Trigonometry

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$
$$\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$
$$\sin\alpha\pm\sin\beta=2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\frac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\frac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\frac{|a|}{a}\sqrt{a^2+b^2}\sin\left(x+\operatorname{atan}\frac{b}{a}\right)$$
$$=\frac{|b|}{b}\sqrt{a^2+b^2}\cos\left(x-\operatorname{atan}\frac{a}{b}\right)$$

Hyperbolic functions

$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\begin{pmatrix}\sinh x\\ \cosh x\end{pmatrix}=\frac{1}{2}\begin{pmatrix}e^x-e^{-x}\\ e^x+e^{-x}\end{pmatrix}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\cosh^2 x=\frac{1}{1-\tanh^2 x}$$
$$\sin x=-i\sinh(ix)$$
$$\cos x=\cosh(ix)$$
$$\operatorname{atanh} x=\frac{1}{2}\log\frac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\left(I+\frac{B}{2}-1\right)A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\frac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\binom{n}{k}=\frac{n!}{k!(n-k)!}\qquad C'_{n,k}=\binom{n+k-1}{k}$$
$$D_{n,k}=\frac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\frac{ABC-AB}{9\times C\quad 0\times B}$$
$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$
$$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$
$$\Gamma(1+z)=\textstyle\int_0^\infty t^z e^{-t}dt$$
$$\tilde{f}(k)=\frac{1}{\sqrt{2\pi}}\textstyle\int_{-\infty}^{\infty}e^{-ikx}f(x)$$

Derivatives

$$\tan' x=1+\tan^2 x$$
$$\cot' x=-1-\cot^2 x$$
$$\operatorname{atan}' x=-\operatorname{acot}' x=\frac{1}{1+x^2}$$
$$\operatorname{asin}' x=-\operatorname{acos}' x=\frac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log'_a x=\frac{1}{x\ln a}$$
$$\cosh' x=\sinh x$$
$$\tanh' x=1-\tanh^2 x$$
$$\operatorname{atanh}' x=\operatorname{acoth}' x=\frac{1}{1-x^2}$$
$$\operatorname{asinh}' x=\frac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}' x=\frac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\frac{1}{f'(f^{-1})}$$
$$\left(\frac{1}{x}\right)'=-\frac{\dot{x}}{x^2}$$
$$\left(\frac{x}{y}\right)'=\frac{\dot{x}y-x\dot{y}}{y^2}$$
$$(x^y)'=x^y\left(\dot{y}\ln x+y\frac{\dot{x}}{x}\right)$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x=\frac{a^x}{\ln a}$$
$$\textstyle\int\frac{1}{x}=\ln|x|$$
$$\textstyle\int\tan x=-\ln|\cos x|$$
$$\textstyle\int\cot x=\ln|\sin x|$$
$$\textstyle\int\frac{1}{\sin x}=\ln\left|\tan\frac{x}{2}\right|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$
$$\textstyle\int\ln x=x(\ln x-1)$$
$$\textstyle\int\tanh x=\ln\cosh x$$
$$\textstyle\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$
$$\textstyle\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy=x\int y-\int(\dot{x}\int y)$$
$$\textstyle\int e^{yx}x=e^{yx}\left(\frac{y}{x}-\frac{1}{y^2}\right)$$

Differential equations

$$\dot{x}+\dot{a}x=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$
$$a\ddot{x}+b\dot{x}+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot{x}=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$
$$x\ddot{x}=k\dot{x}^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot{x}+ax^2=b:x=\sqrt{\frac{b}{a}}\tanh\left(\sqrt{ab}(c_1+t)\right)$$

