Trigonometric functions

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\sin(2\alpha)=2\sin\alpha\cos\alpha$; $\tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$

$=2\cos^2\alpha-1=1-2\sin^2\alpha$

$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$

$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$

$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$

$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\quad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}$ $\quad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}$ $\quad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

$a\sin x+b\cos x=$

$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\left(x+\operatorname{atan}\frac{b}{a}\right)$

$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\left(x-\operatorname{atan}\frac{a}{b}\right)$

$\operatorname{acos}x+\operatorname{asin}x=\frac{\pi}{2}$

Hyperbolic functions

$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$

$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$

$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$

$\binom{\sinh x}{\cosh x}=\frac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$

$\cosh^2x-\sinh^2x=1$

$\cosh^2x=\frac{1}{1-\tanh^2x}$

$\sin x=-i\sinh(ix)$

$\cos x=\cosh(ix)$

$\operatorname{asinh}x=\log\left(x+\sqrt{x^2+1}\right)$

$\operatorname{acosh}x=\log\left(x+\sqrt{x^2-1}\right)$

$\operatorname{atanh}x=\frac{1}{2}\log\frac{1+x}{1-x}$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$

Combinatorics

$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}$ $\quad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}$ $\quad C'_{n,k}=\binom{n+k-1}{k}$

$D_{n,k}=\frac{n!}{(n-k)!}$

Miscellaneous

$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$

$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$

$\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$

$\sum_{x=1}^n x^3=\left(\sum_{x=1}^n x\right)^2=\frac{1}{4}n^2(n+1)^2$

$\sum_{x=1}^n x^2=\frac{1}{6}n(n+1)(2n+1)$

$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$

$(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$

$e^{i\theta}=\cos\theta+i\sin\theta$

$\Gamma(1+z)=\int_0^\infty t^ze^{-t}dt=z!$

$n!\approx\left(\frac{n}{e}\right)^n\sqrt{2\pi n}$

Fourier: $c_n=\frac{2}{T}\int_0^T f(t)\cos\left(n\frac{t}{T}\right)\mathrm{d}t$

$F[f]=\hat{f}(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ikx}f(x)$

$\langle\hat{f}|\hat{g}\rangle=\langle f|g\rangle$

$F\left[\frac{\sin x}{x}\right]=\sqrt{\frac{\pi}{2}}\chi_{[-1;1]}$

$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$

$\pm\sqrt{z}=\sqrt{\frac{\operatorname{Re}z+|z|}{2}}+\frac{i\operatorname{Im}z}{\sqrt{2(\operatorname{Re}z+|z|)}}$

$\delta(g(x))=\frac{\delta(x-x_i)}{|g'(x_i)|}$; $g(x_i)=0$

$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\frac{1}{2}\operatorname{Re}(a\bar{b})$

Derivatives

$\tan'x=1+\tan^2x$

$\cot'x=-1-\cot^2x$

$\operatorname{atan}'x=-\operatorname{acot}'x=\frac{1}{1+x^2}$

$\operatorname{asin}'x=-\operatorname{acos}'x=\frac{1}{\sqrt{1-x^2}}$

$(a^x)'=a^x\ln a$

$\log_a'x=\frac{1}{x\ln a}$

$\cosh'x=\sinh x$

$\tanh'x=1-\tanh^2x$

$\operatorname{atanh}'x=\operatorname{acoth}'x=\frac{1}{1-x^2}$

$\operatorname{asinh}'x=\frac{1}{\sqrt{x^2+1}}$

$\operatorname{acosh}'x=\frac{1}{\sqrt{x^2-1}}$

$(f^{-1})'=\frac{1}{f'(f^{-1})}$

$\left(\frac{1}{x}\right)'=-\frac{\dot{x}}{x^2}$

$\left(\frac{x}{y}\right)'=\frac{\dot{x}y-x\dot{y}}{y^2}$

$(x^y)'=x^y\left(\dot{y}\ln x+y\frac{\dot{x}}{x}\right)$

$\frac{\partial(x,y)}{\partial(u,v)}:=\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}-\frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$

$\frac{\partial(x,y)}{\partial(u,y)}=\frac{\partial x}{\partial u}\Big|_y=-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x$

$\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_x\frac{\partial u}{\partial x}\Big|_y=-1$

$\frac{\partial x}{\partial u}\Big|_y=\frac{\partial x}{\partial u}\Big|_v-\frac{\partial x}{\partial y}\Big|_u\frac{\partial y}{\partial u}\Big|_v$

$\frac{\partial x}{\partial u}\Big|_v=\frac{\partial x}{\partial y}\Big|_v\frac{\partial y}{\partial u}\Big|_v$

