Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\tfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\tfrac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$
$$\tan\tfrac{\alpha}{2}=\tfrac{\sin\alpha}{1+\cos\alpha}=\tfrac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\tfrac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\tfrac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$
$$\operatorname{acos}x+\operatorname{asin}x=\tfrac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\tfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\left(\begin{smallmatrix}\sinh x\\ \cosh x\end{smallmatrix}\right)=\tfrac{1}{2}\left(\begin{smallmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{smallmatrix}\right)$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\cosh^2 x=\tfrac{1}{1-\tanh^2 x}$$
$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\left(\begin{smallmatrix}\operatorname{asinh}x\\ \operatorname{acosh}x\end{smallmatrix}\right)=\log\Big(x+\sqrt{x^2+\left(\begin{smallmatrix}1\\ -1\end{smallmatrix}\right)}\Big)$$
$$\operatorname{atanh}x=\tfrac{1}{2}\log\tfrac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\tfrac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\tbinom{n}{k}=\tfrac{n!}{k!(n-k)!}\qquad C'_{n,k}=\tbinom{n+k-1}{k}$$
$$D_{n,k}=\tfrac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\tfrac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\tfrac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\tfrac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{x=1}^n x^3=\left(\sum_{x=1}^n x\right)^2=\tfrac{1}{4}n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2=\tfrac{1}{6}n(n+1)(2n+1)$$
$$\tbinom{n}{k}=\tbinom{n-1}{k-1}+\tbinom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\textstyle\int_0^\infty t^z e^{-t}dt=z!$$
$$n!\approx(\tfrac{n}{e})^n\sqrt{2\pi n}$$
Fourier: $c_n=\frac{2}{T}\int_0^T f(t)\cos\big(n\frac{t}{T}\big)\mathrm{d}t$
$$F[f]=\hat{f}(k)=\tfrac{1}{\sqrt{2\pi}}\textstyle\int_{-\infty}^\infty e^{-ikx}f(x)$$
$$\langle\hat{f}|\hat{g}\rangle=\langle f|g\rangle$$
$$F\left[\tfrac{\sin x}{x}\right]=\sqrt{\tfrac{\pi}{2}}\chi_{[-1;1]}$$
$$\tfrac{\mathrm{d}}{\mathrm{d}x}\textstyle\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\tfrac{\operatorname{Re}z+|z|}{2}}+\tfrac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$
$$\delta(g(x))=\tfrac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\tfrac{1}{2}\operatorname{Re}(ab^*)$$
$$\operatorname{erf}(x)=\tfrac{2}{\sqrt{\pi}}\textstyle\int_0^x e^{-t^2}\mathrm{d}t$$
$$f^{(n)}(z_0)=\tfrac{n!}{2\pi i}\textstyle\oint\frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$
$$f(z)=\textstyle\sum_{k=-\infty}^\infty\left(\frac{1}{2\pi i}\oint\frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\right)(z-z_0)^k$$

Derivatives

$$\tan' x=1+\tan^2 x$$
$$\cot' x=-1-\cot^2 x$$
$$\operatorname{atan}' x=-\operatorname{acot}' x=\tfrac{1}{1+x^2}$$
$$\operatorname{asin}' x=-\operatorname{acos}' x=\tfrac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log'_a x=\tfrac{1}{x\ln a}$$
$$\cosh' x=\sinh x$$
$$\tanh' x=1-\tanh^2 x$$
$$\operatorname{atanh}' x=\operatorname{acoth}' x=\tfrac{1}{1-x^2}$$
$$\operatorname{asinh}' x=\tfrac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}' x=\tfrac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\tfrac{1}{f'(f^{-1})}$$
$$\left(\tfrac{1}{x}\right)'=-\tfrac{\dot{x}}{x^2}$$
$$\left(\tfrac{x}{y}\right)'=\tfrac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)'=x^y\big(\dot{y}\ln x+y\tfrac{\dot{x}}{x}\big)$$
$$\tfrac{\partial(x,y)}{\partial(u,v)}:=\tfrac{\partial x}{\partial u}\tfrac{\partial y}{\partial v}-\tfrac{\partial x}{\partial v}\tfrac{\partial y}{\partial u}$$
$$\frac{\partial(x,y)}{\partial(u,y)}=\tfrac{\partial x}{\partial u}\Big|_y=-\tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_x$$
$$\tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_x\tfrac{\partial u}{\partial x}\Big|_y=-1$$
$$\tfrac{\partial x}{\partial u}\Big|_y=\tfrac{\partial x}{\partial u}\Big|_v-\tfrac{\partial x}{\partial y}\Big|_u\tfrac{\partial y}{\partial u}\Big|_v$$
$$\tfrac{\partial x}{\partial u}\Big|_v=\tfrac{\partial x}{\partial y}\Big|_v\tfrac{\partial y}{\partial u}\Big|_v$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac{1}{x}=\ln|x|$$
$$\textstyle\int\tan x=-\ln|\cos x|$$
$$\textstyle\int\cot x=\ln|\sin x|$$
$$\textstyle\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$
$$\textstyle\int\ln x=x(\ln x-1)$$
$$\textstyle\int\tanh x=\ln\cosh x$$
$$\textstyle\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\textstyle\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy=x\int y-\int(\dot{x}\int y)$$
$$\textstyle\int_{-\infty}^\infty e^{-x^2}=\sqrt{\pi}$$
$$\textstyle\int_{-\infty}^\infty e^{i\omega t}\mathrm{d}t=2\pi\delta(\omega)$$

Differential equations

$$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$
$$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot{x}+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot{x}+\gamma\dot{x}+\omega_0^2x=fe^{-i\omega t}:x=\tfrac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac{1}{x}+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac{1}{x}-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$
$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac{1}{x}-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac{1}{x}+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac{5}{6}x^4-\tfrac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\left(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\right)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i=j \vee j=k \vee k=i \\ 1 & i+1\equiv j \wedge j+1\equiv k \\ -1 & i\equiv j+1 \wedge j\equiv k+1 \end{cases}$$

$$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

$$\vec a \times \vec b = \varepsilon_{ijk} a_j b_k \hat e_i$$

$$(\vec a \otimes \vec b)_{ij} = a_i b_j$$

$$(\vec a \times \vec b)\vec c = (\vec c \times \vec a)\vec b$$

$$(\vec a \times \vec b) \times \vec c = -(\vec b\vec c)\vec a + (\vec a\vec c)\vec b$$

$$(\vec a \times \vec b)(\vec c \times \vec d) = (\vec a\vec c)(\vec b\vec d) - (\vec a\vec d)(\vec b\vec c)$$

