UNIT - 1
INTRODUCTION TO DATA STRUCTURES

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Contents

- Introduction to Data Structures
- Classification and Operations on Data Structures
- Preliminaries of Algorithm
- Algorithm Analysis and Complexity

Introduction to Data Structures

- A data structure is a way of storing data in a computer so that it can be used efficiently and it will allow the most efficient algorithm to be used.

- A data structure should be seen as a logical concept that must address two fundamental concerns.

  I.   First, how the data will be stored, and

  II. Second, what operations will be performed on it.

Classification of Data Structures

- Data structures can be classified as
  - i. Simple data structure
  - ii. Compound data structure
  - iii. Linear data structure
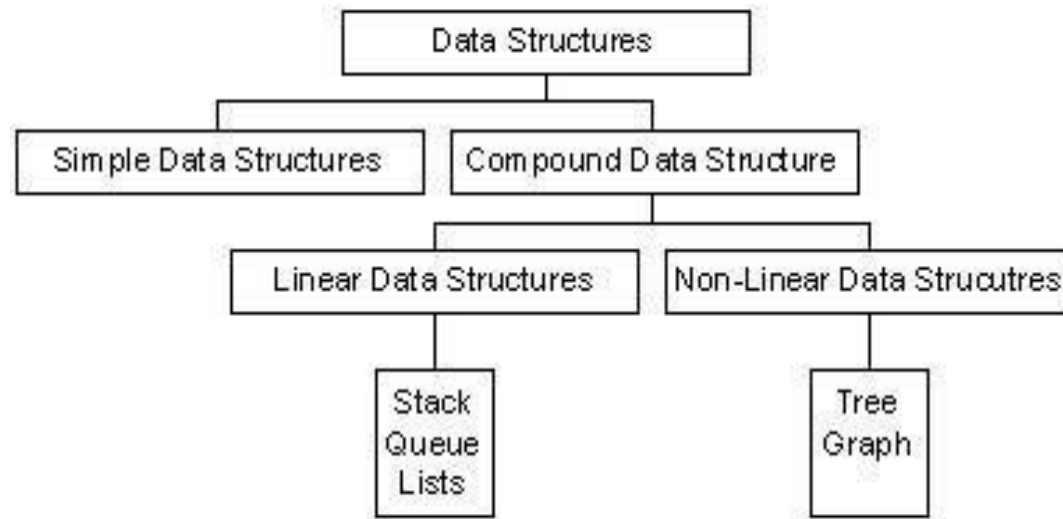 - iv. Non linear data structure

Fig. Classification of Data Structures

Simple and Compound Data Structures

- **Simple Data Structure:** Simple data structure can be constructed with the help of primitive data structure. A primitive data structure used to represent the standard data types of any one of the computer languages. Variables, arrays, pointers, structures, unions, etc. are examples of primitive data structures. **Compound Data structure:** Compound data structure can be constructed with the help of any one of the primitive data structure and it is having a specific functionality. It can be designed by user. It can be classified as

  i.   Linear data structure

  ii.   Non-linear data structure

Linear and Non-linear Data Structures

- **Linear Data Structure:** Linear data structures can be constructed as a continuous arrangement of data elements in the memory. It can be constructed by using array data type. In the linear Data Structures the relationship of adjacency is maintained between the data elements.

- **Non-linear Data Structure:** Non-linear data structure can be constructed as a collection of randomly distributed set of data item joined together by using a special pointer (tag). In non-linear Data structure the relationship of adjacency is not maintained between the data items.

Operations on Data Structures

i. Add an element

ii. Delete an element

iii. Traverse

iv. Sort the list of elements

v. Search for a data element

Abstract Data Type

- An abstract data type, sometimes abbreviated ADT, is a logical description of how we view the data and the operations that are allowed without regard to how they will be implemented.

- By providing this level of abstraction, we are creating an encapsulation around the data. The idea is that by encapsulating the details of the implementation, we are hiding them from the user's view. This is called information hiding.

Algorithm Definition

- An **Algorithm** may be defined as a finite sequence of instructions each of which has a clear meaning and can be performed with a finite amount of effort in a finite length of time.

- The word algorithm originates from the Arabic word **Algorism** which is linked to the name of the Arabic Mathematician **AI Khwarizmi**.

- **AI Khwarizmi** is considered to be the first algorithm designer for adding numbers.

Structure of an Algorithm

- An algorithm has the following structure:
    - Input Step
    - Assignment Step
    - Decision Step
    - Repetitive Step
    - Output Step

Properties of an Algorithm

- **Finiteness:-** An algorithm must terminate after finite number of steps.
- **Definiteness:-** The steps of the algorithm must be precisely defined.
- **Generality:-** An algorithm must be generic enough to solve all problems of a particular class.
- **Effectiveness:-** The operations of the algorithm must be basic enough to be put down on pencil and paper.

Input-Output:- The algorithm must have certain initial and precise inputs, and outputs that may be generated both at its intermediate and final steps

Algorithm Analysis and Complexity

- The performances of algorithms can be measured on the scales of **Time and Space**.

- The **Time Complexity** of an algorithm or a program is a function of the running time of the algorithm or a program.

- The **Space Complexity** of an algorithm or a program is a function of the space needed by the algorithm or program to run to completion.

Algorithm Analysis and Complexity

- The Time Complexity of an algorithm can be computed either by an
  - **Empirical or Posteriori Testing**
  - **Theoretical or Apriori Approach**
- The **Empirical or Posteriori Testing** approach calls for implementing the complete algorithm and executes them on a computer for various instances of the problem.
- The **Theoretical or Apriori** Approach calls for mathematically determining the resources such as time and space needed by the algorithm, as a function of parameter related to the instances of the problem considered.

Algorithm Analysis and Complexity

- Apriori analysis computed the efficiency of the program as a function of the total frequency count of the statements comprising the program.

- **Example:** Let us estimate the frequency count of the statement x = x+2 occurring in the following three program segments A, B andC.

Total Frequency Count of Program Segment A

- Program Statements

  …………………………

  $$x = x + 2$$

  …………………………

  Total Frequency Count

- Frequency Count

  1

  1

Time Complexity of Program Segment A is **O(1)**.

Total Frequency Count of Program Segment B

- Program Statements

  ....

      for k = 1 to n do

          x = x+ 2;

      end

  ....

  Total Frequency Count

- Frequency Count

          (n+1)

          n

          n

  ....

          3n+1

Time Complexity of Program Segment B is **O(n)**.

Total Frequency Count of Program Segment C

- Program Statements

- Frequency Count

```
……………………
for j = 1 to n do
      for k = 1 to n do
             x = x+ 2;
      end
end
…………………….
Total Frequency Count
```

$$(n+1)$$
$$(n+1)n$$
$$n^2$$
$$n^2$$
$$n$$
………………………..
$$3n^2+3n+1$$

Time Complexity of Program Segment C is **O(n²)**.

Asymptotic Notations

- **Big oh(O):** $f(n) = O(g(n))$ (read as f of n is big oh of g of n), if there exists a positive integer n_0 and a positive number c such that $|f(n)| \le c |g(n)|$ for all $n \ge n_0$.

f(n)	g(n)	
$16n^3 + 45n^2 + 12n$	n^3	$f(n) = O(n^3)$
$34n - 40$	n	$f(n) = O(n)$
50	1	$f(n) = O(1)$

- Here g(n) is the upper bound of the function f(n).

Asymptotic Notations

- **Omega(Ω)**: f(n) = Ω(g(n)) (read as f of n is omega of g of n), if there exists a positive integer n_0 and a positive number c such that $|f(n)| \geq c |g(n)|$ for all n $\geq n_0$.

f(n)	g(n)	
$16n^3 + 8n^2 + 2$	n^3	f(n) = Ω (n^3)
24n +9	n	f(n) = Ω (n)

- Here g(n) is the lower bound of the function f(n).

Asymptotic Notations

- **Theta(Θ)**: $f(n) = \Theta(g(n))$ (read as f of n is theta of g of n), if there exists a positive integer n_0 and two positive constants c_1 and c_2 such that $c_1 |g(n)| \leq |f(n)| \leq c_2 |g(n)|$ for all $n \geq n_0$.

f(n)	g(n)	
$16n^3 + 30n^2 - 90$	n^2	$f(n) = \Theta(n^2)$
7. $2^n + 30n$	2^n	$f(n) = \Theta(2^n)$

- The function g(n) is both an upper bound and a lower bound for the function f(n) for all values of n, $n \geq n_0$

Asymptotic Notations

- **Little oh(o):** f(n) = O(g(n)) (read as f of n is little oh of g of n), if f(n) = O(g(n)) and f(n) ≠ Ω(g(n)).

f(n)	g(n)	
18n + 9	n^2	f(n) = o(n^2) since f(n) = O(n^2) and f(n) ≠ Ω(n^2) however f(n) ≠ O(n).

Time Complexity

Complexity	Notation	Description
Constant	$O(1)$	Constant number of operations, not depending on the input data size.
Logarithmic	$O(\log n)$	Number of operations proportional of $\log(n)$ where n is the size of the input data.
Linear	$O(n)$	Number of operations proportional to the input data size.
Quadratic	$O(n^2)$	Number of operations proportional to the square of the size of the input data.
Cubic	$O(n^3)$	Number of operations proportional to the cube of the size of the input data.
Exponential	$O(2^n)$	Exponential number of operations, fast growing.
	$O(k^n)$	
	$O(n!)$	

Time Complexities of various Algorithms
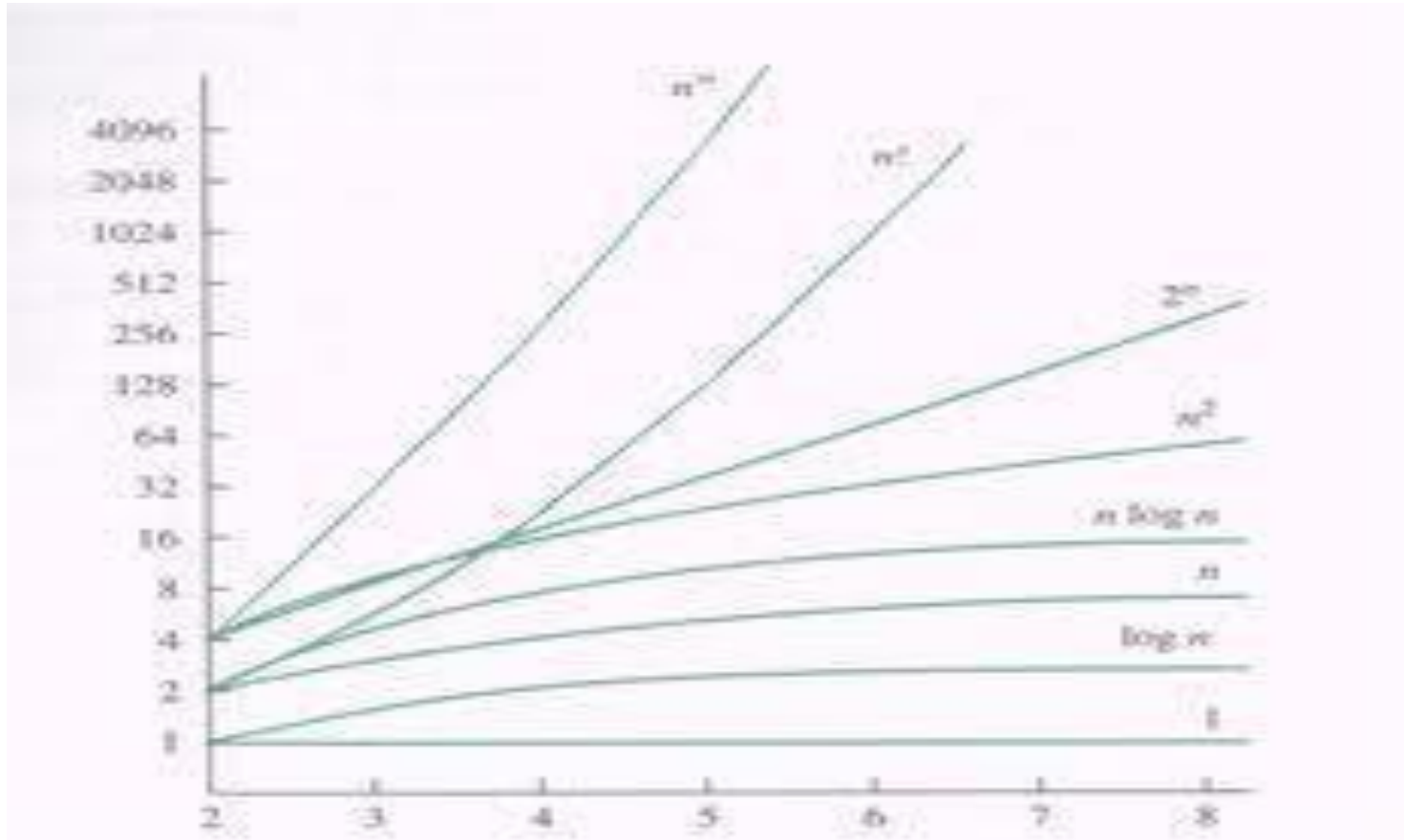
UNIT - 2
STACK

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Contents

- Stack - Primitive Operations and Implementation
- Stack Operations: Concept and Algorithms
- Applications of Stack

Stacks

- A stack is a list of elements in which an element may be inserted or deleted only at one end, called the top of the stack.

- The elements are removed from a stack in the reverse order of that in which they were inserted into the stack.

- Stack is also known as a LIFO (Last in Fast out) list or Push down list.

Basic Stack Operations

PUSH: It is the term used to insert an element into a stack.
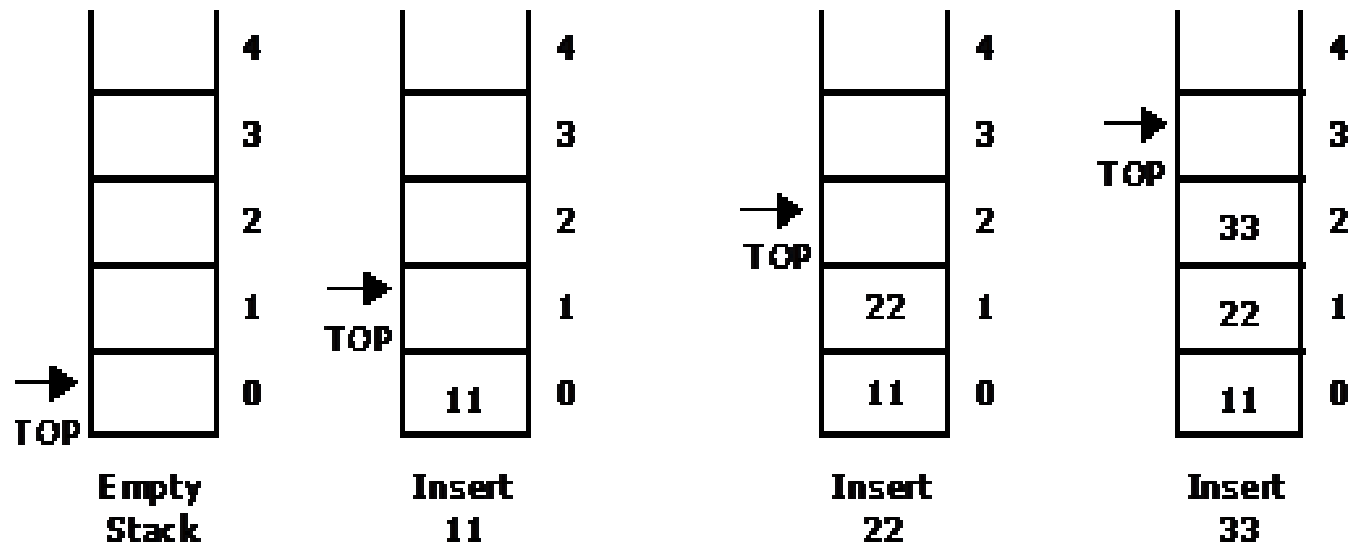
PUSH operations on stack

Basic Stack Operations

POP: It is the term used to delete an element from a stack.
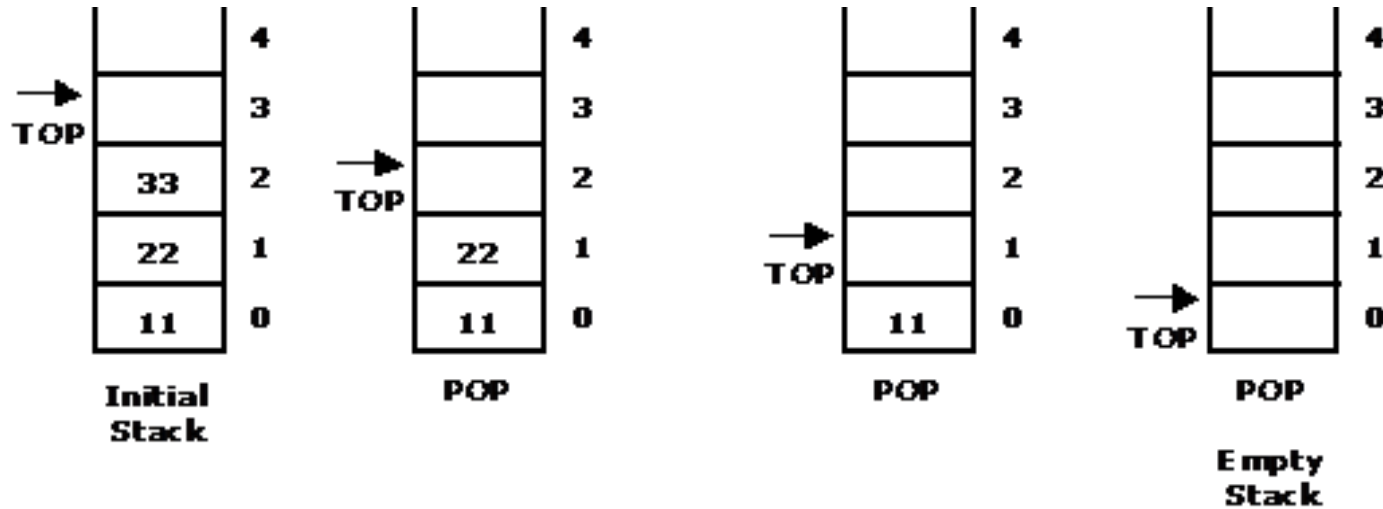
POP operation from a stack

Standard Error Messages in Stack

- Two standard error messages of stack are
  - **Stack Overflow**: If we attempt to add new element beyond the maximum size, we will encounter a *stack overflow* condition.
  - **Stack Underflow**: If we attempt to remove elements beyond the base of the stack, we will encounter a *stack underflow* condition.

Stack Operations

- PUSH (STACK, TOP, MAXSTR, ITEM): This procedure pushes an ITEM onto a stack

  1. If TOP = MAXSIZE, then Print: OVERFLOW, and Return.

  2. Set TOP := TOP + 1 [Increases TOP by 1]

  3. Set STACK [TOP] := ITEM. [Insert ITEM in TOP position]

  4. Return

- POP (STACK, TOP, ITEM): This procedure deletes the top element of STACK and assign it to the variable ITEM

  1. If TOP = 0, then Print: UNDERFLOW, and Return.

  2. Set ITEM := STACK[TOP]

  3. Set TOP := TOP - 1 [Decreases TOP by 1]

  4. Return

Applications of Stack

- Converting algebraic expressions from one form to another. E.g. Infix to Postfix, Infix to Prefix, Prefix to Infix, Prefix to Postfix, Postfix to Infix and Postfix to prefix.

- Evaluation of Postfix expression.

- Parenthesis Balancing in Compilers.

- Depth First Search Traversal of Graph.

- Recursive Applications.

Algebraic Expressions

- **Infix:** It is the form of an arithmetic expression in which we fix (place) the arithmetic operator in between the two operands. E.g.: (A + B) * (C - D)

- **Prefix:** It is the form of an arithmetic notation in which we fix (place) the arithmetic operator before (pre) its two operands. The prefix notation is called as polish notation. E.g.: * + A B – C D

- **Postfix:** It is the form of an arithmetic expression in which we fix (place) the arithmetic operator after (post) its two operands. The postfix notation is called as *suffix notation* and is also referred to *reverse polish notation*. E.g: A B + C D - *

Conversion from Infix to Postfix

Convert the following infix expression A + B * C – D / E * H into its equivalent postfix expression.

Symbol	Postfix string	Stack	Remarks
A	A		
+	A	+	
B	A B	+	
*	A B	+ *	
C	A B C	-	
-	A B C* +	-	
D	A B C* + D	-	
/	A B C* + D	- /	
E	A B C* + D E	- /	
*	A B C* + D E/	- *	
H	A B C* + D E/ H	- *	
End of string	A B C* + D E/ H * -	The input is now empty. Pop the output symbols from the stack until it is empty.

Evaluation of Postfix Expression

Postfix expression: 6 5 2 3 + 8 * + 3 + *

Symbol	Operand 1	Operand 2	Value	Stack	Remarks
6				6	
5				6, 5	
2				6, 5, 2	
3				6, 5, 2, 3	The first four symbols are placed on the stack.
+	2	3	5	6, 5, 5	Next a '+' is read, so 3 and 2 are popped from the stack and their sum 5, is pushed
8	2	3	5	6, 5, 5, 8	Next 8 is pushed
*	5	8	40	6, 5, 40	Now a '*' is seen, so 8 and 5 are popped as 8 * 5 = 40 is pushed
+	5	40	45	6, 45	Next, a '+' is seen, so 40 and 5 are popped and 40 + 5 = 45 is pushed
3	5	40	45	6, 45, 3	Now, 3 is pushed
+	45	3	48	6, 48	Next, '+' pops 3 and 45 and pushes 45 + 3 = 48 is pushed
*	6	48	288	**288**	Finally, a '* is seen and 48 and 6 are popped, the result 6 * 48 = 288 is pushed

UNIT - 3
QUEUE

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Content

- Queue - Primitive Operations and Implementation
- Linear Queue operations
- Applications of Linear Queue
- Circular Queue operations and algorithm
- Linear Queue vs Circular queue
- Priority Queue
- Double Ended Queue (Deque)

- A queue is a data structure where items are inserted at one end called the rear and deleted at the other end called the front.

- Another name for a queue is a FIFO or
  —First-in-first-out

- Operations of a Queue:
  - enqueue:which inserts an element at the end of the queue.
  - dequeue:which deletes an element at the front of the queue.

Representation of Queue

Initially the queue is empty.


```
      0    1    2    3    4
    ┌────┬────┬────┬────┬────┐
    │    │    │    │    │    │
    └────┴────┴────┴────┴────┘
     ↑↑
     F R
```

Queue Empty
FRONT = REAR = 0
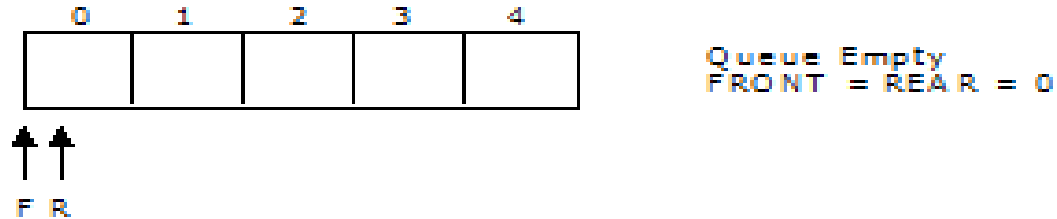
Now, insert 11 to the queue. Then queue status will be:

```
      0    1    2    3    4
    ┌────┬────┬────┬────┬────┐
    │ 11 │    │    │    │    │
    └────┴────┴────┴────┴────┘
      ↑    ↑
      F    R
```

REAR = REAR + 1 = 1
FRONT = 0
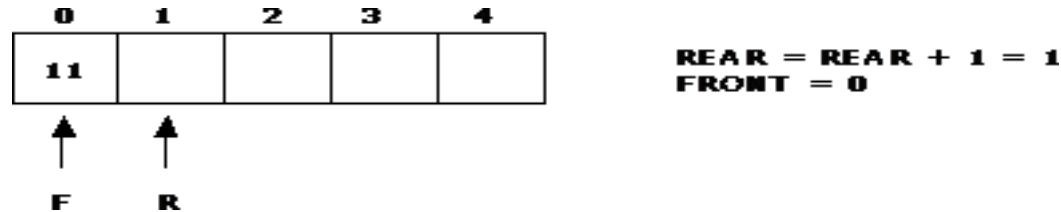
Next, insert 22 to the queue. Then the queue status is:

```
      0    1    2    3    4
    ┌────┬────┬────┬────┬────┐
    │ 11 │ 22 │    │    │    │
    └────┴────┴────┴────┴────┘
      ↑         ↑
      F         R
```

REAR = REAR + 1 = 2
FRONT = 0
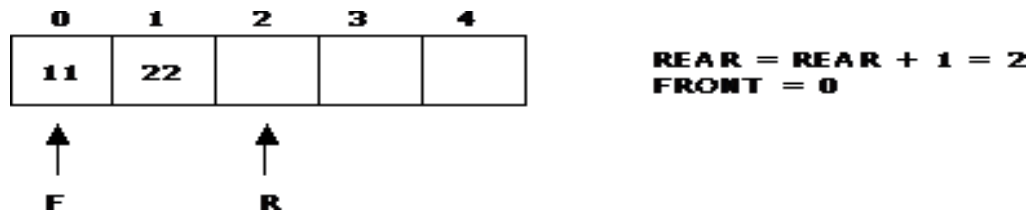
Representation of Queue

Now, delete an element 11.

```
     0     1     2     3     4
   ┌─────┬─────┬─────┬─────┬─────┐
   │     │ 22  │ 33  │     │     │
   └─────┴─────┴─────┴─────┴─────┘
            ↑           ↑
            F           R
```

REAR = 3
FRONT = FRONT + 1 = 1
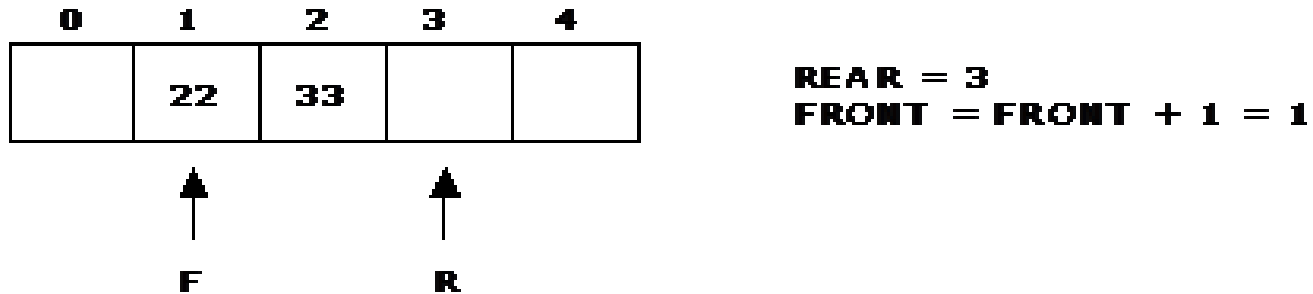
Next insert another element, say 66 to the queue. We cannot insert 66 to the queue as it signals queue is full. The queue status is as follows:

```
     0     1     2     3     4
   ┌─────┬─────┬─────┬─────┬─────┐
   │     │     │ 33  │ 44  │ 55  │
   └─────┴─────┴─────┴─────┴─────┘
                  ↑           ↑
                  F           R
```

REAR = 5
FRONT = 2
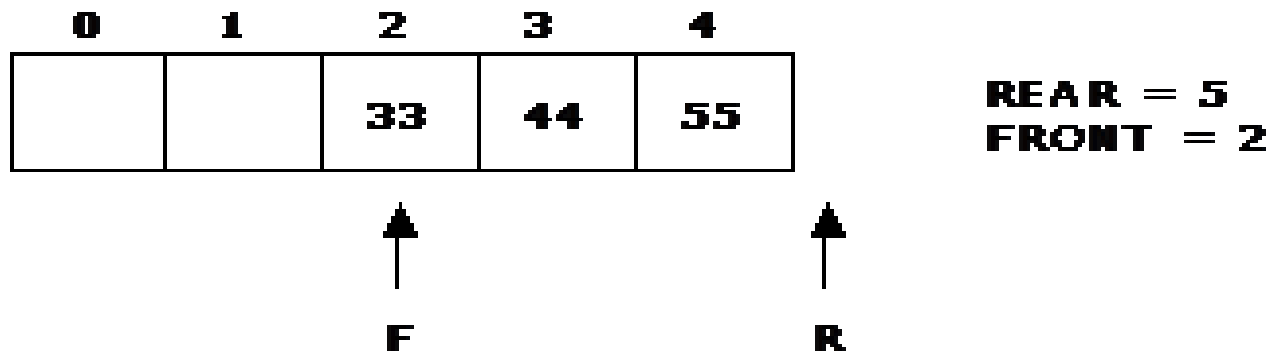
Queue Operations using Array

- Various operations of Queue are:
  - ➢ **insertQ():** inserts an element at the end of queue Q.
  - ➢ **deleteQ():** deletes the first element of Q.
  - ➢ **displayQ():** displays the elements in the queue.
- There are two problems associated with linear queue. They are:
  - ➢ **Time consuming**: linear time to be spent in shifting the elements to the beginning of the queue.
  - ➢ **Signaling queue full**: even if the queue is having vacant position.

