Trigonometric functions

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha;\ \tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = $$

$$= 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\sin^2\alpha = \frac{\tan^2\alpha}{1+\tan^2\alpha} \qquad \sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$\cos^2\alpha = \frac{1}{1+\tan^2\alpha} \qquad \cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$a\sin x + b\cos x =$$

$$= \frac{|a|}{a}\sqrt{a^2+b^2}\sin\left(x + \operatorname{atan}\frac{b}{a}\right)$$

$$= \frac{|b|}{b}\sqrt{a^2+b^2}\cos\left(x - \operatorname{atan}\frac{a}{b}\right)$$

$$\operatorname{acos} x + \operatorname{asin} x = \frac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$$

$$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$$

$$\tanh(x+y) = \frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$

$$\binom{\sinh x}{\cosh x} = \frac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x = \frac{1}{1-\tanh^2 x}$$

$$\sin x = -i\sinh(ix)$$

$$\cos x = \cosh(ix)$$

$$\binom{\operatorname{asinh} x}{\operatorname{acosh} x} = \log\left(x + \sqrt{x^2 + \binom{1}{-1}}\right)$$

$$\operatorname{atanh} x = \frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d) - abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A = \left(I + \frac{B}{2} - 1\right)A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)} = \frac{n!}{m_1!m_2!\dots} \qquad C_{n,k} = \binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad C'_{n,k} = \binom{n+k-1}{k}$$

$$D_{n,k} = \frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C} = \frac{ABC-AB}{9\times C \quad 0\times B}$$

$$\sqrt{a\pm\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} \pm \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\textstyle\sum_{i=0}^n a^i = \frac{1-a^{n+1}}{1-a}$$

$$\textstyle\sum_{x=1}^n x^3 = \left(\sum_{x=1}^n x\right)^2 = \frac{1}{4}n^2(n+1)^2$$

$$\textstyle\sum_{x=1}^n x^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$(a+b)^n = \textstyle\sum_{k=0}^n \binom{n}{k}a^{n-k}b^k$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\Gamma(1+z) = \int_0^\infty t^z e^{-t}dt = z!$$

$$n! \approx \left(\frac{n}{e}\right)^n\sqrt{2\pi n}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y = \int_0^x \frac{\partial g}{\partial x}(x,y)\mathrm{d}y + g(x,x)$$

$$\pm\sqrt{z} = \sqrt{\frac{\operatorname{Re} z+|z|}{2}} + \frac{i\operatorname{Im} z}{\sqrt{2(\operatorname{Re} z+|z|)}}$$

$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle = \frac{1}{2}\operatorname{Re}(ab^*)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$$

$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint\frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$

$$f(z) = \textstyle\sum_{k=-\infty}^\infty\left(\frac{1}{2\pi i}\oint\frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\right)(z-z_0)^k$$

$$\operatorname{sinc} x := \frac{\sin x}{x}$$

Derivatives

$$\tan' x = 1 + \tan^2 x$$

$$\cot' x = -1 - \cot^2 x$$

$$\operatorname{atan}' x = -\operatorname{acot}' x = \frac{1}{1+x^2}$$

$$\operatorname{asin}' x = -\operatorname{acos}' x = \frac{1}{\sqrt{1-x^2}}$$

$$(a^x)' = a^x\ln a$$

$$\log_a' x = \frac{1}{x\ln a}$$

$$\cosh' x = \sinh x$$

$$\tanh' x = 1 - \tanh^2 x$$

$$\operatorname{atanh}' x = \operatorname{acoth}' x = \frac{1}{1-x^2}$$

$$\operatorname{asinh}' x = \frac{1}{\sqrt{x^2+1}}$$

$$\operatorname{acosh}' x = \frac{1}{\sqrt{x^2-1}}$$

$$(f^{-1})' = \frac{1}{f'(f^{-1})}$$

$$\left(\frac{1}{x}\right)' = -\frac{\dot{x}}{x^2}$$

$$\left(\frac{x}{y}\right)' = \frac{\dot{x}y - x\dot{y}}{y^2}$$

$$(x^y)' = x^y\left(\dot{y}\ln x + y\frac{\dot{x}}{x}\right)$$

$$\frac{\partial(x,y)}{\partial(u,v)} := \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\frac{\partial(x,y)}{\partial(u,y)} = \frac{\partial x}{\partial u}\Big|_y = -\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x$$

$$\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x\frac{\partial u}{\partial x}\Big|_y = -1$$

$$\frac{\partial x}{\partial u}\Big|_y = \frac{\partial x}{\partial u}\Big|_v - \frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_v$$

$$\frac{\partial x}{\partial u}\Big|_v = \frac{\partial x}{\partial y}\Big|_v\frac{\partial y}{\partial u}\Big|_v$$

Integrals

$$\int x^a = \frac{x^{a+1}}{a+1}$$

$$\int a^x = \frac{a^x}{\ln a}$$

$$\int\frac{1}{x} = \ln|x|$$

$$\int\tan x = -\ln|\cos x|$$

$$\int\cot x = \ln|\sin x|$$

$$\int\frac{1}{\sin x} = \ln\left|\tan\frac{x}{2}\right|$$

$$\int\frac{1}{\cos x} = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

$$\int\ln x = x(\ln x - 1)$$

$$\int\tanh x = \ln\cosh x$$

$$\int\coth x = \ln|\sinh x|$$

$$\int\frac{1}{\sqrt{a^2-x^2}} = \operatorname{asin}\frac{x}{a}$$

$$\int\frac{1}{a^2+x^2} = \frac{1}{a}\operatorname{atan}\frac{x}{a}$$

$$\int xy = x\int y - \int\left(\dot{x}\int y\right)$$

$$\int_{-\infty}^\infty e^{-x^2} = \sqrt{\pi}$$

$$\int_{-\infty}^\infty e^{i\omega t}\mathrm{d}t = 2\pi\delta(\omega)$$

Differential equations

$$\dot{x} + \dot{a}x = b : x = e^{-a}\left(\int be^a + c_1\right)$$

$$a\ddot{x} + b\dot{x} + cx = 0 : x = c_1e^{z_1t} + c_2e^{z_2t}$$

$$x\ddot{x} = k\dot{x}^2 : x = c_2\sqrt[1-k]{(1-k)t + c_1}$$

$$\dot{x} + ax^2 = b : x = \sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2x = fe^{-i\omega t} : x = \frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \mathrm{O}(x^9)$$

$$\frac{1}{\sin x} = \frac{1}{x} + \frac{x}{6} + \frac{7}{360}x^3 + \frac{31}{15120}x^5 + \mathrm{O}(x^7)$$

$$\frac{1}{\cos x} = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \mathrm{O}(x^{10})$$

$$\frac{1}{\tan x} = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2}{945}x^5 + \mathrm{O}(x^7)$$

$$\operatorname{asin} x = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \mathrm{O}(x^9)$$

$$\operatorname{atan} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \mathrm{O}(x^9)$$

$$\frac{1}{\sinh x} = \frac{1}{x} - \frac{x}{6} + \frac{7}{360}x^3 - \frac{31}{15120}x^5 + \mathrm{O}(x^7)$$

$$\frac{1}{\cosh x} = 1 - \frac{x^2}{2} + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \mathrm{O}(x^{10})$$

$$\frac{1}{\tanh x} = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2}{945}x^5 + \mathrm{O}(x^7)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \mathrm{O}(x^3)$$

$$(1+x)^x = 1 + x^2 - \frac{x^3}{2} + \frac{5}{6}x^4 - \frac{3}{4}x^5 + \mathrm{O}(x^6)$$

$$x! = 1 - \gamma x + \left(\frac{\gamma^2}{2} + \frac{\pi^2}{12}\right)x^2 + \mathrm{O}(x^3)$$