$$|\vec u \times \vec v|^2 = u^2v^2 - (\vec u\vec v)^2$$

$$\vec\nabla = \left(\tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z}\right);\ \Box = \tfrac{\partial^2}{\partial t^2} - \nabla^2$$

$$\vec\nabla V = \tfrac{\partial V}{\partial\rho}\hat\rho + \tfrac{1}{\rho}\tfrac{\partial V}{\partial\phi}\hat\phi + \tfrac{\partial V}{\partial z}\hat z$$

$$\vec\nabla\vec v = \tfrac{1}{\rho}\tfrac{\partial(\rho v_\rho)}{\partial\rho} + \tfrac{1}{\rho}\tfrac{\partial v_\phi}{\partial\phi} + \tfrac{\partial v_z}{\partial z}$$

$$\vec\nabla \times \vec v = \left(\tfrac{1}{\rho}\tfrac{\partial v_z}{\partial\phi} - \tfrac{\partial v_\phi}{\partial z}\right)\hat\rho + \\ +\left(\tfrac{\partial v_\rho}{\partial z} - \tfrac{\partial v_z}{\partial\rho}\right)\hat\phi + \tfrac{1}{\rho}\left(\tfrac{\partial(\rho v_\phi)}{\partial\rho} - \tfrac{\partial v_\rho}{\partial\phi}\right)\hat z$$

$$\nabla^2 V = \tfrac{1}{\rho}\tfrac{\partial}{\partial\rho}\left(\rho\tfrac{\partial V}{\partial\rho}\right) + \tfrac{1}{\rho^2}\tfrac{\partial^2 V}{\partial\phi^2} + \tfrac{\partial^2 V}{\partial z^2}$$

$$\vec\nabla V = \tfrac{\partial V}{\partial r}\hat r + \tfrac{1}{r}\tfrac{\partial V}{\partial\theta}\hat\theta + \tfrac{1}{r\sin\theta}\tfrac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v = \tfrac{1}{r^2}\tfrac{\partial(r^2 v_r)}{\partial r} + \tfrac{1}{r\sin\theta}\tfrac{\partial(v_\theta\sin\theta)}{\partial\theta} + \tfrac{1}{r\sin\theta}\tfrac{\partial v_\varphi}{\partial\varphi}$$

$$\vec\nabla \times \vec v = \tfrac{1}{r\sin\theta}\left(\tfrac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \tfrac{\partial v_\theta}{\partial\varphi}\right)\hat r + \\ + \tfrac{1}{r}\left(\tfrac{1}{\sin\theta}\tfrac{\partial v_r}{\partial\varphi} - \tfrac{\partial(r v_\varphi)}{\partial r}\right)\hat\theta + \tfrac{1}{r}\left(\tfrac{\partial(r v_\theta)}{\partial r} - \tfrac{\partial v_r}{\partial\theta}\right)\hat\varphi$$

$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\tfrac{1}{r^2}\tfrac{\partial}{\partial r}\left(r^2\tfrac{\partial V}{\partial r}\right) = \tfrac{1}{r}\tfrac{\partial^2}{\partial r^2}(rV) = \tfrac{2}{r}\tfrac{\partial V}{\partial r} + \tfrac{\partial^2 V}{\partial r^2}$$

$$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$$

$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$

$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$

$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$

$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$

$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\vec\nabla)\vec w$$

$$\tfrac{1}{2}\vec\nabla v^2 = (\vec v\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$

$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\tfrac{\partial g}{\partial n} - g\tfrac{\partial f}{\partial n}\right)\mathrm{d}S$$

$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$

$$\delta(\vec r - \vec r_0) = \tfrac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r_0^2\sin\theta_0}$$

$$\nabla^2\tfrac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$

$$\Delta x_{\text{hist}} \approx \tfrac{x_{\max}-x_{\min}}{\sqrt N}$$

$$P(x\le k) = F(k) = \textstyle\int_{-\infty}^k p(x)$$

$$\text{median} = F^{-1}(\tfrac12)$$

$$E[f(x)] = \textstyle\int_{-\infty}^\infty f(x)p(x)$$

$$\mu = E[x] = \textstyle\int_{-\infty}^\infty xp(x)$$

$$\alpha_n = E[x^n]$$

$$M_n = E[(x-\mu)^n]$$

$$\sigma^2 = M_2 = E[x^2] - \mu^2$$

$$\text{FWHM} \approx 2\sigma$$

$$\gamma_1 = \tfrac{M_3}{\sigma^3},\ \gamma_2 = \tfrac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$

$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$

$$\alpha_n = i^{-n}\tfrac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$

$$h\ge 0 : P(h\ge k) \le \tfrac{E[h]}{k}$$

$$P(|x-\mu| > k\sigma) \le \tfrac{1}{k^2}$$

$$B(k;n,p) = \tbinom{n}{k}p^k(1-p)^{n-k}$$

$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$

$$P(k;\mu) = \tfrac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$

$$u(x;a,b) = \tfrac{1}{b-a},\ x\in[a;b]$$

$$\mu_u = \tfrac{b+a}{2},\ \sigma_u^2 = \tfrac{(b-a)^2}{12}$$

$$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \tfrac{1}{\lambda},\ \sigma_\varepsilon^2 = \tfrac{1}{\lambda^2}$$

$$g(x;\mu,\sigma) = \tfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac12\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$g(\vec x;\vec\mu,V) = \frac{e^{-\frac12(\vec x-\vec\mu)^{\mathrm T}V^{-1}(\vec x-\vec\mu)}}{\sqrt{\det(2\pi V)}}$$

$$\text{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$

$$z = \tfrac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$

$$\chi^2 = \textstyle\sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$$

$$\wp(x;n) = \tfrac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$$

$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$

$$n\ge 30 : \wp(x;n) \approx g(x;n,\sqrt{2n})$$

$$n\ge 8 : p[\sqrt{2\chi^2}] \approx g(;\sqrt{2n-1},1)$$

$$S(x;n) = \tfrac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac n2)}\left(1+\tfrac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

$$\mu_S = 0,\ \sigma_S^2 = \tfrac{n}{n-2}$$

$$p\left[z\sqrt{\tfrac{n}{\chi^2}}\right] = S(,n)$$

$$n\ge 35 : S(x;n) \approx g(x;0,1)$$

$$c(x;a) = \tfrac{a}{\pi}\tfrac{1}{a^2+x^2}$$

$$\sigma_{xy} = E[xy] - \mu_x\mu_y \le \sigma_x\sigma_y$$

$$\rho_{xy} = \tfrac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}|\le 1$$

