Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\left(x+\operatorname{atan}\frac{b}{a}\right)$$
$$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\left(x-\operatorname{atan}\frac{a}{b}\right)$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$

$$\binom{\sinh x}{\cosh x}=\frac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\frac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix)$$

$$\cos x=\cosh(ix)$$
$$\operatorname{asinh}x=\log\left(x+\sqrt{x^2+1}\right)$$
$$\operatorname{acosh}x=\log\left(x+\sqrt{x^2-1}\right)$$
$$\operatorname{atanh}x=\frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\sum_{x=1}^n x^3=\left(\sum_{x=1}^n x\right)^2=\frac{1}{4}n^2(n+1)^2$$
$$\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$$

$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\int_0^\infty t^ze^{-t}dt=z!$$
$$n!=\left(\frac{n}{e}\right)^n\sqrt{2\pi n}$$
Fourier: $c_n=\frac{2}{T}\int_0^T f(t)\cos\left(n\frac{t}{T}\right)\mathrm{d}t$

$$F[f]=\hat{f}(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ikx}f(x)$$
$$\langle\hat{f}|\hat{g}\rangle=\langle f|g\rangle$$
$$F\left[\frac{\sin x}{x}\right]=\sqrt{\frac{\pi}{2}}\chi_{[-1;1]}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\pm\sqrt{z}=\sqrt{\frac{\operatorname{Re}z+|z|}{2}}+\frac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$$

Derivatives

$$\tan'x=1+\tan^2x$$
$$\cot'x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\frac{1}{1+x^2}$$

$$\operatorname{asin}'x=-\operatorname{acos}'x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a'x=\frac{1}{x\ln a}$$

$$\cosh'x=\sinh x$$
$$\tanh'x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\frac{1}{1-x^2}$$

$$\operatorname{asinh}'x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}'x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$

$$\left(\frac{1}{x}\right)'=-\frac{\dot{x}}{x^2}$$
$$\left(\frac{x}{y}\right)'=\frac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)'=x^y\left(\dot{y}\ln x+y\frac{\dot{x}}{x}\right)$$

Integrals

$$\int x^a=\frac{x^{a+1}}{a+1}\qquad\int a^x=\frac{a^x}{\ln a}$$
$$\int\frac{1}{x}=\ln|x|\qquad\int\tan x=-\ln|\cos x|\qquad\int\cot x=\ln|\sin x|$$
$$\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|\qquad\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\qquad\int\ln x=x(\ln x-1)$$
$$\int\tanh x=\ln\cosh x\qquad\int\coth x=\ln|\sinh x|\qquad\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}\qquad\int xy=x\int y-\int\left(\dot{x}\int y\right)\qquad\int e^{yx}x=e^{yx}\left(\frac{y}{x}-\frac{1}{y^2}\right)$$

Differential equations

$$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\int be^a+c_1\right)$$
$$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$

$$\ddot{x}=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$

$$\dot{x}+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$
$$\ddot{x}+\gamma\dot{x}+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$
$$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$
$$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+\mathrm{O}(x^9)$$

$$\operatorname{acos}x=\frac{\pi}{2}-\operatorname{asin}x$$
$$\operatorname{atan}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$
$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$
$$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\left(\frac{\gamma^2}{2}+\frac{\pi^2}{12}\right)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$

$$\vec{a}\times\vec{b}=\varepsilon_{ijk}a_jb_k\hat{e}_i$$
$$(\vec{a}\otimes\vec{b})_{ij}=a_ib_j$$
$$(\vec{a}\times\vec{b})\vec{c}=(\vec{c}\times\vec{a})\vec{b}$$
$$(\vec{a}\times\vec{b})\times\vec{c}=-(\vec{b}\vec{c})\vec{a}+(\vec{a}\vec{c})\vec{b}$$

$$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})=(\vec{a}\vec{c})(\vec{b}\vec{d})-(\vec{a}\vec{d})(\vec{b}\vec{c})$$
$$|\vec{u}\times\vec{v}|^2=u^2v^2-(\vec{u}\vec{v})^2$$
$$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right);\ \Box=\frac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec{\nabla}V=\frac{\partial V}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{\phi}+\frac{\partial V}{\partial z}\hat{z}$$

$$\vec\nabla\vec v=\frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$

$$\vec\nabla\times\vec v=\Big(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\Big)\hat\rho+$$

$$+\Big(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\Big)\hat\phi+\frac{1}{\rho}\Big(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\Big)$$

$$\nabla^2V=\frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial V}{\partial\rho}\Big)+\frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2}+\frac{\partial^2V}{\partial z^2}$$

$$\vec\nabla V=\frac{\partial V}{\partial r}\hat r+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$

$$\vec\nabla\vec v=\frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$

$$\vec\nabla\times\vec v=\frac{1}{r\sin\theta}\Big(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta}-\frac{\partial v_\theta}{\partial\varphi}\Big)\hat r+$$

$$+\frac{1}{r}\Big(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(rv_\varphi)}{\partial r}\Big)\hat\theta+\frac{1}{r}\Big(\frac{\partial(rv_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}\Big)\hat\varphi$$

