Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$

$$\sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$

$$\tan\tfrac{\alpha}{2}=\tfrac{\sin\alpha}{1+\cos\alpha}=\tfrac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\tfrac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\tfrac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$

$$\begin{pmatrix}\sinh x\\ \cosh x\end{pmatrix}=\frac{1}{2}\begin{pmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{pmatrix}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\cosh^2 x=\frac{1}{1-\tanh^2 x}$$
$$\sin x=-i\sinh(ix)$$

$$\cos x=\cosh(ix)$$
$$\operatorname{asinh}x=\log\big(x+\sqrt{x^2+1}\big)$$
$$\operatorname{acosh}x=\log\big(x+\sqrt{x^2-1}\big)$$
$$\operatorname{atanh}x=\tfrac{1}{2}\log\tfrac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\big(I+\frac{B}{2}-1\big)\,A_{\text{check}}$

Combinatorics

$$D_{n,k}=\frac{n!}{(n-k)!}\qquad P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\tfrac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{x=1}^n x^3=\big(\sum_{x=1}^n x\big)^2=\frac{1}{4}n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$$

$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\textstyle\int_0^\infty t^z e^{-t}dt=z!$$
$$n!=(\tfrac{n}{e})^n\sqrt{2\pi n}$$
$$\text{Fourier: } c_n=\tfrac{2}{T}\textstyle\int_0^T f(t)\cos\big(n\frac{t}{T}\big)\mathrm{d}t$$

$$F[f]=\hat{f}(k)=\tfrac{1}{\sqrt{2\pi}}\textstyle\int_{-\infty}^{\infty}e^{-ikx}f(x)$$
$$\langle\hat{f}|\hat{g}\rangle=\langle f|g\rangle$$
$$F\big[\tfrac{\sin x}{x}\big]=\sqrt{\tfrac{\pi}{2}}\chi_{[-1;1]}$$
$$\tfrac{\mathrm{d}}{\mathrm{d}x}\textstyle\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\tfrac{\operatorname{Re}z+|z|}{2}}+\tfrac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$

Derivatives

$$\tan' x=1+\tan^2 x$$
$$\cot' x=-1-\cot^2 x$$
$$\operatorname{atan}' x=-\operatorname{acot}' x=\tfrac{1}{1+x^2}$$
$$\operatorname{asin}' x=-\operatorname{acos}' x=\tfrac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a' x=\tfrac{1}{x\ln a}$$
$$\cosh' x=\sinh x$$
$$\tanh' x=1-\tanh^2 x$$
$$\operatorname{atanh}' x=\operatorname{acoth}' x=\tfrac{1}{1-x^2}$$
$$\operatorname{asinh}' x=\tfrac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}' x=\tfrac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\tfrac{1}{f'(f^{-1})}$$
$$\big(\tfrac{1}{x}\big)'=-\tfrac{\dot{x}}{x^2}$$
$$\big(\tfrac{x}{y}\big)'=\tfrac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)'=x^y\big(\dot{y}\ln x+y\tfrac{\dot{x}}{x}\big)$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac{1}{x}=\ln|x|$$
$$\textstyle\int\tan x=-\ln|\cos x|$$
$$\textstyle\int\cot x=\ln|\sin x|$$
$$\textstyle\int\frac{1}{\sin x}=\ln\big|\tan\frac{x}{2}\big|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\big|\tan\big(\frac{x}{2}+\frac{\pi}{4}\big)\big|$$
$$\textstyle\int\ln x=x(\ln x-1)$$
$$\textstyle\int\tanh x=\ln\cosh x$$
$$\textstyle\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\textstyle\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy=x\int y-\int(\dot{x}\int y)$$
$$\textstyle\int e^{yx}x=e^{yx}\big(\frac{y}{x}-\frac{1}{y^2}\big)$$

Differential equations

$$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$
$$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot{x}=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot{x}+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot{x}+\gamma\dot{x}+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac{1}{x}+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac{1}{x}-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$

$$\operatorname{acos}x=\tfrac{\pi}{2}-\operatorname{asin}x$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$