Taylor

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$
$$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$
$$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sin x}=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cos x}=1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tan x}=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin} x=x+\frac{x^3}{6}+\frac{3}{40}x^5+\frac{5}{112}x^7+\mathrm{O}(x^9)$$
$$\operatorname{acos} x=\frac{\pi}{2}-\operatorname{asin} x$$
$$\operatorname{atan} x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$
$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$
$$\cosh x=1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$
$$\tanh x=x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\frac{1}{\sinh x}=\frac{1}{x}-\frac{x}{6}+\frac{7}{360}x^3-\frac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\frac{1}{\cosh x}=1-\frac{x^2}{2}+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\frac{1}{\tanh x}=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\frac{x^3}{2}+\frac{5}{6}x^4-\frac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\left(\frac{\gamma^2}{2}+\frac{\pi^2}{12}\right)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$
$$\vec{a}\times\vec{b}=\varepsilon_{ijk}a_jb_k\hat{e}_i$$
$$(\vec{a}\times\vec{b})\vec{c}=(\vec{c}\times\vec{a})\vec{b}$$
$$(\vec{a}\times\vec{b})\times\vec{c}=-(\vec{b}\vec{c})\vec{a}+(\vec{a}\vec{c})\vec{b}$$
$$(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})=(\vec{a}\vec{c})(\vec{b}\vec{d})-(\vec{a}\vec{d})(\vec{b}\vec{c})$$
$$|\vec{u}\times\vec{v}|^2=u^2v^2-(\vec{u}\vec{v})^2$$
$$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right);\ \Box=\frac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec{\nabla}V=\frac{\partial V}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{\phi}+\frac{\partial V}{\partial z}\hat{z}$$
$$\vec{\nabla}\vec{v}=\frac{1}{\rho}\frac{\partial(\rho v_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$
$$\vec{\nabla}\times\vec{v}=\left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{\rho}+$$
$$+\left(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\right)\hat{\phi}+\frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\right)$$
$$\nabla^2V=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2}+\frac{\partial^2V}{\partial z^2}$$
$$\vec{\nabla}V=\frac{\partial V}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}\hat{\varphi}$$
$$\vec{\nabla}\vec{v}=\frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(v_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$
$$\vec{\nabla}\times\vec{v}=\frac{1}{r\sin\theta}\left(\frac{\partial(v_\varphi\sin\theta)}{\partial\theta}-\frac{\partial v_\theta}{\partial\varphi}\right)\hat{r}+$$
$$+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi}-\frac{\partial(rv_\varphi)}{\partial r}\right)\hat{\theta}+\frac{1}{r}\left(\frac{\partial(rv_\theta)}{\partial r}-\frac{\partial v_r}{\partial\theta}\right)\hat{\varphi}$$
$$\nabla^2V=\frac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2}+\frac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta}+\frac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\vec{\nabla}(f\vec{v})=(\vec{\nabla}f)\vec{v}+f\vec{\nabla}\vec{v}$$
$$\vec{\nabla}\times(f\vec{v})=\vec{\nabla}f\times\vec{v}+f\vec{\nabla}\times\vec{v}$$
$$\vec{\nabla}\times(\vec{\nabla}\times\vec{v})=-\nabla^2\vec{v}+\vec{\nabla}(\vec{\nabla}\vec{v})$$
$$\vec{\nabla}(\vec{v}\times\vec{w})=\vec{w}(\vec{\nabla}\times\vec{v})-\vec{v}(\vec{\nabla}\times\vec{w})$$
$$\vec{\nabla}\times(\vec{v}\times\vec{w})=(\vec{\nabla}\vec{w}+\vec{w}\vec{\nabla})\vec{v}-(\vec{\nabla}\vec{v}+\vec{v}\vec{\nabla})\vec{w}$$

$$\tfrac{1}{2}\vec\nabla v^2 = (\vec v\,\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$
$$\int\vec\nabla v\,\mathrm{d}^3x = \oint \vec v\,\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\,\mathrm{d}\vec l$$
$$\int(f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}^3x = \oint_S\left(f\tfrac{\partial g}{\partial n} - g\tfrac{\partial f}{\partial n}\right)\mathrm{d}S$$
$$\oint\vec v\times\mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(\vec r-\vec r_0) = \tfrac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\nabla^2\tfrac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$$
$$\delta(g(x)) = \tfrac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle = \tfrac{1}{2}\,\mathrm{Re}(a\bar b)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\text{hist}} \approx \tfrac{x_{\max}-x_{\min}}{\sqrt N}$$
$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$
$$\text{median} = F^{-1}(\tfrac{1}{2})$$
$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$
$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$
$$\alpha_n = E[x^n]$$
$$M_n = E[(x-\mu)^n]$$
$$\sigma^2 = M_2 = E[x^2]-\mu^2$$
$$\text{FWHM}\approx 2\sigma$$
$$\gamma_1 = \tfrac{M_3}{\sigma^3},\ \gamma_2 = \tfrac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$
$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n = i^{-n}\tfrac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge 0: P(h\ge k)\le\tfrac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\tfrac{1}{k^2}$$
$$B(n,p,k) = \tbinom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$
$$P(\mu,k) = \tfrac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$
$$u(x,a,b) = \tfrac{1}{b-a},\ x\in[a;b]$$
$$\mu_u = \tfrac{b+a}{2},\ \sigma_u^2 = \tfrac{(b-a)^2}{12}$$
$$\varepsilon(x,\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \tfrac{1}{\lambda},\ \sigma_\varepsilon^2 = \tfrac{1}{\lambda^2}$$
$$g(x,\mu,\sigma) = \tfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
$$\text{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$
$$z = \tfrac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$
$$\chi^2 = \sum_{i=1}^n z_i^2$$
$$\wp_n(x) = \tfrac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$
$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$
$$n\ge 30: \wp_n(x)\approx g(x,n,\sqrt{2n})$$
$$n\ge 8: p[\sqrt{2\chi^2}]\approx g(,\sqrt{2n-1},1)$$
$$S(x,n) = \tfrac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\tfrac{x^2}{n}\right)^{-\frac{n+1}{2}}$$
$$\mu_S = 0,\ \sigma_S^2 = \tfrac{n}{n-2}$$

