

Trigonometric functions

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$\sin(2\alpha) = 2 \sin \alpha \cos \alpha; \tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$

$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha =$

$= 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$

Hyperbolic functions

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

$\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d) - abcd \cos^2 \frac{\alpha + \gamma}{2}}$

Pick: $A = \left(I + \frac{B}{2} - 1\right) A_{\text{check}}$

Combinatorics

$P_n^{(m_1, m_2, \dots)} = \frac{n!}{m_1! m_2! \dots}$

$C_{n,k} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

$C'_{n,k} = \binom{n+k-1}{k}$

$D_{n,k} = \frac{n!}{(n-k)!}$

Miscellaneous

$A.BC = \frac{ABC - AB}{9 \times C} - \frac{AB}{0 \times B}$

$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} \quad \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

$\sum_{i=0}^n a^i = \frac{1 - a^{n+1}}{1 - a}$

$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$

$e^{i\theta} = \cos \theta + i \sin \theta$

$\Gamma(1 + z) = \int_0^\infty t^z e^{-t} dt$

$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-ikx} f(x)$

Derivatives

$\tan' x = 1 + \tan^2 x$

$\cot' x = -1 - \cot^2 x$

$\operatorname{atan}' x = -\operatorname{acot}' x = \frac{1}{1+x^2}$

$\operatorname{asin}' x = -\operatorname{acos}' x = \frac{1}{\sqrt{1-x^2}}$

$(a^x)' = a^x \ln a$

$\log'_a x = \frac{1}{x \ln a}$

$\cosh' x = \sinh x$

$\tanh' x = 1 - \tanh^2 x$

$\operatorname{atanh}' x = \operatorname{acoth}' x = \frac{1}{1-x^2}$

$\operatorname{asinh}' x = \frac{1}{\sqrt{x^2+1}}$

$\operatorname{acosh}' x = \frac{1}{\sqrt{x^2-1}}$

$(f^{-1})' = \frac{1}{f'(f^{-1})}$

$\left(\frac{1}{x}\right)' = -\frac{\dot{x}}{x^2}$

$\left(\frac{x}{y}\right)' = \frac{\dot{x}y - x\dot{y}}{y^2}$

$(x^y)' = x^y \left(\dot{y} \ln x + y \frac{\dot{x}}{x}\right)$

Integrals

$\int x^a = \frac{x^{a+1}}{a+1}$

$\int a^x = \frac{a^x}{\ln a}$

$\int \frac{1}{x} = \ln |x|$

$\int \tan x = -\ln |\cos x|$

$\int \cot x = \ln |\sin x|$

$\int \frac{1}{\sin x} = \ln \left| \tan \frac{x}{2} \right|$

$\int \frac{1}{\cos x} = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right|$

$\int \ln x = x(\ln x - 1)$

$\int \tanh x = \ln \cosh x$

$\int \coth x = \ln |\sinh x|$

$\int \frac{1}{\sqrt{a^2-x^2}} = \operatorname{asin} \frac{x}{a}$

$\int \frac{1}{a^2+x^2} = \frac{1}{a} \operatorname{atan} \frac{x}{a}$

$\int xy = x \int y - \int (\dot{x} \int y)$

$\int e^{yx} x = e^{yx} \left(\frac{y}{x} - \frac{1}{y^2}\right)$

Differential equations

$\dot{x} + \dot{a}x = b : x = e^{-a} \left(\int b e^a + c_1\right)$

$a\ddot{x} + b\dot{x} + cx = 0 : x = c_1 e^{z_1 t} + c_2 e^{z_2 t}$

$\ddot{x} = -\omega^2 x : x = c_1 \sin(\omega t) + c_2 \cos(\omega t)$

$x\ddot{x} = k\dot{x}^2 : x = c_2 \sqrt[1-k]{(1-k)t + c_1}$

$\dot{x} + ax^2 = b : x = \sqrt{\frac{b}{a}} \tanh\left(\sqrt{ab}(c_1 + t)\right)$

Taylor

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \operatorname{O}(x^9)$

$\frac{1}{\sin x} = \frac{1}{x} + \frac{x}{6} + \frac{7}{360}x^3 + \frac{31}{15120}x^5 + \operatorname{O}(x^7)$

$\frac{1}{\cos x} = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \operatorname{O}(x^{10})$

$\frac{1}{\tan x} = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2}{945}x^5 + \operatorname{O}(x^7)$

$\operatorname{asin} x = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \operatorname{O}(x^9)$

$\operatorname{acos} x = \frac{\pi}{2} - \operatorname{asin} x$

$\operatorname{atan} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$

$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$

$\tanh x = x - \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \operatorname{O}(x^9)$

$\frac{1}{\sinh x} = \frac{1}{x} - \frac{x}{6} + \frac{7}{360}x^3 - \frac{31}{15120}x^5 + \operatorname{O}(x^7)$

$\frac{1}{\cosh x} = 1 - \frac{x^2}{2} + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \operatorname{O}(x^{10})$

$\frac{1}{\tanh x} = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2}{945}x^5 + \operatorname{O}(x^7)$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \operatorname{O}(x^3)$

$(1+x)^x = 1 + x^2 - \frac{x^3}{2} + \frac{5}{6}x^4 - \frac{3}{4}x^5 + \operatorname{O}(x^6)$

$x! = 1 - \gamma x + \left(\frac{\gamma^2}{2} + \frac{\pi^2}{12}\right)x^2 + \operatorname{O}(x^3)$

Vectors

$\varepsilon_{ijk} = \begin{cases} 0 & i = j \vee j = k \vee k = i \\ 1 & i + 1 \equiv j \wedge j + 1 \equiv k \\ -1 & i \equiv j + 1 \wedge j \equiv k + 1 \end{cases}$