$$\nabla^2V=\frac{\frac{\partial}{\partial r}\big(r^2\frac{\partial V}{\partial r}\big)}{r^2}+\frac{\frac{\partial}{\partial\theta}\big(\sin\theta\frac{\partial V}{\partial\theta}\big)}{r^2\sin\theta}+\frac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\vec\nabla\Big(\vec\nabla\times\vec v\Big)=\vec\nabla\times\vec\nabla\vec V=0$$

$$\vec\nabla(f\vec v)=(\vec\nabla f)\vec v+f\vec\nabla\vec v$$

$$\vec\nabla\times(f\vec v)=\vec\nabla f\times\vec v+f\vec\nabla\times\vec v$$

$$\vec\nabla\times(\vec\nabla\times\vec v)=-\nabla^2\vec v+\vec\nabla(\vec\nabla\vec v)$$

$$\vec\nabla(\vec v\times\vec w)=\vec w(\vec\nabla\times\vec v)-\vec v(\vec\nabla\times\vec w)$$

$$\vec\nabla\times(\vec v\times\vec w)=(\vec\nabla\vec w+\vec w\,\vec\nabla)\vec v-(\vec\nabla\vec v+\vec v\,\vec\nabla)\vec w$$

$$\tfrac12\vec\nabla v^2=(\vec v\,\vec\nabla)\vec v+\vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm d^3x=\oint\vec v\,\mathrm d\vec S;\ \int(\vec\nabla\times\vec v)\mathrm d\vec S=\oint\vec v\,\mathrm d\vec l$$

$$\int(f\nabla^2g-g\nabla^2f)\,\mathrm d^3x=\oint_S\Big(f\frac{\partial g}{\partial n}-g\frac{\partial f}{\partial n}\Big)\,\mathrm dS$$

$$\oint\vec v\times\mathrm d\vec S=-\int(\vec\nabla\times\vec v)\mathrm d^3x$$

$$\delta(\vec r-\vec r_0)=\frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$

$$\nabla^2\frac{1}{|\vec r-\vec r_0|}=-4\pi\delta(\vec r-\vec r_0)$$

$$\delta(g(x))=\frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$

$$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle=\tfrac12\,\mathrm{Re}(a\bar b)$$

Statistics

$$P(E\cap E_1)=P(E_1)\cdot P(E|E_1)$$

$$\Delta x_{\text{hist}}\approx\frac{x_{\max}-x_{\min}}{\sqrt N}$$

$$P(x\le k)=F(k)=\int_{-\infty}^k p(x)$$

$$\text{median}=F^{-1}(\tfrac12)$$

$$E[f(x)]=\int_{-\infty}^{\infty}f(x)p(x)$$

$$\mu=E[x]=\int_{-\infty}^{\infty}xp(x)$$

$$\alpha_n=E[x^n]$$

$$M_n=E[(x-\mu)^n]$$

$$\sigma^2=M_2=E[x^2]-\mu^2$$

$$\text{FWHM}\approx2\sigma$$

$$\gamma_1=\frac{M_3}{\sigma^3},\ \gamma_2=\frac{M_4}{\sigma^4}$$

$$\phi[y](t)=E[e^{ity}]$$

$$\phi[y_1+\lambda y_2]=\phi[y_1]\phi[\lambda y_2]$$

$$\alpha_n=i^{-n}\frac{\partial^nt}{\partial\phi[x]^n}\Big|_{t=0}$$

$$h\ge0:P(h\ge k)\le\frac{E[h]}{k}$$

$$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$$

$$B(n,p,k)=\binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu_B=np,\ \sigma_B^2=np(1-p)$$

$$P(\mu,k)=\frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2=\mu$$

$$u(x,a,b)=\frac{1}{b-a},\ x\in[a;b]$$

$$\mu_u=\frac{b+a}{2},\ \sigma_u^2=\frac{(b-a)^2}{12}$$

$$\varepsilon(x,\lambda)=\lambda e^{-\lambda x},\ x\ge0$$

$$\mu_\varepsilon=\frac1\lambda,\ \sigma_\varepsilon^2=\frac{1}{\lambda^2}$$

$$g(x,\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac12(\frac{x-\mu}{\sigma})^2}$$

$$\text{FWHM}_g=2\sigma\sqrt{2\ln2}$$

$$z=\frac{x-\mu}{\sigma};\ \mu,\sigma[z]=0,1$$

$$\chi^2=\sum_{i=1}^n z_i^2$$

$$\wp_n(x)=\frac{2^{-\frac n2}}{\Gamma(\frac n2)}x^{\frac n2-1}e^{-\frac x2}$$

$$\mu_\wp=n,\ \sigma_\wp^2=2n$$

$$n\ge30:\wp_n(x)\approx g(x,n,\sqrt{2n})$$

$$n\ge8:p[\sqrt{2\chi^2}]\approx g(,\sqrt{2n-1},1)$$

$$S(x,n)=\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac n2)}\Big(1+\frac{x^2}{n}\Big)^{-\frac{n+1}{2}}$$