Algorithm for ENQUEUE operation
1.Check if the queue is full or not.
2.If the queue is full, then print overflow error and exit the program.
3.If the queue is not full, then increment the tail and add the element.

Algorithm for DEQUEUE operation

1.Check if the queue is empty or not.
2.If the queue is empty, then print underflow error and exit the program.
3.If the queue is not empty, then print the element at the head and increment the head.

Algorithm of Queue

Insertion Algorithm

1. If (rear>MAXSIZE)
   Print ("Queue is Full") GO TO Step 4
2. Else
   Queue [rear] =item;
3. rear=rear+1
4. Exit

Algorithm of Queue

Deletion Algorithm (Dequeue)

1. If (font>rear)
   Print ("Queue is empty") GO TO Step 5
2. Else
   item= queue(font);
3. If (font ==rear)
   Set front =0 , rear = -1;
4. Else
   front=front+1;
4. Exit

Applications of Queue

- It is used to schedule the jobs to be processed by the CPU..

- When multiple users send print jobs to a printer, each printing job is kept in the printing queue. Then the printer prints those jobs according to first in first out (FIFO) basis.

- Breadth first search uses a queue data structure to find an element from a graph.

Circular Queue

- A circular queue is one in which the insertion of new element is done at the very first location of the queue if the last location of the queue is full.

- Suppose if we have a Queue of n elements then after adding the element at the last index i.e. (n-1)th , as queue is starting with 0 index, the next element will be inserted at the very first location of the queue which was not possible in the simple linear queue.

Circular Queue operations

- The Basic Operations of a circular queue are
  - ➢ **InsertionCQ**: Inserting an element into a circular queue results in Rear = (Rear + 1) % MAX, where MAX is the maximum size of the array.
  - ➢ **DeletionCQ** : Deleting an element from a circular queue results in Front = (Front + 1) % MAX, where MAX is the maximum size of the array.
  - ➢ **TraversCQ**: Displaying the elements of a circular Queue.
- Circular Queue Empty: Front=Rear=0.

Circular Queue Representation using Arrays

Let us consider a circular queue, which can hold maximum (MAX) of six elements. Initially the queue is empty.
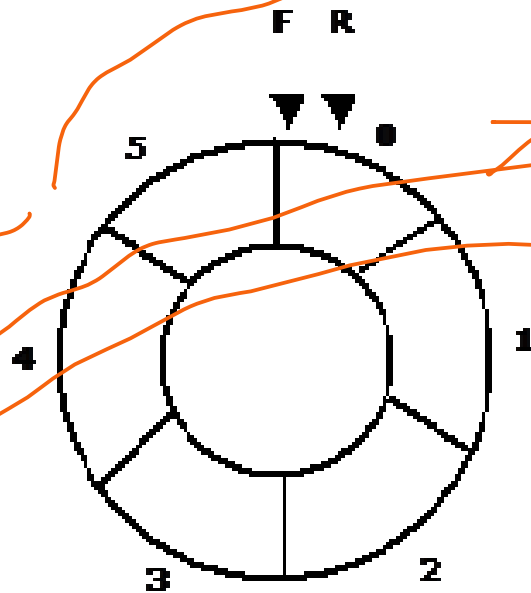
Queue Empty
MAX = 6
FRONT = REAR = 0
COUNT = 0

Circular Queue
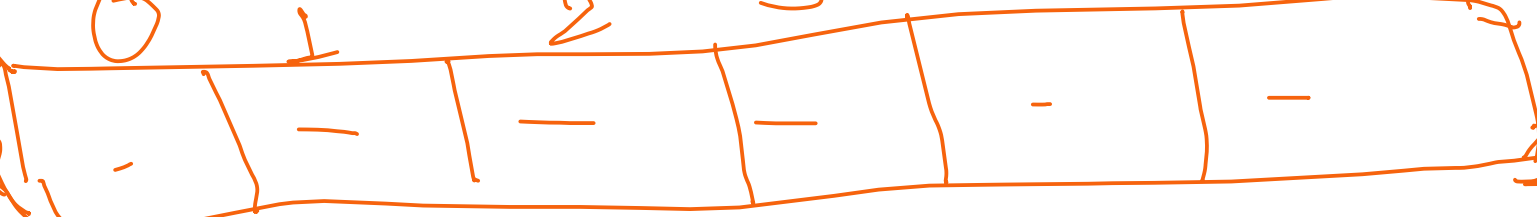
Insertion and Deletion operations on a Circular Queue

Insert new elements 11, 22, 33, 44 and 55 into the circular queue. The circular queue status is:
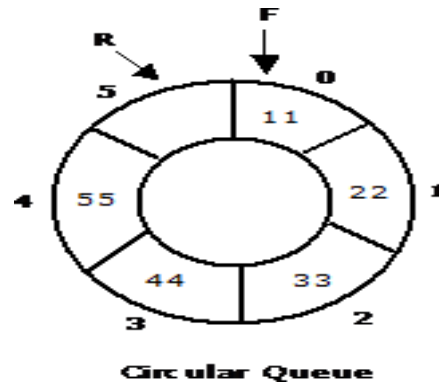
Circular Queue

FRONT = 0, REAR = 5
REAR = REAR % 6 = 5
COUNT = 5

Now, delete two elements 11, 22 from the circular queue. The circular queue status is as follows:
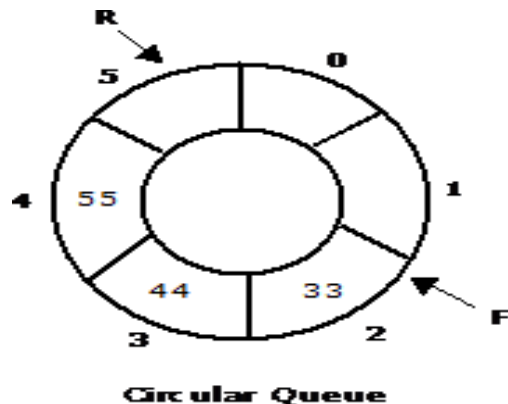
Circular Queue

FRONT = (FRONT + 1) % 6 = 2
REAR = 5
COUNT = COUNT - 1 = 3

Insertion and Deletion operations on a Circular Queue

Again, insert another element 66 to the circular queue. The status of the circular queue is:
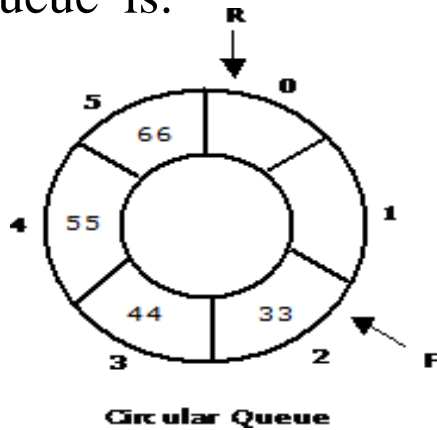
FRONT = 2
REAR = (REAR + 1) % 6 = 0
COUNT = COUNT + 1 = 4

Circular Queue

Again, insert 77 and 88 to the circular queue. The status of the Circular queue is:
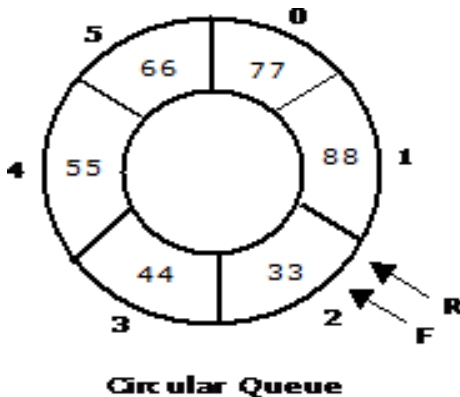
FRONT = 2, REAR = 2
REAR = REAR % 6 = 2
COUNT = 6

Circular Queue

Algorithm to insert an element in a circular queue

Step 1: IF (REAR+1)%MAX = FRONT

Write " OVERFLOW "

Goto step 4

[End OF IF]

Step 2: IF FRONT = -1 and REAR = -1

SET FRONT = REAR = 0

ELSE IF REAR = MAX - 1 and FRONT ! = 0

SET REAR = 0

ELSE

SET REAR = (REAR + 1) % MAX

[END OF IF]

Step 3: SET QUEUE[REAR] = VAL

Step 4: EXIT

Algorithm to delete an element from the circular queue
Step 1: IF FRONT = -1

Write " UNDERFLOW "Goto Step 4

[END of IF]

Step 2: SET VAL = QUEUE[FRONT]

Step 3: IF FRONT = REAR

SET FRONT = REAR = -1

ELSE

IF FRONT = MAX -1

SET FRONT = 0

ELSE

SET FRONT = FRONT + 1

[END of IF]

[END OF IF]

Step 4: EXIT

Linear vs Circular Queue

Basis of comparison	Linear Queue	Circular Queue
Meaning	The linear queue is a type of linear data structure that contains the elements in a sequential manner.	The circular queue is also a linear data structure in which the last element of the Queue is connected to the first element, thus creating a circle.
Insertion and Deletion	In linear queue, insertion is done from the rear end, and deletion is done from the front end.	In circular queue, the insertion and deletion can take place from any end.
Memory space	The memory space occupied by the linear queue is more than the circular queue.	It requires less memory as compared to linear queue.
Memory utilization	The usage of memory is inefficient.	The memory can be more efficiently utilized.
Order of execution	It follows the FIFO principle in order to perform the tasks.	It has no specific order for execution.

Double Ended Queue (DEQUE)

- It is a special queue like data structure that supports insertion and deletion at both the front and the rear of the queue.

- Such an extension of a queue is called a **double-ended queue, or deque,** which is usually pronounced **"deck"** to avoid confusion with the dequeue method of the regular queue, which is pronounced like the abbreviation **"D.Q."**

- It is also often called a **head-tail linked list**.

DEQUE Representation using arrays

| 11 | 22 |

enqueue_front(33) →

| 33 | 11 | 22 |

enqueue_rear(44) →

| 33 | 11 | 22 | 44 |

dequeue_front(33)

| 55 | 11 | 22 |

← enqueue_front(55)

| 11 | 22 |

← dequeue_rear(44)

| 11 | 22 | 44 |
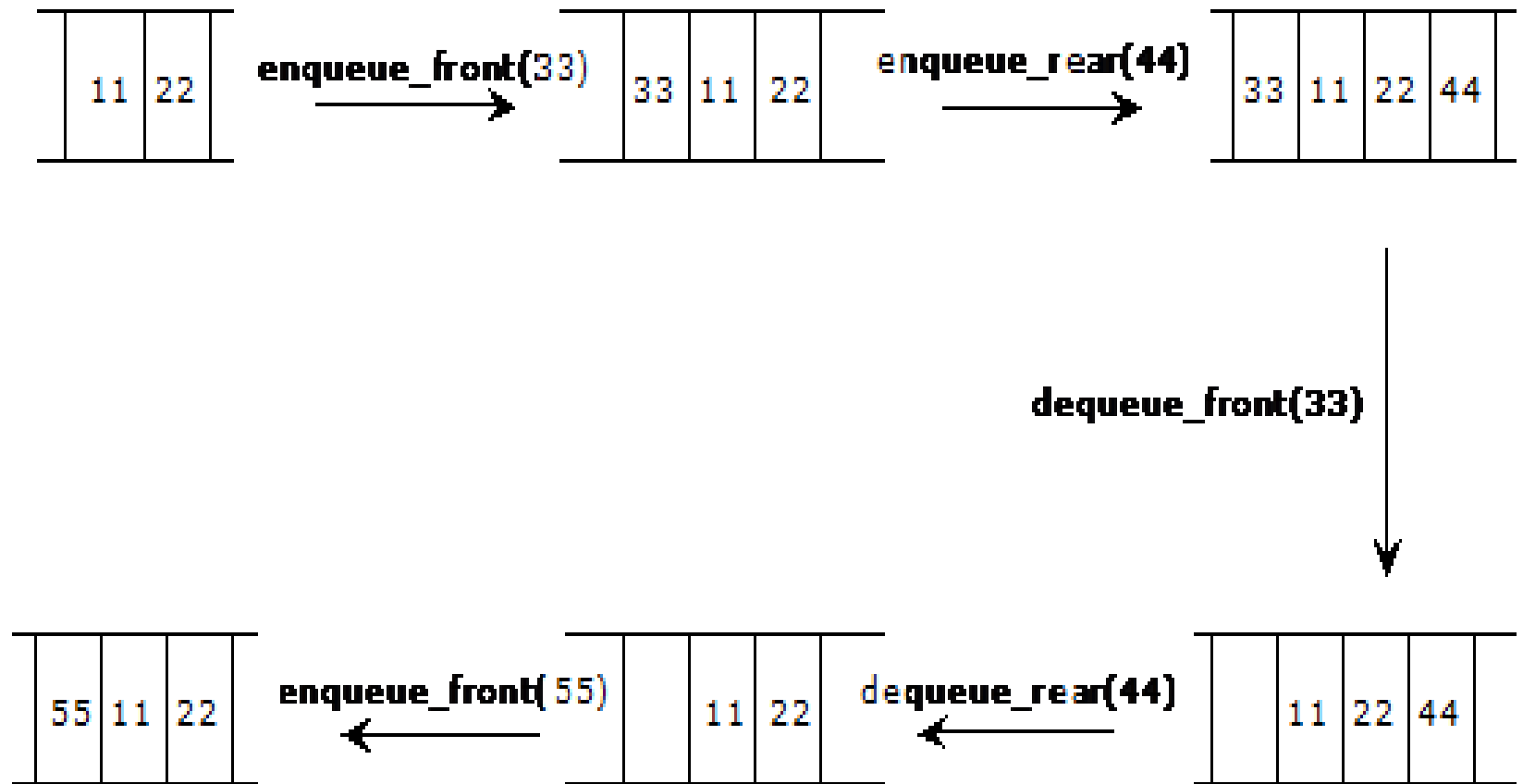
Types of DEQUE

- There are two variations of deque. They are:
  - Input restricted deque (IRD)
  - Output restricted deque (ORD)

- An Input restricted deque is a deque, which allows insertions at one end but allows deletions at both ends of the list.

- An output restricted deque is a deque, which allows deletions at one end but allows insertions at both ends of the list.

Priority Queue

- A priority queue is a collection of elements that each element has been assigned a priority and such that order in which elements are deleted and processed comes from the following riles:

  – An element of higher priority is processed before any element of lower priority.

  – Two element with the same priority are processed according to the order in which they were added to the queue.
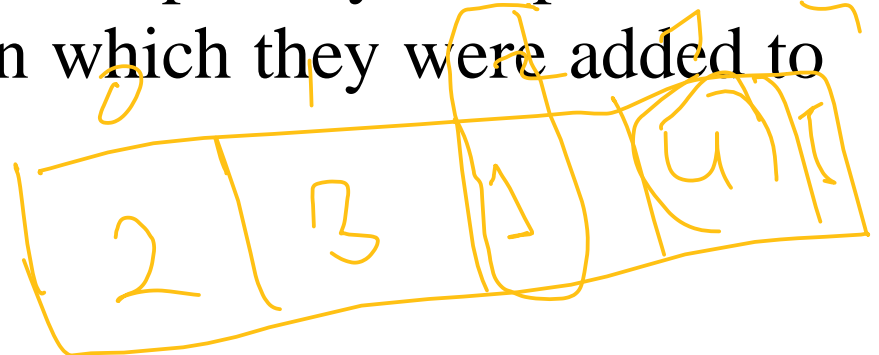
Priority Queue Operations and Usage

- Inserting new elements.

- Removing the largest or smallest element.

- Priority Queue Usages are:

  Simulations: Events are ordered by the time at which they should be executed.

  Job scheduling in computer systems: Higher priority jobs should be executed first.

  Constraint systems: Higher priority constraints should be satisfied before lower priority constraints.

Important Questions

1. Define Queue. Differentiate between Enqueue and dequeue algorithm with suitable example. [2+6=8] Regular/Back Exam - 2073, Falgun

2. What is Queue ? What are the Application of Queue (1+4) 2077 Unit Test PEC

3. What is Circular Queue ? Differentiate Linear Queue and Circular Queue.

4. Define Circular Queue . Write the Enqueue and Dequeue algorithm of Circular queue .

5. Write Short Note on :
   a. Priority Queue
   • b. Queue as ADT

EXAM

UNIT - 4
RECURCION

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Introduction

The process in which a function calls itself directly or indirectly is called recursion and the corresponding function is called as recursive function.

Using recursive algorithm, certain problems can be solved quite easily. Examples of such problems are Towers of Hanoi (TOH), Inorder/Preorder/Postorder Tree Traversals, DFS of Graph, etc.

In programming languages, if a program allows you to call a function inside the same function, then it is called a recursive call of the function.

Recursive Function

A recursive function performs the tasks by dividing it into the subtasks. There is a termination condition defined in the function which is satisfied by some specific subtask.

After this, the recursion stops and the final result is returned from the function.

Recursion Vs Iteration

Criteria	Iteration	Recursion
Mode of implementation	Implemented using loops	Function calls itself
State	Defined by the control variable's value	Defined by the parameter values stored in stack
Progression	The value of control variable moves towards the value in condition	The function state converges towards the base case
Termination	Loop ends when control variable's value satisfies the condition	Recursion ends when base case becomes true
Code Size	Iterative code tends to be bigger in size	Recursion decrease the size of code
No Termination State	Infinite Loops uses CPU Cycles	Infinite Recursion may cause Stack Overflow error or it might crash the system
Execution	Execution is faster	Execution is slower

Factorial

Factorial of n

Factorial of any number **n** is denoted as **n!** and is equal to

n! = 1 x 2 x 3 x ... x (n – 2) x (n – 1) x n

Factorial of 3 3! = 1 x 2 x 3 = 6

Factorial Function using recursion

F(n) = 1 when n = 0 or 1 = F(n-1) when n > 1

So, if the value of n is either 0 or 1 then the factorial
 returned is 1.

If the value of n is greater than 1 then we call the function
with (n - 1) value.

Factorial

Example

Find 3!

We know, F(n) = 1 when n = 0 or 1 = n x F(n-1) when n > 1 So, F(3) = 3 x F(3-1) = 3 x F(2) and

 F(2) = 2 x F(2-1) = 2 x F(1) We know, F(1) = 1 Therefore, F(2) = 2 x F(1) = 2 x 1 = 2 Similarly, F(3) = 3 x F(2) = 3 x 2 = 6

Factorial Pseudo Code

```
Fact(n)
 Begin if n == 0 or 1 then
Return 1;
else
Return n*Call Fact(n-1);
 endif
 End
```

Fibonacci Pseudo Code

A **Fibonacci Series** in Java is a series of numbers in which the next number is the sum of the previous two numbers.

The first two numbers of the Fibonacci series are 0 and 1.

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

Fibonacci

0 + 1 = 1
1 + 1 = 2
1 + 2 = 3
2 + 3 = 5
3 + 5 = 8
5 + 8 = 13
....

55

34

8

5

13

21
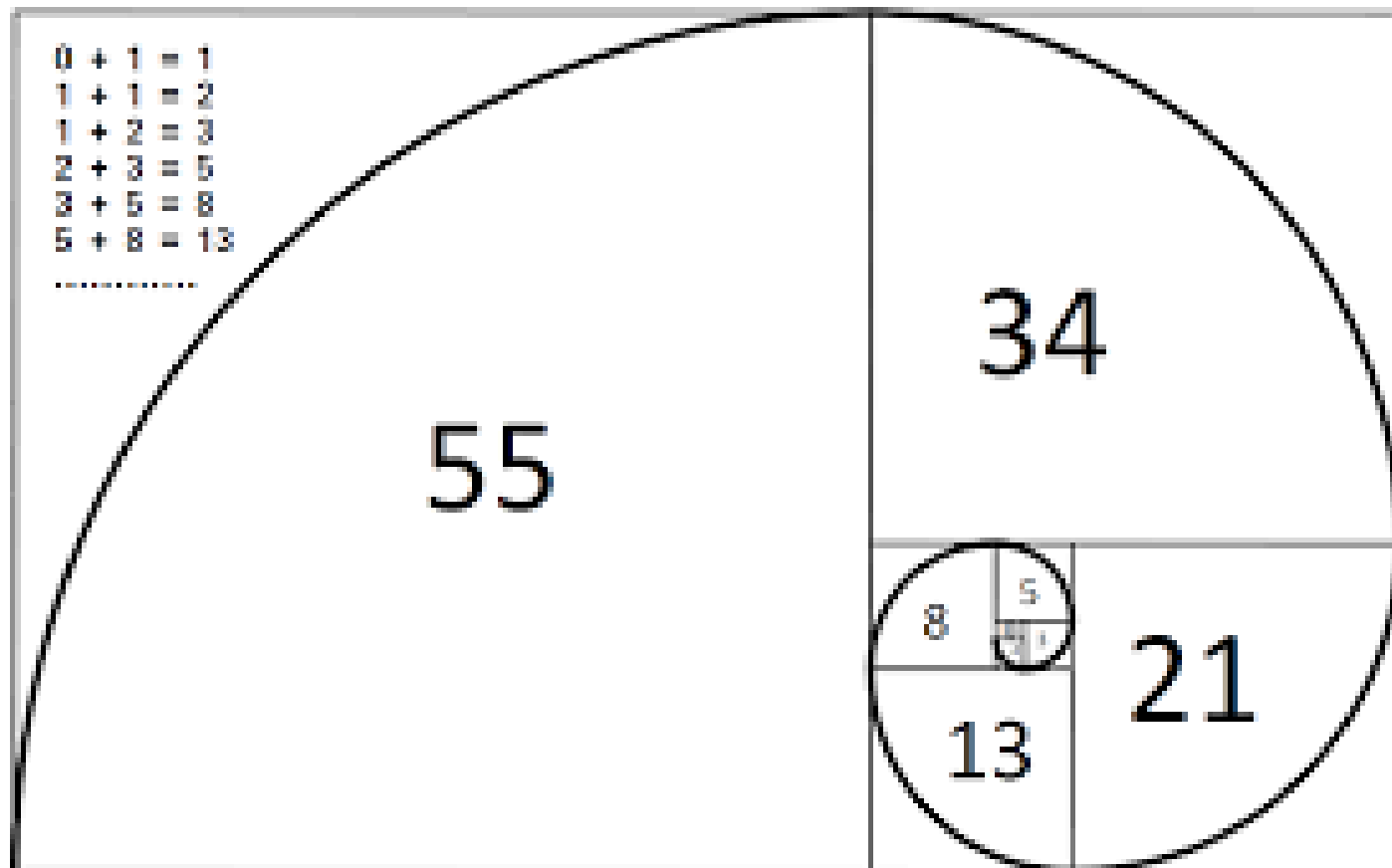
Fibonacci Pseudo Code

Fibo(n)
 Begin if n <= 1 then
Return n;
else
Return Call Fibo(n-1) + Call Fibo(n-2);
endif
 End

UNIT - 5
LINKED LISTS

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Contents

- Introduction to Linked list
- Advantages and Disadvantages of Linked List
- Types of Linked List
- Single Linked List
- Applications of Linked List
- Circular Single Linked list
- Double Linked List

Introduction to Linked List

A linked list is a collection of data in which each element contains the location of the next element—that is, each element contains two parts: data and link.
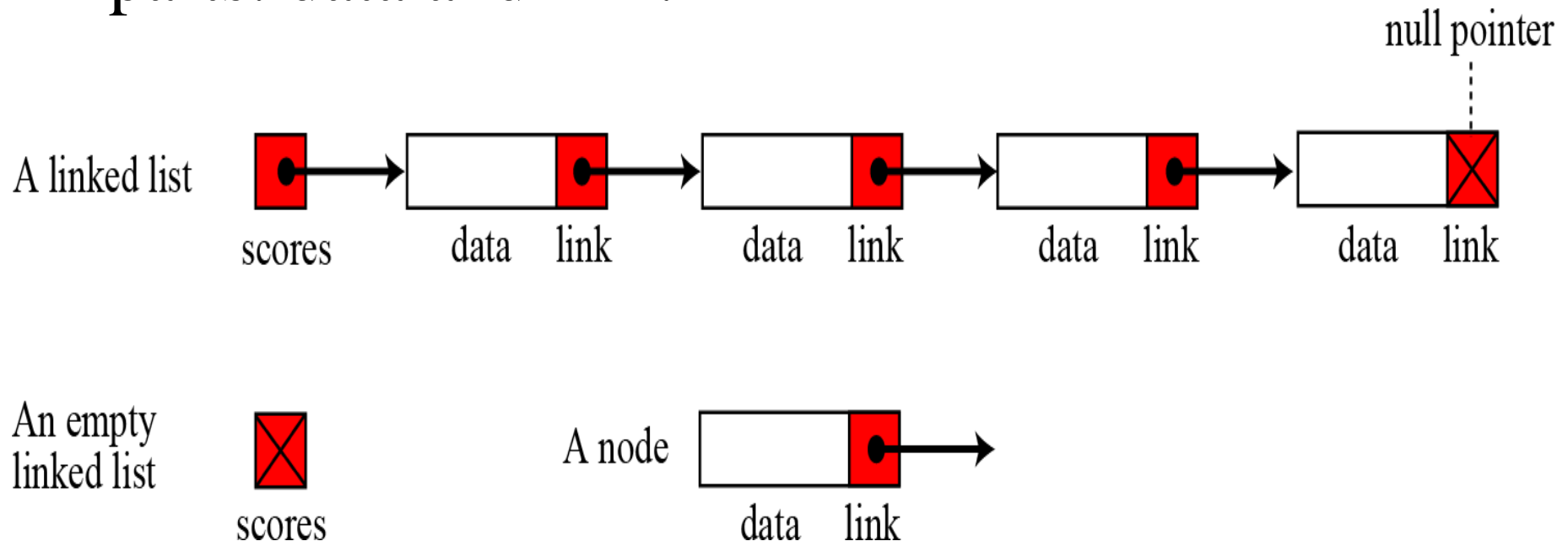
Arrays versus Linked Lists

- Both an array and a linked list are representations of a list of items in memory. The only difference is the way in which the items are linked together. The Figure below compares the two representations for a list of five integers.

	scores
scores [1]	66
scores [2]	72
scores [3]	74
scores [4]	85
scores [5]	96

a. Array representation
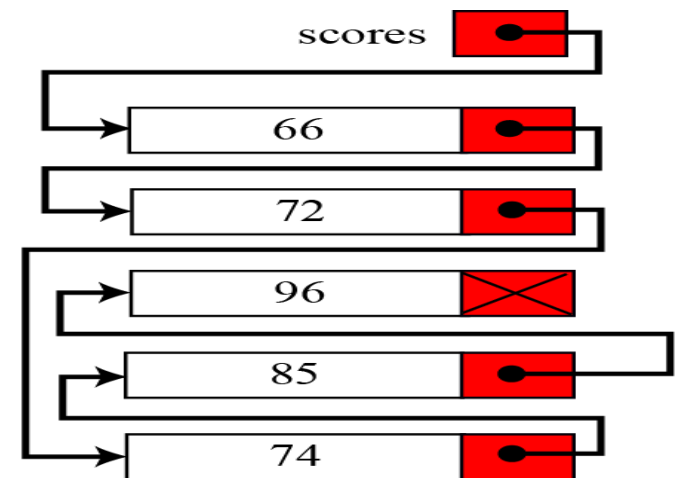
b. Linked list representation

Linked List: A Dynamic Data Structure

- A data structure that can shrink or grow during program execution.
- The size of a dynamic data structure is not necessarily known at compilation time, in most programming languages.
- Efficient insertion and deletion of elements.
- The data in a dynamic data structure can be stored in non-contiguous (arbitrary) locations.
- Linked list is an example of a dynamic data structure.