$$\mu_{f(x)} \approx f(\mu_x)$$

$$\sigma_{fg} \approx \sigma_{x_ix_j}\tfrac{\partial f}{\partial x_i}\Big|_{\mu_{x_i}}\tfrac{\partial g}{\partial x_j}\Big|_{\mu_{x_j}}$$

$$\mu \approx m = \tfrac1n\textstyle\sum_{i=1}^n x_i$$

$$\sigma^2 \approx s^2 = \tfrac{1}{n-1}\textstyle\sum_{i=1}^n(x_i-m)^2$$

$$s_m^2 = \tfrac{s^2}{n}$$

$$p\left[\tfrac{m-\mu}{s_m}\right] = S(;n)$$

Fit (ML)

$$f(x) = mx+q,\quad f(x) = a,$$

$$f(x) = bx,\quad f(x;\theta) = \theta_i h_i(x)$$

$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta mq = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$

$$\mathbf a = (\textstyle\sum V_{\mathbf y}^{-1})^{-1}(\sum V_{\mathbf y}^{-1}\mathbf y)$$

$$\Delta\mathbf a^2 = (\textstyle\sum V_{\mathbf y}^{-1})^{-1}$$

$$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

$$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$$

$$\chi^2 = (y-f(x;\theta))^T V^{-1}(y-f(x;\theta))$$

$$\theta = (H^TV^{-1}H)^{-1}H^TV^{-1}y$$

$$\Delta\theta\theta = (H^TV^{-1}H)^{-1}$$

Kinematics

$$\tfrac{1}{R} = \left|\tfrac{v_xa_y - v_ya_x}{v^3}\right|$$

$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$$

$$\dot{\vec w} = \tfrac{\mathrm d(\vec w\hat r)}{\mathrm dt}\hat r + \tfrac{\mathrm d(\vec w\hat\theta)}{\mathrm dt}\hat\theta + \tfrac{\mathrm d(\vec w\hat\varphi)}{\mathrm dt}\hat\varphi + \vec\omega\times\vec w$$

$$\theta\equiv\tfrac{\pi}{2} \to \dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$

$$\theta\equiv\tfrac{\pi}{2} \to \ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi$$

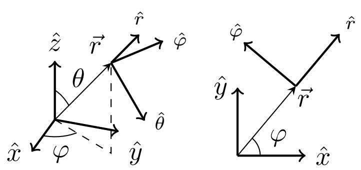
$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$

$$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$

$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$

$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$

$$\vec A = \ddot{\vec r} + \vec A_{\mathrm T} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$



Mechanics

$$\dot\alpha = \tfrac{\mathrm d}{\mathrm dt}\alpha(\beta,t) = \tfrac{\partial\alpha}{\partial\beta}\dot\beta + \tfrac{\partial\alpha}{\partial t}$$

$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \tfrac{\mathrm d(mT)}{\mathrm dt} = \vec F\vec p$$

$$M := \textstyle\sum_i m_i;\ \vec R := \tfrac{m_i\vec r_i}{M}$$

$$T = \tfrac12 M\dot{\vec R}^2 + \tfrac12 m_i(\dot{\vec r}_i - \dot{\vec R})^2$$

$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$$

$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$

$$\tau_1 = I_1\dot\omega_1 + (I_3 - I_2)\omega_3\omega_2$$

$$\mathcal L(q,\dot q,t) = T - V + \tfrac{\mathrm d}{\mathrm dt}f(q,t)$$

$$S[q] = \textstyle\int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$$

$$\tfrac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$

$$p := \tfrac{\partial\mathcal L}{\partial\dot q};\ \dot p = \tfrac{\partial\mathcal L}{\partial q}$$

$$\mathcal H(q,p,t) = \dot qp - \mathcal L$$

$$\dot q = \tfrac{\partial\mathcal H}{\partial p};\ \dot p = -\tfrac{\partial\mathcal H}{\partial q}$$

$$\tfrac{\mathrm d\mathcal H}{\mathrm dt} = \tfrac{\partial\mathcal H}{\partial t} = -\tfrac{\partial\mathcal L}{\partial t}$$

$$\{u,v\} = \tfrac{\partial u}{\partial q}\tfrac{\partial v}{\partial p} - \tfrac{\partial u}{\partial p}\tfrac{\partial v}{\partial q}$$

$$\tfrac{\mathrm du}{\mathrm dt} = \{u,\mathcal H\} + \tfrac{\partial u}{\partial t}$$

$$\eta = (q,p);\ \Gamma = \left(\begin{smallmatrix}0 & 1\\ -1 & 0\end{smallmatrix}\right)$$

$$\dot\eta = \Gamma\tfrac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \tfrac{\partial u}{\partial\eta}\Gamma\tfrac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac12 mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac23 mr^2$

ball: $\frac25 mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2 + \frac34 r^2\right)$

ellipsoid: $I_a = \frac15 m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$\langle U\rangle = -2\langle T\rangle$

$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$

$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}\mu\dot{\vec r}^2$

$\vec L = \vec R \times M\dot{\vec R} + \vec r \times \mu\dot{\vec r}$

$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}}$

$r = \frac{k}{1+\varepsilon\cos\theta}$

$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$

$a^3\omega^2 = G(m_1+m_2) = \frac{\alpha}{\mu}$

$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$

Inequalities

$|a| - |b| \le |a+b| \le |a| + |b|$

$x > -1 : 1 + nx \le (1+x)^n$

$\frac{|a^n - b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$

$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$

$\sum a_ib_i \le \left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^py^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$

$\sum\left(\frac{a_1+\ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p\sum a_i^p$

$x \ge 0, |\ddot x| \le M : |\dot x| \le \sqrt{2Mx}$

$\frac{1}{1+x} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$

$\dim V = \dim \ell(V) + \dim(V \cap \ker\ell)$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772\cdot 10^{-1}$

$G = 6.674\cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206\cdot 10^{-2}\ \frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$

$N_{\mathrm{A}} = 6.022\cdot 10^{23}\ \frac{1}{\mathrm{mol}}$

$k_{\mathrm{B}} = 1.381\cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$

$k_{\mathrm{B}} = 8.617\cdot 10^{-5}\ \frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998\cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$

