Trigonometry

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$
$$\tan(\alpha\pm\beta)=\tfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$
$$\sin(2\alpha)=2\sin\alpha\cos\alpha;\ \tan(2\alpha)=\tfrac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=$$
$$=2\cos^2\alpha-1=1-2\sin^2\alpha$$

$$\sin\alpha\pm\sin\beta=2\sin\tfrac{\alpha\pm\beta}{2}\cos\tfrac{\alpha\mp\beta}{2}$$
$$\cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$
$$2\sin\alpha\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$$
$$\sin\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1-\cos\alpha}{2}}\qquad\cos\tfrac{\alpha}{2}=\pm\sqrt{\tfrac{1+\cos\alpha}{2}}$$

$$\tan\tfrac{\alpha}{2}=\tfrac{\sin\alpha}{1+\cos\alpha}=\tfrac{1-\cos\alpha}{\sin\alpha}=\pm\sqrt{\tfrac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\sin^2\alpha=\tfrac{\tan^2\alpha}{1+\tan^2\alpha}\qquad\sin\alpha=\tfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\cos^2\alpha=\tfrac{1}{1+\tan^2\alpha}\qquad\cos\alpha=\tfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$a\sin x+b\cos x=$$
$$=\tfrac{|a|}{a}\sqrt{a^2+b^2}\sin\big(x+\operatorname{atan}\tfrac{b}{a}\big)$$
$$=\tfrac{|b|}{b}\sqrt{a^2+b^2}\cos\big(x-\operatorname{atan}\tfrac{a}{b}\big)$$

Hyperbolic functions

$$\cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$
$$\binom{\sinh x}{\cosh x}=\tfrac{1}{2}\binom{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\cosh^2x-\sinh^2x=1$$
$$\cosh^2x=\tfrac{1}{1-\tanh^2x}$$
$$\sin x=-i\sinh(ix)$$

$$\cos x=\cosh(ix)$$
$$\operatorname{atanh}x=\tfrac{1}{2}\log\tfrac{1+x}{1-x}$$

Areas

triangle: $\sqrt{p(p-a)(p-b)(p-c)}$

quad: $\sqrt{(p-a)(p-b)(p-c)(p-d)-abcd\cos^2\frac{\alpha+\gamma}{2}}$

Pick: $A=\big(I+\frac{B}{2}-1\big)\,A_{\text{check}}$

Combinatorics

$$P_n^{(m_1,m_2,\dots)}=\tfrac{n!}{m_1!m_2!\dots}\qquad C_{n,k}=\tbinom{n}{k}=\tfrac{n!}{k!(n-k)!}\qquad C'_{n,k}=\tbinom{n+k-1}{k}$$
$$D_{n,k}=\tfrac{n!}{(n-k)!}$$

Miscellaneous

$$A.B\overline{C}=\tfrac{ABC-AB}{9\times C\quad 0\times B}$$

$$\sqrt{a\pm\sqrt{b}}=\sqrt{\tfrac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\tfrac{a-\sqrt{a^2-b}}{2}}$$
$$\textstyle\sum_{i=0}^n a^i=\frac{1-a^{n+1}}{1-a}$$

$$\tbinom{n}{k}=\tbinom{n-1}{k-1}+\tbinom{n-1}{k}$$
$$(a+b)^n=\textstyle\sum_{k=0}^n\binom{n}{k}a^{n-k}b^k$$
$$e^{i\theta}=\cos\theta+i\sin\theta$$

$$\Gamma(1+z)=\textstyle\int_0^\infty t^ze^{-t}dt$$
$$\tilde f(k)=\tfrac{1}{\sqrt{2\pi}}\textstyle\int_{-\infty}^\infty e^{-ikx}f(x)$$

Derivatives

$$\tan'x=1+\tan^2x$$
$$\cot'x=-1-\cot^2x$$
$$\operatorname{atan}'x=-\operatorname{acot}'x=\tfrac{1}{1+x^2}$$

$$\operatorname{asin}'x=-\operatorname{acos}'x=\tfrac{1}{\sqrt{1-x^2}}$$
$$(a^x)'=a^x\ln a$$
$$\log_a'x=\tfrac{1}{x\ln a}$$

$$\cosh'x=\sinh x$$
$$\tanh'x=1-\tanh^2x$$
$$\operatorname{atanh}'x=\operatorname{acoth}'x=\tfrac{1}{1-x^2}$$

$$\operatorname{asinh}'x=\tfrac{1}{\sqrt{x^2+1}}$$
$$\operatorname{acosh}'x=\tfrac{1}{\sqrt{x^2-1}}$$
$$(f^{-1})'=\tfrac{1}{f'(f^{-1})}$$

$$\big(\tfrac{1}{x}\big)'=-\tfrac{\dot x}{x^2}$$
$$\big(\tfrac{x}{y}\big)'=\tfrac{\dot xy-x\dot y}{y^2}$$
$$(x^y)'=x^y\big(\dot y\ln x+y\tfrac{\dot x}{x}\big)$$

Integrals

$$\textstyle\int x^a=\frac{x^{a+1}}{a+1}$$
$$\textstyle\int a^x=\frac{a^x}{\ln a}$$

$$\textstyle\int\frac{1}{x}=\ln|x|$$
$$\textstyle\int\tan x=-\ln|\cos x|$$
$$\textstyle\int\cot x=\ln|\sin x|$$

$$\textstyle\int\frac{1}{\sin x}=\ln\big|\tan\frac{x}{2}\big|$$
$$\textstyle\int\frac{1}{\cos x}=\ln\big|\tan\big(\frac{x}{2}+\frac{\pi}{4}\big)\big|$$
$$\textstyle\int\ln x=x(\ln x-1)$$

$$\textstyle\int\tanh x=\ln\cosh x$$
$$\textstyle\int\coth x=\ln|\sinh x|$$
$$\textstyle\int\frac{1}{\sqrt{a^2-x^2}}=\operatorname{asin}\frac{x}{a}$$

$$\textstyle\int\frac{1}{a^2+x^2}=\frac{1}{a}\operatorname{atan}\frac{x}{a}$$
$$\textstyle\int xy=x\int y-\int(\dot x\int y)$$
$$\textstyle\int e^{yx}x=e^{yx}\big(\frac{y}{x}-\frac{1}{y^2}\big)$$