Integrals

$\int x^a=\frac{x^{a+1}}{a+1}$

$\int a^x=\frac{a^x}{\ln a}$

$\int\frac{1}{x}=\ln|x|$

$\int\tan x=-\ln|\cos x|$

$\int\cot x=\ln|\sin x|$

$\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|$

$\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$

$\int\ln x=x(\ln x-1)$

$\int\tanh x=\ln\cosh x$

$\int\coth x=\ln|\sinh x|$

$\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$

$\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$

$\int xy=x\int y-\int(\dot{x}\int y)$

$\int e^{yx}x=e^{yx}\left(\frac{y}{x}-\frac{1}{y^2}\right)$

$\int e^{-x^2}=\sqrt{\pi}$

Differential equations

$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\int be^a+c_1\right)$

$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$

$\ddot{x}=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$

$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$

$\dot{x}+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$

$\ddot{x}+\gamma\dot{x}+\omega_0^2x=fe^{-i\omega t}:x=\frac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$

Taylor

$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$

$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\mathrm{O}(x^9)$

$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\mathrm{O}(x^7)$

$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$

$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+\mathrm{O}(x^7)$

$\operatorname{asin}x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+\mathrm{O}(x^9)$

$\operatorname{atan}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$

$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$

$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$

$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$

$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$

$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$

$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$

$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+\mathrm{O}(x^9)$

$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+\mathrm{O}(x^7)$

$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$

$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+\mathrm{O}(x^7)$

$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$

$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+\mathrm{O}(x^6)$

$x!=1-\gamma x+\left(\frac{\gamma^2}{2}+\frac{\pi^2}{12}\right)x^2+\mathrm{O}(x^3)$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$

$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$

$\vec{a}\times\vec{b}=\varepsilon_{ijk}a_jb_k\hat{e}_i$

$(\vec{a}\otimes\vec{b})_{ij}=a_ib_j$

$(\vec{a}\times\vec{b})\vec{c}=(\vec{c}\times\vec{a})\vec{b}$

$(\vec{a}\times\vec{b})\times\vec{c}=-(\vec{b}\vec{c})\vec{a}+(\vec{a}\vec{c})\vec{b}$

$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})=(\vec{a}\vec{c})(\vec{b}\vec{d})-(\vec{a}\vec{d})(\vec{b}\vec{c})$

$|\vec{u}\times\vec{v}|^2=u^2v^2-(\vec{u}\vec{v})^2$

$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$; $\Box=\frac{\partial^2}{\partial t^2}-\nabla^2$

$\vec{\nabla}V=\frac{\partial V}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{\phi}+\frac{\partial V}{\partial z}\hat{z}$

$\vec{\nabla}\vec{v}=\frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$

$\vec{\nabla}\times\vec{v}=\left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{\rho}+$

$+\left(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\right)\hat{\phi}+\frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\right)$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\vec\nabla V = \frac{\partial V}{\partial r}\hat r + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat\varphi$$
$$\vec\nabla\vec v = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec\nabla\times\vec v = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\varphi}\right)\hat r +$$
$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(rv_\varphi)}{\partial r}\right)\hat\theta + \frac{1}{r}\left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat\varphi$$
$$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial\varphi^2}}{r^2\sin^2\theta}$$

$$\vec\nabla(\vec\nabla\times\vec v) = \vec\nabla\times\vec\nabla V = 0$$
$$\vec\nabla(f\vec v) = (\vec\nabla f)\vec v + f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v) = \vec\nabla f\times\vec v + f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v) = -\nabla^2\vec v + \vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w) = \vec w(\vec\nabla\times\vec v) - \vec v(\vec\nabla\times\vec w)$$
$$\vec\nabla\times(\vec v\times\vec w) = (\vec\nabla\vec w + \vec w\,\vec\nabla)\vec v - (\vec\nabla\vec v + \vec v\,\vec\nabla)\vec w$$
$$\frac{1}{2}\vec\nabla v^2 = (\vec v\,\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$

$$\int\vec\nabla\vec v\,\mathrm{d}^3x = \oint\vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$
$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$$
$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(\vec r - \vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\nabla^2\frac{1}{|\vec r - \vec r_0|} = -4\pi\delta(\vec r - \vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}} \approx \frac{x_{\max} - x_{\min}}{\sqrt N}$$
$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$
$$\text{median} = F^{-1}(\tfrac{1}{2})$$
$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$
$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$
$$\alpha_n = E[x^n]$$
$$M_n = E[(x-\mu)^n]$$
$$\sigma^2 = M_2 = E[x^2] - \mu^2$$
$$\text{FWHM} \approx 2\sigma$$
$$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$
$$\phi[y_1 + \lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge 0 : P(h\ge k) \le \frac{E[h]}{k}$$
$$P(|x-\mu| > k\sigma) \le \frac{1}{k^2}$$
$$B(k;n,p) = \binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$
$$P(k;\mu) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$
$$u(x;a,b) = \frac{1}{b-a},\ x\in[a;b]$$
$$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$$
$$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \frac{1}{\lambda},\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$$
$$g(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
$$\text{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$
$$z = \frac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$
$$\chi^2 = \sum_{i=1}^n z_i^2;\ \wp := p[\chi^2]$$
$$\wp(x;n) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$
$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$
$$n\ge 30 : \wp(x;n) \approx g(x;n,\sqrt{2n})$$
$$n\ge 8 : p[\sqrt{2\chi^2}] \approx g(;\sqrt{2n-1},1)$$
$$S(x;n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$
$$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$$

