Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\tfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\tfrac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$
$$\tan\tfrac{\alpha}{2}=\tfrac{\sin\alpha}{1+\cos\alpha}=\tfrac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\tfrac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\tfrac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$
$$\operatorname{acos}x+\operatorname{asin}x=\tfrac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\tfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\begin{pmatrix}\sinh x\\ \cosh x\end{pmatrix}=\tfrac12\begin{pmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{pmatrix}$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\tfrac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\operatorname{asinh}x=\log\big(x+\sqrt{x^2+1}\big)$$
$$\operatorname{acosh}x=\log\big(x+\sqrt{x^2-1}\big)$$
$$\operatorname{atanh}x=\tfrac12\log\tfrac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\big(I+\frac{B}{2}-1\big)\,A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\tfrac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\tbinom{n}{k}=\tfrac{n!}{k!(n-k)!}\qquad C'_{n,k}=\tbinom{n+k-1}{k}$$
$$D_{n,k}=\tfrac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\tfrac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\tfrac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\tfrac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{x=1}^n x^3=\big(\sum_{x=1}^n x\big)^2=\tfrac14 n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2=\tfrac16 n(n+1)(2n+1)$$
$$\tbinom{n}{k}=\tbinom{n-1}{k-1}+\tbinom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\textstyle\int_0^\infty t^ze^{-t}dt=z!$$
$$n!\approx(\tfrac{n}{e})^n\sqrt{2\pi n}$$
$$\text{Fourier: }c_n=\tfrac{2}{T}\textstyle\int_0^T f(t)\cos\big(n\frac{t}{T}\big)\mathrm{d}t$$
$$F[f]=\hat f(k)=\tfrac{1}{\sqrt{2\pi}}\textstyle\int_{-\infty}^\infty e^{-ikx}f(x)$$
$$\langle\hat f|\hat g\rangle=\langle f|g\rangle$$
$$F\big[\tfrac{\sin x}{x}\big]=\sqrt{\tfrac{\pi}{2}}\chi_{[-1;1]}$$
$$\tfrac{\mathrm{d}}{\mathrm{d}x}\textstyle\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\tfrac{\operatorname{Re}z+|z|}{2}}+\tfrac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$
$$\delta(g(x))=\tfrac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\tfrac12\operatorname{Re}(a\bar b)$$

Derivatives

$$\tan'x=1+\tan^2x\qquad\cot'x=-1-\cot^2x\qquad\operatorname{atan}'x=-\operatorname{acot}'x=\tfrac{1}{1+x^2}$$
$$\operatorname{asin}'x=-\operatorname{acos}'x=\tfrac{1}{\sqrt{1-x^2}}\qquad(a^x)'=a^x\ln a\qquad\log_a'x=\tfrac{1}{x\ln a}$$
$$\cosh'x=\sinh x\qquad\tanh'x=1-\tanh^2x\qquad\operatorname{atanh}'x=\operatorname{acoth}'x=\tfrac{1}{1-x^2}$$
$$\operatorname{asinh}'x=\tfrac{1}{\sqrt{x^2+1}}\qquad\operatorname{acosh}'x=\tfrac{1}{\sqrt{x^2-1}}\qquad(f^{-1})'=\tfrac{1}{f'(f^{-1})}$$
$$\big(\tfrac1x\big)'=-\tfrac{\dot x}{x^2}\qquad\big(\tfrac{x}{y}\big)'=\tfrac{\dot xy-x\dot y}{y^2}\qquad(x^y)'=x^y\big(\dot y\ln x+y\tfrac{\dot x}{x}\big)$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}\qquad\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac1x=\ln|x|\qquad\int\tan x=-\ln|\cos x|\qquad\int\cot x=\ln|\sin x|\qquad\int\frac{1}{\sin x}=\ln\big|\tan\frac{x}{2}\big|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\big|\tan\big(\frac{x}{2}+\frac{\pi}{4}\big)\big|\qquad\int\ln x=x(\ln x-1)\qquad\int\tanh x=\ln\cosh x\qquad\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}\qquad\int\frac{1}{a^2+x^2}=\frac1a\operatorname{atan}\frac{x}{a}\qquad\int xy=x\int y-\int(\dot x\int y)$$
$$\textstyle\int e^{yx}x=e^{yx}\big(\frac{y}{x}-\frac{1}{y^2}\big)\qquad\int e^{-x^2}=\sqrt{\pi}$$

