Trigonometric functions

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\tfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\tfrac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$
$$\tan\tfrac{\alpha}{2}=\tfrac{\sin\alpha}{1+\cos\alpha}=\tfrac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\tfrac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\tfrac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$
$$\operatorname{acos}x+\operatorname{asin}x=\tfrac{\pi}{2}$$

Hyperbolic functions

$$\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$
$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\tanh(x+y)=\tfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$$
$$\left(\begin{smallmatrix}\sinh x\\ \cosh x\end{smallmatrix}\right)=\tfrac12\left(\begin{smallmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{smallmatrix}\right)$$
$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\tfrac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix);\ \cos x=\cosh(ix)$$
$$\left(\begin{smallmatrix}\operatorname{asinh}x\\ \operatorname{acosh}x\end{smallmatrix}\right)=\log\Big(x+\sqrt{x^2+\left(\begin{smallmatrix}1\\-1\end{smallmatrix}\right)}\Big)$$
$$\operatorname{atanh}x=\tfrac12\log\tfrac{1+x}{1-x}$$

Areas

$$\text{triangle: }\sqrt{p(p-a)(p-b)(p-c)}$$
$$\text{quad: }\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\tfrac{\alpha+\gamma}{2}}$$
$$\text{Pick: }A=\big(I+\tfrac{B}{2}-1\big)\,A_{\text{check}}$$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\tfrac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\tfrac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\tfrac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\tfrac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\tfrac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\textstyle\sum_{x=1}^n x^3=\big(\sum_{x=1}^n x\big)^2=\frac14 n^2(n+1)^2$$
$$\textstyle\sum_{x=1}^n x^2=\frac16 n(n+1)(2n+1)$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$\textstyle(a+b)^n=\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\textstyle\Gamma(1+z)=\int_0^\infty t^ze^{-t}dt=z!$$
$$n!\approx(\tfrac{n}{e})^n\sqrt{2\pi n}$$
$$\textstyle\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x g(x,y)\mathrm{d}y=\int_0^x\frac{\partial g}{\partial x}(x,y)\mathrm{d}y+g(x,x)$$
$$\sqrt{z}=\sqrt{\tfrac{|z|+\operatorname{Re}z}{2}}\pm\sqrt{\tfrac{|z|-\operatorname{Re}z}{2}}$$
$$\langle\operatorname{Re}(ae^{-i\omega t})\operatorname{Re}(be^{-i\omega t})\rangle=\tfrac12\operatorname{Re}(ab^*)$$
$$\textstyle\operatorname{erf}(x)=\frac{2}{\sqrt\pi}\int_0^x e^{-t^2}\mathrm{d}t$$
$$\textstyle f^{(n)}(z_0)=\frac{n!}{2\pi i}\oint\frac{f(z)\mathrm{d}z}{(z-z_0)^{n+1}}$$
$$\textstyle f(z)=\sum_{k=-\infty}^\infty\Big(\frac{1}{2\pi i}\oint\frac{f(z')\mathrm{d}z'}{(z'-z_0)^{k+1}}\Big)(z-z_0)^k$$
$$\operatorname{sinc}x:=\tfrac{\sin x}{x}$$

Derivatives

$$\tan'x=1+\tan^2x\qquad\cot'x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\tfrac{1}{1+x^2}\qquad\operatorname{asin}'x=-\operatorname{acos}'x=\tfrac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a\qquad\log_a'x=\tfrac{1}{x\ln a}\qquad\cosh'x=\sinh x\qquad\tanh'x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\tfrac{1}{1-x^2}$$
$$\operatorname{asinh}'x=\tfrac{1}{\sqrt{x^2+1}}\qquad\operatorname{acosh}'x=\tfrac{1}{\sqrt{x^2-1}}\qquad(f^{-1})'=\tfrac{1}{f'(f^{-1})}\qquad\big(\tfrac1x\big)'=-\tfrac{\dot x}{x^2}$$
$$\big(\tfrac{x}{y}\big)'=\tfrac{\dot xy-x\dot y}{y^2}\qquad(x^y)'=x^y\big(\dot y\ln x+y\tfrac{\dot x}{x}\big)$$
$$\tfrac{\partial(x,y)}{\partial(u,v)}:=\tfrac{\partial x}{\partial u}\tfrac{\partial y}{\partial v}-\tfrac{\partial x}{\partial v}\tfrac{\partial y}{\partial u}\qquad\tfrac{\partial(x,y)}{\partial(u,y)}=\tfrac{\partial x}{\partial u}\big|_y=-\tfrac{\partial x}{\partial y}\big|_u\tfrac{\partial y}{\partial u}\big|_x$$
$$\tfrac{\partial x}{\partial y}\big|_u\tfrac{\partial y}{\partial u}\big|_x\tfrac{\partial u}{\partial x}\big|_y=-1$$
$$\tfrac{\partial x}{\partial u}\big|_y=\tfrac{\partial x}{\partial u}\big|_v-\tfrac{\partial x}{\partial y}\big|_u\tfrac{\partial y}{\partial u}\big|_v$$
$$\tfrac{\partial x}{\partial u}\big|_v=\tfrac{\partial x}{\partial y}\big|_v\tfrac{\partial y}{\partial u}\big|_v$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}\qquad\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac1x=\ln|x|\qquad\int\tan x=-\ln|\cos x|\qquad\int\cot x=\ln|\sin x|\qquad\int\frac{1}{\sin x}=\ln\big|\tan\frac{x}{2}\big|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\big|\tan(\frac{x}{2}+\frac{\pi}{4})\big|\qquad\int\ln x=x(\ln x-1)\qquad\int\tanh x=\ln\cosh x\qquad\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}\qquad\int\frac{1}{a^2+x^2}=\frac1a\operatorname{atan}\frac{x}{a}\qquad\int xy=x\int y-\int(\dot x\int y)$$
$$\textstyle\int_{-\infty}^\infty e^{-x^2}=\sqrt\pi\qquad\int_{-\infty}^\infty e^{i\omega t}\mathrm{d}t=2\pi\delta(\omega)$$

Differential equations

$$\textstyle\dot x+\dot ax=b:x=e^{-a}\left(\int be^a+c_1\right)$$
$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$
$$\ddot x+\gamma\dot x+\omega_0^2x=fe^{-i\omega t}:x=\tfrac{fe^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega}$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac1x+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac1x-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$
$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac1x-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac1x+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac56x^4-\tfrac34x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Fourier

$$\textstyle\text{Fourier: }c_n=\frac2T\int_0^T f(t)\cos\big(n\frac{t}{T}\big)\mathrm{d}t$$
$$\textstyle\mathcal{F}[f](\omega)=\hat f(\omega)=\int\mathrm{d}te^{i\omega t}f(t)$$
$$f,g\in L^2:(\hat f,\hat g)=2\pi(f,g)$$
$$\mathcal{F}\big[\tfrac{\sin t}{t}\big]=\sqrt{\tfrac{\pi}{2}}\chi_{[-1;1]}(\omega)$$
$$t^{k\le n}f(t)\in L^1:\mathcal{F}[t^nf(t)]=(-i)^n\hat f^{(n)}$$
$$f^{(k\le n)}\in L^1:\mathcal{F}[f^{(n)}]=(-i\omega)^n\hat f$$
$$\mathcal{F}^2f=2\pi f(-t);\ (\omega\hat f)'=-\mathcal{F}[tf']$$
$$f\star g=g\star f;\ \hat f\star\hat g=2\pi\mathcal{F}[fg]$$
$$f\in L^1,\ g\in L^p:\mathcal{F}[f\star g]=\hat f\hat g$$
$$\textstyle f\star g(x)=\int f(x-y)g(y)\mathrm{d}y$$
$$(f\star g)'=f'\star g=f\star g'$$
$$f(x+\Delta)\star g=f\star g(x+\Delta)$$
$$f\in L^1,\ g\in L^p\ \Rightarrow\ f\star g\in L^p$$
$$\textstyle f,g\in L^2:f\star g=\frac{1}{2\pi}\int\hat f\hat ge^{-i\omega t}\mathrm{d}\omega$$
$$\|f\|=1:\Delta\omega\Delta t\ge\tfrac12$$
$$\Delta\omega\Delta t=\tfrac12:f(t)=g(t;\bar t,\Delta t)e^{-i\bar\omega t}$$

Distributions

$\mathcal{D} := \{f \in C^\infty \mid \exists K \text{ compact} : f(\mathscr{C} K) = 0\}$

$\mathcal{S} := \{f \in C^\infty \mid |x^n f^{(k)}| \le A_{nk}\} \supset \mathcal{D}$

$\langle 1, f\rangle := \int f;\ \langle gT, f\rangle := \langle T, gf\rangle$

$T \in \mathcal{S}' : \langle \mathcal{F}T, f\rangle := \langle T, \mathcal{F}f\rangle$