$$p\left[z\sqrt{\frac{n}{\chi^2}}\right] = S(,n)$$
$$n\ge 35 : S(x;n) \approx g(x;0,1)$$
$$c(x;a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$$
$$\sigma_{xy} = E[xy] - \mu_x\mu_y \le \sigma_x\sigma_y$$
$$\rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le 1$$
$$\mu[f(x_1,...)] \approx f(\mu_1,...)$$
$$\sigma^2[f(x_1,...)] \approx \sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$
$$\mu \approx m = \frac{1}{n}\sum_{i=1}^n x_i$$
$$\sigma^2 \approx s^2 = \frac{1}{n-1}\sum_{i=1}^n(x_i - m)^2$$
$$s_m^2 = \frac{s^2}{n}$$
$$p\left[\frac{m-\mu}{s_m}\right] = S(;n)$$

Fit

$$f(x) = mx + q,\quad f(x) = a,$$
$$f(x) = bx,\quad f(x;\theta) = \theta_i h_i(x)$$
$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$
$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$

$$\Delta mq = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$$
$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$$

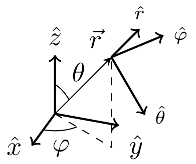
$$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$$
$$\chi^2 = (y - f(x;\theta))^T V^{-1}(y - f(x;\theta))$$
$$\theta = (H^T V^{-1} H)^{-1} H^T V^{-1} y$$
$$\Delta\theta\theta = (H^T V^{-1} H)^{-1}$$

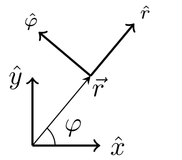
Kinematics

$$\theta \equiv \frac{\pi}{2} \to \ddot{\vec r} = (\ddot r - r\dot\varphi^2)\hat r + (r\ddot\varphi + 2\dot r\dot\varphi)\hat\varphi \qquad \vec A = \ddot{\vec r} + \vec A_{\mathrm{T}} + \vec\omega\times(\vec\omega\times\vec r) + \dot{\vec\omega}\times\vec r + 2\vec\omega\times\dot{\vec r}$$

$$\frac{1}{R} = \left|\frac{v_x a_y - v_y a_x}{v^3}\right|$$
$$\vec\omega = \dot\varphi\cos\theta\hat r - \dot\varphi\sin\theta\hat\theta + \dot\theta\hat\varphi$$
$$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r + \frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta + \frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi + \vec\omega\times\vec w$$
$$\theta \equiv \frac{\pi}{2} \to \dot{\vec r} = \dot r\hat r + r\dot\varphi\hat\varphi$$

$$\dot{\vec r} = \dot r\hat r + r\dot\theta\hat\theta + r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r - r\dot\theta^2 - r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta$$





Mechanics

$$\dot\alpha = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta + \frac{\partial\alpha}{\partial t}$$
$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec F\vec p$$
$$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$$
$$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}m_i(\dot{\vec r}_i - \dot{\vec R})^2$$

$$\vec L = \vec R\times M\dot{\vec R} + (\vec r_i - \vec R)\times m_i(\dot{\vec r}_i - \dot{\vec R})$$
$$\vec\tau_O = \dot{\vec L}_O + \vec v_O\times\vec p$$
$$\tau_1 = I_1\dot\omega_1 + (I_3 - I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t) = T - V + \frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$
$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm{d}t$$

$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$
$$p := \frac{\partial\mathcal L}{\partial\dot q};\ \dot p = \frac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t) = \dot q p - \mathcal L$$
$$\dot q = \frac{\partial\mathcal H}{\partial p};\ \dot p = -\frac{\partial\mathcal H}{\partial q}$$
$$\frac{\mathrm{d}\mathcal H}{\mathrm{d}t} = \frac{\partial\mathcal H}{\partial t} = -\frac{\partial\mathcal L}{\partial t}$$

$$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal H\} + \frac{\partial u}{\partial t}$$
$$\eta = (q,p);\ \Gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$\dot\eta = \Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

rod: $\frac{1}{12}mL^2$ octahedron: $\frac{1}{10}ms^2$ cone: $\frac{3}{10}mr^2$ rectangulus: $\frac{1}{12}m(a^2+b^2)$

point: mr^2 disk: $\frac{1}{2}mr^2$ sphere: $\frac{2}{3}mr^2$ torus: $m\left(R^2 + \frac{3}{4}r^2\right)$

two points: μd^2 tetrahedron: $\frac{1}{20}ms^2$ ball: $\frac{2}{5}mr^2$ ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$

