Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\left(x+\operatorname{atan}\frac{b}{a}\right)$$
$$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\left(x-\operatorname{atan}\frac{a}{b}\right)$$
$$\operatorname{acos}x+\operatorname{asin}x=\frac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\begin{pmatrix}\sinh x\\ \cosh x\end{pmatrix}=\frac{1}{2}\begin{pmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{pmatrix}$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\frac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\begin{pmatrix}\operatorname{asinh}x\\ \operatorname{acosh}x\end{pmatrix}=\log\left(x+\sqrt{x^2+\begin{pmatrix}1\\ -1\end{pmatrix}}\right)$$
$$\operatorname{atanh}x=\frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

$$\text{triangle: }\sqrt{p(p-a)(p-b)(p-c)}$$
$$\text{quad: }\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$$
$$\text{Pick: }A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\sum_{x=1}^n x^3=\left(\sum_{x=1}^n x\right)^2=\frac{1}{4}n^2(n+1)^2$$
$$\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\int_0^\infty t^ze^{-t}dt=z!$$
$$n!\approx\left(\frac{n}{e}\right)^n\sqrt{2\pi n}$$
$$\text{Fourier: }c_n=\frac{2}{T}\int_0^T f(t)\cos\left(n\frac{t}{T}\right)\mathrm{d}t$$
$$F[f]=\hat{f}(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ikx}f(x)$$
$$\langle\hat{f}|\hat{g}\rangle=\langle f|g\rangle$$
$$F\left[\frac{\sin x}{x}\right]=\sqrt{\frac{\pi}{2}}\chi_{[-1;1]}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\frac{\operatorname{Re}z+|z|}{2}}+\frac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$
$$\delta(g(x))=\frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\frac{1}{2}\operatorname{Re}(a\bar{b})$$
$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$$
$$f^{(n)}(z_0)=\frac{n!}{2\pi i}\oint\frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$
$$f(z)=\sum_{k=-\infty}^{\infty}\left(\frac{1}{2\pi i}\oint\frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\right)(z-z_0)^k$$

Derivatives

$$\tan'x=1+\tan^2x$$
$$\cot'x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\frac{1}{1+x^2}$$
$$\operatorname{asin}'x=-\operatorname{acos}'x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a'x=\frac{1}{x\ln a}$$
$$\cosh'x=\sinh x$$
$$\tanh'x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\frac{1}{1-x^2}$$
$$\operatorname{asinh}'x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}'x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$
$$\left(\frac{1}{x}\right)'=-\frac{\dot{x}}{x^2}$$
$$\left(\frac{x}{y}\right)'=\frac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)'=x^y\left(\dot{y}\ln x+y\frac{\dot{x}}{x}\right)$$
$$\frac{\partial(x,y)}{\partial(u,v)}:=\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}-\frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$
$$\frac{\partial(x,y)}{\partial(u,y)}=\frac{\partial x}{\partial u}\Big|_y=-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x$$
$$\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x\frac{\partial u}{\partial x}\Big|_y=-1$$
$$\frac{\partial x}{\partial u}\Big|_y=\frac{\partial x}{\partial u}\Big|_v-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_v$$
$$\frac{\partial x}{\partial u}\Big|_v=\frac{\partial x}{\partial y}\Big|_v\frac{\partial y}{\partial u}\Big|_v$$

Integrals

$$\int x^a=\frac{x^{a+1}}{a+1}$$
$$\int a^x=\frac{a^x}{\ln a}$$
$$\int\frac{1}{x}=\ln|x|$$
$$\int\tan x=-\ln|\cos x|$$
$$\int\cot x=\ln|\sin x|$$
$$\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|$$
$$\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$
$$\int\ln x=x(\ln x-1)$$
$$\int\tanh x=\ln\cosh x$$
$$\int\coth x=\ln|\sinh x|$$
$$\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\int xy=x\int y-\int\left(\dot{x}\int y\right)$$
$$\int e^{yx}x=e^{yx}\left(\frac{y}{x}-\frac{1}{y^2}\right)$$
$$\int_{-\infty}^{\infty}e^{-x^2}=\sqrt{\pi}$$

Differential equations

$$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\int be^a+c_1\right)$$
$$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot{x}=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot{x}+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$
$$\ddot{x}+\gamma\dot{x}+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$
$$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$
$$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{atan}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$
$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$
$$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$
$$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\left(\frac{\gamma^2}{2}+\frac{\pi^2}{12}\right)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i=j \lor j=k \lor k=i \\ 1 & i+1\equiv j \land j+1 \equiv k \\ -1 & i \equiv j+1 \land j \equiv k+1 \end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$
$$\vec a \times \vec b = \varepsilon_{ijk} a_j b_k \hat e_i$$
$$(\vec a \otimes \vec b)_{ij} = a_i b_j$$
$$(\vec a \times \vec b)\vec c = (\vec c \times \vec a)\vec b$$
$$(\vec a \times \vec b)\times \vec c = -(\vec b \vec c)\vec a + (\vec a \vec c)\vec b$$
$$(\vec a\times\vec b)(\vec c\times\vec d) = (\vec a\vec c)(\vec b\vec d) - (\vec a\vec d)(\vec b\vec c)$$
$$|\vec u\times\vec v|^2 = u^2v^2 - (\vec u\vec v)^2$$
$$\vec\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right); \Box = \frac{\partial^2}{\partial t^2} - \nabla^2$$