Differential equations

$$\dot x+\dot ax=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$
$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot x=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot x+\gamma\dot x+\omega_0^2x=fe^{-i\omega t}:x=\tfrac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac1x+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac1x-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$
$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac1x-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac1x+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac56x^4-\tfrac34x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$
$$\vec a\times\vec b=\varepsilon_{ijk}a_jb_k\hat e_i$$
$$(\vec a\otimes\vec b)_{ij}=a_ib_j$$
$$(\vec a\times\vec b)\vec c=(\vec c\times\vec a)\vec b$$
$$(\vec a\times\vec b)\times\vec c=-(\vec b\vec c)\vec a+(\vec a\vec c)\vec b$$
$$(\vec a\times\vec b)(\vec c\times\vec d)=(\vec a\vec c)(\vec b\vec d)-(\vec a\vec d)(\vec b\vec c)$$
$$|\vec u\times\vec v|^2=u^2v^2-(\vec u\vec v)^2$$
$$\vec\nabla=\big(\tfrac{\partial}{\partial x},\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}\big);\ \Box=\tfrac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec\nabla V=\tfrac{\partial V}{\partial\rho}\hat\rho+\tfrac1\rho\tfrac{\partial V}{\partial\phi}\hat\phi+\tfrac{\partial V}{\partial z}\hat z$$
$$\vec\nabla\vec v=\tfrac1\rho\tfrac{\partial(\rho v_\rho)}{\partial\rho}+\tfrac1\rho\tfrac{\partial v_\phi}{\partial\phi}+\tfrac{\partial v_z}{\partial z}$$
$$\vec\nabla\times\vec v=\big(\tfrac1\rho\tfrac{\partial v_z}{\partial\phi}-\tfrac{\partial v_\phi}{\partial z}\big)\hat\rho+$$
$$+\big(\tfrac{\partial v_\rho}{\partial z}-\tfrac{\partial v_z}{\partial\rho}\big)\hat\phi+\tfrac1\rho\big(\tfrac{\partial(\rho v_\phi)}{\partial\rho}-\tfrac{\partial v_\rho}{\partial\phi}\big)$$
$$\nabla^2V=\tfrac1\rho\tfrac{\partial}{\partial\rho}\big(\rho\tfrac{\partial V}{\partial\rho}\big)+\tfrac{1}{\rho^2}\tfrac{\partial^2V}{\partial\phi^2}+\tfrac{\partial^2V}{\partial z^2}$$
$$\vec\nabla V=\tfrac{\partial V}{\partial r}\hat r+\tfrac1r\tfrac{\partial V}{\partial\theta}\hat\theta+\tfrac{1}{r\sin\theta}\tfrac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec\nabla\times\vec v = \frac{1}{r\sin\theta}\Big(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\Big)\hat r +$$
$$+\frac{1}{r}\Big(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(r v_\varphi)}{\partial r}\Big)\hat\theta + \frac{1}{r}\Big(\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\Big)\hat\varphi$$
$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\big(r^2\frac{\partial V}{\partial r}\big)}{r^2} + \frac{\frac{\partial}{\partial\theta}\big(\sin\theta\frac{\partial V}{\partial\theta}\big)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$$

$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$
$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w+\vec w\,\vec\nabla)\vec v - (\vec\nabla\vec v+\vec v\,\vec\nabla)\vec w$$
$$\tfrac{1}{2}\vec\nabla v^2 = (\vec v\,\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$
$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\Big(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\Big)\mathrm{d}S$$
$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(\vec r-\vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\nabla^2\frac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}} \approx \frac{x_{\max}-x_{\min}}{\sqrt N}$$
$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$
$$\text{median} = F^{-1}(\tfrac12)$$
$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$
$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$
$$\alpha_n = E[x^n]$$
$$M_n = E[(x-\mu)^n]$$
$$\sigma^2 = M_2 = E[x^2]-\mu^2$$
$$\text{FWHM}\approx 2\sigma$$
$$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$
$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge 0: P(h\ge k)\le\frac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$$
$$B(k;n,p) = \binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$
$$P(k;\mu) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$
$$u(x;a,b) = \frac{1}{b-a},\ x\in[a;b]$$
$$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$$
$$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \frac{1}{\lambda},\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$$
$$g(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac12(\frac{x-\mu}{\sigma})^2}$$
$$\text{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$
$$z = \frac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$
$$\chi^2 = \sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$$
$$\wp(x;n) = \frac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$$
$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$
$$n\ge 30: \wp(x;n)\approx g(x;n,\sqrt{2n})$$
$$n\ge 8: p[\sqrt{2\chi^2}]\approx g(;\sqrt{2n-1},1)$$
$$S(x;n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\,\Gamma(\frac n2)}\Big(1+\frac{x^2}{n}\Big)^{-\frac{n+1}{2}}$$
$$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$$