Kepler

$$\langle U\rangle = -2\langle T\rangle$$
$$U_{\text{eff}} = U + \frac{L^2}{2mr^2}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$
$$\vec r = \vec r_1 - \vec r_2,\ \alpha = Gm_1m_2$$
$$T = \frac{1}{2}M\dot{\vec R}^2 + \frac{1}{2}\mu\dot{\vec r}^2$$

$$\vec L = \vec R\times M\dot{\vec R} + \vec r\times\mu\dot{\vec r}$$
$$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}}$$

$$r = \frac{k}{1+\varepsilon\cos\theta}$$
$$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$$
$$a^3\omega^2 = G(m_1+m_2) = \frac{\alpha}{\mu}$$

$$\vec A = \mu\dot{\vec r}\times\vec L - \mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$$|a| - |b| \le |a+b| \le |a| + |b|$$
$$x > -1 : 1 + nx \le (1+x)^n$$

$$\frac{|a^n - b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$$
$$\sum a_ib_i \le \left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$$

$$x^py^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$$
$$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$$

$$\sum\left(\frac{a_1+\ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p\sum a_i^p$$
$$x\ge 0, |\ddot x| \le M : |\dot x| \le \sqrt{2Mx}$$
$$\frac{1}{1+x} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}$$

Linear algebra

$$\dim V = \dim\ell(V) + \dim(V\cap\ker\ell)$$
$$\dim(U+V) = \dim U + \dim V - \dim(U\cap V)$$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\, \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\, \frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\, \frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022 \cdot 10^{23}\, \frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381 \cdot 10^{-23}\, \frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617 \cdot 10^{-5}\, \frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\, \frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602 \cdot 10^{-19}\,\mathrm{A\,s}$

$m_\mathrm{e} = 9.109 \cdot 10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673 \cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675 \cdot 10^{-27}\,\mathrm{kg}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\,\mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\,\mathrm{J\,s}$

$h = 4.136 \cdot 10^{-15}\,\mathrm{eV\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\, \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988 \cdot 10^{9}\, \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257 \cdot 10^{-6}\, \frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274 \cdot 10^{-24}\,\mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{barn} = 1 \cdot 10^{-28}\,\mathrm{m}^2$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i \mathrm{A}_i \to b_j \mathrm{B}_j$

$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$

$\forall\, i, j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$

$k = A e^{-\frac{E_\mathrm{a}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0}$, $[\mathrm{X}]_0 = 1\, \frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma \frac{p}{p_0}$, $p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}}$, $K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}}$, $K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}}$, $\chi = \frac{n}{n_\mathrm{tot}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_\chi = K_n n_\mathrm{tot}^{\sum a_i - \sum b_j}$

$\Delta G^\mathrm{o}_\mathrm{r} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln \frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu \mathrm{d}N$

$C_{V,N} = \frac{\partial Q}{\partial T}\big|_{V,N} = \frac{\partial U}{\partial T}\big|_{V,N}$

$C_{p,N} = \frac{\partial Q}{\partial T}\big|_{p,N} = \frac{\partial U}{\partial T}\big|_{p,N} + p\frac{\partial V}{\partial T}\big|_{p,N}$

$\gamma := \frac{C_p}{C_V}$

$\mu_J := \frac{\partial T}{\partial V}\big|_{U,N}$

$\lambda U = U(\lambda(S, V, N)) \Rightarrow U = ST - pV + \mu N$

$\Rightarrow S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix S, V, N : $\min U$ at equilibrium

Fix T, V, N : $\min F = U - TS$

Fix T, p, N : $\min G = F + pV$

Fix S, p, N : $\min H = U + pV$

$$\begin{array}{ccc} V & F & T \\ U & & G \\ S & H & p \end{array}$$

$\frac{\partial}{\partial T}\frac{G}{T}\big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\big|_V = -\frac{U}{T^2}$

$\Omega = U - TS - \mu N$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n}$, $c_V = \frac{\mathrm{dof}}{2}R$, $c_p = c_V + R$

$c_V = \frac{R}{\gamma - 1}$, $c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int \mathrm{d}q_1 \cdots \mathrm{d}q_N \int \mathrm{d}p_1 \cdots \mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z$; $\beta = \frac{1}{k_\mathrm{B}T}$; $C = \frac{\partial U}{\partial T}$

$F(T, V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\binom{V}{I} = \binom{V_0}{I_0} e^{i\omega t}$

$Z = \frac{V}{I}$

$Z_R = R$

$Z_C = -i\frac{1}{\omega C}$

$Z_L = i\omega L$

$Z_\mathrm{series} = \sum_k Z_k$

$\frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$

$\sum_\mathrm{loop} V_k = 0$

$\sum_\mathrm{node} I_k = 0$

$\mathcal{E} = -L\dot{I}$

$L = \frac{\Phi_B}{I}$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec{p} = \gamma m \vec{v}$

$\mathcal{E} = \gamma m c^2$

free particle: $\mathcal{L} = \frac{mc^2}{\gamma}$

$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec{v}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$

$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\left(1 - \frac{vV_\parallel}{c^2}\right)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct, \vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = m v^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$

$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(\frac{\vec{v}}{c}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}, \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu} x^\nu$

$\partial_\mu \partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

$M \to \sum_i m_i$

$E_1^\mathrm{max} = \frac{M^2 + m_1^2 - \sum_{i\neq 1} m_i^2}{2M} c^2$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$

$\mathrm{SO}_{1,3} = \left\{ \Lambda \,\middle|\, \begin{smallmatrix} \Lambda^\mathrm{T} g \Lambda = g \\ \det\Lambda \geq 0 \end{smallmatrix} \right\}$