$\langle T', f\rangle := -\langle T, f'\rangle;\ \langle \delta, f\rangle := f(0)$

$\langle T \otimes S, \phi\rangle := \langle T(x), \langle S(y), \phi(x+y)\rangle\rangle$

$\langle T \star S, \phi\rangle := \langle T \otimes S, \phi(x+y)\rangle$

$\mathcal{F}1 = 2\pi\delta(\omega);\ \mathcal{F}\,\mathrm{sgn} = 2i\mathcal{P}\frac{1}{\omega}$

$\mathcal{F}\theta = i\mathcal{P}\frac{1}{\omega} + \pi\delta(\omega)$

$x^n T = 0 \ \Rightarrow\ T = \sum_{k=0}^{n-1} a_k \delta^{(k)}$

$xT = S \ \Rightarrow\ T = S/x + k\delta$

$T, S \in \mathcal{D}' : T \otimes S = S \otimes T$

$\sum_{n=0}^{\infty} e^{inx} = \mathcal{P}\frac{1}{1-e^{ix}} + \pi\sum_{n=-\infty}^{\infty}\delta(x-2n\pi)$

$\delta^{(n)} \star f = f^{(n)}$

$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i) = 0$

Bessel functions

sol. of $x^2\partial_x^2 f + x\partial_x f + (x^2 - \alpha^2) f = 0$

$\alpha =$ "order"

$J_\alpha =$ "first kind, normal"

$\alpha \in \mathbb{Z}_0 \vee \alpha > 0 : J_\alpha(0) = 0$

$J_0(0) = 1;$ otherwise $|J_\alpha(0)| = \infty$

$\alpha \notin \mathbb{Z} : J_\alpha, J_{-\alpha}$ indep.

$\alpha \in \mathbb{Z} : J_{-\alpha} = (-1)^\alpha J_\alpha$

$Y_\alpha =$ "second kind, normal" (also N_α)

$\alpha \notin \mathbb{Z} : Y_\alpha = \frac{\cos(\alpha\pi) J_\alpha - J_{-\alpha}}{\sin(\alpha\pi)}$

$\alpha \in \mathbb{Z} : Y_\alpha = \lim_{\alpha' \to \alpha} Y_{\alpha'}$

$\alpha \in \mathbb{Z} : Y_\alpha, J_\alpha$ indep.

$\alpha \in \mathbb{Z} : Y_{-\alpha} = (-1)^\alpha Y_\alpha$

$\frac{2\alpha}{x} J_\alpha(x) = J_{\alpha-1}(x) + J_{\alpha+1}(x)$

$2J'_\alpha(x) = J_{\alpha-1}(x) - J_{\alpha+1}(x)$

$\int_0^1 \mathrm{d}x\, x J_\alpha(x u_{\alpha,m}) J_\alpha(x u_{\alpha,n}) = \frac{\delta_{mn}}{2} J^2_{\alpha+1}(u_{\alpha,m})$

$u_{\alpha,n} = n$th. zero of J_α

Cylindrical harmonics

$V(\rho,\phi,z) = \sum_{n=0}^{\infty} \int \mathrm{d}k A_{nk} P_{nk}(\rho)\Phi_n(\phi) Z_k(z)$

$P_{nk}(\rho) =$ comb. of $J_n(k\rho), Y_n(k\rho)$

$\Phi_n(\phi) =$ comb. of $e^{\pm in\phi}$

$Z_k(z) =$ comb. of $e^{\pm kz}$

Inequalities

$|a| - |b| \le |a+b| \le |a| + |b|$

$x > -1 : 1 + nx \le (1+x)^n$

$\frac{|a^n - b^n|}{|a-b|<1} \le n(1+|b|)^{n-1}$

$\sqrt[p]{\sum (a_i + b_i)^p} \le \sqrt[p]{\sum a_i^p} + \sqrt[p]{\sum b_i^p}$

$\sum a_i b_i \le \left(\sum a_i^p\right)^{\frac{1}{p}} \left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$

$x^p y^q \le \left(\frac{px+qy}{p+q}\right)^{p+q}$

$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}} \le \sqrt[q]{\frac{1}{n}\sum a_i^q}$

$\sum\left(\frac{a_1 + \ldots a_i}{i}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum a_i^p$

$x \ge 0, |\ddot{x}| \le M : |\dot{x}| \le \sqrt{2Mx}$

$\frac{1}{1+x} < \ln\left(1 + \frac{1}{x}\right) < \frac{1}{x}$

Linear algebra

$\dim(U+V) = \dim U + \dim V - \dim(U \cap V)$

$\dim V = \dim \ell(V) + \dim(V \cap \ker \ell)$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ
N	Ξ	O	Π	P	Σ	T	Υ	Φ	X	Ψ	Ω
ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$

$e = 2.718$

$\gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$

$R = 8.206 \cdot 10^{-2}\,\frac{1\,\mathrm{atm}}{\mathrm{mol\,K}}$

$N_\mathrm{A} = 6.022 \cdot 10^{23}\,\frac{1}{\mathrm{mol}}$

$k_\mathrm{B} = 1.381 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$k_\mathrm{B} = 8.617 \cdot 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}}$

$c = 2.998 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$

$q_\mathrm{e} = 1.602 \cdot 10^{-19}\,\mathrm{A\,s}$

$m_\mathrm{e} = 9.109 \cdot 10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673 \cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675 \cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{e} = 5.110 \cdot 10^{-1}\,\mathrm{MeV}$

$m_\mathrm{p} = 9.383 \cdot 10^{2}\,\mathrm{MeV}$

$m_\mathrm{n} = 9.396 \cdot 10^{2}\,\mathrm{MeV}$

$m_\mathrm{n} - m_\mathrm{p} = 1.293\,\mathrm{MeV}$

$\mathrm{amu} = 1.661 \cdot 10^{-27}\,\mathrm{kg}$

$h = 6.626 \cdot 10^{-34}\,\mathrm{J\,s}$

$h = 4.136 \cdot 10^{-15}\,\mathrm{eV\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\,m}^2}$

$\frac{1}{4\pi\varepsilon_0} = 8.988 \cdot 10^{9}\,\frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$

$\mu_0 = 1.257 \cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

$\mu_\mathrm{B} = 9.274 \cdot 10^{-24}\,\mathrm{A\,m}^2$

$\alpha = 7.297 \cdot 10^{-3}$

$\mathrm{barn} = 1 \cdot 10^{-28}\,\mathrm{m}^2$

$\mathrm{cd}_{555\,\mathrm{nm}} = 1.464 \cdot 10^{-3}\,\frac{\mathrm{W}}{\mathrm{sr}}$

$r_B = 5.292 \cdot 10^{-11}\,\mathrm{m}$

$\mathrm{Rydberg} = 1.361 \cdot 10^{1}\,\mathrm{eV}$

$r_e = 2.818 \cdot 10^{-15}\,\mathrm{m}$

Vectors

$$\varepsilon_{ijk} = \begin{cases} 0 & i = j \vee j = k \vee k = i \\ 1 & i+1 \equiv j \wedge j+1 \equiv k \\ -1 & i \equiv j+1 \wedge j \equiv k+1 \end{cases}$$

$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

$\vec{a} \times \vec{b} = \varepsilon_{ijk} a_j b_k \hat{e}_i;\ (\vec{a} \otimes \vec{b})_{ij} = a_i b_j$

$(\vec{a} \times \vec{b})\vec{c} = (\vec{c} \times \vec{a})\vec{b}$

$(\vec{a} \times \vec{b}) \times \vec{c} = -(\vec{b}\vec{c})\vec{a} + (\vec{a}\vec{c})\vec{b}$

$(\vec{a} \times \vec{b})(\vec{c} \times \vec{d}) = (\vec{a}\vec{c})(\vec{b}\vec{d}) - (\vec{a}\vec{d})(\vec{b}\vec{c})$

$|\vec{u} \times \vec{v}|^2 = u^2 v^2 - (\vec{u}\vec{v})^2$

$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right);\ \square = \frac{\partial^2}{\partial t^2} - \nabla^2$

$\vec{\nabla} V = \frac{\partial V}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$

$\vec{\nabla}\vec{v} = \frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

$\vec{\nabla} \times \vec{v} = \left(\frac{1}{\rho}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\rho} +$

$+\left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial \rho} - \frac{\partial v_\rho}{\partial \phi}\right)\hat{z}$

$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

$\vec{\nabla} V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}$

$\vec{\nabla}\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$

$\vec{\nabla} \times \vec{v} = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi \sin\theta)}{\partial \theta} - \frac{\partial v_\theta}{\partial \varphi}\right)\hat{r} +$

$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial(r v_\varphi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right)\hat{\varphi}$

$\nabla^2 V = \frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2} + \frac{\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right)}{r^2\sin\theta} + \frac{\frac{\partial^2 V}{\partial \varphi^2}}{r^2\sin^2\theta}$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rV) = \frac{2}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial r^2}$