$$\vec\nabla V = \frac{\partial V}{\partial\rho}\hat\rho + \frac1\rho\frac{\partial V}{\partial\phi}\hat\phi + \frac{\partial V}{\partial z}\hat z$$
$$\vec\nabla\vec v = \frac1\rho\frac{\partial(\rho v_\rho)}{\partial\rho} + \frac1\rho\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$
$$\vec\nabla\times\vec v = \left(\frac1\rho\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat\rho +$$
$$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right)\hat\phi + \frac1\rho\left(\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\phi}\right)$$
$$\nabla^2 V = \frac1\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac1r\frac{\partial V}{\partial\theta}\hat\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$
$$\vec\nabla\vec v = \frac1{r^2}\frac{\partial(r^2v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac1{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec\nabla\times\vec v = \frac1{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\right)\hat r +$$
$$+\frac1r\left(\frac1{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(rv_\varphi)}{\partial r}\right)\hat\theta + \frac1r\left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat\varphi$$
$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$$
$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$
$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\vec\nabla)\vec w$$
$$\frac12\vec\nabla v^2 = (\vec v\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$
$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$
$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$$
$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(\vec r - \vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\nabla^2\frac{1}{|\vec r - \vec r_0|} = -4\pi\delta(\vec r - \vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}} \approx \frac{x_{\max}-x_{\min}}{\sqrt N}$$
$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$
$$\text{median} = F^{-1}\left(\tfrac12\right)$$
$$E[f(x)] = \int_{-\infty}^\infty f(x)p(x)$$
$$\mu = E[x] = \int_{-\infty}^\infty xp(x)$$
$$\alpha_n = E[x^n]$$
$$M_n = E[(x-\mu)^n]$$
$$\sigma^2 = M_2 = E[x^2] - \mu^2$$
$$\text{FWHM} \approx 2\sigma$$
$$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$
$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge0: P(h\ge k)\le\frac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\frac1{k^2}$$
$$B(k;n,p) = \binom nk p^k(1-p)^{n-k}$$
$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$
$$P(k;\mu) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$
$$u(x;a,b) = \frac1{b-a},\ x\in[a;b]$$
$$\mu_u = \frac{b+a}2,\ \sigma_u^2 = \frac{(b-a)^2}{12}$$
$$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge0$$
$$\mu_\varepsilon = \frac1\lambda,\ \sigma_\varepsilon^2 = \frac1{\lambda^2}$$

$$g(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac12\left(\frac{x-\mu}{\sigma}\right)^2}$$
$$g(\vec x;\vec\mu,V) = \frac{e^{-\frac12(\vec x-\vec\mu)^{\mathrm{T}} V^{-1}(\vec x-\vec\mu)}}{\sqrt{\det(2\pi V)}}$$
$$\text{FWHM}_g = 2\sigma\sqrt{2\ln2}$$
$$z = \frac{x-\mu}\sigma;\ \mu,\sigma[z] = 0,1$$
$$\chi^2 = \sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$$
$$\wp(x;n) = \frac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$$
$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$
$$n\ge30: \wp(x;n)\approx g(x;n,\sqrt{2n})$$
$$n\ge8: p[\sqrt{2\chi^2}]\approx g(;\sqrt{2n-1},1)$$
$$S(x;n) = \frac{\Gamma(\frac{n+1}2)}{\sqrt{n\pi}\,\Gamma(\frac n2)}\left(1+\frac{x^2}n\right)^{-\frac{n+1}2}$$
$$\mu_S = 0,\ \sigma_S^2 = \frac n{n-2}$$

$$p\left[z\sqrt{\frac n{\chi^2}}\right] = S(,n)$$
$$n\ge35: S(x;n)\approx g(x;0,1)$$
$$c(x;a) = \frac a\pi\frac1{a^2+x^2}$$
$$\sigma_{xy} = E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}|\le1$$
$$\mu_{f(x)}\approx f(\mu_x)$$
$$\sigma_{fg}\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_{x_i}}\frac{\partial g}{\partial x_j}\Big|_{\mu_{x_j}}$$
$$\mu\approx m = \frac1n\sum_{i=1}^n x_i$$
$$\sigma^2\approx s^2 = \frac1{n-1}\sum_{i=1}^n(x_i-m)^2$$
$$s_m^2 = \frac{s^2}n$$
$$p\left[\frac{m-\mu}{s_m}\right] = S(;n)$$

