Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$
$$\tan\tfrac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$
$$\operatorname{acos}x+\operatorname{asin}x=\tfrac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\binom{\sinh x}{\cosh x}=\frac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\frac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\binom{\operatorname{asinh}x}{\operatorname{acosh}x}=\log\Big(x+\sqrt{x^2+\binom{1}{-1}}\Big)$$
$$\operatorname{atanh}x=\tfrac{1}{2}\log\frac{1+x}{1-x}$$

Areas

$$\text{triangle: }\sqrt{p(p-a)(p-b)(p-c)}$$
$$\text{quad: }\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\tfrac{\alpha+\gamma}{2}}$$
$$\text{Pick: }A=\big(I+\tfrac{B}{2}-1\big)\,A_{\text{check}}$$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{x=1}^n x^3=\big(\sum_{x=1}^n x\big)^2=\frac{1}{4}n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\textstyle\int_0^\infty t^ze^{-t}dt=z!$$
$$n!\approx(\tfrac{n}{e})^n\sqrt{2\pi n}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\frac{\operatorname{Re}z+|z|}{2}}+\frac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$
$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\tfrac{1}{2}\operatorname{Re}(ab^*)$$
$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$$
$$f^{(n)}(z_0)=\frac{n!}{2\pi i}\oint\frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$
$$f(z)=\sum_{k=-\infty}^\infty\Big(\frac{1}{2\pi i}\oint\frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\Big)(z-z_0)^k$$
$$\operatorname{sinc}x:=\frac{\sin x}{x}$$

Derivatives

$$\tan' x=1+\tan^2x$$
$$\cot' x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\frac{1}{1+x^2}$$
$$\operatorname{asin}'x=-\operatorname{acos}'x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a' x=\frac{1}{x\ln a}$$
$$\cosh' x=\sinh x$$
$$\tanh' x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\frac{1}{1-x^2}$$
$$\operatorname{asinh}'x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}'x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$
$$\big(\tfrac{1}{x}\big)'=-\frac{\dot x}{x^2}$$
$$\big(\tfrac{x}{y}\big)'=\frac{\dot xy-x\dot y}{y^2}$$
$$(x^y)'=x^y\big(\dot y\ln x+y\tfrac{\dot x}{x}\big)$$
$$\frac{\partial(x,y)}{\partial(u,v)}:=\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}-\frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$
$$\frac{\partial(x,y)}{\partial(u,y)}=\frac{\partial x}{\partial u}\Big|_y=-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x$$
$$\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x\frac{\partial u}{\partial x}\Big|_y=-1$$
$$\frac{\partial x}{\partial u}\Big|_y=\frac{\partial x}{\partial u}\Big|_v-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_v$$
$$\frac{\partial x}{\partial u}\Big|_v=\frac{\partial x}{\partial y}\Big|_v\frac{\partial y}{\partial u}\Big|_v$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac{1}{x}=\ln|x|$$
$$\textstyle\int\tan x=-\ln|\cos x|$$
$$\textstyle\int\cot x=\ln|\sin x|$$
$$\textstyle\int\frac{1}{\sin x}=\ln\big|\tan\frac{x}{2}\big|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\big|\tan\big(\frac{x}{2}+\frac{\pi}{4}\big)\big|$$
$$\textstyle\int\ln x=x(\ln x-1)$$
$$\textstyle\int\tanh x=\ln\cosh x$$
$$\textstyle\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\textstyle\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy=x\int y-\int(\dot x\int y)$$
$$\textstyle\int_{-\infty}^\infty e^{-x^2}=\sqrt{\pi}$$
$$\textstyle\int_{-\infty}^\infty e^{i\omega t}\mathrm{d}t=2\pi\delta(\omega)$$

Differential equations

$$\dot x+\dot ax=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$
$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot x+\gamma\dot x+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac{1}{x}+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac{1}{x}-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$
$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac{1}{x}-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac{1}{x}+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac{5}{6}x^4-\tfrac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Fourier

$$\text{Fourier: }c_n=\tfrac{2}{T}\textstyle\int_0^T f(t)\cos\big(n\frac{t}{T}\big)\mathrm{d}t$$
$$\mathcal{F}[f](\omega)=\hat f(\omega)=\textstyle\int\mathrm{d}te^{i\omega t}f(t)$$
$$f,g\in L^2:(\hat f,\hat g)=2\pi(f,g)$$
$$\mathcal{F}\big[\tfrac{\sin t}{t}\big]=\sqrt{\tfrac{\pi}{2}}\chi_{[-1;1]}(\omega)$$
$$t^{k\le n}f(t)\in L^1:\mathcal{F}[t^nf(t)]=(-i)^n\hat f^{(n)}$$
$$f^{(k\le n)}\in L^1:\mathcal{F}[f^{(n)}]=(-i\omega)^n\hat f$$
$$\mathcal{F}^2f=2\pi f(-t);\ (\omega\hat f)'=-\mathcal{F}[tf']$$
$$f\star g=g\star f;\ \hat f\star\hat g=2\pi\mathcal{F}[fg]$$
$$f\in L^1,\ g\in L^p:\mathcal{F}[f\star g]=\hat f\hat g$$
$$f\star g(x)=\textstyle\int f(x-y)g(y)\mathrm{d}y$$
$$(f\star g)'=f'\star g=f\star g'$$
$$f(x+\Delta)\star g=f\star g(x+\Delta)$$
$$f\in L^1,\ g\in L^p\ \Rightarrow\ f\star g\in L^p$$
$$f,g\in L^2:f\star g=\tfrac{1}{2\pi}\textstyle\int\hat f\hat ge^{-i\omega t}\mathrm{d}\omega$$
$$\|f\|=1:\Delta\omega\Delta t\ge\tfrac{1}{2}$$
$$\Delta\omega\Delta t=\tfrac{1}{2}:f(t)=g(t;\bar t,\Delta t)e^{-i\bar\omega t}$$

Distributions

$\mathcal{D} := \{f \in C^\infty \mid \exists K \text{ compact} : f(\complement K) = 0\}$

$\mathcal{S} := \{f \in C^\infty \mid |x^n f^{(k)}| \le A_{nk}\} \supset \mathcal{D}$

$\langle 1, f\rangle := \int f;\ \langle gT, f\rangle := \langle T, gf\rangle$

$T \in \mathcal{S}' : \langle \mathcal{F}T, f\rangle := \langle T, \mathcal{F}f\rangle$

$\langle T', f\rangle := -\langle T, f'\rangle;\ \langle \delta, f\rangle := f(0)$

$\langle T\otimes S,\phi\rangle := \langle T(x), \langle S(y), \phi(x+y)\rangle\rangle$

$\langle T\star S, \phi\rangle := \langle T\otimes S, \phi(x+y)\rangle$

$\mathcal{F}1 = 2\pi\delta(\omega);\ \mathcal{F}\,\mathrm{sgn} = 2i\mathcal{P}\frac{1}{\omega}$

$\mathcal{F}\theta = i\mathcal{P}\frac{1}{\omega} + \pi\delta(\omega)$

$x^nT = 0 \ \Rightarrow\ T = \sum_{k=0}^{n-1} a_k\delta^{(k)}$

$xT = S \ \Rightarrow\ T = S/x + k\delta$

$T,S\in\mathcal{D}' : T\otimes S = S\otimes T$

$\sum_{n=0}^\infty e^{inx} = \mathcal{P}\frac{1}{1-e^{ix}} + \pi\sum_{n=-\infty}^{\infty}\delta(x-2n\pi)$

$\delta^{(n)}\star f = f^{(n)}$

$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$

Bessel functions

sol. of $x^2\partial_x^2 f + x\partial_x f + (x^2-\alpha^2)f = 0$

$\alpha =$ "order"

$J_\alpha =$ "first kind, normal"

$\alpha\in\mathbb{Z}_0 \vee \alpha > 0 : J_\alpha(0) = 0$

$J_0(0) = 1;$ otherwise $|J_\alpha(0)| = \infty$

$\alpha\notin\mathbb{Z} : J_\alpha, J_{-\alpha}$ indep.

