Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\frac{b}{a}\big)$$
$$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\frac{a}{b}\big)$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$

$$\begin{pmatrix}\sinh x\\ \cosh x\end{pmatrix}=\frac{1}{2}\begin{pmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{pmatrix}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\cosh^2 x=\frac{1}{1-\tanh^2 x}$$
$$\sin x=-i\sinh(ix)$$

$$\cos x=\cosh(ix)$$
$$\operatorname{asinh}x=\log\big(x+\sqrt{x^2+1}\big)$$
$$\operatorname{acosh}x=\log\big(x+\sqrt{x^2-1}\big)$$
$$\operatorname{atanh}x=\frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\big(I+\frac{B}{2}-1\big)\,A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\sum_{x=1}^n x^3=\Big(\sum_{x=1}^n x\Big)^2=\frac{1}{4}n^2(n+1)^2$$

$$\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\int_0^\infty t^z e^{-t}dt=z!$$
$$n!=\big(\tfrac{n}{e}\big)^n\sqrt{2\pi n}$$

$$\text{Fourier: } c_n=\frac{2}{T}\int_0^T f(t)\cos\big(n\tfrac{t}{T}\big)\mathrm{d}t$$
$$F[f]=\hat f(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ikx}f(x)$$
$$\langle\hat f|\hat g\rangle=\langle f|g\rangle$$
$$F\Big[\frac{\sin x}{x}\Big]=\sqrt{\frac{\pi}{2}}\chi_{[-1;1]}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$

Derivatives

$$\tan' x=1+\tan^2 x$$
$$\cot' x=-1-\cot^2 x$$
$$\operatorname{atan}' x=-\operatorname{acot}' x=\frac{1}{1+x^2}$$
$$\operatorname{asin}' x=-\operatorname{acos}' x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a' x=\frac{1}{x\ln a}$$
$$\cosh' x=\sinh x$$
$$\tanh' x=1-\tanh^2 x$$
$$\operatorname{atanh}' x=\operatorname{acoth}' x=\frac{1}{1-x^2}$$
$$\operatorname{asinh}' x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}' x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$
$$\Big(\frac{1}{x}\Big)'=-\frac{\dot x}{x^2}$$
$$\Big(\frac{x}{y}\Big)'=\frac{\dot x y-x\dot y}{y^2}$$
$$(x^y)'=x^y\big(\dot y\ln x+y\tfrac{\dot x}{x}\big)$$

Integrals

$$\int x^a=\frac{x^{a+1}}{a+1}\qquad\int a^x=\frac{a^x}{\ln a}$$
$$\int\frac{1}{x}=\ln|x|\qquad\int\tan x=-\ln|\cos x|\qquad\int\cot x=\ln|\sin x|$$
$$\int\frac{1}{\sin x}=\ln\big|\tan\tfrac{x}{2}\big|\qquad\int\frac{1}{\cos x}=\ln\big|\tan\big(\tfrac{x}{2}+\tfrac{\pi}{4}\big)\big|\qquad\int\ln x=x(\ln x-1)$$
$$\int\tanh x=\ln\cosh x\qquad\int\coth x=\ln|\sinh x|\qquad\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}\qquad\int xy=x\int y-\int\big(\dot x\int y\big)\qquad\int e^{yx}x=e^{yx}\big(\tfrac{y}{x}-\tfrac{1}{y^2}\big)$$

Differential equations

$$\dot x+\dot a x=b:x=e^{-a}\left(\int be^a+c_1\right)$$
$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot x=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot x+\gamma\dot x+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$
$$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$
$$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+\mathrm{O}(x^9)$$

$$\operatorname{acos}x=\frac{\pi}{2}-\operatorname{asin}x$$
$$\operatorname{atan}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$
$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$
$$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$

$$\vec a\times\vec b=\varepsilon_{ijk}a_jb_k\hat e_i$$
$$(\vec a\otimes\vec b)_{ij}=a_ib_j$$
$$(\vec a\times\vec b)\vec c=(\vec c\times\vec a)\vec b$$
$$(\vec a\times\vec b)\times\vec c=-(\vec b\vec c)\vec a+(\vec a\vec c)\vec b$$