Advantages of linked list

- Unused locations in array is often a wastage of space

- Linked lists offer an efficient use of memory
  - Create nodes when they are required
  - Delete nodes when they are not required anymore
  - We don't have to know in advance how long the list should be

Applications of linked list

- Linked lists are used to represent and manipulate polynomial. Polynomials are expression containing terms with non zero coefficient and exponents. For example:

$$P(x) = a_0 X^n + a_1 X^{n-1} + \ldots\ldots + a_{n-1} X + a_n$$

- Represent very large numbers and operations of the large number such as addition, multiplication and division.

- Linked lists are to implement stack, queue, trees and graphs.

- Implement the symbol table in compiler construction.

Types of linked lists

- There are four types of Linked lists:
  - Single linked list
    - Begins with a pointer to the first node
    - Terminates with a null pointer
    - Only traversed in one direction
  - Circular single linked list
    - Pointer in the last node points back to the first node
  - Doubly linked list
    - Two —start pointers‖ – first element and last element
    - Each node has a forward pointer and a backward pointer
    - Allows traversals both forwards and backwards
  - Circular double linked list
    - Forward pointer of the last node points to the first node and backward pointer of the first node points to the last node

Singly Linked Lists

- A singly linked list is a concrete data structure consisting of a sequence of nodes
- Each node stores
  - element
  - link to the next node
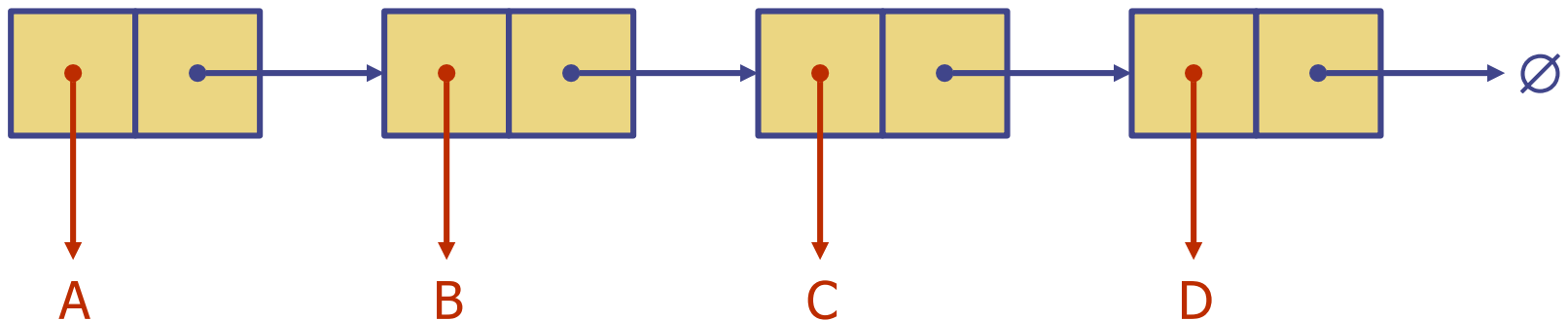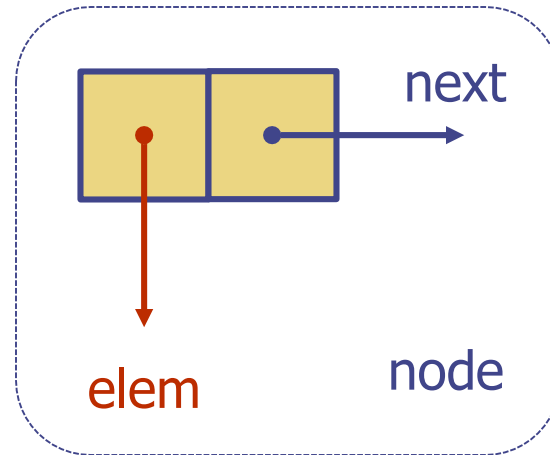
Singly Linked Lists

- A linked list allocates space for each element separately in its own block of memory called a "node".

- Each node contains two fields; a "**data**" field to store whatever element, and a "**next**" field which is a pointer used to link to the next node.

- Each node is allocated in the heap using malloc(), so the node memory continues to exist until it is explicitly de-allocated using free().

- The front of the list is a pointer to the ―start‖ node.

Single Linked List

STACK

HEAP

```
100
```
start

The start pointer holds the address of the first node of the list.

```
10 | 200      20 | 300      30 | 400      40 | X
```
100   200   300   400

Each node stores the data.

Stores the next node address.

The next field of the last node is NULL.
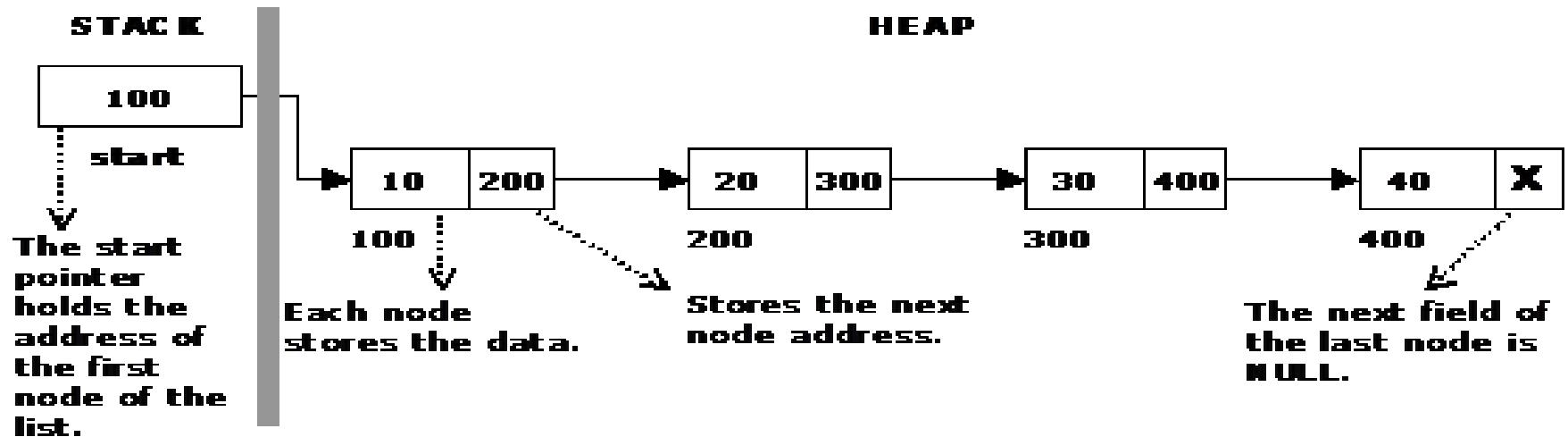
```
struct slinklist
{
        int data;
        struct slinklist* next;
};

typedef struct slinklist node;

node *start = NULL;
```

node: | data | next |

Empty list: start | NULL |

Operations on Linked Lists

- The basic operations of a single linked list are
  - Creation
  - Insertion
  - Deletion
  - Traversing

Creating a node for Single Linked List:

Sufficient memory has to be allocated for creating a node. The information is stored in the memory, allocated by using the malloc() function. The function getnode(), is used for creating a node, after allocating memory for the structure of type node, the information for the item (i.e., data) has to be read from the user, set next field to NULL and finally returns the address of the node.

newnode

10	X

100
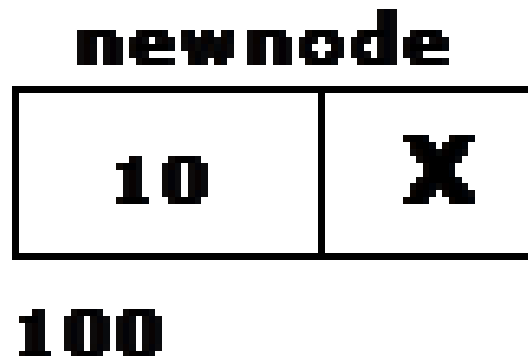
```python
class Node:
        def __init__(self,data,nextNode=None):
                self.data = data
                self.nextNode = nextNode
        def getData(self):
                return self.data
        def setData(self,val):
                self.data = val
        def getNextNode(self):
                return self.nextNode
        def setNextNode(self,val):
                self.nextNode = val
```

```python
class LinkedList:
        def __init__(self,head = None):
                self.head = head
                self.size = 0
        def getSize(self):
                return self.size
        def addNode(self,data):
                newNode = Node(data,self.head)
                self.head = newNode
                self.size+=1
                return True
        def printNode(self):
                curr = self.head
                while curr:
                        print(curr.data)
                        curr = curr.getNextNode()
```

Creating a single linked list with N nodes
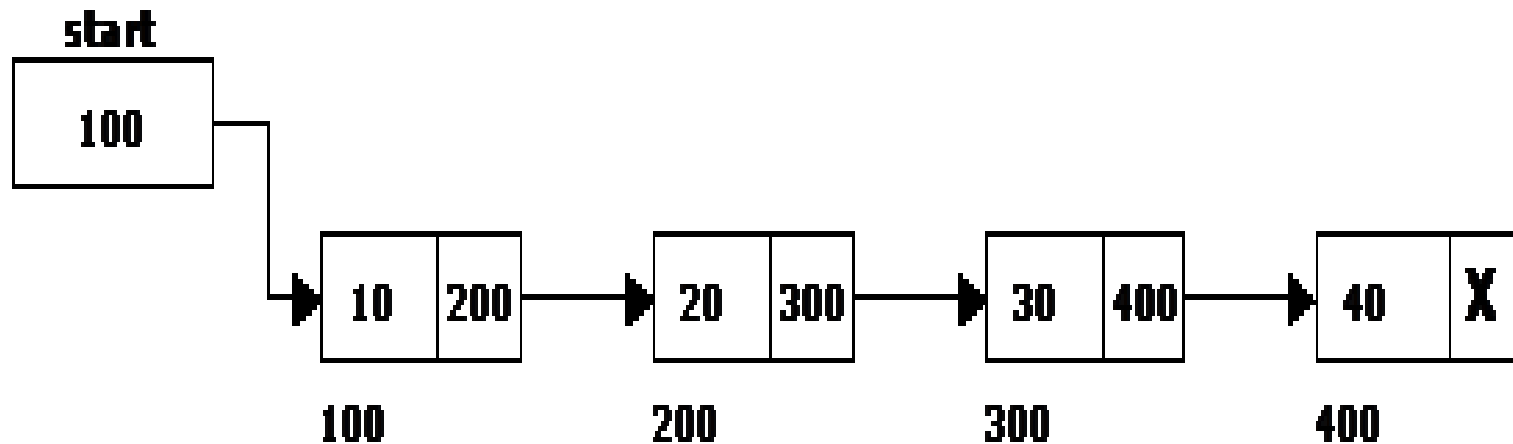
Inserting a node

- Inserting a node into a single linked list can be done at
  - Inserting into an empty list.
  - Insertion at the beginning of the list.
  - Insertion at the end of the list.
  - Insertion in the middle of the list.

Inserting a node at the beginning

The following steps are to be followed to insert a new node at the beginning of the list:

#Function to insert a new node at the   beginning

def push(self, new_data):

    # Allocate the Node & Put in the data

    new_node = Node(new_data)

#Make next of new Node as head

    new_node.next = self.head

    # Move the head to point to new Node

    self.head = new_node

Inserting a node at the beginning
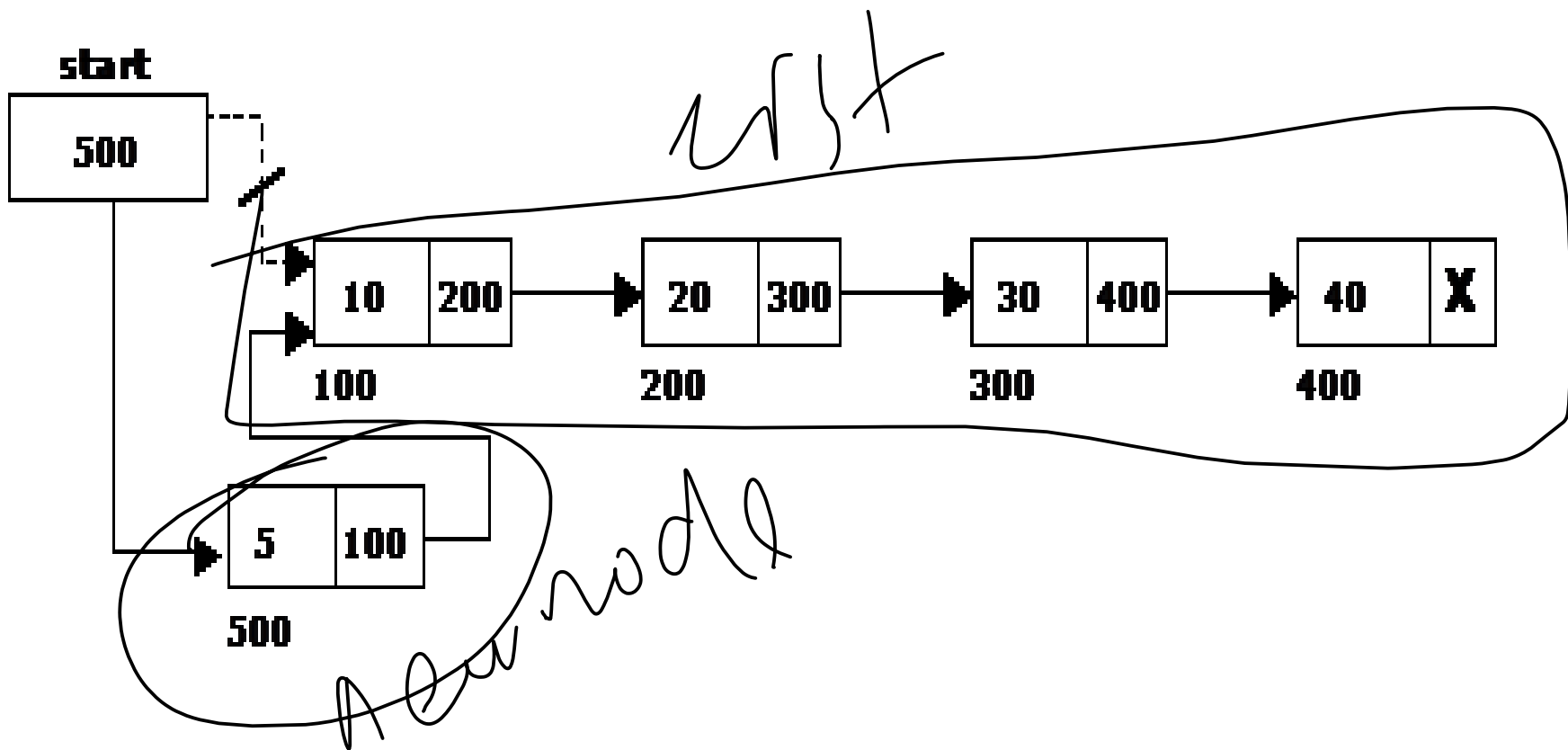
Inserting a node at the end

- The following steps are followed to insert a new node at the end of the list:
  ```
  # This function is defined in Linked List class
  # Appends a new node at the end.  This method
  is defined inside LinkedList class shown above
  def append(self, new_data):
      # Create a new node, Put in the data, Set next
        as None
      new_node = Node(new_data)
  ```

Inserting a node at the end

```python
# If the Linked List is empty, then make the
    new node as head
if self.head is None:
        self.head = new_node
        return
#Else traverse till the last node
last = self.head
while last.next:
        last = last.next
# Change the next of last node
last.next =  new_node
```

Inserting a node at the end
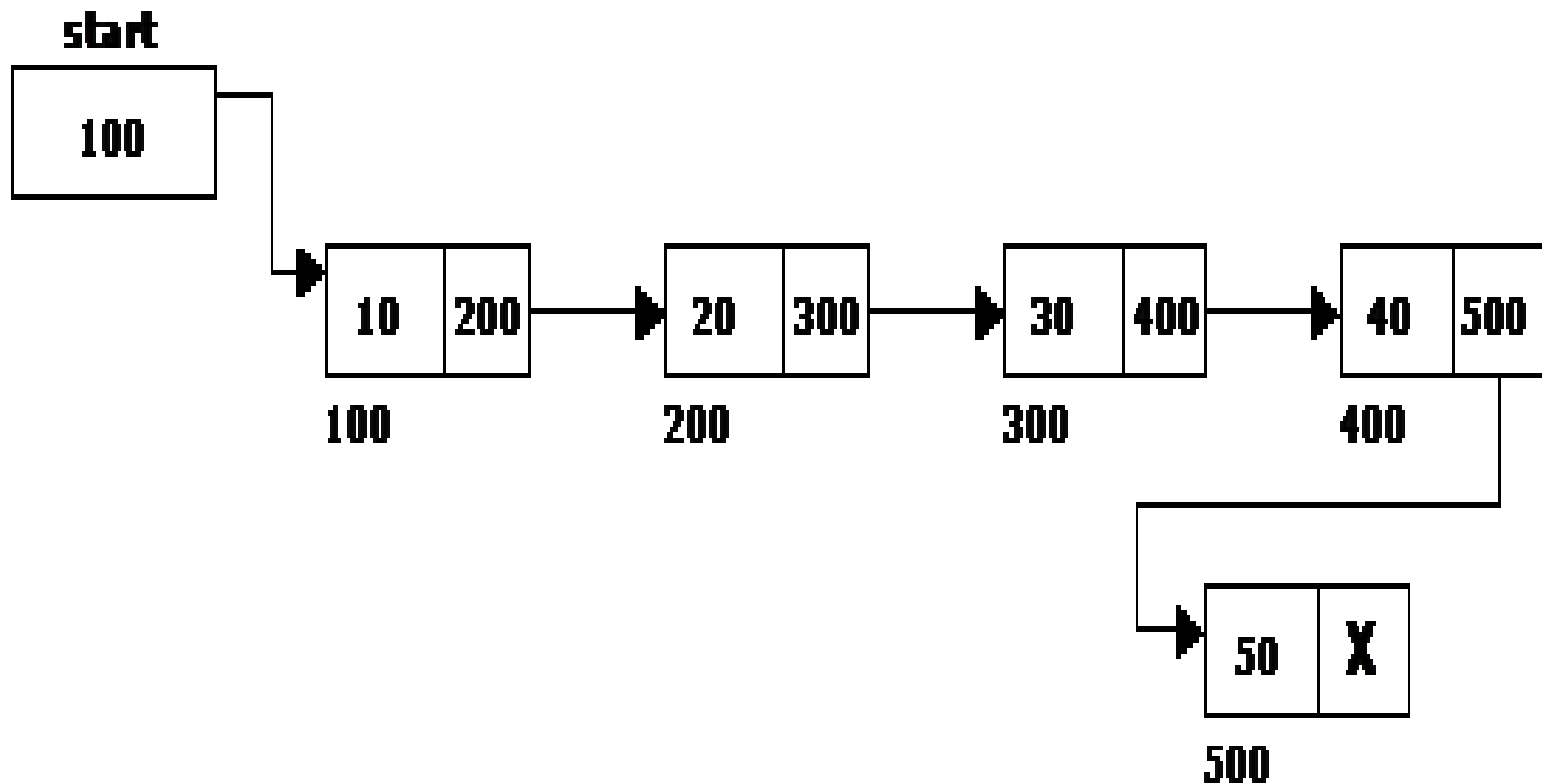
Inserting a node at intermediate position

- The following steps are followed, to insert a new node after the given previous node in the list:

  def insertAfter(self, prev_node, new_data):

  #check if the given prev_node exists

  if prev_node is None:

  print(―The given previous node must in Linked List.‖)

  return

  #Create new node & Put in the data

  new_node = Node(new_data)

  # Make next of new Node as next of prev_node

  new_node.next = prev_node.next

```
#Make next of prev_node as new_node
prev_node.next = new_node
```

Inserting a node at intermediate position
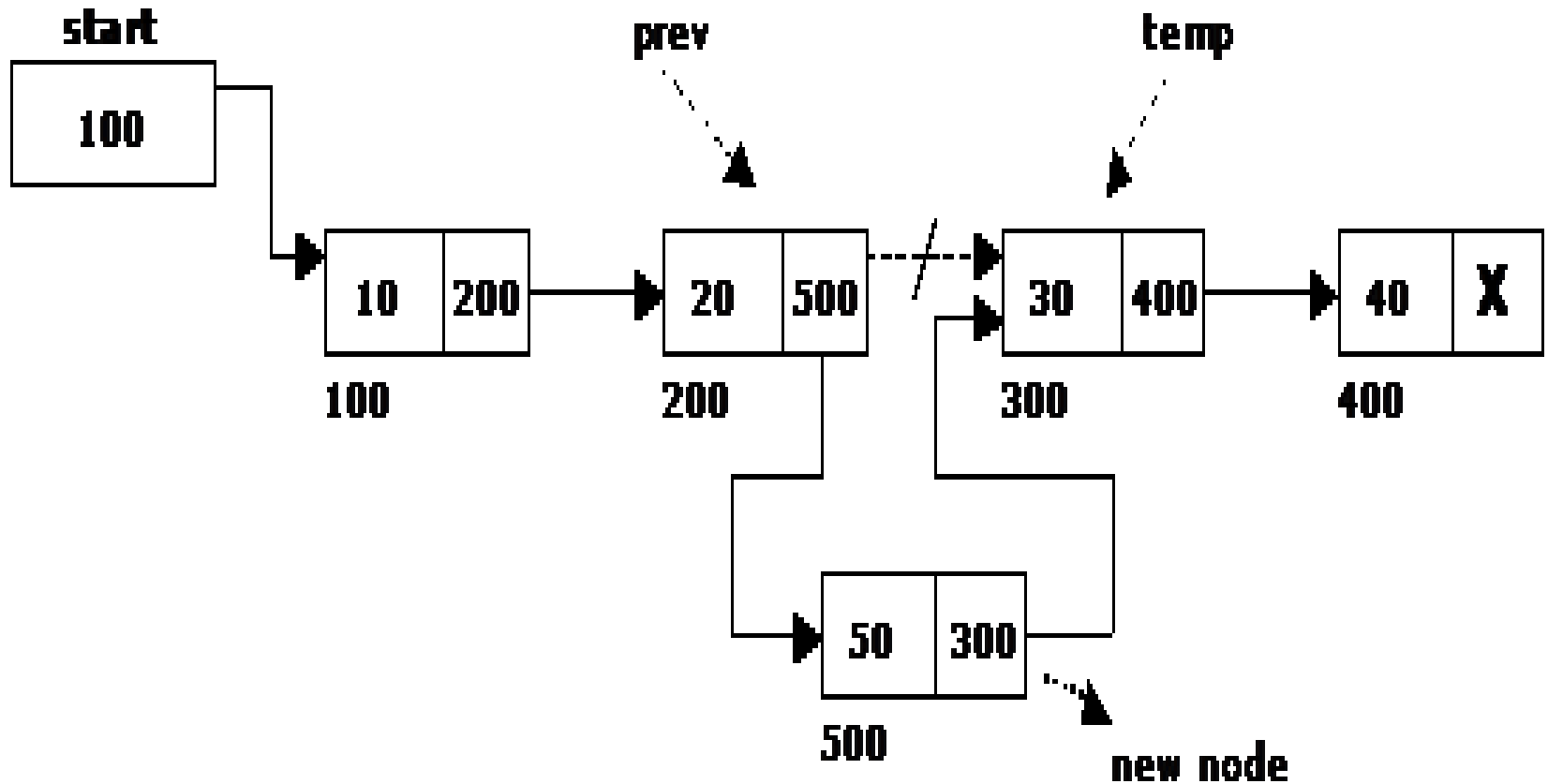
Deletion of a node

- Another primitive operation that can be done in a singly linked list is the deletion of a node. Memory is to be released for the node to be deleted. A node can be deleted from the list from three different places namely.
  - Deleting a node at the beginning.
  - Deleting a node at the end.
  - Deleting a node at intermediate position.