$\vec{\nabla}(\vec{\nabla} \times \vec{v}) = \vec{\nabla} \times \vec{\nabla} V = 0$

$\vec{\nabla}(f\vec{v}) = (\vec{\nabla} f)\vec{v} + f\vec{\nabla}\vec{v}$

$\vec{\nabla} \times (f\vec{v}) = \vec{\nabla} f \times \vec{v} + f\vec{\nabla} \times \vec{v}$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = -\nabla^2\vec{v} + \vec{\nabla}(\vec{\nabla}\vec{v})$

$\vec{\nabla}(\vec{v} \times \vec{w}) = \vec{w}(\vec{\nabla} \times \vec{v}) - \vec{v}(\vec{\nabla} \times \vec{w})$

$\vec{\nabla} \times (\vec{v} \times \vec{w}) = (\vec{\nabla}\vec{w} + \vec{w}\vec{\nabla})\vec{v} - (\vec{\nabla}\vec{v} + \vec{v}\vec{\nabla})\vec{w}$

$\frac{1}{2}\vec{\nabla} v^2 = (\vec{v}\vec{\nabla})\vec{v} + \vec{v} \times (\vec{\nabla} \times \vec{v})$

$\int \vec{\nabla}\vec{v}\,\mathrm{d}^3x = \oint \vec{v}\,\mathrm{d}\vec{S};\ \int(\vec{\nabla} \times \vec{v})\mathrm{d}\vec{S} = \oint \vec{v}\,\mathrm{d}\vec{l}$

$\int (f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S \left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)\mathrm{d}S$

$\oint \vec{v} \times \mathrm{d}\vec{S} = -\int(\vec{\nabla} \times \vec{v})\mathrm{d}^3x$

$\delta(\vec{r} - \vec{r}_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r_0^2\sin\theta_0}$

$\nabla^2\frac{1}{|\vec{r}-\vec{r}_0|} = -4\pi\delta(\vec{r} - \vec{r}_0)$

Statistics

$P(E \cap E_1) = P(E_1) \cdot P(E|E_1)$

$\Delta x_\mathrm{hist} \approx \frac{x_\mathrm{max} - x_\mathrm{min}}{\sqrt{N}}$

$P(x \le k) = F(k) = \int_{-\infty}^{k} p(x)$

$\mathrm{median} = F^{-1}(\frac{1}{2})$

$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$

$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$

$\alpha_n = E[x^n]$

$M_n = E[(x-\mu)^n]$

$\sigma^2 = M_2 = E[x^2] - \mu^2$

$\mathrm{FWHM} \approx 2\sigma$

$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$

$\phi[y](t) = E[e^{ity}]$

$\phi[y_1 + \lambda y_2] = \phi[y_1]\phi[\lambda y_2]$

$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$

$h \ge 0 : P(h \ge k) \le \frac{E[h]}{k}$

$P(|x - \mu| > k\sigma) \le \frac{1}{k^2}$

$B(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$

$\mu_B = np,\ \sigma_B^2 = np(1-p)$

$P(k;\mu) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$

$u(x; a, b) = \frac{1}{b-a},\ x \in [a; b]$

$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$

$\varepsilon(x;\lambda) = \lambda e^{-\lambda x},\ x \ge 0$

$\mu_\varepsilon = \frac{1}{\lambda},\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$

$g(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$g(\vec{x};\vec{\mu},V) = \frac{e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^\mathrm{T} V^{-1}(\vec{x}-\vec{\mu})}}{\sqrt{\det(2\pi V)}}$

$\mathrm{FWHM}_g = 2\sigma\sqrt{2\ln 2}$

$z = \frac{x-\mu}{\sigma};\ \mu, \sigma[z] = 0, 1$

$\chi^2 = \sum_{i=1}^{n} z_i^2;\ \wp := p[\chi^2]$

$\wp(x; n) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$

$\mu_\wp = n,\ \sigma_\wp^2 = 2n$

$n \geq 30 : \wp(x;n) \approx g(x;n,\sqrt{2n})$

$n \geq 8 : p[\sqrt{2\chi^2}] \approx g(;\sqrt{2n-1},1)$

$S(x;n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\big(1+\frac{x^2}{n}\big)^{-\frac{n+1}{2}}$

$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$

$p\big[z\sqrt{\frac{n}{\chi^2}}\big] = S(,n)$

$n \geq 35 : S(x;n) \approx g(x;0,1)$

$c(x;a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$

$\sigma_{xy} = E[xy] - \mu_x\mu_y \leq \sigma_x\sigma_y$

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho_{xy}| \leq 1$

$\mu_{f(x)} \approx f(\mu_x)$

$\sigma_{fg} \approx \sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\big|_{\mu_{x_i}}\frac{\partial g}{\partial x_j}\big|_{\mu_{x_j}}$

$\mu \approx m = \frac{1}{n}\sum_{i=1}^n x_i$

$\sigma^2 \approx s^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - m)^2$

$s_m^2 = \frac{s^2}{n}$

$p\big[\frac{m-\mu}{s_m}\big] = S(;n)$

Fit (ML)

$f(x) = mx + q, \quad f(x) = a,$

$f(x) = bx, \quad f(x;\theta) = \theta_i h_i(x)$

$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - \sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$\Delta mq = \frac{-\sum\frac{x}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2} - (\sum\frac{x}{\Delta y^2})^2}$

$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$

$\mathbf{a} = (\sum V_{\mathbf{y}}^{-1})^{-1}(\sum V_{\mathbf{y}}^{-1}\mathbf{y})$

$\Delta\mathbf{a}^2 = (\sum V_{\mathbf{y}}^{-1})^{-1}$

$b = \frac{\sum\frac{xy}{\Delta y^2}}{\sum\frac{x^2}{\Delta y^2}},\ \Delta b^2 = \frac{1}{\sum\frac{x^2}{\Delta y^2}}$

$H_{ij} := h_j(x_i);\ V_{ij} := \Delta y_i y_j$

$\chi^2 = (y - f(x;\theta))^T V^{-1}(y - f(x;\theta))$

$\theta = (H^T V^{-1} H)^{-1} H^T V^{-1} y$

$\Delta\theta\theta = (H^T V^{-1} H)^{-1}$

Kinematics

$\theta \equiv \frac{\pi}{2} \to \ddot{\vec{r}} = (\ddot{r} - r\dot{\varphi}^2)\hat{r} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{\varphi}$

$\vec{A} = \ddot{\vec{r}} + \vec{A}_{\mathrm{T}} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r} + 2\vec{\omega}\times\dot{\vec{r}}$

$\frac{1}{R} = \big|\frac{v_xa_y - v_ya_x}{v^3}\big|$

$\vec{\omega} = \dot{\varphi}\cos\theta\hat{r} - \dot{\varphi}\sin\theta\hat{\theta} + \dot{\theta}\hat{\varphi}$

$\dot{\vec{w}} = \frac{\mathrm{d}(\vec{w}\hat{r})}{\mathrm{d}t}\hat{r} + \frac{\mathrm{d}(\vec{w}\hat{\theta})}{\mathrm{d}t}\hat{\theta} + \frac{\mathrm{d}(\vec{w}\hat{\varphi})}{\mathrm{d}t}\hat{\varphi} + \vec{\omega}\times\vec{w}$

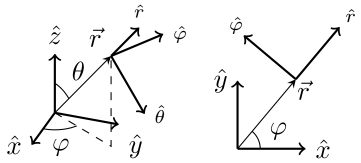
$\theta \equiv \frac{\pi}{2} \to \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi}$

$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\varphi}\sin\theta\hat{\varphi}$

$\langle\ddot{\vec{r}},\hat{r}\rangle = \ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta$

$\langle\ddot{\vec{r}},\hat{\theta}\rangle = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta$

$\langle\ddot{\vec{r}},\hat{\varphi}\rangle = r\ddot{\varphi}\sin\theta + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta$



Mechanics

$\dot{\alpha} = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot{\beta} + \frac{\partial\alpha}{\partial t}$

$\vec{p} := m\dot{\vec{r}};\ \vec{F} = \dot{\vec{p}};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec{F}\vec{p}$

$M := \sum_i m_i;\ \vec{R} := \frac{m_i\vec{r}_i}{M}$

$T = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}m_i(\dot{\vec{r}}_i - \dot{\vec{R}})^2$

$\vec{L} = \vec{R}\times M\dot{\vec{R}} + (\vec{r}_i - \vec{R})\times m_i(\dot{\vec{r}}_i - \dot{\vec{R}})$

$\vec{\tau}_O = \dot{\vec{L}}_O + \vec{v}_O\times\vec{p}$

$\tau_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_3\omega_2$

$\mathcal{L}(q,\dot{q},t) = T - V + \frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$

$S[q] = \int_{t_1}^{t_2}\mathcal{L}(q,\dot{q},t)\,\mathrm{d}t$

$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\big|_{\epsilon\equiv 0}^{\epsilon(t_1)=\epsilon(t_2)=0} = 0$