$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

$\vec{a} \times \vec{b} = \varepsilon_{ijk} a_j b_k \hat{e}_i$

$(\vec{a} \times \vec{b})\vec{c} = (\vec{c} \times \vec{a})\vec{b}$

$(\vec{a} \times \vec{b}) \times \vec{c} = -(\vec{b}\vec{c})\vec{a} + (\vec{a}\vec{c})\vec{b}$

$(\vec{a} \times \vec{b})(\vec{c} \times \vec{d}) = (\vec{a}\vec{c})(\vec{b}\vec{d}) - (\vec{a}\vec{d})(\vec{b}\vec{c})$

$|\vec{u} \times \vec{v}|^2 = u^2 v^2 - (\vec{u}\vec{v})^2$

$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right); \square = \frac{\partial^2}{\partial t^2} - \nabla^2$

$\vec{\nabla} V = \frac{\partial V}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{\phi} + \frac{\partial V}{\partial z} \hat{z}$

$\vec{\nabla} \vec{v} = \frac{1}{\rho} \frac{\partial(\rho v_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

$\vec{\nabla} \times \vec{v} = \left(\frac{1}{\rho} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right) \hat{\rho} +$

$+ \left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho}\right) \hat{\phi} + \frac{1}{\rho} \left(\frac{\partial(\rho v_\phi)}{\partial \rho} - \frac{\partial v_\rho}{\partial \phi}\right)$

$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

$\vec{\nabla} V = \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \varphi} \hat{\varphi}$

$\vec{\nabla} \vec{v} = \frac{1}{r^2} \frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\varphi}{\partial \varphi}$

$\vec{\nabla} \times \vec{v} = \frac{1}{r \sin \theta} \left(\frac{\partial(v_\varphi \sin \theta)}{\partial \theta} - \frac{\partial v_\theta}{\partial \varphi}\right) \hat{r} +$

$+ \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \varphi} - \frac{\partial(r v_\varphi)}{\partial r}\right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right) \hat{\varphi}$

$\nabla^2 V = \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r}\right) + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta}\right) + \frac{\partial^2 V}{r^2 \sin^2 \theta}$

$\vec{\nabla}(f\vec{v}) = (\vec{\nabla} f)\vec{v} + f\vec{\nabla} \vec{v}$

$\vec{\nabla} \times (f\vec{v}) = \vec{\nabla} f \times \vec{v} + f\vec{\nabla} \times \vec{v}$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = -\nabla^2 \vec{v} + \vec{\nabla}(\vec{\nabla} \cdot \vec{v})$

$\vec{\nabla}(\vec{v} \times \vec{w}) = \vec{w}(\vec{\nabla} \cdot \vec{v}) - \vec{v}(\vec{\nabla} \cdot \vec{w})$

$$\vec{\nabla} \times (\vec{v} \times \vec{w}) = (\vec{\nabla} \vec{w} + \vec{w} \vec{\nabla}) \vec{v} - (\vec{\nabla} \vec{v} + \vec{v} \vec{\nabla}) \vec{w}$$

$$\frac{1}{2} \vec{\nabla} v^2 = (\vec{v} \vec{\nabla}) \vec{v} + \vec{v} \times (\vec{\nabla} \times \vec{v})$$

$$\int \vec{\nabla} \vec{v} d^3x = \oint \vec{v} d\vec{S}; \int (\vec{\nabla} \times \vec{v}) d\vec{S} = \oint \vec{v} d\vec{l}$$

$$\int (f \nabla^2 g - g \nabla^2 f) d^3x = \oint_S (f \frac{\partial g}{\partial n} - g \frac{\partial f}{\partial n}) dS$$

$$\oint \vec{v} \times d\vec{S} = - \int (\vec{\nabla} \times \vec{v}) d^3x$$

$$\delta(\vec{r} - \vec{r}_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2 \sin \theta_0}$$

$$\nabla^2 \frac{1}{|\vec{r}-\vec{r}_0|} = -4\pi \delta(\vec{r}-\vec{r}_0)$$

$$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|}; g(x_i) = 0$$

$$\langle \text{Re}(ae^{-i\omega t}) \text{Re}(be^{-i\omega t}) \rangle = \frac{1}{2} \text{Re}(a\bar{b})$$

Statistics

$$P(E \cap E_1) = P(E_1) \cdot P(E|E_1)$$

$$\Delta x_{\text{hist}} \approx \frac{x_{\text{max}} - x_{\text{min}}}{\sqrt{N}}$$

$$P(x \leq k) = F(k) = \int_{-\infty}^k p(x)$$

$$\text{median} = F^{-1}(\frac{1}{2})$$

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$

$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$

$$\alpha_n = E[x^n]$$

$$M_n = E[(x - \mu)^n]$$

$$\sigma^2 = M_2 = E[x^2] - \mu^2$$

$$\text{FWHM} \approx 2\sigma$$

$$\gamma_1 = \frac{M_3}{\sigma^3}, \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$

$$\phi[y_1 + \lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$

$$\alpha_n = i^{-n} \frac{\partial^n t}{\partial \phi[x]^n} \Big|_{t=0}$$

$$h \geq 0 : P(h \geq k) \leq \frac{E[h]}{k}$$

$$P(|x - \mu| > k\sigma) \leq \frac{1}{k^2}$$

$$B(n,p,k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_B = np, \sigma_B^2 = np(1-p)$$

$$P(\mu,k) = \frac{\mu^k}{k!} e^{-\mu}, \sigma_P^2 = \mu$$

$$u(x,a,b) = \frac{1}{b-a}, x \in [a;b]$$

$$\mu_u = \frac{b+a}{2}, \sigma_u^2 = \frac{(b-a)^2}{12}$$

$$\varepsilon(x,\lambda) = \lambda e^{-\lambda x}, x \geq 0$$

$$\mu_\varepsilon = \frac{1}{\lambda}, \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$$