$q_{\mathrm{e}} = 1.602\cdot 10^{-19}\ \mathrm{A\,s}$

$m_{\mathrm{e}} = 9.109\cdot 10^{-31}\ \mathrm{kg}$

$m_{\mathrm{p}} = 1.673\cdot 10^{-27}\ \mathrm{kg}$

$m_{\mathrm{n}} = 1.675\cdot 10^{-27}\ \mathrm{kg}$

$\mathrm{amu} = 1.661\cdot 10^{-27}\ \mathrm{kg}$

$h = 6.626\cdot 10^{-34}\ \mathrm{J\,s}$

$h = 4.136\cdot 10^{-15}\ \mathrm{eV\,s}$

$\varepsilon_0 = 8.854\cdot 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988\cdot 10^{9}\ \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257\cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_{\mathrm{B}} = 9.274\cdot 10^{-24}\ \mathrm{A\,m}^2$

$\alpha = 7.297\cdot 10^{-3}$

$\mathrm{barn} = 1\cdot 10^{-28}\ \mathrm{m}^2$

$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464\cdot 10^{-3}\ \frac{\mathrm{W}}{\mathrm{sr}}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i\mathrm{A}_i \rightarrow b_j\mathrm{B}_j$

$\Delta H^{\mathrm{o}}_{\mathrm{r}} = b_j\Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{B}_j) - a_i\Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{A}_i)$

$\forall\, i,j : v_{\mathrm{r}} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_{\mathrm{r}} = k[\mathrm{A}_i]^{m_i}$

$k = Ae^{-\frac{E_{\mathrm{a}}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod[\mathrm{B}_j]^{b_j}}{\prod[\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_{\mathrm{tot}}}$

$K_c = K_p(RT)^{\sum a_i - \sum b_j}$

$K_c = K_nV^{\sum a_i - \sum b_j}$

$K_\chi = K_nn_{\mathrm{tot}}^{\sum a_i - \sum b_j}$

$\Delta G^{\mathrm{o}}_{\mathrm{r}} = -RT\ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT\ln\frac{Q}{K}$

$\ln\frac{K_2}{K_1} = -\frac{\Delta H^{\mathrm{o}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_{\mathrm{w}} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^{\mathrm{o}} - \frac{RT}{n_{\mathrm{e}}N_Aq_{\mathrm{e}}}\ln Q$ (Nerst)

(std) $\Delta E = \Delta E^{\mathrm{o}} - \frac{0.059}{n_{\mathrm{e}}}\log_{10}Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu\mathrm{d}N$

$C_{V,N} = \frac{\partial Q}{\partial T}\Big|_{V,N} = \frac{\partial U}{\partial T}\Big|_{V,N}$

$C_{p,N} = \frac{\partial Q}{\partial T}\Big|_{p,N} = \frac{\partial U}{\partial T}\Big|_{p,N} + p\frac{\partial V}{\partial T}\Big|_{p,N}$

$\gamma := \frac{C_p}{C_V}$

$\mu_J := \frac{\partial T}{\partial V}\Big|_{U,N}$

$\lambda U = U(\lambda(S,V,N)) \Rightarrow U = ST - pV + \mu N$

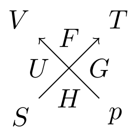
$\Rightarrow S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix $S,V,N : \min U$ at equilibrium

Fix $T,V,N : \min F = U - TS$

Fix $T,p,N : \min G = F + pV$

Fix $S,p,N : \min H = U + pV$



$\frac{\partial}{\partial T}\frac{G}{T}\Big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\Big|_V = -\frac{U}{T^2}$

$\Omega = U - TS - \mu N$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int\mathrm{d}q_1\cdots\mathrm{d}q_N\int\mathrm{d}p_1\cdots\mathrm{d}p_Ne^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_{\mathrm{B}}T};\ C = \frac{\partial U}{\partial T}$

$F(T,V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t},\ Z = \frac{V}{I}$

$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$

$Z_{\text{series}} = \sum_k Z_k,\ \frac{1}{Z_{\text{parallel}}} = \sum_k\frac{1}{Z_k}$

$\sum_{\text{loop}} V_k = 0,\ \sum_{\text{node}} I_k = 0$

$\mathcal{E} = -L\dot I,\ L = \frac{\Phi_B}{I}$

$I_{A\to C} = I_0\left(e^{\frac{V_{AC}}{V_T}} - 1\right),\ V_T = \eta\frac{k_{\mathrm{B}}T}{q_{\mathrm{e}}}$

$I_{E,\text{out}} = I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right) - \alpha_RI_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$

$I_{C,\text{in}} = -I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right) + \alpha_FI_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec p = \gamma m\vec v$

$\mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \frac{mc^2}{\gamma}$

$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec v\frac{\mathrm{d}\vec p}{\mathrm{d}t}$

$\binom{ct'}{x'} = \gamma\begin{pmatrix}1 & -\beta\\ -\beta & 1\end{pmatrix}\binom{ct}{x}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct, \vec x)$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec v)$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec v)}{\mathrm{d}t}\right)$

$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec p\right)$

$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\vec v}{c}\frac{\mathrm{d}\vec p}{\mathrm{d}t}, \frac{\mathrm{d}\vec p}{\mathrm{d}t}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec\nabla\right)$

$g_{\mu\nu} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu\partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \rightarrow \sum_i m_i$

$E_1^{\max} = \frac{M^2 + m_1^2 - \sum_{i\ne1}m_i^2}{2M}c^2$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$

$\mathrm{SO}_{1,3} = \left\{\Lambda\ \middle|\ \begin{matrix}\Lambda^{\mathrm{T}}g\Lambda = g\\ \det\Lambda \ge 0\end{matrix}\right\}$

$({\Lambda^0}_0)^2 \ge 1$

$\Lambda = \begin{pmatrix}\gamma & -\gamma\vec\beta\\ -\gamma\vec\beta & I + \frac{\gamma-1}{\beta^2}\vec\beta\otimes\vec\beta\end{pmatrix}$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$ $\quad \vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$ $\quad \vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{H} \times \sqrt{4\pi\mu_0}$ $\quad \vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \varepsilon/\varepsilon_0$ $\quad \mu/\mu_0$ $\quad R, Z \times 4\pi\varepsilon_0$ $\quad L \times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$$

$$\oint \vec{E} \mathrm{d}\vec{S} = 4\pi \int \rho \, \mathrm{d}^3 x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla} \times \vec{E} = 0$$

$$U = \frac{1}{8\pi} \int E^2 \, \mathrm{d}^3 x;\ \tilde{U} = \frac{1}{2} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi} \sum_{i \neq j} \int \vec{E}_i \vec{E}_j \, \mathrm{d}^3 x$$