Deleting a node at the beginning

- The following steps are followed, to delete a node at the beginning of the list:

Deleting a node at the beginning
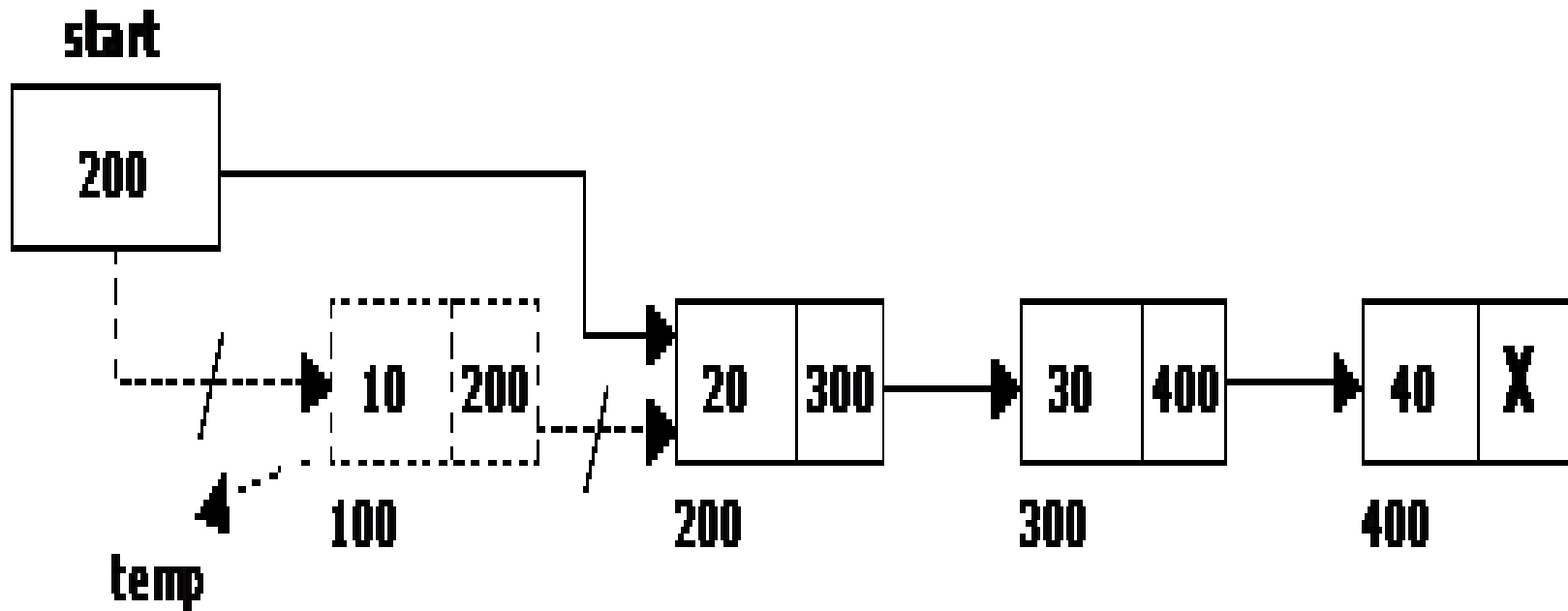
Deleting a node at the end

- The following steps are followed to delete a node at the end of the list:
  - If list is empty then display ‗Empty List'message.
  - If the list is not empty, follow the steps given below:

    temp = prev = start;
    while(temp -> next != NULL)
    {
           prev = temp;
           temp = temp -> next;
    }
    prev -> next = NULL;
    free(temp);

Deleting a node at the end
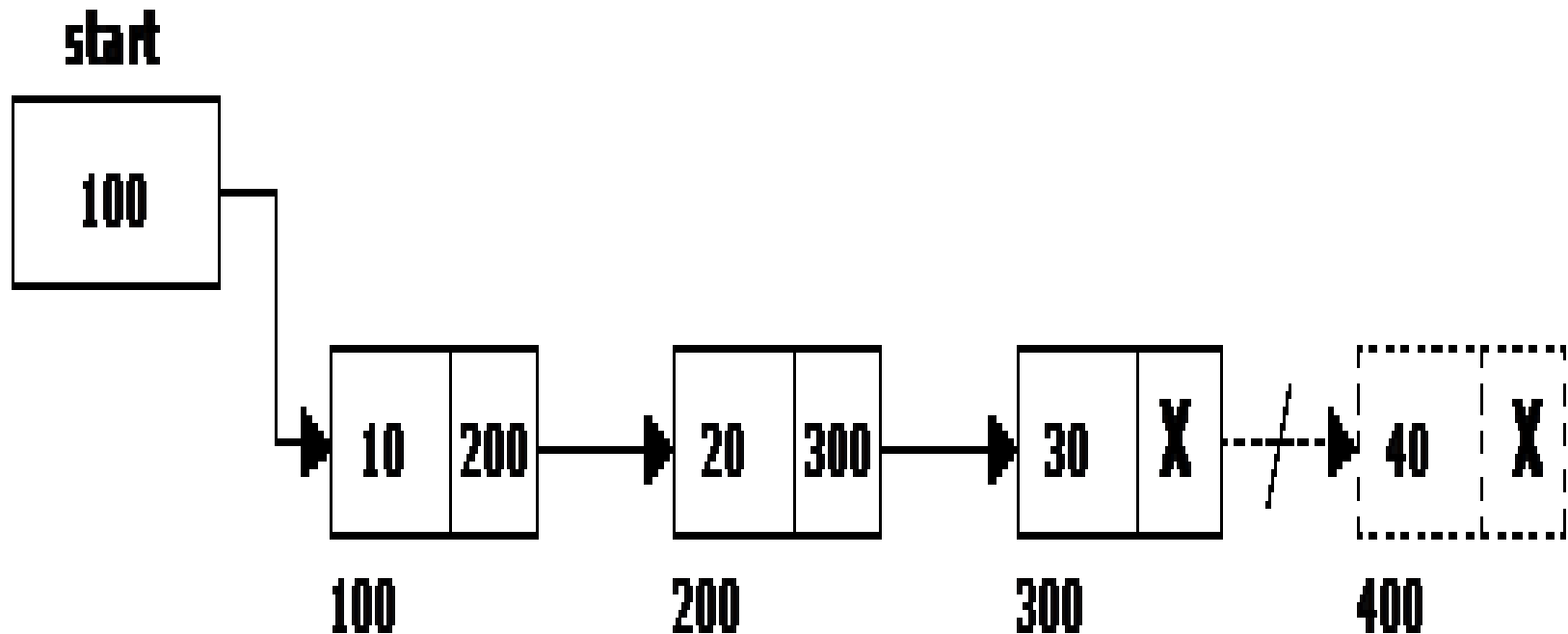
Deleting a node at Intermediate position

- The following steps are followed, to delete a node from an intermediate position in the list:

  # Given a reference to the head of a list and a

  position, delete the node at a given position

  def deleteNode(self, position):

      # If linked list is empty

      if self.head == None:

        return

      # Store head node

      temp = self.head

```python
    # If head needs to be removed
    if position == 0:
                    self.head = temp.next
            temp = None
            return
 # Find previous node of the node to be
 deleted
for i in range(position -1 ):
    temp = temp.next
    if temp is None:
        break
```

```python
    # If position is more than number of nodes
    if temp is None:
        return
    if temp.next is None:
        return
    # Node temp.next is the node to be deleted
    # store pointer to the next of node to be deleted
    next = temp.next.next
    # Unlink the node from linked list
    temp.next = None
    temp.next=next
```

```python
# Find previous node of the node to be deleted
    for i in range(position -1 ):
        temp = temp.next
        if temp is None:
            break

    # If position is more than number of nodes
    if temp is None:
        return
    if temp.next is None:
        return
```

Deleting a node at Intermediate position
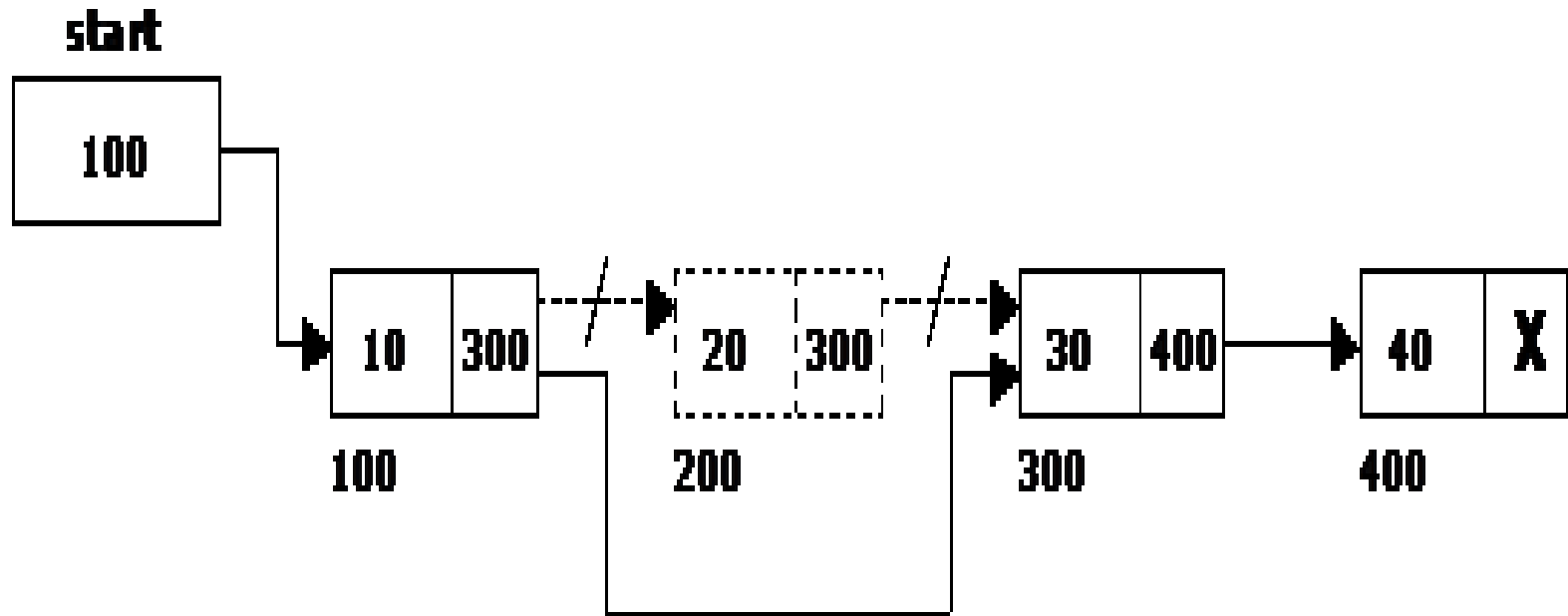
Traversal and displaying a list

- To display the information, you have to traverse (move) a linked list, node by node from the first node, until the end of the list is reached. Traversing a list involves the following steps:

  - Assign the address of start pointer to a temp pointer.

  - Display the information from the data field of each node.

Double Linked List

- A double linked list is a two-way list in which all nodes will have two links. This helps in accessing both successor node and predecessor node from the given node position. It provides bi-directional traversing. Each node contains three fields:
  - Left link.
  - Data.
  - Right link.

- The left link points to the predecessor node and the right link points to the successor node. The data field stores the required data.

A Double Linked List

STACK

HEAP

Stores the previous node address.

100

start

The start pointer holds the address of the first node of the list.

X	10	200

100

100	20	300

200

200	30	X

300

Stores the data.

Stores the next node address.

The right field of the last node is NULL.
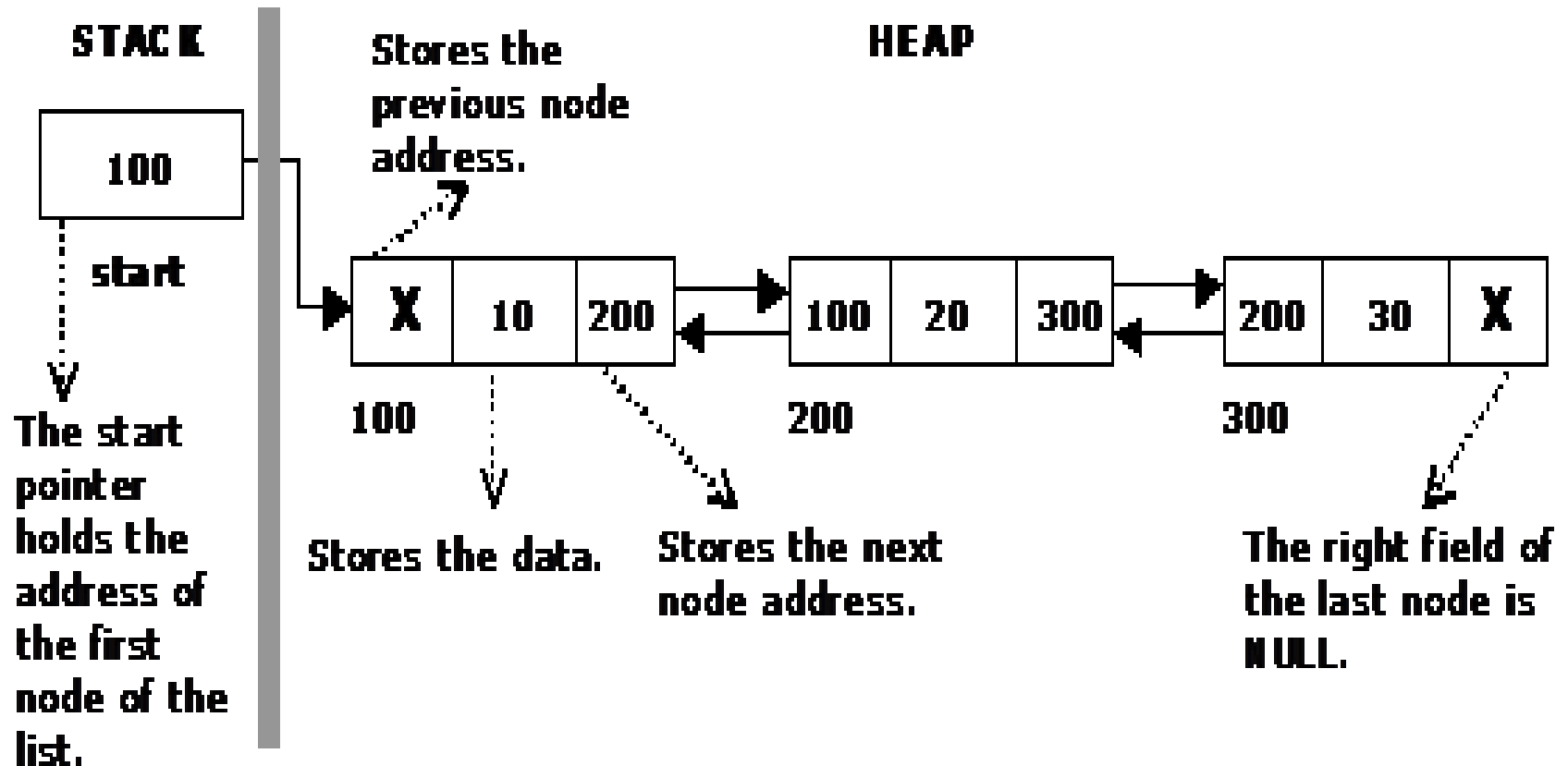
Basic operations in a double linked list

- Creation
- Insertion
- Deletion
- Traversing

The beginning of the double linked list is stored in a "**start**" pointer which points to the first node. The first node's left link and last node's right link is set to NULL.

Structure of a Double Linked List

```
struct dlinklist
{
        struct dlinklist *left;
        int data;
        struct dlinklist *right;

};

typedef struct dlinklist node;
node *start = NULL;
```
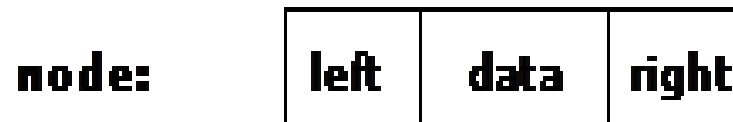
node:

left	data	right

Empty list:

start

NULL

Creating a Double Linked List with N number of nodes

- The following steps are to be followed to create _n' number of nodes:

```
class Node(object):
        def __init__(self, data, prev, next):
            self.data = data
            self.prev = prev
            self.next = next


    class DoubleList(object):
        head = None
        tail = None
```

Creating a Double Linked List with N number of nodes
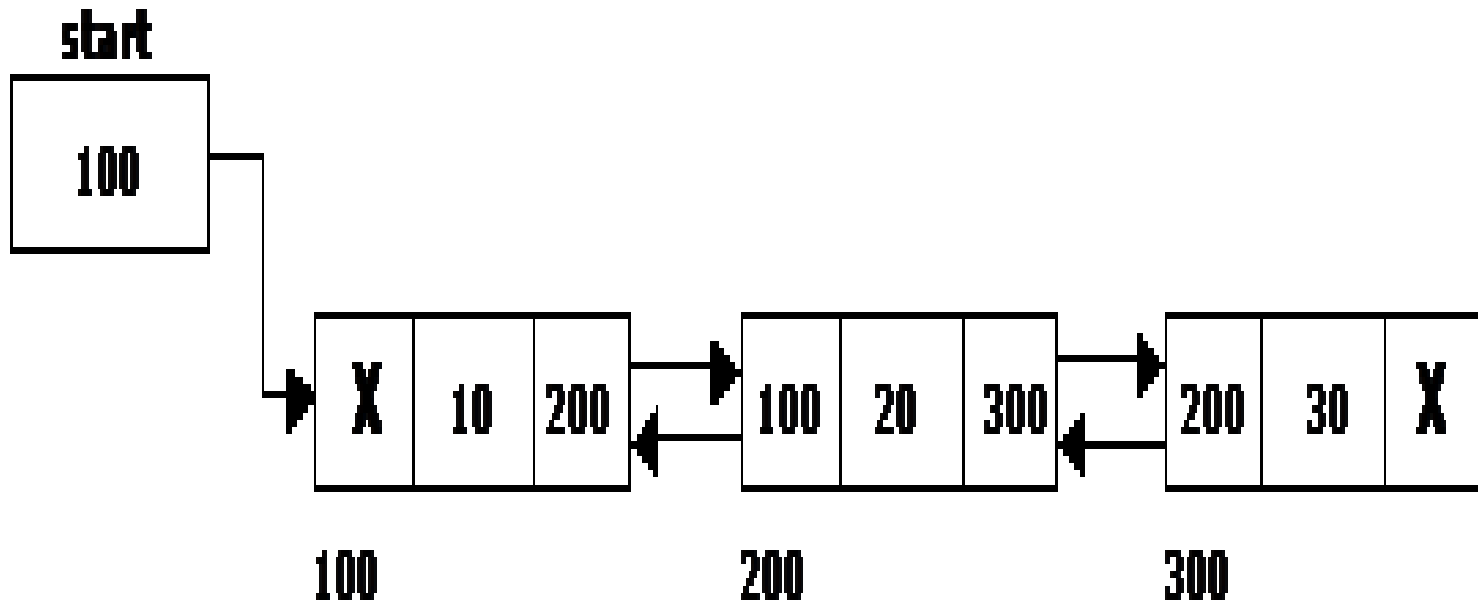
Inserting a node at the beginning

- The following steps are to be followed to insert a new node at the beginning of the list:
- Get the new node using getnode().

  getnode().

  newnode=getnode();
- If the list is empty then *start = newnode.*
- If the list is not empty, follow the steps given below:

  newnode -> right = start;

  start -> left = newnode;

  start = newnode;

Inserting a node at the beginning
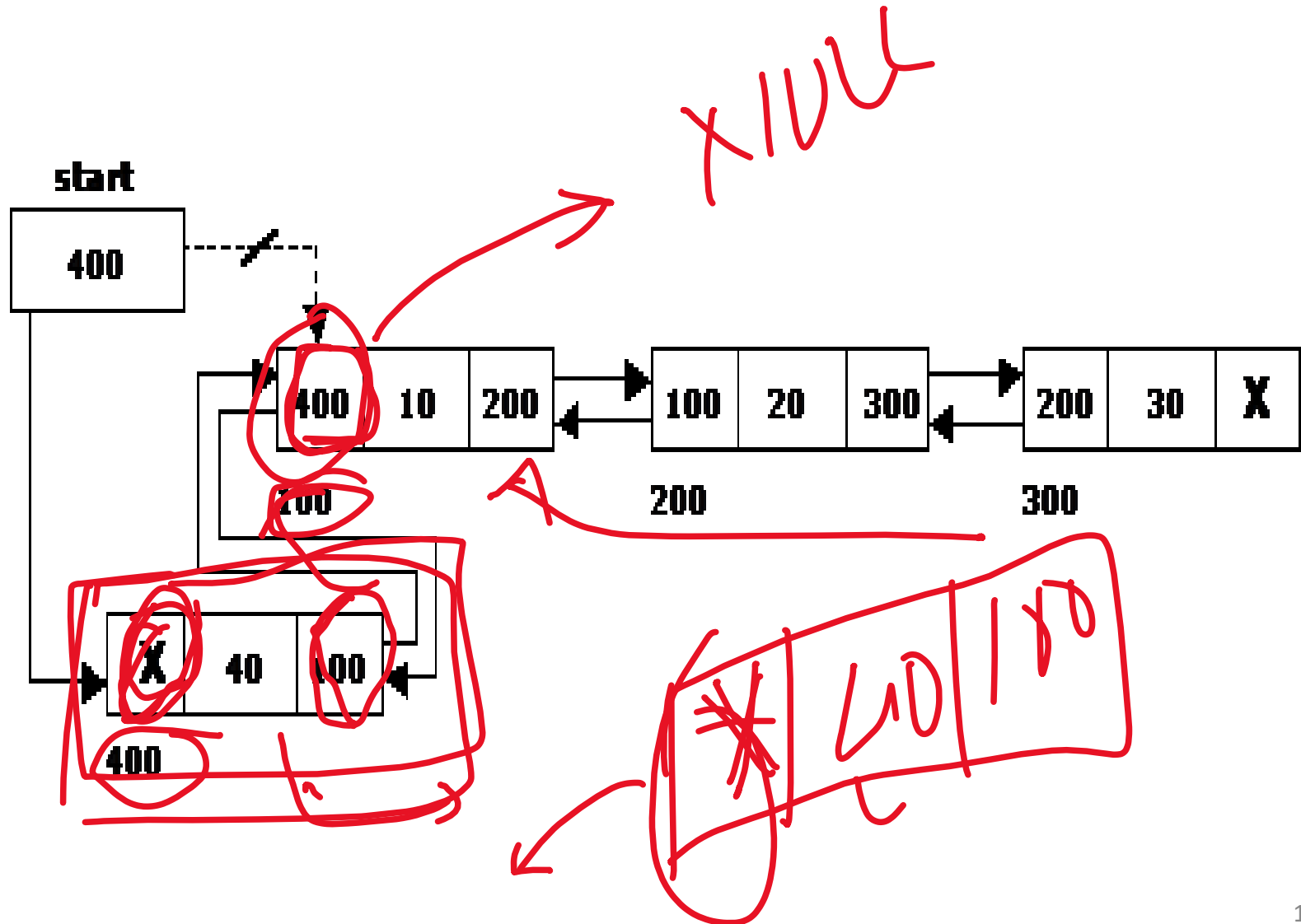
Inserting a node at the end

- The following steps are followed to insert a new node at the end of the list:
- Get the new node using getnode()
  newnode=getnode();
- If the list is empty then *start = newnode.*
- If the list is not empty follow the steps given
  below: temp = start;
  while(temp -> right != NULL)
          temp = temp -> right;
  temp -> right = newnode;
  newnode -> left = temp;

Inserting a node at the end
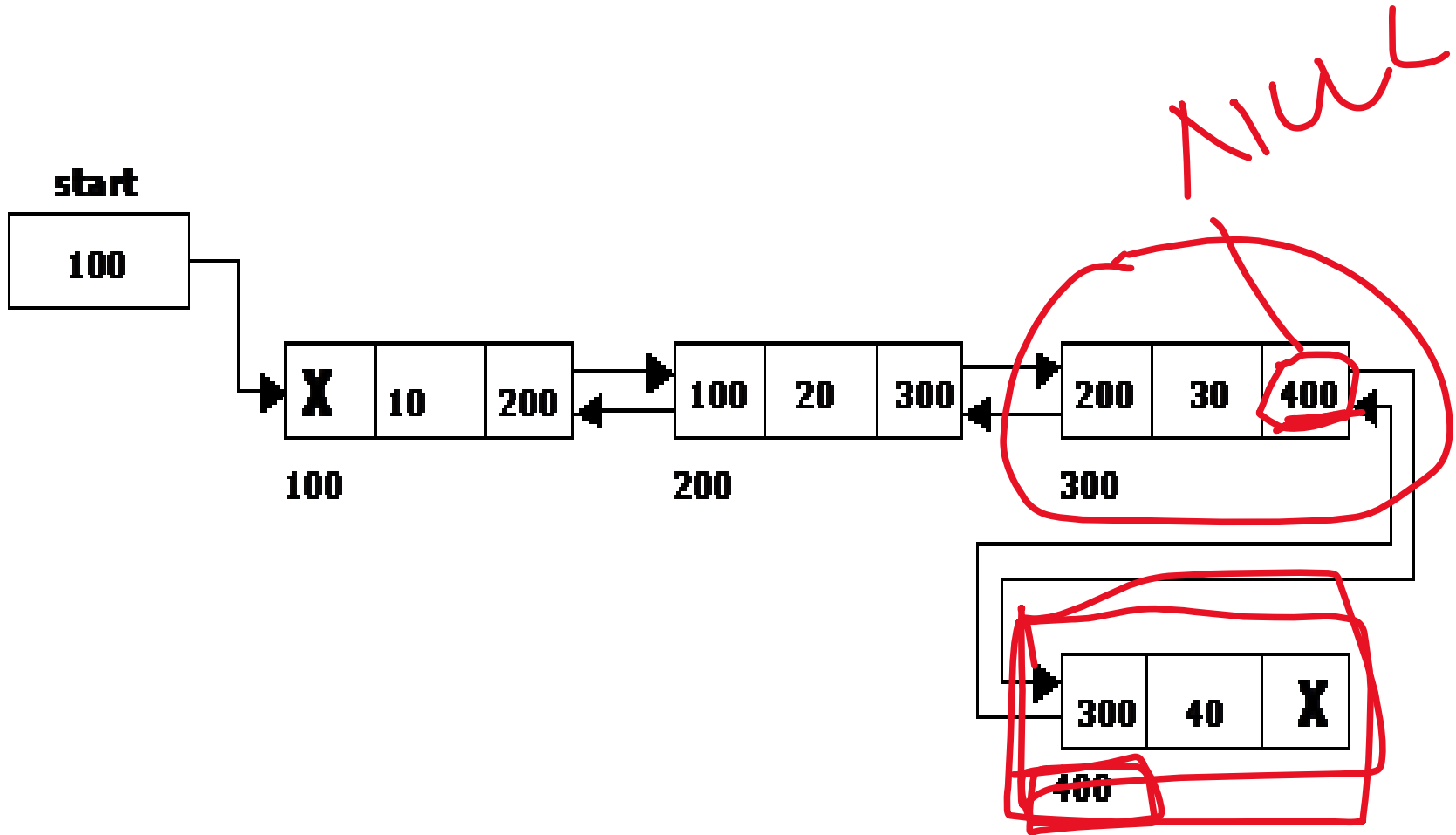
Inserting a node at an intermediate position

- The following steps are followed, to insert a new node in an intermediate position in the list:

-       Get the new node using

  getnode().

  newnode=getnode();

- Ensure that the specified position is in between first node and last node. If not, specified position is invalid. This is done by countnode() function.

- Store the starting address (which is in start pointer) in temp and prev pointers. Then traverse the temp pointer upto the specified position followed by prev pointer.

-       After reaching the specified position, follow the steps given

  below: newnode -> left = temp;

  newnode -> right = temp -> right;

  temp -> right -> left = newnode;

  temp -> right = newnode;

Inserting a node at an intermediate position
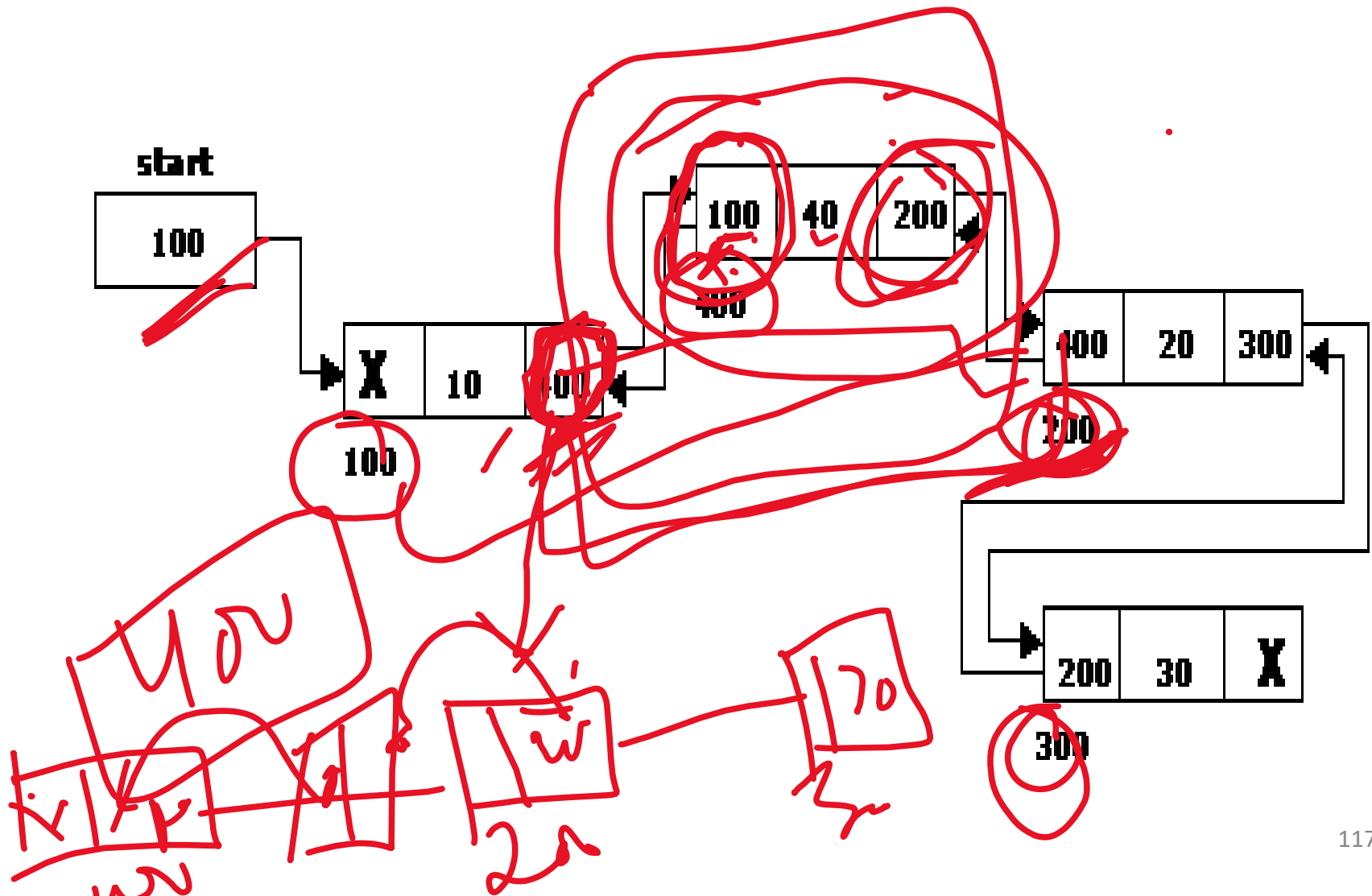
Deleting a node at the beginning

- The following steps are followed, to delete a node at the beginning of the list:

- If list is empty then display ‗Empty List‘ message.