$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac{1}{x}-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac{1}{x}+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac{5}{6}x^4-\tfrac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\lor j=k\lor k=i\\ 1 & i+1\equiv j\land j+1\equiv k\\ -1 & i\equiv j+1\land j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$

$$\vec{a}\times\vec{b}=\varepsilon_{ijk}a_jb_k\hat{e}_i$$
$$(\vec{a}\otimes\vec{b})_{ij}=a_ib_j$$
$$(\vec{a}\times\vec{b})\vec{c}=(\vec{c}\times\vec{a})\vec{b}$$
$$(\vec{a}\times\vec{b})\times\vec{c}=-(\vec{b}\vec{c})\vec{a}+(\vec{a}\vec{c})\vec{b}$$

$$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})=(\vec{a}\vec{c})(\vec{b}\vec{d})-(\vec{a}\vec{d})(\vec{b}\vec{c})$$
$$|\vec{u}\times\vec{v}|^2=u^2v^2-(\vec{u}\vec{v})^2$$
$$\vec{\nabla}=\big(\tfrac{\partial}{\partial x},\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}\big);\ \Box=\tfrac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec{\nabla}V=\tfrac{\partial V}{\partial\rho}\hat{\rho}+\tfrac{1}{\rho}\tfrac{\partial V}{\partial\phi}\hat{\phi}+\tfrac{\partial V}{\partial z}\hat{z}$$

$$\vec\nabla\vec v=\frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$
$$\vec\nabla\times\vec v=\Big(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\Big)\hat\rho+$$
$$+\Big(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\Big)\hat\phi+\frac{1}{\rho}\Big(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\Big)$$
$$\nabla^2V=\frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial V}{\partial\rho}\Big)+\frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2}+\frac{\partial^2V}{\partial z^2}$$
$$\vec\nabla V=\frac{\partial V}{\partial r}\hat r+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$
$$\vec\nabla\vec v=\frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec\nabla\times\vec v=\frac{1}{r\sin\theta}\Big(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta}-\frac{\partial v_\theta}{\partial\varphi}\Big)\hat r+$$
$$+\frac{1}{r}\Big(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(rv_\varphi)}{\partial r}\Big)\hat\theta+\frac{1}{r}\Big(\frac{\partial(rv_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}\Big)\hat\varphi$$
$$\nabla^2V=\frac{\frac{\partial}{\partial r}\big(r^2\frac{\partial V}{\partial r}\big)}{r^2}+\frac{\frac{\partial}{\partial\theta}\big(\sin\theta\frac{\partial V}{\partial\theta}\big)}{r^2\sin\theta}+\frac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\vec\nabla\Big(\vec\nabla\times\vec v\Big)=\vec\nabla\times\vec\nabla\vec V=0$$
$$\vec\nabla(f\vec v)=(\vec\nabla f)\vec v+f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v)=\vec\nabla f\times\vec v+f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v)=-\nabla^2\vec v+\vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w)=\vec w(\vec\nabla\times\vec v)-\vec v(\vec\nabla\times\vec w)$$
$$\vec\nabla\times(\vec v\times\vec w)=(\vec\nabla\vec w+\vec w\vec\nabla)\vec v-(\vec\nabla\vec v+\vec v\vec\nabla)\vec w$$
$$\tfrac{1}{2}\vec\nabla v^2=(\vec v\vec\nabla)\vec v+\vec v\times(\vec\nabla\times\vec v)$$
$$\int\vec\nabla\vec v\,\mathrm{d}^3x=\oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S=\oint\vec v\,\mathrm{d}\vec l$$
$$\int(f\nabla^2g-g\nabla^2f)\,\mathrm{d}^3x=\oint_S\Big(f\frac{\partial g}{\partial n}-g\frac{\partial f}{\partial n}\Big)\,\mathrm{d}S$$
$$\oint\vec v\times\mathrm{d}\vec S=-\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(\vec r-\vec r_0)=\frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\nabla^2\frac{1}{|\vec r-\vec r_0|}=-4\pi\delta(\vec r-\vec r_0)$$
$$\delta(g(x))=\frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle=\tfrac{1}{2}\,\mathrm{Re}(a\bar b)$$

Statistics

$$P(E\cap E_1)=P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}}\approx\frac{x_{\max}-x_{\min}}{\sqrt N}$$
$$P(x\le k)=F(k)=\int_{-\infty}^k p(x)$$
$$\text{median}=F^{-1}(\tfrac{1}{2})$$
$$E[f(x)]=\int_{-\infty}^{\infty}f(x)p(x)$$
$$\mu=E[x]=\int_{-\infty}^{\infty}xp(x)$$
$$\alpha_n=E[x^n]$$
$$M_n=E[(x-\mu)^n]$$
$$\sigma^2=M_2=E[x^2]-\mu^2$$
$$\text{FWHM}\approx2\sigma$$
$$\gamma_1=\frac{M_3}{\sigma^3},\ \gamma_2=\frac{M_4}{\sigma^4}$$