Differential equations

$$\dot x+\dot ax=b:x=e^{-a}\left(\textstyle\int be^a+c_1\right)$$

$$a\ddot x+b\dot x+cx=0:x=c_1e^{z_1t}+c_2e^{z_2t}$$
$$\ddot x=-\omega^2x:x=c_1\sin(\omega t)+c_2\cos(\omega t)$$

$$x\ddot x=k\dot x^2:x=c_2\sqrt[1-k]{(1-k)t+c_1}$$
$$\dot x+ax^2=b:x=\sqrt{\tfrac{b}{a}}\tanh\Big(\sqrt{ab}(c_1+t)\Big)$$

Taylor

$$\sin x=x-\tfrac{x^3}{3!}+\tfrac{x^5}{5!}-\tfrac{x^7}{7!}+\cdots$$
$$\cos x=1-\tfrac{x^2}{2}+\tfrac{x^4}{4!}-\tfrac{x^6}{6!}+\cdots$$
$$\tan x=x+\tfrac{x^3}{3}+\tfrac{2}{15}x^5+\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sin x}=\tfrac{1}{x}+\tfrac{x}{6}+\tfrac{7}{360}x^3+\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cos x}=1+\tfrac{x^2}{2}+\tfrac{5}{24}x^4+\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tan x}=\tfrac{1}{x}-\tfrac{x}{3}-\tfrac{x^3}{45}-\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$\operatorname{asin}x=x+\tfrac{x^3}{6}+\tfrac{3}{40}x^5+\tfrac{5}{112}x^7+\mathrm{O}(x^9)$$

$$\operatorname{acos}x=\tfrac{\pi}{2}-\operatorname{asin}x$$
$$\operatorname{atan}x=x-\tfrac{x^3}{3}+\tfrac{x^5}{5}-\tfrac{x^7}{7}+\cdots$$
$$\tfrac{1}{1-x}=1+x+x^2+x^3+\cdots$$
$$\tfrac{1}{1+x}=1-x+x^2-x^3+\cdots$$
$$e^x=1+x+\tfrac{x^2}{2}+\tfrac{x^3}{3!}+\cdots$$
$$\ln(1+x)=x-\tfrac{x^2}{2}+\tfrac{x^3}{3}-\tfrac{x^4}{4}+\cdots$$
$$\sinh x=x+\tfrac{x^3}{3!}+\tfrac{x^5}{5!}+\tfrac{x^7}{7!}+\cdots$$
$$\cosh x=1+\tfrac{x^2}{2}+\tfrac{x^4}{4!}+\tfrac{x^6}{6!}+\cdots$$

$$\tanh x=x-\tfrac{x^3}{3}+\tfrac{2}{15}x^5-\tfrac{17}{315}x^7+\mathrm{O}(x^9)$$
$$\tfrac{1}{\sinh x}=\tfrac{1}{x}-\tfrac{x}{6}+\tfrac{7}{360}x^3-\tfrac{31}{15120}x^5+\mathrm{O}(x^7)$$
$$\tfrac{1}{\cosh x}=1-\tfrac{x^2}{2}+\tfrac{5}{24}x^4-\tfrac{61}{720}x^6+\tfrac{277}{8064}x^8+\mathrm{O}(x^{10})$$
$$\tfrac{1}{\tanh x}=\tfrac{1}{x}+\tfrac{x}{3}-\tfrac{x^3}{45}+\tfrac{2}{945}x^5+\mathrm{O}(x^7)$$
$$(1+x)^n=1+nx+\tfrac{n(n-1)}{2}x^2+\mathrm{O}(x^3)$$
$$(1+x)^x=1+x^2-\tfrac{x^3}{2}+\tfrac{5}{6}x^4-\tfrac{3}{4}x^5+\mathrm{O}(x^6)$$
$$x!=1-\gamma x+\big(\tfrac{\gamma^2}{2}+\tfrac{\pi^2}{12}\big)x^2+\mathrm{O}(x^3)$$

Vectors

$$\varepsilon_{ijk}=\begin{cases}0 & i=j\vee j=k\vee k=i\\ 1 & i+1\equiv j\wedge j+1\equiv k\\ -1 & i\equiv j+1\wedge j\equiv k+1\end{cases}$$
$$\varepsilon_{ijk}\varepsilon_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$
$$\vec a\times\vec b=\varepsilon_{ijk}a_jb_k\hat e_i$$
$$(\vec a\times\vec b)\vec c=(\vec c\times\vec a)\vec b$$
$$(\vec a\times\vec b)\times\vec c=-(\vec b\vec c)\vec a+(\vec a\vec c)\vec b$$
$$(\vec a\times\vec b)(\vec c\times\vec d)=(\vec a\vec c)(\vec b\vec d)-(\vec a\vec d)(\vec b\vec c)$$
$$|\vec u\times\vec v|^2=u^2v^2-(\vec u\vec v)^2$$

$$\vec\nabla=\big(\tfrac{\partial}{\partial x},\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}\big);\ \Box=\tfrac{\partial^2}{\partial t^2}-\nabla^2$$
$$\vec\nabla V=\tfrac{\partial V}{\partial\rho}\hat\rho+\tfrac{1}{\rho}\tfrac{\partial V}{\partial\phi}\hat\phi+\tfrac{\partial V}{\partial z}\hat z$$
$$\vec\nabla\vec v=\tfrac{1}{\rho}\tfrac{\partial(\rho v_\rho)}{\partial\rho}+\tfrac{1}{\rho}\tfrac{\partial v_\phi}{\partial\phi}+\tfrac{\partial v_z}{\partial z}$$
$$\vec\nabla\times\vec v=\big(\tfrac{1}{\rho}\tfrac{\partial v_z}{\partial\phi}-\tfrac{\partial v_\phi}{\partial z}\big)\hat\rho+$$
$$+\big(\tfrac{\partial v_\rho}{\partial z}-\tfrac{\partial v_z}{\partial\rho}\big)\hat\phi+\tfrac{1}{\rho}\big(\tfrac{\partial(\rho v_\phi)}{\partial\rho}-\tfrac{\partial v_\rho}{\partial\phi}\big)$$
$$\nabla^2V=\tfrac{1}{\rho}\tfrac{\partial}{\partial\rho}\big(\rho\tfrac{\partial V}{\partial\rho}\big)+\tfrac{1}{\rho^2}\tfrac{\partial^2V}{\partial\phi^2}+\tfrac{\partial^2V}{\partial z^2}$$
$$\vec\nabla V=\tfrac{\partial V}{\partial r}\hat r+\tfrac{1}{r}\tfrac{\partial V}{\partial\theta}\hat\theta+\tfrac{1}{r\sin\theta}\tfrac{\partial V}{\partial\varphi}\hat\varphi$$
$$\vec\nabla\vec v=\tfrac{1}{r^2}\tfrac{\partial(r^2v_r)}{\partial r}+\tfrac{1}{r\sin\theta}\tfrac{\partial(v_\theta\sin\theta)}{\partial\theta}+\tfrac{1}{r\sin\theta}\tfrac{\partial v_\varphi}{\partial\varphi}$$