- If the list is not empty, follow the steps given below:

        temp = start;
        start = start -> right;
        start -> left = NULL;
        free(temp);

Deleting a node at the beginning
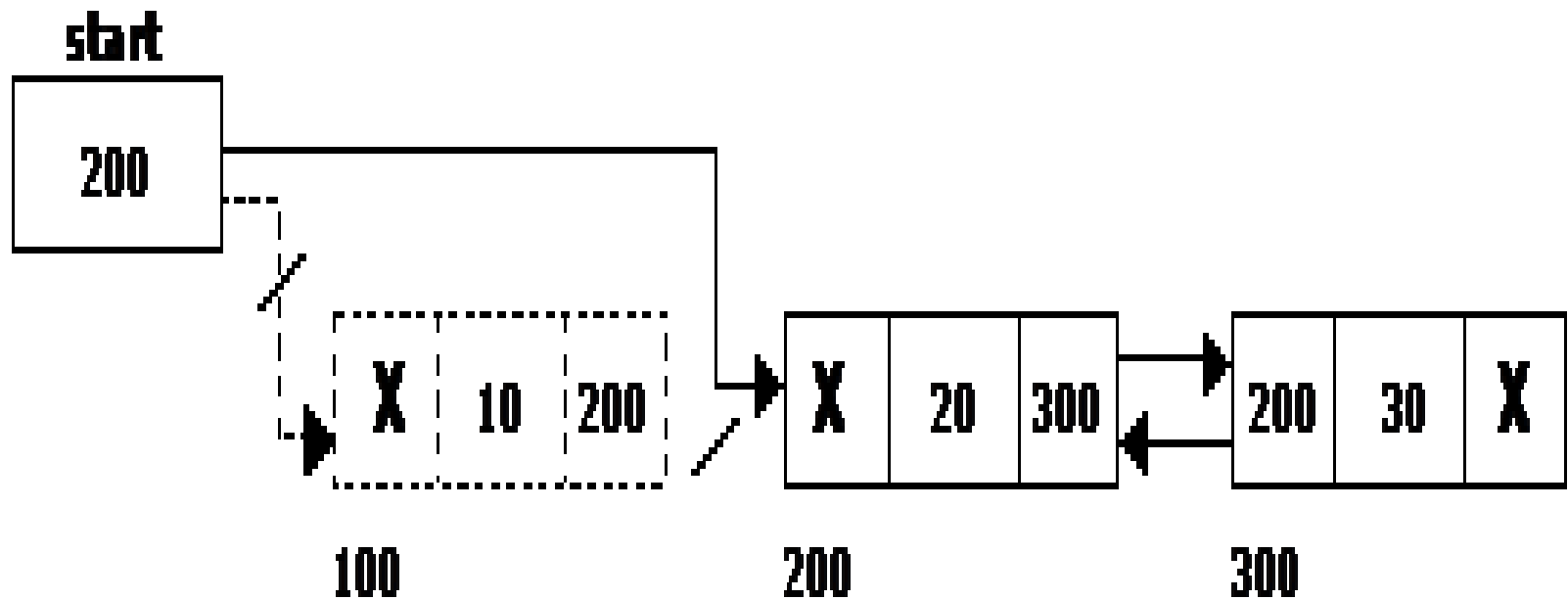
Deleting a node at the end

- The following steps are followed to delete a node at the end of the list:
  - If list is empty then display ‗Empty List' message
  - If the list is not empty, follow the steps given below:

    temp = start;
    while(temp -> right != NULL)
    {
            temp = temp -> right;
    }
    temp -> left -> right = NULL;
    free(temp);

Deleting a node at the end
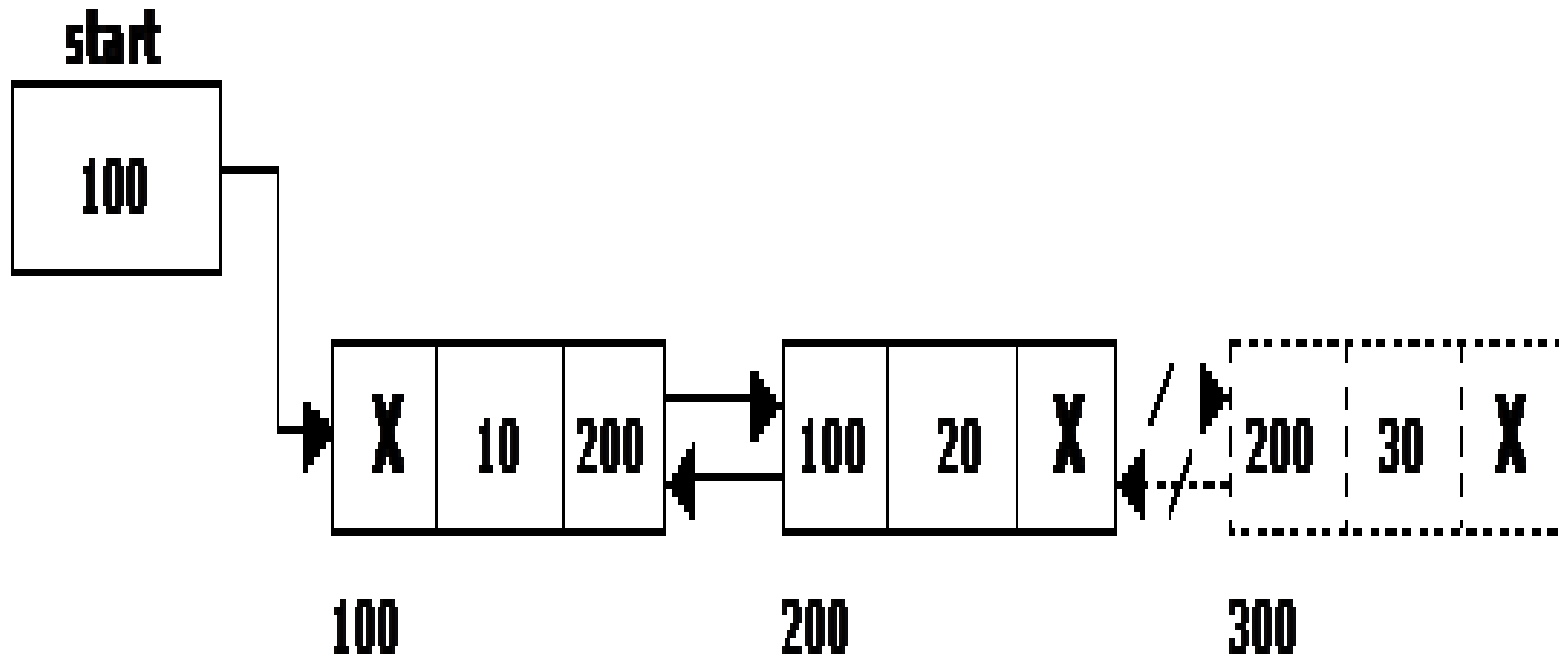
Deleting a node at Intermediate position

- The following steps are followed, to delete a node from an intermediate position in the list.
- If list is empty then display _Empty List' message.
- If the list is not empty, follow the steps given below:
  - Get the position of the node to delete.
  - Ensure that the specified position is in between first node and last node. If not, specified position is invalid.
- Then perform the following steps:

```
if(pos > 1 && pos < nodectr)
{
        temp = start;
        i = 1;
        while(i < pos)
        {
                temp = temp -> right;
                i++;
        }
        temp -> right -> left = temp -> left;
        temp -> left -> right = temp -> right;
        free(temp);
        printf("\n node deleted..");
}
```

Deleting a node at Intermediate position
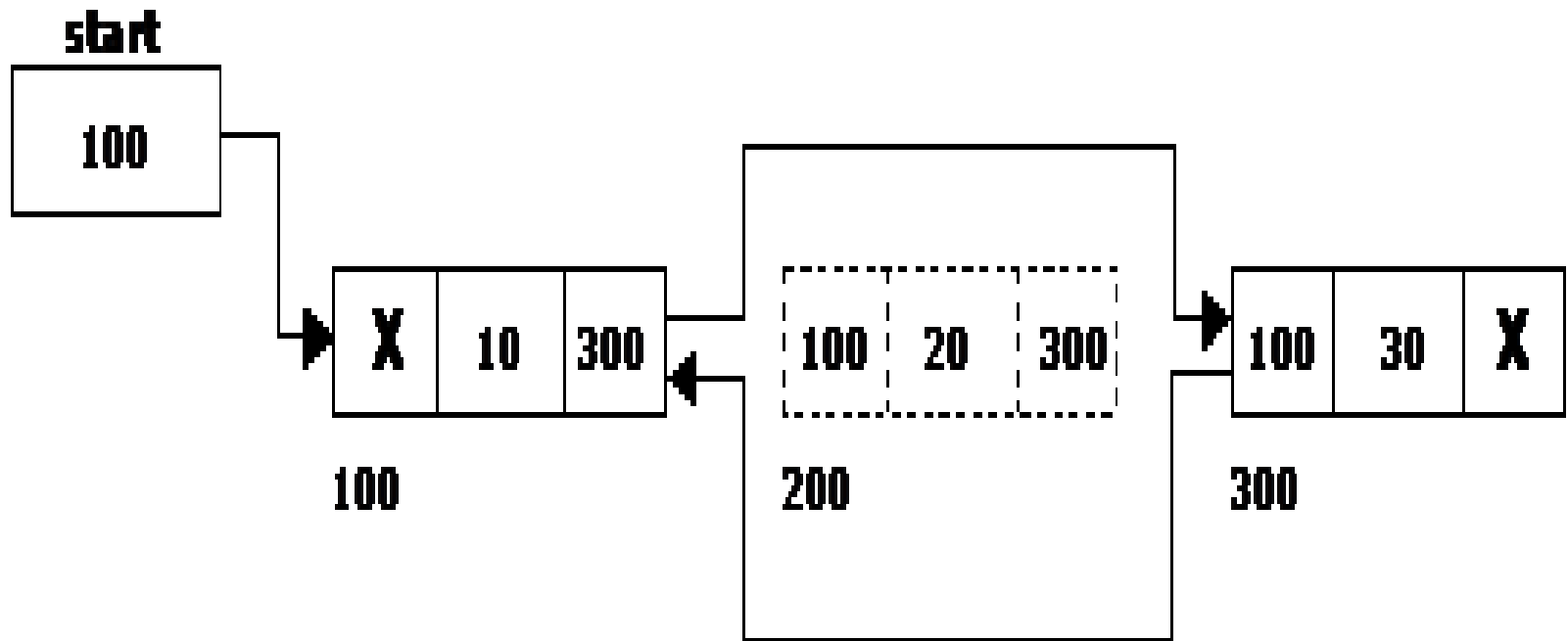
Traversal and displaying a list (Left to Right)

- The following steps are followed, to traverse a list from left to right:

- If list is empty then display ‗Empty List' message.

-        If the list is not empty, follow the steps given

below: temp = start;

while(temp != NULL)

{

        print temp -> data;

        temp = temp -> right;

}

Traversal and displaying a list (Right to Left)

- The following steps are followed, to traverse a list from right to left:

- If list is empty then display ‗Empty List' message.

- If the list is not empty, follow the steps given
  below: temp = start;

  ```
  while(temp -> right != NULL)
  temp = temp -> right;
  while(temp != NULL)
  {
  print temp -> data;
  temp = temp -> left;
  }
  ```

Advantages and Disadvantages of Double Linked List

- The **major disadvantage** of doubly linked lists (over singly linked lists) is that they require more space (every node has two pointer fields instead of one). Also, the code to manipulate doubly linked lists needs to maintain the *prev* fields as well as the *next* fields; the more fields that have to be maintained, the more chance there is for errors.

- The **major advantage** of doubly linked lists is that they make some operations (like the removal of a given node, or a right-to-left traversal of the list) more efficient.

Circular Single Linked List

- It is just a single linked list in which the link field of the last node points back to the address of the first node.

- A circular linked list has no beginning and no end. It is necessary to establish a special pointer called *start* pointer always pointing to the first node of the list.

- Circular linked lists are frequently used instead of ordinary linked list because many operations are much easier to implement. In circular linked list no null pointers are used, hence all pointers contain valid address.

Circular Single Linked List and its basic operations
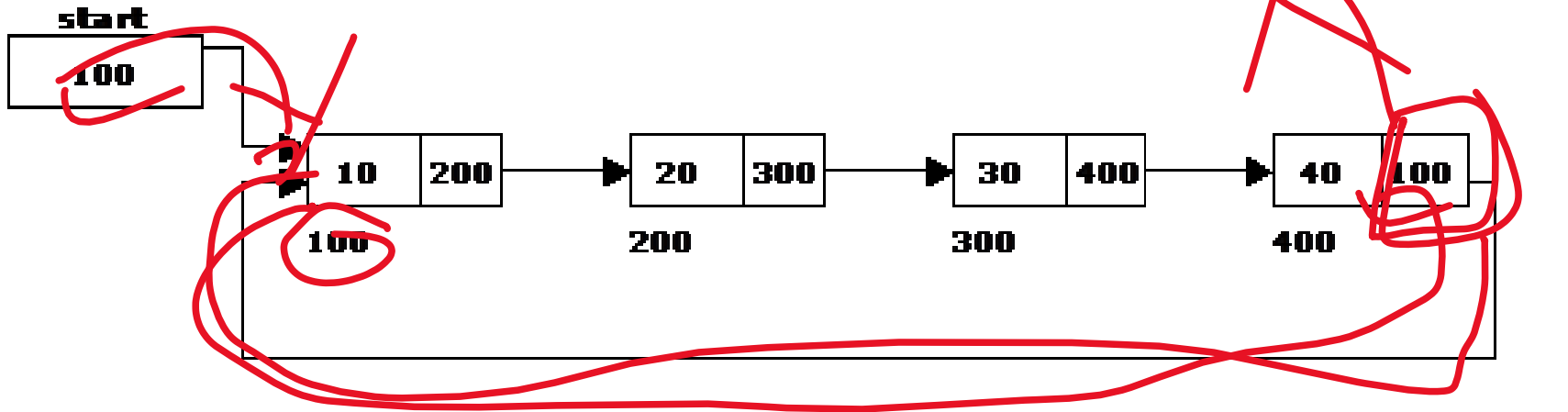
The basic operations in a circular single linked list are:
- Creation
- Insertion
- Deletion
- Traversing

Creating a circular single Linked List with N number of nodes

- The following steps are to be followed to create _n' number of nodes:

- Get the new node using getnode().
     newnode = getnode();

-      If the list is empty, assign new node as
     start. start = newnode;

-   If the list is not empty, follow the steps given below:
  temp = start;
     while(temp -> next != NULL)
             temp = temp -> next;
  temp -> next = newnode;

-      Repeat the above steps _n'
     times. newnode -> next =
     start;

Inserting a node at the beginning

- The following steps are to be followed to insert a new node at the beginning of the circular list:
- Get the new node using getnode().

  newnode = getnode();

-      If the list is empty, assign new node as

  start. start = newnode;

  newnode -> next = start;

-      If the list is not empty, follow the steps given

  below: last = start;

  while(last -> next != start)

          last = last -> next;

  newnode -> next = start;

  start = newnode;

  last -> next = start;

Inserting a node at the beginning
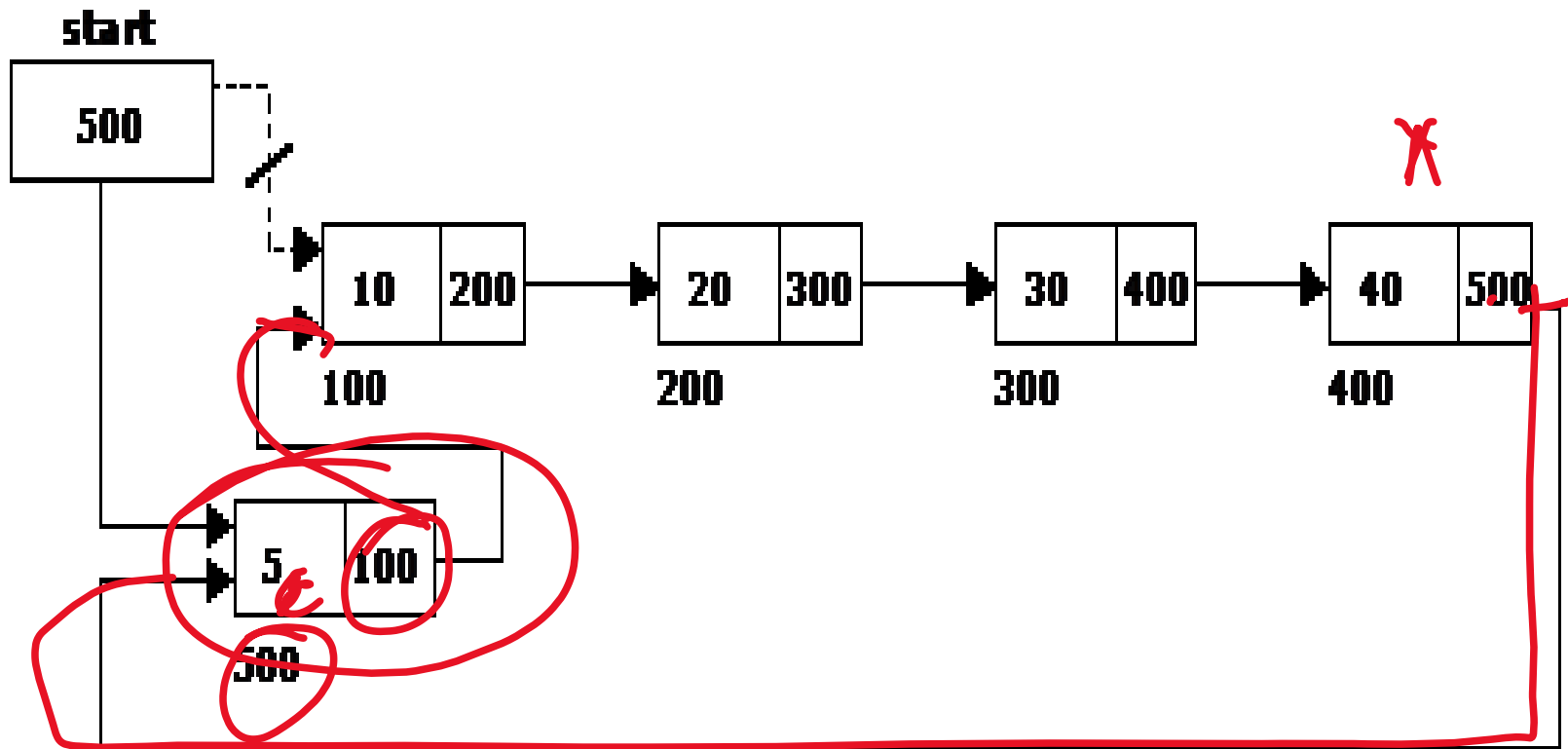
Inserting a node at the end

- The following steps are followed to insert a new node at the end of the list:
- Get the new node using
  getnode(). newnode =
  getnode();
- If the list is empty, assign new node as
  start. start = newnode;
  newnode -> next = start;
- If the list is not empty follow the steps given
  below: temp = start;
  while(temp -> next != start)
          temp = temp -> next;
  temp -> next = newnode;
  newnode -> next = start;
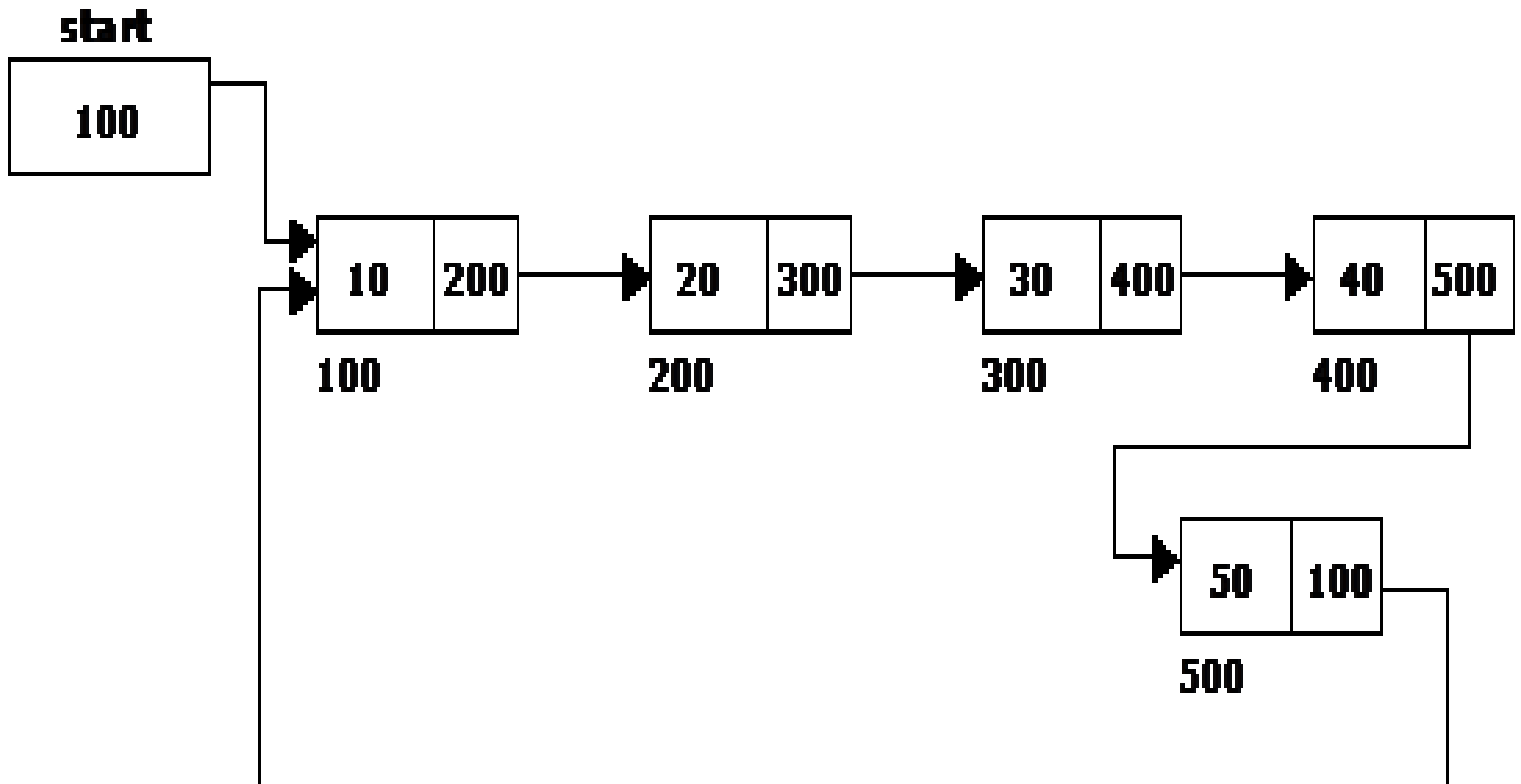
Inserting a node at the end

Deleting a node at the beginning

- The following steps are followed, to delete a node at the beginning of the list:
- If the list is empty, display a message 'Empty List'.
- If the list is not empty, follow the steps given

    below: last = temp = start;

    while(last -> next != start)

    last = last -> next;

    start = start -> next;

    last -> next = start;

- After deleting the node, if the list is empty then *start = NULL.*

Deleting a node at the beginning
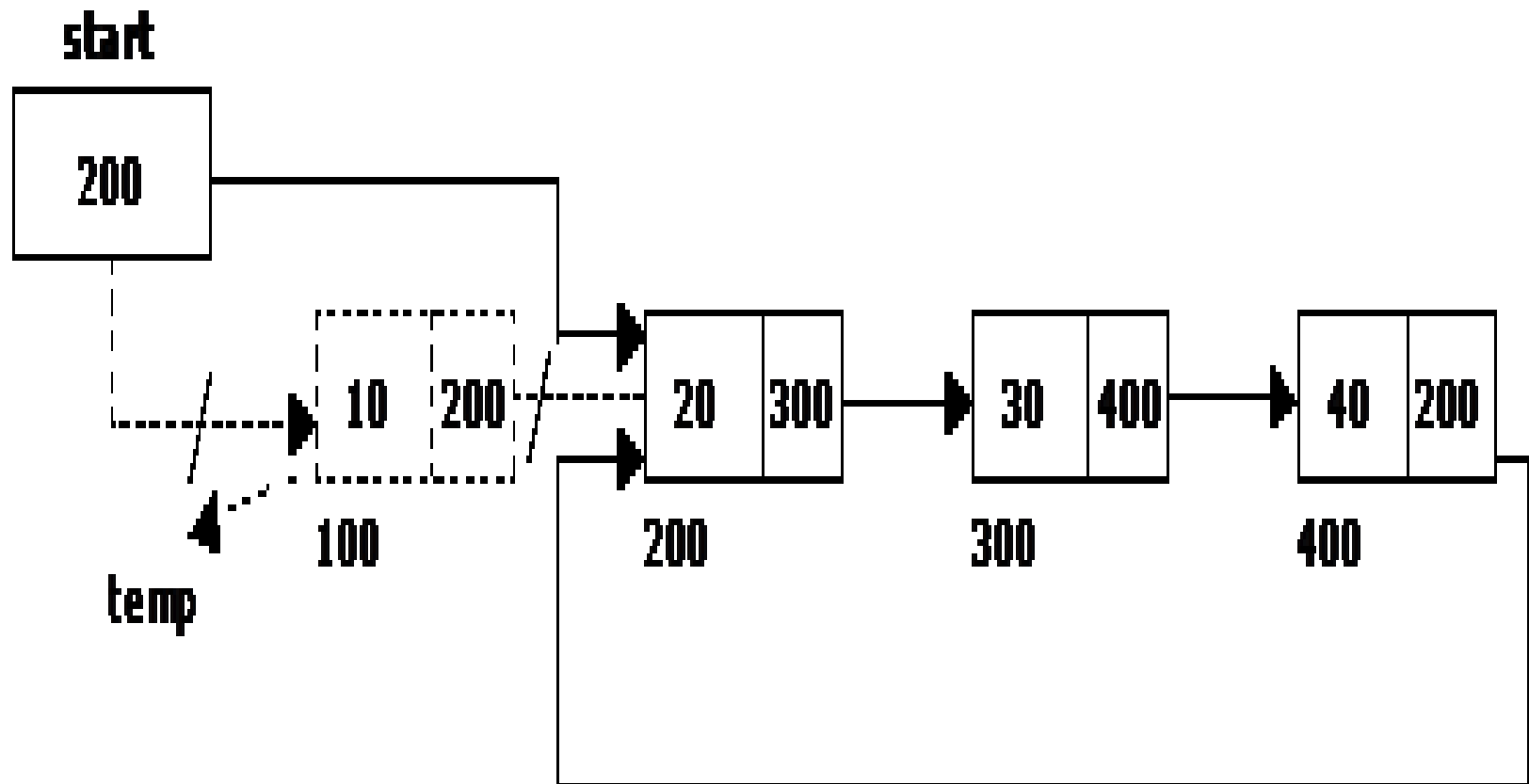
Deleting a node at the end

- The following steps are followed to delete a node at the end of the list:
- If the list is empty, display a message ‗Empty List'.
- If the list is not empty, follow the steps given

  below: temp = start;

  prev = start;
  while(temp -> next != start)
  {
         prev = temp;
         temp = temp -> next;
  }
  prev -> next = start;