$p := \frac{\partial\mathcal{L}}{\partial\dot{q}};\ \dot{p} = \frac{\partial\mathcal{L}}{\partial q}$

$\mathcal{H}(q,p,t) = \dot{q}p - \mathcal{L}$

$\dot{q} = \frac{\partial\mathcal{H}}{\partial p};\ \dot{p} = -\frac{\partial\mathcal{H}}{\partial q}$

$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = \frac{\partial\mathcal{H}}{\partial t} = -\frac{\partial\mathcal{L}}{\partial t}$

$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$

$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal{H}\} + \frac{\partial u}{\partial t}$

$\eta = (q,p);\ \Gamma = \left(\begin{smallmatrix}0 & 1\\ -1 & 0\end{smallmatrix}\right)$

$\dot{\eta} = \Gamma\frac{\partial\mathcal{H}}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac{1}{2}mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac{2}{3}mr^2$

ball: $\frac{2}{5}mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\big(R^2 + \frac{3}{4}r^2\big)$

ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$\langle U\rangle = -2\langle T\rangle$

$U_{\mathrm{eff}} = U + \frac{L^2}{2mr^2}$

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$\vec{r} = \vec{r}_1 - \vec{r}_2,\ \alpha = Gm_1m_2$

$T = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2$

$\vec{L} = \vec{R}\times M\dot{\vec{R}} + \vec{r}\times\mu\dot{\vec{r}}$

$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}}$

$r = \frac{k}{1+\varepsilon\cos\theta}$

$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$

$a^3\omega^2 = G(m_1+m_2) = \frac{\alpha}{\mu}$

$\vec{A} = \mu\dot{\vec{r}}\times\vec{L} - \mu\alpha\hat{r},\ \dot{\vec{A}} = 0$

Relativity

$\beta = \frac{v}{c} = \tanh\chi$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \cosh\chi$

$\vec{p} = \gamma m\vec{v};\ \mathcal{E} = \gamma mc^2$

free particle: $\mathcal{L} = \frac{mc^2}{\gamma}$

$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \vec{v}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t};\ \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x}$

$\left(\begin{smallmatrix}ct'\\ x'\end{smallmatrix}\right) = \gamma\left(\begin{smallmatrix}1 & -\beta\\ -\beta & 1\end{smallmatrix}\right)\left(\begin{smallmatrix}ct\\ x\end{smallmatrix}\right)$

$\chi'' = \chi' + \chi$

$V'_\parallel = \frac{V_\parallel - v}{1 - \frac{vV_\parallel}{c^2}}$

$V'_\perp = \frac{1}{\gamma}\frac{V_\perp}{1 - \frac{vV_\parallel}{c^2}}$

$\frac{V'}{c} = 1 - \frac{(1-\frac{V^2}{c^2})(1-\frac{v^2}{c^2})}{\big(1-\frac{vV_\parallel}{c^2}\big)^2}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$

$x^\mu = (ct,\vec{x})$

$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c,\vec{v})$

$a^\mu = \frac{\mathrm{d}v^\mu}{\mathrm{d}\tau} = \gamma\big(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\big)$

$p^\mu = mv^\mu = \big(\frac{\mathcal{E}}{c},\vec{p}\big)$

$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\big(\frac{\vec{v}}{c}\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}, \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\big)$

$\partial_\mu = \frac{\partial}{\partial x^\mu} = \big(\frac{1}{c}\frac{\partial}{\partial t},\vec{\nabla}\big)$

$g_{\mu\nu} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$

$x_\mu = g_{\mu\nu}x^\nu$

$\partial_\mu\partial^\mu = \Box$

$p^\mu p_\mu = (mc)^2$

$v^\mu a_\mu = 0$

doppler: $\sqrt{\frac{1+\beta}{1-\beta}} \approx 1 + \beta$

$\mathrm{SO}_{1,3} = \Big\{\Lambda\ \Big|\ \begin{smallmatrix}\Lambda^{\mathrm{T}}g\Lambda = g\\ \det\Lambda \geq 0\end{smallmatrix}\Big\}$

$(\Lambda^0{}_0)^2 \geq 1$

$\Lambda = \begin{pmatrix}\gamma & -\gamma\vec{\beta}\\ -\gamma\vec{\beta} & I + \frac{\gamma-1}{\beta^2}\vec{\beta}\otimes\vec{\beta}\end{pmatrix}$

$M \rightarrow \sum_i m_i$

$E_1^{\max} = \frac{M^2 + m_1^2 - \sum_{i\neq 1}m_i^2}{2M}$

$A + B_{\mathrm{still}} \rightarrow \sum_i m_i$

$E_A^{\min} = \frac{(\sum_i m_i)^2 - m_A^2 - m_B^2}{2m_B}$

m, M_{still} 1D coll.

$E'_m = \frac{(M+m)^2E_m + 2Mm^2}{M^2 + m^2 + 2ME_m}$

Thermodynamics

$\mathrm{d}Q = T\mathrm{d}S = \mathrm{d}U + \mathrm{d}L = \mathrm{d}U + p\mathrm{d}V - \mu\mathrm{d}N$

$C_{V,N} = \frac{\partial Q}{\partial T}\big|_{V,N} = \frac{\partial U}{\partial T}\big|_{V,N}$

$C_{p,N} = \frac{\partial Q}{\partial T}\big|_{p,N} = \frac{\partial U}{\partial T}\big|_{p,N} + p\frac{\partial V}{\partial T}\big|_{p,N}$

$\gamma := \frac{C_p}{C_V}$

$\mu_J := \frac{\partial T}{\partial V}\big|_{U,N}$

$\lambda U = U(\lambda(S,V,N)) \Rightarrow U = ST - pV + \mu N$

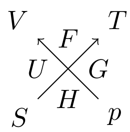
$\Rightarrow S\mathrm{d}T - V\mathrm{d}p + N\mathrm{d}\mu = 0$

Fix S,V,N : $\min U$ at equilibrium

Fix T,V,N : $\min F = U - TS$

Fix T,p,N : $\min G = F + pV$

Fix S,p,N : $\min H = U + pV$



$\frac{\partial}{\partial T}\frac{G}{T}\big|_p = -\frac{H}{T^2}$

$\frac{\partial}{\partial T}\frac{F}{T}\big|_V = -\frac{U}{T^2}$

$\Omega = U - TS - \mu N$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$

$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

Statistical mechanics

$Z = \frac{1}{h^N}\int\mathrm{d}q_1\cdots\mathrm{d}q_N\int\mathrm{d}p_1\cdots\mathrm{d}p_N e^{-\beta\mathcal{H}}$

$U = -\frac{\partial}{\partial\beta}\log Z;\ \beta = \frac{1}{k_{\mathrm{B}}T};\ C = \frac{\partial U}{\partial T}$

$F(T,V) = U - TS = -\frac{\log Z}{\beta}$

$S = -\frac{\partial F}{\partial T}$

Electronics (MKS)

$\left(\begin{smallmatrix}V\\ I\end{smallmatrix}\right) = \left(\begin{smallmatrix}V_0\\ I_0\end{smallmatrix}\right)e^{i\omega t},\ Z = \frac{V}{I}$

$Z_R = R,\ Z_C = -i\frac{1}{\omega C},\ Z_L = i\omega L$

$Z_{\mathrm{series}} = \sum_k Z_k,\ \frac{1}{Z_{\mathrm{parallel}}} = \sum_k\frac{1}{Z_k}$

$\sum_{\mathrm{loop}}V_k = 0,\ \sum_{\mathrm{node}}I_k = 0$

$\mathcal{E} = -L\dot{I},\ L = \frac{\Phi_B}{I}$

$I_{A\to C} = I_0\big(e^{\frac{V_{AC}}{V_T}} - 1\big),\ V_T = \eta\frac{k_{\mathrm{B}}T}{q_{\mathrm{e}}}$

$I_{E,\mathrm{out}} = I_0^E\big(e^{\frac{V_{BE}}{V_T}} - 1\big) - \alpha_R I_0^C\big(e^{\frac{V_{BC}}{V_T}} - 1\big)$

$I_{C,\mathrm{in}} = -I_0^C\big(e^{\frac{V_{BC}}{V_T}} - 1\big) + \alpha_F I_0^E\big(e^{\frac{V_{BE}}{V_T}} - 1\big)$

