}}{c} \times \vec{B} = \mathrm{d}^3 x \frac{\vec{J}}{c} \times \vec{B}$; $\vec{F}_q = q\frac{\dot{\vec{r}}}{c} \times \vec{B}$

$\mathrm{d}\vec{B} = \frac{I\mathrm{d}\vec{l}}{c} \times \frac{\vec{r}}{r^3}$; $\vec{B}_q = q\frac{\dot{\vec{r}}}{c} \times \frac{\vec{r}}{r^3}$

$\vec{B} = \vec{\nabla} \times \vec{A}$; $\vec{A} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c}\frac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla}A_0$

$\vec{B} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c} \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$

$\vec{\nabla}\vec{A} = 0 \to \nabla^2 \vec{A} = -4\pi\frac{\vec{J}}{c}$

$\vec{\nabla}\vec{B} = 0$; $\vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c}$; $\oint \vec{B}\mathrm{d}\vec{l} = 4\pi\frac{I}{c}$

$\vec{m} = \frac{1}{2}\int \mathrm{d}^3 r' \left(\vec{r}' \times \frac{\vec{J}'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec{L}$

$\vec{A} \approx \frac{\vec{m} \times \vec{r}}{r^3}$; $\vec{\tau} = \vec{m} \times \vec{B}$

Electromagnetism (CGS)

Faraday: $\mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$

$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$; $\vec{\nabla}\vec{E} = 4\pi\rho$; $\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$

$\vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$; $\vec{\nabla}\vec{B} = 0$

$\mathrm{d}\vec{F} = \mathrm{d}^3 x\left(\rho\vec{E} + \frac{\vec{J}}{c} \times \vec{B}\right)$; $\vec{F}_q = q\left(\vec{E} + \frac{\dot{\vec{r}}}{c} \times \vec{B}\right)$

$u = \frac{E^2 + B^2}{8\pi}$; $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$; $\vec{g} = \frac{\vec{S}}{c^2}$

$$T^E_{ij} = \frac{1}{4\pi}\big(E_iE_j - \tfrac{1}{2}\delta_{ij}E^2\big);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ \frac{\partial \vec{g}}{\partial t} = -\vec{f} + \partial_j T_{ij}\hat{x}_i$$

$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\big(\vec{\nabla}\vec{A} + \tfrac{1}{c}\tfrac{\partial\phi}{\partial t}\big) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$

$$(\phi,\vec{A}) \cong \big(\phi - \tfrac{1}{c}\tfrac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\big)$$

$$(\phi,\vec{A}) = \int \mathrm{d}^3r'\,\frac{\left(\rho,\frac{\vec{J}}{c}\right)\left(\vec{r}',t-\frac{1}{c}\left|\vec{r}-\vec{r}'\right|\right)}{|\vec{r}-\vec{r}'|}$$

Coulomb gauge: $\vec{\nabla}\vec{A} = 0$

Lorenz gauge: $\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$

$$\vec{E}'\hat{v} = \vec{E}\hat{v};\ \vec{B}'\hat{v} = \vec{B}\hat{v}$$

$$\vec{E}'\times\hat{v} = \gamma\big(\vec{E} + \tfrac{\vec{v}}{c}\times\vec{B}\big)\times\hat{v}$$

$$\vec{B}'\times\hat{v} = \gamma\big(\vec{B} - \tfrac{\vec{v}}{c}\times\vec{E}\big)\times\hat{v}$$

$$\text{plane wave: } \begin{cases}\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}\\ \vec{B} = \hat{k}\times\vec{E}\\ \omega = ck\end{cases}$$

dipole: $\vec{B}\big|_{r\gg\frac{c}{\omega}} \approx \frac{1}{c^2}\frac{\ddot{\vec{p}}\times\hat{r}}{r};\ \vec{E}\approx\vec{B}\times\hat{r}$

$$A^\mu = (\phi,\vec{A});\ J^\mu = (c\rho,\vec{J})$$

Lorenz gauge: $\partial_\mu A^\mu = 0$

$$\partial_\mu F^{\mu\nu} = 4\pi\frac{J^\nu}{c};\ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}$$

$$\mathscr{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$F^{\mu\nu}F_{\mu\nu} = E^2 - B^2;\ F^{\mu\nu}\mathscr{F}_{\mu\nu} = 4\vec{E}\vec{B}$$