Fit (ML)

$$f(x) = mx+q, \quad f(x) = a,$$
$$f(x) = bx, \quad f(x;\theta) = \theta_i h_i(x)$$
$$m = \frac{\sum\frac1{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac x{\Delta y^2}\cdot\sum\frac y{\Delta y^2}}{\sum\frac1{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac x{\Delta y^2})^2}$$

$$\Delta m^2 = \frac{\sum\frac1{\Delta y^2}}{\sum\frac1{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac x{\Delta y^2})^2}$$
$$q = \frac{\sum\frac y{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac x{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac1{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac x{\Delta y^2})^2}$$
$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac1{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac x{\Delta y^2})^2}$$

$$\Delta mq = \frac{-\sum\frac x{\Delta y^2}}{\sum\frac1{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac x{\Delta y^2})^2}$$
$$a = \frac{\sum\frac y{\Delta y^2}}{\sum\frac1{\Delta y^2}},\ \Delta a^2 = \frac1{\sum\frac1{\Delta y^2}}$$
$$\mathbf a = (\textstyle\sum V_{\mathbf y}^{-1})^{-1}(\sum V_{\mathbf y}^{-1}\mathbf y)$$
$$\Delta\mathbf a^2 = (\textstyle\sum V_{\mathbf y}^{-1})^{-1}$$

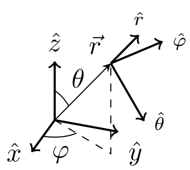
$$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac1{\sum\frac{x^2}{\Delta y^2}}$$
$$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$$
$$\chi^2 = (y-f(x;\theta))^T V^{-1}(y-f(x;\theta))$$
$$\theta = (H^TV^{-1}H)^{-1}H^TV^{-1}y$$
$$\Delta\theta\theta = (H^TV^{-1}H)^{-1}$$

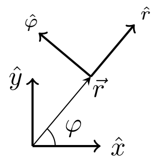
Kinematics

$$\frac1R = \left|\frac{v_xa_y - v_ya_x}{v^3}\right|$$
$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$$
$$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$$
$$\theta\equiv\frac\pi2\to\dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$

$$\theta\equiv\frac\pi2\to\ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi$$
$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$

$$\vec A = \ddot{\vec r} + \vec A_{\mathrm{T}} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta + \frac{\partial\alpha}{\partial t}$$
$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec F\vec p$$
$$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$$
$$T = \frac12M\dot{\vec R}^2 + \frac12 m_i(\dot{\vec r}_i - \dot{\vec R})^2$$

$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$$
$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$
$$\tau_1 = I_1\dot\omega_1 + (I_3-I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t) = T - V + \frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$
$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm{d}t$$

$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$
$$p := \frac{\partial\mathcal L}{\partial\dot q};\ \dot p = \frac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t) = \dot qp - \mathcal L$$
$$\dot q = \frac{\partial\mathcal H}{\partial p};\ \dot p = -\frac{\partial\mathcal H}{\partial q}$$
$$\frac{\mathrm{d}\mathcal H}{\mathrm{d}t} = \frac{\partial\mathcal H}{\partial t} = -\frac{\partial\mathcal L}{\partial t}$$

$$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal H\} + \frac{\partial u}{\partial t}$$
$$\eta = (q,p);\ \Gamma = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$$
$$\dot\eta = \Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2
two points: μd^2
rod: $\frac1{12}mL^2$
disk: $\frac12mr^2$
tetrahedron: $\frac1{20}ms^2$
octahedron: $\frac1{10}ms^2$
sphere: $\frac23mr^2$
ball: $\frac25mr^2$
cone: $\frac3{10}mr^2$
torus: $m\left(R^2+\frac34r^2\right)$
ellipsoid: $I_a = \frac15m(b^2+c^2)$
rectangulus: $\frac1{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle = -2\langle T\rangle$$
$$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$$

$$\frac1\mu = \frac1{m_1}+\frac1{m_2}$$
$$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$$
$$T = \frac12M\dot{\vec R}^2 + \frac12\mu\dot{\vec r}^2$$

$$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$$
$$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$

$$r = \frac{k}{1+\varepsilon\cos\theta}$$
$$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$$
$$a^3\omega^2 = G(m_1+m_2) = \frac\alpha\mu$$

$$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$|a|-|b| \le |a+b| \le |a|+|b|$

$x>-1: 1+nx \le (1+x)^n$

$\frac{|a^n-b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$

$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$

$\sum a_i b_i \le \left(\sum a_i^p\right)^{\frac{1}{p}} \left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^p y^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$

$\sum\left(\frac{a_1+\ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum a_i^p$

$x \ge 0, |\ddot{x}| \le M : |\dot{x}| \le \sqrt{2Mx}$

$\frac{1}{1+x} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U \cap V)$

$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\ \frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022 \cdot 10^{23}\ \frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617 \cdot 10^{-5}\ \frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998 \cdot 10^8\ \frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602 \cdot 10^{-19}\ \mathrm{A\,s}$

$m_\mathrm{e} = 9.109 \cdot 10^{-31}\ \mathrm{kg}$

$m_\mathrm{p} = 1.673 \cdot 10^{-27}\ \mathrm{kg}$

$m_\mathrm{n} = 1.675 \cdot 10^{-27}\ \mathrm{kg}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\ \mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}$