$$g(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$\text{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$

$$z = \frac{x-\mu}{\sigma}; \mu, \sigma[z] = 0, 1$$

$$\chi^2 = \sum_{i=1}^n z_i^2$$

$$\wp_n(x) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$$

$$\mu_\wp = n, \sigma_\wp^2 = 2n$$

$$n \geq 30 : \wp_n(x) \approx g(x,n,\sqrt{2n})$$

$$n \geq 8 : p[\sqrt{2\chi^2}] \approx g(\sqrt{2n-1},1)$$

$$S(x,n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})} (1+\frac{x^2}{n})^{-\frac{n+1}{2}}$$

$$\mu_S = 0, \sigma_S^2 = \frac{n}{n-2}$$

$$p[z\sqrt{\frac{n}{\chi^2}}] = S(,n)$$

$$n \geq 35 : S(x,n) \approx g(x,0,1)$$

$$c(x,a) = \frac{a}{\pi} \frac{1}{a^2+x^2}$$

$$\sigma_{xy} = E[xy] - \mu_x \mu_y \leq \sigma_x \sigma_y$$

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}, |\rho| \leq 1$$

$$\mu[f(x_1,...)] \approx f(\mu_1,...)$$

$$\sigma^2[f(x_1,...)] \approx \sigma_{x_i x_j} \frac{\partial f}{\partial x_i} \Big|_{\mu_i} \frac{\partial f}{\partial x_j} \Big|_{\mu_j}$$

$$\mu \approx m = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\sigma^2 \approx s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - m)^2$$

$$s_m^2 = \frac{s^2}{n}$$

$$p[\frac{m-\mu}{s_m}] = S(,n)$$

Fit

$$f(x) = mx + q, \quad f(x) = a$$

$$f(x) = bx$$

$$m = \frac{\frac{\sum \frac{1}{\Delta y^2} \cdot \sum \frac{xy}{\Delta y^2} - \sum \frac{x}{\Delta y^2} \cdot \sum \frac{y}{\Delta y^2}}{\sum \frac{1}{\Delta y^2} \cdot \sum \frac{x^2}{\Delta y^2} - (\sum \frac{x}{\Delta y^2})^2}}{}$$

$$q = \frac{\frac{\sum \frac{y}{\Delta y^2} \cdot \sum \frac{x^2}{\Delta y^2} - \sum \frac{x}{\Delta y^2} \cdot \sum \frac{xy}{\Delta y^2}}{\sum \frac{1}{\Delta y^2} \cdot \sum \frac{x^2}{\Delta y^2} - (\sum \frac{x}{\Delta y^2})^2}}{}$$

$$a = \frac{\sum \frac{y}{\Delta y^2}}{\sum \frac{1}{\Delta y^2}}, \Delta a^2 = \frac{1}{\sum \frac{1}{\Delta y^2}}$$

$$b = \frac{\sum \frac{xy}{x^2}}{\sum \frac{1}{x^2}}, \Delta b^2 = \frac{\Delta y^2}{\sum x^2}$$

$$\Delta m^2 = \frac{\sum \frac{1}{\Delta y^2}}{\sum \frac{1}{\Delta y^2} \cdot \sum \frac{x^2}{\Delta y^2} - (\sum \frac{x}{\Delta y^2})^2}$$

$$\Delta q^2 = \frac{\sum \frac{x^2}{\Delta y^2}}{\sum \frac{1}{\Delta y^2} \cdot \sum \frac{x^2}{\Delta y^2} - (\sum \frac{x}{\Delta y^2})^2}$$

Kinematics

$$\frac{1}{R} = \Big| \frac{v_x a_y - v_y a_x}{v^3} \Big|$$

$$\vec{\omega} = \dot{\varphi} \cos \theta \hat{r} - \dot{\varphi} \sin \theta \hat{\theta} + \dot{\theta} \hat{\varphi}$$

$$\dot{\vec{w}} = \frac{d(\vec{w}\hat{r})}{dt} \hat{r} + \frac{d(\vec{w}\hat{\theta})}{dt} \hat{\theta} + \frac{d(\vec{w}\hat{\varphi})}{dt} \hat{\varphi} + \vec{\omega} \times \vec{w}$$

$$\theta \equiv \frac{\pi}{2} \rightarrow \dot{\vec{r}} = \dot{r} \hat{r} + r \dot{\varphi} \hat{\varphi}$$

$$\theta \equiv \frac{\pi}{2} \rightarrow \ddot{\vec{r}} = (\ddot{r} - r\dot{\varphi}^2) \hat{r} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) \hat{\varphi}$$

$$\dot{\vec{r}} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta} + r \dot{\varphi} \sin \theta \hat{\varphi}$$

$$\langle \dot{\vec{r}}, \hat{r} \rangle = \dot{r} - r \dot{\theta}^2 - r \dot{\varphi}^2 \sin^2 \theta$$

$$\langle \dot{\vec{r}}, \hat{\theta} \rangle = r \ddot{\theta} + 2\dot{r}\dot{\theta} - r \dot{\varphi}^2 \sin \theta \cos \theta$$

$$\langle \dot{\vec{r}}, \hat{\varphi} \rangle = r \ddot{\varphi} \sin \theta + 2\dot{r}\dot{\varphi} \sin \theta + 2r\dot{\theta}\dot{\varphi} \cos \theta$$