$\alpha\in\mathbb{Z} : J_{-\alpha} = (-1)^\alpha J_\alpha$

$Y_\alpha =$ "second kind, normal" (also N_α)

$\alpha\notin\mathbb{Z} : Y_\alpha = \frac{\cos(\alpha\pi)J_\alpha - J_{-\alpha}}{\sin(\alpha\pi)}$

$\alpha\in\mathbb{Z} : Y_\alpha = \lim_{\alpha'\to\alpha} Y_{\alpha'}$

$\alpha\in\mathbb{Z} : Y_\alpha, J_\alpha$ indep.

$\alpha\in\mathbb{Z} : Y_{-\alpha} = (-1)^\alpha Y_\alpha$

$\frac{2\alpha}{x}J_\alpha(x) = J_{\alpha-1}(x) + J_{\alpha+1}(x)$

$2J'_\alpha(x) = J_{\alpha-1}(x) - J_{\alpha+1}(x)$

$\int_0^1 \mathrm{d}x\, x J_\alpha(xu_{\alpha,m})J_\alpha(xu_{\alpha,n}) = \frac{\delta_{mn}}{2}J^2_{\alpha+1}(u_{\alpha,m})$

$u_{\alpha,n} = n$th. zero of J_α

Cylindrical harmonics

$V(\rho,\phi,z) = \sum_{n=0}^\infty \int \mathrm{d}k A_{nk}P_{nk}(\rho)\Phi_n(\phi)Z_k(z)$

$P_{nk}(\rho) =$ comb. of $J_n(k\rho), Y_n(k\rho)$

$\Phi_n(\phi) =$ comb. of $e^{\pm in\phi}$

$Z_k(z) =$ comb. of $e^{\pm kz}$

Inequalities

$|a| - |b| \le |a+b| \le |a| + |b|$

$x > -1 : 1 + nx \le (1+x)^n$

$\frac{|a^n-b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$

$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$

$\sum a_ib_i \le \left(\sum a_i^p\right)^{\frac1p}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^py^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac1n\sum a_i^{p\le q}} \le \sqrt[q]{\frac1n\sum a_i^q}$

$\sum\left(\frac{a_1+\dots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p\sum a_i^p$

$x\ge0, |\ddot x|\le M : |\dot x| \le \sqrt{2Mx}$

$\frac{1}{1+x} < \ln\left(1+\frac1x\right) < \frac1x$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$

$\dim V = \dim\ell(V) + \dim(V\cap\ker\ell)$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772\cdot10^{-1}$

$G = 6.674\cdot10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206\cdot10^{-2}\,\frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022\cdot10^{23}\,\frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381\cdot10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617\cdot10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998\cdot10^8\,\frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602\cdot10^{-19}\,\mathrm{A\,s}$

$m_\mathrm{e} = 9.109\cdot10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673\cdot10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675\cdot10^{-27}\,\mathrm{kg}$

$m_\mathrm{e} = 5.110\cdot10^{-1}\,\mathrm{MeV}$

$m_\mathrm{p} = 9.383\cdot10^{2}\,\mathrm{MeV}$

$m_\mathrm{n} = 9.396\cdot10^{2}\,\mathrm{MeV}$

$m_\mathrm{n} - m_\mathrm{p} = 1.293\,\mathrm{MeV}$

$\mathrm{amu} = 1.661\cdot10^{-27}\,\mathrm{kg}$

$h = 6.626\cdot10^{-34}\,\mathrm{J\,s}$

$h = 4.136\cdot10^{-15}\,\mathrm{eV\,s}$

$\varepsilon_0 = 8.854\cdot10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988\cdot10^{9}\,\frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257\cdot10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274\cdot10^{-24}\,\mathrm{A\,m}^2$

$\alpha = 7.297\cdot10^{-3}$

$\mathrm{barn} = 1\cdot10^{-28}\,\mathrm{m}^2$

$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464\cdot10^{-3}\,\frac{\mathrm{W}}{\mathrm{sr}}$

$r_B = 5.292\cdot10^{-11}\,\mathrm{m}$

$\mathrm{Rydberg} = 1.361\cdot10^{1}\,\mathrm{eV}$

$r_e = 2.818\cdot10^{-15}\,\mathrm{m}$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i=j\vee j=k\vee k=i \\ 1 & i+1\equiv j\wedge j+1\equiv k \\ -1 & i\equiv j+1\wedge j\equiv k+1 \end{cases}$$

$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

$\vec a\times\vec b = \varepsilon_{ijk}a_jb_k\hat e_i;\ (\vec a\otimes\vec b)_{ij} = a_ib_j$

$(\vec a\times\vec b)\vec c = (\vec c\times\vec a)\vec b$

$(\vec a\times\vec b)\times\vec c = -(\vec b\vec c)\vec a + (\vec a\vec c)\vec b$

$(\vec a\times\vec b)(\vec c\times\vec d) = (\vec a\vec c)(\vec b\vec d) - (\vec a\vec d)(\vec b\vec c)$

$|\vec u\times\vec v|^2 = u^2v^2 - (\vec u\vec v)^2$

$\vec\nabla = \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right);\ \Box = \frac{\partial^2}{\partial t^2} - \nabla^2$

$\vec\nabla V = \frac{\partial V}{\partial\rho}\hat\rho + \frac1\rho\frac{\partial V}{\partial\phi}\hat\phi + \frac{\partial V}{\partial z}\hat z$

$\vec\nabla\vec v = \frac1\rho\frac{\partial(\rho v_\rho)}{\partial\rho} + \frac1\rho\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

$\vec\nabla\times\vec v = \left(\frac1\rho\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat\rho +$

$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right)\hat\phi + \frac1\rho\left(\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\phi}\right)\hat z$

$\nabla^2V = \frac1\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2} + \frac{\partial^2V}{\partial z^2}$

$\vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac1r\frac{\partial V}{\partial\theta}\hat\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$

$\vec\nabla\vec v = \frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$

$\vec\nabla\times\vec v = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\right)\hat r +$

$+\frac1r\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(rv_\varphi)}{\partial r}\right)\hat\theta + \frac1r\left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat\varphi$

$\nabla^2V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = \frac1r\frac{\partial^2}{\partial r^2}(rV) = \frac2r\frac{\partial V}{\partial r} + \frac{\partial^2V}{\partial r^2}$

$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$

$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$

$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$

$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$

$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$

$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\vec\nabla)\vec w$

$\frac12\vec\nabla v^2 = (\vec v\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$

$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$

$\int(f\nabla^2g - g\nabla^2f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$

$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$

$\delta(\vec r-\vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r_0^2\sin\theta_0}$