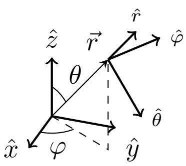
$$p\Big[z\sqrt{\frac{n}{\chi^2}}\Big] = S(,n)$$
$$n\ge 35: S(x;n)\approx g(x;0,1)$$
$$c(x;a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$$
$$\sigma_{xy} = E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le 1$$
$$\mu[f(x_1,\ldots)]\approx f(\mu_1,\ldots)$$
$$\sigma^2[f(x_1,\ldots)]\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$
$$\mu\approx m = \frac1n\sum_{i=1}^n x_i$$
$$\sigma^2\approx s^2 = \frac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$
$$s_m^2 = \frac{s^2}{n}$$
$$p\Big[\frac{m-\mu}{s_m}\Big] = S(;n)$$

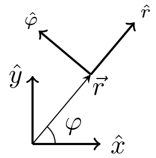
Fit

$$f(x) = mx+q,\quad f(x) = a,$$
$$f(x) = bx,\quad f(x;\theta) = \theta_i h_i(x)$$
$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta mq = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$$
$$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$$
$$\chi^2 = (y-f(x;\theta))^TV^{-1}(y-f(x;\theta))$$
$$\theta = (H^TV^{-1}H)^{-1}H^TV^{-1}y$$
$$\Delta\theta\theta = (H^TV^{-1}H)^{-1}$$

Kinematics

$$\frac1R = \Big|\frac{v_xa_y-v_ya_x}{v^3}\Big|$$
$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$$
$$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$$
$$\theta\equiv\frac\pi2\to\dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$
$$\theta\equiv\frac\pi2\to\ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$$
$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$
$$\vec A = \ddot{\vec r} + \vec A_{\mathrm T} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \frac{\mathrm d}{\mathrm dt}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta + \frac{\partial\alpha}{\partial t}$$
$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm d(mT)}{\mathrm dt} = \vec F\vec p$$
$$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$$
$$T = \tfrac12 M\dot{\vec R}^2 + \tfrac12 m_i(\dot{\vec r}_i - \dot{\vec R})^2$$
$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$$
$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$
$$\tau_1 = I_1\dot\omega_1 + (I_3-I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t) = T - V + \frac{\mathrm d}{\mathrm dt}f(q,t)$$
$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$$
$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$
$$p := \frac{\partial\mathcal L}{\partial\dot q};\ \dot p = \frac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t) = \dot qp - \mathcal L$$
$$\dot q = \frac{\partial\mathcal H}{\partial p};\ \dot p = -\frac{\partial\mathcal H}{\partial q}$$
$$\frac{\mathrm d\mathcal H}{\mathrm dt} = \frac{\partial\mathcal H}{\partial t} = -\frac{\partial\mathcal L}{\partial t}$$
$$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm du}{\mathrm dt} = \{u,\mathcal H\} + \frac{\partial u}{\partial t}$$
$$\eta = (q,p);\ \Gamma = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$$
$$\dot\eta = \Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2 · two points: μd^2 · rod: $\frac{1}{12}mL^2$ · disk: $\frac12 mr^2$ · tetrahedron: $\frac{1}{20}ms^2$ · octahedron: $\frac{1}{10}ms^2$ · sphere: $\frac23 mr^2$ · ball: $\frac25 mr^2$ · cone: $\frac{3}{10}mr^2$ · torus: $m\big(R^2+\frac34 r^2\big)$ · ellipsoid: $I_a = \frac15 m(b^2+c^2)$ · rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle = -2\langle T\rangle$$
$$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$$
$$\frac1\mu = \frac{1}{m_1}+\frac{1}{m_2}$$
$$\vec r = \vec r_1-\vec r_2,\ \alpha = Gm_1m_2$$
$$T = \tfrac12 M\dot{\vec R}^2 + \tfrac12\mu\dot{\vec r}^2$$
$$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$$
$$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$
$$r = \frac{k}{1+\varepsilon\cos\theta}$$
$$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$$
$$a^3\omega^2 = G(m_1+m_2) = \frac\alpha\mu$$
$$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$$|a|-|b|\le|a+b|\le|a|+|b|$$
$$x>-1: 1+nx\le(1+x)^n$$
$$\frac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$$
$$\sum a_ib_i\le\Big(\sum a_i^p\Big)^{\frac1p}\Big(\sum b_i^{\frac{p}{p-1}}\Big)^{\frac{p-1}{p}}$$
$$x^py^q\le\Big(\frac{px+qy}{p+q}\Big)^{p+q}$$
$$\sqrt[p]{\frac1n\sum a_i^{p\le q}}\le\sqrt[q]{\frac1n\sum a_i^q}$$
$$\sum\Big(\frac{a_1+\ldots a_i}{i}\Big)^p\le\Big(\frac{p}{p-1}\Big)^p\sum a_i^p$$
$$x\ge 0, |\ddot x|\le M: |\dot x|\le\sqrt{2Mx}$$
$$\frac{1}{1+x}<\ln\Big(1+\frac1x\Big)<\frac1x$$