$$\phi[y](t)=E[e^{ity}]$$
$$\phi[y_1+\lambda y_2]=\phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n=i^{-n}\frac{\partial^nt}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge0:P(h\ge k)\le\frac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$$
$$B(n,p,k)=\binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B=np,\ \sigma_B^2=np(1-p)$$
$$P(\mu,k)=\frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2=\mu$$
$$u(x,a,b)=\frac{1}{b-a},\ x\in[a;b]$$
$$\mu_u=\frac{b+a}{2},\ \sigma_u^2=\frac{(b-a)^2}{12}$$
$$\varepsilon(x,\lambda)=\lambda e^{-\lambda x},\ x\ge0$$

$$\mu_\varepsilon=\frac{1}{\lambda},\ \sigma_\varepsilon^2=\frac{1}{\lambda^2}$$
$$g(x,\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
$$\text{FWHM}_g=2\sigma\sqrt{2\ln2}$$
$$z=\frac{x-\mu}{\sigma};\ \mu,\sigma[z]=0,1$$
$$\chi^2=\sum_{i=1}^nz_i^2$$
$$\wp_n(x)=\frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$
$$\mu_\wp=n,\ \sigma_\wp^2=2n$$
$$n\ge30:\wp_n(x)\approx g(x,n,\sqrt{2n})$$
$$n\ge8:p[\sqrt{2\chi^2}]\approx g(,\sqrt{2n-1},1)$$
$$S(x,n)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\Big(1+\frac{x^2}{n}\Big)^{-\frac{n+1}{2}}$$
$$\mu_S=0,\ \sigma_S^2=\frac{n}{n-2}$$

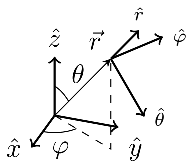
$$p\Big[z\sqrt{\frac{n}{\chi^2}}\Big]=S(,n)$$
$$n\ge35:S(x,n)\approx g(x,0,1)$$
$$c(x,a)=\frac{a}{\pi}\frac{1}{a^2+x^2}$$
$$\sigma_{xy}=E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho=\frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le1$$
$$\mu[f(x_1,\ldots)]\approx f(\mu_1,\ldots)$$
$$\sigma^2[f(x_1,\ldots)]\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$
$$\mu\approx m=\frac{1}{n}\sum_{i=1}^nx_i$$
$$\sigma^2\approx s^2=\frac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$
$$s_m^2=\frac{s^2}{n}$$
$$p\Big[\frac{m-\mu}{s_m}\Big]=S(,n)$$

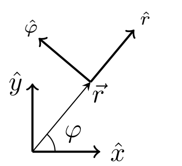
Fit

$$f(x)=mx+q,\quad f(x)=a$$
$$f(x)=bx$$
$$m=\frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}\qquad q=\frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$a=\frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2=\frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$\Delta m^2=\frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}\qquad\Delta q^2=\frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$b=\frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2=\frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

Kinematics

$$\frac{1}{R}=\Big|\frac{v_xa_y-v_ya_x}{v^3}\Big|$$
$$\vec\omega=\dot\varphi\cos\theta\hat r-\dot\varphi\sin\theta\hat\theta+\dot\theta\hat\varphi$$
$$\dot{\vec w}=\frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r+\frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta+\frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi+\vec\omega\times\vec w$$
$$\theta\equiv\frac{\pi}{2}\to\dot{\vec r}=\dot r\hat r+r\dot\varphi\hat\varphi$$
$$\theta\equiv\frac{\pi}{2}\to\ddot{\vec r}=(\ddot r-r\dot\varphi^2)\hat r+(r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$$
$$\dot{\vec r}=\dot r\hat r+r\dot\theta\hat\theta+r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle=\ddot r-r\dot\theta^2-r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle=r\ddot\theta+2\dot r\dot\theta-r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle=r\ddot\varphi\sin\theta+2\dot r\dot\varphi\sin\theta+2r\dot\theta\dot\varphi\cos\theta$$
$$\vec A=\ddot{\vec r}+\vec A_{\mathrm T}+\vec\omega\times(\vec\omega\times\vec r)+\dot{\vec\omega}\times\vec r+2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha=\frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t)=\frac{\partial\alpha}{\partial\beta}\dot\beta+\frac{\partial\alpha}{\partial t}$$
$$\vec p:=m\dot{\vec r};\ \vec F=\dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t}=\vec F\vec p$$
$$M:=\sum_im_i;\ \vec R:=\frac{m_i\vec r_i}{M}$$
$$T=\tfrac{1}{2}M\dot{\vec R}^2+\tfrac{1}{2}m_i(\dot{\vec r}_i-\dot{\vec R})^2$$
$$\vec L=\vec R\times M\dot{\vec R}+(\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$$
$$\vec\tau_O=\dot{\vec L}_O+\vec v_O\times\vec p$$
$$\tau_1=I_1\dot\omega_1+(I_3-I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t)=T-V+\frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$
$$S[q]=\int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm{d}t$$
$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv0}^{\epsilon(t_1)=\epsilon(t_2)=0}=0$$
$$p:=\frac{\partial\mathcal L}{\partial\dot q};\ \dot p=\frac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t)=\dot qp-\mathcal L$$
$$\dot q=\frac{\partial\mathcal H}{\partial p};\ \dot p=-\frac{\partial\mathcal H}{\partial q}$$
$$\frac{\mathrm{d}\mathcal H}{\mathrm{d}t}=\frac{\partial\mathcal H}{\partial t}=-\frac{\partial\mathcal L}{\partial t}$$
$$\{u,v\}=\frac{\partial u}{\partial q}\frac{\partial v}{\partial p}-\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t}=\{u,\mathcal H\}+\frac{\partial u}{\partial t}$$
$$\eta=(q,p);\ \Gamma=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$$
$$\dot\eta=\Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\}=\frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