$$(\vec a\times\vec b)(\vec c\times\vec d)=(\vec a\vec c)(\vec b\vec d)-(\vec a\vec d)(\vec b\vec c)$$
$$|\vec u\times\vec v|^2=u^2v^2-(\vec u\vec v)^2$$
$$\vec\nabla=\big(\tfrac{\partial}{\partial x},\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}\big);\ \Box=\frac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec\nabla V=\frac{\partial V}{\partial\rho}\hat\rho+\frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat\phi+\frac{\partial V}{\partial z}\hat z$$

$$\vec\nabla\vec v = \frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

$$\vec\nabla\times\vec v = \left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat\rho +$$

$$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right)\hat\phi + \frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\phi}\right)$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$

$$\vec\nabla\times\vec v = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\right)\hat r +$$

$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(r v_\varphi)}{\partial r}\right)\hat\theta + \frac{1}{r}\left(\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat\varphi$$

$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\vec\nabla\left(\vec\nabla\times\vec v\right) = \vec\nabla\times\vec\nabla\vec V = 0$$

$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$

$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$

$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$

$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$

$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\,\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\,\vec\nabla)\vec w$$

$$\frac{1}{2}\vec\nabla v^2 = (\vec v\,\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$

$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$$

$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$

$$\delta(\vec r - \vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$

$$\nabla^2\frac{1}{|\vec r - \vec r_0|} = -4\pi\delta(\vec r - \vec r_0)$$

$$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i) = 0$$

$$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle = \frac{1}{2}\mathrm{Re}(a\bar b)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$

$$\Delta x_{\text{hist}} \approx \frac{x_{\max} - x_{\min}}{\sqrt N}$$

$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$

$$\text{median} = F^{-1}\left(\frac{1}{2}\right)$$

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$

$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$

$$\alpha_n = E[x^n]$$

$$M_n = E[(x-\mu)^n]$$

$$\sigma^2 = M_2 = E[x^2] - \mu^2$$

$$\mathrm{FWHM} \approx 2\sigma$$

$$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$

$$\phi[y_1 + \lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$

$$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$

$$h\ge 0 : P(h\ge k) \le \frac{E[h]}{k}$$

$$P(|x-\mu| > k\sigma) \le \frac{1}{k^2}$$

$$B(n,p,k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$

$$P(\mu,k) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$

$$u(x,a,b) = \frac{1}{b-a},\ x\in[a;b]$$

$$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$$

$$\varepsilon(x,\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \frac{1}{\lambda},\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$$

$$g(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$\mathrm{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$

$$z = \frac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$

$$\chi^2 = \sum_{i=1}^n z_i^2$$

$$\wp_n(x) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$

$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$

$$n\ge 30 : \wp_n(x) \approx g(x,n,\sqrt{2n})$$

$$n\ge 8 : p[\sqrt{2\chi^2}] \approx g(,\sqrt{2n-1},1)$$

$$S(x,n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

$$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$$

$$p\left[z\sqrt{\frac{n}{\chi^2}}\right] = S(,n)$$

$$n\ge 35 : S(x,n) \approx g(x,0,1)$$

$$c(x,a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$$

$$\sigma_{xy} = E[xy] - \mu_x\mu_y \le \sigma_x\sigma_y$$

$$\rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le 1$$

$$\mu[f(x_1,\ldots)] \approx f(\mu_1,\ldots)$$

$$\sigma^2[f(x_1,\ldots)] \approx \sigma_{x_i x_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$

$$\mu \approx m = \frac{1}{n}\sum_{i=1}^n x_i$$

$$\sigma^2 \approx s^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - m)^2$$

$$s_m^2 = \frac{s^2}{n}$$

$$p\left[\frac{m-\mu}{s_m}\right] = S(,n)$$

Fit

$$f(x) = mx + q,\quad f(x) = a$$

$$f(x) = bx$$

$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$

$$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

Kinematics

$$\frac{1}{R} = \left|\frac{v_x a_y - v_y a_x}{v^3}\right|$$

$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$$

$$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$$

$$\theta\equiv\frac{\pi}{2} \to \dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$

$$\theta\equiv\frac{\pi}{2} \to \ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi$$