Mechanics

$$\dot{\alpha} = \frac{d}{dt} \alpha(\beta,t) = \frac{\partial \alpha}{\partial \beta} \dot{\beta} + \frac{\partial \alpha}{\partial t}$$

$$\vec{p} := m \dot{\vec{r}}; \vec{F} = \dot{\vec{p}}; \frac{d(mT)}{dt} = \vec{F} \vec{p}$$

$$M := \sum_i m_i; \vec{R} := \frac{m_i \vec{r}_i}{M}$$

$$T = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} m_i (\dot{\vec{r}}_i - \dot{\vec{R}})^2$$

$$\vec{L} = \vec{R} \times M \dot{\vec{R}} + (\vec{r}_i - \vec{R}) \times m_i (\dot{\vec{r}}_i - \dot{\vec{R}})$$

$$\vec{\tau}_O = \dot{\vec{L}}_O + \vec{v}_O \times \vec{p}$$

$$\tau_1 = I_1 \dot{\omega}_1 + (I_3 - I_2) \omega_3 \omega_2$$

$$\mathcal{L}(q,\dot{q},t) = T - V + \frac{d}{dt} f(q,t)$$

$$S[q] = \int_{t_1}^{t_2} \mathcal{L}(q,\dot{q},t) dt$$

$$\frac{\partial}{\partial \epsilon} S[q + \epsilon] \Big|_{\epsilon(t_1) = \epsilon(t_2) = 0} = 0$$

$$p := \frac{\partial \mathcal{L}}{\partial \dot{q}}; \dot{p} = \frac{\partial \mathcal{L}}{\partial q}$$

$$\mathcal{H}(q,p,t) = \dot{q} p - \mathcal{L}$$

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}; \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

$$\frac{d\mathcal{H}}{dt} = \frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}$$

$$\{u,v\} = \frac{\partial u}{\partial q} \frac{\partial v}{\partial p} - \frac{\partial u}{\partial p} \frac{\partial v}{\partial q}$$

$$\frac{du}{dt} = \{u,\mathcal{H}\} + \frac{\partial u}{\partial t}$$

$$\eta = (q,p); \Gamma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\dot{\eta} = \Gamma \frac{\partial \mathcal{H}}{\partial \eta}; \{u,v\} = \frac{\partial u}{\partial \eta} \Gamma \frac{\partial v}{\partial \eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12} m L^2$

disk: $\frac{1}{2} m r^2$

tetrahedron: $\frac{1}{20} m s^2$

octahedron: $\frac{1}{10} m s^2$

sphere: $\frac{2}{3} m r^2$

ball: $\frac{2}{5} m r^2$

cone: $\frac{3}{10} m r^2$

torus: $m(R^2 + \frac{3}{4} r^2)$

ellipsoid: $I_a = \frac{1}{5} m(b^2 + c^2)$

Kepler

$$\langle U \rangle \approx -2 \langle T \rangle$$

$$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2, \alpha = Gm_1 m_2$$

$$T = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \mu \dot{\vec{r}}^2$$

$$\vec{L} = \vec{R} \times M \dot{\vec{R}} + \vec{r} \times \mu \dot{\vec{r}}$$

$$k = \frac{L^2}{\mu \alpha}, \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu \alpha^2}}$$

$$a^3 \omega^2 = G(m_1 + m_2)$$

$$r = \frac{k}{1 + \varepsilon \cos \theta}$$

$$a = \frac{k}{|1 - \varepsilon^2|} = \frac{\alpha}{2|E|}$$

$$\vec{A} = \mu \dot{\vec{r}} \times \vec{L} - \mu \alpha \hat{r}, \dot{\vec{A}} = 0$$

Inequalities

$$|a| - |b| \leq |a + b| \leq |a| + |b|$$

$$x > -1 : 1 + nx \leq (1 + x)^n$$

$$\frac{|a^n - b^n|}{|a - b|^{<1}} \leq n(1 + |b|)^{n-1}$$

$$\sqrt[p]{\sum (a_i + b_i)^p} \leq \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$$

$$\sum a_i b_i \leq (\sum a_i^p)^{\frac{1}{p}} (\sum b_i^{\frac{p}{p-1}})^{\frac{p-1}{p}}$$

$$x^p y^q \leq (\frac{px + qy}{p+q})^{p+q}$$

$$\sqrt[p]{\frac{1}{n} \sum a_i^{p \leq q}} \leq \sqrt[q]{\frac{1}{n} \sum a_i^q}$$

$$\sum (\frac{a_1 + \dots + a_i}{i})^p \leq (\frac{p}{p-1})^p \sum a_i^p$$

$$x \geq 0, |\ddot{x}| \leq M : |\dot{x}| \leq \sqrt{2Mx}$$

$$\frac{1}{1+x} < \ln(1 + \frac{1}{x}) < \frac{1}{x}$$

Vector spaces

$(V, \mathbb{K}, +, \cdot)$ vector space; \mathbb{K} field

$$\exists \vec{0} \in V : \vec{v} + \vec{0} = \vec{v}$$

$$\cdot : \mathbb{K} \times V \rightarrow V; \quad \lambda \cdot (\vec{v} + \vec{w}) = \lambda \vec{v} + \lambda \vec{w}$$

$$0_{\mathbb{K}} \cdot \vec{v} = \vec{0}, 1_{\mathbb{K}} \cdot \vec{v} = \vec{v}$$

$$\lambda \in \mathbb{K}, \vec{v}, \vec{w} \in V \Rightarrow \vec{v} + \vec{w} \in V, \lambda \vec{v} \in V$$

$$\dim(U + V) = \dim U + \dim V - \dim(U \cap V)$$

$$\ell \text{ linear} : \ell(\vec{v} + \lambda \vec{w}) = \ell(\vec{v}) + \lambda \ell(\vec{w})$$

$$\ker \ell = \{ \vec{v} \in V \mid \ell(\vec{v}) = 0 \}$$

$$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$$

$$\langle , \rangle : V \times V \rightarrow \mathbb{K}; \quad \langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$$

$$\langle \vec{v} + \lambda \vec{w}, \vec{u} \rangle = \langle \vec{v}, \vec{u} \rangle + \lambda \langle \vec{w}, \vec{u} \rangle$$

$$\| \cdot \| : V \rightarrow \mathbb{K}; \quad \| \vec{v} \| = 0 \rightarrow \vec{v} = \vec{0}$$

$$\| \lambda \vec{v} \| = |\lambda| \| \vec{v} \|; \quad \| \vec{v} + \vec{w} \| \leq \| \vec{v} \| + \| \vec{w} \|^$$