Linear algebra

$$\dim(U+V) = \dim U+\dim V-\dim(U\cap V)$$
$$\dim V = \dim\ell(V) + \dim(V\cap\ker\ell)$$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\ \frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022 \cdot 10^{23}\ \frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617 \cdot 10^{-5}\ \frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602 \cdot 10^{-19}\ \mathrm{A\,s}$

$m_\mathrm{e} = 9.109 \cdot 10^{-31}\ \mathrm{kg}$

$m_\mathrm{p} = 1.673 \cdot 10^{-27}\ \mathrm{kg}$

$m_\mathrm{n} = 1.675 \cdot 10^{-27}\ \mathrm{kg}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\ \mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}$

$h = 4.136 \cdot 10^{-15}\ \mathrm{eV\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988 \cdot 10^{9}\ \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257 \cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274 \cdot 10^{-24}\ \mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{barn} = 1 \cdot 10^{-28}\ \mathrm{m}^2$

Chemistry

$$H = U + pV$$

$$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$$

$$G = H - TS$$

$$a_i \mathrm{A}_i \rightarrow b_j \mathrm{B}_j$$

$$\Delta H^\circ_\mathrm{r} = b_j \Delta H^\circ_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\circ_\mathrm{f}(\mathrm{A}_i)$$

$$\forall\, i, j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$$

$$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$$

$$k = Ae^{-\frac{E_\mathrm{a}}{RT}}\ \text{(Arrhenius)}$$

$$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$$

$$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$$

$$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$$

$$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$$

$$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\mathrm{tot}}$$

$$K_c = K_p(RT)^{\sum a_i - \sum b_j}$$

$$K_c = K_n V^{\sum a_i - \sum b_j}$$

$$K_\chi = K_n n_\mathrm{tot}^{\sum a_i - \sum b_j}$$

$$\Delta G^\circ_\mathrm{r} = -RT \ln K$$

$$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$$

$$\Delta G = RT \ln \frac{Q}{K}$$

$$\ln\frac{K_2}{K_1} = -\frac{\Delta H^\circ}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$$

$$\Delta E = \Delta E^\circ - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q\ \text{(Nerst)}$$

$$\text{(std)}\ \Delta E = \Delta E^\circ - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$$

$$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$$

$$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$$

Thermodynamics

$$\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L \qquad \mathrm{d}S = \frac{\mathrm{d}Q}{T} \qquad C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V \qquad C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p \qquad \gamma = \frac{C_p}{C_V}$$

$$\mathrm{d}L = p\mathrm{d}V$$

Ideal gas

$$pV = nRT$$

$$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$$

$$c_V = \frac{R}{\gamma - 1},\ c_p = \frac{\gamma}{\gamma - 1}R$$

$$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T\ \text{const.}$$

Statistical mechanics

$$Z = \frac{1}{h^N}\int \mathrm{d}q_1 \cdots \mathrm{d}q_N \int \mathrm{d}p_1 \cdots \mathrm{d}p_N e^{-\beta\mathcal{H}}$$

$$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_\mathrm{B}T};\ C = \frac{\partial U}{\partial T}$$

$$F(T,V) = U - TS = -\frac{\log Z}{\beta}$$

$$S = -\frac{\partial F}{\partial T}$$

Electronics (MKS)

$$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t}$$

$$Z = \frac{V}{I} \qquad Z_C = -i\frac{1}{\omega C} \qquad Z_\mathrm{series} = \textstyle\sum_k Z_k \qquad \sum_\mathrm{loop} V_k = 0 \qquad \mathcal{E} = -L\dot{I}$$

$$Z_R = R \qquad Z_L = i\omega L \qquad \frac{1}{Z_\mathrm{parallel}} = \textstyle\sum_k \frac{1}{Z_k} \qquad \sum_\mathrm{node} I_k = 0 \qquad L = \frac{\Phi_B}{I}$$