$\nabla^2\frac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$

Statistics

$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$

$\Delta x_\mathrm{hist} \approx \frac{x_\mathrm{max}-x_\mathrm{min}}{\sqrt N}$

$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$

$\mathrm{median} = F^{-1}(\frac12)$

$E[f(x)] = \int_{-\infty}^\infty f(x)p(x)$

$\mu = E[x] = \int_{-\infty}^\infty xp(x)$

$\alpha_n = E[x^n]$

$M_n = E[(x-\mu)^n]$

$\sigma^2 = M_2 = E[x^2] - \mu^2$

$\mathrm{FWHM} \approx 2\sigma$

$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$

$\phi[y](t) = E[e^{ity}]$

$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$

$\alpha_n = i^{-n}\frac{\partial^nt}{\partial\phi[x]^n}\Big|_{t=0}$

$h\ge0 : P(h\ge k) \le \frac{E[h]}{k}$

$P(|x-\mu| > k\sigma) \le \frac{1}{k^2}$

$B(k;n,p) = \binom{n}{k}p^k(1-p)^{n-k}$

$\mu_B = np,\ \sigma_B^2 = np(1-p)$

$P(k;\mu) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$

$u(x;a,b) = \frac{1}{b-a},\ x\in[a;b]$

$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$

$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge0$

$\mu_\varepsilon = \frac1\lambda,\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$

$g(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac12\left(\frac{x-\mu}{\sigma}\right)^2}$

$g(\vec x;\vec\mu,V) = \frac{e^{-\frac12(\vec x-\vec\mu)^\mathrm{T}V^{-1}(\vec x-\vec\mu)}}{\sqrt{\det(2\pi V)}}$

$\mathrm{FWHM}_g = 2\sigma\sqrt{2\ln2}$

$z = \frac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$

$\chi^2 = \sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$

$\wp(x;n) = \frac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$

$\mu_\wp = n,\ \sigma_\wp^2 = 2n$

$n \ge 30 : \wp(x;n) \approx g(x;n,\sqrt{2n})$

$n \ge 8 : p[\sqrt{2\chi^2}] \approx g(;\sqrt{2n-1},1)$

$S(x;n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$

$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$

$p\left[z\sqrt{\frac{n}{\chi^2}}\right] = S(,n)$

$n \ge 35 : S(x;n) \approx g(x;0,1)$

$c(x;a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$

$\sigma_{xy} = E[xy] - \mu_x\mu_y \le \sigma_x\sigma_y$

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}| \le 1$

$\mu_{f(x)} \approx f(\mu_x)$

$\sigma_{fg} \approx \sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_{x_i}}\frac{\partial g}{\partial x_j}\Big|_{\mu_{x_j}}$

$\mu \approx m = \frac{1}{n}\sum_{i=1}^n x_i$

$\sigma^2 \approx s^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - m)^2$

$s_m^2 = \frac{s^2}{n}$

$p\left[\frac{m-\mu}{s_m}\right] = S(;n)$

Fit (ML)

$f(x) = mx + q, \quad f(x) = a,$

$f(x) = bx, \quad f(x;\theta) = \theta_i h_i(x)$

$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta mq = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$

$\mathbf{a} = (\sum V_{\mathbf{y}}^{-1})^{-1}(\sum V_{\mathbf{y}}^{-1}\mathbf{y})$

$\Delta\mathbf{a}^2 = (\sum V_{\mathbf{y}}^{-1})^{-1}$

$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$

$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$

$\chi^2 = (y - f(x;\theta))^T V^{-1}(y - f(x;\theta))$

$\theta = (H^T V^{-1} H)^{-1} H^T V^{-1} y$

$\Delta\theta\theta = (H^T V^{-1} H)^{-1}$

Kinematics

$\frac{1}{R} = \left|\frac{v_x a_y - v_y a_x}{v^3}\right|$

$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$

$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$

$\theta \equiv \frac{\pi}{2} \to \dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$

$\theta \equiv \frac{\pi}{2} \to \ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi$

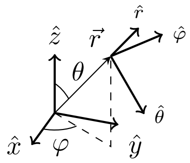
$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$

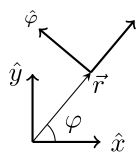
$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$

$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$

$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$

$\vec A = \ddot{\vec r} + \vec A_{\mathrm{T}} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$





Mechanics

$\dot\alpha = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta + \frac{\partial\alpha}{\partial t}$

$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec F\vec p$

$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$

$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}m_i(\dot{\vec r}_i - \dot{\vec R})^2$

$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$

$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$

$\tau_1 = I_1\dot\omega_1 + (I_3 - I_2)\omega_3\omega_2$

$\mathcal{L}(q,\dot q,t) = T - V + \frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$

$S[q] = \int_{t_1}^{t_2}\mathcal{L}(q,\dot q,t)\,\mathrm{d}t$

$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$

$p := \frac{\partial\mathcal{L}}{\partial\dot q};\ \dot p = \frac{\partial\mathcal{L}}{\partial q}$

$\mathcal{H}(q,p,t) = \dot q p - \mathcal{L}$

$\dot q = \frac{\partial\mathcal{H}}{\partial p};\ \dot p = -\frac{\partial\mathcal{H}}{\partial q}$

$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = \frac{\partial\mathcal{H}}{\partial t} = -\frac{\partial\mathcal{L}}{\partial t}$

$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$

$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal{H}\} + \frac{\partial u}{\partial t}$

$\eta = (q,p);\ \Gamma = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

$\dot\eta = \Gamma\frac{\partial\mathcal{H}}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac{1}{2}mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac{2}{3}mr^2$

ball: $\frac{2}{5}mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2 + \frac{3}{4}r^2\right)$

ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$\langle U\rangle = -2\langle T\rangle$

$U_{\mathrm{eff}} = U + \frac{L^2}{2mr^2}$

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$

$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}\mu\dot{\vec r}^2$

$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$

$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}}$

$r = \frac{k}{1+\varepsilon\cos\theta}$

$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$

$a^3\omega^2 = G(m_1+m_2) = \frac{\alpha}{\mu}$

$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec p = \gamma m\vec v;\ \mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \frac{mc^2}{\gamma}$

$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec v\frac{\mathrm{d}\vec p}{\mathrm{d}t};\ \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x}$

$\begin{pmatrix}ct'\\ x'\end{pmatrix} = \gamma\begin{pmatrix}1 & -\beta\\ -\beta & 1\end{pmatrix}\begin{pmatrix}ct\\ x\end{pmatrix}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct,\vec x)$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c,\vec v)$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec v)}{\mathrm{d}t}\right)$

$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c},\vec p\right)$

$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\vec v}{c}\frac{\mathrm{d}\vec p}{\mathrm{d}t}, \frac{\mathrm{d}\vec p}{\mathrm{d}t}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t},\vec\nabla\right)$

$g_{\mu\nu} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu\partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$

$\mathrm{SO}_{1,3} = \left\{\Lambda\ \middle|\ \begin{smallmatrix}\Lambda^{\mathrm{T}}g\Lambda = g\\ \det\Lambda \ge 0\end{smallmatrix}\right\}$

$({\Lambda^0}_0)^2 \ge 1$

$\Lambda = \begin{pmatrix}\gamma & -\gamma\vec\beta\\ -\gamma\vec\beta & I + \frac{\gamma-1}{\beta^2}\vec\beta\otimes\vec\beta\end{pmatrix}$

$M \to \sum_i m_i$

$E_1^{\max} = \frac{M^2 + m_1^2 - \sum_{i\ne 1}m_i^2}{2M}$

$A + B_{\mathrm{still}} \to \sum_i m_i$

$E_A^{\min} = \frac{(\sum_i m_i)^2 - m_A^2 - m_B^2}{2m_B}$

m, M_{still} 1D coll.