$$\mu_S=0,\ \sigma_S^2=\frac{n}{n-2}$$

$$p\Big[z\sqrt{\frac{n}{\chi^2}}\Big]=S(,n)$$

$$n\ge35:S(x,n)\approx g(x,0,1)$$

$$c(x,a)=\frac a\pi\frac{1}{a^2+x^2}$$

$$\sigma_{xy}=E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$

$$\rho=\frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le1$$

$$\mu[f(x_1,...)]\approx f(\mu_1,...)$$

$$\sigma^2[f(x_1,...)]\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$

$$\mu\approx m=\frac1n\sum_{i=1}^n x_i$$

$$\sigma^2\approx s^2=\frac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$

$$s_m^2=\frac{s^2}{n}$$

$$p\Big[\frac{m-\mu}{s_m}\Big]=S(,n)$$

Fit

$$f(x)=mx+q,\quad f(x)=a$$

$$f(x)=bx$$

$$m=\frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$q=\frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$a=\frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2=\frac{1}{\sum\frac{1}{\Delta y^2}}$$

$$\Delta m^2=\frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta q^2=\frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$

$$b=\frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2=\frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

Kinematics

$$\frac1R=\Big|\frac{v_xa_y-v_ya_x}{v^3}\Big|$$

$$\vec\omega=\dot\varphi\cos\theta\hat r-\dot\varphi\sin\theta\hat\theta+\dot\theta\hat\varphi$$

$$\dot{\vec w}=\frac{\mathrm d(\vec w\hat r)}{\mathrm dt}\hat r+\frac{\mathrm d(\vec w\hat\theta)}{\mathrm dt}\hat\theta+\frac{\mathrm d(\vec w\hat\varphi)}{\mathrm dt}\hat\varphi+\vec\omega\times\vec w$$

$$\theta\equiv\frac\pi2\to\dot{\vec r}=\dot r\hat r+r\dot\varphi\hat\varphi$$

$$\theta\equiv\frac\pi2\to\ddot{\vec r}=(\ddot r-r\dot\varphi^2)\hat r+(r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$$

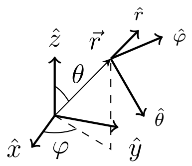
$$\dot{\vec r}=\dot r\hat r+r\dot\theta\hat\theta+r\dot\varphi\sin\theta\hat\varphi$$

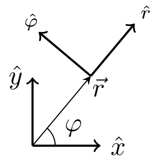
$$\langle\ddot{\vec r},\hat r\rangle=\ddot r-r\dot\theta^2-r\dot\varphi^2\sin^2\theta$$

$$\langle\ddot{\vec r},\hat\theta\rangle=r\ddot\theta+2\dot r\dot\theta-r\dot\varphi^2\sin\theta\cos\theta$$

$$\langle\ddot{\vec r},\hat\varphi\rangle=r\ddot\varphi\sin\theta+2\dot r\dot\varphi\sin\theta+2r\dot\theta\dot\varphi\cos\theta$$

$$\vec A=\ddot{\vec r}+\vec A_{\mathrm T}+\vec\omega\times(\vec\omega\times\vec r)+\dot{\vec\omega}\times\vec r+2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha=\frac{\mathrm d}{\mathrm dt}\alpha(\beta,t)=\frac{\partial\alpha}{\partial\beta}\dot\beta+\frac{\partial\alpha}{\partial t}$$

$$\vec p:=m\dot{\vec r};\ \vec F=\dot{\vec p};\ \frac{\mathrm d(mT)}{\mathrm dt}=\vec F\vec p$$

$$M:=\sum_i m_i;\ \vec R:=\frac{m_i\vec r_i}{M}$$

$$T=\tfrac12M\dot{\vec R}^2+\tfrac12m_i(\dot{\vec r}_i-\dot{\vec R})^2$$

$$\vec L=\vec R\times M\dot{\vec R}+(\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$$

$$\vec\tau_O=\dot{\vec L}_O+\vec v_O\times\vec p$$

$$\tau_1=I_1\dot\omega_1+(I_3-I_2)\omega_3\omega_2$$

$$\mathcal L(q,\dot q,t)=T-V+\frac{\mathrm d}{\mathrm dt}f(q,t)$$

$$S[q]=\int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$$

$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv0}^{\epsilon(t_1)=\epsilon(t_2)=0}=0$$

$$p:=\frac{\partial\mathcal L}{\partial\dot q};\ \dot p=\frac{\partial\mathcal L}{\partial q}$$

$$\mathcal H(q,p,t)=\dot qp-\mathcal L$$

$$\dot q=\frac{\partial\mathcal H}{\partial p};\ \dot p=-\frac{\partial\mathcal H}{\partial q}$$

$$\frac{\mathrm d\mathcal H}{\mathrm dt}=\frac{\partial\mathcal H}{\partial t}=-\frac{\partial\mathcal L}{\partial t}$$

$$\{u,v\}=\frac{\partial u}{\partial q}\frac{\partial v}{\partial p}-\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$

$$\frac{\mathrm du}{\mathrm dt}=\{u,\mathcal H\}+\frac{\partial u}{\partial t}$$

$$\eta=(q,p);\ \Gamma=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$$

$$\dot\eta=\Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\}=\frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac12mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac23mr^2$

ball: $\frac25mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\big(R^2+\frac34r^2\big)$

ellipsoid: $I_a=\frac15m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle\approx-2\langle T\rangle$$