Relativity

$$\beta = \frac{v}{c} = \tanh\chi$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$$

$$\vec{p} = \gamma m\vec{v}$$

$$\mathcal{E} = \gamma mc^2$$

$$\text{free particle: } \mathcal{L} = \frac{mc^2}{\gamma}$$

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec{v}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\binom{ct'}{x'} = \gamma\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}\binom{ct}{x}$$

$$\chi'' = \chi' + \chi$$

$$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$$

$$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$$

$$\frac{V'}{c} = 1 - \frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$$

$$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$$

$$x^\mu = (ct, \vec{x})$$

$$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$$

$$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$$

$$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$$

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\vec{v}}{c}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}, \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x_\mu = g_{\mu\nu}x^\nu$$

$$\partial_\mu\partial^\mu = \Box$$

$$p^\mu p_\mu = (mc)^2$$

$$v^\mu a_\mu = 0$$

$$M \rightarrow \textstyle\sum_i m_i$$

$$E_1^\mathrm{max} = \frac{M^2 + m_1^2 - \sum_{i\neq 1} m_i^2}{2M}c^2$$

$$\text{doppler: } \sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$$

$$\mathrm{SO}_{1,3} = \left\{\Lambda \,\middle|\, \begin{smallmatrix}\Lambda^\mathrm{T} g\Lambda = g \\ \det\Lambda \geq 0\end{smallmatrix}\right\}$$

$$({\Lambda^0}_0)^2 \geq 1$$

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\vec{\beta} \\ -\gamma\vec{\beta} & I + \frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta} \end{pmatrix}$$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$ $\quad \vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$ $\quad \vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{H} \times \sqrt{4\pi\mu_0}$ $\quad \vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \varepsilon/\varepsilon_0$ $\quad \mu/\mu_0$ $\quad R, Z \times 4\pi\varepsilon_0$ $\quad L \times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec{F}_{12} = q_1 q_2\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3r'\,\frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$$

$$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\times\vec{E} = 0$$

$$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int \vec{E}_i\vec{E}_j\,\mathrm{d}^3x$$

$$V(\vec{r}) = \int\rho G_\mathrm{D}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_\mathrm{D}}{\partial n}\,\mathrm{d}S$$

$$V(\vec{r}) = \langle V\rangle_S + \int\rho G_\mathrm{N}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n}G_\mathrm{N}(\vec{r})\,\mathrm{d}S$$

$$\nabla^2_y G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_\mathrm{D}(\vec{x}, \vec{y})|_{\vec{y}\in S} = 0;\ \frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y}\in S} = -\frac{4\pi}{S}$$

$$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R};\ \vec{p} = \int\mathrm{d}^3r\,\rho\vec{r};\ \vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_\mathrm{dip} = \frac{\vec{p}\hat{r}}{r^2}$$

$$\text{force on a dipole: } \vec{F}_\mathrm{dip} = (\vec{p}\vec{\nabla})\vec{E}$$

$$Q_{ij} = \int\mathrm{d}^3r\,\rho(\vec{r})(3r_i r_j - \delta_{ij}r^2);\ V_\mathrm{quad} = \frac{1}{6r^5}Q_{ij}(3r_i r_j - \delta_{ij}r^2)$$

$$V(r,\theta) = \sum_{l=0}^\infty\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$V(r,\theta,\varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l\left(A_{lm}r^l + \frac{B_{lm}}{r^{l+1}}\right)Y_{lm}(\theta,\varphi)$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty\frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$$

$$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$$

$$P_l(1) = 1;\ \langle P_n|P_m\rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm}|Y_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}$$

$$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l,\ |m| \leq l$$

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m\overline{Y}_{lm}$$

$$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$$

$$V(r > \operatorname{diam}\operatorname{supp}\rho, \theta, \varphi) = \sum_{l=0}^\infty\frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\,r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0;\ I = \int\vec{J}\mathrm{d}\vec{S}$$

$$\text{solenoid: } B = 4\pi\frac{j_\mathrm{s}}{c}$$

$$\mathrm{d}\vec{F} = \frac{I\mathrm{d}\vec{l}}{c}\times\vec{B} = \mathrm{d}^3x\frac{\vec{J}}{c}\times\vec{B};\ \vec{F}_q = q\frac{\dot{\vec{r}}}{c}\times\vec{B}$$

$$\mathrm{d}\vec{B} = \frac{I\mathrm{d}\vec{l}}{c}\times\frac{\vec{r}}{r^3};\ \vec{B}_q = q\frac{\dot{\vec{r}}}{c}\times\frac{\vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{A} = \int\mathrm{d}^3r'\,\frac{\vec{J}'}{c}\frac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla}A_0$$

$$\vec{B} = \int\mathrm{d}^3r'\,\frac{\vec{J}'}{c}\times\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\varphi = \frac{I}{c}\Omega,\ \vec{B} = -\vec{\nabla}\varphi$$