- After deleting the node, if the list is empty then *start = NULL.*

Deleting a node at the end

start

100

| 10 | 200 |
100

| 20 | 300 |
200

| 30 | 100 |
300

| 40 | 100 |
400
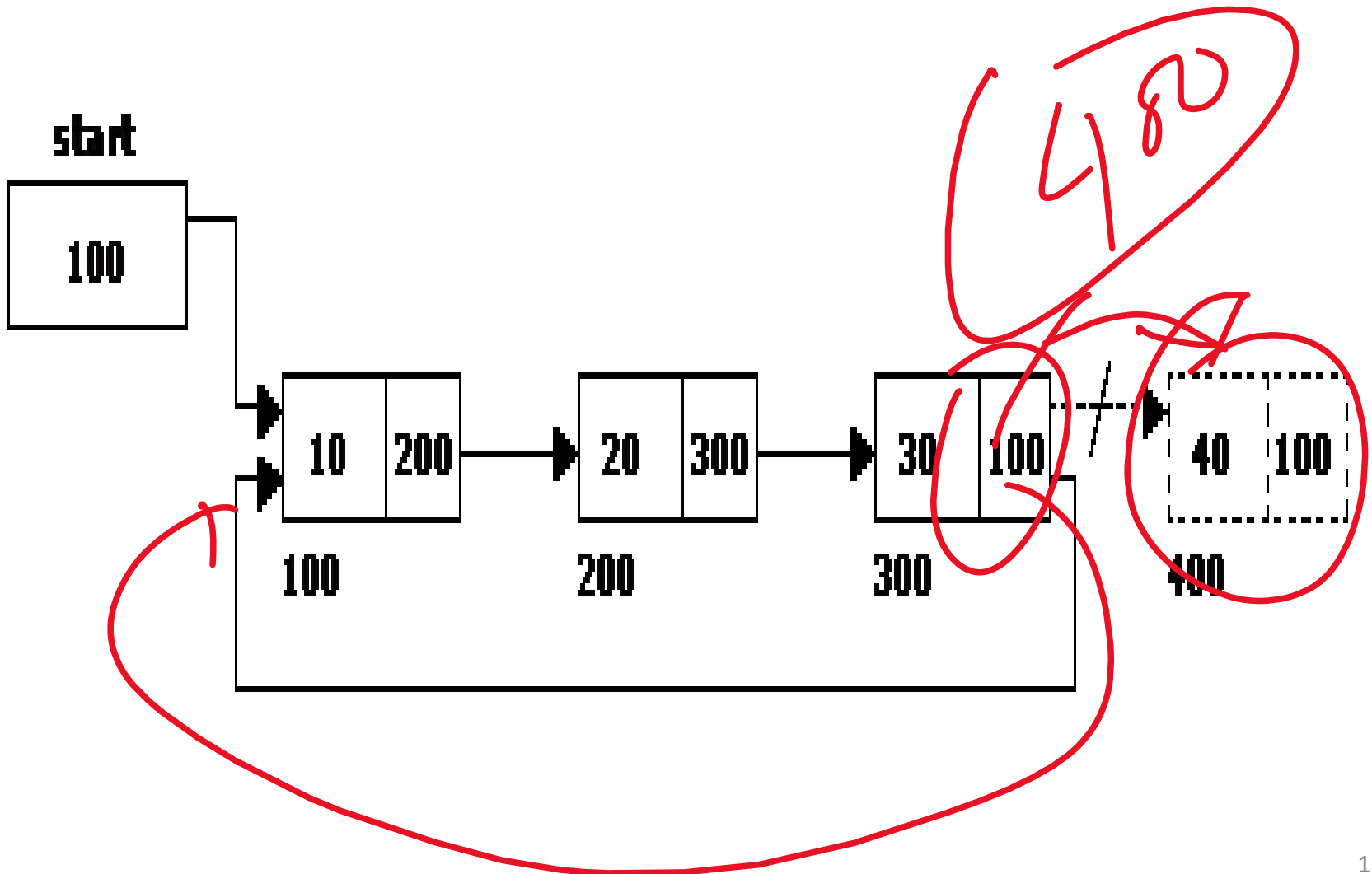
Traversing a circular single linked list from left to right

- The following steps are followed, to traverse a list from left to right:
- If list is empty then display _Empty List' message.
- If the list is not empty, follow the steps given below:

temp = start;

do

    {

        printf("%d ", temp -> data);

        temp = temp -> next;

    } while(temp != start);

Advantages of Circular Lists

- The major advantage of circular lists (over non-circular lists) is that they eliminate some extra-case code for some operations (like deleting last node).

- Also, some applications lead naturally to circular list representations.

- For example, a computer network might best be modeled using a circular list.

Application of Linked List

- Implementation of stacks and queues

- Implementation of graphs : Adjacency list representation of graphs is most popular which is uses linked list to store adjacent vertices.

- Dynamic memory allocation : We use linked list of free blocks.
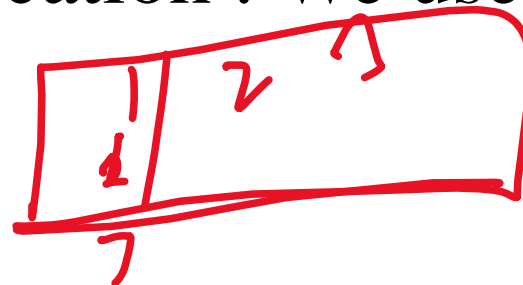
Application of Linked List

- Maintaining directory of names

- Performing arithmetic operations on long integers

- Manipulation of polynomials by storing constants in the node of linked list

- representing sparse matrices

Applications of linked list in real world

1.Image viewer – Previous and next images are linked, hence can be accessed by next and previous button.

2.Previous and next page in web browser – We can access previous and next url searched in web browser by pressing back and next button since, they are linked as linked list.

3.Music Player – Songs in music player are linked to previous and next song. you can play songs either from starting or ending of the list.

Applications of Linked Lists: Representing Polynomials

A polynomial is of the form: $\sum_{i=0}^{n} c_i\, x^i$

Where, c_i is the coefficient of the ith term and n is the degree of the polynomial

Some examples are:

$5x^2 + 3x + 1$

$5x^4 - 8x^3 + 2x^2 + 4x^1 + 9x^0$

The computer implementation requires implementing polynomials as a list of pairs of coefficient and exponent. Each of these pairs will constitute a structure, so a polynomial will be represented as a list of structures. A linked list structure that represents polynomials $5x^4 - 8x^3 + 2x^2 + 4x^1 + 9x^0$ illustrated.
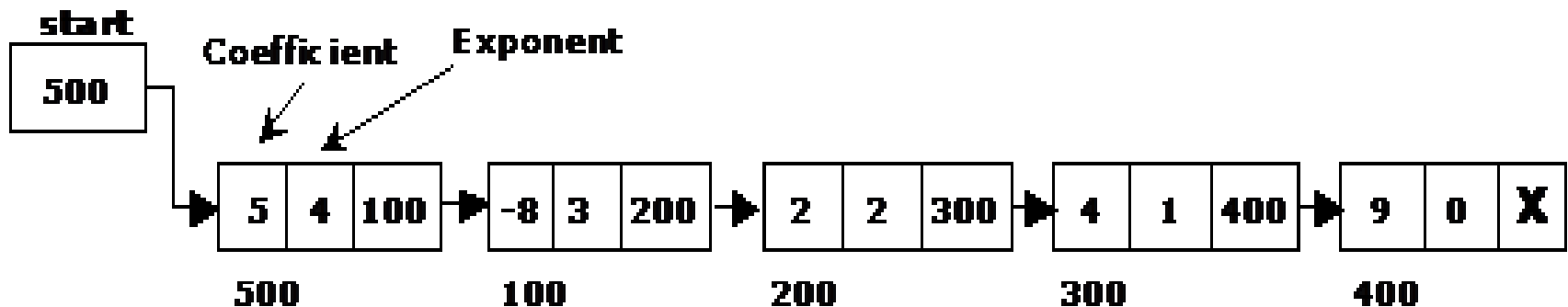
Addition of Polynomials

- To add two polynomials, if we find terms with the same exponent in the two polynomials, then we add the coefficients; otherwise, we copy the term of larger exponent into the sum and go on. When we reach at the end of one of the polynomial, then remaining part of the other is copied into the sum.

- To add two polynomials follow the following steps:
  - Read two polynomials.
  - Add them.
  - Display the resultant polynomial.

UNIT – 6
TREES

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Lecturer
Pokhara Engineering College
Soch College of IT
Infomax College of IT and Management

CONTENTS

- Concept and Definition :Concept of level, depth, number of nodes

- Binary Tree and Binary Search Tree

- Insertion, Deletion, and Traversal of BST

- Applications of Tree, Concept of Balanced Trees

Tree – a Hierarchical Data Structure

- Trees are non linear data structure that can be represented in a hierarchical manner.

  – A tree contains a finite non-empty set of elements.

  – Any two nodes in the tree are connected with a relationship of parent-child.

  – Every individual elements in a tree can have any number of sub trees.

An Example of a Tree
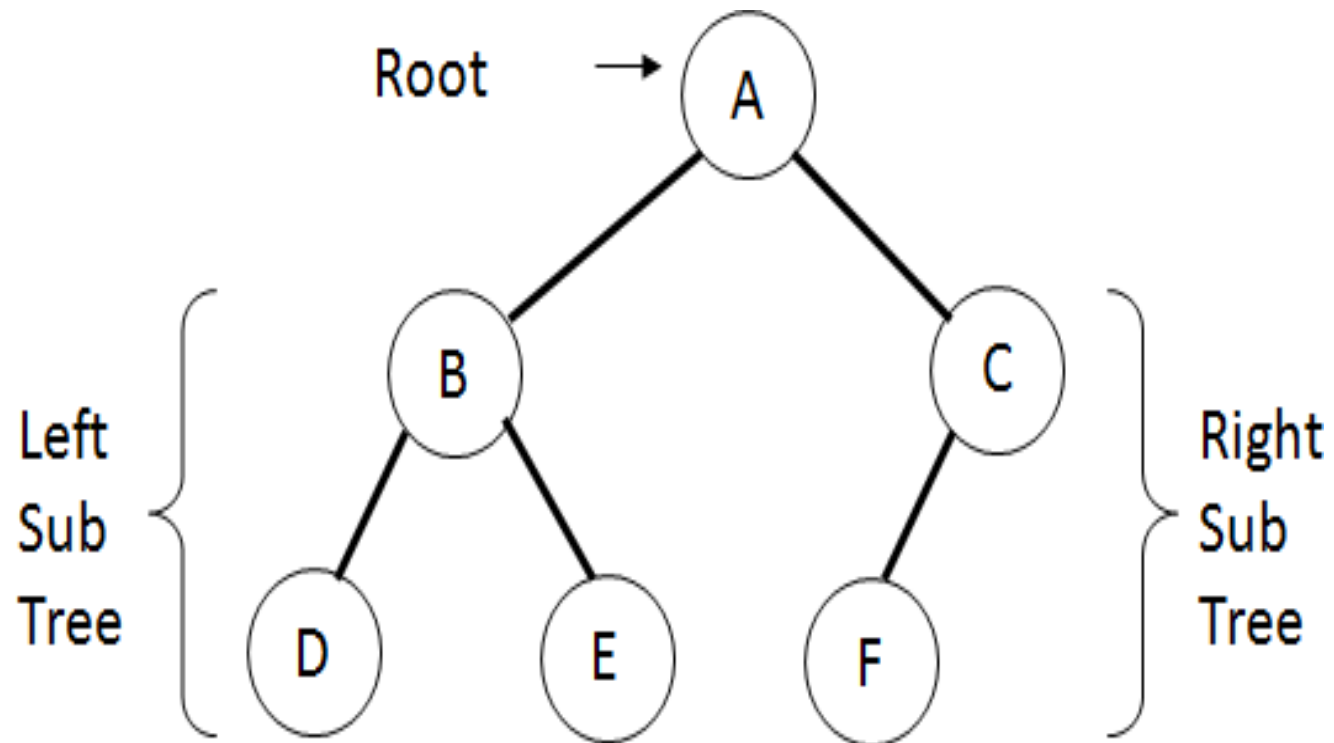
Tree – Basic Terminology

- **Root :** The basic node of all nodes in the tree. All operations on the tree are performed with passing root node to the functions.
- **Child :** a successor node connected to a node is called child. A node in binary tree may have at most two children.
- **Parent :** a node is said to be parent node to all its child nodes.
- **Leaf :** a node that has no child nodes.
- **Siblings :** Two nodes are siblings if they are children to the same parent node.

Tree – Basic Terminology Contd…

- **Ancestor :** a node which is parent of parent node (A is ancestor node to D,E and F).
- **Descendent :** a node which is child of child node (D, E and F are descendent nodes of node A)
- **Level** : The distance of a node from the root node, The root is at level – 0,(B and C are at Level 1 and D, E, F have Level 2 (highest level of tree is called **height** of tree)
- **Degree :** The number of nodes connected to a particular parent node.

Binary Tree

- A binary tree is a hierarchy of nodes, where every parent node has at most two child nodes. There is a unique node, called the root, that does not have a parent.

- A binary tree can be defined recursively as

- **Root node**

- **Left subtree:** left child and all its descendants

- **Right subtree:** right child and all its descendants

Binary Tree
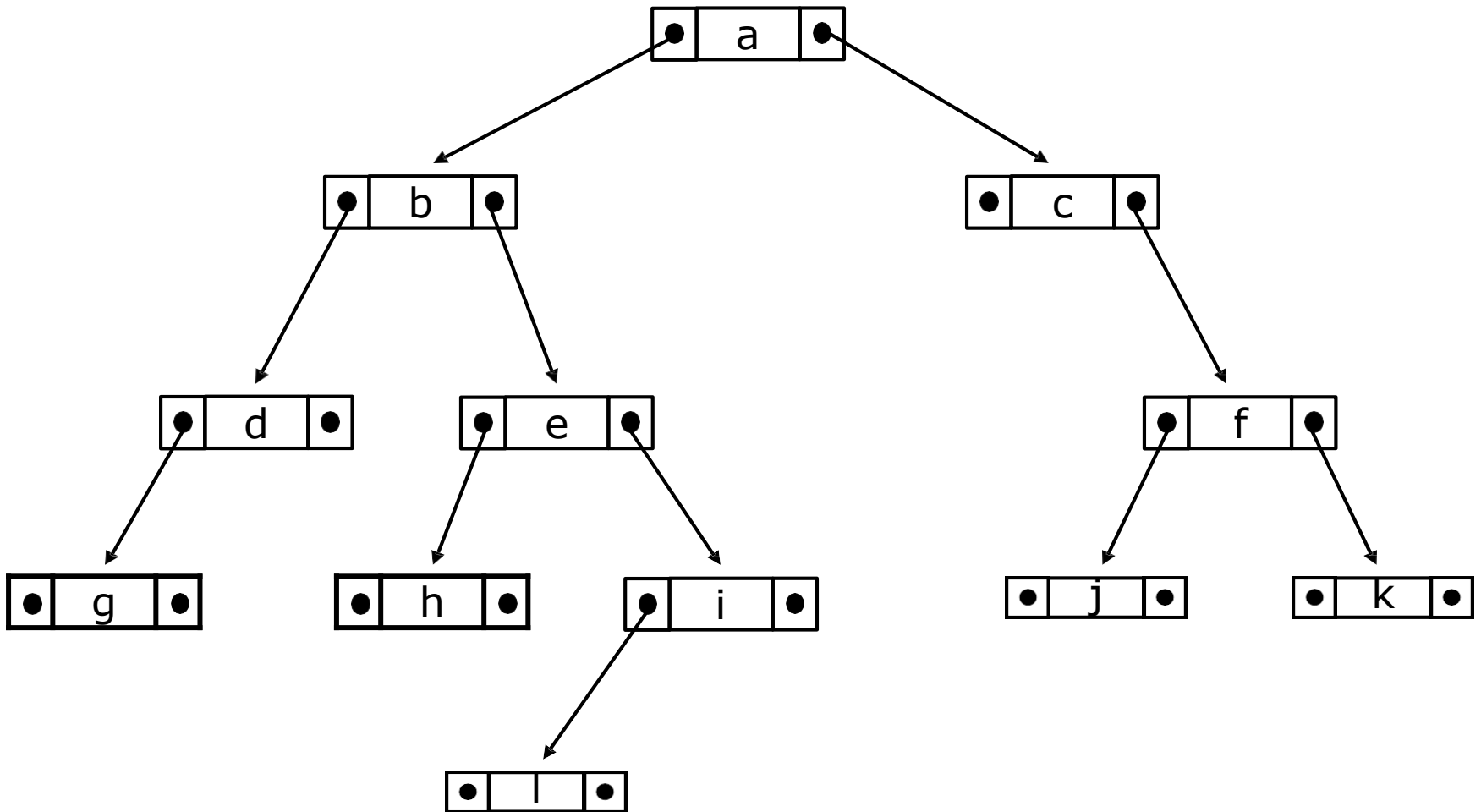
Full and Complete Binary Trees

- **A full tree is a binary tree in which**
  - The full binary tree is also known as a strict binary tree. The tree can only be considered as the full binary tree if each node must contain either 0 or 2 children.
  - The full binary tree can also be defined as the tree in which each node must contain 2 children except the leaf nodes.

Full and Complete Binary Trees

– A **complete** *is a binary tree in Which*

– The complete binary tree is a tree in which all the nodes are completely filled except the last level.

– In the last level, all the nodes must be as left as possible. In a complete binary tree, the nodes should be added from the left.

Full and Complete Binary Trees
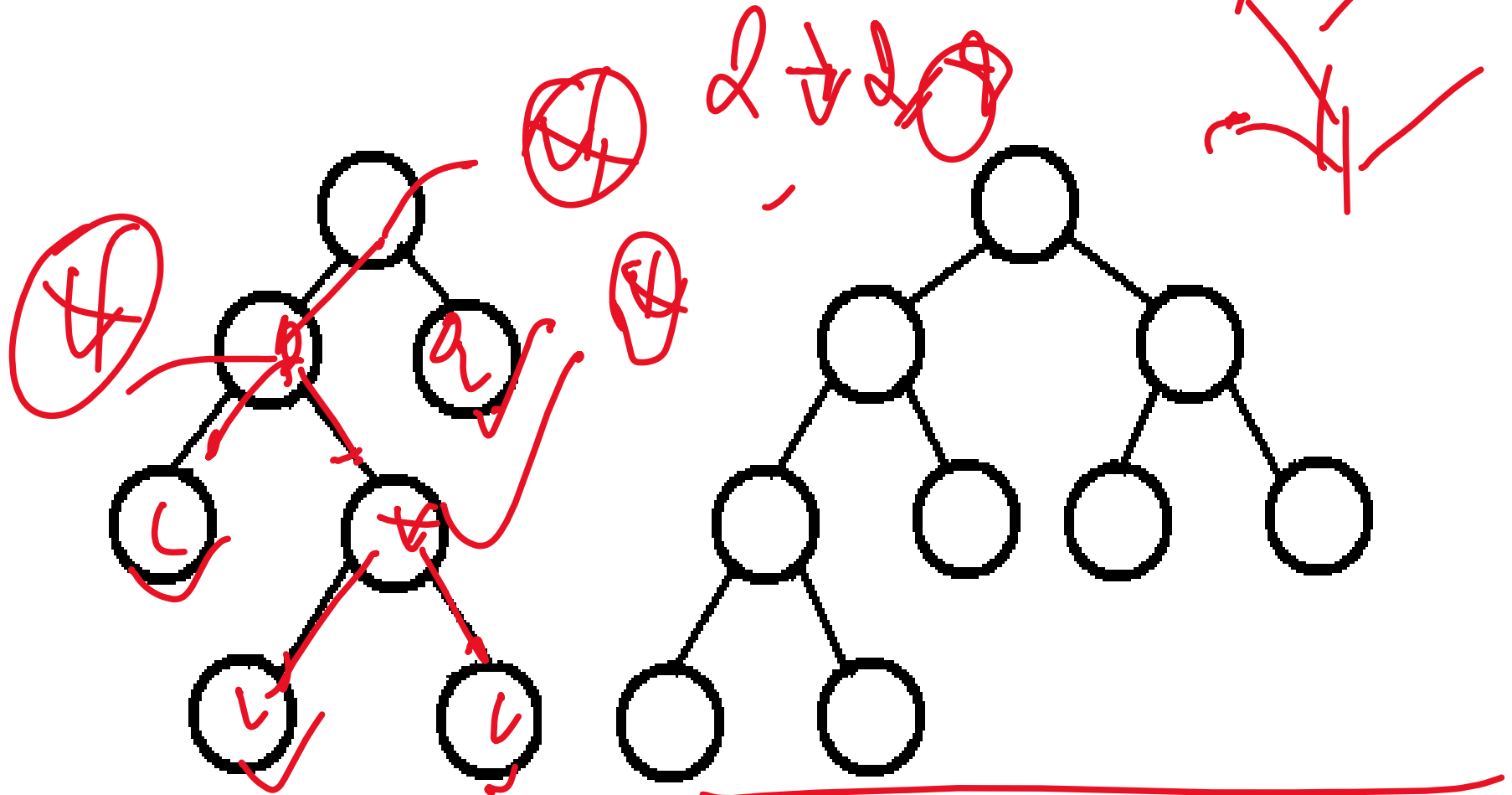
full tree

complete tree

Binary Search Tree

Binary Search Tree is a special type of binary tree that has a specific order of elements in it. It follows three basic properties:-

- All elements in the **left subtree** of a node should have a value **lesser than** the node's value.
- All elements in the **right subtree** of a node should have a value **greater than** the node's value
- Both the left and right subtrees should be binary search trees too.

Binary Search Tree
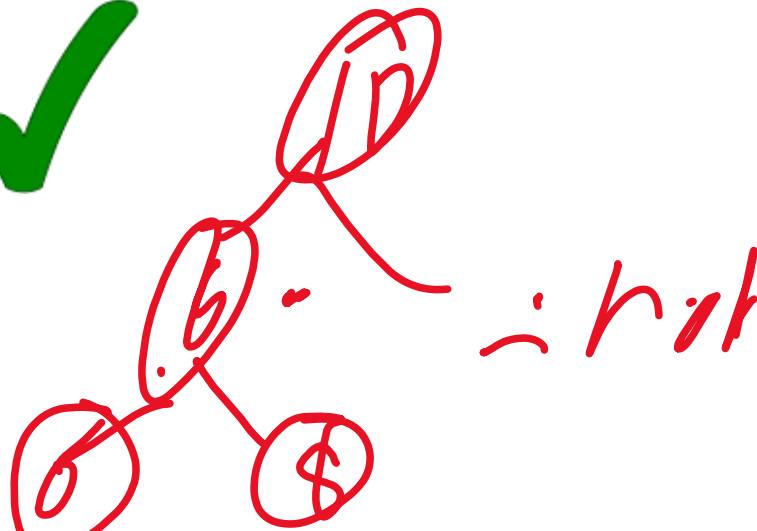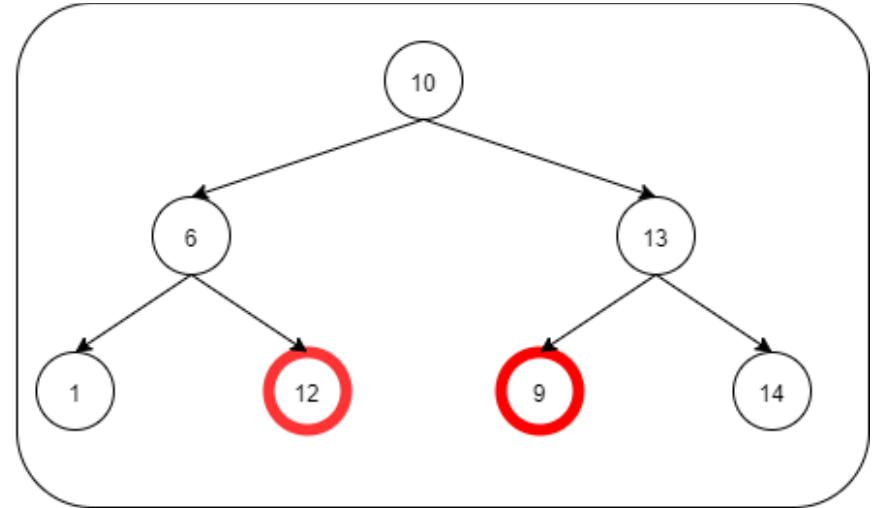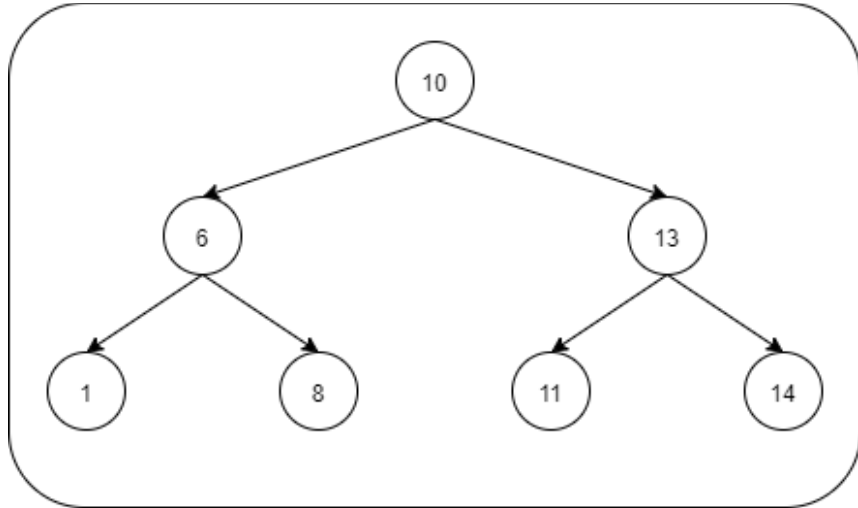
BST Basic Operations

The basic operations that can be performed on a binary search tree data structure, are the following −

•**Insert** − Inserts an element in a tree/create a tree.

•**Search** − Searches an element in a tree.

•**Preorder Traversal** − Traverses a tree in a pre-order manner.

•**Inorder Traversal** − Traverses a tree in an in-order manner.

•**Postorder Traversal** − Traverses a tree in a post-order manner.

Tree Traversals

- A binary tree is defined recursively: it consists of a root, a left subtree, and a right subtree.

- To traverse (or walk) the binary tree is to visit each node in the binary tree exactly once.

- Tree traversals are naturally recursive.

- Standard traversal orderings:
  - **preorder**
  - **inorder**
  - **postorder**
  - **level-order**

Preoder, Inorder, Postorder

- In Preorder, the root is visited before (pre) the subtrees traversals.
- In Inorder, the root is visited in-between left and right subtree traversal.
- In Preorder, the root is visited after (pre) the subtrees traversals.