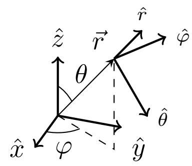
$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$

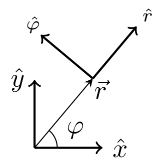
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$

$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$

$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$

$$\vec A = \ddot{\vec r} + \vec A_{\mathrm T} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta + \frac{\partial\alpha}{\partial t}$$

$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec F\vec p$$

$$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$$

$$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}m_i(\dot{\vec r}_i - \dot{\vec R})^2$$

$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$$

$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$

$$\tau_1 = I_1\dot\omega_1 + (I_3 - I_2)\omega_3\omega_2$$

$$\mathcal L(q,\dot q,t) = T - V + \frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$

$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm{d}t$$

$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$

$$p := \frac{\partial\mathcal L}{\partial\dot q};\ \dot p = \frac{\partial\mathcal L}{\partial q}$$

$$\mathcal H(q,p,t) = \dot q p - \mathcal L$$

$$\dot q = \frac{\partial\mathcal H}{\partial p};\ \dot p = -\frac{\partial\mathcal H}{\partial q}$$

$$\frac{\mathrm{d}\mathcal H}{\mathrm{d}t} = \frac{\partial\mathcal H}{\partial t} = -\frac{\partial\mathcal L}{\partial t}$$

$$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal H\} + \frac{\partial u}{\partial t}$$

$$\eta = (q,p);\ \Gamma = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$\dot\eta = \Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac{1}{2}mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac{2}{3}mr^2$

ball: $\frac{2}{5}mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2 + \frac{3}{4}r^2\right)$

ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle \approx -2\langle T\rangle$$

$$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$$

$$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}\mu\dot{\vec r}^2$$

$$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$$

$$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}}$$

$$r = \frac{k}{1+\varepsilon\cos\theta}$$

$$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$$

$$a^3\omega^2 = G(m_1+m_2) = \frac{\alpha}{\mu}$$

$$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$$|a| - |b| \le |a+b| \le |a| + |b|$$

$$x > -1 : 1 + nx \le (1+x)^n$$

$$\frac{|a^n - b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$$

$$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$$

$$\sum a_i b_i \le \left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$$

$$x^p y^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$$

$$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$$

$$\sum\left(\frac{a_1 + \ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p\sum a_i^p$$

$$x\ge 0, |\ddot x| \le M : |\dot x| \le \sqrt{2Mx}$$

$$\frac{1}{1+x} < \ln\left(1 + \frac{1}{x}\right) < \frac{1}{x}$$

Vector spaces

$(V, \mathbb{K}, +, \cdot)$ vector space; $\mathbb{K}$ field

$\exists \vec{0} \in V : \vec{v} + \vec{0} = \vec{v}$

$\cdot : \mathbb{K} \times V \to V; \quad \lambda \cdot (\vec{v} + \vec{w}) = \lambda\vec{v} + \lambda\vec{w}$

$0_{\mathbb{K}} \cdot \vec{v} = \vec{0},\ 1_{\mathbb{K}} \cdot \vec{v} = \vec{v}$

$\lambda \in \mathbb{K},\ \vec{v}, \vec{w} \in V \ \Rightarrow\ \vec{v} + \vec{w} \in V,\ \lambda\vec{v} \in V$

$\dim(U+V) = \dim U + \dim V - \dim(U \cap V)$

ℓ linear : $\ell(\vec{v} + \lambda\vec{w}) = \ell(\vec{v}) + \lambda\ell(\vec{w})$

$\ker \ell = \{\vec{v} \in V \mid \ell(\vec{v}) = 0\}$

$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$

$\langle , \rangle : V \times V \to \mathbb{K}; \quad \langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$

$\langle \vec{v} + \lambda\vec{w}, \vec{u} \rangle = \langle \vec{v}, \vec{u} \rangle + \lambda \langle \vec{w}, \vec{u} \rangle$

