A Study on Simplex Method in Linear Programming Problem

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ABSTRACT:

"The purpose of this investigation is to explore the Simplex Method within the realm of Linear Programming Problems. The Simplex Method represents a manual methodology for addressing linear programming models, employing elements like slack variables, tableaus, and pivot variables to discern the optimal solution to an optimization challenge. This inquiry delves into the historical backdrop of the Simplex Method and its utilization across diverse domains. Furthermore, it conducts a comprehensive assessment of the merits and demerits associated with the Simplex Method relative to alternative optimization techniques. In summation, this study establishes that the Simplex Method stands as a potent instrument for resolving linear programming predicaments and boasts a multitude of practical applications."

OBJECTIVES:

1. **Examine the Evolution of the Simplex Method:** Delve into the historical evolution of the Simplex Method and its progressive development over time.
2. **Evaluate the Pros and Cons of the Simplex Method:** Conduct a comprehensive analysis of the advantages and disadvantages inherent to the Simplex Method in contrast to other optimization techniques.
3. **Survey the Scope of Linear Programming Applications:** Explore the diverse array of linear programming problems that can be effectively addressed and resolved through the application of the Simplex Method.

CHAPTER 1

History and background of simplex method:

The simplex method, pioneered by George Dantzig in the late 1940s, stands as a foundational optimization tool. It emerged in the realm of linear programming, with the primary aim of optimizing linear objective functions while abiding by linear constraints. Its origins can be attributed to the endeavors of mathematicians and economists grappling with resource allocation and production planning dilemmas. Dantzig's innovative insight transformed this into a pragmatic algorithm. The approach systematically traverses the feasible region, pinpointing pivotal elements to enhance the objective function's value. Despite initial skepticism about its efficiency, it has undergone refinements and extensions to tackle various challenges. Despite its limitations, it remains an indispensable instrument, exerting a profound impact across diverse domains, encompassing operations research, economics, and engineering. Thanks to Dantzig's genius, the simplex method facilitated the transition of optimization from theory to practical application, forging an enduring legacy in its historical evolution.

Introduction to Simplex Method:

The simplex method, crafted by George Dantzig in 1947, presents a potent algorithm tailored for tackling linear programming quandaries. Linear programming constitutes a branch of mathematical optimization, where the primary objective is to maximize or minimize a linear objective function while adhering to a set of linear constraints. Over the years, this method has evolved into one of the most extensively utilized techniques across a spectrum of disciplines, spanning operations research, economics, engineering, and other arenas grappling with optimization challenges. The simplex method's core mission revolves around identifying the optimal solution for linear programming predicaments. It accomplishes this task by meticulously tracing the edges of a geometric figure, shaped by the governing constraints, in a systematic fashion. This approach maintains simplicity by adhering to a methodical procedure in pursuit of the optimal solution.

Basic Terms:

1. Constraints: Constraints represent the conditions imposed on decision variables. For instance, if 'x' and 'y' denote the quantities of articles manufactured by a factory, then '$x \leq 500$' and '$y \geq 725$' embody the constraints restricting the article production.

2. Feasible Solution: Feasible solutions encompass the values of decision variables that satisfy all prescribed conditions within the objective function.

3. Tableau: The simplex method employs a tableau, which serves as a tabular representation of the underlying linear programming problem. This tableau plays a pivotal role in tracking the current solution, pivot operations, and the value of the objective function.

4. Pivoting: Pivoting stands as a recurring operation within the algorithm, executed to enhance the current solution while ensuring its continued feasibility. Pivoting involves selecting an entering variable and a leaving variable, effectively transitioning from one vertex of the feasible region to an adjacent vertex along an edge.

5. Optimality Test: The simplex method rigorously examines optimality at each iteration. It culminates when further improvement in the objective function becomes unattainable, signifying the attainment of an optimal solution.

6. Slack Variable: A slack variable is a nonnegative variable introduced when adding to the left side of an inequality of the form '$ax \leq b$,' transforming the inequality into an equation of the form '$ax + s = b$,' where '$s \geq 0$' signifies the slack variable.