Chemistry

$$H = U + pV$$

$$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$$

$$G = H - TS$$

$$a_i \mathrm{A}_i \rightarrow b_j \mathrm{B}_j$$

$$\Delta H_\mathrm{r}^\mathrm{o} = b_j \Delta H_\mathrm{f}^\mathrm{o}(\mathrm{B}_j) - a_i \Delta H_\mathrm{f}^\mathrm{o}(\mathrm{A}_i)$$

$$\forall\, i, j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$$

$$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$$

$$k = Ae^{-\frac{E_\mathrm{a}}{RT}} \text{ (Arrhenius)}$$

$$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$$

$$a_{(g)} = \gamma \frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$$

$$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$$

$$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$$

$$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\mathrm{tot}}$$

$$K_c = K_p (RT)^{\sum a_i - \sum b_j}$$

$$K_c = K_n V^{\sum a_i - \sum b_j}$$

$$K_\chi = K_n n_\mathrm{tot}^{\sum a_i - \sum b_j}$$

$$\Delta G_\mathrm{r}^\mathrm{o} = -RT \ln K$$

$$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$$

$$\Delta G = RT \ln \frac{Q}{K}$$

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$$

$$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q \text{ (Nerst)}$$

$$\text{(std) } \Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$$

$$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$$

$$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$$

CGS→MKS

Substitutions: $\vec{E}, V \times \sqrt{4\pi\varepsilon_0}$ $\quad \vec{D} \times \sqrt{\frac{4\pi}{\varepsilon_0}}$ $\quad \rho, \vec{J}, I, \vec{P}/\sqrt{4\pi\varepsilon_0}$ $\quad \vec{B}, \vec{A} \times \sqrt{\frac{4\pi}{\mu_0}}$ $\quad \vec{H} \times \sqrt{4\pi\mu_0}$ $\quad \vec{M} \times \sqrt{\frac{\mu_0}{4\pi}}$ $\quad \sigma$ (cond.)$/4\pi\varepsilon_0$ $\quad \varepsilon/\varepsilon_0$ $\quad \mu/\mu_0$ $\quad R, Z \times 4\pi\varepsilon_0$ $\quad L \times 4\pi\varepsilon_0$ $\quad C/4\pi\varepsilon_0$

Electrostatics (CGS)

$$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$$

$$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi \int \rho\, \mathrm{d}^3 x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla} \times \vec{E} = 0$$

$$U = \frac{1}{8\pi}\int E^2\, \mathrm{d}^3 x;\ \tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{i \neq j}\int \vec{E}_i \vec{E}_j\, \mathrm{d}^3 x$$

$$V(\vec{r}) = \int \rho G_\mathrm{D}(\vec{r})\, \mathrm{d}^3 x - \frac{1}{4\pi}\oint_S V \frac{\partial G_\mathrm{D}}{\partial n}\, \mathrm{d}S$$

$$V(\vec{r}) = \langle V \rangle_S + \int \rho G_\mathrm{N}(\vec{r})\, \mathrm{d}^3 x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n} G_\mathrm{N}(\vec{r})\, \mathrm{d}S$$

$$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_\mathrm{D}(\vec{x}, \vec{y})\big|_{\vec{y} \in S} = 0;\ \frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y} \in S} = -\frac{4\pi}{S}$$

$$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R};\ \vec{p} = \int \mathrm{d}^3 r \rho \vec{r};\ \vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_\mathrm{dip} = \frac{\vec{p}\hat{r}}{r^2}$$

$$\text{force on a dipole: } \vec{F}_\mathrm{dip} = (\vec{p}\vec{\nabla})\vec{E}$$

$$Q_{ij} = \int \mathrm{d}^3 r \rho(\vec{r})(3r_i r_j - \delta_{ij} r^2);\ V_\mathrm{quad} = \frac{1}{6r^5} Q_{ij}(3r_i r_j - \delta_{ij} r^2)$$

$$V(r, \theta) = \sum_{l=0}^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

$$V(r, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$$

$$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 f P_l$$

$$P_l(1) = 1;\ (P_n, P_m) = \frac{2\delta_{nm}}{2n+1};\ (Y_{lm}, Y_{l'm'}) = \delta_{ll'}\delta_{mm'}$$

$$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$

$$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l,\ 0 \leq m \leq l$$

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m Y_{lm}^*$$

$$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l Y_{lm}^*(\theta', \varphi') Y_{lm}(\theta, \varphi)$$

$$V(r > \operatorname{diam}\operatorname{supp}\rho, \theta, \varphi) = \sum_{l=0}^\infty \frac{4\pi}{2l+1}\sum_{m=-l}^l q_{lm}[\rho]\frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$$

$$q_{lm}[\rho] = \int_0^\infty r^2 \mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \int_0^\pi \sin\theta\, \mathrm{d}\theta\, r^l \rho(r, \theta, \varphi) Y_{lm}^*(\theta, \varphi)$$

Magnetostatics (CGS)

$$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0;\ I = \int \vec{J}\mathrm{d}\vec{S}$$

$$\text{solenoid: } B = 4\pi\frac{j_\mathrm{s}}{c}$$

$$\mathrm{d}\vec{F} = \frac{I\mathrm{d}\vec{l}}{c} \times \vec{B} = \mathrm{d}^3 x \frac{\vec{J}}{c} \times \vec{B};\ \vec{F}_q = q\frac{\dot{\vec{r}}}{c} \times \vec{B}$$

$$\mathrm{d}\vec{B} = \frac{I\mathrm{d}\vec{l}}{c} \times \frac{\vec{r}}{r^3};\ \vec{B}_q = q\frac{\dot{\vec{r}}}{c} \times \frac{\vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{A} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c}\frac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla} A_0$$

$$\vec{B} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c} \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\varphi = \frac{I}{c}\Omega,\ \vec{B} = -\vec{\nabla}\varphi$$

$$\vec{\nabla}\vec{A} = 0 \to \nabla^2 \vec{A} = -4\pi\frac{\vec{J}}{c}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c};\ \oint \vec{B}\mathrm{d}\vec{l} = 4\pi\frac{I}{c}$$

$$\vec{m} = \frac{1}{2}\int \mathrm{d}^3 r' \left(\vec{r}' \times \frac{\vec{J}'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec{L} = \frac{SI}{c}$$

$$\vec{A}_\mathrm{dm} = \frac{\vec{m} \times \vec{r}}{r^3};\ \vec{\tau} = \vec{m} \times \vec{B}$$

$$\vec{F}_\mathrm{dmdm} = -\vec{\nabla}_R \frac{\vec{m}\vec{m}' - 3(\vec{m}\hat{R})(\vec{m}'\hat{R})}{R^3}$$

$$\text{loop axis: } \vec{B} = \hat{z}\frac{2\pi R^2}{(z^2 + R^2)^{3/2}}\frac{I}{c}$$

Electromagnetism (CGS)

$$\text{Faraday: } \mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t};\ \int \mathrm{d}^3 x \vec{J} = \dot{\vec{p}}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$$

$$\mathrm{d}\vec{F} = \mathrm{d}^3 x\left(\rho\vec{E} + \frac{\vec{J}}{c} \times \vec{B}\right);\ \vec{F}_q = q\left(\vec{E} + \frac{\dot{\vec{r}}}{c} \times \vec{B}\right)$$

$$u = \frac{E^2 + B^2}{8\pi};\ \vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B};\ \vec{g} = \frac{\vec{S}}{c^2}$$

$$\mathbf{T}^E = \frac{1}{4\pi}\left(\vec{E} \otimes \vec{E} - \frac{1}{2}E^2\right);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ -\frac{\partial\vec{g}}{\partial t} = \rho\vec{E} + \frac{\vec{J}}{c} \times \vec{B} - \vec{\nabla}\mathbf{T}$$

$$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\left(\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$

$$(\phi, \vec{A}) \cong \left(\phi - \frac{1}{c}\frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\right)$$

$$(\phi, \vec{A}) = \int \mathrm{d}^3 r' \frac{\left(\rho, \frac{\vec{J}}{c}\right)\left(\vec{r}', t - \frac{1}{c}\left|\vec{r} - \vec{r}'\right|\right)}{|\vec{r} - \vec{r}'|}$$

$$\vec{\nabla}\vec{A} = 0 \to \Box\vec{A} = \frac{4\pi}{c}(\vec{J} - \vec{J}_L) =: \frac{4\pi}{c}\vec{J}_T$$

$$\vec{J}_L = \frac{1}{4\pi}\vec{\nabla}\frac{\partial\phi}{\partial t} = -\frac{1}{4\pi}\vec{\nabla}\int \frac{\vec{\nabla}'\vec{J}'}{|\vec{r} - \vec{r}'|}\mathrm{d}^3 r'$$

$$\vec{E}'_\parallel = \vec{E}_\parallel;\ \vec{B}'_\parallel = \vec{B}_\parallel$$

$$\vec{E}'_\perp = \gamma\left(\vec{E}_\perp + \frac{\vec{v}}{c} \times \vec{B}\right)$$

$$\vec{B}'_\perp = \gamma\left(\vec{B}_\perp - \frac{\vec{v}}{c} \times \vec{E}\right)$$