$\| \| : V \to \mathbb{K}; \quad \|\vec{v}\| = 0 \to \vec{v} = \vec{0}$

$\|\lambda\vec{v}\| = |\lambda| \|\vec{v}\|; \quad \|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\ \frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_{\mathrm{A}} = 6.022 \cdot 10^{23}\ \frac{1}{\mathrm{mol}}$

$k_{\mathrm{B}} = 1.381 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$

$q_{\mathrm{e}} = 1.602 \cdot 10^{-19}\ \mathrm{A\,s}$

$m_{\mathrm{e}} = 9.109 \cdot 10^{-31}\ \mathrm{kg}$

$m_{\mathrm{p}} = 1.673 \cdot 10^{-27}\ \mathrm{kg}$

$m_{\mathrm{n}} = 1.675 \cdot 10^{-27}\ \mathrm{kg}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\ \mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{\mathrm{C}^2\,\mathrm{s}^2}{\mathrm{kg\,m}^3}$

$\mu_0 = 1.257 \cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_{\mathrm{B}} = 9.274 \cdot 10^{-24}\ \mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{eV} = 1 \cdot 10^{-12}\ \mathrm{erg}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H =$ heat transfer

$G = H - TS$

$a_i \mathrm{A}_i \to b_j \mathrm{B}_j$

$\Delta H^{\mathrm{o}}_{\mathrm{r}} = b_j \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{B}_j) - a_i \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{A}_i)$

$\forall\, i, j : v_{\mathrm{r}} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_{\mathrm{r}} = k[\mathrm{A}_i]^{m_i}$

$k = Ae^{-\frac{E_{\mathrm{a}}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma \frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_{\mathrm{tot}}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_{\mathrm{tot}}^{\sum a_i - \sum b_j}$

$\Delta G^{\mathrm{o}}_{\mathrm{r}} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln \frac{K_2}{K_1} = -\frac{\Delta H^{\mathrm{o}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_{\mathrm{w}} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^{\mathrm{o}} - \frac{RT}{n_{\mathrm{e}} N_A q_{\mathrm{e}}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^{\mathrm{o}} - \frac{0.059}{n_{\mathrm{e}}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$

$\mathrm{d}L = p\mathrm{d}V$

$\mathrm{d}S = \frac{\mathrm{d}Q}{T}$ $\quad C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V$ $\quad C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p$ $\quad \gamma = \frac{C_p}{C_V}$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma - 1},\ c_p = \frac{\gamma}{\gamma - 1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int \mathrm{d}q_1 \cdots \mathrm{d}q_N \int \mathrm{d}p_1 \cdots \mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_{\mathrm{B}}T};\ C = \frac{\partial U}{\partial T}$

$F(T, V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0} e^{i\omega t}$

$Z = \frac{V}{I}$ $\quad Z_R = R$

$Z_C = -i\frac{1}{\omega C}$ $\quad Z_L = i\omega L$

$Z_{\mathrm{series}} = \sum_k Z_k$ $\quad \frac{1}{Z_{\mathrm{parallel}}} = \sum_k \frac{1}{Z_k}$

$\sum_{\mathrm{loop}} V_k = 0$ $\quad \sum_{\mathrm{node}} I_k = 0$

$\mathcal{E} = -L\dot{I}$ $\quad L = \frac{\Phi_B}{I}$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec{p} = \gamma m\vec{v}$

$\mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \mathcal{E}$

$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$

$\binom{ct'}{x'} = \gamma\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}\binom{ct}{x}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{(1 - \frac{V^2}{c^2})(1 - \frac{v^2}{c^2})}{\left(1 - \frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct, \vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu \partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \to \sum_i m_i$

$E_1^{\max} = \frac{M^2 + m_1^2 - \sum_{i\neq 1} m_i^2}{2M}c^2$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\times\vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int \vec{E}_i\vec{E}_j\,\mathrm{d}^3x$

$V(\vec{r}) = \int \rho G_{\mathrm{D}}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_{\mathrm{D}}}{\partial n}\,\mathrm{d}S$

$V(\vec{r}) = \langle V\rangle_S + \int \rho G_{\mathrm{N}}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n}G_{\mathrm{N}}(\vec{r})\,\mathrm{d}S$