rod: $\frac{1}{12}mL^2$ octahedron: $\frac{1}{10}ms^2$ cone: $\frac{3}{10}mr^2$ rectangulus: $\frac{1}{12}m(a^2+b^2)$

point: mr^2 disk: $\frac{1}{2}mr^2$ sphere: $\frac{2}{3}mr^2$ torus: $m\big(R^2+\frac{3}{4}r^2\big)$

two points: μd^2 tetrahedron: $\frac{1}{20}ms^2$ ball: $\frac{2}{5}mr^2$ ellipsoid: $I_a=\frac{1}{5}m(b^2+c^2)$

Kepler

$$\langle U\rangle\approx-2\langle T\rangle$$
$$U_{\text{eff}}=U+\frac{L^2}{2mr^2}$$
$$\frac{1}{\mu}=\frac{1}{m_1}+\frac{1}{m_2}$$
$$\vec r=\vec r_1-\vec r_2,\ \alpha=Gm_1m_2$$
$$T=\tfrac{1}{2}M\dot{\vec R}^2+\tfrac{1}{2}\mu\dot{\vec r}^2$$
$$\vec L=\vec R\times M\dot{\vec R}+\vec r\times\mu\dot{\vec r}$$
$$k=\frac{L^2}{\mu\alpha},\ \varepsilon=\sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$
$$r=\frac{k}{1+\varepsilon\cos\theta}$$
$$a=\frac{k}{|1-\varepsilon^2|}=\frac{\alpha}{2|E|}$$
$$a^3\omega^2=G(m_1+m_2)=\frac{\alpha}{\mu}$$
$$\vec A=\mu\dot{\vec r}\times\vec L-\mu\alpha\hat r,\ \dot{\vec A}=0$$

Inequalities

$$|a|-|b|\le|a+b|\le|a|+|b|$$
$$x>-1:1+nx\le(1+x)^n$$
$$\frac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$$
$$\sum a_ib_i\le\Big(\sum a_i^p\Big)^{\frac{1}{p}}\Big(\sum b_i^{\frac{p}{p-1}}\Big)^{\frac{p-1}{p}}$$
$$x^py^q\le\Big(\frac{px+qy}{p+q}\Big)^{p+q}$$
$$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}}\le\sqrt[q]{\frac{1}{n}\sum a_i^q}$$
$$\sum\Big(\frac{a_1+\ldots a_i}{i}\Big)^p\le\Big(\frac{p}{p-1}\Big)^p\sum a_i^p$$
$$x\ge0,|\ddot x|\le M:|\dot x|\le\sqrt{2Mx}$$
$$\frac{1}{1+x}<\ln\Big(1+\frac{1}{x}\Big)<\frac{1}{x}$$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$

$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$

Groups

stabilizer: $\mathrm{Stab}_G\, x = \{g \in G \,|\, gx = x\}$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772\cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206\cdot 10^{-2}\,\frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022\cdot 10^{23}\,\frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381\cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$c = 2.998\cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602\cdot 10^{-19}\,\mathrm{A\,s}$

$m_\mathrm{e} = 9.109\cdot 10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673\cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675\cdot 10^{-27}\,\mathrm{kg}$