Symbols													N	Ξ	O	Π	P	Σ	T	Y	Φ	X	Ψ	Ω							
A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M		ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	v	ϕ/φ	χ	ψ	ω							
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ																				
Constants, units				$R = 8.314 \frac{\text{J}}{\text{mol K}}$				$c = 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}}$				$m_{\text{n}} = 1.675 \cdot 10^{-27} \text{ kg}$				$\mu_0 = 1.257 \cdot 10^{-6} \frac{\text{N}}{\text{A}^2}$															
$\pi = 3.142$				$R = 8.206 \cdot 10^{-2} \frac{1 \text{atm}}{\text{mol K}}$				$q_{\text{e}} = 1.602 \cdot 10^{-19} \text{ A s}$				$\text{amu} = 1.661 \cdot 10^{-27} \text{ kg}$				$\mu_{\text{B}} = 9.274 \cdot 10^{-24} \text{ A m}^2$															
$e = 2.718$				$N_{\text{A}} = 6.022 \cdot 10^{23} \frac{1}{\text{mol}}$				$m_{\text{e}} = 9.109 \cdot 10^{-31} \text{ kg}$				$h = 6.626 \cdot 10^{-34} \text{ J s}$				$\alpha = 7.297 \cdot 10^{-3}$															
$\gamma = 5.772 \cdot 10^{-1}$				$k = 1.381 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$				$m_{\text{p}} = 1.673 \cdot 10^{-27} \text{ kg}$				$\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{\text{C}^2 \text{s}^2}{\text{kg m}^3}$				$\text{eV} = 1 \cdot 10^{-12} \text{ erg}$															
$G = 6.674 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$																															
Chemistry				$\exists k, (m_i) : v_{\text{r}} = k[\text{A}_i]^{m_i}$								$K_{\chi} = \frac{\prod \chi_{\text{B}_j}^{b_j}}{\prod \chi_{\text{A}_i}^{a_i}}, \chi = \frac{n}{n_{\text{tot}}}$								$\Delta G = RT \ln \frac{Q}{K}$											
$H = U + pV$				$k = Ae^{-\frac{E_{\text{a}}}{RT}}$ (Arrhenius)																$\ln \frac{K_2}{K_1} = -\frac{\Delta H^{\circ}}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$											
$\text{d}p = 0 \rightarrow \Delta H = \text{heat transfer}$				$a_{(\ell)} = \gamma \frac{[\text{X}]}{[\text{X}]_0}, [\text{X}]_0 = 1 \frac{\text{mol}}{1}$								$K_{\text{c}} = K_p(RT)^{\sum a_i - \sum b_j}$								$K_{\text{w}} = [\text{H}_3\text{O}^+][\text{OH}^-] = 10^{-14}$											
$G = H - TS$				$a_{(\text{g})} = \gamma \frac{p}{p_0}, p_0 = 1 \text{ atm}$								$K_{\text{c}} = K_{\text{n}} V^{\sum a_i - \sum b_j}$								$\Delta E = \Delta E^{\circ} - \frac{RT}{n_{\text{e}} N_{\text{A}} q_{\text{e}}} \ln Q$ (Nerst)											
$a_i \text{A}_i \rightarrow b_j \text{B}_j$												$K_{\text{n}} = K_{\chi} n_{\text{tot}}^{\sum b_j - \sum a_i}$								$(\text{std}) \Delta E = \Delta E^{\circ} - \frac{0.059}{n_{\text{e}}} \log_{10} Q$											
$\Delta H_{\text{r}}^{\circ} = b_j \Delta H_{\text{f}}^{\circ}(\text{B}_j) - a_i \Delta H_{\text{f}}^{\circ}(\text{A}_i)$				$K = \frac{\prod a_{\text{B}_j}^{b_j}}{\prod a_{\text{A}_i}^{a_i}}, K_{\text{c}} = \frac{\prod [\text{B}_j]^{b_j}}{\prod [\text{A}_i]^{a_i}}$								$\Delta G_{\text{r}}^{\circ} = -RT \ln K$								$\text{pH} = -\log_{10} [\text{H}_3\text{O}^+]$											
$\forall i, j : v_{\text{r}} = -\frac{1}{a_i} \frac{\Delta[\text{A}_i]}{\Delta t} = \frac{1}{b_j} \frac{\Delta[\text{B}_j]}{\Delta t}$				$K_p = \frac{\prod p_{\text{B}_j}^{b_j}}{\prod p_{\text{A}_i}^{a_i}}, K_{\text{n}} = \frac{\prod n_{\text{B}_j}^{b_j}}{\prod n_{\text{A}_i}^{a_i}}$								$Q = K(t) = \frac{\prod a_{\text{B}_j}^{b_j}(t)}{\prod a_{\text{A}_i}^{a_i}(t)}$								$K_{\text{a}} = \frac{[\text{A}^-][\text{H}_3\text{O}^+]}{[\text{AH}]}$											
Thermodynamics				$\text{d}Q = \text{d}U + \text{d}L$								$\text{d}S = \frac{\text{d}Q}{T}$								$C_V = \left(\frac{\text{d}Q}{\text{d}T} \right)_V$				$C_p = \left(\frac{\text{d}Q}{\text{d}T} \right)_p$				$\gamma = \frac{C_p}{C_V}$			
$\text{d}L = p\text{d}V$																															
Ideal gas				$c_V, c_p = \frac{C_V, C_p}{n}, c_V = \frac{\text{dof}}{2} R, c_p = c_V + R$								$\text{d}Q = 0 : pV^{\gamma}, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1} T \text{ const.}$																			
$pV = nRT$				$c_V = \frac{R}{\gamma-1}, c_p = \frac{\gamma}{\gamma-1} R$																											
Electronics				$Z = \frac{V}{I}$								$Z_C = -i \frac{1}{\omega C}$								$Z_{\text{series}} = \sum_k Z_k$				$\sum_{\text{loop}} V_k = 0$				$\mathcal{E} = -L\dot{I}$			
(MKS)				$Z_R = R$								$Z_L = i\omega L$								$\frac{1}{Z_{\text{parallel}}} = \sum_k \frac{1}{Z_k}$				$\sum_{\text{node}} I_k = 0$				$L = \frac{\Phi_B}{I}$			
$\left(\frac{V}{I} \right) = \left(\frac{V_0}{I_0} \right) e^{i\omega t}$																															
Relativity				$\mathcal{E} = \gamma mc^2$								$\text{d}\tau = \frac{1}{\gamma} \text{d}t$								$p^{\mu} = mv^{\mu} = \left(\frac{\mathcal{E}}{c}, \vec{p} \right)$				$x_{\mu} = g_{\mu\nu} x^{\nu}$							
$\beta = \frac{v}{c}$				$\frac{\text{d}\vec{p}}{\text{d}t} = \vec{F}$								$x^{\mu} = (ct, \vec{x})$								$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}} = \left(\frac{1}{c} \frac{\partial}{\partial t}, \vec{\nabla} \right)$				$\partial_{\mu} \partial^{\mu} = \square$							
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$				$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$								$v^{\mu} = \frac{\text{d}x^{\mu}}{\text{d}\tau} = \gamma(c, \vec{v})$								$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$				$p^{\mu} p_{\mu} = (mc)^2$							