Fourier

$$\text{Fourier: } c_n = \frac{2}{T}\int_0^T f(t)\cos\!\left(n\frac{t}{T}\right)\mathrm{d}t$$
$$\mathcal{F}[f](\omega) = \hat{f}(\omega) = \int \mathrm{d}t e^{i\omega t} f(t)$$
$$f,g \in L^2 : (\hat{f},\hat{g}) = 2\pi(f,g)$$
$$\mathcal{F}\left[\frac{\sin t}{t}\right] = \sqrt{\frac{\pi}{2}}\chi_{[-1;1]}(\omega)$$
$$t^{k\le n} f(t) \in L^1 : \mathcal{F}[t^n f(t)] = (-i)^n \hat{f}^{(n)}$$

$$f^{(k\le n)} \in L^1 : \mathcal{F}[f^{(n)}] = (-i\omega)^n \hat{f}$$
$$\mathcal{F}^2 f = 2\pi f(-t);\ (\omega\hat{f})' = -\mathcal{F}[tf']$$
$$f \star g = g \star f;\ \hat{f}\star\hat{g} = 2\pi\mathcal{F}[fg]$$
$$f \in L^1,\ g \in L^p : \mathcal{F}[f\star g] = \hat{f}\hat{g}$$
$$f\star g(x) = \int f(x-y)g(y)\mathrm{d}y$$
$$(f\star g)' = f'\star g = f \star g'$$

$$f(x+\Delta)\star g = f\star g(x+\Delta)$$
$$f \in L^1,\ g\in L^p \ \Rightarrow\ f\star g \in L^p$$
$$f,g \in L^2 : f\star g = \frac{1}{2\pi}\int \hat{f}\hat{g}e^{-i\omega t}\mathrm{d}\omega$$
$$\|f\| = 1 : \Delta\omega\Delta t \ge \frac{1}{2}$$
$$\Delta\omega\Delta t = \frac{1}{2} : f(t) = g(t;\bar{t},\Delta t)e^{-i\bar{\omega}t}$$

Distributions

$$\mathcal{D} := \{f \in C^\infty \mid \exists K \text{ compact} : f(\mathscr{C} K) = 0\}$$
$$\mathcal{S} := \{f \in C^\infty \mid |x^n f^{(k)}| \le A_{nk}\} \supset \mathcal{D}$$
$$\langle 1, f\rangle := \int f;\ \langle gT, f\rangle := \langle T, gf\rangle$$
$$T \in \mathcal{S}' : \langle \mathcal{F}T, f\rangle := \langle T, \mathcal{F}f\rangle$$
$$\langle T', f\rangle := -\langle T, f'\rangle;\ \langle \delta, f\rangle := f(0)$$

$$\langle T\otimes S, \phi\rangle := \langle T(x), \langle S(y), \phi(x+y)\rangle\rangle$$
$$\langle T\star S, \phi\rangle := \langle T\otimes S, \phi(x+y)\rangle$$
$$\mathcal{F}1 = 2\pi\delta(\omega);\ \mathcal{F}\,\mathrm{sgn} = 2i\mathcal{P}\frac{1}{\omega}$$
$$\mathcal{F}\theta = i\mathcal{P}\frac{1}{\omega} + \pi\delta(\omega)$$
$$x^n T = 0 \ \Rightarrow\ T = \sum_{k=0}^{n-1} a_k\delta^{(k)}$$

$$xT = S \ \Rightarrow\ T = S/x + k\delta$$
$$T,S \in \mathcal{D}' : T\otimes S = S\otimes T$$
$$\sum_{n=0}^\infty e^{inx} = \mathcal{P}\frac{1}{1-e^{ix}} + \pi\sum_{n=-\infty}^\infty \delta(x-2n\pi)$$
$$\delta^{(n)}\star f = f^{(n)}$$
$$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i) = 0$$

Bessel functions

$$\text{sol. of } x^2\partial_x^2 f + x\partial_x f + (x^2-\alpha^2)f = 0$$
$$\alpha = \text{“order”}$$
$$J_\alpha = \text{“first kind, normal”}$$
$$\alpha \in \mathbb{Z}_0 \vee \alpha > 0 : J_\alpha(0) = 0$$
$$J_0(0) = 1;\ \text{otherwise } |J_\alpha(0)| = \infty$$

$$\alpha \notin \mathbb{Z} : J_\alpha, J_{-\alpha} \text{ indep.}$$
$$\alpha \in \mathbb{Z} : J_{-\alpha} = (-1)^\alpha J_\alpha$$
$$Y_\alpha = \text{“second kind, normal” (also } N_\alpha)$$
$$\alpha \notin \mathbb{Z} : Y_\alpha = \frac{\cos(\alpha\pi)J_\alpha - J_{-\alpha}}{\sin(\alpha\pi)}$$
$$\alpha \in \mathbb{Z} : Y_\alpha = \lim_{\alpha'\to\alpha} Y_{\alpha'}$$

$$\alpha \in \mathbb{Z} : Y_\alpha, J_\alpha \text{ indep.}$$
$$\alpha \in \mathbb{Z} : Y_{-\alpha} = (-1)^\alpha Y_\alpha$$
$$\frac{2\alpha}{x}J_\alpha(x) = J_{\alpha-1}(x) + J_{\alpha+1}(x)$$
$$2J'_\alpha(x) = J_{\alpha-1}(x) - J_{\alpha+1}(x)$$
$$\int_0^1 \mathrm{d}x x J_\alpha(xu_{\alpha,m})J_\alpha(xu_{\alpha,n}) = \frac{\delta_{mn}}{2}J^2_{\alpha+1}(u_{\alpha,m})$$
$$u_{\alpha,n} = n\text{th. zero of } J_\alpha$$

Cylindrical harmonics

$$V(\rho,\phi,z) = \sum_{n=0}^\infty \int \mathrm{d}k A_{nk}P_{nk}(\rho)\Phi_n(\phi)Z_k(z)$$
$$P_{nk}(\rho) = \text{comb. of } J_n(k\rho), Y_n(k\rho)$$
$$\Phi_n(\phi) = \text{comb. of } e^{\pm in\phi}$$
$$Z_k(z) = \text{comb. of } e^{\pm kz}$$

Inequalities

$$|a| - |b| \le |a+b| \le |a| + |b|$$
$$x > -1 : 1 + nx \le (1+x)^n$$

$$\frac{|a^n - b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$$
$$\sum a_i b_i \le \left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$$

$$x^p y^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$$
$$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$$

$$\sum\left(\frac{a_1+\ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum a_i^p$$
$$x \ge 0, |\ddot{x}| \le M : |\dot{x}| \le \sqrt{2Mx}$$
$$\frac{1}{1+x} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}$$

Linear algebra

$$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$$
$$\dim V = \dim \ell(V) + \dim(V\cap\ker\ell)$$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$$\pi = 3.142$$
$$e = 2.718$$
$$\gamma = 5.772\cdot 10^{-1}$$
$$G = 6.674\cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$$
$$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$$

$$R = 8.206\cdot 10^{-2}\ \frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$$
$$N_\mathrm{A} = 6.022\cdot 10^{23}\ \frac{1}{\mathrm{mol}}$$
$$k_\mathrm{B} = 1.381\cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$$
$$k_\mathrm{B} = 8.617\cdot 10^{-5}\ \frac{\mathrm{eV}}{\mathrm{K}}$$
$$c = 2.998\cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$$
$$q_\mathrm{e} = 1.602\cdot 10^{-19}\ \mathrm{A\,s}$$

$$m_\mathrm{e} = 9.109\cdot 10^{-31}\ \mathrm{kg}$$
$$m_\mathrm{p} = 1.673\cdot 10^{-27}\ \mathrm{kg}$$
$$m_\mathrm{n} = 1.675\cdot 10^{-27}\ \mathrm{kg}$$
$$m_\mathrm{e} = 5.110\cdot 10^{-1}\ \mathrm{MeV}$$
$$m_\mathrm{p} = 9.383\cdot 10^{2}\ \mathrm{MeV}$$
$$m_\mathrm{n} = 9.396\cdot 10^{2}\ \mathrm{MeV}$$

$$m_\mathrm{n} - m_\mathrm{p} = 1.293\ \mathrm{MeV}$$
$$\mathrm{amu} = 1.661\cdot 10^{-27}\ \mathrm{kg}$$
$$h = 6.626\cdot 10^{-34}\ \mathrm{J\,s}$$
$$h = 4.136\cdot 10^{-15}\ \mathrm{eV\,s}$$
$$\varepsilon_0 = 8.854\cdot 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$$
$$\frac{1}{4\pi\varepsilon_0} = 8.988\cdot 10^{9}\ \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$$