$$\vec\nabla\times\vec v=\tfrac{1}{r\sin\theta}\big(\tfrac{\partial(v_\varphi\sin\theta)}{\partial\theta}-\tfrac{\partial v_\theta}{\partial\varphi}\big)\hat r+$$
$$+\tfrac{1}{r}\big(\tfrac{1}{\sin\theta}\tfrac{\partial v_r}{\partial\varphi}-\tfrac{\partial(rv_\varphi)}{\partial r}\big)\hat\theta+\tfrac{1}{r}\big(\tfrac{\partial(rv_\theta)}{\partial r}-\tfrac{\partial v_r}{\partial\theta}\big)\hat\varphi$$
$$\nabla^2V=\tfrac{\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}{r^2}+\tfrac{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}{r^2\sin\theta}+\tfrac{\frac{\partial^2V}{\partial\varphi^2}}{r^2\sin^2\theta}$$
$$\vec\nabla(f\vec v)=(\vec\nabla f)\vec v+f\vec\nabla\vec v$$
$$\vec\nabla\times(f\vec v)=\vec\nabla f\times\vec v+f\vec\nabla\times\vec v$$
$$\vec\nabla\times(\vec\nabla\times\vec v)=-\nabla^2\vec v+\vec\nabla(\vec\nabla\vec v)$$
$$\vec\nabla(\vec v\times\vec w)=\vec w(\vec\nabla\times\vec v)-\vec v(\vec\nabla\times\vec w)$$
$$\vec\nabla\times(\vec v\times\vec w)=(\vec\nabla\vec w+\vec w\vec\nabla)\vec v-(\vec\nabla\vec v+\vec v\vec\nabla)\vec w$$

$$\frac{1}{2}\vec\nabla v^2 = (\vec v\vec\nabla)\vec v + \vec v\times(\vec\nabla\times\vec v)$$
$$\oint \vec v\times \mathrm{d}\vec S = -\int(\vec\nabla\times\vec v)\mathrm{d}^3x$$
$$\delta(g(x)) = \frac{\delta(x-x_i)}{|g'(x_i)|};\ g(x_i)=0$$
$$\int\vec\nabla\vec v\mathrm{d}^3x = \oint\vec v\mathrm{d}\vec S;\ \int(\vec\nabla\times\vec v)\mathrm{d}\vec S = \oint\vec v\mathrm{d}\vec l$$
$$\delta(\vec r-\vec r_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)}{r^2\sin\theta_0}$$
$$\langle\mathrm{Re}(ae^{-i\omega t})\,\mathrm{Re}(be^{-i\omega t})\rangle = \frac{1}{2}\mathrm{Re}(a\bar b)$$
$$\int(f\nabla^2g-g\nabla^2f)\,\mathrm{d}^3x = \oint_S\left(f\frac{\partial g}{\partial n}-g\frac{\partial f}{\partial n}\right)\mathrm{d}S$$
$$\nabla^2\frac{1}{|\vec r-\vec r_0|} = -4\pi\delta(\vec r-\vec r_0)$$

Statistics

$$P(E\cap E_1) = P(E_1)\cdot P(E|E_1)$$
$$\Delta x_{\mathrm{hist}} \approx \frac{x_{\max}-x_{\min}}{\sqrt N}$$
$$P(x\le k) = F(k) = \int_{-\infty}^k p(x)$$
$$\mathrm{median} = F^{-1}(\tfrac{1}{2})$$
$$E[f(x)] = \int_{-\infty}^{\infty} f(x)p(x)$$
$$\mu = E[x] = \int_{-\infty}^{\infty} xp(x)$$
$$\alpha_n = E[x^n]$$
$$M_n = E[(x-\mu)^n]$$
$$\sigma^2 = M_2 = E[x^2]-\mu^2$$
$$\mathrm{FWHM}\approx 2\sigma$$
$$\gamma_1 = \frac{M_3}{\sigma^3},\ \gamma_2 = \frac{M_4}{\sigma^4}$$

$$\phi[y](t) = E[e^{ity}]$$
$$\phi[y_1+\lambda y_2] = \phi[y_1]\phi[\lambda y_2]$$
$$\alpha_n = i^{-n}\frac{\partial^n t}{\partial\phi[x]^n}\Big|_{t=0}$$
$$h\ge 0: P(h\ge k)\le\frac{E[h]}{k}$$
$$P(|x-\mu|>k\sigma)\le\frac{1}{k^2}$$
$$B(n,p,k) = \binom{n}{k}p^k(1-p)^{n-k}$$
$$\mu_B = np,\ \sigma_B^2 = np(1-p)$$
$$P(\mu,k) = \frac{\mu^k}{k!}e^{-\mu},\ \sigma_P^2 = \mu$$
$$u(x,a,b) = \frac{1}{b-a},\ x\in[a;b]$$
$$\mu_u = \frac{b+a}{2},\ \sigma_u^2 = \frac{(b-a)^2}{12}$$
$$\varepsilon(x,\lambda) = \lambda e^{-\lambda x},\ x\ge 0$$