$E'_m = \frac{(M+m)^2E_m + 2Mm^2}{M^2 + m^2 + 2ME_m}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu\mathrm{d}N$

$C_{V,N} = \frac{\partial Q}{\partial T}\Big|_{V,N} = \frac{\partial U}{\partial T}\Big|_{V,N}$

$C_{p,N} = \frac{\partial Q}{\partial T}\Big|_{p,N} = \frac{\partial U}{\partial T}\Big|_{p,N} + p\frac{\partial V}{\partial T}\Big|_{p,N}$

$\gamma := \frac{C_p}{C_V}$

$\mu_J := \frac{\partial T}{\partial V}\Big|_{U,N}$

$\lambda U = U(\lambda(S,V,N)) \Rightarrow U = ST - pV + \mu N$

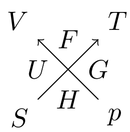
$\Rightarrow\ S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix S,V,N : $\min U$ at equilibrium

Fix T,V,N : $\min F = U - TS$

Fix T,p,N : $\min G = F + pV$

Fix S,p,N : $\min H = U + pV$



$\frac{\partial}{\partial T}\frac{G}{T}\Big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\Big|_V = -\frac{U}{T^2}$

$\Omega = U - TS - \mu N$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int\mathrm{d}q_1\cdots\mathrm{d}q_N\int\mathrm{d}p_1\cdots\mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_{\mathrm{B}}T};\ C = \frac{\partial U}{\partial T}$

$F(T,V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\begin{pmatrix}V\\ I\end{pmatrix} = \begin{pmatrix}V_0\\ I_0\end{pmatrix}e^{i\omega t},\ Z = \frac{V}{I}$

$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$

$Z_{\mathrm{series}} = \sum_k Z_k,\ \frac{1}{Z_{\mathrm{parallel}}} = \sum_k\frac{1}{Z_k}$

$\sum_{\mathrm{loop}}V_k = 0,\ \sum_{\mathrm{node}}I_k = 0$

$\mathcal{E} = -L\dot I,\ L = \frac{\Phi_B}{I}$

$I_{A\to C} = I_0\left(e^{\frac{V_{AC}}{V_T}} - 1\right),\ V_T = \eta\frac{k_{\mathrm{B}}T}{q_{\mathrm{e}}}$

$I_{E,\mathrm{out}} = I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right) - \alpha_R I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$

$I_{C,\mathrm{in}} = -I_0^C\left(e^{\frac{V_{BC}}{V_T}} - 1\right) + \alpha_F I_0^E\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$

Chemistry

$$H = U + pV$$

$$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$$

$$G = H - TS$$

$$a_i \mathrm{A}_i \;\rightarrow\; b_j \mathrm{B}_j$$

$$\Delta H^{\mathrm{o}}_{\mathrm{r}} = b_j \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{B}_j) - a_i \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{A}_i)$$

$$\forall\, i, j : v_{\mathrm{r}} = -\tfrac{1}{a_i}\tfrac{\Delta[\mathrm{A}_i]}{\Delta t} = \tfrac{1}{b_j}\tfrac{\Delta[\mathrm{B}_j]}{\Delta t}$$

$$\exists\, k, (m_i) : v_{\mathrm{r}} = k[\mathrm{A}_i]^{m_i}$$

$$k = A e^{-\frac{E_{\mathrm{a}}}{RT}} \text{ (Arrhenius)}$$

$$a_{(\ell)} = \gamma \tfrac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\,\tfrac{\mathrm{mol}}{\mathrm{l}}$$

$$a_{(g)} = \gamma \tfrac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$$

$$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$$

$$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$$

$$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \tfrac{n}{n_{\mathrm{tot}}}$$

$$K_c = K_p (RT)^{\sum a_i - \sum b_j}$$

$$K_c = K_n V^{\sum a_i - \sum b_j}$$

$$K_\chi = K_n n_{\mathrm{tot}}^{\sum a_i - \sum b_j}$$

$$\Delta G^{\mathrm{o}}_{\mathrm{r}} = -RT \ln K$$

$$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$$

$$\Delta G = RT \ln \tfrac{Q}{K}$$

$$\ln \tfrac{K_2}{K_1} = -\tfrac{\Delta H^{\mathrm{o}}}{R}\left(\tfrac{1}{T_2} - \tfrac{1}{T_1}\right)$$

$$K_{\mathrm{w}} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$$

$$\Delta E = \Delta E^{\mathrm{o}} - \tfrac{RT}{n_{\mathrm{e}} N_A q_{\mathrm{e}}} \ln Q \text{ (Nerst)}$$

$$\text{(std) } \Delta E = \Delta E^{\mathrm{o}} - \tfrac{0.059}{n_{\mathrm{e}}} \log_{10} Q$$

$$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$$

$$K_a = \tfrac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$ $\quad \vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \rho, \vec{J}, I, \vec{P} / \sqrt{4\pi\varepsilon_0}$ $\quad \vec{H} \times \sqrt{4\pi\mu_0}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \mu/\mu_0$ $\quad L \times 4\pi\varepsilon_0$

$\vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$ $\quad \varepsilon/\varepsilon_0$ $\quad R, Z \times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec{F}_{12} = q_1 q_2 \tfrac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \tfrac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \tfrac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$$

$$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi \int \rho\, \mathrm{d}^3 x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla} \times \vec{E} = 0$$

$$U = \tfrac{1}{8\pi}\int E^2\, \mathrm{d}^3 x;\ \tilde{U} = \tfrac{1}{2}\tfrac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \tfrac{1}{8\pi}\sum_{i\neq j}\int \vec{E}_i \vec{E}_j\, \mathrm{d}^3 x$$

$$V(\vec{r}) = \int \rho G_{\mathrm{D}}(\vec{r})\, \mathrm{d}^3 x - \tfrac{1}{4\pi}\oint_S V \tfrac{\partial G_{\mathrm{D}}}{\partial n}\, \mathrm{d}S$$

$$V(\vec{r}) = \langle V \rangle_S + \int \rho G_{\mathrm{N}}(\vec{r})\, \mathrm{d}^3 x + \tfrac{1}{4\pi}\oint_S \tfrac{\partial V}{\partial n} G_{\mathrm{N}}(\vec{r})\, \mathrm{d}S$$

$$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_{\mathrm{D}}(\vec{x}, \vec{y})\big|_{\vec{y}\in S} = 0;\ \tfrac{\partial G_{\mathrm{N}}}{\partial n}\big|_{\vec{y}\in S} = -\tfrac{4\pi}{S}$$

$$U_{\mathrm{sphere}} = \tfrac{3}{5}\tfrac{Q^2}{R};\ \vec{p} = \int \mathrm{d}^3 r \rho \vec{r};\ \vec{E}_{\mathrm{dip}} = \tfrac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_{\mathrm{dip}} = \tfrac{\vec{p}\hat{r}}{r^2}$$

$$\text{force on a dipole: } \vec{F}_{\mathrm{dip}} = (\vec{p}\vec{\nabla})\vec{E}$$