$\mathrm{amu} = 1.661\cdot 10^{-27}\,\mathrm{kg}$

$h = 6.626\cdot 10^{-34}\,\mathrm{J\,s}$

$\varepsilon_0 = 8.854\cdot 10^{-12}\,\frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}$

$\mu_0 = 1.257\cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274\cdot 10^{-24}\,\mathrm{A\,m^2}$

$\alpha = 7.297\cdot 10^{-3}$

$\mathrm{eV} = 1.602\cdot 10^{-12}\,\mathrm{erg}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i \mathrm{A}_i \to b_j \mathrm{B}_j$

$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$

$\forall\, i,j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$

$k = Ae^{-\frac{E_\mathrm{a}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0}$, $[\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma\frac{p}{p_0}$, $p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}}$, $K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}}$, $K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}}$, $\chi = \frac{n}{n_\mathrm{tot}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_\mathrm{tot}^{\sum a_i - \sum b_j}$

$\Delta G^\mathrm{o}_\mathrm{r} = -RT\ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln\frac{Q}{K}$

$\ln\frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}}\ln Q$ (Nerst)

(std) $\Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}}\log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$

$\mathrm{d}L = p\mathrm{d}V$

$\mathrm{d}S = \frac{\mathrm{d}Q}{T}$

$C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V$

$C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p$

$\gamma = \frac{C_p}{C_V}$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n}$, $c_V = \frac{\mathrm{dof}}{2}R$, $c_p = c_V + R$

$c_V = \frac{R}{\gamma-1}$, $c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int \mathrm{d}q_1\cdots\mathrm{d}q_N \int \mathrm{d}p_1\cdots\mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z$; $\beta = \frac{1}{k_\mathrm{B}T}$; $C = \frac{\partial U}{\partial T}$

$F(T,V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t}$

$Z = \frac{V}{I}$

$Z_R = R$

$Z_C = -i\frac{1}{\omega C}$

$Z_L = i\omega L$

$Z_\mathrm{series} = \sum_k Z_k$

$\frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$

$\sum_\mathrm{loop} V_k = 0$

$\sum_\mathrm{node} I_k = 0$

$\mathcal{E} = -L\dot{I}$

$L = \frac{\Phi_B}{I}$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec{p} = \gamma m\vec{v}$

$\mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \mathcal{E}$

$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$

$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma\begin{pmatrix} 1 & -\beta \\ -\beta & 1\end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1-\frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{\left(1-\frac{V^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct, \vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu\partial^\mu = \square$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \to \sum_i m_i$

$E_1^\mathrm{max} = \frac{M^2 + m_1^2 - \sum_{i\neq 1} m_i^2}{2M}c^2$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3}$; $\vec{E}_1 = \frac{\vec{F}_{12}}{q_2}$; $V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$; $\rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x$; $-\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho$; $\vec{\nabla}\times\vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x$; $\tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int \vec{E}_i\vec{E}_j\,\mathrm{d}^3x$

$V(\vec{r}) = \int\rho G_\mathrm{D}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_\mathrm{D}}{\partial n}\,\mathrm{d}S$

$V(\vec{r}) = \langle V\rangle_S + \int\rho G_\mathrm{N}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n}G_\mathrm{N}(\vec{r})\,\mathrm{d}S$

$\nabla_y^2 G(\vec{x},\vec{y}) = -4\pi\delta(\vec{x}-\vec{y})$; $G_\mathrm{D}(\vec{x},\vec{y})|_{\vec{y}\in S} = 0$; $\frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y}\in S} = -\frac{4\pi}{S}$

$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R}$; $\vec{p} = \int \mathrm{d}^3 r\rho\vec{r}$; $\vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3}$; $V_\mathrm{dip} = \frac{\vec{p}\hat{r}}{r^2}$

force on a dipole: $\vec{F}_\mathrm{dip} = (\vec{p}\,\vec{\nabla})\vec{E}$

$Q_{ij} = \int \mathrm{d}^3 r\rho(\vec{r})(3r_ir_j - \delta_{ij}r^2)$; $V_\mathrm{quad} = \frac{1}{6r^5}Q_{ij}(3r_ir_j - \delta_{ij}r^2)$

$V(r,\theta) = \sum_{l=0}^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$

$V(r,\theta,\varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l \left(A_{lm}r^l + \frac{B_{lm}}{r^{l+1}}\right)Y_{lm}(\theta,\varphi)$

$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2-1\right)^l$; $f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$

$P_l(1) = 1$; $\langle P_n|P_m\rangle = \frac{2\delta_{nm}}{2n+1}$; $\langle Y_{lm}|Y_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}$