$$\mu_\varepsilon = \frac{1}{\lambda},\ \sigma_\varepsilon^2 = \frac{1}{\lambda^2}$$
$$g(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$
$$\mathrm{FWHM}_g = 2\sigma\sqrt{2\ln 2}$$
$$z = \frac{x-\mu}{\sigma};\ \mu,\sigma[z] = 0,1$$
$$\chi^2 = \sum_{i=1}^n z_i^2$$
$$\wp_n(x) = \frac{2^{-\frac{n}{2}}}{\Gamma(\frac{n}{2})}x^{\frac{n}{2}-1}e^{-\frac{x}{2}}$$
$$\mu_\wp = n,\ \sigma_\wp^2 = 2n$$
$$n\ge 30: \wp_n(x)\approx g(x,n,\sqrt{2n})$$
$$n\ge 8: p[\sqrt{2\chi^2}]\approx g(,\sqrt{2n-1},1)$$
$$S(x,n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$
$$\mu_S = 0,\ \sigma_S^2 = \frac{n}{n-2}$$

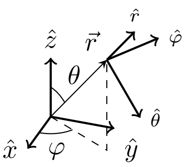
$$p\Big[z\sqrt{\frac{n}{\chi^2}}\Big] = S(,n)$$
$$n\ge 35: S(x,n)\approx g(x,0,1)$$
$$c(x,a) = \frac{a}{\pi}\frac{1}{a^2+x^2}$$
$$\sigma_{xy} = E[xy]-\mu_x\mu_y\le\sigma_x\sigma_y$$
$$\rho = \frac{\sigma_{xy}}{\sigma_x\sigma_y},\ |\rho|\le 1$$
$$\mu[f(x_1,\ldots)]\approx f(\mu_1,\ldots)$$
$$\sigma^2[f(x_1,\ldots)]\approx\sigma_{x_ix_j}\frac{\partial f}{\partial x_i}\Big|_{\mu_i}\frac{\partial f}{\partial x_j}\Big|_{\mu_j}$$
$$\mu\approx m = \frac{1}{n}\sum_{i=1}^n x_i$$
$$\sigma^2\approx s^2 = \frac{1}{n-1}\sum_{i=1}^n(x_i-m)^2$$
$$s_m^2 = \frac{s^2}{n}$$
$$p\Big[\frac{m-\mu}{s_m}\Big] = S(,n)$$

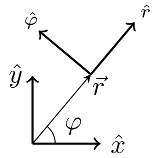
Fit

$$f(x) = mx+q,\quad f(x) = a$$
$$f(x) = bx$$
$$m = \frac{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$q = \frac{\sum\frac{y}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-\sum\frac{x}{\Delta y^2}\cdot\sum\frac{xy}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$a = \frac{\sum\frac{y}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}},\ \Delta a^2 = \frac{1}{\sum\frac{1}{\Delta y^2}}$$
$$\Delta m^2 = \frac{\sum\frac{1}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$\Delta q^2 = \frac{\sum\frac{x^2}{\Delta y^2}}{\sum\frac{1}{\Delta y^2}\cdot\sum\frac{x^2}{\Delta y^2}-(\sum\frac{x}{\Delta y^2})^2}$$
$$b = \frac{\sum xy}{\sum x^2},\ \Delta b^2 = \frac{\Delta y^2}{\sum x^2}$$

Kinematics

$$\frac{1}{R} = \Big|\frac{v_xa_y-v_ya_x}{v^3}\Big|$$
$$\vec\omega = \dot\varphi\cos\theta\hat r-\dot\varphi\sin\theta\hat\theta+\dot\theta\hat\varphi$$
$$\dot{\vec w} = \frac{\mathrm{d}(\vec w\hat r)}{\mathrm{d}t}\hat r+\frac{\mathrm{d}(\vec w\hat\theta)}{\mathrm{d}t}\hat\theta+\frac{\mathrm{d}(\vec w\hat\varphi)}{\mathrm{d}t}\hat\varphi+\vec\omega\times\vec w$$
$$\theta\equiv\frac{\pi}{2}\to\dot{\vec r} = \dot r\hat r+r\dot\varphi\hat\varphi$$
$$\theta\equiv\frac{\pi}{2}\to\ddot{\vec r} = (\ddot r-r\dot\varphi^2)\hat r+(r\ddot\varphi+2\dot r\dot\varphi)\hat\varphi$$
$$\dot{\vec r} = \dot r\hat r+r\dot\theta\hat\theta+r\dot\varphi\sin\theta\hat\varphi$$
$$\langle\ddot{\vec r},\hat r\rangle = \ddot r-r\dot\theta^2-r\dot\varphi^2\sin^2\theta$$
$$\langle\ddot{\vec r},\hat\theta\rangle = r\ddot\theta+2\dot r\dot\theta-r\dot\varphi^2\sin\theta\cos\theta$$
$$\langle\ddot{\vec r},\hat\varphi\rangle = r\ddot\varphi\sin\theta+2\dot r\dot\varphi\sin\theta+2r\dot\theta\dot\varphi\cos\theta$$
$$\vec A = \ddot{\vec r}+\vec A_{\mathrm{T}}+\vec\omega\times(\vec\omega\times\vec r)+\dot{\vec\omega}\times\vec r+2\vec\omega\times\dot{\vec r}$$