$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \hat{k} \times \vec{E} \\ \omega = ck \end{cases}$$

$$\vec{B}_\mathrm{diprad} = \frac{1}{c^2}\frac{\ddot{\vec{p}} \times \hat{r}}{r}\Big|_{t_\mathrm{rit}};\ \vec{E}_\mathrm{diprad} = \vec{B}_\mathrm{diprad} \times \hat{r}$$

$$\text{Larmor: } P = \frac{2}{3c^3}|\ddot{\vec{p}}|^2$$

$$\text{Rel. Larmor: } P = \frac{2}{3c^3}q^2\gamma^6(a^2 - (\vec{a} \times \vec{\beta})^2)$$

$$\vec{A}_\mathrm{dm} = \frac{1}{c}\frac{\dot{\vec{m}} \times \hat{r}}{r}\Big|_{t_\mathrm{rit}}$$

$$\text{L.W.: } (\phi, \vec{A}) = \frac{q(1, \frac{\vec{v}}{c})}{[r - \frac{\vec{v}\vec{r}}{c}]_{t_\mathrm{rit}}};\ t_\mathrm{rit} = t - \frac{r}{c}\Big|_{t_\mathrm{rit}}$$

$$A^\mu = (\phi, \vec{A});\ J^\mu = (c\rho, \vec{J})$$

$$\text{Lorenz gauge: } \partial_\alpha A^\alpha = 0$$

$$\text{Temporal gauge: } \phi = 0$$

$$\text{Axial gauge: } A_3 = 0$$

$$\text{Coulomb gauge: } \vec{\nabla}\vec{A} = 0$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu;\ \mathscr{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\partial_\alpha F^{\alpha\nu} = 4\pi\frac{J^\nu}{c};\ \partial_\alpha \mathscr{F}^{\alpha\nu} = 0;\ \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = qF^{\mu\alpha}\frac{v_\alpha}{c}$$

$$\partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0;\ \det F = (\vec{E}\vec{B})^2$$

$$F^{\alpha\beta}F_{\alpha\beta} = 2(B^2 - E^2);\ F^{\alpha\beta}\mathscr{F}_{\alpha\beta} = 4\vec{E}\vec{B}$$

$$\Theta^{\mu\nu} = \frac{1}{4\pi}\left(F^\mu{}_\alpha F^{\alpha\nu} + \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\Theta^{\mu\nu} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T} \end{pmatrix};\ \partial_\alpha\Theta^{\alpha\nu} = \frac{J_\alpha}{c}F^{\alpha\nu} = -G^\nu$$

$$\mathcal{L} = \frac{mc^2}{\gamma} - q\vec{A}\frac{\vec{v}}{c} + q\phi;\ \mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{q\vec{A}}{c}\right)^2 + q\phi$$

$$\text{plane wave: } \mathbf{T} = -u\hat{k} \otimes \hat{k};\ \Theta^{\mu\nu} = u\hat{k}^\mu\hat{k}^\nu$$

E.M. in matter (CGS)

$$\vec{\nabla}\vec{D} = 4\pi\rho_\mathrm{ext};\ \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{H} = 4\pi\frac{\vec{J}_\mathrm{ext}}{c} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

$$\vec{P} = \frac{\mathrm{d}\langle\vec{p}\rangle}{\mathrm{d}V};\ \vec{M} = \frac{\mathrm{d}\langle\vec{m}\rangle}{\mathrm{d}V}$$

$$\rho_\mathrm{pol} = -\vec{\nabla}\vec{P};\ \sigma_\mathrm{pol} = \hat{n}\vec{P};\ \frac{\vec{J}_\mathrm{mag}}{c} = \vec{\nabla} \times \vec{M}$$

$$\vec{D}_\mathrm{pol} = \vec{E} + 4\pi\vec{P};\ \vec{H}_\mathrm{mag} = \vec{B} - 4\pi\vec{M}$$

$$\text{static linear isotropic: } \vec{P} = \chi\vec{E}$$

$$\text{static linear: } P_i = \chi_{ij}E_j$$

$$\text{static linear: } \varepsilon = 1 + 4\pi\chi$$

$$\text{static: } \Delta D_\perp = 4\pi\sigma_\mathrm{ext};\ \Delta E_\parallel = 0$$

$$\text{static linear: } u = \tfrac{1}{8\pi}\vec{E}\vec{D}$$
$$\Delta U_{\text{dielectric}} = -\tfrac{1}{2}\int \mathrm{d}^3 r \vec{P}\vec{E}_0$$
$$\text{plane capacitor: } C = \tfrac{\varepsilon}{4\pi}\tfrac{S}{d}$$
$$\text{cilindric capacitor: } C = \tfrac{L}{2\log\frac{R}{r}}$$
$$\text{atomic polarizability: } \vec{p} = \alpha\vec{E}_{\text{loc}}$$
$$\text{non-interacting gas: } \vec{p} = \alpha\vec{E}_0;\ \chi = n\alpha$$
$$\text{hom. cubic isotropic: } \chi = \tfrac{1}{\frac{1}{n\alpha}-\frac{4\pi}{3}}$$
$$\text{Clausius-Mossotti: } \tfrac{\varepsilon-1}{\varepsilon+2} = \tfrac{4\pi}{3}n\alpha$$
$$\text{perm. dipole: } \chi = \tfrac{1}{3}\tfrac{np_0^2}{kT}$$
$$\text{local field: } \vec{E}_{\text{loc}} = \vec{E} + \tfrac{4\pi}{3}\vec{P}$$
$$\vec{J}\vec{E} = -\vec{\nabla}\Big(\tfrac{c}{4\pi}\vec{E}\times\vec{H}\Big) - \tfrac{1}{4\pi}\Big(\vec{E}\tfrac{\partial\vec{D}}{\partial t} + \vec{H}\tfrac{\partial\vec{B}}{\partial t}\Big)$$
$$n = \sqrt{\varepsilon\mu};\ k = n\tfrac{\omega}{c}$$

Quantum mechanics (CGS)

$$r_e = \tfrac{e^2}{mc^2};\ \alpha = \tfrac{e^2}{\hbar c};\ \lambda_{\text{Broglie}} = \tfrac{h}{p}$$
$$\text{Planck: } \tfrac{8\pi\hbar}{c^3}\tfrac{\nu^3}{e^{\frac{h\nu}{kT}}-1}\mathrm{d}\nu$$
$$i\hbar\tfrac{\partial\mathcal{U}}{\partial t} = \mathcal{H}\mathcal{U};\ \tfrac{\partial\mathcal{H}}{\partial t} = 0 \ \Rightarrow\ \mathcal{U}(t) = e^{-\frac{i\mathcal{H}t}{\hbar}}$$
$$[\mathcal{H}(t),\mathcal{H}(t')] = 0 \ \Rightarrow\ \mathcal{U}(t) = e^{-\frac{i\int_0^t \mathrm{d}t\mathcal{H}(t)}{\hbar}}$$
$$\mathcal{U}(t) = \big(\tfrac{-i}{\hbar}\big)^k \int_0^t \mathrm{d}t_1\cdots\int_0^{t_{k-1}}\mathrm{d}t_k\mathcal{H}(t_1)\cdots\mathcal{H}(t_k)$$
$$A_H(t) = \mathcal{U}(t)^\dagger A\mathcal{U}(t)$$
$$\tfrac{\partial\mathcal{H}}{\partial t} = 0 \ \Rightarrow\ \tfrac{\mathrm{d}A_H}{\mathrm{d}t} = \tfrac{[A_H,\mathcal{H}]}{i\hbar}$$
$$(A\otimes B)(|a\rangle\otimes|b\rangle) = A|a\rangle\otimes B|b\rangle$$
$$(\langle a|\otimes\langle b|)(|c\rangle\otimes|d\rangle) = \langle a|c\rangle\langle b|d\rangle$$
$$\mathcal{H}_{1\otimes 2} = \mathcal{H}^{(1)}\otimes\mathbb{1}^{(2)} + \mathbb{1}^{(1)}\otimes\mathcal{H}^{(2)}$$
$$H = H_0 + V_\lambda : \tfrac{\partial E_n}{\partial\lambda}\big|_{\lambda=0} = \langle\psi_n|\tfrac{\partial V_\lambda}{\partial\lambda}|\psi_n\rangle\big|_{\lambda=0}$$
$$[A,BC] = [A,B]C + B[A,C]$$
$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$
$$[X,P] = i\hbar;\ \langle x|p\rangle = \tfrac{1}{\sqrt{2\pi\hbar}}e^{\frac{ipx}{\hbar}}$$
$$\langle x|X|\psi\rangle = x\langle x|\psi\rangle;\ \langle x|P|\psi\rangle = \tfrac{\hbar}{i}\tfrac{\partial}{\partial x}\langle x|\psi\rangle$$
$$\big\langle(A-\langle A\rangle)^2\big\rangle\big\langle(B-\langle B\rangle)^2\big\rangle \geq \tfrac{1}{4}|\langle[A,B]\rangle|^2$$
$$e^B A e^{-B} = A + [B,A] + \tfrac{1}{2!}[B,[B,A]] + \cdots$$