$$V(\vec{r}) = \int \rho G_{\mathrm{D}}(\vec{r}) \, \mathrm{d}^3 x - \frac{1}{4\pi} \oint_S V \frac{\partial G_{\mathrm{D}}}{\partial n} \, \mathrm{d}S$$

$$V(\vec{r}) = \langle V \rangle_S + \int \rho G_{\mathrm{N}}(\vec{r}) \, \mathrm{d}^3 x + \frac{1}{4\pi} \oint_S \frac{\partial V}{\partial n} G_{\mathrm{N}}(\vec{r}) \, \mathrm{d}S$$

$$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_{\mathrm{D}}(\vec{x}, \vec{y})\big|_{\vec{y} \in S} = 0;\ \frac{\partial G_{\mathrm{N}}}{\partial n}\Big|_{\vec{y} \in S} = -\frac{4\pi}{S}$$

$$U_{\text{sphere}} = \frac{3}{5} \frac{Q^2}{R};\ \vec{p} = \int \mathrm{d}^3 r \rho \vec{r};\ \vec{E}_{\text{dip}} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_{\text{dip}} = \frac{\vec{p}\vec{r}}{r^2}$$

force on a dipole: $\vec{F}_{\text{dip}} = (\vec{p}\vec{\nabla})\vec{E}$

$$Q_{ij} = \int \mathrm{d}^3 r \rho(\vec{r})(3 r_i r_j - \delta_{ij} r^2);\ V_{\text{quad}} = \frac{1}{6r^5} Q_{ij}(3 r_i r_j - \delta_{ij} r^2)$$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

$$V(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^{\infty} \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$$

$$P_l(x) = \frac{1}{2^l l!} \frac{\mathrm{d}^l}{\mathrm{d}x^l} \left(x^2 - 1\right)^l;\ f = \sum_{l=0}^{\infty} c_l P_l : c_l = \frac{2l+1}{2} \int_{-1}^{1} f P_l$$

$$P_l(1) = 1;\ \langle P_n | P_m \rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm} | Y_{l'm'} \rangle = \delta_{ll'} \delta_{mm'}$$

$$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}} \sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x) = \frac{(-1)^m}{2^l l!} (1 - x^2)^{\frac{m}{2}} \frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}} \left(x^2 - 1\right)^l,\ 0 \leq m \leq l$$

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m Y_{lm}^*$$

$$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}^*(\theta', \varphi') Y_{lm}(\theta, \varphi)$$

$$V(r > \operatorname{diam} \operatorname{supp} \rho, \theta, \varphi) = \sum_{l=0}^{\infty} \frac{4\pi}{2l+1} \sum_{m=-l}^{l} q_{lm}[\rho] \frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^{\infty} r^2 \mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, r^l \rho(r, \theta, \varphi) Y_{lm}^*(\theta, \varphi)$$

Magnetostatics (CGS)

$$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0;\ I = \int \vec{J} \mathrm{d}\vec{S}$$

solenoid: $B = 4\pi \frac{j_{\mathrm{s}}}{c}$

$$\mathrm{d}\vec{F} = \frac{I \mathrm{d}\vec{l}}{c} \times \vec{B} = \mathrm{d}^3 x \frac{\vec{J}}{c} \times \vec{B};\ \vec{F}_q = q \frac{\dot{\vec{r}}}{c} \times \vec{B}$$

$$\mathrm{d}\vec{B} = \frac{I \mathrm{d}\vec{l}}{c} \times \frac{\vec{r}}{r^3};\ \vec{B}_q = q \frac{\dot{\vec{r}}}{c} \times \frac{\vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{A} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c} \frac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla} A_0$$

$$\vec{B} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c} \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\varphi = \frac{I}{c}\Omega,\ \vec{B} = -\vec{\nabla}\varphi$$

$$\vec{\nabla}\vec{A} = 0 \to \nabla^2 \vec{A} = -4\pi \frac{\vec{J}}{c}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{B} = 4\pi \frac{\vec{J}}{c};\ \oint \vec{B} \mathrm{d}\vec{l} = 4\pi \frac{I}{c}$$

$$\vec{m} = \frac{1}{2} \int \mathrm{d}^3 r' \left(\vec{r}' \times \frac{\vec{J}'}{c}\right) = \frac{1}{2c} \frac{q}{m} \vec{L} = \frac{SI}{c}$$

$$\vec{A}_{\mathrm{dm}} = \frac{\vec{m} \times \vec{r}}{r^3};\ \vec{\tau} = \vec{m} \times \vec{B}$$

$$\vec{F}_{\mathrm{dmdm}} = -\vec{\nabla}_R \frac{\vec{m}\vec{m}' - 3(\vec{m}\hat{R})(\vec{m}'\hat{R})}{R^3}$$

loop axis: $\vec{B} = \hat{z} \frac{2\pi R^2}{(z^2 + R^2)^{3/2}} \frac{I}{c}$

Electromagnetism (CGS)

Faraday: $\mathcal{E} = -\frac{1}{c} \frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int \mathrm{d}^3 x \vec{J} = \dot{\vec{p}}$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = 4\pi \frac{\vec{J}}{c} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$$

$$\mathrm{d}\vec{F} = \mathrm{d}^3 x \left(\rho \vec{E} + \frac{\vec{J}}{c} \times \vec{B}\right);\ \vec{F}_q = q\left(\vec{E} + \frac{\dot{\vec{r}}}{c} \times \vec{B}\right)$$

$$u = \frac{E^2 + B^2}{8\pi};\ \vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B};\ \vec{g} = \frac{\vec{S}}{c^2}$$

$$\mathbf{T}^E = \frac{1}{4\pi}\left(\vec{E} \otimes \vec{E} - \frac{1}{2} E^2\right);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ -\frac{\partial \vec{g}}{\partial t} = \rho \vec{E} + \frac{\vec{J}}{c} \times \vec{B} - \vec{\nabla}\mathbf{T}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

$$-\nabla^2 \phi - \frac{1}{c} \frac{\partial}{\partial t} \vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\left(\vec{\nabla}\vec{A} + \frac{1}{c} \frac{\partial\phi}{\partial t}\right) - \nabla^2 \vec{A} + \frac{1}{c} \frac{\partial^2 \vec{A}}{\partial t^2} = 4\pi \frac{\vec{J}}{c}$$

$$(\phi, \vec{A}) \cong \left(\phi - \frac{1}{c} \frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\right)$$

$$(\phi, \vec{A}) = \int \mathrm{d}^3 r' \frac{\left(\rho, \frac{\vec{J}}{c}\right)\left(\vec{r}', t - \frac{1}{c}\left|\vec{r} - \vec{r}'\right|\right)}{|\vec{r} - \vec{r}'|}$$

$$\vec{\nabla}\vec{A} = 0 \to \Box \vec{A} = \frac{4\pi}{c}(\vec{J} - \vec{J}_L) =: \frac{4\pi}{c} \vec{J}_T$$

$$\vec{J}_L = \frac{1}{4\pi} \vec{\nabla} \frac{\partial\phi}{\partial t} = -\frac{1}{4\pi} \vec{\nabla} \int \frac{\vec{\nabla}' \vec{J}'}{|\vec{r} - \vec{r}'|} \mathrm{d}^3 r'$$