$({\Lambda^0}_0)^2 \geq 1$

$\Lambda = \begin{pmatrix} \gamma & -\gamma\vec{\beta} \\ -\gamma\vec{\beta} & I + \frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta} \end{pmatrix}$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$

$\vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$

$\rho, \vec{J}, I, \vec{P} / \sqrt{4\pi\varepsilon_0}$

$\vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$

$\vec{H} \times \sqrt{4\pi\mu_0}$

$\vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$

σ (cond.)$/4\pi\varepsilon_0$

$\varepsilon/\varepsilon_0$

μ/μ_0

$R, Z \times 4\pi\varepsilon_0$

$L \times 4\pi\varepsilon_0$

$C/4\pi\varepsilon_0$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3}$; $\vec{E}_1 = \frac{\vec{F}_{12}}{q_2}$; $V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$; $\rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi\int\rho\,\mathrm{d}^3x$; $-\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho$; $\vec{\nabla}\times\vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x$; $\tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{i\neq j}\int \vec{E}_i \vec{E}_j\,\mathrm{d}^3x$

$V(\vec{r}) = \int \rho G_\mathrm{D}(\vec{r})\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_\mathrm{D}}{\partial n}\,\mathrm{d}S$

$V(\vec{r}) = \langle V\rangle_S + \int\rho G_\mathrm{N}(\vec{r})\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n} G_\mathrm{N}(\vec{r})\,\mathrm{d}S$

$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y})$; $G_\mathrm{D}(\vec{x}, \vec{y})|_{\vec{y}\in S} = 0$; $\frac{\partial G_\mathrm{N}}{\partial n}\big|_{\vec{y}\in S} = -\frac{4\pi}{S}$

$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R}$; $\vec{p} = \int\mathrm{d}^3r\,\rho\vec{r}$; $\vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3}$; $V_\mathrm{dip} = \frac{\vec{p}\vec{r}}{r^2}$

force on a dipole: $\vec{F}_\mathrm{dip} = (\vec{p}\vec{\nabla})\vec{E}$

$Q_{ij} = \int\mathrm{d}^3r\,\rho(\vec{r})(3r_i r_j - \delta_{ij}r^2)$; $V_\mathrm{quad} = \frac{1}{6r^5}Q_{ij}(3r_i r_j - \delta_{ij}r^2)$

$V(r, \theta) = \sum_{l=0}^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$

$V(r, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$

$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l$; $f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 f P_l$

$P_l(1) = 1$; $\langle P_n | P_m\rangle = \frac{2\delta_{nm}}{2n+1}$; $\langle Y_{lm} | Y_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}$

$P_0 = 1$; $P_1 = x$; $P_2 = \frac{3x^2 - 1}{2}$; $Y_{00} = \frac{1}{\sqrt{4\pi}}$; $Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi}$; $Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi}$; $Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l$, $|m| \leq l$

$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos\theta)$; $Y_{l,-m} = (-1)^m \overline{Y}_{lm}$

$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l \overline{Y}_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi)$

$V(r > \mathrm{diam\,supp}\,\rho, \theta, \varphi) = \sum_{l=0}^\infty \frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$

$q_{lm}[\rho] = \int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi \sin\theta\,\mathrm{d}\theta\, r^l \rho(r, \theta, \varphi)\overline{Y}_{lm}(\theta, \varphi)$

Magnetostatics (CGS)

$$\vec\nabla\vec J = -\frac{\partial\rho}{\partial t} = 0;\ I = \int \vec J \mathrm{d}\vec S$$

$$\text{solenoid: } B = 4\pi\frac{j_\mathrm{s}}{c}$$

$$\mathrm{d}\vec F = \frac{I\mathrm{d}\vec l}{c}\times\vec B = \mathrm{d}^3x\frac{\vec J}{c}\times\vec B;\ \vec F_q = q\frac{\dot{\vec r}}{c}\times\vec B$$

$$\mathrm{d}\vec B = \frac{I\mathrm{d}\vec l}{c}\times\frac{\vec r}{r^3};\ \vec B_q = q\frac{\dot{\vec r}}{c}\times\frac{\vec r}{r^3}$$

$$\vec B = \vec\nabla\times\vec A;\ \vec A = \int\mathrm{d}^3r'\frac{\vec J'}{c}\frac{1}{|\vec r-\vec r'|}+\vec\nabla A_0$$

$$\vec B = \int\mathrm{d}^3r'\frac{\vec J'}{c}\times\frac{\vec r-\vec r'}{|\vec r-\vec r'|^3}$$

$$\varphi = \frac{I}{c}\Omega,\ \vec B = -\vec\nabla\varphi$$

$$\vec\nabla\vec A = 0\to\nabla^2\vec A = -4\pi\frac{\vec J}{c}$$

$$\vec\nabla\vec B = 0;\ \vec\nabla\times\vec B = 4\pi\frac{\vec J}{c};\ \oint\vec B\mathrm{d}\vec l = 4\pi\frac{I}{c}$$

$$\vec m = \frac12\int\mathrm{d}^3r'\left(\vec r'\times\frac{\vec J'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec L = \frac{SI}{c}$$

$$\vec A_\mathrm{dm} = \frac{\vec m\times\vec r}{r^3};\ \vec\tau = \vec m\times\vec B$$

$$\vec F_\mathrm{dmdm} = -\vec\nabla_R\frac{\vec m\vec m'-3(\vec m\hat R)(\vec m'\hat R)}{R^3}$$

$$\text{loop axis: } \vec B = \hat z\frac{2\pi R^2}{(z^2+R^2)^{3/2}}\frac{I}{c}$$