$P_0 = 1$; $P_1 = x$; $P_2 = \frac{3x^2-1}{2}$; $Y_{00} = \frac{1}{\sqrt{4\pi}}$; $Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi}$; $Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi}$; $Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2-1\right)^l$, $|m| \le l$

$$Y_{lm}(\theta,\varphi)=\sqrt{\tfrac{2l+1}{4\pi}\tfrac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m}=(-1)^m\overline{Y}_{lm}$$

$$P_l\left(\tfrac{\vec r\vec r'}{rr'}\right)=\tfrac{4\pi}{2l+1}\textstyle\sum_{m=-l}^{l}\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$$

$$V(r>\operatorname{diam}\operatorname{supp}\rho,\theta,\varphi)=\textstyle\sum_{l=0}^{\infty}\tfrac{4\pi}{2l+1}\sum_{m=-l}^{l}q_{lm}[\rho]\tfrac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho]=\textstyle\int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\,r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec\nabla\vec J=-\tfrac{\partial\rho}{\partial t}=0;\ I=\textstyle\int\vec J\mathrm{d}\vec S$$

solenoid: $B=4\pi\frac{j_\mathrm{s}}{c}$

$$\mathrm{d}\vec F=\tfrac{I\mathrm{d}\vec l}{c}\times\vec B=\mathrm{d}^3x\tfrac{\vec J}{c}\times\vec B;\ \vec F_q=q\tfrac{\dot{\vec r}}{c}\times\vec B$$

$$\mathrm{d}\vec B=\tfrac{I\mathrm{d}\vec l}{c}\times\tfrac{\vec r}{r^3};\ \vec B_q=q\tfrac{\dot{\vec r}}{c}\times\tfrac{\vec r}{r^3}$$

$$\vec B=\vec\nabla\times\vec A;\ \vec A=\textstyle\int\mathrm{d}^3r'\tfrac{\vec J'}{c}\tfrac{1}{|\vec r-\vec r'|}+\vec\nabla A_0$$

$$\vec B=\textstyle\int\mathrm{d}^3r'\tfrac{\vec J'}{c}\times\tfrac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$$

$$\varphi=\tfrac{I}{c}\Omega,\ \vec B=-\vec\nabla\varphi$$

$$\vec\nabla\vec A=0\to\nabla^2\vec A=-4\pi\tfrac{\vec J}{c}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec B=4\pi\tfrac{\vec J}{c};\ \textstyle\oint\vec B\mathrm{d}\vec l=4\pi\tfrac{I}{c}$$

$$\vec m=\tfrac12\textstyle\int\mathrm{d}^3r'\left(\vec r'\times\tfrac{\vec J'}{c}\right)=\tfrac{1}{2c}\tfrac{q}{m}\vec L=\tfrac{SI}{c}$$

$$\vec A_\mathrm{dm}=\tfrac{\vec m\times\vec r}{r^3};\ \vec\tau=\vec m\times\vec B$$

$$\vec F_\mathrm{dmdm}=-\vec\nabla_R\tfrac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$$

loop axis: $\vec B=\hat z\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac{I}{c}$

Electromagnetism (CGS)

Faraday: $\mathcal{E}=-\frac1c\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int\mathrm{d}^3x\vec J=\dot{\vec p}$

$$\vec\nabla\times\vec E=-\tfrac1c\tfrac{\partial\vec B}{\partial t};\ \vec\nabla\vec E=4\pi\rho;\ \vec\nabla\vec J=-\tfrac{\partial\rho}{\partial t}$$

$$\vec\nabla\times\vec B=4\pi\tfrac{\vec J}{c}+\tfrac1c\tfrac{\partial\vec E}{\partial t};\ \vec\nabla\vec B=0$$

$$\mathrm{d}\vec F=\mathrm{d}^3x\left(\rho\vec E+\tfrac{\vec J}{c}\times\vec B\right);\ \vec F_q=q\left(\vec E+\tfrac{\dot{\vec r}}{c}\times\vec B\right)$$

$$u=\tfrac{E^2+B^2}{8\pi};\ \vec S=\tfrac{c}{4\pi}\vec E\times\vec B;\ \vec g=\tfrac{\vec S}{c^2}$$

$$\mathbf{T}^E=\tfrac{1}{4\pi}\left(\vec E\otimes\vec E-\tfrac12E^2\right);\ \mathbf{T}=\mathbf{T}^E+\mathbf{T}^B$$

$$-\tfrac{\partial u}{\partial t}=\vec J\vec E+\vec\nabla\vec S;\ -\tfrac{\partial\vec g}{\partial t}=\rho\vec E+\tfrac{\vec J}{c}\times\vec B-\vec\nabla\mathbf{T}$$