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel};\ \vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

$$\vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

$$\vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\vec{v}}{c} \times \vec{E}\right)$$

$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \hat{k} \times \vec{E} \\ \omega = ck \end{cases}$$

$$\vec{B}_{\text{diprad}} = \frac{1}{c^2} \frac{\ddot{\vec{p}} \times \hat{r}}{r}\Big|_{t_{\text{rit}}};\ \vec{E}_{\text{diprad}} = \vec{B}_{\text{diprad}} \times \hat{r}$$

Larmor: $P = \frac{2}{3c^3} |\ddot{\vec{p}}|^2$

Rel. Larmor: $P = \frac{2}{3c^3} q^2 \gamma^6 (a^2 - (\vec{a} \times \vec{\beta})^2)$

$$\vec{A}_{\mathrm{dm}} = \frac{1}{c} \frac{\dot{\vec{m}} \times \hat{r}}{r}\Big|_{t_{\text{rit}}}$$

L.W.: $(\phi, \vec{A}) = \frac{q\left(1, \frac{\vec{v}}{c}\right)}{\left[r - \frac{\vec{v}\vec{r}}{c}\right]_{t_{\text{rit}}}};\ t_{\text{rit}} = t - \frac{r}{c}\Big|_{t_{\text{rit}}}$

$$A^\mu = (\phi, \vec{A});\ J^\mu = (c\rho, \vec{J})$$

Lorenz gauge: $\partial_\alpha A^\alpha = 0$

Temporal gauge: $\phi = 0$

Axial gauge: $A_3 = 0$

Coulomb gauge: $\vec{\nabla}\vec{A} = 0$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi \frac{J^\nu}{c};\ \partial_\alpha \mathscr{F}^{\alpha\nu} = 0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = q F^{\mu\alpha} \frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0;\ \det F = (\vec{E}\vec{B})^2$$

$$F^{\alpha\beta} F_{\alpha\beta} = 2(B^2 - E^2);\ F^{\alpha\beta} \mathscr{F}_{\alpha\beta} = 4\vec{E}\vec{B}$$

$$\Theta^{\mu\nu} = \frac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T} \end{pmatrix};\ \partial_\alpha \Theta^{\alpha\nu} = \frac{J_\alpha}{c} F^{\alpha\nu} (-?)$$

$$\mathcal{L} = \frac{mc^2}{\gamma} - q\vec{A}\frac{\vec{v}}{c} + q\phi;\ \mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{q\vec{A}}{c}\right)^2 + q\phi$$

E.M. in matter (CGS)

$$\vec{\nabla}\vec{D} = 4\pi\rho_{\text{ext}};\ \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{H} = 4\pi \frac{\vec{J}_{\text{ext}}}{c} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

$$\vec{P} = \frac{\mathrm{d}\langle\vec{p}\rangle}{\mathrm{d}V};\ \vec{M} = \frac{\mathrm{d}\langle\vec{m}\rangle}{\mathrm{d}V}$$

$$\rho_{\text{pol}} = -\vec{\nabla}\vec{P};\ \sigma_{\text{pol}} = \hat{n}\vec{P};\ \frac{\vec{J}_{\text{mag}}}{c} = \vec{\nabla} \times \vec{M}$$

$$\vec{D}_{\text{pol}} = \vec{E} + 4\pi\vec{P};\ \vec{H}_{\text{mag}} = \vec{B} - 4\pi\vec{M}$$

static linear isotropic: $\vec{P} = \chi\vec{E}$

static linear: $P_i = \chi_{ij} E_j$

static linear: $\varepsilon = 1 + 4\pi\chi$

static: $\Delta D_\perp = 4\pi\sigma_{\text{ext}};\ \Delta E_\parallel = 0$

static linear: $u = \frac{1}{8\pi} \vec{E}\vec{D}$

$$\Delta U_{\text{dielectric}} = -\frac{1}{2} \int \mathrm{d}^3 r \vec{P} \vec{E}_0$$

plane capacitor: $C = \frac{\varepsilon}{4\pi} \frac{S}{d}$

cilindric capacitor: $C = \frac{L}{2 \log \frac{R}{r}}$

atomic polarizability: $\vec{p} = \alpha\vec{E}_{\text{loc}}$

non-interacting gas: $\vec{p} = \alpha\vec{E}_0;\ \chi = n\alpha$

hom. cubic isotropic: $\chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi}{3} n\alpha$

perm. dipole: $\chi = \frac{1}{3} \frac{n p_0^2}{kT}$

local field: $\vec{E}_{\text{loc}} = \vec{E} + \frac{4\pi}{3} \vec{P}$

$$\vec{J}\vec{E} = -\vec{\nabla}\left(\frac{c}{4\pi} \vec{E} \times \vec{H}\right) - \frac{1}{4\pi}\left(\vec{E} \frac{\partial \vec{D}}{\partial t} + \vec{H} \frac{\partial \vec{B}}{\partial t}\right)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\frac{\omega}{c}$$

plane wave: $B = nE$

$$\vec{J}_{\mathrm{c}} = \sigma\vec{E};\ \varepsilon_\sigma = 1 + i\frac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm{p}}^2 = 4\pi \frac{n_{\text{vol}} q^2}{m};\ \omega_{\text{cyclo}} = \frac{qB}{mc}$$

I: $u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$

I: $\langle S_z \rangle = \frac{c}{n} \langle u \rangle$

II: $u = \frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega) E^2 + \frac{\partial}{\partial\omega}(\mu\omega) H^2\right)$

II: $\langle S_z \rangle = v_{\mathrm{g}} \langle u \rangle;\ v_{\mathrm{g}} = \frac{\partial\omega}{\partial k} = \frac{c}{n + \omega \frac{\partial n}{\partial\omega}}$

III: $\langle W \rangle = \frac{\omega}{4\pi}\left(\operatorname{Im}\varepsilon \langle E^2 \rangle + \operatorname{Im}\mu \langle H^2 \rangle\right)$