Electromagnetism (CGS)

$$\text{Faraday: } \mathcal{E} = -\frac1c\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int\mathrm{d}^3x\vec J = \dot{\vec p}$$

$$\vec\nabla\times\vec E = -\frac1c\frac{\partial\vec B}{\partial t};\ \vec\nabla\vec E = 4\pi\rho;\ \vec\nabla\vec J = -\frac{\partial\rho}{\partial t}$$

$$\vec\nabla\times\vec B = 4\pi\frac{\vec J}{c}+\frac1c\frac{\partial\vec E}{\partial t};\ \vec\nabla\vec B = 0$$

$$\mathrm{d}\vec F = \mathrm{d}^3x\left(\rho\vec E+\frac{\vec J}{c}\times\vec B\right);\ \vec F_q = q\left(\vec E+\frac{\dot{\vec r}}{c}\times\vec B\right)$$

$$u = \frac{E^2+B^2}{8\pi};\ \vec S = \frac{c}{4\pi}\vec E\times\vec B;\ \vec g = \frac{\vec S}{c^2}$$

$$\mathbf{T}^E = \frac{1}{4\pi}\left(\vec E\otimes\vec E-\frac12E^2\right);\ \mathbf{T} = \mathbf{T}^E+\mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec J\vec E+\vec\nabla\vec S;\ -\frac{\partial\vec g}{\partial t} = \rho\vec E+\frac{\vec J}{c}\times\vec B-\vec\nabla\mathbf{T}$$

$$\vec B = \nabla\times\vec A;\ \vec E = -\vec\nabla\phi-\frac1c\frac{\partial\vec A}{\partial t}$$

$$-\nabla^2\phi-\frac1c\frac{\partial}{\partial t}\vec\nabla\vec A = 4\pi\rho$$

$$\vec\nabla\left(\vec\nabla\vec A+\frac1c\frac{\partial\phi}{\partial t}\right)-\nabla^2\vec A+\frac1c\frac{\partial^2\vec A}{\partial t^2} = 4\pi\frac{\vec J}{c}$$

$$(\phi,\vec A)\cong\left(\phi-\frac1c\frac{\partial\chi}{\partial t},\vec A+\vec\nabla\chi\right)$$

$$(\phi,\vec A) = \int\mathrm{d}^3r'\frac{\left(\rho,\frac{\vec J}{c}\right)\left(\vec r',t-\frac1c\left|\vec r-\vec r'\right|\right)}{|\vec r-\vec r'|}$$

$$\vec\nabla\vec A = 0\to\Box\vec A = \frac{4\pi}{c}(\vec J-\vec J_L) =: \frac{4\pi}{c}\vec J_T$$

$$\vec J_L = \frac{1}{4\pi}\vec\nabla\frac{\partial\phi}{\partial t} = -\frac{1}{4\pi}\vec\nabla\int\frac{\vec\nabla'\vec J'}{|\vec x-\vec x'|}\mathrm{d}^3x'$$

$$\vec E'_\parallel = \vec E_\parallel;\ \vec B'_\parallel = \vec B_\parallel$$

$$\vec E'_\perp = \gamma\left(\vec E_\perp+\frac{\vec v}{c}\times\vec B\right)$$

$$\vec B'_\perp = \gamma\left(\vec B_\perp-\frac{\vec v}{c}\times\vec E\right)$$

$$\text{plane wave: }\begin{cases}\vec E = \vec E_0e^{i(\vec k\vec r-\omega t)}\\ \vec B = \hat k\times\vec E\\ \omega = ck\end{cases}$$

$$\vec B_\mathrm{diprad} = \frac{1}{c^2}\frac{\ddot{\vec p}\times\hat r}{r}\Big|_{t_\mathrm{rit}};\ \vec E_\mathrm{diprad} = \vec B_\mathrm{diprad}\times\hat r$$

$$\text{Larmor: } P = \frac{2}{3c^3}|\ddot{\vec p}|^2$$

$$\text{Rel. Larmor: } P = \frac{2}{3c^3}q^2\gamma^6(a^2-(\vec a\times\vec\beta)^2)$$

$$\vec A_\mathrm{dm} = \frac1c\frac{\dot{\vec m}\times\vec r}{r}\Big|_{t_\mathrm{rit}}$$

$$\text{L.W.: } (\phi,\vec A) = \frac{q(1,\frac{\vec v}{c})}{[r-\frac{\vec v\vec r}{c}]_{t_\mathrm{rit}}};\ t_\mathrm{rit} = t-\frac{r}{c}\Big|_{t_\mathrm{rit}}$$