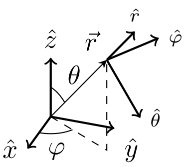
$$p\Big[z\sqrt{\tfrac{n}{\chi^2}}\Big] = S(,n)$$
$$n\ge 35: S(x,n)\approx g(x,0,1)$$
$$c(x,a) = \tfrac{a}{\pi}\tfrac{1}{a^2+x^2}$$
$$\sigma_{xy} = E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho = \tfrac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le 1$$
$$\mu[f(x_1,...)]\approx f(\mu_1,...)$$
$$\sigma^2[f(x_1,...)]\approx\sigma_{x_ix_j}\tfrac{\partial f}{\partial x_i}\Big|_{\mu_i}\tfrac{\partial f}{\partial x_j}\Big|_{\mu_j}$$
$$\mu\approx m = \tfrac{1}{n}\sum_{i=1}^n x_i$$
$$\sigma^2\approx s^2 = \tfrac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$
$$s_m^2 = \tfrac{s^2}{n}$$
$$p\Big[\tfrac{m-\mu}{s_m}\Big] = S(,n)$$

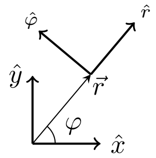
Fit

$$f(x) = mx+q,\quad f(x) = a$$
$$f(x) = bx$$
$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$b = \frac{\sum xy}{\sum x^2},\ \Delta b^2 = \frac{\Delta y^2}{\sum x^2}$$

Kinematics

$$\tfrac{1}{R} = \left|\tfrac{v_xa_y-v_ya_x}{v^3}\right|$$
$$\vec\omega = \dot\varphi\cos\theta\hat r-\dot\varphi\sin\theta\hat\theta+\dot\theta\hat\varphi$$
$$\dot{\vec w} = \tfrac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r+\tfrac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta+\tfrac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi+\vec\omega\times\vec w$$
$$\theta\equiv\tfrac{\pi}{2}\to\dot{\vec r} = \dot r\hat r+r\dot\varphi\hat\varphi$$
$$\theta\equiv\tfrac{\pi}{2}\to\ddot{\vec r} = (\ddot r-r\dot\varphi^2)\hat r+(r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$$
$$\dot{\vec r} = \dot r\hat r+r\dot\theta\hat\theta+r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r-r\dot\theta^2-r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta+2\dot r\dot\theta-r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta+2\dot r\dot\varphi\sin\theta+2r\dot\theta\dot\varphi\cos\theta$$
$$\vec A = \ddot{\vec r}+\vec A_{\mathrm T}+\vec\omega\times(\vec\omega\times\vec r)+\dot{\vec\omega}\times\vec r+2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \tfrac{\mathrm d}{\mathrm dt}\alpha(\beta,t) = \tfrac{\partial\alpha}{\partial\beta}\dot\beta+\tfrac{\partial\alpha}{\partial t}$$
$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \tfrac{\mathrm d(mT)}{\mathrm dt} = \vec F\vec p$$
$$M := \sum_i m_i;\ \vec R := \tfrac{m_i\vec r_i}{M}$$
$$T = \tfrac{1}{2}M\dot{\vec R}^2+\tfrac{1}{2}m_i(\dot{\vec r}_i-\dot{\vec R})^2$$
$$\vec L = \vec R\times M\dot{\vec R}+(\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$$
$$\vec\tau_O = \dot{\vec L}_O+\vec v_O\times\vec p$$
$$\tau_1 = I_1\dot\omega_1+(I_3-I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t) = T-V+\tfrac{\mathrm d}{\mathrm dt}f(q,t)$$
$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm dt$$
$$\tfrac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$
$$p := \tfrac{\partial\mathcal L}{\partial\dot q};\ \dot p = \tfrac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t) = \dot qp-\mathcal L$$
$$\dot q = \tfrac{\partial\mathcal H}{\partial p};\ \dot p = -\tfrac{\partial\mathcal H}{\partial q}$$
$$\tfrac{\mathrm d\mathcal H}{\mathrm dt} = \tfrac{\partial\mathcal H}{\partial t} = -\tfrac{\partial\mathcal L}{\partial t}$$
$$\{u,v\} = \tfrac{\partial u}{\partial q}\tfrac{\partial v}{\partial p}-\tfrac{\partial u}{\partial p}\tfrac{\partial v}{\partial q}$$
$$\tfrac{\mathrm du}{\mathrm dt} = \{u,\mathcal H\}+\tfrac{\partial u}{\partial t}$$
$$\eta = (q,p);\ \Gamma = \left(\begin{smallmatrix}0&1\\-1&0\end{smallmatrix}\right)$$
$$\dot\eta = \Gamma\tfrac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \tfrac{\partial u}{\partial\eta}\Gamma\tfrac{\partial v}{\partial\eta}$$