QM solutions

$$\mathcal{H}_{\text{box}} = \tfrac{P^2}{2m} + \begin{cases} 0 & 0<x<L \\ \infty & \text{otherwise}\end{cases}$$
$$E_n = \tfrac{n^2\pi^2\hbar^2}{2mL^2},\ n\geq 1$$
$$\psi_n(x) = \sqrt{\tfrac{2}{L}}\sin\big(n\pi\tfrac{x}{L}\big) = \sqrt{\tfrac{2}{L}}\sin\Big(\sqrt{\tfrac{2mE}{\hbar^2}}x\Big)$$
$$\Delta x^2 = L^2\big(\tfrac{1}{12} - \tfrac{1}{2n^2\pi^2}\big);\ \Delta p = \tfrac{\hbar n\pi}{L}$$
$$\mathcal{H}_{\text{harm}} = \tfrac{P^2}{2m} + \tfrac{m\omega^2 X^2}{2}$$
$$A = \sqrt{\tfrac{m\omega}{2\hbar}}\big(X + \tfrac{iP}{m\omega}\big);\ A^\dagger = \sqrt{\tfrac{m\omega}{2\hbar}}\big(X - \tfrac{iP}{m\omega}\big)$$
$$N = A^\dagger A = \tfrac{\mathcal{H}}{\hbar\omega} - \tfrac{1}{2};\ \mathcal{H} = \hbar\omega\big(N + \tfrac{1}{2}\big)$$
$$\big[A,A^\dagger\big] = 1;\ [N,A] = -A;\ \big[N,A^\dagger\big] = A^\dagger$$
$$A^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle;\ A|n\rangle = \sqrt{n}|n-1\rangle$$
$$|n\rangle = \tfrac{(A^\dagger)^n}{\sqrt{n!}}|0\rangle,\ n = 0,1,\ldots$$
$$\psi_n(x) = \tfrac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}}\big(\tfrac{x}{x_0} - x_0\tfrac{\mathrm{d}}{\mathrm{d}x}\big)^n e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$
$$\psi_n(x) = \tfrac{1}{\pi^{\frac{1}{4}}\sqrt{2^n n! x_0}}H_n\big(\tfrac{x}{x_0}\big)e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$$

$$\text{plane wave: } B = nE$$
$$\vec{J}_{\text{c}} = \sigma\vec{E};\ \varepsilon_\sigma = 1 + i\tfrac{4\pi\sigma}{\omega}$$
$$\omega_{\text{p}}^2 = 4\pi\tfrac{n_{\text{vol}}q^2}{m};\ \omega_{\text{cyclo}} = \tfrac{qB}{mc}$$
$$\text{I: } u = \tfrac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$$
$$\text{I: } \langle S_z\rangle = \tfrac{c}{n}\langle u\rangle$$
$$\text{II: } u = \tfrac{1}{8\pi}\big(\tfrac{\partial}{\partial\omega}(\varepsilon\omega)E^2 + \tfrac{\partial}{\partial\omega}(\mu\omega)H^2\big)$$
$$\text{II: } \langle S_z\rangle = v_{\text{g}}\langle u\rangle;\ v_{\text{g}} = \tfrac{\partial\omega}{\partial k} = \tfrac{c}{n+\omega\frac{\partial n}{\partial\omega}}$$
$$\text{III: } \langle W\rangle = \tfrac{\omega}{4\pi}\big(\operatorname{Im}\varepsilon\langle E^2\rangle + \operatorname{Im}\mu\langle H^2\rangle\big)$$
$$\text{Fresnel TE (S): } \tfrac{E_{\text{t}}}{E_{\text{i}}} = \tfrac{2}{1+\frac{k_{tz}}{k_{iz}}};\ \tfrac{E_{\text{r}}}{E_{\text{i}}} = \tfrac{1-\frac{k_{tz}}{k_{iz}}}{1+\frac{k_{tz}}{k_{iz}}}$$
$$\text{TM (P): } \tfrac{E_{\text{t}}}{E_{\text{i}}} = \tfrac{2}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}};\ \tfrac{E_{\text{r}}}{E_{\text{i}}} = \tfrac{\frac{n_2}{n_1}-\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}{\frac{n_2}{n_1}+\frac{n_1}{n_2}\frac{k_{tz}}{k_{iz}}}$$
$$\text{Fresnel: } k_{tz} = \pm\sqrt{\varepsilon_2\big(\tfrac{\omega}{c}\big)^2 - k_x^2},\ \operatorname{Im}k_{tz} > 0$$
$$[A,B]\propto I \ \Rightarrow\ [A,f(B)] = [A,B]f'(B)$$
$$[A,B]\propto I \ \Rightarrow\ e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$$
$$e^{ip'X}|p\rangle = |p+p'\rangle;\ e^{-iPx'}|x\rangle = |x+x'\rangle$$
$$\psi(x) = \langle x|\psi\rangle;\ \rho = |\psi|^2;\ \psi = \sqrt{\rho}e^{\frac{iS}{\hbar}}$$
$$\mathcal{H} = \tfrac{\vec{P}^2}{2m} + V(\vec{X}) : \vec{j} = \tfrac{\hbar}{m}\operatorname{Im}(\psi^*\vec{\nabla}\psi) = \tfrac{\rho\vec{\nabla}S}{m}$$
$$\tfrac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j};\ \int\mathrm{d}^3x\vec{j} = \tfrac{\langle\vec{p}\rangle}{m}$$
$$\psi(x,t) = \int\mathrm{d}x' K(x,t;x')\psi(x',t=0)$$
$$K(x,t;x') = \textstyle\sum_E \psi_E(x')^*\psi_E(x)e^{-\frac{iEt}{\hbar}} =$$
$$= \langle x|e^{-\frac{i\mathcal{H}t}{\hbar}}|x'\rangle$$
$$\big(\mathcal{H} - i\hbar\tfrac{\partial}{\partial t}\big)K(x,t;x') = -i\hbar\delta(x-x')\delta(t)$$
$$\sigma_1 = \left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right);\ \sigma_2 = \left(\begin{smallmatrix}0&-i\\i&0\end{smallmatrix}\right);\ \sigma_3 = \left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$$
$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$$
$$[\sigma_i,\sigma_j] = 2i\varepsilon_{ijk}\sigma_k;\ \{\sigma_i,\sigma_j\} = 2\delta_{ij}$$
$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = \vec{a}\vec{b} + i\vec{\sigma}(\vec{a}\times\vec{b})$$
$$e^{-\frac{i\vec{\sigma}\hat{n}\phi}{2}} = \cos\tfrac{\phi}{2} - i(\vec{\sigma}\hat{n})\sin\tfrac{\phi}{2}$$
$$|\vec{\sigma}\hat{n},1\rangle = \cos\tfrac{\theta}{2}|\sigma_3,1\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|\sigma_3,-1\rangle$$
$$R(\hat{n},\phi) = \exp\Big(-\tfrac{i\vec{J}\hat{n}\phi}{\hbar}\Big)$$
$$[J_i,J_j] = i\hbar\varepsilon_{ijk}J_k;\ J_\pm := J_x \pm iJ_y$$

$$x_0 = \sqrt{\tfrac{\hbar}{m\omega}}$$
$$\textstyle\sum_{n=0}^\infty H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$$
$$H_n(-x) = (-1)^n H_n(x)$$
$$n \text{ even: } H_n(0) = (-1)^{\frac{n}{2}}\tfrac{n!}{(n/2)!}$$
$$H_n'(x) = 2nH_{n-1}(x);\ H_0 = 1$$
$$H_1 = 2x;\ H_2 = 4x^2 - 2;\ H_3 = 8x^3 - 12x$$
$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$
$$H_n''(x) = 2xH_n'(x) - 2nH_n(x)$$
$$\int_{-\infty}^\infty \mathrm{d}xH_n(x)H_m(x)e^{-x^2} = \sqrt{\pi}2^n n!\delta_{nm}$$
$$\mathcal{H}_{\text{delta}} = \tfrac{P^2}{2m} - \lambda\delta(x),\ \lambda > 0$$
$$\psi_{\text{bounded}}(x) = \tfrac{1}{\sqrt{x_0}}e^{-\frac{|x|}{x_0}},\ x_0 = \tfrac{\hbar^2}{\lambda m}$$
$$E_{\text{bounded}} = -\tfrac{\lambda}{2x_0}$$
$$\mathcal{H}_{\text{step}} = \tfrac{P^2}{2m} + \begin{cases}0 & x<0\\ V_0>0 & x>0\end{cases}$$