$$Q_{ij} = \int \mathrm{d}^3 r \rho(\vec{r})(3 r_i r_j - \delta_{ij} r^2);\ V_{\mathrm{quad}} = \tfrac{1}{6r^5} Q_{ij}(3 r_i r_j - \delta_{ij} r^2)$$

$$V(r, \theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \tfrac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

$$V(r, \theta, \varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(A_{lm} r^l + \tfrac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$$

$$\tfrac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^{\infty}\tfrac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\tfrac{\vec{r}\vec{r}'}{rr'}\right)$$

$$P_l(x) = \tfrac{1}{2^l l!}\tfrac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l;\ f = \sum_{l=0}^{\infty} c_l P_l : c_l = \tfrac{2l+1}{2}\int_{-1}^{1} f P_l$$

$$P_l(1) = 1;\ (P_n, P_m) = \tfrac{2\delta_{nm}}{2n+1};\ (Y_{lm}, Y_{l'm'}) = \delta_{ll'}\delta_{mm'}$$

$$P_0 = 1;\ P_1 = x;\ P_2 = \tfrac{3x^2 - 1}{2};\ Y_{00} = \tfrac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\tfrac{3}{4\pi}}\cos\theta$$

$$Y_{11} = -\sqrt{\tfrac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\tfrac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_{21} = -\sqrt{\tfrac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\tfrac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x) = \tfrac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\tfrac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l,\ 0 \le m \le l$$

$$Y_{lm}(\theta, \varphi) = \sqrt{\tfrac{2l+1}{4\pi}\tfrac{(l-m)!}{(l+m)!}}\, e^{im\varphi} P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m Y_{lm}^*$$

$$P_l\left(\tfrac{\vec{r}\vec{r}'}{rr'}\right) = \tfrac{4\pi}{2l+1}\sum_{m=-l}^{l} Y_{lm}^*(\theta', \varphi') Y_{lm}(\theta, \varphi)$$

$$V(r > \operatorname{diam}\operatorname{supp}\rho, \theta, \varphi) = \sum_{l=0}^{\infty}\tfrac{4\pi}{2l+1}\sum_{m=-l}^{l} q_{lm}[\rho]\tfrac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^\infty r^2 \mathrm{d}r \int_0^{2\pi}\mathrm{d}\varphi \int_0^\pi \sin\theta\, \mathrm{d}\theta\, r^l \rho(r, \theta, \varphi) Y_{lm}^*(\theta, \varphi)$$

Magnetostatics (CGS)

$$\vec{\nabla}\vec{J} = -\tfrac{\partial\rho}{\partial t} = 0;\ I = \int \vec{J}\mathrm{d}\vec{S}$$

$$\text{solenoid: } B = 4\pi\tfrac{j_{\mathrm{s}}}{c}$$

$$\mathrm{d}\vec{F} = \tfrac{I\mathrm{d}\vec{l}}{c} \times \vec{B} = \mathrm{d}^3 x \tfrac{\vec{J}}{c} \times \vec{B};\ \vec{F}_q = q\tfrac{\dot{\vec{r}}}{c} \times \vec{B}$$

$$\mathrm{d}\vec{B} = \tfrac{I\mathrm{d}\vec{l}}{c} \times \tfrac{\vec{r}}{r^3};\ \vec{B}_q = q\tfrac{\dot{\vec{r}}}{c} \times \tfrac{\vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{A} = \int \mathrm{d}^3 r' \tfrac{\vec{J}'}{c}\tfrac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla} A_0$$

$$\vec{B} = \int \mathrm{d}^3 r' \tfrac{\vec{J}'}{c} \times \tfrac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\varphi = \tfrac{I}{c}\Omega,\ \vec{B} = -\vec{\nabla}\varphi$$

$$\vec{\nabla}\vec{A} = 0 \to \nabla^2\vec{A} = -4\pi\tfrac{\vec{J}}{c}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{B} = 4\pi\tfrac{\vec{J}}{c};\ \oint \vec{B}\mathrm{d}\vec{l} = 4\pi\tfrac{I}{c}$$

$$\vec{m} = \tfrac{1}{2}\int \mathrm{d}^3 r' \left(\vec{r}' \times \tfrac{\vec{J}'}{c}\right) = \tfrac{1}{2c}\tfrac{q}{m}\vec{L} = \tfrac{SI}{c}$$

$$\vec{A}_{\mathrm{dm}} = \tfrac{\vec{m} \times \vec{r}}{r^3};\ \vec{\tau} = \vec{m} \times \vec{B}$$

$$\vec{F}_{\mathrm{dmdm}} = -\vec{\nabla}_R \tfrac{\vec{m}\vec{m}' - 3(\vec{m}\hat{R})(\vec{m}'\hat{R})}{R^3}$$

$$\text{loop axis: } \vec{B} = \hat{z}\tfrac{2\pi R^2}{(z^2 + R^2)^{3/2}}\tfrac{I}{c}$$

Electromagnetism (CGS)

$$\text{Faraday: } \mathcal{E} = -\tfrac{1}{c}\tfrac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int \mathrm{d}^3 x \vec{J} = \dot{\vec{p}}$$

$$\vec{\nabla} \times \vec{E} = -\tfrac{1}{c}\tfrac{\partial\vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\tfrac{\partial\rho}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = 4\pi\tfrac{\vec{J}}{c} + \tfrac{1}{c}\tfrac{\partial\vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$$

$$\mathrm{d}\vec{F} = \mathrm{d}^3 x\left(\rho\vec{E} + \tfrac{\vec{J}}{c} \times \vec{B}\right);\ \vec{F}_q = q\left(\vec{E} + \tfrac{\dot{\vec{r}}}{c} \times \vec{B}\right)$$

$$u = \tfrac{E^2 + B^2}{8\pi};\ \vec{S} = \tfrac{c}{4\pi}\vec{E} \times \vec{B};\ \vec{g} = \tfrac{\vec{S}}{c^2}$$

$$\mathbf{T}^E = \tfrac{1}{4\pi}\left(\vec{E} \otimes \vec{E} - \tfrac{1}{2}E^2\right);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\tfrac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ -\tfrac{\partial\vec{g}}{\partial t} = \rho\vec{E} + \tfrac{\vec{J}}{c} \times \vec{B} - \vec{\nabla}\mathbf{T}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{E} = -\vec{\nabla}\phi - \tfrac{1}{c}\tfrac{\partial\vec{A}}{\partial t}$$

$$-\nabla^2\phi - \tfrac{1}{c}\tfrac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\left(\vec{\nabla}\vec{A} + \tfrac{1}{c}\tfrac{\partial\phi}{\partial t}\right) - \nabla^2\vec{A} + \tfrac{1}{c}\tfrac{\partial^2\vec{A}}{\partial t^2} = 4\pi\tfrac{\vec{J}}{c}$$

$$(\phi, \vec{A}) \cong \left(\phi - \tfrac{1}{c}\tfrac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\right)$$

$$(\phi, \vec{A}) = \int \mathrm{d}^3 r' \tfrac{\left(\rho, \frac{\vec{J}}{c}\right)\left(\vec{r}', t - \frac{1}{c}|\vec{r} - \vec{r}'|\right)}{|\vec{r} - \vec{r}'|}$$

$$\vec{\nabla}\vec{A} = 0 \to \Box\vec{A} = \tfrac{4\pi}{c}(\vec{J} - \vec{J}_L) =: \tfrac{4\pi}{c}\vec{J}_T$$

$$\vec{J}_L = \tfrac{1}{4\pi}\vec{\nabla}\tfrac{\partial\phi}{\partial t} = -\tfrac{1}{4\pi}\vec{\nabla}\int \tfrac{\vec{\nabla}'\vec{J}'}{|\vec{r} - \vec{r}'|}\mathrm{d}^3 r'$$

$$\vec{E}'_\parallel = \vec{E}_\parallel;\ \vec{B}'_\parallel = \vec{B}_\parallel$$

$$\vec{E}'_\perp = \gamma\left(\vec{E}_\perp + \tfrac{\vec{v}}{c} \times \vec{B}\right)$$

$$\vec{B}'_\perp = \gamma\left(\vec{B}_\perp - \tfrac{\vec{v}}{c} \times \vec{E}\right)$$