$$\mu_0 = 1.257\cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$$
$$\mu_\mathrm{B} = 9.274\cdot 10^{-24}\ \mathrm{A\,m}^2$$
$$\alpha = 7.297\cdot 10^{-3}$$
$$\mathrm{barn} = 1\cdot 10^{-28}\ \mathrm{m}^2$$
$$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464\cdot 10^{-3}\ \frac{\mathrm{W}}{\mathrm{sr}}$$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i=j \vee j=k \vee k=i \\ 1 & i+1\equiv j \wedge j+1 \equiv k \\ -1 & i\equiv j+1 \wedge j\equiv k+1 \end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$
$$\vec{a}\times\vec{b} = \varepsilon_{ijk}a_j b_k\hat{e}_i$$
$$(\vec{a}\otimes\vec{b})_{ij} = a_i b_j$$
$$(\vec{a}\times\vec{b})\vec{c} = (\vec{c}\times\vec{a})\vec{b}$$
$$(\vec{a}\times\vec{b})\times\vec{c} = -(\vec{b}\vec{c})\vec{a} + (\vec{a}\vec{c})\vec{b}$$
$$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d}) = (\vec{a}\vec{c})(\vec{b}\vec{d}) - (\vec{a}\vec{d})(\vec{b}\vec{c})$$
$$|\vec{u}\times\vec{v}|^2 = u^2v^2 - (\vec{u}\vec{v})^2$$
$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right);\ \Box = \frac{\partial^2}{\partial t^2} - \nabla^2$$
$$\vec{\nabla}V = \frac{\partial V}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$$

$$\vec{\nabla}\vec{v} = \frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$
$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\rho} +$$
$$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\phi}\right)\hat{z}$$
$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\vec{\nabla}V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat{\varphi}$$
$$\vec{\nabla}\vec{v} = \frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\right)\hat{r} +$$
$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(rv_\varphi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat{\varphi}$$
$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rV) = \frac{2}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial r^2}$$

$$\vec{\nabla}(\vec{\nabla}\times\vec{v}) = \vec{\nabla}\times\vec{\nabla}V = 0$$
$$\vec{\nabla}(f\vec{v}) = (\vec{\nabla}f)\vec{v} + f\vec{\nabla}\vec{v}$$
$$\vec{\nabla}\times(f\vec{v}) = \vec{\nabla}f\times\vec{v} + f\vec{\nabla}\times\vec{v}$$
$$\vec{\nabla}\times(\vec{\nabla}\times\vec{v}) = -\nabla^2\vec{v} + \vec{\nabla}(\vec{\nabla}\vec{v})$$
$$\vec{\nabla}(\vec{v}\times\vec{w}) = \vec{w}(\vec{\nabla}\times\vec{v}) - \vec{v}(\vec{\nabla}\times\vec{w})$$
$$\vec{\nabla}\times(\vec{v}\times\vec{w}) = (\vec{\nabla}\vec{w} + \vec{w}\vec{\nabla})\vec{v} - (\vec{\nabla}\vec{v} + \vec{v}\vec{\nabla})\vec{w}$$
$$\frac{1}{2}\vec{\nabla}v^2 = (\vec{v}\vec{\nabla})\vec{v} + \vec{v}\times(\vec{\nabla}\times\vec{v})$$
$$\int\vec{\nabla}\vec{v}\mathrm{d}^3x = \oint\vec{v}\mathrm{d}\vec{S};\ \int(\vec{\nabla}\times\vec{v})\mathrm{d}\vec{S} = \oint\vec{v}\mathrm{d}\vec{l}$$
$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$$
$$\oint\vec{v}\times\mathrm{d}\vec{S} = -\int(\vec{\nabla}\times\vec{v})\mathrm{d}^3x$$
$$\delta(\vec{r}-\vec{r}_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r_0^2\sin\theta_0}$$
$$\nabla^2\frac{1}{|\vec{r}-\vec{r}_0|} = -4\pi\delta(\vec{r}-\vec{r}_0)$$

Statistics

$$P(E\cap E_1)=P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}}\approx\frac{x_{\max}-x_{\min}}{\sqrt{N}}$$
$$P(x\le k)=F(k)=\int_{-\infty}^{k}p(x)$$
$$\text{median}=F^{-1}(\tfrac{1}{2})$$
$$E[f(x)]=\int_{-\infty}^{\infty}f(x)p(x)$$
$$\mu=E[x]=\int_{-\infty}^{\infty}xp(x)$$
$$\alpha_n=E[x^n]$$
$$M_n=E[(x-\mu)^n]$$
$$\sigma^2=M_2=E[x^2]-\mu^2$$
$$\text{FWHM}\approx 2\sigma$$
$$\gamma_1=\frac{M_3}{\sigma^3},\ \gamma_2=\frac{M_4}{\sigma^4}$$

$$\phi[y](t)=E[e^{ity}]$$
$$\phi[y_1+\lambda y_2]=\phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n=i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge 0: P(h\ge k)\le\frac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$$
$$B(k;n,p)=\binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B=np,\ \sigma_B^2=np(1-p)$$
$$P(k;\mu)=\frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2=\mu$$
$$u(x;a,b)=\frac{1}{b-a},\ x\in[a;b]$$
$$\mu_u=\frac{b+a}{2},\ \sigma_u^2=\frac{(b-a)^2}{12}$$
$$\varepsilon(x;\lambda)=\lambda e^{-\lambda x},\ x\ge 0$$
$$\mu_\varepsilon=\frac{1}{\lambda},\ \sigma_\varepsilon^2=\frac{1}{\lambda^2}$$

$$g(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$
$$g(\vec{x};\vec{\mu},V)=\frac{e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^{\mathrm{T}}V^{-1}(\vec{x}-\vec{\mu})}}{\sqrt{\det(2\pi V)}}$$
$$\text{FWHM}_g=2\sigma\sqrt{2\ln 2}$$
$$z=\frac{x-\mu}{\sigma};\ \mu,\sigma[z]=0,1$$
$$\chi^2=\sum_{i=1}^{n}z_i^2;\ \wp:=p[\chi^2]$$
$$\wp(x;n)=\frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$
$$\mu_\wp=n,\ \sigma_\wp^2=2n$$
$$n\ge 30:\wp(x;n)\approx g(x;n,\sqrt{2n})$$
$$n\ge 8: p[\sqrt{2\chi^2}]\approx g(;\sqrt{2n-1},1)$$
$$S(x;n)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$
$$\mu_S=0,\ \sigma_S^2=\frac{n}{n-2}$$

$$p\Big[z\sqrt{\frac{n}{\chi^2}}\Big]=S(,n)$$
$$n\ge 35: S(x;n)\approx g(x;0,1)$$
$$c(x;a)=\frac{a}{\pi}\frac{1}{a^2+x^2}$$
$$\sigma_{xy}=E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho_{xy}=\frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}|\le 1$$
$$\mu_{f(x)}\approx f(\mu_x)$$
$$\sigma_{fg}\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_{x_i}}\frac{\partial g}{\partial x_j}\Big|_{\mu_{x_j}}$$
$$\mu\approx m=\frac{1}{n}\sum_{i=1}^{n}x_i$$
$$\sigma^2\approx s^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-m)^2$$
$$s_m^2=\frac{s^2}{n}$$
$$p\Big[\frac{m-\mu}{s_m}\Big]=S(;n)$$

Fit (ML)

$$f(x)=mx+q,\quad f(x)=a,$$
$$f(x)=bx,\quad f(x;\theta)=\theta_i h_i(x)$$
$$m=\frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta m^2=\frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$q=\frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta q^2=\frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta mq=\frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$a=\frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2=\frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$\mathbf{a}=(\sum V_{\mathbf{y}}^{-1})^{-1}(\sum V_{\mathbf{y}}^{-1}\mathbf{y})$$
$$\Delta\mathbf{a}^2=(\sum V_{\mathbf{y}}^{-1})^{-1}$$

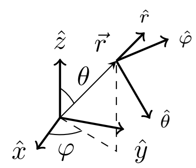
$$b=\frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2=\frac{1}{\sum\frac{x^2}{\Delta y^2}}$$
$$H_{ij}:=h_j(x_i);\ V_{ij}:=\Delta y_iy_j$$
$$\chi^2=(y-f(x;\theta))^TV^{-1}(y-f(x;\theta))$$
$$\theta=(H^TV^{-1}H)^{-1}H^TV^{-1}y$$
$$\Delta\theta\theta=(H^TV^{-1}H)^{-1}$$