Inertia

point: mr^2

two points: μd^2

rod: $\frac{1}{12}mL^2$

disk: $\frac{1}{2}mr^2$

tetrahedron: $\frac{1}{20}ms^2$

octahedron: $\frac{1}{10}ms^2$

sphere: $\frac{2}{3}mr^2$

ball: $\frac{2}{5}mr^2$

cone: $\frac{3}{10}mr^2$

torus: $m\left(R^2+\frac{3}{4}r^2\right)$

ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$

rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle\approx -2\langle T\rangle$$
$$U_{\text{eff}} = U+\tfrac{L^2}{2mr^2}$$
$$\tfrac{1}{\mu} = \tfrac{1}{m_1}+\tfrac{1}{m_2}$$
$$\vec r = \vec r_1-\vec r_2,\ \alpha = Gm_1m_2$$
$$T = \tfrac{1}{2}M\dot{\vec R}^2+\tfrac{1}{2}\mu\dot{\vec r}^2$$
$$\vec L = \vec R\times M\dot{\vec R}+\vec r\times\mu\dot{\vec r}$$
$$k = \tfrac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1+\tfrac{2EL^2}{\mu\alpha^2}}$$
$$r = \tfrac{k}{1+\varepsilon\cos\theta}$$
$$a = \tfrac{k}{|1-\varepsilon^2|} = \tfrac{\alpha}{2|E|}$$
$$a^3\omega^2 = G(m_1+m_2)$$
$$\vec A = \mu\dot{\vec r}\times\vec L-\mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$$|a|-|b|\le|a+b|\le|a|+|b|$$
$$x>-1: 1+nx\le(1+x)^n$$
$$\tfrac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$$
$$\sum a_ib_i\le\left(\sum a_i^p\right)^{\frac{1}{p}}\left(\sum b_i^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}}$$
$$x^py^q\le\left(\tfrac{px+qy}{p+q}\right)^{p+q}$$
$$\sqrt[p]{\tfrac{1}{n}\sum a_i^{p\le q}}\le\sqrt[q]{\tfrac{1}{n}\sum a_i^q}$$
$$\sum\left(\tfrac{a_1+\dots a_i}{i}\right)^p\le\left(\tfrac{p}{p-1}\right)^p\sum a_i^p$$
$$x\ge 0,|\ddot x|\le M: |\dot x|\le\sqrt{2Mx}$$
$$\tfrac{1}{1+x}<\ln\left(1+\tfrac{1}{x}\right)<\tfrac{1}{x}$$

Vector spaces

$(V,\mathbb K,+,\cdot)$ vector space; $\quad\mathbb K$ field

$$\exists\vec 0\in V: \vec v+\vec 0 = \vec v$$
$$\cdot:\mathbb K\times V\to V;\quad\lambda\cdot(\vec v+\vec w) = \lambda\vec v+\lambda\vec w$$
$$0_{\mathbb K}\cdot\vec v = \vec 0,\ 1_{\mathbb K}\cdot\vec v = \vec v$$
$$\lambda\in\mathbb K,\ \vec v,\vec w\in V\ \Rightarrow\ \vec v+\vec w\in V,\ \lambda\vec v\in V$$
$$\dim(U+V) = \dim U+\dim V-\dim(U\cap V)$$
$$\ell\text{ linear}: \ell(\vec v+\lambda\vec w) = \ell(\vec v)+\lambda\ell(\vec w)$$
$$\ker\ell = \{\vec v\in V\,|\,\ell(\vec v) = 0\}$$
$$\dim V = \dim\ell(V)+\dim(V\cap\ker\ell)$$
$$\langle,\rangle: V\times V\to\mathbb K;\quad\langle\vec v,\vec w\rangle = \langle\vec w,\vec v\rangle$$
$$\langle\vec v+\lambda\vec w,\vec u\rangle = \langle\vec v,\vec u\rangle+\lambda\langle\vec w,\vec u\rangle$$
$$\|\|: V\to\mathbb K;\quad\|\vec v\| = 0\to\vec v = \vec 0$$
$$\|\lambda\vec v\| = |\lambda|\|\vec v\|;\quad\|\vec v+\vec w\|\le\|\vec v\|+\|\vec w\|$$