Mechanics

$$\dot\alpha = \frac{\mathrm{d}}{\mathrm{d}t}\alpha(\beta,t) = \frac{\partial\alpha}{\partial\beta}\dot\beta+\frac{\partial\alpha}{\partial t}$$
$$\vec p := m\dot{\vec r};\ \vec F = \dot{\vec p};\ \frac{\mathrm{d}(mT)}{\mathrm{d}t} = \vec F\vec p$$
$$M := \sum_i m_i;\ \vec R := \frac{m_i\vec r_i}{M}$$
$$T = \frac{1}{2}M\dot{\vec R}^2+\frac{1}{2}m_i(\dot{\vec r}_i-\dot{\vec R})^2$$
$$\vec L = \vec R\times M\dot{\vec R}+(\vec r_i-\vec R)\times m_i(\dot{\vec r}_i-\dot{\vec R})$$
$$\vec\tau_O = \dot{\vec L}_O+\vec v_O\times\vec p$$
$$\tau_1 = I_1\dot\omega_1+(I_3-I_2)\omega_3\omega_2$$
$$\mathcal L(q,\dot q,t) = T-V+\frac{\mathrm{d}}{\mathrm{d}t}f(q,t)$$
$$S[q] = \int_{t_1}^{t_2}\mathcal L(q,\dot q,t)\,\mathrm{d}t$$
$$\frac{\partial}{\partial\epsilon}S[q+\epsilon]\Big|_{\epsilon(t_1)=\epsilon(t_2)=0} = 0$$
$$p := \frac{\partial\mathcal L}{\partial\dot q};\ \dot p = \frac{\partial\mathcal L}{\partial q}$$
$$\mathcal H(q,p,t) = \dot qp-\mathcal L$$
$$\dot q = \frac{\partial\mathcal H}{\partial p};\ \dot p = -\frac{\partial\mathcal H}{\partial q}$$
$$\frac{\mathrm{d}\mathcal H}{\mathrm{d}t} = \frac{\partial\mathcal H}{\partial t} = -\frac{\partial\mathcal L}{\partial t}$$
$$\{u,v\} = \frac{\partial u}{\partial q}\frac{\partial v}{\partial p}-\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}$$
$$\frac{\mathrm{d}u}{\mathrm{d}t} = \{u,\mathcal H\}+\frac{\partial u}{\partial t}$$
$$\eta = (q,p);\ \Gamma = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$
$$\dot\eta = \Gamma\frac{\partial\mathcal H}{\partial\eta};\ \{u,v\} = \frac{\partial u}{\partial\eta}\Gamma\frac{\partial v}{\partial\eta}$$

Inertia

point: mr^2
two points: μd^2
rod: $\frac{1}{12}mL^2$
disk: $\frac{1}{2}mr^2$
tetrahedron: $\frac{1}{20}ms^2$
octahedron: $\frac{1}{10}ms^2$
sphere: $\frac{2}{3}mr^2$
ball: $\frac{2}{5}mr^2$
cone: $\frac{3}{10}mr^2$
torus: $m\big(R^2+\frac{3}{4}r^2\big)$
ellipsoid: $I_a = \frac{1}{5}m(b^2+c^2)$
rectangulus: $\frac{1}{12}m(a^2+b^2)$

Kepler

$$\langle U\rangle\approx -2\langle T\rangle$$
$$U_{\mathrm{eff}} = U+\frac{L^2}{2mr^2}$$
$$\frac{1}{\mu} = \frac{1}{m_1}+\frac{1}{m_2}$$
$$\vec r = \vec r_1-\vec r_2,\ \alpha = Gm_1m_2$$
$$T = \frac{1}{2}M\dot{\vec R}^2+\frac{1}{2}\mu\dot{\vec r}^2$$
$$\vec L = \vec R\times M\dot{\vec R}+\vec r\times\mu\dot{\vec r}$$
$$k = \frac{L^2}{\mu\alpha},\ \varepsilon = \sqrt{1+\frac{2EL^2}{\mu\alpha^2}}$$
$$r = \frac{k}{1+\varepsilon\cos\theta}$$
$$a = \frac{k}{|1-\varepsilon^2|} = \frac{\alpha}{2|E|}$$
$$a^3\omega^2 = G(m_1+m_2)$$
$$\vec A = \mu\dot{\vec r}\times\vec L-\mu\alpha\hat r,\ \dot{\vec A} = 0$$

Inequalities

$$|a|-|b|\le|a+b|\le|a|+|b|$$
$$x>-1: 1+nx\le(1+x)^n$$
$$\frac{|a^n-b^n|}{|a-b|<1}\le n(1+|b|)^{n-1}$$
$$\sqrt[p]{\sum(a_i+b_i)^p}\le\sqrt[p]{\sum a_i^p}+\sqrt[p]{\sum b_i^p}$$
$$\sum a_ib_i\le\Big(\sum a_i^p\Big)^{\frac{1}{p}}\Big(\sum b_i^{\frac{p}{p-1}}\Big)^{\frac{p-1}{p}}$$
$$x^py^q\le\Big(\frac{px+qy}{p+q}\Big)^{p+q}$$
$$\sqrt[p]{\frac{1}{n}\sum a_i^{p\le q}}\le\sqrt[q]{\frac{1}{n}\sum a_i^q}$$
$$\sum\Big(\frac{a_1+\ldots a_i}{i}\Big)^p\le\Big(\frac{p}{p-1}\Big)^p\sum a_i^p$$
$$x\ge 0, |\ddot x|\le M: |\dot x|\le\sqrt{2Mx}$$
$$\frac{1}{1+x}<\ln\Big(1+\frac{1}{x}\Big)<\frac{1}{x}$$

Vector spaces

$(V,\mathbb K,+,\cdot)$ vector space; $\quad\mathbb K$ field
$$\exists\vec 0\in V: \vec v+\vec 0 = \vec v$$
$$\cdot:\mathbb K\times V\to V;\quad\lambda\cdot(\vec v+\vec w) = \lambda\vec v+\lambda\vec w$$
$$0_{\mathbb K}\cdot\vec v = \vec 0,\ 1_{\mathbb K}\cdot\vec v = \vec v$$
$$\lambda\in\mathbb K,\ \vec v,\vec w\in V\ \Rightarrow\ \vec v+\vec w\in V,\ \lambda\vec v\in V$$
$$\dim(U+V) = \dim U+\dim V-\dim(U\cap V)$$
$$\ell\ \text{linear}: \ell(\vec v+\lambda\vec w) = \ell(\vec v)+\lambda\ell(\vec w)$$
$$\ker\ell = \{\vec v\in V\,|\,\ell(\vec v) = 0\}$$
$$\dim V = \dim\ell(V)+\dim(V\cap\ker\ell)$$
$$\langle,\rangle: V\times V\to\mathbb K;\quad\langle\vec v,\vec w\rangle = \langle\vec w,\vec v\rangle$$
$$\langle\vec v+\lambda\vec w,\vec u\rangle = \langle\vec v,\vec u\rangle+\lambda\langle\vec w,\vec u\rangle$$
$$\|\|: V\to\mathbb K;\quad\|\vec v\| = 0\to\vec v = \vec 0$$
$$\|\lambda\vec v\| = |\lambda|\|\vec v\|;\quad\|\vec v+\vec w\|\le\|\vec v\|+\|\vec w\|$$