$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_{\mathrm{D}}(\vec{x}, \vec{y})|_{\vec{y}\in S} = 0;\ \frac{\partial G_{\mathrm{N}}}{\partial n}\Big|_{\vec{y}\in S} = -\frac{4\pi}{S}$

$U_{\mathrm{sphere}} = \frac{3}{5}\frac{Q^2}{R};\ \vec{p} = \int \mathrm{d}^3 r\rho\vec{r};\ \vec{E}_{\mathrm{dip}} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_{\mathrm{dip}} = \frac{\vec{p}\hat{r}}{r^2}$

$Q_{ij} = \int \mathrm{d}^3 r\rho(\vec{r})(3r_i r_j - \delta_{ij}r^2);\ V_{\mathrm{quad}} = \frac{1}{6r^5}Q_{ij}(3r_i r_j - \delta_{ij}r^2)$

$V(r, \theta) = \sum_{l=0}^\infty \left(A_l r^l - \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$

$V(r, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l \left(A_{lm}r^l + \frac{B_{lm}}{r^{l+1}}\right)Y_{lm}(\theta, \varphi)$

$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r, r')^l}{\max(r, r')^{l+1}}P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$

$P_l(1) = 1;\ \langle P_n | P_m\rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm} | Y_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}$

$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l,\ |m| \leq l$

$Y_{lm}(\theta,\varphi)=\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m}=(-1)^m\overline{Y}_{lm}$

$P_l\left(\frac{\vec r\vec r'}{rr'}\right)=\frac{4\pi}{2l+1}\sum_{m=-l}^{l}\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$

$V(r>\operatorname{diam}\operatorname{supp}\rho,\theta,\varphi)=\sum_{l=0}^{\infty}\frac{4\pi}{2l+1}\sum_{m=-l}^{l}q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$

$q_{lm}[\rho]=\int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\,r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$

Magnetostatics (CGS)

$\vec\nabla\vec J=-\frac{\partial\rho}{\partial t}=0;\ I=\int\vec J\mathrm{d}\vec S$

solenoid: $B=4\pi\frac{j_s}{c}$

$\mathrm{d}\vec F=\frac{I\mathrm{d}\vec l}{c}\times\vec B=\mathrm{d}^3x\frac{\vec J}{c}\times\vec B;\ \vec F_q=q\frac{\dot{\vec r}}{c}\times\vec B$

$\mathrm{d}\vec B=\frac{I\mathrm{d}\vec l}{c}\times\frac{\vec r}{r^3};\ \vec B_q=q\frac{\dot{\vec r}}{c}\times\frac{\vec r}{r^3}$

$\vec B=\vec\nabla\times\vec A;\ \vec A=\int\mathrm{d}^3r'\frac{\vec J'}{c}\frac{1}{|\vec r-\vec r'|}+\vec\nabla A_0$

$\vec B=\int\mathrm{d}^3r'\frac{\vec J'}{c}\times\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$

$\varphi=\frac{I}{c}\Omega,\ \vec B=-\vec\nabla\varphi$

$\vec\nabla\vec A=0\to\nabla^2\vec A=-4\pi\frac{\vec J}{c}$

$\vec\nabla\vec B=0;\ \vec\nabla\times\vec B=4\pi\frac{\vec J}{c};\ \oint\vec B\mathrm{d}\vec l=4\pi\frac{I}{c}$

$\vec m=\frac12\int\mathrm{d}^3r'\left(\vec r'\times\frac{\vec J'}{c}\right)=\frac{1}{2c}\frac{q}{m}\vec L=\frac{SI}{c}$

$\vec A_{\mathrm{dm}}=\frac{\vec m\times\vec r}{r^3};\ \vec\tau=\vec m\times\vec B$

$\vec F_{\mathrm{dmdm}}=-\vec\nabla_R\frac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$

loop axis: $\vec B=\hat z\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac{I}{c}$

Electromagnetism (CGS)

Faraday: $\mathcal{E}=-\frac1c\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int\mathrm{d}^3x\vec J=\dot{\vec p}$