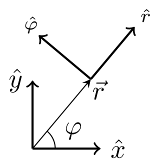
Kinematics

$$\frac{1}{R}=\left|\frac{v_xa_y-v_ya_x}{v^3}\right|$$
$$\vec{\omega}=\dot{\varphi}\cos\theta\hat{r}-\dot{\varphi}\sin\theta\hat{\theta}+\dot{\theta}\hat{\varphi}$$
$$\dot{\vec{w}}=\frac{\mathrm{d}(\vec{w}\hat{r})}{\mathrm{d}t}\hat{r}+\frac{\mathrm{d}(\vec{w}\hat{\theta})}{\mathrm{d}t}\hat{\theta}+\frac{\mathrm{d}(\vec{w}\hat{\varphi})}{\mathrm{d}t}\hat{\varphi}+\vec{\omega}\times\vec{w}$$
$$\theta\equiv\frac{\pi}{2}\to\dot{\vec{r}}=\dot{r}\hat{r}+r\dot{\varphi}\hat{\varphi}$$

$$\theta\equiv\frac{\pi}{2}\to\ddot{\vec{r}}=(\ddot{r}-r\dot{\varphi}^2)\hat{r}+(r\ddot{\varphi}+2\dot{r}\dot{\varphi})\hat{\varphi}$$
$$\dot{\vec{r}}=\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}+r\dot{\varphi}\sin\theta\hat{\varphi}$$
$$\langle\ddot{\vec{r}},\hat{r}\rangle=\ddot{r}-r\dot{\theta}^2-r\dot{\varphi}^2\sin^2\theta$$
$$\langle\ddot{\vec{r}},\hat{\theta}\rangle=r\ddot{\theta}+2\dot{r}\dot{\theta}-r\dot{\varphi}^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec{r}},\hat{\varphi}\rangle=r\ddot{\varphi}\sin\theta+2\dot{r}\dot{\varphi}\sin\theta+2r\dot{\theta}\dot{\varphi}\cos\theta$$

$$\vec{A}=\ddot{\vec{r}}+\vec{A}_{\mathrm{T}}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+\dot{\vec{\omega}}\times\vec{r}+2\vec{\omega}\times\dot{\vec{r}}$$





Mechanics

$$\dot{\alpha}=\frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t)=\frac{\partial\alpha}{\partial\beta}\dot{\beta}+\frac{\partial\alpha}{\partial t}$$
$$\vec{p}:=m\dot{\vec{r}};\ \vec{F}=\dot{\vec{p}};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t}=\vec{F}\vec{p}$$
$$M:=\sum_i m_i;\ \vec{R}:=\frac{m_i\vec{r}_i}{M}$$
$$T=\frac{1}{2}M\dot{\vec{R}}^2+\frac{1}{2}m_i(\dot{\vec{r}}_i-\dot{\vec{R}})^2$$

$$\vec{L}=\vec{R}\times M\dot{\vec{R}}+(\vec{r}_i-\vec{R})\times m_i(\dot{\vec{r}}_i-\dot{\vec{R}})$$
$$\vec{\tau}_O=\dot{\vec{L}}_O+\vec{v}_O\times\vec{p}$$
$$\tau_1=I_1\dot{\omega}_1+(I_3-I_2)\omega_3\omega_2$$
$$\mathcal{L}(q,\dot{q},t)=T-V+\frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$
$$S[q]=\int_{t_1}^{t_2}\mathcal{L}(q,\dot{q},t)\,\mathrm{d}t$$

$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0}=0$$
$$p:=\frac{\partial\mathcal{L}}{\partial\dot{q}};\ \dot{p}=\frac{\partial\mathcal{L}}{\partial q}$$
$$\mathcal{H}(q,p,t)=\dot{q}p-\mathcal{L}$$
$$\dot{q}=\frac{\partial\mathcal{H}}{\partial p};\ \dot{p}=-\frac{\partial\mathcal{H}}{\partial q}$$
$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t}=\frac{\partial\mathcal{H}}{\partial t}=-\frac{\partial\mathcal{L}}{\partial t}$$

$$\{u,v\}=\frac{\partial u}{\partial q}\frac{\partial v}{\partial p}-\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t}=\{u,\mathcal{H}\}+\frac{\partial u}{\partial t}$$
$$\eta=(q,p);\ \Gamma=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$$
$$\dot{\eta}=\Gamma\frac{\partial\mathcal{H}}{\partial\eta};\ \{u,v\}=\frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac{1}{2}mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac{2}{3}mr^2$

ball: $\frac{2}{5}mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2+\frac{3}{4}r^2\right)$

ellipsoid: $I_a=\frac{1}{5}m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle=-2\langle T\rangle$$
$$U_{\text{eff}}=U+\frac{L^2}{2mr^2}$$

$$\frac{1}{\mu}=\frac{1}{m_1}+\frac{1}{m_2}$$
$$\vec{r}=\vec{r}_1-\vec{r}_2,\ \alpha=Gm_1m_2$$
$$T=\frac{1}{2}M\dot{\vec{R}}^2+\frac{1}{2}\mu\dot{\vec{r}}^2$$

$$\vec{L}=\vec{R}\times M\dot{\vec{R}}+\vec{r}\times\mu\dot{\vec{r}}$$
$$k=\frac{L^2}{\mu\alpha},\ \varepsilon=\sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$

$$r=\frac{k}{1+\varepsilon\cos\theta}$$
$$a=\frac{k}{|1-\varepsilon^2|}=\frac{\alpha}{2|E|}$$
$$a^3\omega^2=G(m_1+m_2)=\frac{\alpha}{\mu}$$

$$\vec{A}=\mu\dot{\vec{r}}\times\vec{L}-\mu\alpha\hat{r},\ \dot{\vec{A}}=0$$

Relativity

$$\beta=\frac{v}{c}=\tanh\chi$$
$$\gamma=\frac{1}{\sqrt{1-\beta^2}}=\cosh\chi$$
$$\vec{p}=\gamma m\vec{v};\ \mathcal{E}=\gamma mc^2$$
$$\text{free particle: }\mathcal{L}=\frac{mc^2}{\gamma}$$
$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t}=\vec{v}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t};\ \frac{\mathrm{d}p}{\mathrm{d}t}=\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x}$$

$$\begin{pmatrix}ct'\\x'\end{pmatrix}=\gamma\begin{pmatrix}1&-\beta\\-\beta&1\end{pmatrix}\begin{pmatrix}ct\\x\end{pmatrix}$$
$$\chi''=\chi'+\chi$$
$$V'_{\parallel}=\frac{V_{\parallel}-v}{1-\frac{vV_{\parallel}}{c^2}}$$
$$V'_{\perp}=\frac{1}{\gamma}\frac{V_{\perp}}{1-\frac{vV_{\parallel}}{c^2}}$$
$$\frac{V'}{c}=1-\frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1-\frac{vV_{\parallel}}{c^2}\right)^2}$$

$$\mathrm{d}\tau=\frac{1}{\gamma}\mathrm{d}t$$
$$x^\mu=(ct,\vec{x})$$
$$v^\mu=\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}=\gamma(c,\vec{v})$$
$$a^\mu=\frac{\mathrm{d}v^\mu}{\mathrm{d}\tau}=\gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c,\frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$$
$$p^\mu=mv^\mu=\left(\frac{\mathcal{E}}{c},\vec{p}\right)$$
$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=\gamma\left(\frac{\vec{v}}{c}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t},\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right)$$
$$\partial_\mu=\frac{\partial}{\partial x^\mu}=\left(\frac{1}{c}\frac{\partial}{\partial t},\vec{\nabla}\right)$$

$$g_{\mu\nu}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$
$$x_\mu=g_{\mu\nu}x^\nu$$
$$\partial_\mu\partial^\mu=\Box$$
$$p^\mu p_\mu=(mc)^2$$
$$v^\mu a_\mu=0$$
$$M\to\sum_i m_i$$

$$E_1^{\max}=\frac{M^2+m_1^2-\sum_{i\neq 1}m_i^2}{2M}c^2$$
$$\text{doppler: }\sqrt{\frac{1+\beta}{1-\beta}}\approx 1+\beta$$
$$\mathrm{SO}_{1,3}=\left\{\Lambda\ \middle|\ \begin{smallmatrix}\Lambda^{\mathrm{T}}g\Lambda=g\\ \det\Lambda\ge 0\end{smallmatrix}\right\}$$
$$({\Lambda^0}_0)^2\ge 1$$
$$\Lambda=\begin{pmatrix}\gamma&-\gamma\vec{\beta}\\-\gamma\vec{\beta}&I+\frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta}\end{pmatrix}$$

Thermodynamics

$$\mathrm{d}Q=T\mathrm{d}S=\mathrm{d}U+\mathrm{d}L=\mathrm{d}U+p\mathrm{d}V-\mu\mathrm{d}N$$
$$C_{V,N}=\frac{\partial Q}{\partial T}\Big|_{V,N}=\frac{\partial U}{\partial T}\Big|_{V,N}$$
$$C_{p,N}=\frac{\partial Q}{\partial T}\Big|_{p,N}=\frac{\partial U}{\partial T}\Big|_{p,N}+p\frac{\partial V}{\partial T}\Big|_{p,N}$$

$$\gamma:=\frac{C_p}{C_V}$$
$$\mu_J:=\frac{\partial T}{\partial V}\Big|_{U,N}$$
$$\lambda U=U(\lambda(S,V,N))\ \Rightarrow\ U=ST-pV+\mu N$$
$$\Rightarrow\ S\mathrm{d}T-V\mathrm{d}p+N\mathrm{d}\mu=0$$