Fresnel TE (S): $\frac{E_\mathrm{t}}{E_\mathrm{i}} = \frac{2}{1+\frac{k_{tz}}{k_{iz}}}$; $\frac{E_\mathrm{r}}{E_\mathrm{i}} = \frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_\mathrm{t}}{E_\mathrm{i}} = \frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$; $\frac{E_\mathrm{r}}{E_\mathrm{i}} = \frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz} = \pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2 - k_x^2}$, $\mathrm{Im}\, k_{tz} > 0$

Drüde-Lorentz: $\varepsilon = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$$P(t) = \int_{-\infty}^{\infty} g(t-t')E(t')\mathrm{d}t'$$
$$P(\omega) = \chi(\omega)E(\omega)$$
$$\chi(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} g(t)\mathrm{d}t;\ \chi(-\omega) = \chi^*(\omega)$$
$$g(t<0) = 0 \implies$$
$$\mathrm{Re}\,\varepsilon(\omega) = 1 + \frac{2}{\pi}\fint_0^\infty \frac{\omega'\left(\mathrm{Im}\,\varepsilon(\omega') - \frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\mathrm{Im}\,\varepsilon(\omega) = -\frac{2\omega}{\pi}\fint_0^\infty \frac{\mathrm{Re}\,\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega' + \frac{4\pi\sigma_0}{\omega}$$

sum rule: $\frac{\pi}{2}\omega_\mathrm{p}^2 = \int_0^\infty \omega\,\mathrm{Im}\,\varepsilon\mathrm{d}\omega$

sum rule: $2\pi^2\sigma_0 = \int_0^\infty (1-\mathrm{Re}\,\varepsilon)\mathrm{d}\omega$

sum rule: $\int_0^\infty (\mathrm{Re}\,n - 1)\mathrm{d}\omega = 0$

Miller rule: $\chi^{(2)}(\omega,\omega) \propto \chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$

Quantum mechanics (CGS)

$$r_B = \frac{\hbar^2}{m_e e^2} = 5.292\cdot 10^{-11}\,\mathrm{m}$$

Rydberg $= \frac{e^2}{2r_B} = 13.61\,\mathrm{eV}$

$$r_e = \frac{e^2}{mc^2} = 2.818\cdot 10^{-15}\,\mathrm{m}$$
$$E_B = -\frac{1}{n^2}\frac{e^2}{2r_B}$$
$$\alpha = \frac{e^2}{\hbar c}$$

Planck: $\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$

$$\lambda_\mathrm{Broglie} = \frac{h}{p}$$
$$\sigma_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix};\ \sigma_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix};\ \sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$
$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$$
$$[\sigma_i,\sigma_j] = 2i\varepsilon_{ijk}\sigma_k$$
$$i\hbar\frac{\partial\mathcal{U}}{\partial t} = \mathcal{H}\mathcal{U}$$
$$\frac{\partial\mathcal{H}}{\partial t} = 0 \ \Rightarrow\ \mathcal{U}(t) = e^{-\frac{i\mathcal{H}t}{\hbar}}$$

$$[\mathcal{H}(t),\mathcal{H}(t')] = 0 \ \Rightarrow\ \mathcal{U}(t) = e^{-\frac{i\int_0^t \mathrm{d}t\mathcal{H}(t)}{\hbar}}$$
$$\mathcal{U}(t) = \left(\frac{-i}{\hbar}\right)^k \int_0^t \mathrm{d}t_1\cdots\mathrm{d}t_k\mathcal{H}(t_1)\cdots\mathcal{H}(t_k)$$
$$H = H_0 + V_\lambda : \frac{\partial E_n}{\partial\lambda}\Big|_{\lambda=0} = \langle\psi_n|\,\frac{\partial V_\lambda}{\partial\lambda}\,|\psi_n\rangle\,\Big|_{\lambda=0}$$
$$[A,BC] = [A,B]C + B[A,C]$$
$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$
$$[X,P] = i\hbar$$
$$\psi(x) = \langle x|\psi\rangle$$
$$\langle x|\,X\,|\psi\rangle = x\,\langle x|\psi\rangle$$
$$\langle x|\,P\,|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x}\,\langle x|\psi\rangle$$
$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$$
$$\left\langle(A-\langle A\rangle)^2\right\rangle\left\langle(B-\langle B\rangle)^2\right\rangle \geq \frac{1}{4}|\langle[A,B]\rangle|^2$$
$$e^B A e^{-B} = A + [B,A] + \frac{1}{2!}[B,[B,A]] + \cdots$$
$$\frac{\partial\mathcal{H}}{\partial t} = 0 \ \Rightarrow\ \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{[A,\mathcal{H}]}{i\hbar}$$
$$[X,f(P)] = i\hbar\frac{\partial f}{\partial P}$$

$$[f(X),P] = i\hbar\frac{\partial f}{\partial X}$$
$$[A,B]\propto I \ \Rightarrow\ e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$$
$$e^{ip'X}\,|p\rangle = |p+p'\rangle$$
$$e^{-iPx'}\,|x\rangle = |x+x'\rangle$$
$$\psi = |\psi|e^{\frac{iS}{\hbar}}$$
$$\vec{j} = \frac{|\psi|^2\vec{\nabla}S}{m}$$
$$\rho = |\psi|^2$$
$$\vec{j} = \frac{\hbar}{m}\,\mathrm{Im}(\psi^*\vec{\nabla}\psi)$$
$$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j}$$
$$\int \mathrm{d}^3x\vec{j} = \frac{\langle\vec{p}\rangle}{m}$$
$$\psi(x,t) = \int \mathrm{d}x' K(x,t;x')\psi(x',t=0)$$
$$K(x,t;x') = \sum_E \psi_E(x')^*\psi_E(x)e^{-\frac{iEt}{\hbar}} =$$
$$= \langle x|\,e^{-\frac{i\mathcal{H}t}{\hbar}}\,|x'\rangle$$
$$\left(\mathcal{H} - i\hbar\frac{\partial}{\partial t}\right)K(x,t;x') = -i\hbar\delta(x-x')\delta(t)$$