$$A^\mu = (\phi,\vec A);\ J^\mu = (c\rho,\vec J)$$

$$\text{Lorenz gauge: } \partial_\alpha A^\alpha = 0$$

$$\text{Temporal gauge: } \phi = 0$$

$$\text{Axial gauge: } A_3 = 0$$

$$\text{Coulomb gauge: } \vec\nabla\vec A = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu-\partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu} = \frac12\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu} = \begin{pmatrix}0&-E_x&-E_y&-E_z\\E_x&0&-B_z&B_y\\E_y&B_z&0&-B_x\\E_z&-B_y&B_x&0\end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi\frac{J^\nu}{c};\ \partial_\alpha\mathscr{F}^{\alpha\nu} = 0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma}+\partial_\nu F_{\sigma\mu}+\partial_\sigma F_{\mu\nu} = 0;\ \det F = (\vec E\vec B)^2$$

$$F^{\alpha\beta}F_{\alpha\beta} = 2(B^2-E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta} = 4\vec E\vec B$$

$$\Theta^{\mu\nu} = \frac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu}+\frac14g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu} = \begin{pmatrix}u&c\vec g\\c\vec g&-\mathbf{T}\end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu} = \frac{J_\alpha}{c}F^{\alpha\nu}(-?)$$

$$\mathcal{L} = \frac{mc^2}{\gamma}-q\vec A\frac{\vec v}{c}+q\phi$$

E.M. in matter (CGS)

$$\vec\nabla\vec D = 4\pi\rho_\mathrm{ext};\ \vec\nabla\times\vec E = -\frac1c\frac{\partial\vec B}{\partial t}$$

$$\vec\nabla\vec B = 0;\ \vec\nabla\times\vec H = 4\pi\frac{\vec J_\mathrm{ext}}{c}+\frac1c\frac{\partial\vec D}{\partial t}$$

$$\vec P = \frac{\mathrm{d}\langle\vec p\rangle}{\mathrm{d}V};\ \vec M = \frac{\mathrm{d}\langle\vec m\rangle}{\mathrm{d}V}$$

$$\rho_\mathrm{pol} = -\vec\nabla\vec P;\ \sigma_\mathrm{pol} = \hat n\vec P;\ \frac{\vec J_\mathrm{mag}}{c} = \vec\nabla\times\vec M$$

$$\vec D_\mathrm{pol} = \vec E+4\pi\vec P;\ \vec H_\mathrm{mag} = \vec B-4\pi\vec M$$

$$\text{static linear isotropic: } \vec P = \chi\vec E$$

$$\text{static linear: } P_i = \chi_{ij}E_j$$

$$\text{static linear: } \varepsilon = 1+4\pi\chi$$

$$\text{static: } \Delta D_\perp = 4\pi\sigma_\mathrm{ext};\ \Delta E_\parallel = 0$$

$$\text{static linear: } u = \frac{1}{8\pi}\vec E\vec D$$

$$\Delta U_\mathrm{dielectric} = -\frac12\int\mathrm{d}^3r\vec P\vec E_0$$

$$\text{plane capacitor: } C = \frac{\varepsilon}{4\pi}\frac{S}{d}$$

$$\text{cilindric capacitor: } C = \frac{L}{2\log\frac{R}{r}}$$

$$\text{atomic polarizability: } \vec p = \alpha\vec E$$

$$\text{non-interacting gas: } \vec p = \alpha\vec E_0;\ \chi = n\alpha$$

$$\text{hom. cubic isotropic: } \chi = \frac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$$

$$\text{Clausius-Mossotti: } \frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}n\alpha$$

$$\text{perm. dipole: } \chi = \frac13\frac{np_0^2}{kT}$$

$$\text{local field: } \vec E_\mathrm{loc} = \vec E+\frac{4\pi}{3}\vec P$$

$$\vec J\vec E = -\vec\nabla\left(\frac{c}{4\pi}\vec E\times\vec H\right)-\frac{1}{4\pi}\left(\vec E\frac{\partial\vec D}{\partial t}+\vec H\frac{\partial\vec B}{\partial t}\right)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\frac{\omega}{c}$$

$$\text{plane wave: } B = nE$$

$$\vec J_\mathrm{c} = \sigma\vec E;\ \varepsilon_\sigma = 1+i\frac{4\pi\sigma}{\omega}$$

$$\omega_\mathrm{p}^2 = 4\pi\frac{nq^2}{m};\ \omega_\mathrm{cyclo} = \frac{qB}{mc}$$

$$\text{I: } u = \frac{1}{8\pi}(\vec E\vec D+\vec H\vec B)$$

$$\text{I: } \langle S_z\rangle = \frac{c}{n}\langle u\rangle$$

$$\text{II: } u = \frac{1}{8\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon\omega)E^2+\frac{\partial}{\partial\omega}(\mu\omega)H^2\right)$$

$$\text{II: } \langle S_z\rangle = v_\mathrm{g}\langle u\rangle;\ v_\mathrm{g} = \frac{\partial\omega}{\partial k} = \frac{c}{n+\omega\frac{\partial n}{\partial\omega}}$$