$\vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t};\ \vec\nabla\vec E=4\pi\rho;\ \vec\nabla\vec J=-\frac{\partial\rho}{\partial t}$

$\vec\nabla\times\vec B=4\pi\frac{\vec J}{c}+\frac1c\frac{\partial\vec E}{\partial t};\ \vec\nabla\vec B=0$

$\mathrm{d}\vec F=\mathrm{d}^3x\left(\rho\vec E+\frac{\vec J}{c}\times\vec B\right);\ \vec F_q=q\left(\vec E+\frac{\dot{\vec r}}{c}\times\vec B\right)$

$u=\frac{E^2+B^2}{8\pi};\ \vec S=\frac{c}{4\pi}\vec E\times\vec B;\ \vec g=\frac{\vec S}{c^2}$

$\mathbf{T}^E=\frac{1}{4\pi}\left(\vec E\otimes\vec E-\frac12E^2\right);\ \mathbf{T}=\mathbf{T}^E+\mathbf{T}^B$

$-\frac{\partial u}{\partial t}=\vec J\vec E+\vec\nabla\vec S;\ -\frac{\partial\vec g}{\partial t}=\rho\vec E+\frac{\vec J}{c}\times\vec B-\vec\nabla\mathbf{T}$

$\vec B=\vec\nabla\times\vec A;\ \vec E=-\vec\nabla\phi-\frac1c\frac{\partial\vec A}{\partial t}$

$-\nabla^2\phi-\frac1c\frac{\partial}{\partial t}\vec\nabla\vec A=4\pi\rho$

$\vec\nabla\left(\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}\right)-\nabla^2\vec A+\frac1c\frac{\partial^2\vec A}{\partial t^2}=4\pi\frac{\vec J}{c}$

$(\phi,\vec A)\cong\left(\phi-\frac1c\frac{\partial\chi}{\partial t},\vec A+\vec\nabla\chi\right)$

$(\phi,\vec A)=\int\mathrm{d}^3r'\frac{\left(\rho,\frac{\vec J}{c}\right)\left(\vec r',t-\frac1c|\vec r-\vec r'|\right)}{|\vec r-\vec r'|}$

Coulomb gauge: $\vec\nabla\vec A=0$

Lorenz gauge: $\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}=0$

$\vec E'_\parallel=\vec E_\parallel;\ \vec B'_\parallel=\vec B_\parallel$

$\vec E'_\perp=\gamma\left(\vec E_\perp+\frac{\vec v}{c}\times\vec B\right)$

$\vec B'_\perp=\gamma\left(\vec B_\perp-\frac{\vec v}{c}\times\vec E\right)$

plane wave: $\begin{cases}\vec E=\vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B=\hat k\times\vec E\\ \omega=ck\end{cases}$

$\vec B_{\mathrm{diprad}}=\frac{1}{c^2}\frac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}};\ \vec E_{\mathrm{diprad}}=\vec B_{\mathrm{diprad}}\times\hat r$

Larmor: $P_{\mathrm{diprad}}=\frac{2}{3c^3}|\ddot{\vec p}|^2$

$\vec A_{\mathrm{dm}}=\frac1c\frac{\dot{\vec m}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}}$

L.W.: $(\phi,\vec A)=\frac{q\left(1,\frac{\vec v}{c}\right)}{\left[r-\frac{\vec v\vec r}{c}\right]_{t_{\mathrm{rit}}}};\ t_{\mathrm{rit}}=t-\frac{r}{c}\Big|_{t_{\mathrm{rit}}}$

$A^\mu=(\phi,\vec A);\ J^\mu=(c\rho,\vec J)$

Lorenz gauge: $\partial_\alpha A^\alpha=0$

$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$

$F^{\mu\nu}=\begin{pmatrix}0&-E_x&-E_y&-E_z\\E_x&0&-B_z&B_y\\E_y&B_z&0&-B_x\\E_z&-B_y&B_x&0\end{pmatrix}$

$\mathscr{F}^{\mu\nu}=\frac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$

$\partial_\alpha F^{\alpha\nu}=4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu}=0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=qF^{\mu\alpha}v_\alpha$