$$U_{\text{eff}}=U+\frac{L^2}{2mr^2}$$

$$\frac1\mu=\frac{1}{m_1}+\frac{1}{m_2}$$

$$\vec r=\vec r_1-\vec r_2,\ \alpha=Gm_1m_2$$

$$T=\tfrac12M\dot{\vec R}^2+\tfrac12\mu\dot{\vec r}^2$$

$$\vec L=\vec R\times M\dot{\vec R}+\vec r\times\mu\dot{\vec r}$$

$$k=\frac{L^2}{\mu\alpha},\ \varepsilon=\sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$

$$r=\frac{k}{1+\varepsilon\cos\theta}$$

$$a=\frac{k}{|1-\varepsilon^2|}=\frac{\alpha}{2|E|}$$

$$a^3\omega^2=G(m_1+m_2)=\frac\alpha\mu$$

$$\vec A=\mu\dot{\vec r}\times\vec L-\mu\alpha\hat r,\ \dot{\vec A}=0$$

Inequalities

$$|a|-|b|\le|a+b|\le|a|+|b|$$

$$x>-1:1+nx\le(1+x)^n$$

$$\frac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$$

$$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$$

$$\sum a_ib_i\le\Big(\sum a_i^p\Big)^{\frac1p}\Big(\sum b_i^{\frac{p}{p-1}}\Big)^{\frac{p-1}{p}}$$

$$x^py^q\le\Big(\frac{px+qy}{p+q}\Big)^{p+q}$$

$$\sqrt[p]{\frac1n\sum a_i^{p\le q}}\le\sqrt[q]{\frac1n\sum a_i^q}$$

$$\sum\Big(\frac{a_1+\dots a_i}{i}\Big)^p\le\Big(\frac{p}{p-1}\Big)^p\sum a_i^p$$

$$x\ge0,|\ddot x|\le M:|\dot x|\le\sqrt{2Mx}$$

$$\frac{1}{1+x}<\ln\Big(1+\frac1x\Big)<\frac1x$$

Vector spaces

$(V, \mathbb{K}, +, \cdot)$ vector space; $\mathbb{K}$ field

$\exists \vec{0} \in V : \vec{v} + \vec{0} = \vec{v}$

$\cdot : \mathbb{K} \times V \rightarrow V; \quad \lambda \cdot (\vec{v} + \vec{w}) = \lambda\vec{v} + \lambda\vec{w}$

$0_{\mathbb{K}} \cdot \vec{v} = \vec{0},\ 1_{\mathbb{K}} \cdot \vec{v} = \vec{v}$

$\lambda \in \mathbb{K},\ \vec{v}, \vec{w} \in V \ \Rightarrow\ \vec{v} + \vec{w} \in V,\ \lambda\vec{v} \in V$

$\dim(U+V) = \dim U + \dim V - \dim(U \cap V)$

ℓ linear : $\ell(\vec{v} + \lambda\vec{w}) = \ell(\vec{v}) + \lambda\ell(\vec{w})$

$\ker \ell = \{\vec{v} \in V \mid \ell(\vec{v}) = 0\}$

$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$

$\langle , \rangle : V \times V \to \mathbb{K}; \quad \langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$

$\langle \vec{v} + \lambda\vec{w}, \vec{u} \rangle = \langle \vec{v}, \vec{u} \rangle + \lambda \langle \vec{w}, \vec{u} \rangle$

$\| \| : V \to \mathbb{K}; \quad \|\vec{v}\| = 0 \to \vec{v} = \vec{0}$

$\|\lambda\vec{v}\| = |\lambda| \|\vec{v}\|; \quad \|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\ \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\ \frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_{\mathrm{A}} = 6.022 \cdot 10^{23}\ \frac{1}{\mathrm{mol}}$

$k_{\mathrm{B}} = 1.381 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$

$q_{\mathrm{e}} = 1.602 \cdot 10^{-19}\ \mathrm{A\,s}$

$m_{\mathrm{e}} = 9.109 \cdot 10^{-31}\ \mathrm{kg}$

$m_{\mathrm{p}} = 1.673 \cdot 10^{-27}\ \mathrm{kg}$

$m_{\mathrm{n}} = 1.675 \cdot 10^{-27}\ \mathrm{kg}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\ \mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\ \mathrm{J\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{\mathrm{C}^2\,\mathrm{s}^2}{\mathrm{kg\,m}^3}$

$\mu_0 = 1.257 \cdot 10^{-6}\ \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_{\mathrm{B}} = 9.274 \cdot 10^{-24}\ \mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{eV} = 1.602 \cdot 10^{-12}\ \mathrm{erg}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H =$ heat transfer

$G = H - TS$

$a_i \mathrm{A}_i \rightarrow b_j \mathrm{B}_j$

$\Delta H^{\mathrm{o}}_{\mathrm{r}} = b_j \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{B}_j) - a_i \Delta H^{\mathrm{o}}_{\mathrm{f}}(\mathrm{A}_i)$

$\forall\, i, j : v_{\mathrm{r}} = -\frac{1}{a_i} \frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j} \frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_{\mathrm{r}} = k[\mathrm{A}_i]^{m_i}$