Symbols

A	B	Γ	Δ	E	Z	H	Θ	I	K	Λ	M	N	Ξ	O	Π	P	Σ	T	Y	Φ	X	Ψ	Ω
α	β	γ	δ	ϵ/ε	ζ	η	θ/ϑ	ι	κ	λ	μ	ν	ξ	o	π/ϖ	ρ/ϱ	σ/ς	τ	υ	ϕ/φ	χ	ψ	ω

Constants, units

$\pi = 3.142$; $e = 2.718$; $\gamma = 5.772\cdot 10^{-1}$; $G = 6.674\cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s^2}}$

$R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$; $R = 8.206\cdot 10^{-2}\,\frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$; $N_\mathrm{A} = 6.022\cdot 10^{23}\,\frac{1}{\mathrm{mol}}$; $k = 1.381\cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$

$c = 2.998\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$; $q_\mathrm{e} = 1.602\cdot 10^{-19}\,\mathrm{A\,s}$; $m_\mathrm{e} = 9.109\cdot 10^{-31}\,\mathrm{kg}$; $m_\mathrm{p} = 1.673\cdot 10^{-27}\,\mathrm{kg}$

$m_\mathrm{n} = 1.675\cdot 10^{-27}\,\mathrm{kg}$; $\mathrm{amu} = 1.661\cdot 10^{-27}\,\mathrm{kg}$; $h = 6.626\cdot 10^{-34}\,\mathrm{J\,s}$; $\varepsilon_0 = 8.854\cdot 10^{-12}\,\frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}$

$\mu_0 = 1.257\cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}$; $\mu_\mathrm{B} = 9.274\cdot 10^{-24}\,\mathrm{A\,m^2}$; $\alpha = 7.297\cdot 10^{-3}$; $\mathrm{eV} = 1\cdot 10^{-12}\,\mathrm{erg}$

Chemistry

$$H = U + pV$$
$$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$$
$$G = H - TS$$
$$a_i \mathrm{A}_i \;\to\; b_j \mathrm{B}_j$$
$$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$$
$$\forall\, i,j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$$

$$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$$
$$k = Ae^{-\frac{E_\mathrm{a}}{RT}} \text{ (Arrhenius)}$$
$$a_{(\ell)} = \gamma\frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$$
$$a_{(g)} = \gamma\frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$$
$$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod[\mathrm{B}_j]^{b_j}}{\prod[\mathrm{A}_i]^{a_i}}$$
$$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$$

$$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\mathrm{tot}}$$
$$K_c = K_p(RT)^{\sum a_i - \sum b_j}$$
$$K_c = K_n V^{\sum a_i - \sum b_j}$$
$$K_n = K_\chi n_\mathrm{tot}^{\sum b_j - \sum a_i}$$
$$\Delta G^\mathrm{o}_\mathrm{r} = -RT\ln K$$
$$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$$

$$\Delta G = RT\ln\frac{Q}{K}$$
$$\ln\frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$
$$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$$
$$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}}\ln Q \text{ (Nerst)}$$
$$\text{(std) } \Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}}\log_{10} Q$$
$$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$$
$$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$; $\mathrm{d}L = p\mathrm{d}V$

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} \qquad C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V \qquad C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p \qquad \gamma = \frac{C_p}{C_V}$$

Ideal gas

$$pV = nRT$$
$$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$$
$$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$$
$$\mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T \text{ const.}$$

Electronics (MKS)

$$\binom{V}{I} = \binom{V_0}{I_0}e^{i\omega t}$$
$$Z = \frac{V}{I} \qquad Z_R = R$$
$$Z_C = -i\frac{1}{\omega C} \qquad Z_L = i\omega L$$
$$Z_\mathrm{series} = \sum_k Z_k \qquad \frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$$
$$\sum\nolimits_\mathrm{loop} V_k = 0 \qquad \sum\nolimits_\mathrm{node} I_k = 0$$
$$\mathcal{E} = -L\dot{I} \qquad L = \frac{\Phi_B}{I}$$