$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \hat{k} \times \vec{E} \\ \omega = ck \end{cases}$$

$$\vec{B}_{\mathrm{diprad}} = \tfrac{1}{c^2}\tfrac{\ddot{\vec{p}} \times \hat{r}}{r}\Big|_{t_{\mathrm{rit}}};\ \vec{E}_{\mathrm{diprad}} = \vec{B}_{\mathrm{diprad}} \times \hat{r}$$

$$\text{Larmor: } P = \tfrac{2}{3c^3}|\ddot{\vec{p}}|^2$$

$$\text{Rel. Larmor: } P = \tfrac{2}{3c^3} q^2 \gamma^6 (a^2 - (\vec{a} \times \vec{\beta})^2)$$

$$\vec{A}_{\mathrm{dm}} = \tfrac{1}{c}\tfrac{\dot{\vec{m}} \times \hat{r}}{r}\Big|_{t_{\mathrm{rit}}}$$

$$\text{L.W.: } (\phi, \vec{A}) = \tfrac{q\left(1, \frac{\vec{v}}{c}\right)}{\left[r - \frac{\vec{v}\vec{r}}{c}\right]_{t_{\mathrm{rit}}}};\ t_{\mathrm{rit}} = t - \tfrac{r}{c}\Big|_{t_{\mathrm{rit}}}$$

$$A^\mu = (\phi, \vec{A});\ J^\mu = (c\rho, \vec{J})$$

$$\text{Lorenz gauge: } \partial_\alpha A^\alpha = 0$$

$$\text{Temporal gauge: } \phi = 0$$

$$\text{Axial gauge: } A_3 = 0$$

$$\text{Coulomb gauge: } \vec{\nabla}\vec{A} = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi\tfrac{J^\nu}{c};\ \partial_\alpha \mathscr{F}^{\alpha\nu} = 0;\ \tfrac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = q F^{\mu\alpha}\tfrac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0;\ \det F = (\vec{E}\vec{B})^2$$

$$F^{\alpha\beta} F_{\alpha\beta} = 2(B^2 - E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta} = 4\vec{E}\vec{B}$$

$$\Theta^{\mu\nu} = \tfrac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu} + \tfrac{1}{4}g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T} \end{pmatrix};\ \partial_\alpha \Theta^{\alpha\nu} = \tfrac{J_\alpha}{c} F^{\alpha\nu} = -G^\nu$$

$$\mathcal{L} = \tfrac{mc^2}{\gamma} - q\vec{A}\tfrac{\vec{v}}{c} + q\phi;\ \mathcal{H} = \tfrac{1}{2m}\left(\vec{p} - \tfrac{q\vec{A}}{c}\right)^2 + q\phi$$

$$\text{plane wave: } \mathbf{T} = -u\hat{k} \otimes \hat{k};\ \Theta^{\mu\nu} = u\hat{k}^\mu\hat{k}^\nu$$

E.M. in matter (CGS)

$$\vec{\nabla}\vec{D} = 4\pi\rho_{\mathrm{ext}};\ \vec{\nabla} \times \vec{E} = -\tfrac{1}{c}\tfrac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{H} = 4\pi\tfrac{\vec{J}_{\mathrm{ext}}}{c} + \tfrac{1}{c}\tfrac{\partial\vec{D}}{\partial t}$$

$$\vec{P} = \tfrac{\mathrm{d}\langle\vec{p}\rangle}{\mathrm{d}V};\ \vec{M} = \tfrac{\mathrm{d}\langle\vec{m}\rangle}{\mathrm{d}V}$$

$$\rho_{\mathrm{pol}} = -\vec{\nabla}\vec{P};\ \sigma_{\mathrm{pol}} = \hat{n}\vec{P};\ \tfrac{\vec{J}_{\mathrm{mag}}}{c} = \vec{\nabla} \times \vec{M}$$

$$\vec{D}_{\mathrm{pol}} = \vec{E} + 4\pi\vec{P};\ \vec{H}_{\mathrm{mag}} = \vec{B} - 4\pi\vec{M}$$

$$\text{static linear isotropic: } \vec{P} = \chi\vec{E}$$

$$\text{static linear: } P_i = \chi_{ij} E_j$$

$$\text{static linear: } \varepsilon = 1 + 4\pi\chi$$

$$\text{static: } \Delta D_\perp = 4\pi\sigma_{\mathrm{ext}};\ \Delta E_\parallel = 0$$

$$\text{static linear: } u = \tfrac{1}{8\pi}\vec{E}\vec{D}$$

$$\Delta U_{\mathrm{dielectric}} = -\tfrac{1}{2}\int \mathrm{d}^3 r \vec{P}\vec{E}_0$$

plane capacitor: $C = \frac{\varepsilon}{4\pi}\frac{S}{d}$

cilindric capacitor: $C = \frac{L}{2\log\frac{R}{r}}$

atomic polarizability: $\vec{p} = \alpha\vec{E}_{\text{loc}}$

non-interacting gas: $\vec{p} = \alpha\vec{E}_0$; $\chi = n\alpha$

hom. cubic isotropic: $\chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}n\alpha$

perm. dipole: $\chi = \frac{1}{3}\frac{np_0^2}{kT}$

local field: $\vec{E}_{\text{loc}} = \vec{E} + \frac{4\pi}{3}\vec{P}$

$\vec{J}\vec{E} = -\vec{\nabla}\left(\frac{c}{4\pi}\vec{E}\times\vec{H}\right) - \frac{1}{4\pi}\left(\vec{E}\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t}\right)$

$n = \sqrt{\varepsilon\mu}$; $k = n\frac{\omega}{c}$

plane wave: $B = nE$

$\vec{J}_{\text{c}} = \sigma\vec{E}$; $\varepsilon_\sigma = 1 + i\frac{4\pi\sigma}{\omega}$

$\omega_{\text{p}}^2 = 4\pi\frac{n_{\text{vol}}q^2}{m}$; $\omega_{\text{cyclo}} = \frac{qB}{mc}$

I: $u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$

I: $\langle S_z\rangle = \frac{c}{n}\langle u\rangle$

II: $u = \frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2 + \frac{\partial}{\partial\omega}(\mu\omega)H^2\right)$

II: $\langle S_z\rangle = v_{\text{g}}\langle u\rangle$; $v_{\text{g}} = \frac{\partial\omega}{\partial k} = \frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$

III: $\langle W\rangle = \frac{\omega}{4\pi}\left(\operatorname{Im}\varepsilon\langle E^2\rangle + \operatorname{Im}\mu\langle H^2\rangle\right)$