$k = A e^{-\frac{E_{\mathrm{a}}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\ \frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma \frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_{\mathrm{tot}}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_{\mathrm{tot}}^{\sum a_i - \sum b_j}$

$\Delta G^{\mathrm{o}}_{\mathrm{r}} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln \frac{K_2}{K_1} = -\frac{\Delta H^{\mathrm{o}}}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$

$K_{\mathrm{w}} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^{\mathrm{o}} - \frac{RT}{n_{\mathrm{e}} N_A q_{\mathrm{e}}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^{\mathrm{o}} - \frac{0.059}{n_{\mathrm{e}}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$

$\mathrm{d}L = p\mathrm{d}V$

$\mathrm{d}S = \frac{\mathrm{d}Q}{T}$

$C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T} \right)_V$

$C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T} \right)_p$

$\gamma = \frac{C_p}{C_V}$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2} R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma - 1},\ c_p = \frac{\gamma}{\gamma - 1} R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma - 1}, p^{\frac{1}{\gamma} - 1} T$ const.

Statistical mechanics

$Z = \frac{1}{h^N} \int \mathrm{d}q_1 \cdots \mathrm{d}q_N \int \mathrm{d}p_1 \cdots \mathrm{d}p_N e^{-\beta \mathcal{H}}$

$U = -\frac{\partial}{\partial \beta} \log Z;\ \beta = \frac{1}{k_{\mathrm{B}} T};\ C = \frac{\partial U}{\partial T}$

$F(T, V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0} e^{i\omega t}$

$Z = \frac{V}{I}$

$Z_R = R$

$Z_C = -i \frac{1}{\omega C}$

$Z_L = i\omega L$

$Z_{\mathrm{series}} = \sum_k Z_k$

$\frac{1}{Z_{\mathrm{parallel}}} = \sum_k \frac{1}{Z_k}$

$\sum_{\mathrm{loop}} V_k = 0$

$\sum_{\mathrm{node}} I_k = 0$

$\mathcal{E} = -L\dot{I}$

$L = \frac{\Phi_B}{I}$

Relativity

$\beta = \frac{v}{c} = \tanh \chi$

$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \cosh \chi$

$\vec{p} = \gamma m \vec{v}$

$\mathcal{E} = \gamma m c^2$

free particle: $\mathcal{L} = \mathcal{E}$

$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$

$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$

$\chi'' = \chi' + \chi$

$V'_{\parallel} = \frac{V_{\parallel} - v}{1 - \frac{v V_{\parallel}}{c^2}}$

$V'_{\perp} = \frac{1}{\gamma} \frac{V_{\perp}}{1 - \frac{v V_{\parallel}}{c^2}}$

$\frac{V'}{c} = 1 - \frac{\left(1 - \frac{V^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{v V_{\parallel}}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma} \mathrm{d}t$

$x^\mu = (ct, \vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma \left(\frac{\mathrm{d}\gamma}{\mathrm{d}t} c, \frac{\mathrm{d}(\gamma \vec{v})}{\mathrm{d}t} \right)$

$p^\mu = m v^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p} \right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c} \frac{\partial}{\partial t}, \vec{\nabla} \right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu} x^\nu$

$\partial_\mu \partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \rightarrow \sum_i m_i$

$E_1^{\max} = \frac{M^2 + m_1^2 - \sum_{i \neq 1} m_i^2}{2M} c^2$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E} \mathrm{d}\vec{S} = 4\pi \int \rho\, \mathrm{d}^3 x;\ -\nabla^2 V = \vec{\nabla} \vec{E} = 4\pi\rho;\ \vec{\nabla} \times \vec{E} = 0$

$U = \frac{1}{8\pi} \int E^2\, \mathrm{d}^3 x;\ \tilde{U} = \frac{1}{2} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi} \sum_{i \neq j} \int \vec{E}_i \vec{E}_j\, \mathrm{d}^3 x$

$V(\vec{r}) = \int \rho G_{\mathrm{D}}(\vec{r})\, \mathrm{d}^3 x - \frac{1}{4\pi} \oint_S V \frac{\partial G_{\mathrm{D}}}{\partial n}\, \mathrm{d}S$

$V(\vec{r}) = \langle V \rangle_S + \int \rho G_{\mathrm{N}}(\vec{r})\, \mathrm{d}^3 x + \frac{1}{4\pi} \oint_S \frac{\partial V}{\partial n} G_{\mathrm{N}}(\vec{r})\, \mathrm{d}S$

$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi \delta(\vec{x} - \vec{y});\ G_{\mathrm{D}}(\vec{x}, \vec{y})|_{\vec{y} \in S} = 0;\ \frac{\partial G_{\mathrm{N}}}{\partial n}\Big|_{\vec{y} \in S} = -\frac{4\pi}{S}$

$U_{\mathrm{sphere}} = \frac{3}{5} \frac{Q^2}{R};\ \vec{p} = \int \mathrm{d}^3 r \rho \vec{r};\ \vec{E}_{\mathrm{dip}} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_{\mathrm{dip}} = \frac{\vec{p}\vec{r}}{r^2}$

force on a dipole: $\vec{F}_{\mathrm{dip}} = (\vec{p} \vec{\nabla}) \vec{E}$

$Q_{ij} = \int \mathrm{d}^3 r \rho(\vec{r})(3 r_i r_j - \delta_{ij} r^2);\ V_{\mathrm{quad}} = \frac{1}{6r^5} Q_{ij}(3 r_i r_j - \delta_{ij} r^2)$

