

Arithmetic of Polynomials

1 Define and display polynomials

- Define polynomials by claiming their coefficients:

```
\PolySet{f}{1,0,4,0,2}
\PolySet{g}{1,6,3}
```

- Display Polynomials

```
\[\PolyPrint{f}\qquad\PolyPrint{g}\]
```

$$T^4 + \overline{4}T^2 + \overline{2} \quad T^2 + \overline{6}T + \overline{3}$$

- Change the variable (globally)

```
\PolyOptions{variable={x}}
\[\PolyPrint{f}\qquad\PolyPrint{g}\]
```

$$x^4 + \overline{4}x^2 + \overline{2} \quad x^2 + \overline{6}x + \overline{3}$$

Or put the option into argument (so change the variable locally),

```
\[\PolyPrint[variable={x}]{f}\qquad\PolyPrint[variable={x}]{g}\]
```

$$x^4 + \overline{4}x^2 + \overline{2} \quad x^2 + \overline{6}x + \overline{3}$$

- One can also change the modulus:

```
\[\PolyPrint[base=5]{f}\qquad\PolyPrint[base=5]{g}\]
\[\PolyPrint[base=4]{f}\qquad\PolyPrint[base=3]{g}\]
```

$$\begin{array}{cc} T^4 + \overline{4}T^2 + \overline{2} & T^2 + T + \overline{3} \\ T^4 + \overline{2} & T^2 \end{array}$$

- Get rid of the bar:

```
\[\PolyPrint[coef=]{f}\qquad\PolyPrint[coef=]{g}\]
```

$$T^4 + 4T^2 + 2 \quad T^2 + 6T + 3$$

2 Arithmetic of polynomials

We keep use polynomials f and g from previous section (with the modulus 7).

- Addition:

$\backslash[\backslash\text{PolyPrint}\{f\}+\backslash\text{PolyPrint}\{g\}=\backslash\text{PolyAdd}\{f\}\{g\}\backslash]$

$$T^4 + 4T^2 + 2 + T^2 + 6T + 3 = T^4 + 5T^2 + 6T + 5$$

- Multiplication

$\backslash[(\backslash\text{PolyPrint}\{f\})(\backslash\text{PolyPrint}\{g\})=\backslash\text{PolyMult}\{f\}\{g\}\backslash]$

$$(T^4 + 4T^2 + 2)(T^2 + 6T + 3) = T^6 + 6T^5 + 3T^3 + 5T + 6$$

- Long division

$\backslash[\backslash\text{PolyLongDiv}\{f\}\{g\}\backslash]$

$$\begin{array}{r} T^2 + T + 2 \\ T^2 + 6T + 3 \overline{) T^4 + 0T^3 + 4T^2 + 0T + 2} \\ \underline{T^4 + 6T^3 + 3T^2} \\ T^3 + T^2 + 0T \\ \underline{T^3 + 6T^2 + 3T} \\ 2T^2 + 4T + 2 \\ \underline{2T^2 + 5T + 6} \\ 6T + 3 \end{array}$$

- Short division

$\backslash[\backslash\text{PolyShortDiv}\{f\}\{g\}\backslash]$

$$T^4 + 4T^2 + 2 = (T^2 + T + 2)(T^2 + 6T + 3) + (6T + 3)$$

- (Euclidean) division algorithm

$\backslash[\backslash\text{PolyEuclid}\{f\}\{g\}\backslash]$

$$\begin{aligned} T^4 + 4T^2 + 2 &= (T^2 + T + 2)(T^2 + 6T + 3) + (6T + 3) \\ T^2 + 6T + 3 &= (6T + 5)(6T + 3) + (2) \end{aligned}$$

Install

You need to manually out the “polydiv.sty” into your working folder in order to use above.

Practices

Try to practice yourself as follows:

1. Choose a modulus and start with any two polynomials f and g .
2. Try to do the long division or Euclidean algorithm by yourself. Then verify your answer by running the corresponding code.