Symbols

$A\ B\ \Gamma\ \Delta\ E\ Z\ H\ \Theta\ I\ K\ \Lambda\ M\ N\ \Xi\ O\ \Pi\ P\ \Sigma\ T\ Y\ \Phi\ X\ \Psi\ \Omega$

$\alpha\ \beta\ \gamma\ \delta\ \epsilon/\varepsilon\ \zeta\ \eta\ \theta/\vartheta\ \iota\ \kappa\ \lambda\ \mu\ \nu\ \xi\ o\ \pi/\varpi\ \rho/\varrho\ \sigma/\varsigma\ \tau\ \upsilon\ \phi/\varphi\ \chi\ \psi\ \omega$

Constants

$\pi = 3.142$ $\quad e = 2.718$ $\quad \gamma = 5.772 \cdot 10^{-1}$

$G = 6.674 \cdot 10^{-11}\,\frac{\mathrm{m^3}}{\mathrm{kg\,s^2}}$ $\quad R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$ $\quad R = 8.206 \cdot 10^{-2}\,\frac{\mathrm{l\,atm}}{\mathrm{mol\,K}}$ $\quad N_\mathrm{A} = 6.022 \cdot 10^{23}\,\frac{1}{\mathrm{mol}}$

$k = 1.381 \cdot 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$ $\quad c = 2.998 \cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$ $\quad q_\mathrm{e} = 1.602 \cdot 10^{-19}\,\mathrm{A\,s}$ $\quad m_\mathrm{e} = 9.109 \cdot 10^{-31}\,\mathrm{kg}$

$m_\mathrm{p} = 1.673 \cdot 10^{-27}\,\mathrm{kg}$ $\quad m_\mathrm{n} = 1.675 \cdot 10^{-27}\,\mathrm{kg}$ $\quad \mathrm{amu} = 1.661 \cdot 10^{-27}\,\mathrm{kg}$ $\quad h = 6.626 \cdot 10^{-34}\,\mathrm{J\,s}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\,\frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}$ $\quad \mu_0 = 1.257 \cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A^2}}$ $\quad \mu_\mathrm{B} = 9.274 \cdot 10^{-24}\,\mathrm{A\,m^2}$ $\quad \alpha = 7.297 \cdot 10^{-3}$

Chemistry

$H = U + pV$

$\mathrm{d}p = 0 \to \Delta H = \text{heat transfer}$

$G = H - TS$

$a_i \mathrm{A}_i \;\to\; b_j \mathrm{B}_j$

$\Delta H^\mathrm{o}_\mathrm{r} = b_j \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{B}_j) - a_i \Delta H^\mathrm{o}_\mathrm{f}(\mathrm{A}_i)$

$\forall\, i,j : v_\mathrm{r} = -\frac{1}{a_i}\frac{\Delta[\mathrm{A}_i]}{\Delta t} = \frac{1}{b_j}\frac{\Delta[\mathrm{B}_j]}{\Delta t}$

$\exists\, k, (m_i) : v_\mathrm{r} = k[\mathrm{A}_i]^{m_i}$

$k = A e^{-\frac{E_\mathrm{a}}{RT}}$ (Arrhenius)

$a_{(\ell)} = \gamma \frac{[\mathrm{X}]}{[\mathrm{X}]_0},\ [\mathrm{X}]_0 = 1\,\frac{\mathrm{mol}}{\mathrm{l}}$

$a_{(g)} = \gamma \frac{p}{p_0},\ p_0 = 1\,\mathrm{atm}$

$K = \frac{\prod a_{\mathrm{B}_j}^{b_j}}{\prod a_{\mathrm{A}_i}^{a_i}},\ K_c = \frac{\prod [\mathrm{B}_j]^{b_j}}{\prod [\mathrm{A}_i]^{a_i}}$

$K_p = \frac{\prod p_{\mathrm{B}_j}^{b_j}}{\prod p_{\mathrm{A}_i}^{a_i}},\ K_n = \frac{\prod n_{\mathrm{B}_j}^{b_j}}{\prod n_{\mathrm{A}_i}^{a_i}}$

$K_\chi = \frac{\prod \chi_{\mathrm{B}_j}^{b_j}}{\prod \chi_{\mathrm{A}_i}^{a_i}},\ \chi = \frac{n}{n_\mathrm{tot}}$

$K_c = K_p (RT)^{\sum a_i - \sum b_j}$

$K_c = K_n V^{\sum a_i - \sum b_j}$

$K_n = K_\chi n_\mathrm{tot}^{\sum b_j - \sum a_i}$

$\Delta G^\mathrm{o}_\mathrm{r} = -RT \ln K$

$Q = K(t) = \frac{\prod a_{\mathrm{B}_j}^{b_j}(t)}{\prod a_{\mathrm{A}_i}^{a_i}(t)}$

$\Delta G = RT \ln \frac{Q}{K}$

$\ln \frac{K_2}{K_1} = -\frac{\Delta H^\mathrm{o}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

$K_\mathrm{w} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = 10^{-14}$

$\Delta E = \Delta E^\mathrm{o} - \frac{RT}{n_\mathrm{e} N_A q_\mathrm{e}} \ln Q$ (Nerst)

(std) $\Delta E = \Delta E^\mathrm{o} - \frac{0.059}{n_\mathrm{e}} \log_{10} Q$

$\mathrm{pH} = -\log_{10}[\mathrm{H_3O^+}]$

$K_a = \frac{[\mathrm{A^-}][\mathrm{H_3O^+}]}{[\mathrm{AH}]}$

Thermodynamics $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}L$ $\quad \mathrm{d}S = \frac{\mathrm{d}Q}{T}$ $\quad C_V = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_V$ $\quad C_p = \left(\frac{\mathrm{d}Q}{\mathrm{d}T}\right)_p$ $\quad \gamma = \frac{C_p}{C_V}$

$\mathrm{d}L = p\mathrm{d}V$

Ideal gas

$pV = nRT$

$c_V, c_p = \frac{C_V, C_p}{n},\ c_V = \frac{\mathrm{dof}}{2}R,\ c_p = c_V + R$ $\quad \mathrm{d}Q = 0 : pV^\gamma, TV^{\gamma-1}, p^{\frac{1}{\gamma}-1}T$ const.