$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$

$V(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}} \right) Y_{lm}(\theta, \varphi)$

$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^{\infty} \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l \left(\frac{\vec{r}\vec{r}'}{r r'} \right)$

$P_l(x) = \frac{1}{2^l l!} \frac{\mathrm{d}^l}{\mathrm{d}x^l} \left(x^2 - 1 \right)^l;\ f = \sum_{l=0}^{\infty} c_l P_l : c_l = \frac{2l+1}{2} \int_{-1}^{1} f P_l$

$P_l(1) = 1;\ \langle P_n | P_m \rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm} | Y_{l'm'} \rangle = \delta_{ll'} \delta_{mm'}$

$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}} \sin^2\theta e^{2i\varphi}$

$$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\big(x^2-1\big)^l,\ |m|\le l$$

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m\overline{Y}_{lm}$$

$$P_l\big(\tfrac{\vec r\vec r'}{rr'}\big) = \tfrac{4\pi}{2l+1}\textstyle\sum_{m=-l}^{l}\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$$

$$V(r>\operatorname{diam}\operatorname{supp}\rho,\theta,\varphi) = \textstyle\sum_{l=0}^{\infty}\tfrac{4\pi}{2l+1}\sum_{m=-l}^{l}q_{lm}[\rho]\tfrac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \textstyle\int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\, r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec\nabla\vec J = -\tfrac{\partial\rho}{\partial t} = 0;\ I = \textstyle\int\vec J\mathrm{d}\vec S$$

$$\text{solenoid: } B = 4\pi\tfrac{j_s}{c}$$

$$\mathrm{d}\vec F = \tfrac{I\mathrm{d}\vec l}{c}\times\vec B = \mathrm{d}^3x\tfrac{\vec J}{c}\times\vec B;\ \vec F_q = q\tfrac{\dot{\vec r}}{c}\times\vec B$$

$$\mathrm{d}\vec B = \tfrac{I\mathrm{d}\vec l}{c}\times\tfrac{\vec r}{r^3};\ \vec B_q = q\tfrac{\dot{\vec r}}{c}\times\tfrac{\vec r}{r^3}$$

$$\vec B = \vec\nabla\times\vec A;\ \vec A = \textstyle\int\mathrm{d}^3r'\tfrac{\vec J'}{c}\tfrac{1}{|\vec r-\vec r'|}+\vec\nabla A_0$$

$$\vec B = \textstyle\int\mathrm{d}^3r'\tfrac{\vec J'}{c}\times\tfrac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$$

$$\varphi = \tfrac{I}{c}\Omega,\ \vec B = -\vec\nabla\varphi$$

$$\vec\nabla\vec A = 0\to\nabla^2\vec A = -4\pi\tfrac{\vec J}{c}$$

$$\vec\nabla\vec B = 0;\ \vec\nabla\times\vec B = 4\pi\tfrac{\vec J}{c};\ \textstyle\oint\vec B\mathrm{d}\vec l = 4\pi\tfrac{I}{c}$$

$$\vec m = \tfrac12\textstyle\int\mathrm{d}^3r'\big(\vec r'\times\tfrac{\vec J'}{c}\big) = \tfrac{1}{2c}\tfrac{q}{m}\vec L = \tfrac{SI}{c}$$

$$\vec A_{\mathrm{dm}} = \tfrac{\vec m\times\vec r}{r^3};\ \vec\tau = \vec m\times\vec B$$

$$\vec F_{\mathrm{dmdm}} = -\vec\nabla_R\tfrac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$$

$$\text{loop axis: }\vec B = \hat z\tfrac{2\pi R^2}{(z^2+R^2)^{3/2}}\tfrac{I}{c}$$

Electromagnetism (CGS)

$$\text{Faraday: }\mathcal{E} = -\tfrac1c\tfrac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \textstyle\int\mathrm{d}^3x\vec J = \dot{\vec p}$$

$$\vec\nabla\times\vec E = -\tfrac1c\tfrac{\partial\vec B}{\partial t};\ \vec\nabla\vec E = 4\pi\rho;\ \vec\nabla\vec J = -\tfrac{\partial\rho}{\partial t}$$

$$\vec\nabla\times\vec B = 4\pi\tfrac{\vec J}{c}+\tfrac1c\tfrac{\partial\vec E}{\partial t};\ \vec\nabla\vec B = 0$$

$$\mathrm{d}\vec F = \mathrm{d}^3x\big(\rho\vec E+\tfrac{\vec J}{c}\times\vec B\big);\ \vec F_q = q\big(\vec E+\tfrac{\dot{\vec r}}{c}\times\vec B\big)$$

$$u = \tfrac{E^2+B^2}{8\pi};\ \vec S = \tfrac{c}{4\pi}\vec E\times\vec B;\ \vec g = \tfrac{\vec S}{c^2}$$

$$\mathbf{T}^E = \tfrac{1}{4\pi}\big(\vec E\otimes\vec E-\tfrac12E^2\big);\ \mathbf{T} = \mathbf{T}^E+\mathbf{T}^B$$