$$\text{Drüde-Lorentz: } \varepsilon = 1 - \tfrac{\omega_{\text{p}}^2}{\omega^2+i\gamma\omega-\omega_0^2}$$
$$P(t) = \int_{-\infty}^\infty g(t-t')E(t')\mathrm{d}t'$$
$$P(\omega) = \chi(\omega)E(\omega)$$
$$\chi(\omega) = \int_{-\infty}^\infty e^{i\omega t}g(t)\mathrm{d}t;\ \chi(-\omega) = \chi^*(\omega)$$
$$g(t<0) = 0 \implies$$
$$\operatorname{Re}\varepsilon(\omega) = 1 + \tfrac{2}{\pi}\fint_0^\infty \tfrac{\omega'\left(\operatorname{Im}\varepsilon(\omega') - \frac{4\pi\sigma_0}{\omega'}\right)}{\omega'^2-\omega^2}\mathrm{d}\omega'$$
$$\operatorname{Im}\varepsilon(\omega) = -\tfrac{2\omega}{\pi}\fint_0^\infty \tfrac{\operatorname{Re}\varepsilon(\omega')-1}{\omega'^2-\omega^2}\mathrm{d}\omega' + \tfrac{4\pi\sigma_0}{\omega}$$
$$\text{sum rule: } \tfrac{\pi}{2}\omega_{\text{p}}^2 = \int_0^\infty \omega\operatorname{Im}\varepsilon\mathrm{d}\omega$$
$$\text{sum rule: } 2\pi^2\sigma_0 = \int_0^\infty(1-\operatorname{Re}\varepsilon)\mathrm{d}\omega$$
$$\text{sum rule: } \int_0^\infty(\operatorname{Re}n - 1)\mathrm{d}\omega = 0$$
$$\text{Miller rule: } \chi^{(2)}(\omega,\omega) \propto \chi^{(1)}(\omega)^2\chi^{(1)}(2\omega)$$

$$[J_+,J_-] = i\hbar J_z;\ [J_z,J_\pm] = \pm\hbar J_\pm$$
$$\big[J^2,J_\pm\big] = \big[J^2,J_z\big] = 0$$
$$J^2|j,m\rangle = j(j+1)\hbar^2|j,m\rangle$$
$$J_z|j,m\rangle = m\hbar|j,m\rangle$$
$$m = -j, j-1, \ldots, j;\ 2j\in\mathbb{N}$$
$$U^\dagger U = 1 : U = \left(\begin{smallmatrix}a & b\\ -b^* & a^*\end{smallmatrix}\right),\ |a|^2 + |b|^2 = 1$$
$$U = e^{-\frac{i\sigma_z\alpha}{2}}e^{-\frac{i\sigma_y\beta}{2}}e^{-\frac{i\sigma_z\gamma}{2}}$$
$$a{=}\cos\tfrac{\phi}{2}{-}in_z\sin\tfrac{\phi}{2}{=}e^{-i\frac{\alpha+\gamma}{2}}\cos\tfrac{\beta}{2}$$
$$b{=}{-}\sin\tfrac{\phi}{2}(n_y{+}in_x){=}{-}e^{-i\frac{\alpha-\gamma}{2}}\sin\tfrac{\beta}{2}$$
$$\vec{L} = \vec{X}\times\vec{P};\ \langle\vec{x}|L_z|\psi\rangle = \tfrac{\hbar}{i}\tfrac{\partial}{\partial\phi}\langle\vec{x}|\psi\rangle$$
$$A = \vec{A} :\leftrightarrow [A_i,J_j] = i\varepsilon_{ijk}\hbar A_k$$
$$T = \mathbf{T} :\leftrightarrow [J_z,T_q] = \hbar qT_q$$
$$\Big[J_\pm,T_q^{(k)}\Big] = \hbar\sqrt{(k\mp q)(k\pm q+1)}T_{q\pm 1}^{(k)}$$
$$\rho[|\alpha_i\rangle,w_i] := \textstyle\sum_i w_i|\alpha_i\rangle\langle\alpha_i|$$
$$\operatorname{tr}\rho = 1;\ [A] := \operatorname{tr}(\rho A)$$
$$\#\{w_i>0\} = 1 \iff tr(\rho^2) = 1$$
$$\#\{w_i>0\} > 1 \iff 0 < \operatorname{tr}\big(\rho^2\big) < 1$$
$$i\hbar\tfrac{\partial\rho}{\partial t} = -[\rho,\mathcal{H}]$$
$$W_\psi(x,p) = \int\tfrac{\mathrm{d}y}{2\pi\hbar}\big\langle x+\tfrac{y}{2}\big|\psi\big\rangle\big\langle\psi\big|x-\tfrac{y}{2}\big\rangle e^{-\frac{ipy}{2}}$$
$$k^2 := \tfrac{2mE}{\hbar^2},\ q^2 := \tfrac{2m(E-V_0)}{\hbar^2}$$
$$\psi_{\text{right}}(x) \propto \begin{cases} e^{ikx} + \frac{k-q}{k+q}e^{-ikx} & x<0\\ \frac{2k}{k+q}e^{iqx} & x>0\end{cases}$$
$$\mathcal{H}_{\text{hydrogen}} = \tfrac{\vec{P}^2}{2M} - \tfrac{e^2}{X}$$
$$a := r_B := \tfrac{\hbar^2}{Me^2};\ \text{Rydberg} = \tfrac{e^2}{2r_B}$$
$$E_n = -\tfrac{1}{n^2}\tfrac{e^2}{2a};\ \text{degen.} = n^2$$
$$\psi_{nlm} = R_{nl}Y_{lm};\ \vec{j} = \tfrac{\hbar}{M}\hat{\varphi}\tfrac{m}{r\sin\theta}|\psi|^2$$
$$R_{nl} = 2\sqrt{\tfrac{(n-l-1)!}{a^3n^4(n+l)!}}e^{-\frac{r}{na}}\big(\tfrac{2r}{na}\big)^l L_{n+l}^{2l+1}\big(\tfrac{2r}{na}\big)$$
$$L_n^{(j)}(x) = \textstyle\sum_{m=0}^{n-j}(-1)^m\binom{n}{n-j-m}\frac{x^m}{m!}$$
$$L_k(x) = e^x\tfrac{\mathrm{d}^k}{\mathrm{d}x^k}\big(x^ke^{-x}\big)$$
$$L_k^{(j)} = (-1)^j\tfrac{\mathrm{d}^j}{\mathrm{d}x^j}L_k(x)$$
$$\mathcal{H}_{\text{harm3D}} = \tfrac{\vec{P}^2}{2m} + \tfrac{m\omega^2\vec{X}^2}{2}$$
$$E_{ql} = \big(2q + l + \tfrac{3}{2}\big)\hbar\omega$$
$$l = 0,1,\ldots;\ q = 0,1,\ldots$$

Particle physics

$$M(A,Z) = Zm_\mathrm{p} + (A-Z)m_\mathrm{n} - B(A,Z)$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_\mathrm{sym} \frac{(A-2Z)^2}{A} + a_p A^{-3/4}\Delta$$

$$\Delta = \begin{cases} 0 & A \text{ odd} \\ \left.\begin{matrix} 1 & Z \text{ even} \\ -1 & Z \text{ odd} \end{matrix}\right\} & A \text{ even} \end{cases}$$

$$a_v = 15.5;\ a_s = 16.8;\ a_c = 0.72;\ a_\mathrm{sym} = 23;\ a_p = 34\ [\mathrm{MeV}]$$

$$\frac{\partial M}{\partial Z} = 0 : Z = \frac{m_\mathrm{n} - m_\mathrm{p} + 4a_\mathrm{sym}}{\frac{2a_c}{A^{1/3}} + \frac{8a_\mathrm{sym}}{A}}$$

$$s_{ab} := (p_a + p_b)^2$$

$$M \to abc : (m_a + m_b)^2 \leq s_{ab} \leq (M - m_c)^2$$

$$M \to abc : s_{ab} + s_{bc} + s_{ac} = M^2 + m_a^2 + m_b^2 + m_c^2$$

$$a_i A_i \to b_j B_j : Q := a_i m_{A_i} - b_j m_{B_j}$$

$$p = qBR$$

$$\frac{\mathrm{d}^3\vec{p}}{2E} = \mathrm{d}^4 p \delta(p^2 - m^2)\theta(p_0)$$

$$\mathrm{d}L_p = \left(\prod_n \frac{\mathrm{d}^3\vec{p}_n}{2E_n}\right)\delta^4(p_\mathrm{in} - \textstyle\sum_n p_n);\ \mathrm{d}\sigma = f_\mathrm{coll}(p_1, \dots, p_n)\mathrm{d}L_p$$

$$\text{two body: } \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1} = f(\Omega_1)\frac{p_1}{4\sqrt{s}};\ \sqrt{s} = \text{c.m. energy}$$

$$\text{Rutherford: } \tan\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Qqm}{p^2 b};\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left|\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|;\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{d_\mathrm{min}^2}{16}\frac{1}{\sin^4\frac{\theta}{2}}$$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_\mathrm{Mott} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_\mathrm{Rutherford} \cdot \cos^2\frac{\theta}{2}$$

$$\text{mass defect} := M - A \cdot \text{amu}$$