$c_V = \frac{R}{\gamma-1},\ c_p = \frac{\gamma}{\gamma-1}R$

Electronics

$\binom{V}{I} = \binom{V_0}{I_0} e^{i\omega t}$

$Z = \frac{V}{I}$ $\quad Z_R = R$ $\quad Z_C = -i\frac{1}{\omega C}$ $\quad Z_L = i\omega L$ $\quad Z_\mathrm{series} = \sum_k Z_k$ $\quad \frac{1}{Z_\mathrm{parallel}} = \sum_k \frac{1}{Z_k}$

$\sum_\mathrm{loop} V_k = 0$ $\quad \sum_\mathrm{node} I_k = 0$

Relativity

$\beta = \frac{v}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\quad \vec{p} = \gamma m \vec{v}$

$\mathcal{E} = \gamma m c^2$ $\quad \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F}$ $\quad \binom{ct'}{x'} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \binom{ct}{x}$

$\mathrm{d}\tau = \frac{1}{\gamma}\mathrm{d}t$ $\quad x^\mu = (ct, \vec{x})$ $\quad v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(c, \vec{v})$ $\quad a^\mu = \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}c, \frac{\mathrm{d}(\gamma\vec{v})}{\mathrm{d}t}\right)$

$p^\mu = m v^\mu = \left(\frac{\mathcal{E}}{c}, \vec{p}\right)$ $\quad \partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$ $\quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$x_\mu = g_{\mu\nu} x^\nu$ $\quad \partial_\mu \partial^\mu = \Box$ $\quad p^\mu p_\mu = (mc)^2$

Electrostatics (CGS)

$\vec{F}_{12} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3};\ \vec{E}_1 = \frac{\vec{F}_{12}}{q_2};\ V(\vec{r}) = \int \mathrm{d}^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|};\ \rho_q = \delta(\vec{r} - \vec{r}_q)$

$\oint \vec{E}\mathrm{d}\vec{S} = 4\pi \int \rho\, \mathrm{d}^3 x;\ -\nabla^2 V = \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla} \times \vec{E} = 0$

$U = \frac{1}{8\pi}\int E^2\, \mathrm{d}^3 x;\ \tilde{U} = \frac{1}{2}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi}\sum_{ij}\int \vec{E}_i \vec{E}_j\, \mathrm{d}^3 x$

$V(\vec{r}) = \int \rho G_\mathrm{D}(\vec{r})\, \mathrm{d}^3 x - \frac{1}{4\pi}\oint_S V \frac{\partial G_\mathrm{D}}{\partial n}\, \mathrm{d}S$

$V(\vec{r}) = \langle V \rangle_S + \int \rho G_\mathrm{N}(\vec{r})\, \mathrm{d}^3 x + \frac{1}{4\pi}\oint_S \frac{\partial V}{\partial n} G_\mathrm{N}(\vec{r})\, \mathrm{d}S$

$\nabla_y^2 G(\vec{x}, \vec{y}) = -4\pi\delta(\vec{x} - \vec{y});\ G_\mathrm{D}(\vec{x}, \vec{y})|_{\vec{y}\in S} = 0;\ \frac{\partial G_\mathrm{N}}{\partial n}\Big|_{\vec{y}\in S} = -\frac{4\pi}{S}$

$U_\mathrm{sphere} = \frac{3}{5}\frac{Q^2}{R};\ \vec{E}_\mathrm{dip} = \frac{3(\vec{p}\hat{r})\hat{r} - \vec{p}}{r^3};\ V_\mathrm{dip} = \frac{\vec{p}\hat{r}}{r^2}$

$V(r, \theta) = \sum_{l=0}^\infty \left(A_l r^l - \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$

$V(r, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}}\right) Y_{lm}(\theta, \varphi)$

$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^\infty \frac{\min(r, r')^l}{\max(r, r')^{l+1}} P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right)$

$P_l(x) = \frac{1}{2^l l!}\frac{\mathrm{d}^l}{\mathrm{d}x^l}\left(x^2 - 1\right)^l;\ f = \sum_{l=0}^\infty c_l P_l : c_l = \frac{2l+1}{2}\int_{-1}^1 f P_l$

$P_l(1) = 1;\ \langle P_n | P_m \rangle = \frac{2\delta_{nm}}{2n+1};\ \langle Y_{lm} | Y_{l'm'} \rangle = \delta_{ll'}\delta_{mm'}$

$P_0 = 1;\ P_1 = x;\ P_2 = \frac{3x^2 - 1}{2};\ Y_{00} = \frac{1}{\sqrt{4\pi}};\ Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi};\ Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

$Y_{21} = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\varphi};\ Y_{22} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{2i\varphi}$

$P_{lm}(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{\frac{m}{2}}\frac{\mathrm{d}^{l+m}}{\mathrm{d}x^{l+m}}\left(x^2 - 1\right)^l,\ |m| \le l$

$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_{lm}(\cos\theta);\ Y_{l,-m} = (-1)^m \overline{Y}_{lm}$

$P_l\left(\frac{\vec{r}\vec{r}'}{rr'}\right) = \frac{4\pi}{2l+1}\sum_{m=-l}^l \overline{Y}_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi)$

$V(r > \mathrm{diam\,supp}\,\rho, \theta, \varphi) = \sum_{l=0}^\infty \sum_{m=-l}^l \frac{4\pi}{2l+1} q_{lm}[\rho] \frac{Y_{lm}(\theta, \varphi)}{r^{l+1}}$

$q_{lm}[\rho] = \int_0^\infty r^2 \mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \int_0^\pi \sin\theta\, \mathrm{d}\theta\, r^l \rho(r, \theta, \varphi) \overline{Y}_{lm}(\theta, \varphi)$