$h = 4.136 \cdot 10^{-15}\ \mathrm{eV\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988 \cdot 10^9\ \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257 \cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274 \cdot 10^{-24}\ \mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{barn} = 1 \cdot 10^{-28}\ \mathrm{m}^2$

$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464 \cdot 10^{-3}\ \frac{\mathrm{W}}{\mathrm{sr}}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i \mathrm{A}_i \to b_j \mathrm{B}_j$

$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$

$\forall\, i,j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$

$k = Ae^{-\frac{E_\mathrm{a}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\mathrm{tot}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_\mathrm{tot}^{\sum a_i - \sum b_j}$

$\Delta G^\mathrm{o}_\mathrm{r} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln\frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu\mathrm{d}N$

$C_{V,N} = \frac{\partial Q}{\partial T}\Big|_{V,N} = \frac{\partial U}{\partial T}\Big|_{V,N}$

$C_{p,N} = \frac{\partial Q}{\partial T}\Big|_{p,N} = \frac{\partial U}{\partial T}\Big|_{p,N} + p\frac{\partial V}{\partial T}\Big|_{p,N}$

$\gamma := \frac{C_p}{C_V}$

$\mu_J := \frac{\partial T}{\partial V}\Big|_{U,N}$

$\lambda U = U(\lambda(S,V,N)) \Rightarrow U = ST - pV + \mu N$

$\Rightarrow S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix S,V,N : $\min U$ at equilibrium

Fix T,V,N : $\min F = U - TS$

Fix T,p,N : $\min G = F + pV$

Fix S,p,N : $\min H = U + pV$

$$\begin{array}{ccc} V & F & T \\ U & & G \\ S & H & p \end{array}$$

$\frac{\partial}{\partial T}\frac{G}{T}\Big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\Big|_V = -\frac{U}{T^2}$

$\Omega = U - TS - \mu N$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int \mathrm{d}q_1 \cdots \mathrm{d}q_N \int \mathrm{d}p_1 \cdots \mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_\mathrm{B}T};\ C = \frac{\partial U}{\partial T}$

$F(T,V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0} e^{i\omega t},\ Z = \frac{V}{I}$

$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$

$Z_\mathrm{series} = \sum_k Z_k,\ \frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$

$\sum_\mathrm{loop} V_k = 0,\ \sum_\mathrm{node} I_k = 0$

$\mathcal{E} = -L\dot{I},\ L = \frac{\Phi_B}{I}$

$I_{A\to C} = I_0\left(e^{\frac{V_{AC}}{V_T}} - 1\right),\ V_T = \eta\frac{k_\mathrm{B}T}{q_\mathrm{e}}$

$I_{E,\mathrm{out}} = I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right) - \alpha_R I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$

$I_{C,\mathrm{in}} = -I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right) + \alpha_F I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec{p} = \gamma m\vec{v}$

$\mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \frac{mc^2}{\gamma}$

$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec{v}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$

$\binom{ct'}{x'} = \gamma\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}\binom{ct}{x}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{\left(1-\frac{V^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct, \vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$

$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\vec{v}}{c}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}, \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu\partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \to \sum_i m_i$

$E_1^\mathrm{max} = \frac{M^2 + m_1^2 - \sum_{i\ne1} m_i^2}{2M}c^2$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$

$\mathrm{SO}_{1,3} = \left\{\Lambda \,\middle|\, \begin{array}{c}\Lambda^\mathrm{T} g\Lambda = g \\ \det\Lambda \ge 0\end{array}\right\}$

$(\Lambda^0{}_0)^2 \ge 1$

$\Lambda = \begin{pmatrix} \gamma & -\gamma\vec{\beta} \\ -\gamma\vec{\beta} & I + \frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta} \end{pmatrix}$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$

$\vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$

$\rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$

$\vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$

$\vec{H} \times \sqrt{4\pi\mu_0}$

$\vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$

σ (cond.)$/4\pi\varepsilon_0$

$\varepsilon/\varepsilon_0$

μ/μ_0

$R, Z \times 4\pi\varepsilon_0$

$L \times 4\pi\varepsilon_0$

$C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec F_{12} = q_1 q_2 \frac{\vec r_2-\vec r_1}{|\vec r_1-\vec r_2|^3};\ \vec E_1 = \frac{\vec F_{12}}{q_2};\ V(\vec r)=\int \mathrm{d}^3r'\, \frac{\rho(\vec r')}{|\vec r-\vec r'|};\ \rho_q=\delta(\vec r-\vec r_q)$$

$$\oint \vec E \mathrm{d}\vec S = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2 V = \vec\nabla\vec E = 4\pi\rho;\ \vec\nabla\times\vec E = 0$$

$$U=\frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde U = \frac12 \frac{q_i q_j}{|\vec r_i-\vec r_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int \vec E_i\vec E_j\,\mathrm{d}^3x$$

$$V(\vec r)=\int\rho G_{\mathrm D}(\vec r)\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_{\mathrm D}}{\partial n}\,\mathrm{d}S$$

$$V(\vec r)=\langle V\rangle_S + \int\rho G_{\mathrm N}(\vec r)\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n} G_{\mathrm N}(\vec r)\,\mathrm{d}S$$

