Arithmetic of Polynomials

1 Define and display polynomials

- Define polynomials by claiming their coefficients:

```
\PolySet{f}{1,0,4,0,2}
\PolySet{g}{1,6,3}
```

- Display Polynomials

```
\[\PolyPrint{f}\qquad\PolyPrint{g}\]
```

$$T^4 + \overline{4}T^2 + \overline{2} \qquad T^2 + \overline{6}T + \overline{3}$$

- Change the variable (globally)

```
\PolyOptions{variable={x}}
\[\PolyPrint{f}\qquad\PolyPrint{g}\]
```

$$x^4 + \overline{4}x^2 + \overline{2} \qquad x^2 + \overline{6}x + \overline{3}$$

Or put the option into argument (so change the variable locally),

```
\[\PolyPrint[variable={x}]{f}\qquad\PolyPrint[variable={x}]{g}\]
```

$$x^4 + \overline{4}x^2 + \overline{2} \qquad x^2 + \overline{6}x + \overline{3}$$

- One can also change the modulus:

```
\[\PolyPrint[base=5]{f}\qquad\PolyPrint[base=5]{g}\]
\[\PolyPrint[base=4]{f}\qquad\PolyPrint[base=3]{g}\]
```

$$T^4 + \overline{4}T^2 + \overline{2} \qquad T^2 + T + \overline{3}$$
$$T^4 + \overline{2} \qquad T^2$$

- Get rid of the bar:

```
\[\PolyPrint[coef=]{f}\qquad\PolyPrint[coef=]{g}\]
```

$$T^4 + 4T^2 + 2 \qquad T^2 + 6T + 3$$

2 Arithmetic of polynomials

We keep use polynomials f and g from previous section (with the modulus 7).

- Addition:

```
\[\PolyPrint{f}+\PolyPrint{g}=\PolyAdd{f}{g}\]
```

$$T^4 + \overline{4}T^2 + \overline{2} + T^2 + \overline{6}T + \overline{3} = T^4 + \overline{5}T^2 + \overline{6}T + \overline{5}$$

- Multiplication

```
\[(\PolyPrint{f})(\PolyPrint{g})=\PolyMult{f}{g}\]
```

$$(T^4 + \overline{4}T^2 + \overline{2})(T^2 + \overline{6}T + \overline{3}) = T^6 + \overline{6}T^5 + \overline{3}T^3 + \overline{5}T + \overline{6}$$

- Long division

```
\[\PolyLongDiv{f}{g}\]
```

$$\begin{array}{r|l} & T^2 + \ T + \overline{2} \\ \hline T^2 + \overline{6}T + \overline{3} & T^4 + \overline{0}T^3 + \overline{4}T^2 + \overline{0}T + \overline{2} \\ & \underline{T^4 + \overline{6}T^3 + \overline{3}T^2} \quad \downarrow \\ & T^3 + \ T^2 + \overline{0}T \\ & \underline{T^3 + \overline{6}T^2 + \overline{3}T} \quad \downarrow \\ & \overline{2}T^2 + \overline{4}T + \overline{2} \\ & \underline{\overline{2}T^2 + \overline{5}T + \overline{6}} \\ & \overline{6}T + \overline{3} \end{array}$$

- Short division

```
\[\PolyShortDiv{f}{g}\]
```

$$T^4 + \overline{4}T^2 + \overline{2} = \left(T^2 + T + \overline{2}\right)\left(T^2 + \overline{6}T + \overline{3}\right) + \left(\overline{6}T + \overline{3}\right)$$

- (Euclidean) division algorithm

```
\[\PolyEuclid{f}{g}\]
```

$$\begin{aligned} T^4 + \overline{4}T^2 + \overline{2} &= \left(T^2 + T + \overline{2}\right)\left(T^2 + \overline{6}T + \overline{3}\right) + \left(\overline{6}T + \overline{3}\right) \\ T^2 + \overline{6}T + \overline{3} &= \left(\overline{6}T + \overline{5}\right)\left(T^2 + \overline{6}T + \overline{3}\right) + \left(\overline{2}\right) \end{aligned}$$

Install

You need to manually out the “polydiv.sty” into your working folder in order to use above.

Practices

Try to practice yourself as follows:

1. Choose a modulus and start with any two polynomials f and g.
2. Try to do the long division or Euclidean algorithm by yourself. Then verify your answer by running the corresponding code.