$\vec{p} = \gamma m \vec{v}$												$a^{\mu} = \frac{\text{d}^2 x^{\mu}}{\text{d}\tau^2} = \gamma \left(\frac{\text{d}\gamma}{\text{d}t} c, \frac{\text{d}(\gamma \vec{v})}{\text{d}t} \right)$																			
Electrostatics (CGS)												$P_l(x) = \frac{1}{2^l l!} \frac{\text{d}^l}{\text{d}x^l} (x^2 - 1)^l; f = \sum_{l=0}^{\infty} c_l P_l : c_l = \frac{2l+1}{2} \int_{-1}^1 f P_l$																			
$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{ \vec{r}_1 - \vec{r}_2 ^3}; \vec{E}_1 = \frac{\vec{F}_{12}}{q_2}; V(\vec{r}) = \int \text{d}^3 r' \frac{\rho(\vec{r}')}{ \vec{r} - \vec{r}' }; \rho_q = \delta(\vec{r} - \vec{r}_q)$												$P_l(1) = 1; \langle P_n P_m \rangle = \frac{2\delta_{nm}}{2n+1}; \langle Y_{lm} Y_{l'm'} \rangle = \delta_{ll'} \delta_{mm'}$																			
$\oint \vec{E} \text{d}\vec{S} = 4\pi \int \rho \text{d}^3 x; -\nabla^2 V = \vec{\nabla} \vec{E} = 4\pi \rho; \vec{\nabla} \times \vec{E} = 0$												$P_0 = 1; P_1 = x; P_2 = \frac{3x^2-1}{2}; Y_{00} = \frac{1}{\sqrt{4\pi}}; Y_{10} = \sqrt{\frac{3}{4\pi}} \cos \theta$																			
$U = \frac{1}{8\pi} \int E^2 \text{d}^3 x; \tilde{U} = \frac{1}{2} \frac{q_i q_j}{ \vec{r}_i - \vec{r}_j } = \frac{1}{8\pi} \sum_{ij} \int \vec{E}_i \vec{E}_j \text{d}^3 x$												$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\varphi}; Y_{20} = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$																			
$V(\vec{r}) = \int \rho G_{\text{D}}(\vec{r}) \text{d}^3 x - \frac{1}{4\pi} \oint_S V \frac{\partial G_{\text{D}}}{\partial n} \text{d}S$												$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\varphi}; Y_{22} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{2i\varphi}$																			
$V(\vec{r}) = \langle V \rangle_S + \int \rho G_{\text{N}}(\vec{r}) \text{d}^3 x + \frac{1}{4\pi} \oint_S \frac{\partial V}{\partial n} G_{\text{N}}(\vec{r}) \text{d}S$												$P_{lm}(x) = \frac{(-1)^m}{2^l l!} (1-x^2)^{\frac{m}{2}} \frac{\text{d}^{l+m}}{\text{d}x^{l+m}} (x^2-1)^l, m \leq l$																			
$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi \delta(\vec{x} - \vec{y}); G_{\text{D}}(\vec{x}, \vec{y}) _{\vec{y} \in S} = 0; \frac{\partial G_{\text{N}}}{\partial n} \Big _{\vec{y} \in S} = -\frac{4\pi}{S}$												$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos \theta); Y_{l,-m} = (-1)^m \bar{Y}_{lm}$																			
$U_{\text{sphere}} = \frac{3}{5} \frac{Q^2}{R}; \vec{E}_{\text{dip}} = \frac{3(\vec{p}\vec{r})\vec{r} - \vec{p}}{r^3}; V_{\text{dip}} = \frac{\vec{p}\vec{r}}{r^2}$												$P_l(\frac{\vec{r}\vec{r}'}{rr'}) = \frac{4\pi}{2l+1} \sum_{m=-l}^l \bar{Y}_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi)$																			
$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l - \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$												$V(r > \text{diam supp } \rho, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{4\pi}{2l+1} q_{lm}[\rho] \frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$																			
$V(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}} \right) Y_{lm}(\theta, \varphi)$								$q_{lm}[\rho] = \int_0^{\infty} r^2 \text{d}r \int_0^{2\pi} \text{d}\varphi \int_0^{\pi} \sin \theta \text{d}\theta r^l \rho(r, \theta, \varphi) \bar{Y}_{lm}(\theta, \varphi)$																							
$\frac{1}{ \vec{r} - \vec{r}' } = \sum_{l=0}^{\infty} \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l(\frac{\vec{r}\vec{r}'}{rr'})$																															
Magnetostatics (CGS)				$\text{d}\vec{B} = \frac{I \text{d}\vec{l}}{c} \times \frac{\vec{r}}{r^3}; \vec{B}_q = q \frac{\dot{\vec{r}}}{c} \times \frac{\vec{r}}{r^3}$				$\vec{\nabla} \vec{A} = 0 \rightarrow \nabla^2 \vec{A} = -4\pi \frac{\vec{J}}{c}$																							
$\vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t} = 0; I = \int \vec{J} \text{d}\vec{S}$				$\vec{B} = \vec{\nabla} \times \vec{A}; \vec{A} = \int \text{d}^3 r' \frac{\vec{J}}{c} \frac{1}{ \vec{r} - \vec{r}' } + \vec{\nabla} A_0$				$\vec{\nabla} \vec{B} = 0; \vec{\nabla} \times \vec{B} = 4\pi \frac{\vec{J}}{c}; \oint \vec{B} \text{d}\vec{l} = 4\pi \frac{I}{c}$																							
solenoid: $B = 4\pi \frac{I_{\text{s}}}{c}$				$\vec{B} = \int \text{d}^3 r' \frac{\vec{J}}{c} \times \frac{\vec{r} - \vec{r}'}{ \vec{r} - \vec{r}' ^3}$				$\vec{m} = \frac{1}{2} \int \text{d}^3 r' (\vec{r}' \times \frac{\vec{J}}{c}) = \frac{1}{2c} \frac{q}{m} \vec{L}$																							
$\text{d}\vec{F} = \frac{I \text{d}\vec{l}}{c} \times \vec{B} = \text{d}^3 x \frac{\vec{J}}{c} \times \vec{B}; \vec{F}_q = q \frac{\dot{\vec{r}}}{c} \times \vec{B}$								$\vec{A} \approx \frac{\vec{m} \times \vec{r}}{r^3}; \vec{\tau} = \vec{m} \times \vec{B}$																							
Electromagnetism (CGS)				$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}; \vec{\nabla} \vec{E} = 4\pi \rho; \vec{\nabla} \vec{J} = -\frac{\partial \rho}{\partial t}$				$\text{d}\vec{F} = \text{d}^3 x (\rho \vec{E} + \frac{\vec{J}}{c} \times \vec{B}); \vec{F}_q = q(\vec{E} + \frac{\dot{\vec{r}}}{c} \times \vec{B})$																							
Faraday: $\mathcal{E} = -\frac{1}{c} \frac{\text{d}\Phi_B}{\text{d}t}$				$\vec{\nabla} \times \vec{B} = 4\pi \frac{\vec{J}}{c} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}; \vec{\nabla} \vec{B} = 0$				$u = \frac{E^2 + B^2}{8\pi}; \vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}; \vec{g} = \frac{\vec{S}}{c^2}$																							