Fresnel TE (S): $\frac{E_{\text{t}}}{E_{\text{i}}} = \frac{2}{1+\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\text{r}}}{E_{\text{i}}} = \frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_{\text{t}}}{E_{\text{i}}} = \frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$; $\frac{E_{\text{r}}}{E_{\text{i}}} = \frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz} = \pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2 - k_x^2}$, $\operatorname{Im}k_{tz} > 0$

Drüde-Lorentz: $\varepsilon = 1 - \frac{\omega_{\text{p}}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$P(t) = \int_{-\infty}^{\infty} g(t-t')E(t')\mathrm{d}t'$

$P(\omega) = \chi(\omega)E(\omega)$

$\chi(\omega) = \int_{-\infty}^{\infty} e^{i\omega t}g(t)\mathrm{d}t$; $\chi(-\omega) = \chi^*(\omega)$

$g(t<0) = 0 \implies$

$\operatorname{Re}\varepsilon(\omega) = 1 + \frac{2}{\pi}\fint_0^\infty \frac{\omega'\left(\operatorname{Im}\varepsilon(\omega') - \frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$

$\operatorname{Im}\varepsilon(\omega) = -\frac{2\omega}{\pi}\fint_0^\infty \frac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega' + \frac{4\pi\sigma_0}{\omega}$

sum rule: $\frac{\pi}{2}\omega_{\text{p}}^2 = \int_0^\infty \omega\operatorname{Im}\varepsilon\mathrm{d}\omega$

sum rule: $2\pi^2\sigma_0 = \int_0^\infty (1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$

sum rule: $\int_0^\infty (\operatorname{Re}n - 1)\mathrm{d}\omega = 0$

Miller rule: $\chi^{(2)}(\omega,\omega) \propto \chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$

Quantum mechanics (CGS)

$r_e = \frac{e^2}{mc^2}$; $\alpha = \frac{e^2}{\hbar c}$; $\lambda_{\text{Broglie}} = \frac{h}{p}$

Planck: $\frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$

$i\hbar\frac{\partial\mathcal{U}}{\partial t} = \mathcal{H}\mathcal{U}$; $\frac{\partial\mathcal{H}}{\partial t} = 0 \Rightarrow \mathcal{U}(t) = e^{-\frac{i\mathcal{H}t}{\hbar}}$

$[\mathcal{H}(t),\mathcal{H}(t')] = 0 \Rightarrow \mathcal{U}(t) = e^{-\frac{i\int_0^t \mathrm{d}t\mathcal{H}(t)}{\hbar}}$

$\mathcal{U}(t) = \left(\frac{-i}{\hbar}\right)^k\int_0^t \mathrm{d}t_1\cdots\int_0^{t_{k-1}}\mathrm{d}t_k\mathcal{H}(t_1)\cdots\mathcal{H}(t_k)$

$A_H(t) = \mathcal{U}(t)^\dagger A\mathcal{U}(t)$

$\frac{\partial\mathcal{H}}{\partial t} = 0 \Rightarrow \frac{\mathrm{d}A_H}{\mathrm{d}t} = \frac{[A_H,\mathcal{H}]}{i\hbar}$

$H = H_0 + V_\lambda : \frac{\partial E_n}{\partial\lambda}\big|_{\lambda=0} = \langle\psi_n|\frac{\partial V_\lambda}{\partial\lambda}|\psi_n\rangle\big|_{\lambda=0}$

$[A,BC] = [A,B]C + B[A,C]$

$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$

$[X,P] = i\hbar$; $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$

$\langle x|X|\psi\rangle = x\langle x|\psi\rangle$; $\langle x|P|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|\psi\rangle$

$\left\langle(A-\langle A\rangle)^2\right\rangle\left\langle(B-\langle B\rangle)^2\right\rangle \geq \frac{1}{4}|\langle[A,B]\rangle|^2$

$e^BAe^{-B} = A + [B,A] + \frac{1}{2!}[B,[B,A]] + \cdots$

$[A,B] \propto I \Rightarrow [A,f(B)] = [A,B]f'(B)$

$[A,B] \propto I \Rightarrow e^Ae^B = e^{A+B+\frac{1}{2}[A,B]}$

$e^{ip'X}|p\rangle = |p+p'\rangle$; $e^{-iPx'}|x\rangle = |x+x'\rangle$

$\psi(x) = \langle x|\psi\rangle$; $\psi = |\psi|e^{\frac{iS}{\hbar}}$

$\rho = |\psi|^2$; $\vec{j} = \frac{\hbar}{m}\operatorname{Im}(\psi^*\vec{\nabla}\psi) = \frac{|\psi|^2\vec{\nabla}S}{m}$

$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j}$; $\int\mathrm{d}^3x\vec{j} = \frac{\langle\vec{p}\rangle}{m}$

$\psi(x,t) = \int\mathrm{d}x'K(x,t;x')\psi(x',t=0)$

$K(x,t;x') = \sum_E \psi_E(x')^*\psi_E(x)e^{-\frac{iEt}{\hbar}} =$

$= \langle x|e^{-\frac{i\mathcal{H}t}{\hbar}}|x'\rangle$

$\left(\mathcal{H} - i\hbar\frac{\partial}{\partial t}\right)K(x,t;x') = -i\hbar\delta(x-x')\delta(t)$

$\sigma_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix}$; $\sigma_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$; $\sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$

$[\sigma_i,\sigma_j] = 2i\varepsilon_{ijk}\sigma_k$; $\{\sigma_i,\sigma_j\} = 2\delta_{ij}$

$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = \vec{a}\vec{b} + i\vec{\sigma}(\vec{a}\times\vec{b})$

$e^{-\frac{i\vec{\sigma}\hat{n}\phi}{2}} = \cos\frac{\phi}{2} - i(\vec{\sigma}\hat{n})\sin\frac{\phi}{2}$

$|\vec{\sigma}\hat{n},1\rangle = \cos\frac{\theta}{2}|\sigma_3,1\rangle + e^{i\varphi}\sin\frac{\theta}{2}|\sigma_3,-1\rangle$

$R(\hat{n},\phi) = \exp\left(-\frac{i\vec{J}\hat{n}\phi}{\hbar}\right)$

$[J_i,J_j] = i\hbar\varepsilon_{ijk}J_k$; $J_\pm := J_x \pm iJ_y$

$[J_+,J_-] = i\hbar J_z$; $[J_z,J_\pm] = \pm\hbar J_\pm$

$\left[J^2,J_\pm\right] = \left[J^2,J_z\right] = 0$

$J^2|j,m\rangle = j(j+1)\hbar^2|j,m\rangle$

$J_z|j,m\rangle = m\hbar|j,m\rangle$

$m = -j, j-1, \ldots, j$; $2j \in \mathbb{N}$

$\vec{L} = \vec{X}\times\vec{P}$; $\langle\vec{x}|L_z|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial\phi}\langle\vec{x}|\psi\rangle$