$$\vec B=\vec\nabla\times\vec A;\ \vec E=-\vec\nabla\phi-\tfrac1c\tfrac{\partial\vec A}{\partial t}$$

$$-\nabla^2\phi-\tfrac1c\tfrac{\partial}{\partial t}\vec\nabla\vec A=4\pi\rho$$

$$\vec\nabla\left(\vec\nabla\vec A+\tfrac1c\tfrac{\partial\phi}{\partial t}\right)-\nabla^2\vec A+\tfrac1c\tfrac{\partial^2\vec A}{\partial t^2}=4\pi\tfrac{\vec J}{c}$$

$$(\phi,\vec A)\cong\left(\phi-\tfrac1c\tfrac{\partial\chi}{\partial t},\vec A+\vec\nabla\chi\right)$$

$$(\phi,\vec A)=\textstyle\int\mathrm{d}^3r'\tfrac{\left(\rho,\frac{\vec J}{c}\right)\left(\vec r',t-\frac1c|\vec r-\vec r'|\right)}{|\vec r-\vec r'|}$$

Coulomb gauge: $\vec\nabla\vec A=0$

Lorenz gauge: $\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}=0$

$$\vec E'_\parallel=\vec E_\parallel;\ \vec B'_\parallel=\vec B_\parallel$$

$$\vec E'_\perp=\gamma\left(\vec E_\perp+\tfrac{\vec v}{c}\times\vec B\right)$$

$$\vec B'_\perp=\gamma\left(\vec B_\perp-\tfrac{\vec v}{c}\times\vec E\right)$$

plane wave: $\begin{cases}\vec E=\vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B=\hat k\times\vec E\\ \omega=ck\end{cases}$

$$\vec B_\mathrm{diprad}=\tfrac{1}{c^2}\tfrac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_\mathrm{rit}};\ \vec E_\mathrm{diprad}=\vec B_\mathrm{diprad}\times\hat r$$

Larmor: $P_\mathrm{diprad}=\frac{2}{3c^3}|\ddot{\vec p}|^2$

$$\vec A_\mathrm{dm}=\tfrac1c\tfrac{\dot{\vec m}\times\hat r}{r}\Big|_{t_\mathrm{rit}}$$

L.W.: $(\phi,\vec A)=\frac{q(1,\frac{\vec v}{c})}{[r-\frac{\vec v\vec r}{c}]_{t_\mathrm{rit}}};\ t_\mathrm{rit}=t-\frac{r}{c}\Big|_{t_\mathrm{rit}}$

$$A^\mu=(\phi,\vec A);\ J^\mu=(c\rho,\vec J)$$

Lorenz gauge: $\partial_\alpha A^\alpha=0$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$$

$$F^{\mu\nu}=\begin{pmatrix}0&-E_x&-E_y&-E_z\\E_x&0&-B_z&B_y\\E_y&B_z&0&-B_x\\E_z&-B_y&B_x&0\end{pmatrix}$$

$$\mathscr{F}^{\mu\nu}=\tfrac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\partial_\alpha F^{\alpha\nu}=4\pi\tfrac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu}=0;\ \tfrac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=qF^{\mu\alpha}v_\alpha$$

$$F^{\alpha\beta}F_{\alpha\beta}=2(B^2-E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta}=4\vec E\vec B$$

$$\Theta^{\mu\nu}=\tfrac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu}+\tfrac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu}=\begin{pmatrix}u&c\vec g\\c\vec g&-\mathbf{T}\end{pmatrix}$$

$$\partial_\alpha\Theta^{\alpha\nu}=\tfrac{J_\alpha}{c}F^{\alpha\nu}$$

E.M. in matter (CGS)

$$\vec\nabla\vec D=4\pi\rho_\mathrm{ext};\ \vec\nabla\times\vec E=-\tfrac1c\tfrac{\partial\vec B}{\partial t}$$

$$\vec\nabla\vec B=0;\ \vec\nabla\times\vec H=4\pi\tfrac{\vec J_\mathrm{ext}}{c}+\tfrac1c\tfrac{\partial\vec D}{\partial t}$$

$$\vec P=\tfrac{\mathrm{d}\langle\vec p\rangle}{\mathrm{d}V};\ \vec M=\tfrac{\mathrm{d}\langle\vec m\rangle}{\mathrm{d}V}$$

$$\rho_\mathrm{pol}=-\vec\nabla\vec P;\ \sigma_\mathrm{pol}=\hat n\vec P;\ \tfrac{\vec J_\mathrm{mag}}{c}=\vec\nabla\times\vec M$$