$$\nabla^2_y G(\vec x,\vec y) = -4\pi\delta(\vec x-\vec y);\ G_{\mathrm D}(\vec x,\vec y)\big|_{\vec y\in S}=0;\ \frac{\partial G_{\mathrm N}}{\partial n}\Big|_{\vec y\in S} = -\frac{4\pi}{S}$$

$$U_{\text{sphere}}=\frac35\frac{Q^2}{R};\ \vec p=\int\mathrm{d}^3r\rho\vec r;\ \vec E_{\text{dip}}=\frac{3(\vec p\hat r)\hat r-\vec p}{r^3};\ V_{\text{dip}}=\frac{\vec p\vec r}{r^2}$$

force on a dipole: $\vec F_{\text{dip}} = (\vec p\vec\nabla)\vec E$

$$Q_{ij}=\int\mathrm{d}^3r\rho(\vec r)(3r_ir_j-\delta_{ij}r^2);\ V_{\text{quad}}=\frac{1}{6r^5}Q_{ij}(3r_ir_j-\delta_{ij}r^2)$$

$$V(r,\theta)=\sum_{l=0}^\infty\big(A_lr^l+\frac{B_l}{r^{l+1}}\big)P_l(\cos\theta)$$

$$V(r,\theta,\varphi)=\sum_{l=0}^\infty\sum_{m=-l}^l\big(A_{lm}r^l+\frac{B_{lm}}{r^{l+1}}\big)Y_{lm}(\theta,\varphi)$$

$$\frac{1}{|\vec r-\vec r'|}=\sum_{l=0}^\infty\frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\big(\frac{\vec r\vec r'}{rr'}\big)$$

$$P_l(x)=\frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\big(x^2-1\big)^l;\ f=\sum_{l=0}^\infty c_lP_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$$

$$P_l(1)=1;\ \langle P_n|P_m\rangle=\frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm}|Y_{l'm'}\rangle=\delta_{ll'}\delta_{mm'}$$

$$P_0=1;\ P_1=x;\ P_2=\frac{3x^2-1}{2};\ Y_{00}=\frac{1}{\sqrt{4\pi}};\ Y_{10}=\sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{11}=-\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20}=\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$$

$$Y_{21}=-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22}=\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x)=\frac{(-1)^m}{2^l l!}(1-x^2)^{\frac m2}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\big(x^2-1\big)^l,\ |m|\le l$$

$$Y_{lm}(\theta,\varphi)=\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m}=(-1)^m\overline{Y}_{lm}$$

$$P_l\big(\frac{\vec r\vec r'}{rr'}\big)=\frac{4\pi}{2l+1}\sum_{m=-l}^l\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$$

$$V(r>\operatorname{diam}\operatorname{supp}\rho,\theta,\varphi)=\sum_{l=0}^\infty\frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho]=\int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\, r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec\nabla\vec J=-\frac{\partial\rho}{\partial t}=0;\ I=\int\vec J\mathrm{d}\vec S$$

solenoid: $B=4\pi\frac{j_{\mathrm s}}{c}$

$$\mathrm{d}\vec F=\frac{I\mathrm{d}\vec l}{c}\times\vec B=\mathrm{d}^3x\frac{\vec J}{c}\times\vec B;\ \vec F_q=q\frac{\dot{\vec r}}{c}\times\vec B$$

$$\mathrm{d}\vec B=\frac{I\mathrm{d}\vec l}{c}\times\frac{\vec r}{r^3};\ \vec B_q=q\frac{\dot{\vec r}}{c}\times\frac{\vec r}{r^3}$$

$$\vec B=\vec\nabla\times\vec A;\ \vec A=\int\mathrm{d}^3r'\,\frac{\vec J'}{c}\frac{1}{|\vec r-\vec r'|}+\vec\nabla A_0$$

$$\vec B=\int\mathrm{d}^3r'\,\frac{\vec J'}{c}\times\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$$

$$\varphi=\frac{I}{c}\Omega,\ \vec B=-\vec\nabla\varphi$$

$$\vec\nabla\vec A=0\to\nabla^2\vec A=-4\pi\frac{\vec J}{c}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec B=4\pi\frac{\vec J}{c};\ \oint\vec B\mathrm{d}\vec l=4\pi\frac{I}{c}$$

$$\vec m=\frac12\int\mathrm{d}^3r'\big(\vec r'\times\frac{\vec J'}{c}\big)=\frac{1}{2c}\frac{q}{m}\vec L=\frac{SI}{c}$$

$$\vec A_{\text{dm}}=\frac{\vec m\times\vec r}{r^3};\ \vec\tau=\vec m\times\vec B$$

$$\vec F_{\text{dmdm}}=-\vec\nabla_R\frac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$$

loop axis: $\vec B=\hat z\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac{I}{c}$

Electromagnetism (CGS)