Relativity

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \vec p = \gamma m\vec v$$
$$\mathcal{E} = \gamma mc^2 \qquad \frac{\mathrm{d}\vec p}{\mathrm{d}t} = \vec F$$
$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma\begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix}$$
$$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t \qquad x^\mu = (ct, \vec x)$$
$$v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec v)$$
$$a^\mu = \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec v)}{\mathrm{d}t}\right)$$
$$p^\mu = mv^\mu = \left(\frac{\mathcal{E}}{c}, \vec p\right) \qquad \partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec\nabla\right)$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$x_\mu = g_{\mu\nu}x^\nu \qquad \partial_\mu\partial^\mu = \Box \qquad p^\mu p_\mu = (mc)^2$$

Electrostatics (CGS)

$$\vec F_{12} = q_1 q_2\frac{\vec r_2 - \vec r_1}{|\vec r_1 - \vec r_2|^3};\ \vec E_1 = \frac{\vec F_{12}}{q_2};\ V(\vec r) = \int \mathrm{d}^3 r'\,\frac{\rho(\vec r')}{|\vec r - \vec r'|};\ \rho_q = \delta(\vec r - \vec r_q)$$
$$\oint \vec E\mathrm{d}\vec S = 4\pi\int\rho\,\mathrm{d}^3x;\ -\nabla^2 V = \vec\nabla\vec E = 4\pi\rho;\ \vec\nabla\times\vec E = 0$$
$$U = \frac{1}{8\pi}\int E^2\,\mathrm{d}^3x;\ \tilde U = \frac{1}{2}\frac{q_i q_j}{|\vec r_i - \vec r_j|} = \frac{1}{8\pi}\sum_{ij}\int\vec E_i\vec E_j\,\mathrm{d}^3x$$
$$V(\vec r) = \int\rho G_\mathrm{D}(\vec r)\,\mathrm{d}^3x - \frac{1}{4\pi}\oint_S V\frac{\partial G_\mathrm{D}}{\partial n}\,\mathrm{d}S$$
$$V(\vec r) = \langle V\rangle_S + \int\rho G_\mathrm{N}(\vec r)\,\mathrm{d}^3x + \frac{1}{4\pi}\oint_S\frac{\partial V}{\partial n}G_\mathrm{N}(\vec r)\,\mathrm{d}S$$
$$\nabla^2_y G(\vec x, \vec y) = -4\pi\delta(\vec x - \vec y);\ G_\mathrm{D}(\vec x, \vec y)|_{\vec y\in S} = 0;\ \frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec y\in S} = -\frac{4\pi}{S}$$
$$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R};\ \vec E_\mathrm{dip} = \frac{3(\vec p\hat r)\hat r - \vec p}{r^3};\ V_\mathrm{dip} = \frac{\vec p\hat r}{r^2}$$
$$V(r,\theta) = \sum_{l=0}^\infty\left(A_l r^l - \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$
$$V(r,\theta,\varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l\left(A_{lm}r^l + \frac{B_{lm}}{r^{l+1}}\right)Y_{lm}(\theta,\varphi)$$
$$\frac{1}{|\vec r - \vec r'|} = \sum_{l=0}^\infty\frac{\min(r,r')^l}{\max(r,r')^{l+1}}P_l\left(\frac{\vec r\vec r'}{rr'}\right)$$

$$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2-1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 fP_l$$
$$P_l(1) = 1;\ \langle P_n|P_m\rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm}|Y_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}$$
$$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2-1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$
$$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$
$$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$$
$$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2-1\right)^l,\ |m| \le l$$
$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}e^{im\varphi}P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m\overline{Y}_{lm}$$
$$P_l\left(\frac{\vec r\vec r'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l\overline{Y}_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$$
$$V(r > \operatorname{diam}\operatorname{supp}\rho, \theta, \varphi) = \sum_{l=0}^\infty\sum_{m=-l}^l\frac{4\pi}{2l+1}q_{lm}[\rho]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$
$$q_{lm}[\rho] = \int_0^\infty r^2\mathrm{d}r\int_0^{2\pi}\mathrm{d}\varphi\int_0^\pi\sin\theta\,\mathrm{d}\theta\, r^l\rho(r,\theta,\varphi)\overline{Y}_{lm}(\theta,\varphi)$$