$$\vec D_\mathrm{pol}=\vec E+4\pi\vec P;\ \vec H_\mathrm{mag}=\vec B-4\pi\vec M$$

static linear isotropic: $\vec P=\chi\vec E$

static linear: $P_i=\chi_{ij}E_j$

static linear: $\varepsilon=1+4\pi\chi$

static: $\Delta D_\perp=4\pi\sigma_\mathrm{ext};\ \Delta E_\parallel=0$

static linear: $u=\frac{1}{8\pi}\vec E\vec D$

$$\Delta U_\mathrm{dielectric}=-\tfrac12\textstyle\int\mathrm{d}^3r\vec P\vec E_0$$

plane capacitor: $C=\frac{\varepsilon}{4\pi}\frac{S}{d}$

cilindric capacitor: $C=\frac{L}{2\log\frac{R}{r}}$

atomic polarizability: $\vec p=\alpha\vec E$

non-interacting gas: $\vec p=\alpha\vec E_0;\ \chi=n\alpha$

hom. cubic isotropic: $\chi=\frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2}=\frac{4\pi}{3}n\alpha$

perm. dipole: $\chi=\frac13\frac{np_0^2}{kT}$

local field: $\vec E_\mathrm{loc}=\vec E+\frac{4\pi}{3}\vec P$

$$\vec J\vec E=-\vec\nabla\left(\tfrac{c}{4\pi}\vec E\times\vec H\right)-\tfrac{1}{4\pi}\left(\vec E\tfrac{\partial\vec D}{\partial t}+\vec H\tfrac{\partial\vec B}{\partial t}\right)$$

$$n=\sqrt{\varepsilon\mu};\ k=n\tfrac{\omega}{c}$$

plane wave: $B=nE$

$$\vec J_\mathrm{c}=\sigma\vec E;\ \varepsilon_\sigma=1+i\tfrac{4\pi\sigma}{\omega}$$

$$\omega_\mathrm{p}^2=4\pi\tfrac{nq^2}{m};\ \omega_\mathrm{cyclo}=\tfrac{qB}{mc}$$

I: $u=\frac{1}{8\pi}(\vec E\vec D+\vec H\vec B)$

I: $\langle S_z\rangle=\frac{c}{n}\langle u\rangle$

II: $u=\frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\frac{\partial}{\partial\omega}(\mu\omega)H^2\right)$

II: $\langle S_z\rangle=v_\mathrm{g}\langle u\rangle;\ v_\mathrm{g}=\frac{\partial\omega}{\partial k}=\frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$

III: $\langle W\rangle=\frac{\omega}{4\pi}\left(\operatorname{Im}\varepsilon\langle E^2\rangle+\operatorname{Im}\mu\langle H^2\rangle\right)$

Fresnel TE (S): $\frac{E_\mathrm{t}}{E_\mathrm{i}}=\frac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \frac{E_\mathrm{r}}{E_\mathrm{i}}=\frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$

TM (P): $\frac{E_\mathrm{t}}{E_\mathrm{i}}=\frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \frac{E_\mathrm{r}}{E_\mathrm{i}}=\frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$

Fresnel: $k_{tz}=\pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2-k_x^2},\ \operatorname{Im}k_{tz}>0$

Drüde-Lorentz: $\varepsilon=1-\frac{\omega_\mathrm{p}^2}{\omega^2+i\gamma\omega-\omega_0^2}$

$$P(t)=\textstyle\int_{-\infty}^{\infty}g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega)=\chi(\omega)E(\omega)$$

$$\chi(\omega)=\textstyle\int_{-\infty}^{\infty}e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega)=\overline{\chi}(\omega)$$

$$g(t<0)=0\implies$$

$$\operatorname{Re}\varepsilon(\omega)=1+\tfrac{2}{\pi}\textstyle\fint_0^\infty\tfrac{\omega'\left(\operatorname{Im}\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\operatorname{Im}\varepsilon(\omega)=-\tfrac{2\omega}{\pi}\textstyle\fint_0^\infty\tfrac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega'+\tfrac{4\pi\sigma_0}{\omega}$$

sum rule: $\frac{\pi}{2}\omega_\mathrm{p}^2=\int_0^\infty\omega\operatorname{Im}\varepsilon\mathrm{d}\omega$

sum rule: $2\pi^2\sigma_0=\int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$

sum rule: $\int_0^\infty(\operatorname{Re}n-1)\mathrm{d}\omega=0$

Miller rule: $\chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$