Faraday: $\mathcal E=-\frac1c\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int\mathrm{d}^3x\vec J=\dot{\vec p}$

$$\vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t};\ \vec\nabla\vec E=4\pi\rho;\ \vec\nabla\vec J=-\frac{\partial\rho}{\partial t}$$

$$\vec\nabla\times\vec B=4\pi\frac{\vec J}{c}+\frac1c\frac{\partial\vec E}{\partial t};\ \vec\nabla\vec B=0$$

$$\mathrm{d}\vec F=\mathrm{d}^3x\big(\rho\vec E+\frac{\vec J}{c}\times\vec B\big);\ \vec F_q=q\big(\vec E+\frac{\dot{\vec r}}{c}\times\vec B\big)$$

$$u=\frac{E^2+B^2}{8\pi};\ \vec S=\frac{c}{4\pi}\vec E\times\vec B;\ \vec g=\frac{\vec S}{c^2}$$

$$\mathbf T^E=\frac{1}{4\pi}\big(\vec E\otimes\vec E-\frac12E^2\big);\ \mathbf T=\mathbf T^E+\mathbf T^B$$

$$-\frac{\partial u}{\partial t}=\vec J\vec E+\vec\nabla\vec S;\ -\frac{\partial\vec g}{\partial t}=\rho\vec E+\frac{\vec J}{c}\times\vec B-\vec\nabla\mathbf T$$

$$\vec B=\vec\nabla\times\vec A;\ \vec E=-\vec\nabla\phi-\frac1c\frac{\partial\vec A}{\partial t}$$

$$-\nabla^2\phi-\frac1c\frac{\partial}{\partial t}\vec\nabla\vec A=4\pi\rho$$

$$\vec\nabla\big(\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}\big)-\nabla^2\vec A+\frac1c\frac{\partial^2\vec A}{\partial t^2}=4\pi\frac{\vec J}{c}$$

$$(\phi,\vec A)\cong\big(\phi-\frac1c\frac{\partial\chi}{\partial t},\vec A+\vec\nabla\chi\big)$$

$$(\phi,\vec A)=\int\mathrm{d}^3r'\,\frac{\big(\rho,\frac{\vec J}{c}\big)\big(\vec r',t-\frac1c|\vec r-\vec r'|\big)}{|\vec r-\vec r'|}$$

$$\vec\nabla\vec A=0\to\Box\vec A=\frac{4\pi}{c}(\vec J-\vec J_L)=:\frac{4\pi}{c}\vec J_T$$

$$\vec J_L=\frac{1}{4\pi}\vec\nabla\frac{\partial\phi}{\partial t}=-\frac{1}{4\pi}\vec\nabla\int\frac{\vec\nabla'\vec J'}{|\vec x-\vec x'|}\mathrm{d}^3x'$$

$$\vec E'_\parallel=\vec E_\parallel;\ \vec B'_\parallel=\vec B_\parallel$$

$$\vec E'_\perp=\gamma\big(\vec E_\perp+\frac{\vec v}{c}\times\vec B\big)$$

$$\vec B'_\perp=\gamma\big(\vec B_\perp-\frac{\vec v}{c}\times\vec E\big)$$

$$\text{plane wave: }\begin{cases}\vec E=\vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B=\hat k\times\vec E\\ \omega=ck\end{cases}$$

$$\vec B_{\text{diprad}}=\frac{1}{c^2}\frac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_{\text{rit}}};\ \vec E_{\text{diprad}}=\vec B_{\text{diprad}}\times\hat r$$

Larmor: $P=\frac{2}{3c^3}|\ddot{\vec p}|^2$

Rel. Larmor: $P=\frac{2}{3c^3}q^2\gamma^6(a^2-(\vec a\times\vec\beta)^2)$

$$\vec A_{\text{dm}}=\frac1c\frac{\dot{\vec m}\times\hat r}{r}\Big|_{t_{\text{rit}}}$$

L.W.: $(\phi,\vec A)=\frac{q(1,\frac{\vec v}{c})}{[r-\frac{\vec v\vec r}{c}]_{t_{\text{rit}}}};\ t_{\text{rit}}=t-\frac rc\Big|_{t_{\text{rit}}}$

$$A^\mu=(\phi,\vec A);\ J^\mu=(c\rho,\vec J)$$

Lorenz gauge: $\partial_\alpha A^\alpha=0$

Temporal gauge: $\phi=0$

Axial gauge: $A_3=0$

Coulomb gauge: $\vec\nabla\vec A=0$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu;\ \mathscr F^{\mu\nu}=\frac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu}=\begin{pmatrix}0&-E_x&-E_y&-E_z\\E_x&0&-B_z&B_y\\E_y&B_z&0&-B_x\\E_z&-B_y&B_x&0\end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu}=4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr F^{\alpha\nu}=0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma}+\partial_\nu F_{\sigma\mu}+\partial_\sigma F_{\mu\nu}=0;\ \det F=(\vec E\vec B)^2$$

$$F^{\alpha\beta}F_{\alpha\beta}=2(B^2-E^2);\ F^{\alpha\beta}\mathscr F_{\alpha\beta}=4\vec E\vec B$$

$$\Theta^{\mu\nu}=\frac{1}{4\pi}\big(F^\mu{}_\alpha F^{\alpha\nu}+\frac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\big)$$