$$\vec{\nabla}\vec{A} = 0 \to \nabla^2\vec{A} = -4\pi\frac{\vec{J}}{c}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla}\times\vec{B} = 4\pi\frac{\vec{J}}{c};\ \oint\vec{B}\mathrm{d}\vec{l} = 4\pi\frac{I}{c}$$

$$\vec{m} = \frac{1}{2}\int\mathrm{d}^3r'\left(\vec{r}'\times\frac{\vec{J}'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec{L} = \frac{SI}{c}$$

$$\vec{A}_\mathrm{dm} = \frac{\vec{m}\times\vec{r}}{r^3};\ \vec{\tau} = \vec{m}\times\vec{B}$$

$$\vec{F}_\mathrm{dmdm} = -\vec{\nabla}_R\frac{\vec{m}\vec{m}' - 3(\vec{m}\hat{R})(\vec{m}'\hat{R})}{R^3}$$

$$\text{loop axis: } \vec{B} = \hat{z}\frac{2\pi R^2}{(z^2 + R^2)^{3/2}}\frac{I}{c}$$

Electromagnetism (CGS)

Faraday: $\mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$; $\int \mathrm{d}^3x \vec{J} = \dot{\vec{p}}$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = 4\pi\frac{\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$$

$$\mathrm{d}\vec{F} = \mathrm{d}^3x\big(\rho\vec{E} + \frac{\vec{J}}{c}\times\vec{B}\big);\ \vec{F}_q = q\big(\vec{E} + \frac{\dot{\vec{r}}}{c}\times\vec{B}\big)$$

$$u = \frac{E^2+B^2}{8\pi};\ \vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B};\ \vec{g} = \frac{\vec{S}}{c^2}$$

$$\mathbf{T}^E = \frac{1}{4\pi}\big(\vec{E}\otimes\vec{E} - \frac{1}{2}E^2\big);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ -\frac{\partial\vec{g}}{\partial t} = \rho\vec{E} + \frac{\vec{J}}{c}\times\vec{B} - \vec{\nabla}\mathbf{T}$$

$$\vec{B} = \nabla\times\vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\big(\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\big) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$

$$(\phi,\vec{A}) \cong \big(\phi - \frac{1}{c}\frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\big)$$

$$(\phi,\vec{A}) = \int \mathrm{d}^3r' \frac{\big(\rho,\frac{\vec{J}}{c}\big)\big(\vec{r}', t - \frac{1}{c}\left|\vec{r}-\vec{r}'\right|\big)}{|\vec{r}-\vec{r}'|}$$

E.M. in matter (CGS)

$$\vec{\nabla}\vec{D} = 4\pi\rho_{\mathrm{ext}};\ \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla}\times\vec{H} = 4\pi\frac{\vec{J}_{\mathrm{ext}}}{c} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

$$\vec{P} = \frac{\mathrm{d}\langle\vec{p}\rangle}{\mathrm{d}V};\ \vec{M} = \frac{\mathrm{d}\langle\vec{m}\rangle}{\mathrm{d}V}$$

$$\rho_{\mathrm{pol}} = -\vec{\nabla}\vec{P};\ \sigma_{\mathrm{pol}} = \hat{n}\vec{P};\ \frac{\vec{J}_{\mathrm{mag}}}{c} = \vec{\nabla}\times\vec{M}$$

$$\vec{D}_{\mathrm{pol}} = \vec{E} + 4\pi\vec{P};\ \vec{H}_{\mathrm{mag}} = \vec{B} - 4\pi\vec{M}$$

static linear isotropic: $\vec{P} = \chi\vec{E}$

static linear: $P_i = \chi_{ij}E_j$

static linear: $\varepsilon = 1 + 4\pi\chi$

static: $\Delta D_\perp = 4\pi\sigma_{\mathrm{ext}}$; $\Delta E_\parallel = 0$

static linear: $u = \frac{1}{8\pi}\vec{E}\vec{D}$

$$\Delta U_{\mathrm{dielectric}} = -\frac{1}{2}\int \mathrm{d}^3r \vec{P}\vec{E}_0$$

plane capacitor: $C = \frac{\varepsilon}{4\pi}\frac{S}{d}$

cilindric capacitor: $C = \frac{L}{2\log\frac{R}{r}}$

atomic polarizability: $\vec{p} = \alpha\vec{E}$

Quantum mechanics (CGS)