Preorder Traversal:
1. Visit the root
2. Traverse left subtree
3. Traverse right subtree

Inorder Traversal:
1. Traverse left subtree
2. Visit the root
3. Traverse right subtree

Postorder Traversal:
1. Traverse left subtree
2. Traverse right subtree
3. Visit the root

Example of Tree Traversal
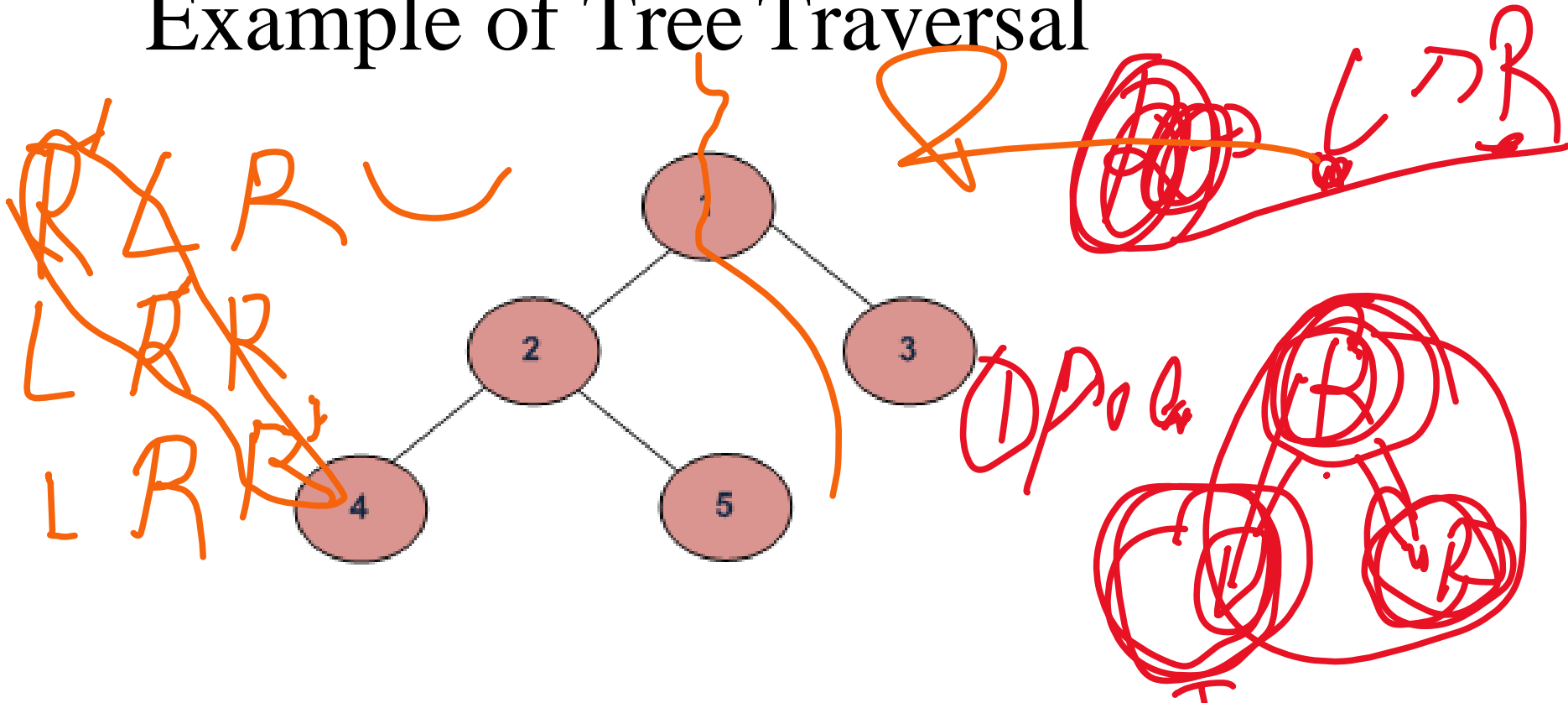
(a) Inorder (Left, Root, Right) : 4 2 5 1 3

(b) Preorder (Root, Left, Right) : 1 2 4 5 3

(c) Postorder (Left, Right, Root) : 4 5 2 3 1

Example of Tree Traversal

- Assume: visiting a node is printing its <u>data</u>
- Preorder: 15 8 2 6 3 7 11 10 12 14 20 27 22 30
- Inorder: 2 3 6 7 8 10 11 12 14 15 20 22 27 30
- Postorder: 3 7 6 2 10 14 12 11 8 22 30 27 20 15
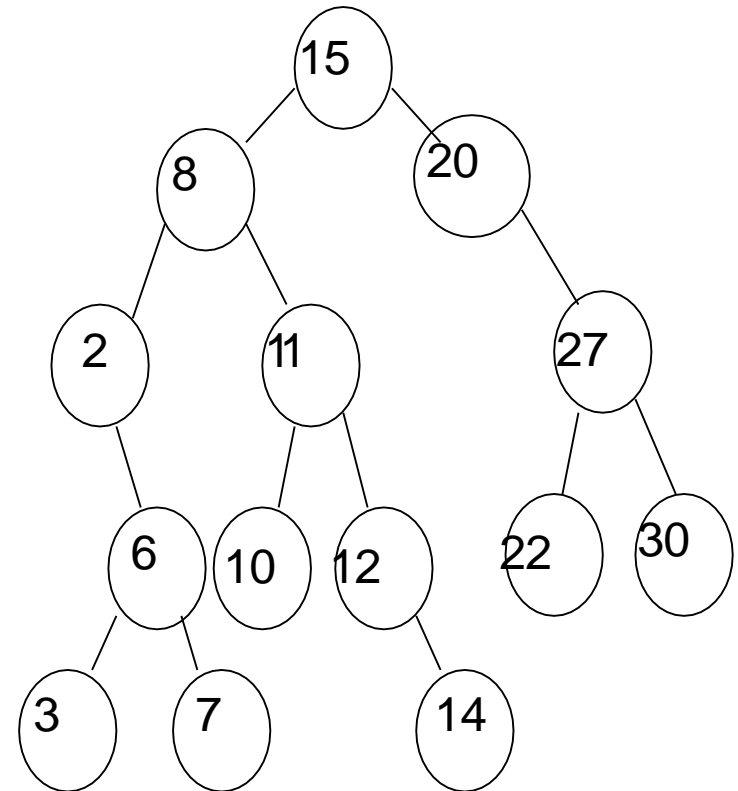
Traversal Techniques

```
    void preorder(tree *tree) {
    if (tree->isEmpty( ))        return;

    visit(tree->getRoot( ));

    preOrder(tree->getLeftSubtree());

    preOrder(tree->getRightSubtree());
}
```
```
void inOrder(Tree *tree){
    if (tree->isEmpty( ))        return;
    inOrder(tree->getLeftSubtree( ));
    visit(tree->getRoot( ));
    inOrder(tree->getRightSubtree( ));
}
```
```
void postOrder(Tree *tree){
    if (tree->isEmpty( ))        return;
    postOrder(tree->getLeftSubtree( ));
    postOrder(tree->getRightSubtree( ));
    visit(tree->getRoot( ));
}
```

Operation on BST:
1. Insertion

• Say we have to build a BST of the keys, 50, 80, 30, 20, 100, 40. It can be clearly seen below, for inserting, first the key is compared is compared with the root, if smaller then go to Left subtree else Right subtree. The same step is repeated until all the keys are inserted.
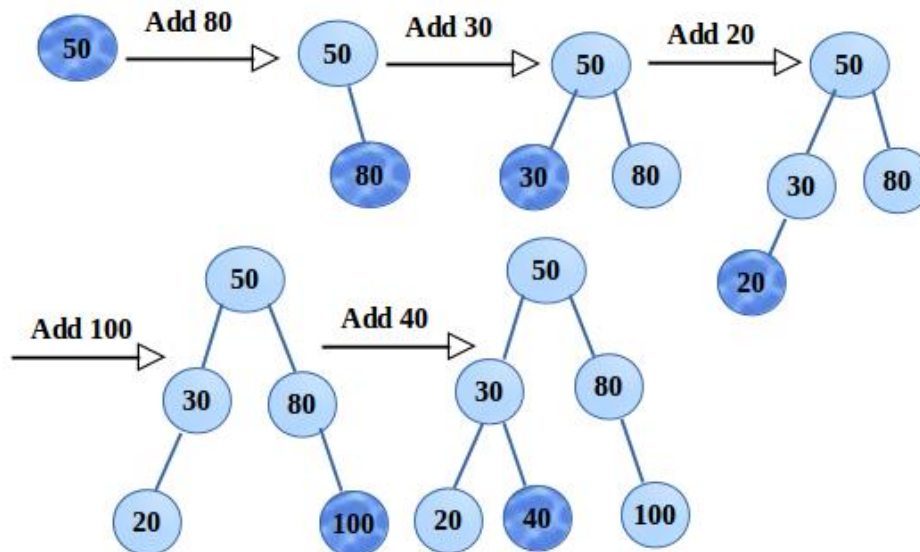
2. Deletion

Deletion of a node is performed as follows,
1.Deleting node has no child, therefore just delete the node(made to point NULL)
2.Deleting node has 1 child, swap the key with the child and delete the child.
3.Deleting node has 2 children, in this case swap the key with *inorder successor* of the deleting node. It should be noted, inorder successor will be the minimum key in the right subtree (of the deleting node).

2. Deletion
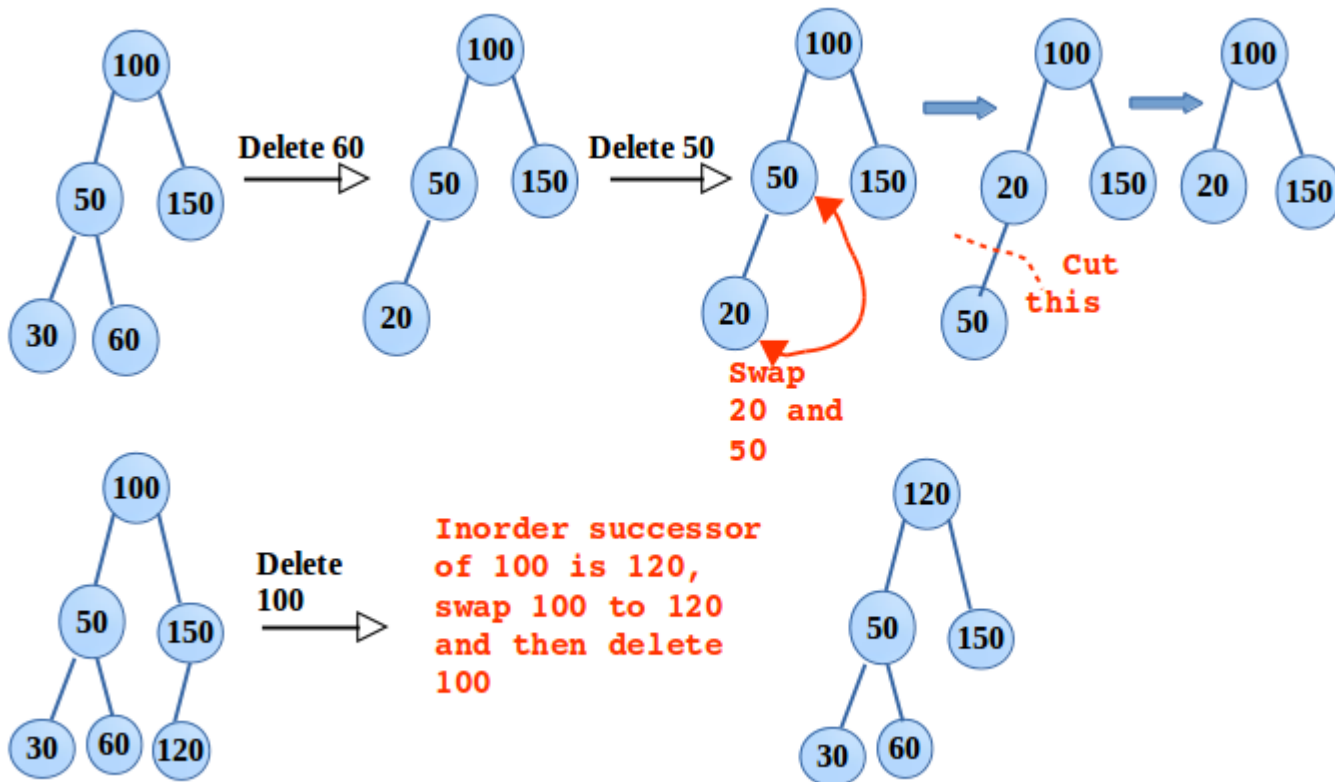
3. Searching

The steps follow in the insertion, are same followed here. Only difference is in comparison, if the key is not matched, repeat the steps till reached NULL. That says, desired key is not available in the BST.
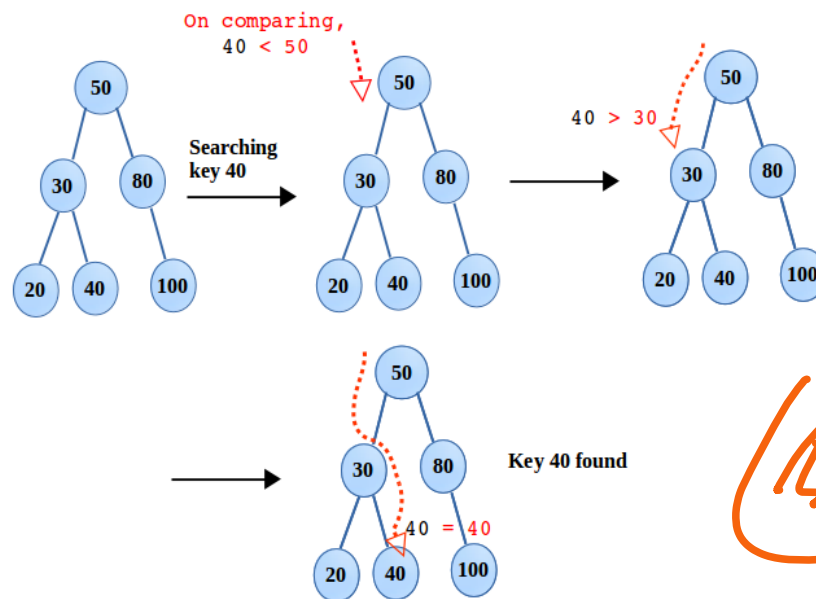
Applications of trees

- Represent organization
- Represent computer file systems
- One reason to use trees might be because you want to store information that naturally forms a hierarchy.
- Networks to find best path in the Internet
- Chemical formulas representation
- Outlines, et

Other Applications :

1.Store hierarchical data, like folder structure, organization structure, XML/HTML data.

2.Binary Search Tree is a tree that allows fast search, insert, delete on a sorted data. It also allows finding closest item

3.Heap is a tree data structure which is implemented using arrays and used to implement priority queues.

4.B-Tree and B+ Tree : They are used to implement indexing in databases.

5.Syntax Tree: Used in Compilers.

6.Suffix Tree : For quick pattern searching in a fixed text.

7.Spanning Trees and shortest path trees are used in routers and bridges respectively in computer networks

8.As a workflow for compositing digital images for visual effects.

Threaded Binary Tree

- A **threaded binary tree** defined as:

- "A binary tree is *threaded* by making all right child pointers that would normally be null point to the inorder successor of the node, and all left child pointers that would normally be null point to the inorder predecessor of the node
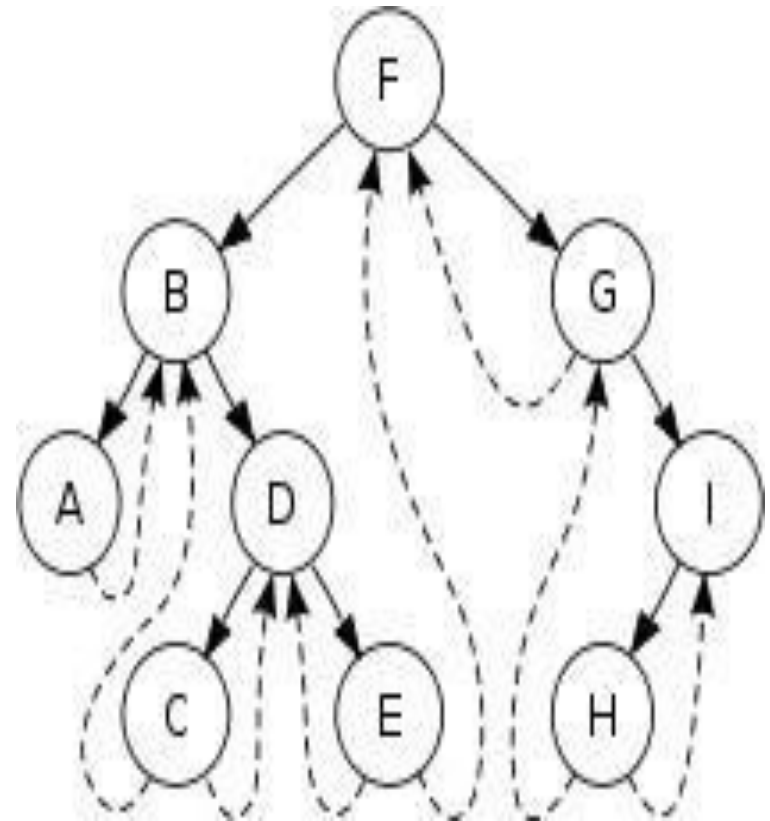
Graph Basics

- Graphs are collections of nodes connected by edges – G = (V,E) where V is a set of nodes and E a set of edges.

- Graphs are useful in a number of applications including

  - Shortest path problems
  - Maximum flow problems

- Graphs unlike trees are more general for they can have connected components.

Graph Types

- **Directed Graphs**: A directed graph edges allow travel in one direction.

- **Undirected Graphs**: An undirected graph edges allow travel in either direction.
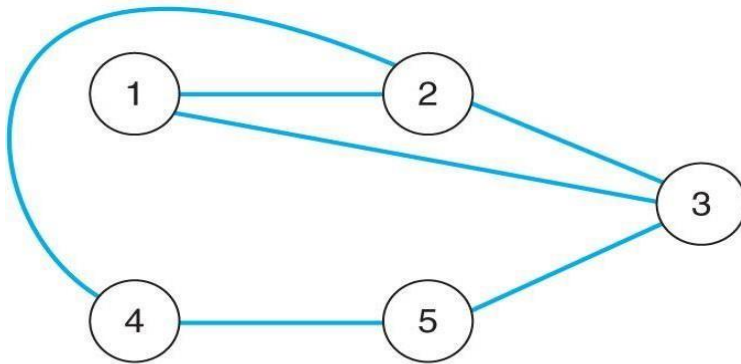
FIGURE 8.1A
The graph G = ({1, 2, 3, 4, 5}, {{1, 2}, {1, 3}, {2, 3}, {2, 4}, {3, 5}, {4, 5}})
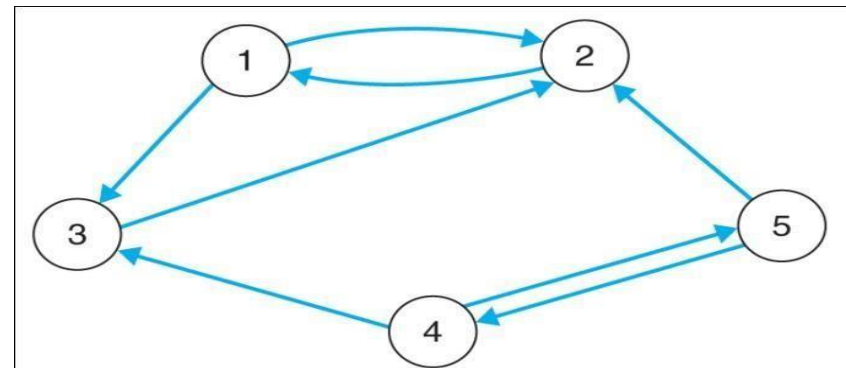
FIGURE 8.1B
The directed graph G = ({1, 2, 3, 4, 5}, {(1, 2), (1, 3), (2, 1), (3, 2), (4, 3), (4, 5), (5, 2), (5,4)})

Graph Terminology

- A graph is an ordered pair G=(V,E) with a set of vertices or nodes and the edges that connect them.

- A subgraph of a graph has a subset of the vertices and edges.

- The edges indicate how we can move through the graph.

- A path is a subset of E that is a series of edges between two nodes.

- A graph is connected if there is at least one path between every pair of nodes.

Graph Terminology

- The length of a path in a graph is the number of edges in the path.

- A complete graph is one that has an edge between every pair of nodes.

- A weighted graph is one where each edge has a cost for traveling between the nodes.

- A cycle is a path that begins and ends at the same node.

- An acyclic graph is one that has no cycles.

- An acyclic, connected graph is also called an unrooted tree

Data Structures for Graphs
An Adjacency Matrix

- For an undirected graph, the matrix will be symmetric along the diagonal.

- For a weighted graph, the adjacency matrix would have the weight for edges in the graph, zeros along the diagonal, and infinity (∞) every place else.

Adjacency Matrix Example 1
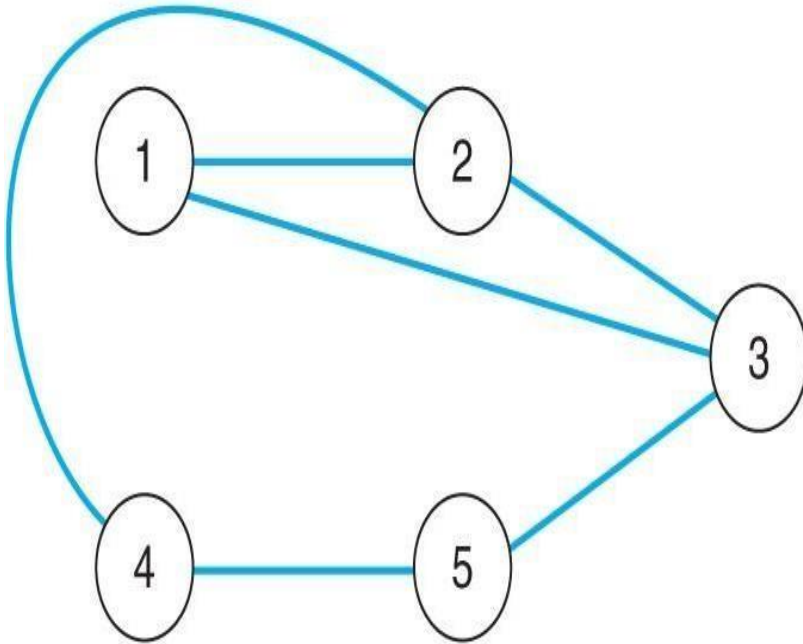
FIGURE 8.1A

The graph G = ({1, 2, 3, 4, 5}, {{1, 2}, {1, 3}, {2, 3}, {2, 4}, {3, 5}, {4, 5}})

|	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	0
3	1	1	0	0	1
4	0	1	0	0	1
5	0	0	1	1	0

FIGURE 8.2A

The adjacency matrix for the graph in Fig. 8.1(a)

Adjacency Matrix Example 2
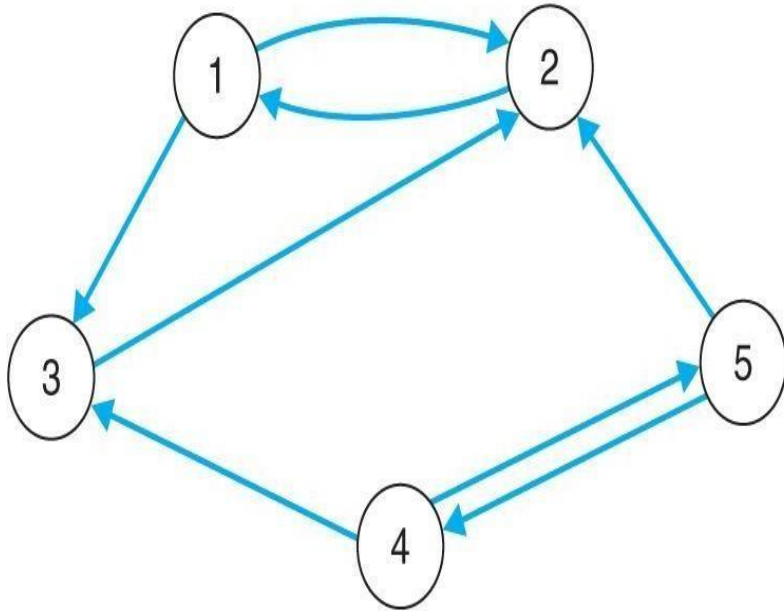
■ FIGURE 8.1B

The directed graph G = ({1, 2, 3, 4, 5}, {(1, 2), (1, 3), (2, 1), (3, 2), (4, 3), (4, 5), (5, 2), (5,4)})

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	0	1	0	1
5	0	1	0	1	0

■ FIGURE 8.2B

The adjacency matrix for the digraph in Fig. 8.1(b)

Data Structures for Graphs
An Adjacency List

- A list of pointers, one for each node of the graph.

- These pointers are the start of a linked list of nodes that can be reached by one edge of the graph.

- For a weighted graph, this list would also include the weight for each edge.

Adjacency List Example 1
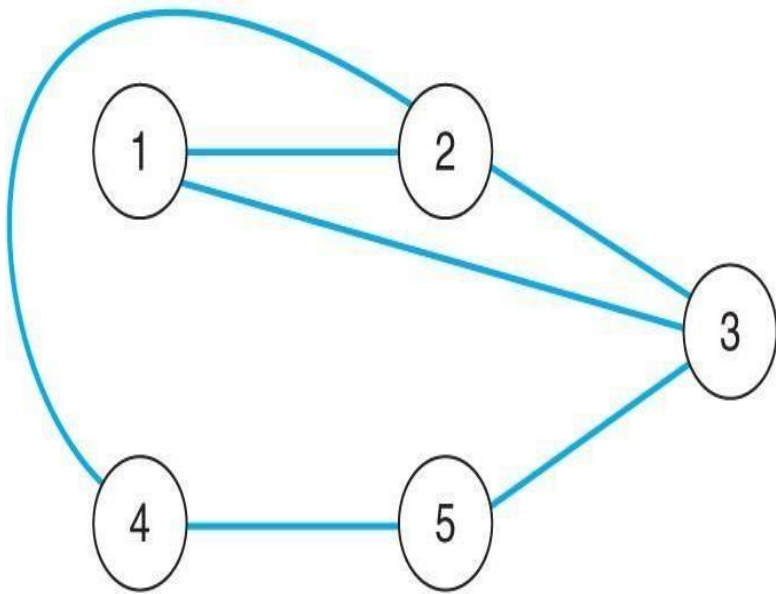
FIGURE 8.1A

The graph G = ({1, 2, 3, 4, 5}, {{1, 2}, {1, 3}, {2, 3}, {2, 4}, {3, 5}, {4, 5}})
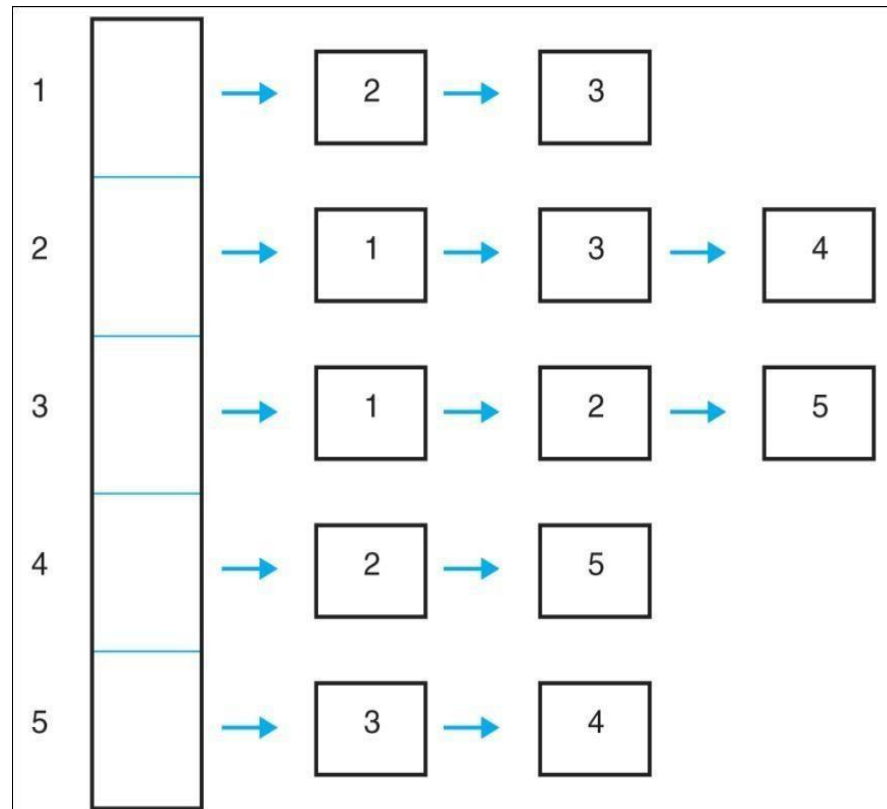
FIGURE 8.3A

The adjacency list for the graph in Fig. 8.1(a)

Adjacency List Example 2
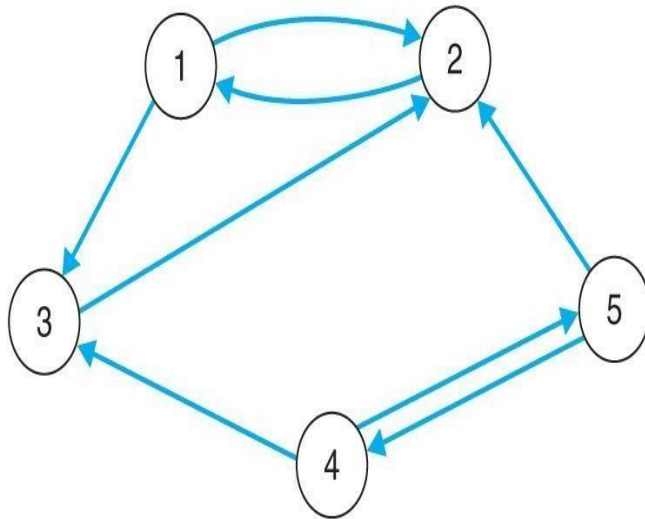
FIGURE 8.1B

The directed graph G = ({1, 2, 3, 4, 5}, {(1, 2), (1, 3), (2, 1), (3, 2), (4, 3), (4, 5), (5, 2), (5,4)})
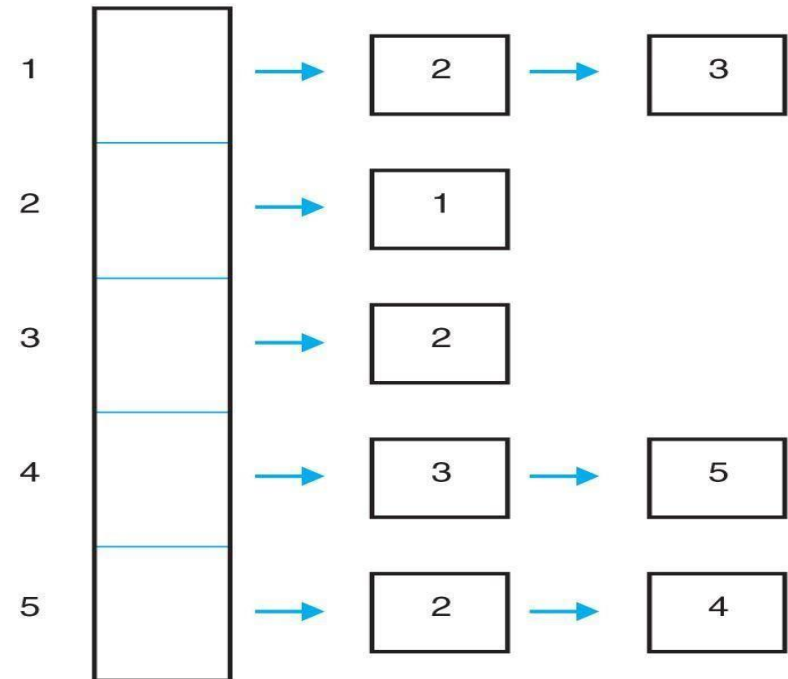
FIGURE 8.3B

The adjacency list for the graph in Fig. 8.1(b)

Graph Traversals

- Some algorithms require that every vertex of a graph be visited exactly once.