$$\Theta^{\mu\nu}=\begin{pmatrix}u&c\vec g\\c\vec g&-\mathbf T\end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu}=\frac{J_\alpha}{c}F^{\alpha\nu}(-?)$$

$$\mathcal L=\frac{mc^2}{\gamma}-q\vec A\frac{\vec v}{c}+q\phi$$

E.M. in matter (CGS)

$$\vec\nabla\vec D=4\pi\rho_{\text{ext}};\ \vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec H=4\pi\frac{\vec J_{\text{ext}}}{c}+\frac1c\frac{\partial\vec D}{\partial t}$$

$$\vec P=\frac{\mathrm{d}\langle\vec p\rangle}{\mathrm{d}V};\ \vec M=\frac{\mathrm{d}\langle\vec m\rangle}{\mathrm{d}V}$$

$$\rho_{\text{pol}}=-\vec\nabla\vec P;\ \sigma_{\text{pol}}=\hat n\vec P;\ \frac{\vec J_{\text{mag}}}{c}=\vec\nabla\times\vec M$$

$$\vec D_{\text{pol}}=\vec E+4\pi\vec P;\ \vec H_{\text{mag}}=\vec B-4\pi\vec M$$

static linear isotropic: $\vec P=\chi\vec E$

static linear: $P_i=\chi_{ij}E_j$

static linear: $\varepsilon=1+4\pi\chi$

static: $\Delta D_\perp=4\pi\sigma_{\text{ext}};\ \Delta E_\parallel=0$

static linear: $u=\frac{1}{8\pi}\vec E\vec D$

$$\Delta U_{\text{dielectric}}=-\frac12\int\mathrm{d}^3r\vec P\vec E_0$$

plane capacitor: $C=\frac{\varepsilon}{4\pi}\frac Sd$

cilindric capacitor: $C=\frac{L}{2\log\frac Rr}$

atomic polarizability: $\vec p=\alpha\vec E$

non-interacting gas: $\vec p=\alpha\vec E_0;\ \chi=n\alpha$

hom. cubic isotropic: $\chi=\frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi}{3}n\alpha$

perm. dipole: $\chi=\frac13\frac{np_0^2}{kT}$

local field: $\vec E_{\text{loc}}=\vec E+\frac{4\pi}{3}\vec P$

$$\vec J\vec E=-\vec\nabla\Big(\frac{c}{4\pi}\vec E\times\vec H\Big)-\frac{1}{4\pi}\Big(\vec E\frac{\partial\vec D}{\partial t}+\vec H\frac{\partial\vec B}{\partial t}\Big)$$

$$n=\sqrt{\varepsilon\mu};\ k=n\frac\omega c$$

plane wave: $B=nE$

$$\vec J_{\mathrm c}=\sigma\vec E;\ \varepsilon_\sigma=1+i\frac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm p}^2=4\pi\frac{n_{\text{vol}}q^2}{m};\ \omega_{\text{cyclo}}=\frac{qB}{mc}$$

I: $u=\frac{1}{8\pi}(\vec E\vec D+\vec H\vec B)$

I: $\langle S_z\rangle=\frac cn\langle u\rangle$

II: $u=\frac{1}{8\pi}\big(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\frac{\partial}{\partial\omega}(\mu\omega)H^2\big)$

II: $\langle S_z\rangle=v_{\mathrm g}\langle u\rangle;\ v_{\mathrm g}=\frac{\partial\omega}{\partial k}=\frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$

III: $\langle W\rangle=\frac{\omega}{4\pi}\big(\operatorname{Im}\varepsilon\langle E^2\rangle+\operatorname{Im}\mu\langle H^2\rangle\big)$

Fresnel TE (S): $\frac{E_{\mathrm t}}{E_{\mathrm i}}=\frac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm r}}{E_{\mathrm i}}=\frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_{\mathrm t}}{E_{\mathrm i}}=\frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm r}}{E_{\mathrm i}}=\frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz}=\pm\sqrt{\varepsilon_2\big(\frac\omega c\big)^2-k_x^2},\ \operatorname{Im}k_{tz}>0$

Drüde-Lorentz: $\varepsilon=1-\frac{\omega_{\mathrm p}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$$P(t)=\int_{-\infty}^\infty g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega)=\chi(\omega)E(\omega)$$

$$\chi(\omega)=\int_{-\infty}^{\infty}e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega)=\overline{\chi}(\omega)$$
$$g(t<0)=0\implies$$
$$\operatorname{Re}\varepsilon(\omega)=1+\frac{2}{\pi}\fint_0^\infty\frac{\omega'\left(\operatorname{Im}\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\operatorname{Im}\varepsilon(\omega)=-\frac{2\omega}{\pi}\fint_0^\infty\frac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega'+\frac{4\pi\sigma_0}{\omega}$$
$$\text{sum rule: }\frac{\pi}{2}\omega_\mathrm{p}^2=\int_0^\infty\omega\operatorname{Im}\varepsilon\mathrm{d}\omega$$
$$\text{sum rule: }2\pi^2\sigma_0=\int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$$

$$\text{sum rule: }\int_0^\infty(\operatorname{Re}n-1)\mathrm{d}\omega=0$$
$$\text{Miller rule: }\chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$$