Fix S,V,N : $\min U$ at equilibrium

Fix T,V,N : $\min F=U-TS$

Fix T,p,N : $\min G=F+pV$

Fix S,p,N : $\min H=U+pV$

$$\begin{array}{ccc} V & & T \\ & F & \\ U & \times & G \\ & H & \\ S & & p \end{array} \qquad \frac{\partial}{\partial T}\frac{G}{T}\Big|_p = -\frac{H}{T^2} \qquad \frac{\partial}{\partial T}\frac{F}{T}\Big|_V = -\frac{U}{T^2}$$

$$\Omega = U - TS - \mu N$$

Ideal gas

$$pV = nRT$$

$$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\text{dof}}{2}R,\ c_p = c_V + R$$

$$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$$

$$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T \text{ const.}$$

Statistical mechanics

$$Z = \frac{1}{h^N}\int \mathrm{d}q_1\cdots\mathrm{d}q_N \int \mathrm{d}p_1\cdots\mathrm{d}p_N e^{-\beta\mathcal{H}}$$

$$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_\mathrm{B}T};\ C = \frac{\partial U}{\partial T}$$

$$F(T,V) = U - TS = -\frac{\log Z}{\beta}$$

$$S = -\frac{\partial F}{\partial T}$$

Electronics (MKS)

$$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t},\ Z = \frac{V}{I}$$

$$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$$

$$Z_\text{series} = \sum_k Z_k,\ \frac{1}{Z_\text{parallel}} = \sum_k \frac{1}{Z_k}$$

$$\sum_\text{loop} V_k = 0,\ \sum_\text{node} I_k = 0$$

$$\mathcal{E} = -L\dot{I},\ L = \frac{\Phi_B}{I}$$

$$I_{A\to C} = I_0\big(e^{\frac{V_{AC}}{V_T}} - 1\big),\ V_T = \eta\frac{k_\mathrm{B}T}{q_\mathrm{e}}$$

$$I_{E,\text{out}} = I_0^E\big(e^{\frac{V_{BE}}{V_T}} - 1\big) - \alpha_R I_0^C\big(e^{\frac{V_{BC}}{V_T}} - 1\big)$$

$$I_{C,\text{in}} = -I_0^C\big(e^{\frac{V_{BC}}{V_T}} - 1\big) + \alpha_F I_0^E\big(e^{\frac{V_{BE}}{V_T}} - 1\big)$$

Chemistry

$$H = U + pV$$

$$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$$

$$G = H - TS$$

$$a_i\mathrm{A}_i \rightarrow b_j\mathrm{B}_j$$

$$\Delta H_\mathrm{r}^\mathrm{o} = b_j\Delta H_\mathrm{f}^\mathrm{o}(\mathrm{B}_j) - a_i\Delta H_\mathrm{f}^\mathrm{o}(\mathrm{A}_i)$$

$$\forall\, i,j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$$

$$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$$

$$k = Ae^{-\frac{E_\mathrm{a}}{RT}} \text{ (Arrhenius)}$$

$$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$$

$$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$$

$$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod[\mathrm{B}_j]^{b_j}}{\prod[\mathrm{A}_i]^{a_i}}$$

$$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$$

$$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\text{tot}}$$

$$K_c = K_p(RT)^{\sum a_i - \sum b_j}$$

$$K_c = K_n V^{\sum a_i - \sum b_j}$$

$$K_\chi = K_n n_\text{tot}^{\sum a_i - \sum b_j}$$

$$\Delta G_\mathrm{r}^\mathrm{o} = -RT\ln K$$

$$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$$

$$\Delta G = RT\ln\frac{Q}{K}$$

$$\ln\frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\Big(\frac{1}{T_2} - \frac{1}{T_1}\Big)$$

$$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$$

$$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e}N_A q_\mathrm{e}}\ln Q \text{ (Nerst)}$$

$$\text{(std) } \Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}}\log_{10} Q$$

$$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$$

$$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$ $\quad \vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$ $\quad \vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{H} \times \sqrt{4\pi\mu_0}$ $\quad \vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \varepsilon/\varepsilon_0$ $\quad \mu/\mu_0$ $\quad R, Z \times 4\pi\varepsilon_0$ $\quad L \times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec{F}_{12} = q_1q_2\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_1-\vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3r'\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|};\ \rho_q = \delta(\vec{r}-\vec{r}_q)$$

$$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\times\vec{E} = 0$$

$$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde{U} = \frac{1}{2}\frac{q_iq_j}{|\vec{r}_i-\vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int\vec{E}_i\vec{E}_j\,\mathrm{d}^3x$$

$$V(\vec{r}) = \int\rho G_\mathrm{D}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_\mathrm{D}}{\partial n}\,\mathrm{d}S$$

$$V(\vec{r}) = \langle V\rangle_S + \int\rho G_\mathrm{N}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S\frac{\partial V}{\partial n}G_\mathrm{N}(\vec{r})\,\mathrm{d}S$$

$$\nabla_y^2 G(\vec{x},\vec{y}) = -4\pi\delta(\vec{x}-\vec{y});\ G_\mathrm{D}(\vec{x},\vec{y})|_{\vec{y}\in S} = 0;\ \frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y}\in S} = -\frac{4\pi}{S}$$

$$U_\text{sphere} = \frac{3}{5}\frac{Q^2}{R};\ \vec{p} = \int\mathrm{d}^3r\rho\vec{r};\ \vec{E}_\text{dip} = \frac{3(\vec{p}\hat{r})\hat{r}-\vec{p}}{r^3};\ V_\text{dip} = \frac{\vec{p}\hat{r}}{r^2}$$

$$\text{force on a dipole: } \vec{F}_\text{dip} = (\vec{p}\vec{\nabla})\vec{E}$$

$$Q_{ij} = \int\mathrm{d}^3r\rho(\vec{r})(3r_ir_j - \delta_{ij}r^2);\ V_\text{quad} = \frac{1}{6r^5}Q_{ij}(3r_ir_j - \delta_{ij}r^2)$$

$$V(r,\theta) = \sum_{l=0}^\infty\big(A_lr^l + \frac{B_l}{r^{l+1}}\big)P_l(\cos\theta)$$

$$V(r,\theta,\varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l\big(A_{lm}r^l + \frac{B_{lm}}{r^{l+1}}\big)Y_{lm}(\theta,\varphi)$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^\infty\frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\big(\frac{\vec{r}\vec{r}'}{rr'}\big)$$

$$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\big(x^2-1\big)^l;\ f = \sum_{l=0}^\infty c_lP_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$$

$$P_l(1) = 1;\ (P_n,P_m) = \frac{2\delta_{nm}}{2n+1};\ (Y_{lm},Y_{l'm'}) = \delta_{ll'}\delta_{mm'}$$

$$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2-1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\big(x^2-1\big)^l,\ 0\le m\le l$$

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^mY_{lm}^*$$

$$P_l\big(\frac{\vec{r}\vec{r}'}{rr'}\big) = \frac{4\pi}{2l+1}\sum_{m=-l}^l Y_{lm}^*(\theta',\varphi')Y_{lm}(\theta,\varphi)$$

$$V(r > \operatorname{diam}\operatorname{supp}\rho,\theta,\varphi) = \sum_{l=0}^\infty\frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\,r^l\rho(r,\theta,\varphi)Y_{lm}^*(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0;\ I = \int\vec{J}\mathrm{d}\vec{S}$$

$$\text{solenoid: } B = 4\pi\frac{j_\mathrm{s}}{c}$$

$$\mathrm{d}\vec{F} = \frac{I\mathrm{d}\vec{l}}{c}\times\vec{B} = \mathrm{d}^3x\frac{\vec{J}}{c}\times\vec{B};\ \vec{F}_q = q\frac{\dot{\vec{r}}}{c}\times\vec{B}$$

$$\mathrm{d}\vec{B} = \frac{I\mathrm{d}\vec{l}}{c}\times\frac{\vec{r}}{r^3};\ \vec{B}_q = q\frac{\dot{\vec{r}}}{c}\times\frac{\vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{A} = \int\mathrm{d}^3r'\frac{\vec{J}'}{c}\frac{1}{|\vec{r}-\vec{r}'|} + \vec{\nabla}A_0$$

$$\vec{B} = \int\mathrm{d}^3r'\frac{\vec{J}'}{c}\times\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\varphi = \frac{I}{c}\Omega,\ \vec{B} = -\vec{\nabla}\varphi$$

$$\vec{\nabla}\vec{A} = 0 \to \nabla^2\vec{A} = -4\pi\frac{\vec{J}}{c}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla}\times\vec{B} = 4\pi\frac{\vec{J}}{c};\ \oint\vec{B}\mathrm{d}\vec{l} = 4\pi\frac{I}{c}$$

$$\vec{m} = \frac{1}{2}\int\mathrm{d}^3r'\big(\vec{r}'\times\frac{\vec{J}'}{c}\big) = \frac{1}{2c}\frac{q}{m}\vec{L} = \frac{SI}{c}$$

$$\vec{A}_\text{dm} = \frac{\vec{m}\times\vec{r}}{r^3};\ \vec{\tau} = \vec{m}\times\vec{B}$$

$$\vec{F}_\text{dmdm} = -\vec{\nabla}_R\frac{\vec{m}\vec{m}' - 3(\vec{m}\hat{R})(\vec{m}'\hat{R})}{R^3}$$

$$\text{loop axis: } \vec{B} = \hat{z}\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac{I}{c}$$