$$r_B = \frac{\hbar^2}{m_e e^2} = 5.292\cdot 10^{-11}\,\mathrm{m}$$

$$\text{Rydberg} = \frac{e^2}{2r_B} = 13.61\,\mathrm{eV}$$

$$r_e = \frac{e^2}{mc^2} = 2.818\cdot 10^{-15}\,\mathrm{m}$$

$$\vec{\nabla}\vec{A} = 0 \to \Box\vec{A} = \frac{4\pi}{c}(\vec{J} - \vec{J}_L) =: \frac{4\pi}{c}\vec{J}_T$$

$$\vec{J}_L = \frac{1}{4\pi}\vec{\nabla}\frac{\partial\phi}{\partial t} = -\frac{1}{4\pi}\vec{\nabla}\int\frac{\vec{\nabla}'\vec{J}'}{|\vec{x}-\vec{x}'|}\mathrm{d}^3x'$$

$$\vec{E}'_\parallel = \vec{E}_\parallel;\ \vec{B}'_\parallel = \vec{B}_\parallel$$

$$\vec{E}'_\perp = \gamma\big(\vec{E}_\perp + \frac{\vec{v}}{c}\times\vec{B}\big)$$

$$\vec{B}'_\perp = \gamma\big(\vec{B}_\perp - \frac{\vec{v}}{c}\times\vec{E}\big)$$

$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)} \\ \vec{B} = \hat{k}\times\vec{E} \\ \omega = ck \end{cases}$$

$$\vec{B}_{\mathrm{diprad}} = \frac{1}{c^2}\frac{\ddot{\vec{p}}\times\hat{r}}{r}\Big|_{t_{\mathrm{rit}}};\ \vec{E}_{\mathrm{diprad}} = \vec{B}_{\mathrm{diprad}}\times\hat{r}$$

Larmor: $P = \frac{2}{3c^3}|\ddot{\vec{p}}|^2$

Rel. Larmor: $P = \frac{2}{3c^3}q^2\gamma^6(a^2 - (\vec{a}\times\vec{\beta})^2)$

$$\vec{A}_{\mathrm{dm}} = \frac{1}{c}\frac{\dot{\vec{m}}\times\hat{r}}{r}\Big|_{t_{\mathrm{rit}}}$$

L.W.: $(\phi,\vec{A}) = \frac{q(1,\frac{\vec{v}}{c})}{[r - \frac{\vec{v}\vec{r}}{c}]_{t_{\mathrm{rit}}}}$; $t_{\mathrm{rit}} = t - \frac{r}{c}\Big|_{t_{\mathrm{rit}}}$

non-interacting gas: $\vec{p} = \alpha\vec{E}_0$; $\chi = n\alpha$

hom. cubic isotropic: $\chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}n\alpha$

perm. dipole: $\chi = \frac{1}{3}\frac{np_0^2}{kT}$

local field: $\vec{E}_{\mathrm{loc}} = \vec{E} + \frac{4\pi}{3}\vec{P}$

$$\vec{J}\vec{E} = -\vec{\nabla}\Big(\frac{c}{4\pi}\vec{E}\times\vec{H}\Big) - \frac{1}{4\pi}\Big(\vec{E}\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t}\Big)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\frac{\omega}{c}$$

plane wave: $B = nE$

$$\vec{J}_{\mathrm{c}} = \sigma\vec{E};\ \varepsilon_\sigma = 1 + i\frac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm{p}}^2 = 4\pi\frac{nq^2}{m};\ \omega_{\mathrm{cyclo}} = \frac{qB}{mc}$$

I: $u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$

I: $\langle S_z\rangle = \frac{c}{n}\langle u\rangle$

II: $u = \frac{1}{8\pi}\big(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2 + \frac{\partial}{\partial\omega}(\mu\omega)H^2\big)$

II: $\langle S_z\rangle = v_{\mathrm{g}}\langle u\rangle$; $v_{\mathrm{g}} = \frac{\partial\omega}{\partial k} = \frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$

III: $\langle W\rangle = \frac{\omega}{4\pi}\big(\operatorname{Im}\varepsilon\langle E^2\rangle + \operatorname{Im}\mu\langle H^2\rangle\big)$

$$E_B = -\frac{1}{n^2}\frac{e^2}{2r_B}$$

$$\alpha = \frac{e^2}{\hbar c}$$

Planck: $\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$

$$\sigma_1 = \left(\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}\right);\ \sigma_2 = \left(\begin{smallmatrix}0 & -i\\ i & 0\end{smallmatrix}\right);\ \sigma_3 = \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$$