Quantum mechanics (CGS)

$$r_B=\frac{\hbar^2}{m_ee^2}=5.292\cdot10^{-11}\,\mathrm{m}$$
$$\text{Rydberg}=\frac{e^2}{2r_B}=13.61\,\mathrm{eV}$$
$$r_e=\frac{e^2}{mc^2}=2.818\cdot10^{-15}\,\mathrm{m}$$
$$E_B=-\frac{1}{n^2}\frac{e^2}{2r_B}$$
$$\alpha=\frac{e^2}{\hbar c}$$
$$\text{Planck: }\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$$
$$\lambda_\text{Broglie}=\frac{h}{p}$$

$$\sigma_1=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right);\ \sigma_2=\left(\begin{smallmatrix}0&-i\\i&0\end{smallmatrix}\right);\ \sigma_3=\left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$$
$$\sigma_i\sigma_j=\delta_{ij}+i\varepsilon_{ijk}\sigma_k$$
$$i\hbar\frac{\partial\mathcal{U}}{\partial t}=\mathcal{H}\mathcal{U}$$
$$\frac{\partial\mathcal{H}}{\partial t}=0\ \Rightarrow\ \mathcal{U}(t)=e^{-\frac{i\mathcal{H}t}{\hbar}}$$
$$[\mathcal{H}(t),\mathcal{H}(t')]=0\ \Rightarrow\ \mathcal{U}(t)=e^{-\frac{i\int_0^t\mathrm{d}t\mathcal{H}(t)}{\hbar}}$$
$$\mathcal{U}(t)=\left(\frac{-i}{\hbar}\right)^k\int_0^t\mathrm{d}t_1\cdots\mathrm{d}t_k\mathcal{H}(t_1)\cdots\mathcal{H}(t_k)$$
$$H=H_0+V_\lambda:\frac{\partial E_n}{\partial\lambda}\Big|_{\lambda=0}=\langle\psi_n|\,\frac{\partial V_\lambda}{\partial\lambda}\,|\psi_n\rangle\,\Big|_{\lambda=0}$$
$$[A,BC]=[A,B]C+B[A,C]$$

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$$
$$[X,P]=i\hbar$$
$$\psi(x)=\langle x|\psi\rangle$$
$$\langle x|\,X\,|\psi\rangle=x\,\langle x|\psi\rangle$$
$$\langle x|\,P\,|\psi\rangle=\frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|\psi\rangle$$
$$\langle x|p\rangle=\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$$
$$\left\langle(A-\langle A\rangle)^2\right\rangle\left\langle(B-\langle B\rangle)^2\right\rangle\geq\frac{1}{4}|\langle[A,B]\rangle|^2$$
$$e^{iH}Ae^{-iH}=A+i[H,A]+\frac{i^2}{2!}[H,[H,A]]+\cdots$$

QM solutions

$$\mathcal{H}_\text{box}=\frac{p^2}{2m}+\begin{cases}0&0<x<L\\\infty&\text{otherwise}\end{cases}$$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(n\pi\frac{x}{L}\right),\ n\geq1$$
$$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}=\frac{n^2h^2}{8mL^2}$$

$$\Delta x^2=L^2\left(\frac{1}{12}-\frac{1}{2n^2\pi^2}\right)$$
$$\Delta p=\frac{\hbar n\pi}{L}=\frac{hn}{2L}$$

Nuclear physics (MKSA)

$$M(A,Z)=Zm_\mathrm{p}+(A-Z)m_\mathrm{n}-B(A,Z)$$
$$B(A,Z)=a_vA-a_sA^{2/3}-a_c\frac{Z(Z-1)}{A^{1/3}}-a_\text{sym}\frac{(A-2Z)^2}{A}+a_pA^{-3/4}\Delta$$
$$\Delta=\begin{cases}0&&A\text{ odd}\\\left.\begin{matrix}1&Z\text{ even}\\-1&Z\text{ odd}\end{matrix}\right\}&&A\text{ even}\end{cases}$$
$$a_v=15.5;\ a_s=16.8;\ a_c=0.72;\ a_\text{sym}=23;\ a_p=34\ [\mathrm{MeV}]$$

$$\frac{\partial M}{\partial Z}=0:Z=\frac{m_\mathrm{n}-m_\mathrm{p}+4a_\text{sym}}{\frac{2a_c}{A^{1/3}}+\frac{8a_\text{sym}}{A}}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$
$$s_{ab}:=|p_a^\mu+p_b^\mu|^2$$
$$M\to abc:(m_a+m_b)^2\leq s_{ab}\leq(M-m_c)^2$$
$$M\to abc:s_{ab}+s_{bc}+s_{ac}=M^2+m_a^2+m_b^2+m_c^2$$
$$a_iA_i\to b_jB_j:Q:=(a_im_{A_i}-b_jm_{B_j})c^2$$
$$p=qBR$$