Electromagnetism (CGS)

$$\text{Faraday: } \mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int\mathrm{d}^3x\vec{J} = \dot{\vec{p}}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = 4\pi\frac{\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$$

$$\mathrm{d}\vec{F} = \mathrm{d}^3x\big(\rho\vec{E} + \frac{\vec{J}}{c}\times\vec{B}\big);\ \vec{F}_q = q\big(\vec{E} + \frac{\dot{\vec{r}}}{c}\times\vec{B}\big)$$

$$u = \frac{E^2+B^2}{8\pi};\ \vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B};\ \vec{g} = \frac{\vec{S}}{c^2}$$

$$\mathbf{T}^E = \frac{1}{4\pi}\big(\vec{E}\otimes\vec{E} - \frac{1}{2}E^2\big);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ -\frac{\partial\vec{g}}{\partial t} = \rho\vec{E} + \frac{\vec{J}}{c}\times\vec{B} - \vec{\nabla}\mathbf{T}$$

$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\big(\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\big) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$

$$(\phi,\vec{A}) \cong \big(\phi - \frac{1}{c}\frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\big)$$

$$(\phi,\vec{A})=\int \mathrm{d}^3r'\,\frac{\left(\rho,\frac{\vec{J}}{c}\right)\left(\vec{r}',t-\frac{1}{c}|\vec{r}-\vec{r}'|\right)}{|\vec{r}-\vec{r}'|}$$

$$\vec{\nabla}\vec{A}=0\to\Box\vec{A}=\frac{4\pi}{c}(\vec{J}-\vec{J}_L)=:\frac{4\pi}{c}\vec{J}_T$$

$$\vec{J}_L=\frac{1}{4\pi}\vec{\nabla}\frac{\partial\phi}{\partial t}=-\frac{1}{4\pi}\vec{\nabla}\int\frac{\vec{\nabla}'\vec{J}'}{|\vec{r}-\vec{r}'|}\mathrm{d}^3r'$$

$$\vec{E}'_\parallel=\vec{E}_\parallel;\ \vec{B}'_\parallel=\vec{B}_\parallel$$

$$\vec{E}'_\perp=\gamma\left(\vec{E}_\perp+\frac{\vec{v}}{c}\times\vec{B}\right)$$

$$\vec{B}'_\perp=\gamma\left(\vec{B}_\perp-\frac{\vec{v}}{c}\times\vec{E}\right)$$

$$\text{plane wave: }\begin{cases}\vec{E}=\vec{E}_0e^{i(\vec{k}\vec{r}-\omega t)}\\ \vec{B}=\hat{k}\times\vec{E}\\ \omega=ck\end{cases}$$

$$\vec{B}_{\mathrm{diprad}}=\frac{1}{c^2}\frac{\ddot{\vec{p}}\times\hat{r}}{r}\Big|_{t_{\mathrm{rit}}};\ \vec{E}_{\mathrm{diprad}}=\vec{B}_{\mathrm{diprad}}\times\hat{r}$$

$$\text{Larmor: }P=\frac{2}{3c^3}|\ddot{\vec{p}}|^2$$

$$\text{Rel. Larmor: }P=\frac{2}{3c^3}q^2\gamma^6(a^2-(\vec{a}\times\vec{\beta})^2)$$

$$\vec{A}_{\mathrm{dm}}=\frac{1}{c}\frac{\dot{\vec{m}}\times\hat{r}}{r}\Big|_{t_{\mathrm{rit}}}$$

$$\text{L.W.: }(\phi,\vec{A})=\frac{q(1,\frac{\vec{v}}{c})}{[r-\frac{\vec{v}\vec{r}}{c}]_{t_{\mathrm{rit}}}};\ t_{\mathrm{rit}}=t-\frac{r}{c}\Big|_{t_{\mathrm{rit}}}$$

$$A^\mu=(\phi,\vec{A});\ J^\mu=(c\rho,\vec{J})$$

$$\text{Lorenz gauge: }\partial_\alpha A^\alpha=0$$

$$\text{Temporal gauge: }\phi=0$$

$$\text{Axial gauge: }A_3=0$$

$$\text{Coulomb gauge: }\vec{\nabla}\vec{A}=0$$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu}=\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu}=\begin{pmatrix}0&-E_x&-E_y&-E_z\\E_x&0&-B_z&B_y\\E_y&B_z&0&-B_x\\E_z&-B_y&B_x&0\end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu}=4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu}=0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma}+\partial_\nu F_{\sigma\mu}+\partial_\sigma F_{\mu\nu}=0;\ \det F=(\vec{E}\vec{B})^2$$

$$F^{\alpha\beta}F_{\alpha\beta}=2(B^2-E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta}=4\vec{E}\vec{B}$$

$$\Theta^{\mu\nu}=\frac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu}+\frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu}=\begin{pmatrix}u&c\vec{g}\\c\vec{g}&-\mathbf{T}\end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu}=\frac{J_\alpha}{c}F^{\alpha\nu}=:-G^\nu$$

$$\mathcal{L}=\frac{mc^2}{\gamma}-q\vec{A}\frac{\vec{v}}{c}+q\phi;\ \mathcal{H}=\frac{1}{2m}\left(\vec{p}-\frac{q\vec{A}}{c}\right)^2+q\phi$$

E.M. in matter (CGS)

$$\vec{\nabla}\vec{D}=4\pi\rho_{\mathrm{ext}};\ \nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B}=0;\ \vec{\nabla}\times\vec{H}=4\pi\frac{\vec{J}_{\mathrm{ext}}}{c}+\frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

$$\vec{P}=\frac{\mathrm{d}\langle\vec{p}\rangle}{\mathrm{d}V};\ \vec{M}=\frac{\mathrm{d}\langle\vec{m}\rangle}{\mathrm{d}V}$$

$$\rho_{\mathrm{pol}}=-\vec{\nabla}\vec{P};\ \sigma_{\mathrm{pol}}=\hat{n}\vec{P};\ \frac{\vec{J}_{\mathrm{mag}}}{c}=\vec{\nabla}\times\vec{M}$$

$$\vec{D}_{\mathrm{pol}}=\vec{E}+4\pi\vec{P};\ \vec{H}_{\mathrm{mag}}=\vec{B}-4\pi\vec{M}$$

$$\text{static linear isotropic: }\vec{P}=\chi\vec{E}$$

$$\text{static linear: }P_i=\chi_{ij}E_j$$

$$\text{static linear: }\varepsilon=1+4\pi\chi$$

$$\text{static: }\Delta D_\perp=4\pi\sigma_{\mathrm{ext}};\ \Delta E_\parallel=0$$

$$\text{static linear: }u=\frac{1}{8\pi}\vec{E}\vec{D}$$

$$\Delta U_{\mathrm{dielectric}}=-\frac{1}{2}\int\mathrm{d}^3r\vec{P}\vec{E}_0$$

$$\text{plane capacitor: }C=\frac{\varepsilon}{4\pi}\frac{S}{d}$$

$$\text{cilindric capacitor: }C=\frac{L}{2\log\frac{R}{r}}$$

$$\text{atomic polarizability: }\vec{p}=\alpha\vec{E}_{\mathrm{loc}}$$

$$\text{non-interacting gas: }\vec{p}=\alpha\vec{E}_0;\ \chi=n\alpha$$

$$\text{hom. cubic isotropic: }\chi=\frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$$

$$\text{Clausius-Mossotti: }\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi}{3}n\alpha$$