7. Surplus Variable: Conversely, a surplus variable is a nonnegative variable introduced when subtracting from the left side of an inequality of the form '$ax \geq b$,' converting it into an equation of the form '$ax - b = t$,' where '$t \geq 0$' denotes the surplus variable.

Advantages and Disadvantages of the Simplex Method:

Advantages of the Simplex Method:

i. Exceptional Efficiency: The Simplex method boasts high efficiency and finds application across diverse sectors, encompassing business, science, and industry, for a multitude of problem scenarios.
ii. Expedited Solutions for Complexity: In scenarios characterized by intricate problems with a multitude of variables, the Simplex method outpaces alternative algorithms in delivering prompt solutions to linear systems.
iii. Feasibility-Driven Approach: This method commences with an initial feasible solution and systematically transitions to subsequent feasible solutions that progressively enhance the objective function's value.

Disadvantages of the Simplex Method:

i. Potential for Prolonged Convergence: The simplex method can exhibit sluggish convergence, necessitating a substantial number of iterations to reach the optimal solution, particularly in complex problem instances.
ii. Lack of Polynomial Time Guarantee: Unlike certain other optimization algorithms, the simplex method lacks a guaranteed polynomial time complexity, which means that in some cases, it may not offer predictable computational speed.
iii. Limited Applicability: Direct application of the simplex method to nonlinear or integer programming problems is infeasible without substantial modification, limiting its versatility in handling a broader range of optimization problems.

CHAPTER 2

Steps in Simplex Method:

Step 1: Formulation of the Mathematical Model:

i. Begin by formulating the mathematical model for the given Linear Programming Problem (L.P.P.).

ii. Determine whether the L.P.P.'s objective function should be maximized or minimized. If minimization is required, convert it into a maximization problem by using the relationship Minimize Z = -Maximize (-Z).

iii. Examine all coefficients of independent variables to ensure they are nonnegative. If any coefficients are negative, negate them to ensure positivity.

iv. Transform all constraint inequalities into equations by introducing slack or surplus variables.

Step 2: Selection of Pivot Column:

i. Identify the non-basic variable corresponding to the largest negative value in the bottom row. This column containing the entry is designated as the pivot column.

ii. In case there are two or more negative entries of equal magnitude in the bottom row, select any one of them.

Step 3: Selection of Pivot Row:

i. Divide the right side values by their corresponding entries in the pivot column.

ii. Choose the row with the smallest non-negative quotient as the pivot row.

iii. If there are multiple smallest non-negative quotients, you can select any of them.

Step 4: Elimination by Row Operations:

i. Implement row operations in all other rows except the pivot row.

ii. Modify the pivot element to become equal to one.

iii. Utilize the Gauss Elimination Method to set all other elements in the pivot column to zero, now that you have the unity element in the pivot position.

Some examples of simplex method:

QN. Maximize Subject Z= 4x+5y

Subject to the constraints

$2x = 3y \leq 12$

$2x + y \leq 8$

$x \geq 0, y \geq 0$

Solution:

The above inequalities are converted into equation by using slack variables s_1 and s_2.

$2x + 3y + s_1 = 12$

$2x + y + s_2 = 9$

$-4x - 5y + Z = 0$

Where $x, y \geq 0, s_1 \geq 0, s_2 \geq 0$

If we put x = 0, y = 0 we get $s_1 = 12$ and $s_2 = 8$ and Z=0 which are the initial basic feasible solution.

Step 1: the above information is given in following table.

B.V.	x	y	S_1	S_2	Z	RHS
S_1	2	3	1	0	0	12
S_2	2	1	0	1	0	8
Z	-4	-5	0	0	1	0

Step 2: The coefficient of x and y in the Z-row are -4 and -5. Since the coefficient -5 of y is the most negative. Therefore, y is entering variable.

Step 3: selection of departing variable

In the first row, the ratio is $\frac{12}{3}$ = 4 and in the second row, $\frac{8}{1}$ = 8

Thus, the smallest non-negative ratio is 4 so s_1 is pivot element which lies on s_1 column.