Magnetostatics (CGS)

$$\vec\nabla\vec J = -\frac{\partial\rho}{\partial t} = 0;\ I = \int\vec J\mathrm{d}\vec S$$
$$\text{solenoid: } B = 4\pi\frac{j_\mathrm{s}}{c}$$
$$\mathrm{d}\vec F = \frac{I\mathrm{d}\vec l}{c}\times\vec B = \mathrm{d}^3x\frac{\vec J}{c}\times\vec B;\ \vec F_q = q\frac{\dot{\vec r}}{c}\times\vec B$$
$$\mathrm{d}\vec B = \frac{I\mathrm{d}\vec l}{c}\times\frac{\vec r}{r^3};\ \vec B_q = q\frac{\dot{\vec r}}{c}\times\frac{\vec r}{r^3}$$
$$\vec B = \vec\nabla\times\vec A;\ \vec A = \int\mathrm{d}^3r'\frac{\vec J'}{c}\frac{1}{|\vec r - \vec r'|} + \vec\nabla A_0$$
$$\vec B = \int\mathrm{d}^3r'\frac{\vec J'}{c}\times\frac{\vec r - \vec r'}{|\vec r - \vec r'|^3}$$
$$\vec\nabla\vec A = 0 \to \nabla^2\vec A = -4\pi\frac{\vec J}{c}$$
$$\vec\nabla\vec B = 0;\ \vec\nabla\times\vec B = 4\pi\frac{\vec J}{c};\ \oint\vec B\mathrm{d}\vec l = 4\pi\frac{I}{c}$$
$$\vec m = \frac{1}{2}\int\mathrm{d}^3r'\left(\vec r'\times\frac{\vec J'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec L$$
$$\vec A \approx \frac{\vec m\times\vec r}{r^3};\ \vec\tau = \vec m\times\vec B$$

Electromagnetism (CGS)

$$\text{Faraday: } \mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$
$$\vec\nabla\times\vec E = -\frac{1}{c}\frac{\partial\vec B}{\partial t};\ \vec\nabla\vec E = 4\pi\rho;\ \vec\nabla\vec J = -\frac{\partial\rho}{\partial t}$$
$$\vec\nabla\times\vec B = 4\pi\frac{\vec J}{c} + \frac{1}{c}\frac{\partial\vec E}{\partial t};\ \vec\nabla\vec B = 0$$
$$\mathrm{d}\vec F = \mathrm{d}^3x\left(\rho\vec E + \frac{\vec J}{c}\times\vec B\right);\ \vec F_q = q\left(\vec E + \frac{\dot{\vec r}}{c}\times\vec B\right)$$
$$u = \frac{E^2+B^2}{8\pi};\ \vec S = \frac{c}{4\pi}\vec E\times\vec B;\ \vec g = \frac{\vec S}{c^2}$$

$$T_{ij}^E = \frac{1}{4\pi}\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$

$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ \frac{\partial \vec{g}}{\partial t} = -\vec{f} + \partial_j T_{ij}\hat{x}_i$$

$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$

$$\vec{\nabla}\left(\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$

$$(\phi,\vec{A}) \cong \left(\phi - \frac{1}{c}\frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\right)$$

$$(\phi,\vec{A}) = \int \mathrm{d}^3r'\, \frac{\left(\rho,\frac{\vec{J}}{c}\right)\left(\vec{r}', t - \frac{1}{c}\left|\vec{r}-\vec{r}'\right|\right)}{|\vec{r}-\vec{r}'|}$$

Coulomb gauge: $\vec{\nabla}\vec{A} = 0$

Lorenz gauge: $\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$

$$\vec{E}'\hat{v} = \vec{E}\hat{v};\ \vec{B}'\hat{v} = \vec{B}\hat{v}$$

$$\vec{E}'\times\hat{v} = \gamma\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)\times\hat{v}$$

$$\vec{B}'\times\hat{v} = \gamma\left(\vec{B} - \frac{\vec{v}}{c}\times\vec{E}\right)\times\hat{v}$$

$$\text{plane wave: } \begin{cases}\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}\\ \vec{B} = \hat{k}\times\vec{E}\\ \omega = ck\end{cases}$$

$$\text{dipole: } \vec{B}\Big|_{r\gg\frac{c}{\omega}} \approx \frac{1}{c^2}\frac{\ddot{\vec{p}}\times\hat{r}}{r};\ \vec{E}\approx\vec{B}\times\hat{r}$$

$$A^\mu = (\phi,\vec{A});\ J^\mu = (c\rho,\vec{J})$$

Lorenz gauge: $\partial_\mu A^\mu = 0$

$$\partial_\mu F^{\mu\nu} = 4\pi\frac{J^\nu}{c};\ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}$$

$$\mathscr{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$F^{\mu\nu}F_{\mu\nu} = E^2 - B^2;\ F^{\mu\nu}\mathscr{F}_{\mu\nu} = 4\vec{E}\vec{B}$$