$$\text{III: } \langle W\rangle = \frac{\omega}{4\pi}\left(\operatorname{Im}\varepsilon\langle E^2\rangle+\operatorname{Im}\mu\langle H^2\rangle\right)$$

$$\text{Fresnel TE (S): } \frac{E_\mathrm{t}}{E_\mathrm{i}} = \frac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \frac{E_\mathrm{r}}{E_\mathrm{i}} = \frac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$$

$$\text{TM (P): } \frac{E_\mathrm{t}}{E_\mathrm{i}} = \frac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \frac{E_\mathrm{r}}{E_\mathrm{i}} = \frac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$$

$$\text{Fresnel: } k_{tz} = \pm\sqrt{\varepsilon_2\left(\frac{\omega}{c}\right)^2-k_x^2},\ \operatorname{Im}k_{tz}>0$$

$$\text{Drüde-Lorentz: } \varepsilon = 1-\frac{\omega_\mathrm{p}^2}{\omega^2+i\gamma\omega-\omega_0^2}$$

$$P(t) = \int_{-\infty}^{\infty}g(t-t')E(t')\mathrm{d}t'$$

$$P(\omega) = \chi(\omega)E(\omega)$$

$$\chi(\omega) = \int_{-\infty}^{\infty}e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega) = \overline{\chi}(\omega)$$

$$g(t<0) = 0\implies$$

$$\operatorname{Re}\varepsilon(\omega) = 1+\frac{2}{\pi}\,⨍_0^\infty\frac{\omega'\left(\operatorname{Im}\varepsilon(\omega')-\frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$

$$\operatorname{Im}\varepsilon(\omega) = -\frac{2\omega}{\pi}\,⨍_0^\infty\frac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega'+\frac{4\pi\sigma_0}{\omega}$$

$$\text{sum rule: } \frac{\pi}{2}\omega_\mathrm{p}^2 = \int_0^\infty\omega\operatorname{Im}\varepsilon\mathrm{d}\omega$$

$$\text{sum rule: } 2\pi^2\sigma_0 = \int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$$

$$\text{sum rule: } \int_0^\infty(\operatorname{Re}n-1)\mathrm{d}\omega = 0$$

$$\text{Miller rule: } \chi^{(2)}(\omega,\omega)\propto\chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$$

Quantum mechanics (CGS)

$$r_B = \frac{\hbar^2}{m_ee^2} = 5.292\cdot10^{-11}\,\mathrm{m}$$

$$\text{Rydberg} = \frac{e^2}{2r_B} = 13.61\,\mathrm{eV}$$

$$r_e = \frac{e^2}{mc^2} = 2.818\cdot10^{-15}\,\mathrm{m}$$

$$E_B = -\frac{1}{n^2}\frac{e^2}{2r_B}$$

$$\alpha = \frac{e^2}{\hbar c}$$

$$\text{Planck: } \frac{8\pi\hbar}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$$

$$\sigma_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix};\ \sigma_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix};\ \sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\text{Schrödinger: } i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

$$U(t) = e^{-\frac{iHt}{\hbar}};\ U^\dagger = U^{-1}$$

$$H = H_0+V_\lambda:\ \frac{\partial E_n(\lambda)}{\partial\lambda} = \langle\psi_n(\lambda)|\,\frac{\partial V_\lambda}{\partial\lambda}\,|\psi_n(\lambda)\rangle$$

$$[A,BC] = [A,B]C+B[A,C]$$

$$[P,Q] = \frac{\hbar}{i}$$

Nuclear physics (MKSA)

$$M(A,Z) = Zm_\mathrm{p}+(A-Z)m_\mathrm{n}-B(A,Z)$$

$$B(A,Z) = a_vA-a_sA^{2/3}-a_c\frac{Z(Z-1)}{A^{1/3}}-a_\mathrm{sym}\frac{(A-2Z)^2}{A}+a_pA^{-3/4}\Delta$$

$$\Delta = \begin{cases}0 & & A\text{ odd}\\ \left.\begin{matrix}1 & Z\text{ even}\\ -1 & Z\text{ odd}\end{matrix}\right\} & & A\text{ even}\end{cases}$$

$$a_v = 15.5;\ a_s = 16.8;\ a_c = 0.72;\ a_\mathrm{sym} = 23;\ a_p = 34\ [\mathrm{MeV}]$$

$$\frac{\partial M}{\partial Z} = 0:\ Z = \frac{m_\mathrm{n}-m_\mathrm{p}+4a_\mathrm{sym}}{\frac{2a_c}{A^{1/3}}+\frac{8a_\mathrm{sym}}{A}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$

$$s_{ab} := |p_a^\mu+p_b^\mu|^2$$

$$M\to abc:\ (m_a+m_b)^2\le s_{ab}\le(M-m_c)^2$$

$$M\to abc:\ s_{ab}+s_{bc}+s_{ac} = M^2+m_a^2+m_b^2+m_c^2$$

$$a_iA_i\to b_jB_j:\ Q := (a_im_{A_i}-b_jm_{B_j})c^2$$