Step 4:

	x	y	S_1	S_2	Z	RHS
S_1	2	3	1	0	0	12
S_2	2	1	0	1	0	8
Z	-4	-5	0	0	1	0

The intersection of the column of entering variable and departing variable gives the pivot entry. Here the pivot entry is 3. The pivot entry is to be converted into 1.

Step 5: Multiply the departing variable row by $\frac{1}{3}$ to get 1 at the pivot entry

	x	y	S_1	S_2	Z	RHS
Y	$\frac{2}{3}$	1	$\frac{1}{3}$	0	0	4
S_1	2	1	0	1	0	8
Z	-4	-5	0	0	1	0

Step 6: Multiply the first row by 5 and add to the third row to get 0 and again subtract the first row from the second row to get 0.

	x	y	S_1	S_2	Z	RHS
y	$\frac{2}{3}$	1	$\frac{1}{3}$	0	0	4
S_2	$\frac{4}{3}$	0	$-\frac{1}{3}$	1	0	4
Z	$-\frac{2}{3}$	0	$\frac{5}{3}$	0	1	20

There is still negative entry in the table. So, we have not obtained optimal solution. Since the coefficient of x in the third row is - $\frac{2}{3}$ the most negative. So, x is new entering variable.

Step 7: Selection of departing variable.

The ratio if y in the first row is $\frac{4}{\frac{2}{3}}$ = 6 and in the second row $\frac{4}{\frac{4}{3}}$ =3.

Here the 3 is smallest non-negative ratio in the s_2 row. Thus, s_2 is the departing variable.

Step 8:

	x	y	S_1	S_2	Z	RHS
y	$\frac{2}{3}$	1	$\frac{1}{3}$	0	0	6
S2	$\frac{4}{3}$	0	$-\frac{1}{3}$	1	0	3
Z	$-\frac{2}{3}$	0	$\frac{5}{3}$	0	1	20

Step 9: Multiply the departing variable by $\frac{3}{4}$ to get 1 at pivot entry.

	x	y	S_1	S_2	Z	RHS
y	$\frac{2}{3}$	1	$\frac{1}{3}$	0	0	4
x	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	3
z	$-\frac{2}{3}$	0	$\frac{5}{3}$	0	1	20

Step 10: Multiply second row by $\frac{2}{3}$ and add to third row and multiply the second row by $-\frac{2}{3}$ and add to first row to get 0.

	x	y	S_1	S_2	Z	RHS
x	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	2
y	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	3
Z	0	0	$\frac{3}{2}$	$\frac{1}{2}$	1	22

As the Z-row of the table has no negative entry in the columns of variable. So, this is the case of optimal solution. From the above table we have,

x = 3, y = 2, s_1 = 0, s_2 = 0 and the maximum value of Z = 22.

Hence, this is the example of simplex method with steps.

CONCLUSION:

In conclusion, the simplex method, originally conceptualized by George Dantzig in the late 1940s, stands as a groundbreaking force within the domain of optimization. This ingenious technique adeptly facilitates the maximization or minimization of linear objectives while adhering to prescribed constraints, with its applicability spanning diverse disciplines such as economics and engineering. Its modus operandi entails the gradual refinement of solutions through systematic adjustments that maintain adherence to constraints. Despite its commendable efficiency and versatility in tackling complex challenges, it may exhibit sluggishness when confronted with exceedingly large-scale tasks, with no assured predictability of computational duration. Furthermore, modifications are requisite when addressing non-linear problems or integral values. Nonetheless, its historical significance and practical utility firmly establish its enduring relevance within the sphere of mathematical optimization.

REFRENCES:

- Gupta, S., Chaudhary, P. K., Kushwaha, N. P., & Shrestha, P. (2080). Conceptual Mathematics: Linear Programming. Green Mathematics.

- Wright, S. J. (2022). Simplex Method. Encyclopedia Britannica.

- Engineeringenotes. (2019). Simplex Method for Solution of L.P.P. with Examples (Operation Research). Engineeringenotes.