$$-\tfrac{\partial u}{\partial t} = \vec J\vec E+\vec\nabla\vec S;\ -\tfrac{\partial\vec g}{\partial t} = \rho\vec E+\tfrac{\vec J}{c}\times\vec B-\vec\nabla\mathbf{T}$$

$$\vec B = \vec\nabla\times\vec A;\ \vec E = -\vec\nabla\phi-\tfrac1c\tfrac{\partial\vec A}{\partial t}$$

$$-\nabla^2\phi-\tfrac1c\tfrac{\partial}{\partial t}\vec\nabla\vec A = 4\pi\rho$$

$$\vec\nabla\big(\vec\nabla\vec A+\tfrac1c\tfrac{\partial\phi}{\partial t}\big)-\nabla^2\vec A+\tfrac1c\tfrac{\partial^2\vec A}{\partial t^2} = 4\pi\tfrac{\vec J}{c}$$

$$(\phi,\vec A)\cong\big(\phi-\tfrac1c\tfrac{\partial\chi}{\partial t},\vec A+\vec\nabla\chi\big)$$

$$(\phi,\vec A) = \textstyle\int\mathrm{d}^3r'\tfrac{\big(\rho,\frac{\vec J}{c}\big)\big(\vec r',t-\frac1c|\vec r-\vec r'|\big)}{|\vec r-\vec r'|}$$

$$\text{Coulomb gauge: }\vec\nabla\vec A = 0$$

$$\text{Lorenz gauge: }\vec\nabla\vec A+\tfrac1c\tfrac{\partial\phi}{\partial t} = 0$$

$$\vec E'_\parallel = \vec E_\parallel;\ \vec B'_\parallel = \vec B_\parallel$$

$$\vec E'_\perp = \gamma\big(\vec E_\perp+\tfrac{\vec v}{c}\times\vec B\big)$$

$$\vec B'_\perp = \gamma\big(\vec B_\perp-\tfrac{\vec v}{c}\times\vec E\big)$$

$$\text{plane wave: }\begin{cases}\vec E = \vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B = \hat k\times\vec E\\ \omega = ck\end{cases}$$

$$\vec B_{\mathrm{diprad}} = \tfrac{1}{c^2}\tfrac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}};\ \vec E_{\mathrm{diprad}} = \vec B_{\mathrm{diprad}}\times\hat r$$

$$\text{Larmor: }P_{\mathrm{diprad}} = \tfrac{2}{3c^3}|\ddot{\vec p}|^2$$

$$\vec A_{\mathrm{dm}} = \tfrac1c\tfrac{\dot{\vec m}\times\hat r}{r}\Big|_{t_{\mathrm{rit}}}$$

$$\text{L.W.: }(\phi,\vec A) = \tfrac{q(1,\frac{\vec v}{c})}{[r-\frac{\vec v\vec r}{c}]_{t_{\mathrm{rit}}}};\ t_{\mathrm{rit}} = t-\tfrac{r}{c}\Big|_{t_{\mathrm{rit}}}$$

$$A^\mu = (\phi,\vec A);\ J^\mu = (c\rho,\vec J)$$

$$\text{Lorenz gauge: }\partial_\alpha A^\alpha = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu-\partial^\nu A^\mu$$

$$F^{\mu\nu} = \begin{pmatrix}0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}$$

$$\mathscr{F}^{\mu\nu} = \tfrac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi\tfrac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu} = 0;\ \tfrac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = qF^{\mu\alpha}v_\alpha$$

$$F^{\alpha\beta}F_{\alpha\beta} = 2(B^2-E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta} = 4\vec E\vec B$$

$$\Theta^{\mu\nu} = \tfrac{1}{4\pi}\big(F^\mu{}_\alpha F^{\alpha\nu}+\tfrac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\big)$$

$$\Theta^{\mu\nu} = \begin{pmatrix}u & c\vec g\\ c\vec g & -\mathbf{T}\end{pmatrix}$$

$$\partial_\alpha\Theta^{\alpha\nu} = \tfrac{J_\alpha}{c}F^{\alpha\nu}$$

E.M. in matter (CGS)

$$\vec\nabla\vec D = 4\pi\rho_{\mathrm{ext}};\ \vec\nabla\times\vec E = -\tfrac1c\tfrac{\partial\vec B}{\partial t}$$

$$\vec\nabla\vec B = 0;\ \vec\nabla\times\vec H = 4\pi\tfrac{\vec J_{\mathrm{ext}}}{c}+\tfrac1c\tfrac{\partial\vec D}{\partial t}$$

$$\vec P = \tfrac{\mathrm{d}\langle\vec p\rangle}{\mathrm{d}V};\ \vec M = \tfrac{\mathrm{d}\langle\vec m\rangle}{\mathrm{d}V}$$

$$\rho_{\mathrm{pol}} = -\vec\nabla\vec P;\ \sigma_{\mathrm{pol}} = \hat n\vec P;\ \tfrac{\vec J_{\mathrm{mag}}}{c} = \vec\nabla\times\vec M$$

$$\vec D_{\mathrm{pol}} = \vec E+4\pi\vec P;\ \vec H_{\mathrm{mag}} = \vec B-4\pi\vec M$$