QM solutions

$$\mathcal{H}_\mathrm{box} = \frac{P^2}{2m} + \begin{cases}0 & 0<x<L\\ \infty & \text{otherwise}\end{cases}$$
$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(n\pi\frac{x}{L}\right),\ n\geq 1$$
$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$$
$$\Delta x^2 = L^2\left(\frac{1}{12} - \frac{1}{2n^2\pi^2}\right)$$
$$\Delta p = \frac{\hbar n\pi}{L} = \frac{hn}{2L}$$
$$\mathcal{H}_\mathrm{harm} = \frac{P^2}{2m} + \frac{m\omega^2X^2}{2}$$
$$A = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{iP}{m\omega}\right)$$
$$A^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X - \frac{iP}{m\omega}\right)$$
$$\left[A,A^\dagger\right] = 1$$

$$N = A^\dagger A = \frac{\mathcal{H}}{\hbar\omega} - \frac{1}{2};\ \mathcal{H} = \hbar\omega\left(N + \frac{1}{2}\right)$$
$$[N,A] = -A$$
$$\left[N,A^\dagger\right] = A^\dagger$$
$$A^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$
$$A\,|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$n = 0,1,\ldots$$
$$|n\rangle = \frac{(A^\dagger)^n}{\sqrt{n!}}\,|0\rangle$$
$$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}}\left(\frac{x}{x_0} - x_0\frac{\mathrm{d}}{\mathrm{d}x}\right)^n e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$
$$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}}H_n\left(\frac{x}{x_0}\right)e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$
$$x_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$\sum_{n=0}^\infty H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$$
$$H_n(-x) = (-1)^n H_n(x)$$
$$n \text{ even: } H_n(0) = (-1)^{\frac{n}{2}}\frac{n!}{(n/2)!}$$
$$H_n'(x) = 2nH_{n-1}(x)$$
$$H_0 = 1;\ H_1 = 2x;\ H_2 = 4x^2-2;\ H_3 = 8x^3-12$$
$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$
$$H_n''(x) = 2xH_n'(x) - 2nH_n(x)$$
$$\int_{-\infty}^\infty \mathrm{d}x H_n(x)H_m(x)e^{-x^2} = \sqrt{\pi}2^n n!\delta_{nm}$$
$$\mathcal{H}_\mathrm{delta} = \frac{P^2}{2m} - \lambda\delta(x),\ \lambda>0$$
$$\psi_\mathrm{bounded}(x) = \frac{1}{\sqrt{x_0}}e^{-\frac{|x|}{x_0}},\ x_0 = \frac{\hbar^2}{\lambda m}$$
$$E_\mathrm{bounded} = -\frac{\lambda}{2x_0}$$

Nuclear physics (MKSA)

$$M(A,Z) = Zm_\mathrm{p} + (A-Z)m_\mathrm{n} - B(A,Z)$$
$$B(A,Z) = a_vA - a_sA^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_\mathrm{sym}\frac{(A-2Z)^2}{A} + a_pA^{-3/4}\Delta$$
$$\Delta = \begin{cases}0 & & A\text{ odd}\\ \left.\begin{matrix}1 & Z\text{ even}\\ -1 & Z\text{ odd}\end{matrix}\right\} & & A\text{ even}\end{cases}$$
$$a_v = 15.5;\ a_s = 16.8;\ a_c = 0.72;\ a_\mathrm{sym} = 23;\ a_p = 34\ [\mathrm{MeV}]$$

$$\frac{\partial M}{\partial Z} = 0 : Z = \frac{m_\mathrm{n} - m_\mathrm{p} + 4a_\mathrm{sym}}{\frac{2a_c}{A^{1/3}} + \frac{8a_\mathrm{sym}}{A}}$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$
$$s_{ab} := |p_a^\mu + p_b^\mu|^2$$
$$M \to abc : (m_a + m_b)^2 \leq s_{ab} \leq (M - m_c)^2$$
$$M \to abc : s_{ab} + s_{bc} + s_{ac} = M^2 + m_a^2 + m_b^2 + m_c^2$$
$$a_iA_i \to b_jB_j : Q := (a_im_{A_i} - b_jm_{B_j})c^2$$
$$p = qBR$$

Fourier

$$\mathcal{F}^2 f = 2\pi f(-t)$$
$$\mathcal{F}[t^n f(t)] = (-i)^n\frac{\mathrm{d}^n}{\mathrm{d}\omega^n}\mathcal{F}[f(t)],\ \ t^{k\leq n}f \in L^1$$
$$\mathcal{F}\left[\frac{\mathrm{d}^k f}{\mathrm{d}t^k}\right] = (-i\omega)^k\mathcal{F}[f],\ \ f^{(k'\leq k)} \in L^1$$

$$\int f^*g = 2\pi\int\mathcal{F}[f]^*\mathcal{F}[g],\ \ f,g \in L^2$$
$$\mathcal{F}[f] = \int \mathrm{d}te^{i\omega t}f(t)$$
$$\mathcal{F}[f\star g\ (\in L^p)] = \mathcal{F}[f]\mathcal{F}[g],\ \ f \in L^1,\ g \in L^p$$
$$f\star g = \frac{1}{2\pi}\int\mathcal{F}[f]\mathcal{F}[g]e^{-i\omega t}\mathrm{d}\omega,\ \ f,g \in L^2$$

$$\|f\| = 1 : \Delta\omega\Delta t \geq \frac{1}{2}$$
$$\Delta\omega\Delta t = \frac{1}{2} : f(t) = g(t;\bar{t},\Delta t)e^{-i\bar{\omega}t}$$
$$(\omega\hat{f})' = -\mathcal{F}(xf')$$

Distributions

$$\mathcal{F}\theta = i\mathcal{P}\frac{1}{\omega} + \pi\delta(\omega)$$
$$\mathcal{F}1 = 2\pi\delta(\omega)$$

$$\mathcal{F}\,\mathrm{sgn} = 2i\mathcal{P}\frac{1}{\omega}$$
$$\langle\mathcal{F}T,f\rangle := \langle T,\mathcal{F}f\rangle$$
$$\langle T',f\rangle := -\langle T,f'\rangle$$

$$\langle gT,f\rangle := \langle T,gf\rangle$$
$$x^nT = 0 \ \Rightarrow\ T = \sum_{k=0}^{n-1}a_k\delta^{(k)}$$
$$xT = S \ \Rightarrow\ T = S/x + k\delta$$

$$\langle T \otimes S, \phi \rangle := \langle T(x), \langle S(y), \phi(x+y) \rangle \rangle \qquad \langle T \star S, \phi \rangle := \langle T \otimes S, \phi(x+y) \rangle$$

$$T \otimes S = S \otimes T,\ T, S \in \mathcal{D}' \qquad \sum_{n=0}^{\infty} e^{inx} = \mathcal{P}\frac{1}{1-e^{ix}} + \pi \sum_{n=-\infty}^{\infty} \delta(x - 2n\pi)$$