$$\begin{aligned}
T_{ij}^E &= \frac{1}{4\pi}(E_i E_j - \tfrac{1}{2}\delta_{ij} E^2); \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B \\
-\frac{\partial u}{\partial t} &= \vec{J}\vec{E} + \vec{\nabla}\vec{S}; \frac{\partial \vec{g}}{\partial t} = -\vec{f} + \partial_j T_{ij} \hat{x}_i \\
\vec{B} &= \vec{\nabla} \times \vec{A}; \vec{E} = -\vec{\nabla}\phi - \tfrac{1}{c}\frac{\partial \vec{A}}{\partial t} \\
&-\nabla^2 \phi - \tfrac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho \\
\vec{\nabla}(\vec{\nabla}\vec{A} + \tfrac{1}{c}\frac{\partial \phi}{\partial t}) - \nabla^2 \vec{A} + \tfrac{1}{c}\frac{\partial^2 \vec{A}}{\partial t^2} &= 4\pi\frac{\vec{J}}{c} \\
(\phi, \vec{A}) &\cong (\phi - \tfrac{1}{c}\frac{\partial \chi}{\partial t}, \vec{A} + \vec{\nabla}\chi) \\
(\phi, \vec{A}) &= \int d^3r' \frac{(\rho, \frac{\vec{J}}{c})(\vec{r}', t - \tfrac{1}{c}|\vec{r} - \vec{r}'|)}{|\vec{r} - \vec{r}'|} \\
\text{Coulomb gauge: } \vec{\nabla}\vec{A} &= 0
\end{aligned}$$