$$\text{static linear isotropic: }\vec P = \chi\vec E$$

$$\text{static linear: }P_i = \chi_{ij}E_j$$

$$\text{static linear: }\varepsilon = 1+4\pi\chi$$

$$\text{static: }\Delta D_\perp = 4\pi\sigma_{\mathrm{ext}};\ \Delta E_\parallel = 0$$

$$\text{static linear: }u = \tfrac{1}{8\pi}\vec E\vec D$$

$$\Delta U_{\mathrm{dielectric}} = -\tfrac12\textstyle\int\mathrm{d}^3r\vec P\vec E_0$$

$$\text{plane capacitor: }C = \tfrac{\varepsilon}{4\pi}\tfrac{S}{d}$$

$$\text{cilindric capacitor: }C = \tfrac{L}{2\log\frac{R}{r}}$$

$$\text{atomic polarizability: }\vec p = \alpha\vec E$$

$$\text{non-interacting gas: }\vec p = \alpha\vec E_0;\ \chi = n\alpha$$

$$\text{hom. cubic isotropic: }\chi = \tfrac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$$

$$\text{Clausius-Mossotti: }\tfrac{\varepsilon-1}{\varepsilon+2} = \tfrac{4\pi}{3}n\alpha$$

$$\text{perm. dipole: }\chi = \tfrac13\tfrac{np_0^2}{kT}$$

$$\text{local field: }\vec E_{\mathrm{loc}} = \vec E+\tfrac{4\pi}{3}\vec P$$

$$\vec J\vec E = -\vec\nabla\Big(\tfrac{c}{4\pi}\vec E\times\vec H\Big)-\tfrac{1}{4\pi}\Big(\vec E\tfrac{\partial\vec D}{\partial t}+\vec H\tfrac{\partial\vec B}{\partial t}\Big)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\tfrac{\omega}{c}$$

$$\text{plane wave: }B = nE$$

$$\vec J_{\mathrm{c}} = \sigma\vec E;\ \varepsilon_\sigma = 1+i\tfrac{4\pi\sigma}{\omega}$$

$$\omega_{\mathrm{p}}^2 = 4\pi\tfrac{nq^2}{m};\ \omega_{\mathrm{cyclo}} = \tfrac{qB}{mc}$$

$$\text{I: }u = \tfrac{1}{8\pi}(\vec E\vec D+\vec H\vec B)$$

$$\text{I: }\langle S_z\rangle = \tfrac{c}{n}\langle u\rangle$$

$$\text{II: }u = \tfrac{1}{8\pi}\big(\tfrac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\tfrac{\partial}{\partial\omega}(\mu\omega)H^2\big)$$

$$\text{II: }\langle S_z\rangle = v_{\mathrm{g}}\langle u\rangle;\ v_{\mathrm{g}} = \tfrac{\partial\omega}{\partial k} = \tfrac{c}{n+\omega\frac{\partial n}{\partial\omega}}$$

$$\text{III: }\langle W\rangle = \tfrac{\omega}{4\pi}\big(\operatorname{Im}\varepsilon\langle E^2\rangle+\operatorname{Im}\mu\langle H^2\rangle\big)$$

$$\text{Fresnel TE (S): }\tfrac{E_{\mathrm{t}}}{E_{\mathrm{i}}} = \tfrac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \tfrac{E_{\mathrm{r}}}{E_{\mathrm{i}}} = \tfrac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$$

$$\text{TM (P): }\tfrac{E_{\mathrm{t}}}{E_{\mathrm{i}}} = \tfrac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \tfrac{E_{\mathrm{r}}}{E_{\mathrm{i}}} = \tfrac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$$

$$\text{Fresnel: }k_{tz} = \pm\sqrt{\varepsilon_2\big(\tfrac{\omega}{c}\big)^2-k_x^2},\ \operatorname{Im}k_{tz}>0$$

$$\text{Drüde-Lorentz: }\varepsilon = 1-\tfrac{\omega_{\mathrm{p}}^2}{\omega^2+i\gamma\omega-\omega_0^2}$$

$$P(t) = \textstyle\int_{-\infty}^{\infty}g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega) = \chi(\omega)E(\omega)$$

$$\chi(\omega) = \textstyle\int_{-\infty}^{\infty}e^{i\omega t}g(t)\mathrm{d}t$$

$$g(t<0) = 0\implies\chi(-\omega) = \overline{\chi}(\omega)$$

$$\operatorname{Re}\varepsilon(\omega) = 1+\tfrac{2}{\pi}\textstyle\fint_0^\infty\tfrac{\omega'\big(\operatorname{Im}\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\big)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\operatorname{Im}\varepsilon(\omega) = -\tfrac{2\omega}{\pi}\textstyle\fint_0^\infty\tfrac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega'+\tfrac{4\pi\sigma_0}{\omega}$$

$$\text{sum rule: }\tfrac{\pi}{2}\omega_{\mathrm{p}}^2 = \textstyle\int_0^\infty\omega\operatorname{Im}\varepsilon\mathrm{d}\omega$$

$$\text{sum rule: }2\pi^2\sigma_0 = \textstyle\int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$$

$$\text{Miller rule: }\chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$$