Part II
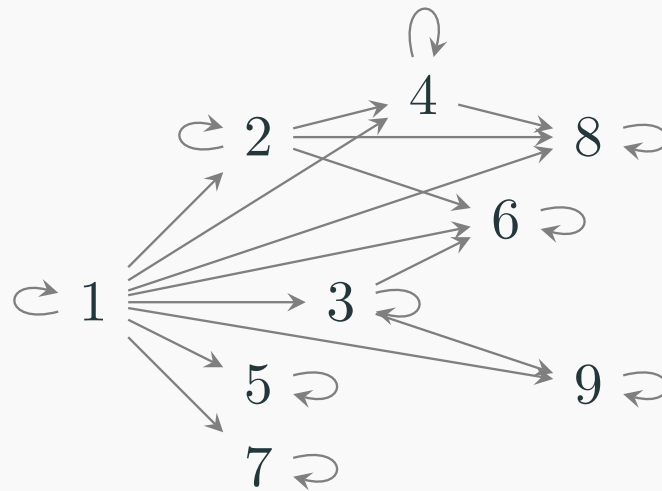
PRIME NUMBERS

Hasse diagram for divisibility

We want to illustrate the divisibility relation between positive integers. The first attempt is to list all the positive integers and whenever $a \mid b$ draw an arrow from a to b. But the result diagram is cluttered and confusing

Divisibility of integers from 1 to 9

To simplify the diagram, we introduce the following omission:

- We may choose a global direction (for example, from left to right) to assemble the integers and omit the heads of arrows.

To simplify the diagram, we introduce the following omission:

- We may choose a global direction (for example, from left to right) to assemble the integers and omit the heads of arrows.
- We may omit the self-loops corresponding to the *reflexivity*: $a \mid a$ ($a \in \mathbb{Z}_+$).

To simplify the diagram, we introduce the following omission:

- We may choose a global direction (for example, from left to right) to assemble the integers and omit the heads of arrows.

- We may omit the self-loops corresponding to the *reflexivity*: $a \mid a$ ($a \in \mathbb{Z}_+$).

- By the *transitivity*, we may only draw the arrows connecting *adjacent* nodes. Here, saying a and b are adjacent means $a \mid b$ and there is no other positive integer c between them in the sense $a \mid c$ and $c \mid b$.

To simplify the diagram, we introduce the following omission:

- We may choose a global direction (for example, from left to right) to assemble the integers and omit the heads of arrows.

- We may omit the self-loops corresponding to the *reflexivity*: $a \mid a$ $(a \in \mathbb{Z}_+)$.

- By the *transitivity*, we may only draw the arrows connecting *adjacent* nodes. Here, saying a and b are adjacent means $a \mid b$ and there is no other positive integer c between them in the sense $a \mid c$ and $c \mid b$.

- By the *antisymmetry*, after above simplifications, the diagram contains no loops and crossings.
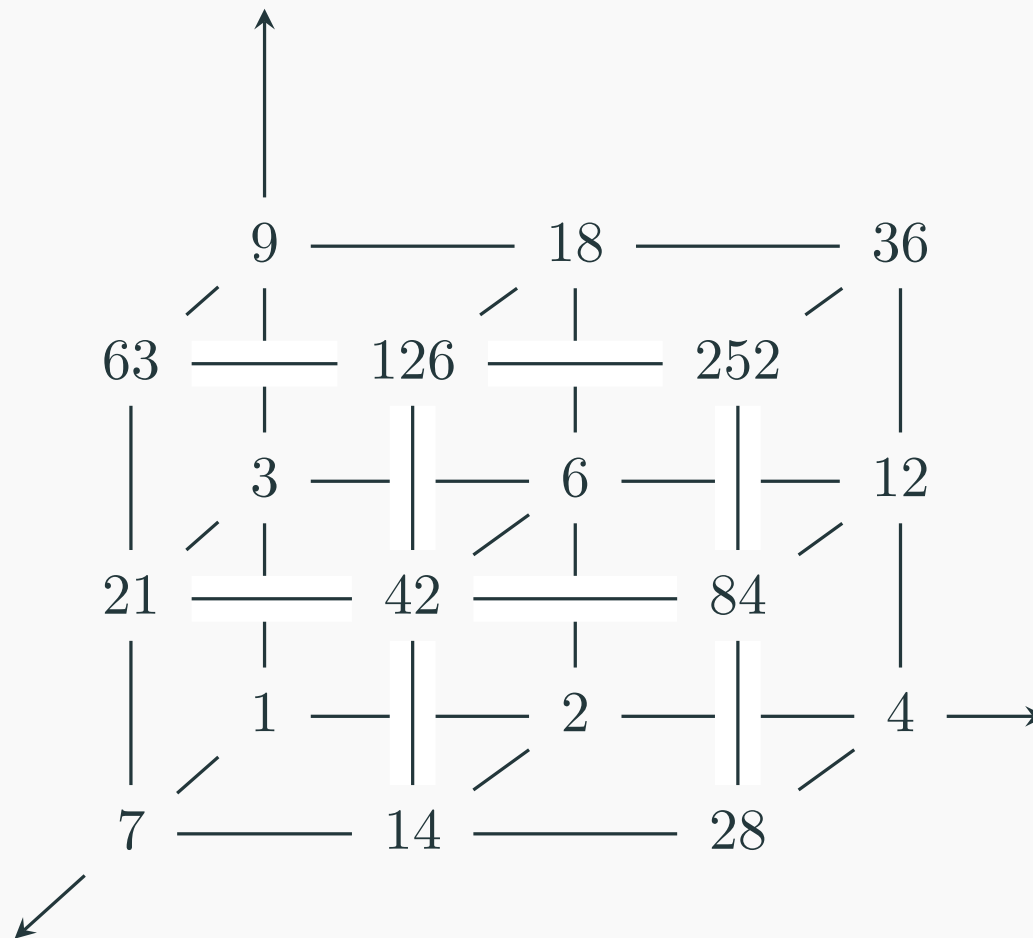
The diagram obtained through previous simplification is called a
Hasse diagram (of divisibility of positive integers).
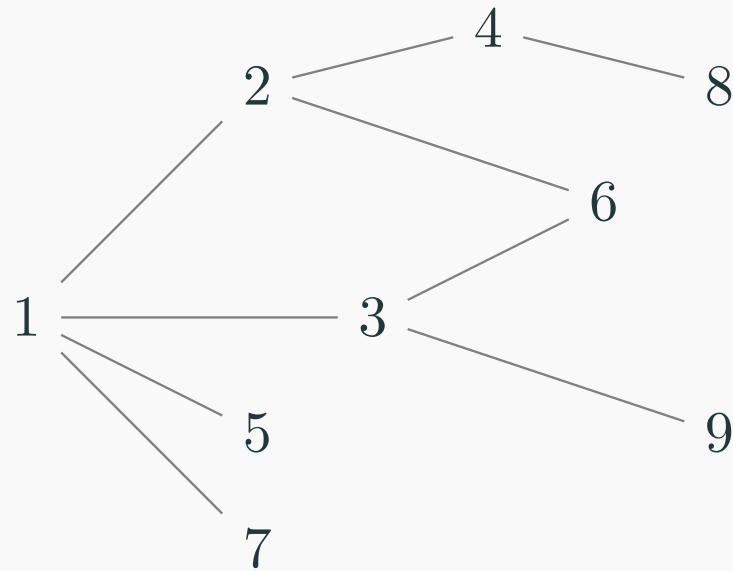
Hasse diagram of integers from 1 to 9 (from left to right)

Hasse diagram of 1 and multiples of 2, 3, 7 (from inner to outer)