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\big(g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} - \tfrac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}\big)$$

$$\Theta^{\alpha\beta} = \begin{pmatrix} u & c\vec{g}\\ c\vec{g} & -\mathbf{T}\end{pmatrix}$$

$$\partial_\mu\Theta^{\mu\nu} = -\tfrac{1}{c}F^{\nu\lambda}J_\lambda = \tfrac{1}{c}J_\lambda F^{\lambda\nu}$$

Electromagnetism in matter (CGS)

$$\vec{P} = \frac{\langle\vec{d}\rangle}{V};\ \vec{M} = \frac{\langle\vec{m}\rangle}{V}$$

$$\rho_{\mathrm{pol}} = -\vec{\nabla}\vec{P};\ \sigma_{\mathrm{pol}} = \hat{n}\vec{P};\ \frac{\vec{J}_{\mathrm{mag}}}{c} = \vec{\nabla}\times\vec{M}$$

$$\vec{D} = \vec{E} + 4\pi\vec{P};\ \vec{H} = \vec{B} - 4\pi\vec{M}$$

$$\vec{\nabla}\vec{D} = 4\pi\rho_{\mathrm{ext}};\ \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla}\times\vec{H} = 4\pi\frac{\vec{J}_{\mathrm{ext}}}{c} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

static linear isotropic: $\vec{P} = \chi\vec{E}$

static linear: $P_i = \chi_{ij}E_j$

static linear: $\varepsilon = 1 + 4\pi\chi$

static: $\Delta D_\perp = 4\pi\sigma_{\mathrm{ext}};\ \Delta E_\parallel = 0$

static linear: $u = \frac{1}{8\pi}\vec{E}\vec{D}$

$$\Delta U_{\mathrm{dielectric}} = \frac{1}{2}\int \mathrm{d}^3r\,\vec{P}\vec{E}_0$$

plane capacitor: $C = \frac{\varepsilon}{4\pi}\frac{S}{d}$

non-interacting gas: $\vec{d} = \alpha\vec{E}_0;\ \chi = n\alpha$

hom. cubic isotropic: $\chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}n\alpha$

$$\chi = \frac{4\pi}{3}\frac{np_0^2}{kT};\ \vec{E}_e = \vec{E} + \frac{4\pi}{3}\vec{P}$$

$$\vec{J}\vec{E} = -\vec{\nabla}\left(\frac{c}{4\pi}\vec{E}\times\vec{H}\right) - \frac{1}{4\pi}\left(\vec{E}\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t}\right)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\frac{\omega}{c}$$

$$\vec{J}_{\mathrm{c}} = \sigma\vec{E};\ \tilde{\varepsilon} = \varepsilon + i\frac{4\pi\sigma}{\omega}$$

I: $u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$

I: $\langle u\rangle = \frac{1}{16\pi}(\varepsilon E^2 + \mu H^2)$

I: $\langle S_z\rangle = \frac{c}{n}\langle u\rangle$

II: $\langle u\rangle = \frac{1}{16\pi}\big(\frac{\partial}{\partial\omega}(\varepsilon_\omega\omega)\langle E^2\rangle + \frac{\partial}{\partial\omega}(\mu_\omega\omega)\langle H^2\rangle\big)$

II: $\langle S_z\rangle = v_{\mathrm{g}}\langle u\rangle;\ v_{\mathrm{g}} = \frac{\partial\omega}{\partial k}$

III: $\langle W\rangle = \frac{\omega}{8\pi}\big(\mathrm{Im}(\varepsilon_\omega)\big|E_0^2\big| + \mathrm{Im}(\mu_\omega)\big|H_0^2\big|\big)$

$(\mu = 1)$ TE: $E_{\mathrm{t}} = \frac{2E_{\mathrm{i}}}{1+\frac{k_{t2}}{k_{i2}}};\ E_{\mathrm{r}} = \frac{1-\frac{k_{t2}}{k_{i2}}}{1+\frac{k_{t2}}{k_{i2}}}E_{\mathrm{i}}$