$A = \vec{A} :\leftrightarrow [A_i,J_j] = i\varepsilon_{ijk}\hbar A_k$

$T = \mathbf{T} :\leftrightarrow [J_z,T_q] = \hbar qT_q$

$\left[J_\pm,T_q^{(k)}\right] = \hbar\sqrt{(k\mp q)(k\pm q+1)}T_{q\pm1}^{(k)}$

$\rho[|\alpha_i\rangle,w_i] := \sum_i w_i|\alpha_i\rangle\langle\alpha_i|$

$\operatorname{tr}\rho = 1$; $[A] := \operatorname{tr}(\rho A)$

$\#\{w_i > 0\} = 1 \iff tr(\rho^2) = 1$

$\#\{w_i > 0\} > 1 \iff 0 < \operatorname{tr}(\rho^2) < 1$

$i\hbar\frac{\partial\rho}{\partial t} = -[\rho,\mathcal{H}]$

$W_\psi(x,p) = \int\frac{\mathrm{d}y}{2\pi\hbar}\left\langle x+\frac{y}{2}\middle|\psi\right\rangle\left\langle\psi\middle|x-\frac{y}{2}\right\rangle e^{-\frac{ipy}{2}}$

QM solutions

$\mathcal{H}_{\text{box}} = \frac{P^2}{2m} + \begin{cases}0 & 0 < x < L\\ \infty & \text{otherwise}\end{cases}$

$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(n\pi\frac{x}{L}\right)$, $n \geq 1$

$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$

$\Delta x^2 = L^2\left(\frac{1}{12} - \frac{1}{2n^2\pi^2}\right)$; $\Delta p = \frac{\hbar n\pi}{L} = \frac{hn}{2L}$

$\mathcal{H}_{\text{harm}} = \frac{P^2}{2m} + \frac{m\omega^2X^2}{2}$

$A = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{iP}{m\omega}\right)$; $A^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X - \frac{iP}{m\omega}\right)$

$N = A^\dagger A = \frac{\mathcal{H}}{\hbar\omega} - \frac{1}{2}$; $\mathcal{H} = \hbar\omega\left(N + \frac{1}{2}\right)$

$\left[A,A^\dagger\right] = 1$; $[N,A] = -A$; $\left[N,A^\dagger\right] = A^\dagger$

$A^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$; $A|n\rangle = \sqrt{n}|n-1\rangle$

$|n\rangle = \frac{(A^\dagger)^n}{\sqrt{n!}}|0\rangle$, $n = 0,1,\ldots$

$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^nn!x_0}}\left(\frac{x}{x_0} - x_0\frac{\mathrm{d}}{\mathrm{d}x}\right)^ne^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$

$\psi_n(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{2^nn!x_0}}H_n\left(\frac{x}{x_0}\right)e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$

$x_0 = \sqrt{\frac{\hbar}{m\omega}}$

$\sum_{n=0}^\infty H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$

$H_n(-x) = (-1)^nH_n(x)$

n even: $H_n(0) = (-1)^{\frac{n}{2}}\frac{n!}{(n/2)!}$

$H_n'(x) = 2nH_{n-1}(x)$; $H_0 = 1$

$H_1 = 2x$; $H_2 = 4x^2 - 2$; $H_3 = 8x^3 - 12x$

$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$

$H_n''(x) = 2xH_n'(x) - 2nH_n(x)$

$\int_{-\infty}^\infty \mathrm{d}xH_n(x)H_m(x)e^{-x^2} = \sqrt{\pi}2^nn!\delta_{nm}$

$\mathcal{H}_{\text{delta}} = \frac{P^2}{2m} - \lambda\delta(x)$, $\lambda > 0$

$\psi_{\text{bounded}}(x) = \frac{1}{\sqrt{x_0}}e^{-\frac{|x|}{x_0}}$, $x_0 = \frac{\hbar^2}{\lambda m}$

$E_{\text{bounded}} = -\frac{\lambda}{2x_0}$

$\mathcal{H}_{\text{hydrogen}} = \frac{\vec{P}^2}{2M} - \frac{e^2}{X}$

$a := r_B := \frac{\hbar^2}{Me^2}$; Rydberg $= \frac{e^2}{2r_B}$

$E_n = -\frac{1}{n^2}\frac{e^2}{2a}$; degen. $= n^2$

$\psi_{nlm} = R_{nl}Y_{lm}$; $\vec{j} = \frac{\hbar}{M}\hat{\varphi}\frac{m}{r\sin\theta}|\psi|^2$

$R_{nl} = 2\sqrt{\frac{(n-l-1)!}{a^3n^4(n+l)!}}e^{-\frac{r}{na}}\left(\frac{2r}{na}\right)^lL_{n+l}^{2l+1}\left(\frac{2r}{na}\right)$

$L_n^{(j)}(x) = \sum_{m=0}^{n-j}(-1)^m\binom{n}{n-j-m}\frac{x^m}{m!}$

$L_k(x) = e^x\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(x^ke^{-x}\right)$

$L_k^{(j)} = (-1)^j\frac{\mathrm{d}^j}{\mathrm{d}x^j}L_k(x)$

$\mathcal{H}_{\text{harm3D}} = \frac{\vec{P}^2}{2m} + \frac{m\omega^2\vec{X}^2}{2}$

$E_{ql} = \left(2q + l + \frac{3}{2}\right)\hbar\omega$

$l = 0,1,\ldots$; $q = 0,1,\ldots$

Particle physics

$M(A,Z) = Zm_{\text{p}} + (A-Z)m_{\text{n}} - B(A,Z)$

$B(A,Z) = a_vA - a_sA^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(A-2Z)^2}{A} + a_pA^{-3/4}\Delta$

$\Delta = \begin{cases}0 & & A\text{ odd}\\ \left.\begin{matrix}1 & Z\text{ even}\\ -1 & Z\text{ odd}\end{matrix}\right\} & & A\text{ even}\end{cases}$

$a_v = 15.5$; $a_s = 16.8$; $a_c = 0.72$; $a_{\text{sym}} = 23$; $a_p = 34$ [MeV]

$\frac{\partial M}{\partial Z} = 0 : Z = \frac{m_{\text{n}} - m_{\text{p}} + 4a_{\text{sym}}}{\frac{2a_c}{A^{1/3}} + \frac{8a_{\text{sym}}}{A}}$

$$s_{ab} := (p_a + p_b)^2$$

$$M \to abc : (m_a + m_b)^2 \leq s_{ab} \leq (M - m_c)^2$$

$$M \to abc : s_{ab} + s_{bc} + s_{ac} = M^2 + m_a^2 + m_b^2 + m_c^2$$

$$a_i A_i \to b_j B_j : Q := a_i m_{A_i} - b_j m_{B_j}$$

$$p = qBR$$

$$\frac{\mathrm{d}^3\vec{p}}{2E} = \mathrm{d}^4 p \delta(p^2 - m^2)\theta(p_0)$$

$$\mathrm{d}L_p = \left(\prod_n \frac{\mathrm{d}^3\vec{p}_n}{2E_n}\right)\delta^4(p_{\mathrm{in}} - \textstyle\sum_n p_n);\ \mathrm{d}\sigma = f_{\mathrm{coll}}(p_1, \ldots, p_n)\mathrm{d}L_p$$

$$\text{two body: } \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1} = f(\Omega_1)\frac{p_1}{4\sqrt{s}};\ \sqrt{s} = \text{c.m. energy}$$

$$\text{Rutherford: } \tan\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Qqm}{p^2 b};\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|;\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{d_{\mathrm{min}}^2}{16}\frac{1}{\sin^4\frac{\theta}{2}}$$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{Mott}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{Rutherford}} \cdot \cos^2\frac{\theta}{2}$$

$$\text{mass defect} := M - A \cdot \text{amu}$$