$F^{\alpha\beta}F_{\alpha\beta}=2(B^2-E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta}=4\vec E\vec B$

$\Theta^{\mu\nu}=\frac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu}+\frac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$

$\Theta^{\mu\nu}=\begin{pmatrix}u&c\vec g\\c\vec g&-\mathbf{T}\end{pmatrix}$

$\partial_\alpha\Theta^{\alpha\nu}=\frac1cJ_\alpha F^{\alpha\nu}$

E.M. in matter (CGS)

$\vec\nabla\vec D=4\pi\rho_{\mathrm{ext}};\ \vec\nabla\times\vec E=-\frac1c\frac{\partial\vec B}{\partial t}$

$\vec\nabla\vec B=0;\ \vec\nabla\times\vec H=4\pi\frac{\vec J_{\mathrm{ext}}}{c}+\frac1c\frac{\partial\vec D}{\partial t}$

$\vec P=\frac{\langle\vec p\rangle}{V};\ \vec M=\frac{\langle\vec m\rangle}{V}$

$\rho_{\mathrm{pol}}=-\vec\nabla\vec P;\ \sigma_{\mathrm{pol}}=\hat n\vec P;\ \frac{\vec J_{\mathrm{mag}}}{c}=\vec\nabla\times\vec M$

$\vec D_{\mathrm{pol}}=\vec E+4\pi\vec P;\ \vec H_{\mathrm{mag}}=\vec B-4\pi\vec M$

static linear isotropic: $\vec P=\chi\vec E$

static linear: $P_i=\chi_{ij}E_j$

static linear: $\varepsilon=1+4\pi\chi$

static: $\Delta D_\perp=4\pi\sigma_{\mathrm{ext}};\ \Delta E_\parallel=0$

static linear: $u=\frac{1}{8\pi}\vec E\vec D$

$\Delta U_{\mathrm{dielectric}}=\frac12\int\mathrm{d}^3r\vec P\vec E_0$

plane capacitor: $C=\frac{\varepsilon}{4\pi}\frac{S}{d}$

atomic polarizability: $\vec p=\alpha\vec E$

non-interacting gas: $\vec p=\alpha\vec E_0;\ \chi=n\alpha$

hom. cubic isotropic: $\chi=\frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi}{3}n\alpha$

$\chi=\frac{4\pi}{3}\frac{np_0^2}{kT};\ \vec E_e=\vec E+\frac{4\pi}{3}\vec P$

$\vec J\vec E=-\vec\nabla\left(\frac{c}{4\pi}\vec E\times\vec H\right)-\frac{1}{4\pi}\left(\vec E\frac{\partial\vec D}{\partial t}+\vec H\frac{\partial\vec B}{\partial t}\right)$

$n=\sqrt{\varepsilon\mu};\ k=n\frac{\omega}{c}$

plane wave: $B=nE$

$\vec J_c=\sigma\vec E;\ \varepsilon_\sigma=1+i\frac{4\pi\sigma}{\omega}$

$\omega_{\mathrm{p}}^2=4\pi\frac{nq^2}{m};\ \omega_{\mathrm{cyclo}}=\frac{qB}{mc}$

I: $u=\frac{1}{8\pi}(\vec E\vec D+\vec H\vec B)$

I: $\langle S_z\rangle=\frac{c}{n}\langle u\rangle$

II: $u=\frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon_\omega\omega)E^2+\frac{\partial}{\partial\omega}(\mu_\omega\omega)H^2\right)$

II: $\langle S_z\rangle=v_{\mathrm{g}}\langle u\rangle;\ v_{\mathrm{g}}=\frac{\partial\omega}{\partial k}$

III: $\langle W\rangle=\frac{\omega}{8\pi}\left(\operatorname{Im}\varepsilon_\omega\left|E_0^2\right|+\operatorname{Im}\mu_\omega\left|H_0^2\right|\right)$

Fresnel TE (S): $\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}}=\frac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm{r}}}{E_{\mathrm{i}}}=\frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}}=\frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm{r}}}{E_{\mathrm{i}}}=\frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz}=\pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2-k_x^2},\ \operatorname{Im}k_{tz}>0$