Electromagnetism in matter (CGS)

$$\begin{aligned}
\vec{P} &= \frac{\langle \vec{d} \rangle}{V}; \vec{M} = \frac{\langle \vec{m} \rangle}{V} \\
\rho_{\text{pol}} &= -\vec{\nabla}\vec{P}; \sigma_{\text{pol}} = \hat{n}\vec{P}; \frac{\vec{J}_{\text{mag}}}{c} = \vec{\nabla} \times \vec{M} \\
\vec{D} &= \vec{E} + 4\pi\vec{P}; \vec{H} = \vec{B} - 4\pi\vec{M} \\
\vec{\nabla}\vec{D} &= 4\pi\rho_{\text{ext}}; \vec{\nabla} \times \vec{E} = -\tfrac{1}{c}\frac{\partial \vec{B}}{\partial t} \\
\vec{\nabla}\vec{B} = 0; \vec{\nabla} \times \vec{H} &= 4\pi\frac{\vec{J}_{\text{ext}}}{c} + \tfrac{1}{c}\frac{\partial \vec{D}}{\partial t} \\
\text{static linear isotropic: } \vec{P} &= \chi\vec{E} \\
\text{static linear: } P_i &= \chi_{ij}E_j \\
\text{static linear: } \varepsilon &= 1 + 4\pi\chi
\end{aligned}$$

$$\text{Lorenz gauge: } \vec{\nabla}\vec{A} + \tfrac{1}{c}\frac{\partial \phi}{\partial t} = 0$$

$$\vec{E}'\hat{v} = \vec{E}\hat{v}; \vec{B}'\hat{v} = \vec{B}\hat{v}$$

$$\vec{E}' \times \hat{v} = \gamma(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}) \times \hat{v}$$

$$\vec{B}' \times \hat{v} = \gamma(\vec{B} - \frac{\vec{v}}{c} \times \vec{E}) \times \hat{v}$$

$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \hat{k} \times \vec{E} \\ \omega = ck \end{cases}$$

$$\text{dipole: } \vec{B}|_{r \gg \frac{c}{\omega}} \approx \frac{1}{c^2} \frac{\ddot{\vec{p}} \times \hat{r}}{r}; \vec{E} \approx \vec{B} \times \hat{r}$$

$$A^\mu = (\phi, \vec{A}); J^\mu = (c\rho, \vec{J})$$

$$\text{static: } \Delta D_\perp = 4\pi\sigma_{\text{ext}}; \Delta E_\parallel = 0$$

$$\text{static linear: } u = \frac{1}{8\pi} \vec{E}\vec{D}$$

$$\Delta U_{\text{dielectric}} = \frac{1}{2} \int d^3r \vec{P}\vec{E}_0$$

$$\text{plane capacitor: } C = \frac{\varepsilon}{4\pi} \frac{S}{d}$$

$$\text{non-interacting gas: } \vec{d} = \alpha\vec{E}_0; \chi = n\alpha$$

$$\text{hom. cubic isotropic: } \chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$$

$$\text{Clausius-Mossotti: } \frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3} n\alpha$$

$$\chi = \frac{4\pi}{3} \frac{np_0^2}{kT}; \vec{E}_e = \vec{E} + \frac{4\pi}{3} \vec{P}$$

$$\vec{J}\vec{E} = -\vec{\nabla}\left(\frac{c}{4\pi}\vec{E} \times \vec{H}\right) - \frac{1}{4\pi}\left(\vec{E}\frac{\partial \vec{D}}{\partial t} + \vec{H}\frac{\partial \vec{B}}{\partial t}\right)$$

$$n = \sqrt{\varepsilon\mu}; k = n\frac{\omega}{c}$$

$$\text{Lorenz gauge: } \partial_\mu A^\mu = 0$$

$$\partial_\mu F^{\mu\nu} = 4\pi\frac{J^\nu}{c}; F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\mathcal{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$F^{\mu\nu}F_{\mu\nu} = E^2 - B^2; F^{\mu\nu}\mathcal{F}_{\mu\nu} = 4\vec{E}\vec{B}$$

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}(g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} - \tfrac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda})$$

$$\Theta^{\alpha\beta} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T} \end{pmatrix}$$

$$\partial_\mu \Theta^{\mu\nu} = -\tfrac{1}{c}F^{\nu\lambda}J_\lambda = \tfrac{1}{c}J_\lambda F^{\lambda\nu}$$

$$\vec{J}_c = \sigma\vec{E}; \tilde{\varepsilon} = \varepsilon + i\frac{4\pi\sigma}{\omega}$$

$$\text{I: } u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$$

$$\text{I: } \langle u \rangle = \frac{1}{16\pi}(\varepsilon E^2 + \mu H^2)$$

$$\text{I: } \langle S_z \rangle = \frac{c}{n} \langle u \rangle$$

$$\text{II: } \langle u \rangle = \frac{1}{16\pi}(\frac{\partial}{\partial\omega}(\varepsilon_\omega\omega)\langle E^2 \rangle + \frac{\partial}{\partial\omega}(\mu_\omega\omega)\langle H^2 \rangle)$$

$$\text{II: } \langle S_z \rangle = v_g \langle u \rangle; v_g = \frac{\partial\omega}{\partial k}$$

$$\text{III: } \langle W \rangle = \frac{\omega}{8\pi}(\text{Im } \varepsilon_\omega |E_0^2| + \text{Im } \mu_\omega |H_0^2|)$$

$$\text{Fresnel TE}_{\mu=1}: E_t = \frac{2E_i}{1 + \frac{k_{tz}}{k_{iz}}}; E_r = \frac{1 - \frac{k_{tz}}{k_{iz}}}{1 + \frac{k_{tz}}{k_{iz}}} E_i$$

$$\text{Fresnel: } k_{tz} = \pm\sqrt{\tilde{\varepsilon}\left(\frac{\omega}{c}\right)^2 - k_x^2}, \text{Im } k_{tz} > 0$$