$\chi = \frac{4\pi}{3}\frac{n p_0^2}{kT};\ \vec{E}_e = \vec{E} + \frac{4\pi}{3}\vec{P};\ \vec{D} = \varepsilon\vec{E};\ \vec{\nabla}\vec{D} = 4\pi\rho$

Magnetostatics (CGS)

$\vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t} = 0;\ I = \int \vec{J}\mathrm{d}\vec{S}$

solenoid: $B = 4\pi\frac{j_s}{c}$

$\mathrm{d}\vec{F} = \frac{I\mathrm{d}\vec{l}}{c} \times \vec{B} = \mathrm{d}^3 x \frac{\vec{J}}{c} \times \vec{B};\ \vec{F}_q = q\frac{\dot{\vec{r}}}{c} \times \vec{B}$

$\mathrm{d}\vec{B} = \frac{I\mathrm{d}\vec{l}}{c} \times \frac{\vec{r}}{r^3};\ \vec{B}_q = q\frac{\dot{\vec{r}}}{c} \times \frac{\vec{r}}{r^3}$

$\vec{B} = \vec{\nabla} \times \vec{A};\ \vec{A} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c}\frac{1}{|\vec{r} - \vec{r}'|} + \vec{\nabla}A_0$

$\vec{B} = \int \mathrm{d}^3 r' \frac{\vec{J}'}{c} \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$

$\vec{\nabla}\vec{A} = 0 \to \nabla^2\vec{A} = -4\pi\frac{\vec{J}}{c}$

$\vec{\nabla}\vec{B} = 0;\ \vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c};\ \oint \vec{B}\mathrm{d}\vec{l} = 4\pi\frac{I}{c}$

$\vec{m} = \frac{1}{2}\int \mathrm{d}^3 r' \left(\vec{r}' \times \frac{\vec{J}'}{c}\right) = \frac{1}{2c}\frac{q}{m}\vec{L}$

$\vec{A} \approx \frac{\vec{m} \times \vec{r}}{r^3};\ \vec{\tau} = \vec{m} \times \vec{B}$

$\vec{H} = \frac{\vec{B}}{\mu} = \vec{B} - 4\pi\vec{M};\ \vec{\nabla} \times \vec{H} = 0$

Electromagnetism (CGS)

Faraday: $\mathcal{E} = -\frac{1}{c}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$

$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t};\ \vec{\nabla}\vec{E} = 4\pi\rho;\ \vec{\nabla}\vec{J} = -\frac{\partial\rho}{\partial t}$

$\vec{\nabla} \times \vec{B} = 4\pi\frac{\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t};\ \vec{\nabla}\vec{B} = 0$

$\mathrm{d}\vec{F} = \mathrm{d}^3 x\left(\rho\vec{E} + \frac{\vec{J}}{c} \times \vec{B}\right);\ \vec{F}_q = q\left(\vec{E} + \frac{\dot{\vec{r}}}{c} \times \vec{B}\right)$

$u = \frac{E^2 + B^2}{8\pi};\ \vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B};\ \vec{g} = \frac{\vec{S}}{c^2}$

$$T_{ij}^E = \frac{1}{4\pi}\big(E_iE_j - \frac{1}{2}\delta_{ij}E^2\big);\ \mathbf{T} = \mathbf{T}^E + \mathbf{T}^B$$
$$-\frac{\partial u}{\partial t} = \vec{J}\vec{E} + \vec{\nabla}\vec{S};\ \frac{\partial \vec{g}}{\partial t} = -\vec{f} + \partial_j T_{ij}\hat{x}_i$$
$$\vec{B} = \vec{\nabla}\times\vec{A};\ \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$
$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\vec{A} = 4\pi\rho$$
$$\vec{\nabla}\big(\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\big) - \nabla^2\vec{A} + \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2} = 4\pi\frac{\vec{J}}{c}$$
$$(\phi,\vec{A}) \cong \big(\phi - \frac{1}{c}\frac{\partial\chi}{\partial t}, \vec{A} + \vec{\nabla}\chi\big)$$
$$(\phi,\vec{A}) = \int \mathrm{d}^3r'\,\frac{\big(\rho,\frac{\vec{J}}{c}\big)\big(\vec{r}', t - \frac{1}{c}\big|\vec{r}-\vec{r}'\big|\big)}{|\vec{r}-\vec{r}'|}$$
Coulomb gauge: $\vec{\nabla}\vec{A} = 0$

Lorenz gauge: $\vec{\nabla}\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$
$$\vec{E}'\hat{v} = \vec{E}\hat{v};\ \vec{B}'\hat{v} = \vec{B}\hat{v}$$
$$\vec{E}'\times\hat{v} = \gamma\big(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\big)\times\hat{v}$$
$$\vec{B}'\times\hat{v} = \gamma\big(\vec{B} - \frac{\vec{v}}{c}\times\vec{E}\big)\times\hat{v}$$
$$\text{plane wave: } \begin{cases} \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)} \\ \vec{B} = \hat{k}\times\vec{E} \\ \omega = ck \end{cases}$$
dipole: $\vec{B}\big|_{r\gg\frac{c}{\omega}} \approx \frac{1}{c^2}\frac{\ddot{\vec{p}}\times\hat{r}}{r};\ \vec{E}\approx\vec{B}\times\hat{r}$
$$A^\mu = (\phi,\vec{A});\ J^\mu = (c\rho,\vec{J})$$

Lorenz gauge: $\partial_\mu A^\mu = 0$
$$\partial_\mu F^{\mu\nu} = 4\pi\frac{J^\nu}{c};\ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$
$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$
$$\mathscr{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$
$$F^{\mu\nu}F_{\mu\nu} = E^2 - B^2;\ F^{\mu\nu}\mathscr{F}_{\mu\nu} = 4\vec{E}\vec{B}$$
$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\big(g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} - \frac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}\big)$$
$$\Theta^{\alpha\beta} = \begin{pmatrix} u & c\vec{g} \\ c\vec{g} & -\mathbf{T} \end{pmatrix}$$
$$\partial_\mu\Theta^{\mu\nu} = -\frac{1}{c}F^{\nu\lambda}J_\lambda = \frac{1}{c}J_\lambda F^{\lambda\nu}$$