$$\text{perm. dipole: }\chi=\frac{1}{3}\frac{np_0^2}{kT}$$

$$\text{local field: }\vec{E}_{\mathrm{loc}}=\vec{E}+\frac{4\pi}{3}\vec{P}$$

$$\vec{J}\vec{E}=-\vec{\nabla}\left(\frac{c}{4\pi}\vec{E}\times\vec{H}\right)-\frac{1}{4\pi}\left(\vec{E}\frac{\partial\vec{D}}{\partial t}+\vec{H}\frac{\partial\vec{B}}{\partial t}\right)$$

$$n=\sqrt{\varepsilon\mu};\ k=n\frac{\omega}{c}$$

$$\text{plane wave: }B=nE$$

$$\vec{J}_{\mathrm{c}}=\sigma\vec{E};\ \varepsilon_\sigma=1+i\frac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm{p}}^2=4\pi\frac{n_{\mathrm{vol}}q^2}{m};\ \omega_{\mathrm{cyclo}}=\frac{qB}{mc}$$

$$\text{I: }u=\frac{1}{8\pi}(\vec{E}\vec{D}+\vec{H}\vec{B})$$

$$\text{I: }\langle S_z\rangle=\frac{c}{n}\langle u\rangle$$

$$\text{II: }u=\frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\frac{\partial}{\partial\omega}(\mu\omega)H^2\right)$$

$$\text{II: }\langle S_z\rangle=v_{\mathrm{g}}\langle u\rangle;\ v_{\mathrm{g}}=\frac{\partial\omega}{\partial k}=\frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$$

$$\text{III: }\langle W\rangle=\frac{\omega}{4\pi}\left(\mathrm{Im}\,\varepsilon\langle E^2\rangle+\mathrm{Im}\,\mu\langle H^2\rangle\right)$$

$$\text{Fresnel TE (S): }\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}}=\frac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm{r}}}{E_{\mathrm{i}}}=\frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$$

$$\text{TM (P): }\frac{E_{\mathrm{t}}}{E_{\mathrm{i}}}=\frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \frac{E_{\mathrm{r}}}{E_{\mathrm{i}}}=\frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$$

$$\text{Fresnel: }k_{tz}=\pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2-k_x^2},\ \mathrm{Im}\,k_{tz}>0$$

$$\text{Drüde-Lorentz: }\varepsilon=1-\frac{\omega_{\mathrm{p}}^2}{\omega^2+i\gamma\omega-\omega_0^2}$$

$$P(t)=\int_{-\infty}^{\infty}g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega)=\chi(\omega)E(\omega)$$

$$\chi(\omega)=\int_{-\infty}^{\infty}e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega)=\chi^*(\omega)$$

$$g(t<0)=0\implies$$

$$\mathrm{Re}\,\varepsilon(\omega)=1+\frac{2}{\pi}\int_0^\infty\frac{\omega'\left(\mathrm{Im}\,\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\mathrm{Im}\,\varepsilon(\omega)=-\frac{2\omega}{\pi}\int_0^\infty\frac{\mathrm{Re}\,\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega'+\frac{4\pi\sigma_0}{\omega}$$

$$\text{sum rule: }\frac{\pi}{2}\omega_{\mathrm{p}}^2=\int_0^\infty\omega\,\mathrm{Im}\,\varepsilon\mathrm{d}\omega$$

$$\text{sum rule: }2\pi^2\sigma_0=\int_0^\infty(1-\mathrm{Re}\,\varepsilon)\mathrm{d}\omega$$

$$\text{sum rule: }\int_0^\infty(\mathrm{Re}\,n-1)\mathrm{d}\omega=0$$

$$\text{Miller rule: }\chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$$

Quantum mechanics (CGS)

$$r_B=\frac{\hbar^2}{m_ee^2}=5.292\cdot10^{-11}\,\mathrm{m}$$

$$\text{Rydberg}=\frac{e^2}{2r_B}=13.61\,\mathrm{eV}$$

$$r_e=\frac{e^2}{mc^2}=2.818\cdot10^{-15}\,\mathrm{m}$$

$$\alpha=\frac{e^2}{\hbar c}$$

$$\text{Planck: }\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$$

$$\lambda_{\mathrm{Broglie}}=\frac{h}{p}$$

$$\sigma_1=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right);\ \sigma_2=\left(\begin{smallmatrix}0&-i\\i&0\end{smallmatrix}\right);\ \sigma_3=\left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$$

$$\sigma_i\sigma_j=\delta_{ij}+i\varepsilon_{ijk}\sigma_k$$

$$[\sigma_i,\sigma_j]=2i\varepsilon_{ijk}\sigma_k$$

$$\{\sigma_i,\sigma_j\}=2\delta_{ij}$$

$$i\hbar\frac{\partial\mathcal{U}}{\partial t}=\mathcal{H}\mathcal{U}$$

$$\frac{\partial\mathcal{H}}{\partial t}=0\ \Rightarrow\ \mathcal{U}(t)=e^{-\frac{i\mathcal{H}t}{\hbar}}$$

$$[\mathcal{H}(t),\mathcal{H}(t')]=0\ \Rightarrow\ \mathcal{U}(t)=e^{-\frac{i\int_0^t\mathrm{d}t\mathcal{H}(t)}{\hbar}}$$

$$\mathcal{U}(t)=\left(\frac{-i}{\hbar}\right)^k\int_0^t\mathrm{d}t_1\cdots\int_0^{t_{k-1}}\mathrm{d}t_k\mathcal{H}(t_1)\cdots\mathcal{H}(t_k)$$

$$H=H_0+V_\lambda:\ \frac{\partial E_n}{\partial\lambda}\Big|_{\lambda=0}=\langle\psi_n|\,\frac{\partial V_\lambda}{\partial\lambda}\,|\psi_n\rangle\,\Big|_{\lambda=0}$$

$$[A,BC]=[A,B]C+B[A,C]$$

$$[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$$

$$[X,P]=i\hbar$$

$$\psi(x)=\langle x|\psi\rangle$$

$$\langle x|\,X\,|\psi\rangle=x\,\langle x|\psi\rangle$$

$$\langle x|\,P\,|\psi\rangle=\frac{\hbar}{i}\frac{\partial}{\partial x}\,\langle x|\psi\rangle$$

$$\langle x|p\rangle=\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$$

$$\left\langle(A-\langle A\rangle)^2\right\rangle\left\langle(B-\langle B\rangle)^2\right\rangle\geq\frac{1}{4}|\langle[A,B]\rangle|^2$$

$$e^BAe^{-B}=A+[B,A]+\frac{1}{2!}[B,[B,A]]+\cdots$$

$$\frac{\partial\mathcal{H}}{\partial t}=0\ \Rightarrow\ \frac{\mathrm{d}A}{\mathrm{d}t}=\frac{[A,\mathcal{H}]}{i\hbar}$$

$$[X,f(P)]=i\hbar\frac{\partial f}{\partial P}$$

$$[f(X),P]=i\hbar\frac{\partial f}{\partial X}$$

$$[A,B]\propto I\ \Rightarrow\ e^Ae^B=e^{A+B+\frac{1}{2}[A,B]}$$

$$e^{ip'X}\,|p\rangle=|p+p'\rangle$$

$$e^{-iPx'}\,|x\rangle=|x+x'\rangle$$

$$\psi=|\psi|e^{\frac{iS}{\hbar}}$$

$$\vec{j}=\frac{|\psi|^2\vec{\nabla}S}{m}$$

$$\rho=|\psi|^2$$

$$\vec{j}=\frac{\hbar}{m}\,\mathrm{Im}(\psi^*\vec{\nabla}\psi)$$

$$\frac{\partial\rho}{\partial t}=-\vec{\nabla}\vec{j}$$

$$\int\mathrm{d}^3x\vec{j}=\frac{\langle\vec{p}\rangle}{m}$$

$$\psi(x,t)=\int\mathrm{d}x'K(x,t;x')\psi(x',t=0)$$

$$K(x,t;x')=\sum_E\psi_E(x')^*\psi_E(x)e^{-\frac{iEt}{\hbar}}=$$

$$=\langle x|\,e^{-\frac{i\mathcal{H}t}{\hbar}}\,|x'\rangle$$

$$\left(\mathcal{H}-i\hbar\frac{\partial}{\partial t}\right)K(x,t;x')=-i\hbar\delta(x-x')\delta(t)$$