$$A^\mu = (\phi,\vec{A});\ J^\mu = (c\rho,\vec{J})$$

Lorenz gauge: $\partial_\alpha A^\alpha = 0$

Temporal gauge: $\phi = 0$

Axial gauge: $A_3 = 0$

Coulomb gauge: $\vec{\nabla}\vec{A} = 0$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu} = 0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0;\ \det F = (\vec{E}\vec{B})^2$$

$$F^{\alpha\beta}F_{\alpha\beta} = 2(B^2 - E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta} = 4\vec{E}\vec{B}$$

$$\Theta^{\mu\nu} = \frac{1}{4\pi}\big(F^\mu{}_\alpha F^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\big)$$

$$\Theta^{\mu\nu} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T}\end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu} = \frac{J_\alpha}{c}F^{\alpha\nu}(-?)$$

$$\mathcal{L} = \frac{mc^2}{\gamma} - q\vec{A}\frac{\vec{v}}{c} + q\phi$$

Fresnel TE (S): $\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}} = \frac{2}{1+\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\mathrm{r}}}{E_{\mathrm{i}}} = \frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}} = \frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\mathrm{r}}}{E_{\mathrm{i}}} = \frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz} = \pm\sqrt{\varepsilon_2\big(\frac{\omega}{c}\big)^2 - k_x^2}$, $\operatorname{Im}k_{tz} > 0$

Drüde-Lorentz: $\varepsilon = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$$P(t) = \int_{-\infty}^{\infty} g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega) = \chi(\omega)E(\omega)$$

$$\chi(\omega) = \int_{-\infty}^{\infty} e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega) = \overline{\chi}(\omega)$$

$$g(t<0) = 0 \implies$$

$$\operatorname{Re}\varepsilon(\omega) = 1 + \frac{2}{\pi}\fint_0^\infty \frac{\omega'\big(\operatorname{Im}\varepsilon(\omega') - \frac{4\pi\sigma_0}{\omega'}\big)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\operatorname{Im}\varepsilon(\omega) = -\frac{2\omega}{\pi}\fint_0^\infty \frac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega' + \frac{4\pi\sigma_0}{\omega}$$

sum rule: $\frac{\pi}{2}\omega_{\mathrm{p}}^2 = \int_0^\infty \omega\operatorname{Im}\varepsilon\mathrm{d}\omega$

sum rule: $2\pi^2\sigma_0 = \int_0^\infty(1 - \operatorname{Re}\varepsilon)\mathrm{d}\omega$

sum rule: $\int_0^\infty(\operatorname{Re}n - 1)\mathrm{d}\omega = 0$

Miller rule: $\chi^{(2)}(\omega,\omega) \propto \chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$

Schrödinger: $i\hbar\frac{\partial\psi}{\partial t} = H\psi$

$$U(t) = e^{-\frac{iHt}{\hbar}};\ U^\dagger = U^{-1}$$

$$H = H_0 + V_\lambda : \frac{\partial E_n(\lambda)}{\partial\lambda} = \langle\psi_n(\lambda)|\frac{\partial V_\lambda}{\partial\lambda}|\psi_n(\lambda)\rangle$$

$$[A,BC] = [A,B]C + B[A,C]$$

$$[P,Q] = \frac{\hbar}{i}$$

Nuclear physics (MKSA)

$$M(A,Z) = Zm_{\mathrm{p}} + (A-Z)m_{\mathrm{n}} - B(A,Z)$$

$$B(A,Z) = a_vA - a_sA^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_{\mathrm{sym}}\frac{(A-2Z)^2}{A} + a_pA^{-3/4}\Delta$$

$$\Delta = \begin{cases} 0 & & A \text{ odd} \\ 1 & Z \text{ even} & A \text{ even} \\ -1 & Z \text{ odd} & A \text{ even} \end{cases}$$

$$a_v = 15.5;\ a_s = 16.8;\ a_c = 0.72;\ a_{\mathrm{sym}} = 23;\ a_p = 34\ [\mathrm{MeV}]$$

$$\frac{\partial M}{\partial Z} = 0 : Z = \frac{m_{\mathrm{n}} - m_{\mathrm{p}} + 4a_{\mathrm{sym}}}{\frac{2a_c}{A^{1/3}} + \frac{8a_{\mathrm{sym}}}{A}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \Big|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\Big|$$

$$s_{ab} := |p_a^\mu + p_b^\mu|^2$$

$$M \to abc : (m_a + m_b)^2 \le s_{ab} \le (M - m_c)^2$$

$$M \to abc : s_{ab} + s_{bc} + s_{ac} = M^2 + m_a^2 + m_b^2 + m_c^2$$

$$a_iA_i \to b_jB_j : Q := (a_im_{A_i} - b_jm_{B_j})c^2$$