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\left(g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} - \frac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}\right)$$

$$\Theta^{\alpha\beta} = \begin{pmatrix} u & c\vec{g}\\ c\vec{g} & -\mathbf{T}\end{pmatrix}$$

$$\partial_\mu\Theta^{\mu\nu} = -\frac{1}{c}F^{\nu\lambda}J_\lambda = \frac{1}{c}J_\lambda F^{\lambda\nu}$$

Electromagnetism in matter (CGS)

$$\vec{P} = \frac{\langle\vec{d}\rangle}{V};\ \vec{M} = \frac{\langle\vec{m}\rangle}{V}$$

$$\rho_{\text{pol}} = -\vec{\nabla}\vec{P};\ \sigma_{\text{pol}} = \hat{n}\vec{P};\ \frac{\vec{J}_{\text{mag}}}{c} = \vec{\nabla}\times\vec{M}$$

$$\vec{D} = \vec{E} + 4\pi\vec{P};\ \vec{H} = \vec{B} - 4\pi\vec{M}$$

$$\vec{\nabla}\vec{D} = 4\pi\rho_{\text{ext}};\ \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla}\times\vec{H} = 4\pi\frac{\vec{J}_{\text{ext}}}{c} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$$

static linear isotropic: $\vec{P} = \chi\vec{E}$

static linear: $P_i = \chi_{ij}E_j$

static linear: $\varepsilon = 1 + 4\pi\chi$

static: $\Delta D_\perp = 4\pi\sigma_{\text{ext}};\ \Delta E_\parallel = 0$

static linear: $u = \frac{1}{8\pi}\vec{E}\vec{D}$

$$\Delta U_{\text{dielectric}} = \frac{1}{2}\int \mathrm{d}^3r\,\vec{P}\vec{E}_0$$

plane capacitor: $C = \frac{\varepsilon}{4\pi}\frac{S}{d}$

non-interacting gas: $\vec{d} = \alpha\vec{E}_0;\ \chi = n\alpha$

hom. cubic isotropic: $\chi = \frac{1}{\frac{1}{n\alpha} - \frac{4\pi}{3}}$

Clausius-Mossotti: $\frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}n\alpha$

$$\chi = \frac{4\pi}{3}\frac{np_0^2}{kT};\ \vec{E}_e = \vec{E} + \frac{4\pi}{3}\vec{P}$$

$$\vec{J}\vec{E} = -\vec{\nabla}\left(\frac{c}{4\pi}\vec{E}\times\vec{H}\right) - \frac{1}{4\pi}\left(\vec{E}\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t}\right)$$

$$n = \sqrt{\varepsilon\mu};\ k = n\frac{\omega}{c}$$

$$\vec{J}_{\text{c}} = \sigma\vec{E};\ \tilde{\varepsilon} = \varepsilon + i\frac{4\pi\sigma}{\omega}$$

I: $u = \frac{1}{8\pi}(\vec{E}\vec{D} + \vec{H}\vec{B})$

I: $\langle u\rangle = \frac{1}{16\pi}(\varepsilon E^2 + \mu H^2)$

I: $\langle S_z\rangle = \frac{c}{n}\langle u\rangle$

II: $\langle u\rangle = \frac{1}{16\pi}\left(\frac{\partial}{\partial\omega}(\varepsilon_\omega\omega)\langle E^2\rangle + \frac{\partial}{\partial\omega}(\mu_\omega\omega)\langle H^2\rangle\right)$

II: $\langle S_z\rangle = v_{\text{g}}\langle u\rangle;\ v_{\text{g}} = \frac{\partial\omega}{\partial k}$

III: $\langle W\rangle = \frac{\omega}{8\pi}\left(\mathrm{Im}(\varepsilon_\omega)\left|E_0^2\right| + \mathrm{Im}(\mu_\omega)\left|H_0^2\right|\right)$

$(\mu = 1)$ TE: $E_{\text{t}} = \frac{2E_{\text{i}}}{1+\frac{k_{t2}}{k_{i2}}};\ E_{\text{r}} = \frac{1-\frac{k_{t2}}{k_{i2}}}{1+\frac{k_{t2}}{k_{i2}}}E_{\text{i}}$