$$[J_i,J_j]=i\hbar\varepsilon_{ijk}J_k$$

$$\left[J^2,J_z\right]=0$$

$$J_\pm:=J_x\pm iJ_y$$

$$[J_+,J_-]=i\hbar J_z$$

$$[J_z,J_\pm]=\pm\hbar J_\pm$$

$$\left[J^2,J_\pm\right]=0$$

$$J^2\,|j,m\rangle=j(j+1)\hbar^2\,|j,m\rangle$$

$$J_z\,|j,m\rangle=m\hbar\,|j,m\rangle$$

$$m=-j,j-1,\ldots,j;\ 2j\in\mathbb{N}$$

$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b})=\vec{a}\vec{b}+i\vec{\sigma}(\vec{a}\times\vec{b})$$

$$e^{-\frac{i\vec{\sigma}\hat{n}\phi}{2}}=\cos\frac{\phi}{2}-i(\vec{\sigma}\hat{n})\sin\frac{\phi}{2}$$

$$\left|\vec{S}\hat{n},\frac{\hbar}{2}\right\rangle=\cos\frac{\theta}{2}\left|S_z,\frac{\hbar}{2}\right\rangle+e^{i\varphi}\sin\frac{\theta}{2}\left|S_z,-\frac{\hbar}{2}\right\rangle$$

$$\langle\vec{x}|\,L_z\,|\alpha\rangle=\frac{\hbar}{i}\frac{\partial}{\partial\phi}\,\langle\vec{x}|\alpha\rangle$$

$$\rho[|\alpha_i\rangle,w_i]:=\sum_i w_i\,|\alpha_i\rangle\!\langle\alpha_i|$$

$$\mathrm{tr}\,\rho=1$$

$$[A]:=\mathrm{tr}(\rho A)$$

$$\#\{w_i>0\}=1\iff tr(\rho^2)=1$$

$$\#\{w_i>0\}>1\iff 0<\mathrm{tr}(\rho^2)<1$$

$$i\hbar\frac{\partial\rho}{\partial t}=-[\rho,\mathcal{H}]$$

$$W_\psi(x,p)=\int\frac{\mathrm{d}y}{2\pi\hbar}\left\langle x+\frac{y}{2}\middle|\psi\right\rangle\left\langle\psi\middle|x-\frac{y}{2}\right\rangle e^{-\frac{ipy}{2}}$$

QM solutions

$$\mathcal{H}_{\text{box}} = \frac{P^2}{2m} + \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise} \end{cases}$$

$$\psi_n(x) = \sqrt{\tfrac{2}{L}} \sin\!\left(n\pi \tfrac{x}{L}\right),\ n \geq 1$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$$

$$\Delta x^2 = L^2\left(\tfrac{1}{12} - \tfrac{1}{2n^2\pi^2}\right)$$

$$\Delta p = \frac{\hbar n \pi}{L} = \frac{hn}{2L}$$

$$\mathcal{H}_{\text{harm}} = \frac{P^2}{2m} + \frac{m\omega^2 X^2}{2}$$

$$A = \sqrt{\tfrac{m\omega}{2\hbar}}\left(X + \tfrac{iP}{m\omega}\right)$$

$$A^\dagger = \sqrt{\tfrac{m\omega}{2\hbar}}\left(X - \tfrac{iP}{m\omega}\right)$$

$$\left[A, A^\dagger\right] = 1$$

$$N = A^\dagger A = \tfrac{\mathcal{H}}{\hbar\omega} - \tfrac{1}{2};\ \mathcal{H} = \hbar\omega\left(N + \tfrac{1}{2}\right)$$

$$[N, A] = -A$$

$$\left[N, A^\dagger\right] = A^\dagger$$

$$A^\dagger \left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

$$A\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle$$

$$n = 0, 1, \ldots$$

$$\left|n\right\rangle = \frac{(A^\dagger)^n}{\sqrt{n!}}\left|0\right\rangle$$

$$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}}\left(\tfrac{x}{x_0} - x_0\tfrac{\mathrm{d}}{\mathrm{d}x}\right)^n e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$

$$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}} H_n\!\left(\tfrac{x}{x_0}\right) e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$

$$x_0 = \sqrt{\tfrac{\hbar}{m\omega}}$$

$$\textstyle\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$$

$$H_n(-x) = (-1)^n H_n(x)$$

$$n \text{ even: } H_n(0) = (-1)^{\frac{n}{2}}\frac{n!}{(n/2)!}$$

$$H_n'(x) = 2nH_{n-1}(x)$$

$$H_0 = 1;\ H_1 = 2x;\ H_2 = 4x^2 - 2;\ H_3 = 8x^3 - 12x$$

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$

$$H_n''(x) = 2xH_n'(x) - 2nH_n(x)$$

$$\textstyle\int_{-\infty}^{\infty} \mathrm{d}x H_n(x) H_m(x) e^{-x^2} = \sqrt{\pi}2^n n! \delta_{nm}$$

$$\mathcal{H}_{\text{delta}} = \frac{P^2}{2m} - \lambda\delta(x),\ \lambda > 0$$

$$\psi_{\text{bounded}}(x) = \frac{1}{\sqrt{x_0}} e^{-\frac{|x|}{x_0}},\ x_0 = \frac{\hbar^2}{\lambda m}$$

$$E_{\text{bounded}} = -\frac{\lambda}{2x_0}$$

$$\mathcal{H}_{\text{hydrogen}} = \frac{P^2}{2M} - \frac{e^2}{r}$$

$$a = \frac{\hbar^2}{Me^2} = r_B$$

$$E_n = -\frac{1}{n^2}\frac{e^2}{2a}$$

$$\text{degen.} = n^2$$

$$\psi_{nlm} = R_{nl} Y_{lm}$$

$$\vec{j} = \frac{\hbar}{M}\hat{\varphi}\frac{m}{r\sin\theta}|\psi|^2$$

$$R_{nl} = 2\sqrt{\frac{(n-l-1)!}{a^3n^4(n+l)!}}\, e^{-\frac{r}{na}}\left(\tfrac{2r}{na}\right)^l L_{n+l}^{2l+1}\left(\tfrac{2r}{na}\right)$$

$$L_n^{(j)}(x) = \textstyle\sum_{m=0}^{n-j}(-1)^m\binom{n}{n-j-m}\frac{x^m}{m!}$$

$$L_k(x) = e^x\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(x^k e^{-x}\right)$$

$$L_k^{(j)} = (-1)^j\frac{\mathrm{d}^j}{\mathrm{d}x^j}L_k(x)$$

Particle physics

$$M(A, Z) = Zm_{\text{p}} + (A - Z)m_{\text{n}} - B(A, Z)$$

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(A-2Z)^2}{A} + a_p A^{-3/4}\Delta$$

$$\Delta = \begin{cases} 0 & & A \text{ odd} \\ \left.\begin{matrix} 1 & Z \text{ even} \\ -1 & Z \text{ odd}\end{matrix}\right\} & & A \text{ even}\end{cases}$$

$$a_v = 15.5;\ a_s = 16.8;\ a_c = 0.72;\ a_{\text{sym}} = 23;\ a_p = 34\ [\text{MeV}]$$

$$\frac{\partial M}{\partial Z} = 0 : Z = \frac{m_{\text{n}} - m_{\text{p}} + 4a_{\text{sym}}}{\frac{2a_c}{A^{1/3}} + \frac{8a_{\text{sym}}}{A}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$

$$s_{ab} := (p_a + p_b)^2$$

$$M \to abc : (m_a + m_b)^2 \leq s_{ab} \leq (M - m_c)^2$$

$$M \to abc : s_{ab} + s_{bc} + s_{ac} = M^2 + m_a^2 + m_b^2 + m_c^2$$

$$a_i A_i \to b_j B_j : Q := a_i m_{A_i} - b_j m_{B_j}$$

$$p = qBR$$

$$\frac{\mathrm{d}^3\vec{p}}{2E} = \mathrm{d}^4 p\,\delta(p^2 - m^2)\theta(p_0)$$

$$\mathrm{d}L_p = \left(\textstyle\prod_n \frac{\mathrm{d}^3\vec{p}_n}{2E_n}\right)\delta^4\!\left(p_{\text{in}} - \textstyle\sum_n p_n\right)$$

$$\mathrm{d}\sigma = f_{\text{coll}}(p_1, \ldots, p_n)\mathrm{d}L_p$$

$$\text{two body: } \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1} = f(\Omega_1)\frac{p_1}{4\sqrt{s}}$$

$$\sqrt{s} = \text{c.m. energy}$$

$$\text{Rutherford: } \tan\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Qqm}{p^2 b};\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{d_{\min}^2}{16}\frac{1}{\sin^4(\frac{\theta}{2})}$$