- The order in which the vertices are visited may be important, and may depend upon the particular algorithm.

- The two common traversals:

  - depth-first
  - breadth-first

Graph Traversals:
Depth First Search Traversal

- We follow a path through the graph until we reach a dead end.

- We then back up until we reach a node with an edge to an unvisited node.

- We take this edge and again follow it until we reach a dead end.

- This process continues until we back up to the starting node and it has no edges to unvisited nodes.

Depth First Search Traversal Example

- Consider the following graph:

FIGURE 8.4
A graph
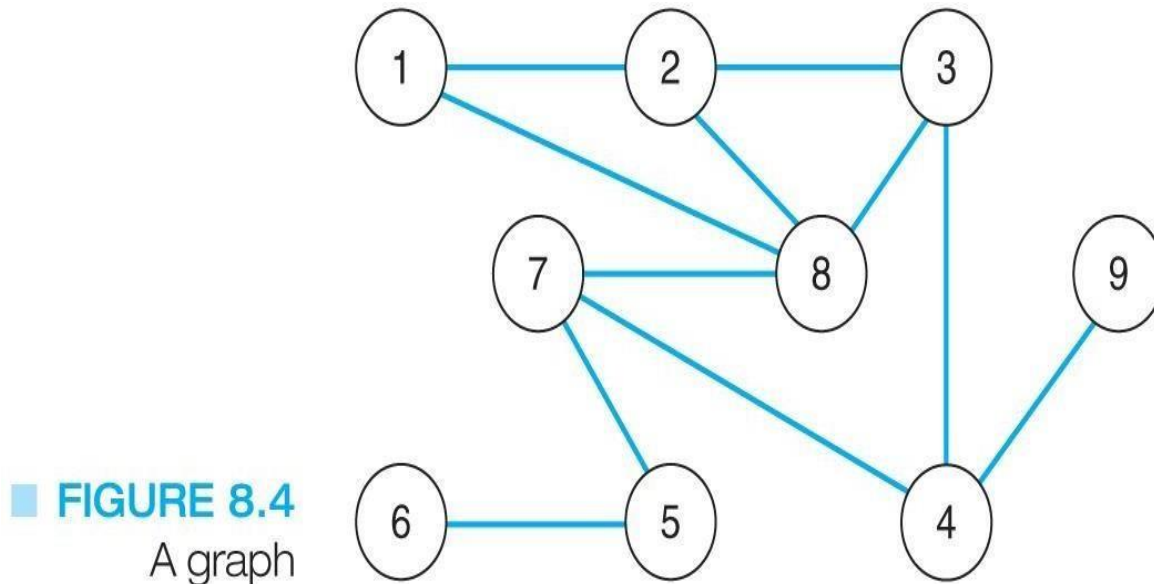
- The order of the depth-first traversal of this graph starting at node 1 would be:
1, 2, 3, 4, 7, 5, 6, 8, 9

Breadth First Search Traversal

- From the starting node, we follow all paths of length one.

- Then we follow paths of length two that go to unvisited nodes.

- We continue increasing the length of the paths until there are no unvisited nodes along any of the paths.

Breadth First Search Traversal Example

- Consider the following graph:

FIGURE 8.4
A graph
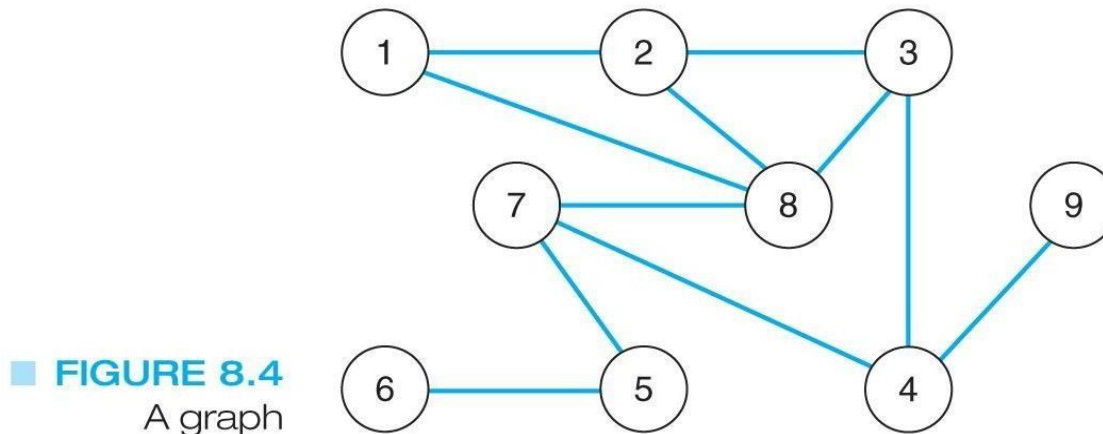
- The order of the breadth-first traversal of this graph starting at node 1 would be: 1, 2, 8, 3, 7, 4, 5, 9, 6

Spanning tree

- Given an undirected and connected graph , a spanning tree of the graph is a tree that spans (that is, it includes every vertex of) and is a subgraph of (every edge in the tree belongs to)

Spanning tree

What is a Minimum Spanning Tree?

- The cost of the spanning tree is the sum of the weights of all the edges in the tree.

- There can be many spanning trees.

- Minimum spanning tree is the spanning tree where the cost is minimum among all the spanning trees.

Spanning tree

Properties of Spanning Tree:

1. There may be several minimum spanning trees of the same weight having the minimum number of edges.

2. If all the edge weights of a given graph are the same, then every spanning tree of that graph is minimum.

3. If each edge has a distinct weight, then there will be only one, unique minimum spanning tree.

Spanning tree

Properties of Spanning Tree:

4. A connected graph G can have more than one spanning trees.

5. A disconnected graph can't have to span the tree, or it can't span all the vertices.
Spanning Tree doesn't contain cycles.

6. Spanning Tree has **(n-1) edges** where n is the number of vertices

Spanning tree

Practical applications are:

1. Cluster Analysis

2. Handwriting recognition

3. Image segmentation

Spanning tree
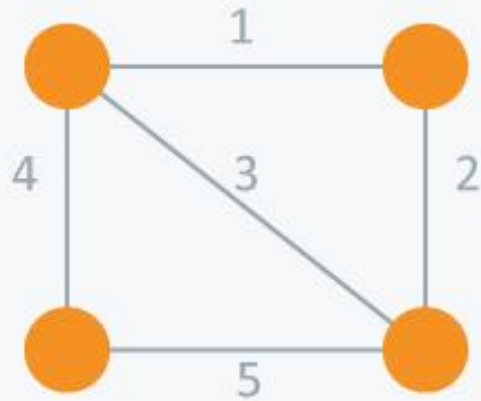
Undirected Graph
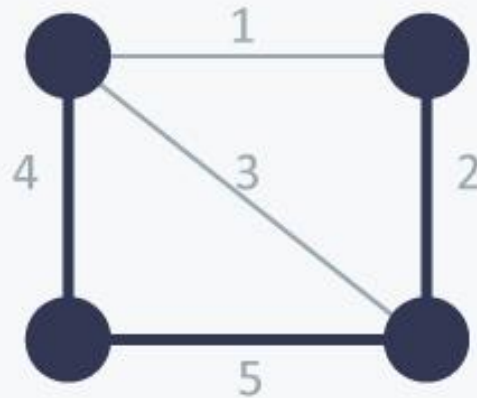
Spanning Tree
Cost = 11(=4+5+2)
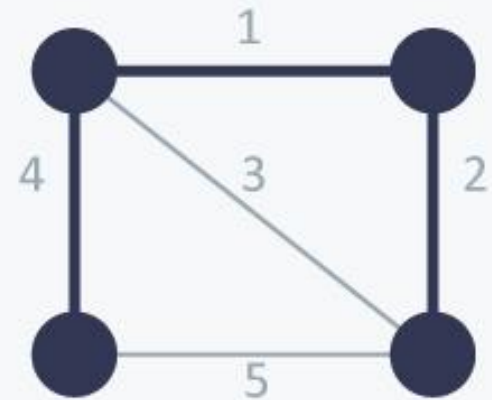
Minimum Spanning Tree
Cost = 7(=4+1+2)

Minimum Spanning tree

In a weighted graph, a minimum spanning tree is a spanning tree that has minimum weight than all other spanning trees of the same graph.

In real-world situations, this weight can be measured as distance, congestion, traffic load or any arbitrary value denoted to the edges

Algorithm :
Prim's Algorithm

Prim's Algorithm

• Prim's and Kruskal's Algorithm are the famous greedy algorithms.

• They are used for finding the Minimum Spanning Tree (MST) of a given graph.

• To apply these algorithms, the given graph must be weighted, connected and undirected.
.
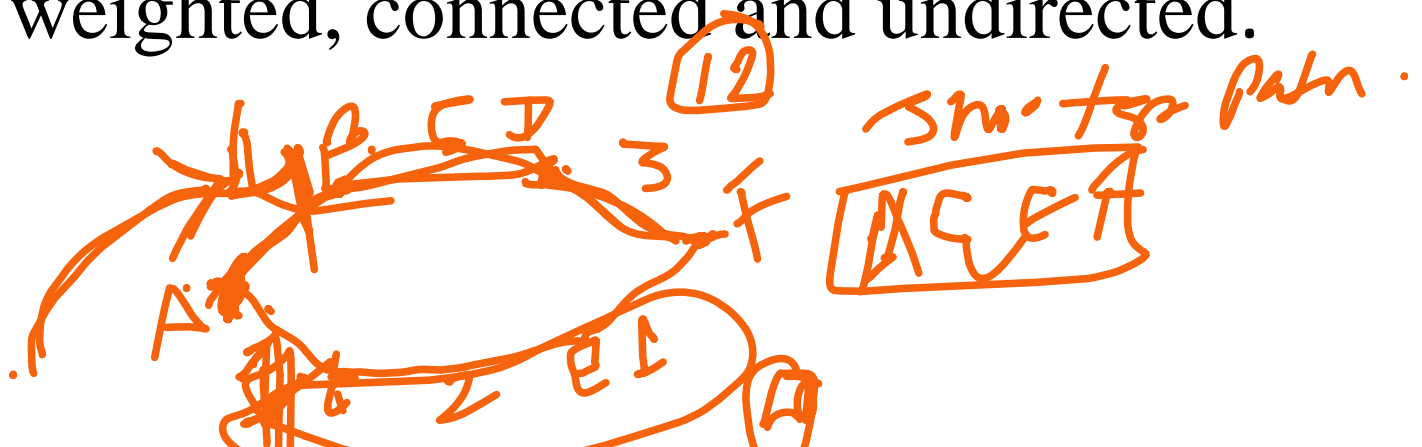
Prim's Algorithm

- Prim's Algorithm also use Greedy approach to find the minimum spanning tree.

- In Prim's Algorithm we grow the spanning tree from a starting position.

- We add **vertex** to the growing spanning tree in Prim's.

Prim's Algorithm

Algorithm Steps:

•Maintain two disjoint sets of vertices. One containing vertices that are in the growing spanning tree and other that are not in the growing spanning tree.

•Select the cheapest vertex that is connected to the growing spanning tree and is not in the growing spanning tree and add it into the growing spanning tree. This can be done using Priority Queues. Insert the vertices, that are connected to growing spanning tree, into the Priority Queue.

•Check for cycles. To do that, mark the nodes which have been already selected and insert only those nodes in the Priority Queue that are not marked.

Prim's Algorithm

$1 + 4 + 2 = 7$
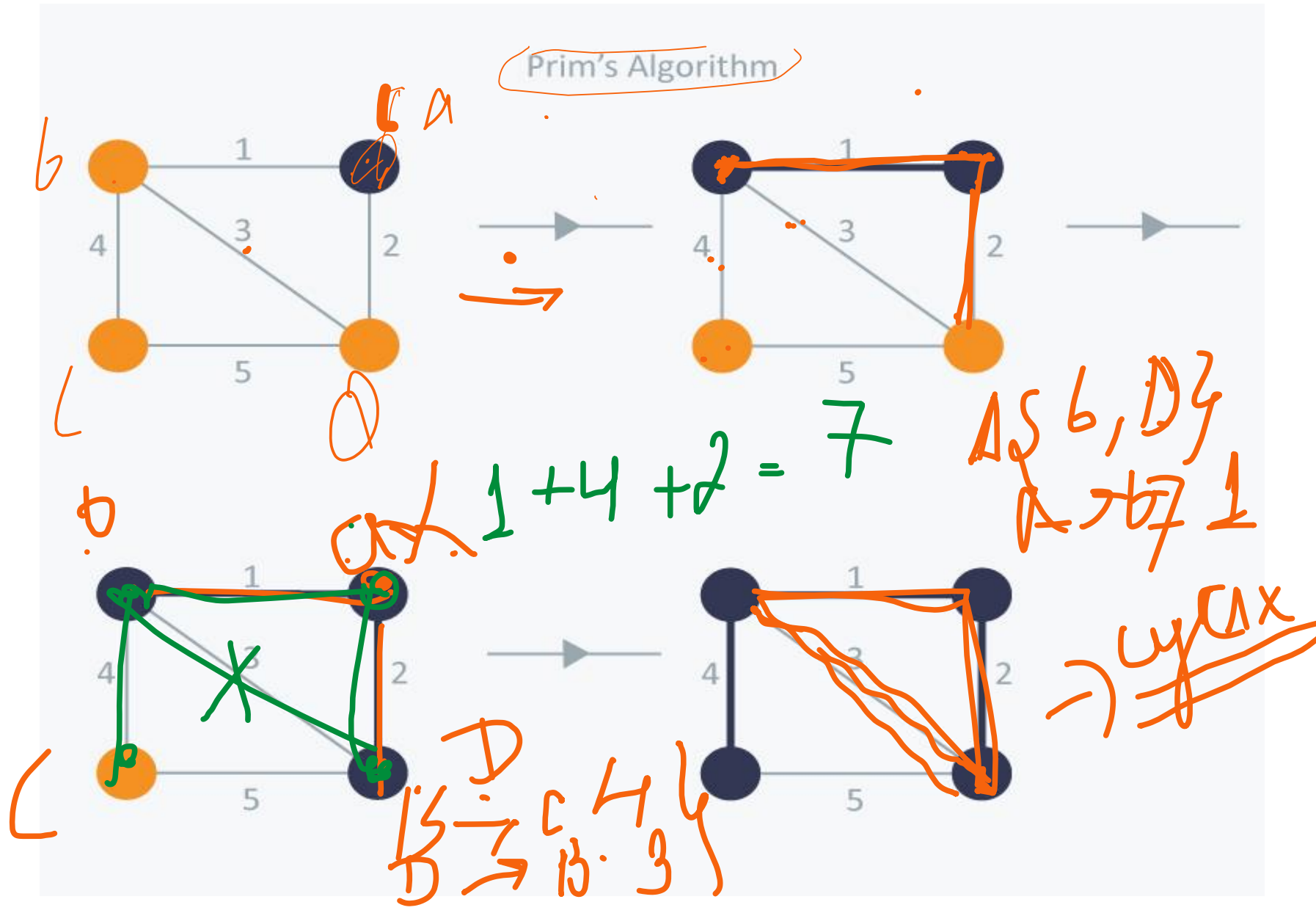
Prim's Algorithm

Example : As a greedy algorithm, Prim's algorithm will select the cheapest edge and mark the vertex. So we will simply choose the edge with weight 1. In the next iteration we have three options, edges with weight 2, 3 and 4. So, we will select the edge with weight 2 and mark the vertex.
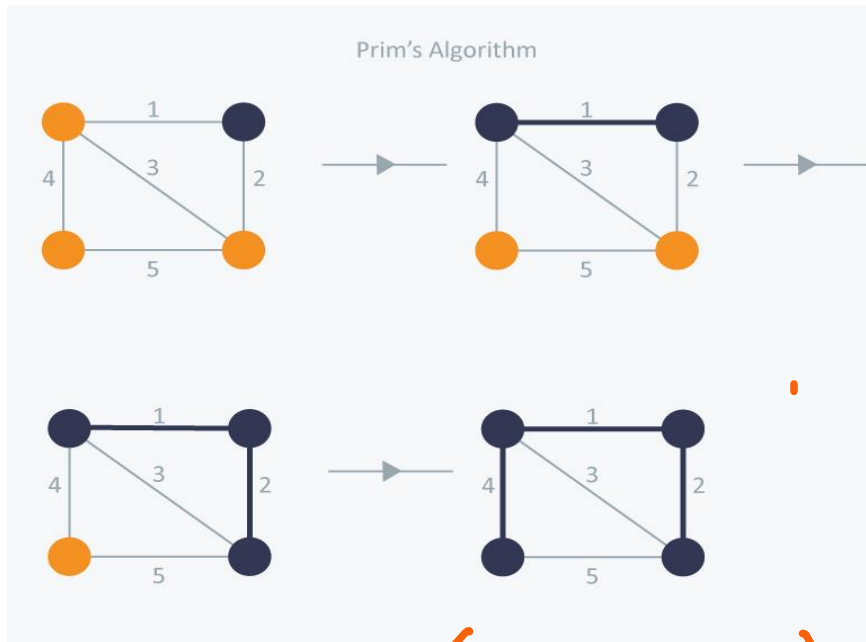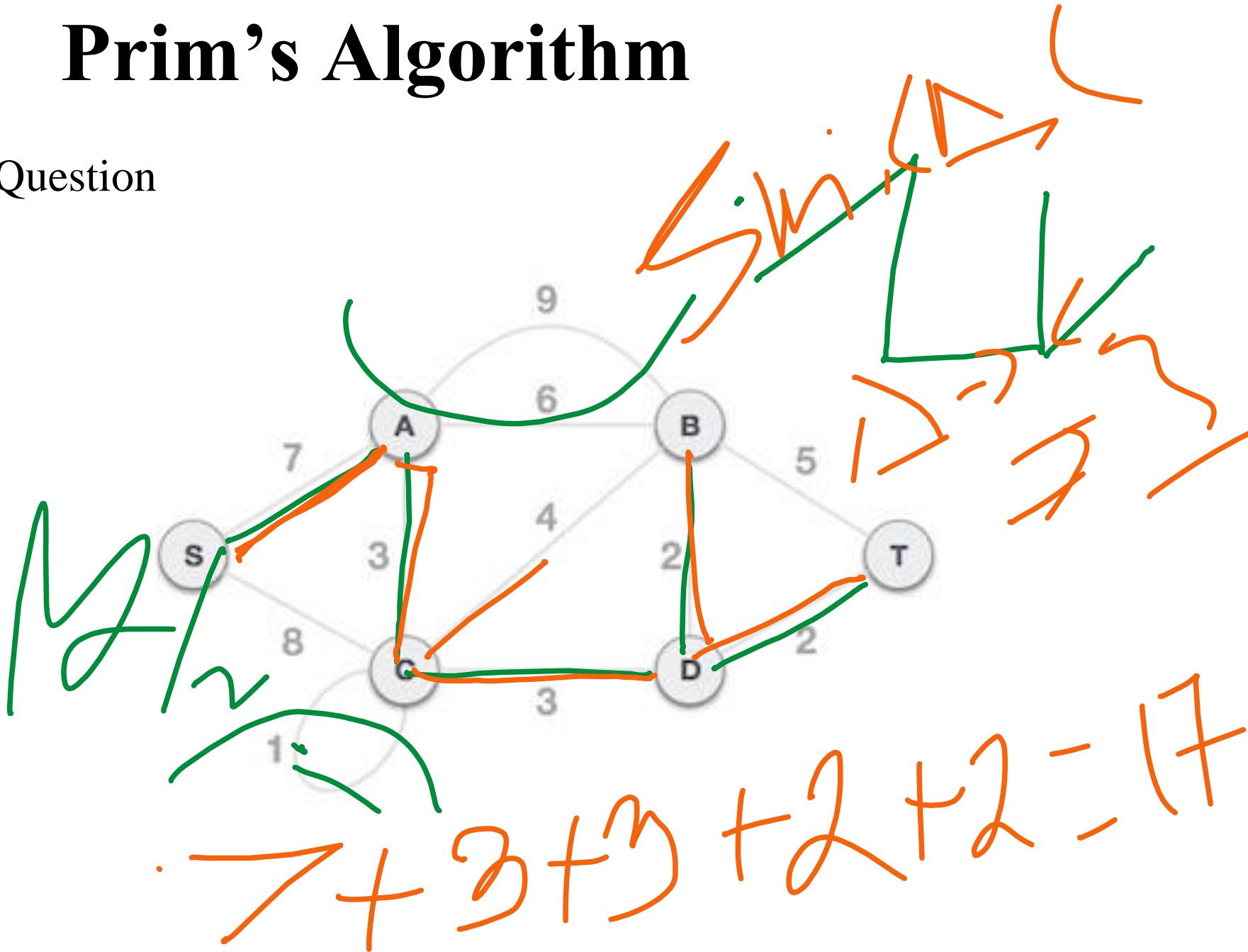

Prim's Algorithm

Now again we have three options, edges with weight 3, 4 and 5. But we can't choose edge with weight 3 as it is creating a cycle. So we will select the edge with weight 4 and we end up with the minimum spanning tree of total cost 7 (= 1 + 2 + 4).

$\text{Min} \{2, 3, 4\} = 2$

Prim's Algorithm

Question

UNIT - 5
BINARY TREES ANDHASHING

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Soch College of IT

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CONTENTS

- Binary Search Trees - Properties and Operations
- Balanced Search Trees – AVLTrees
- M – way Search Trees
- B Trees
- Hashing – Hash Table, Hash Function
- Collisions
- Applications of Hashing

Binary Search Trees

- In a BST, each node stores some information including a unique **key value**, and perhaps some associated data. A binary tree is a BST iff, for every node n in the tree:

- All keys in n's left subtree are less than the key in n, and

- All keys in n's right subtree are greater than the key in n.

- In other words, binary search trees are binary trees in which all values in the node's left subtree are less than node value all values in the node's right subtree are greater than node value.

BST Example

Here are some BSTs in which each node just stores an integer key:
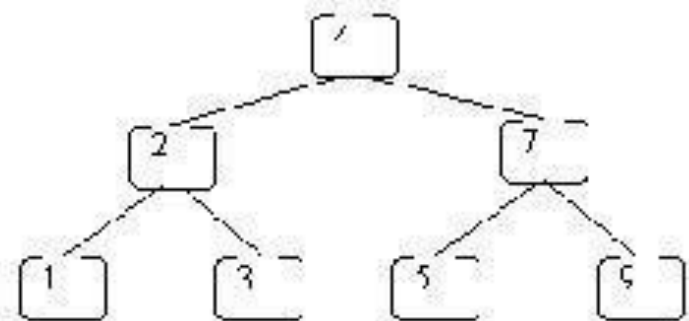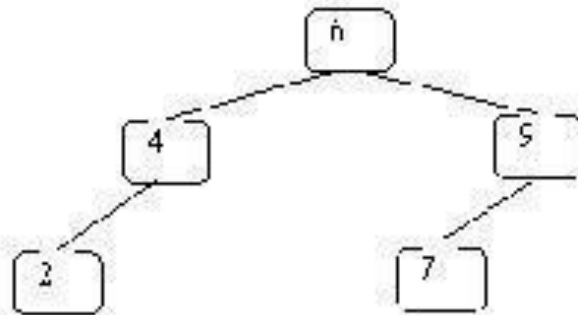
These are not BSTs:
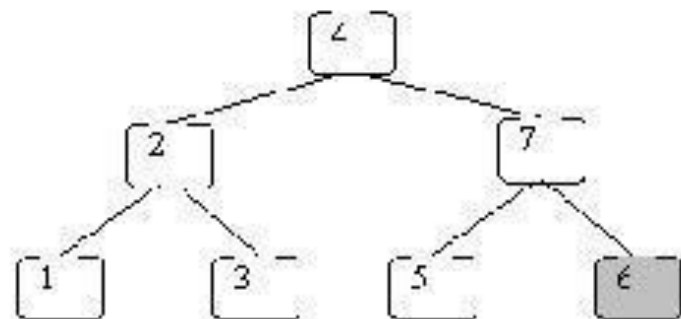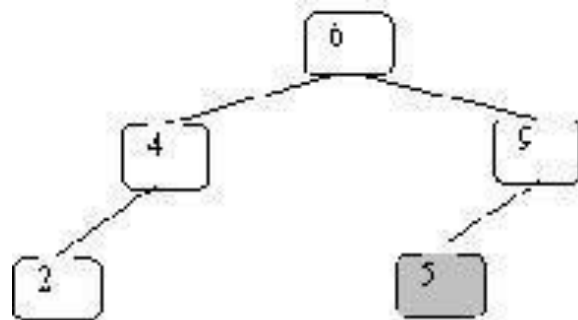
In the left one 5 is not greater than 6. In the right one 6 is not greater than 7.

Properties and Operations

A BST is a binary tree of nodes ordered in the following way:

i.   Each node contains one key (also unique)

ii. The keys in the left subtree are < (less) than the key in its parent node

iii. The keys in the right subtree > (greater) than the key in its parent node

iv. Duplicate node keys are not allowed.

Inserting a node

- A naïve algorithm for inserting a node into a BST is that, we start from the root node, if the node to insert is less than the root, we go to left child, and otherwise we go to the right child of the root.

- We then insert the node as a left or right child of the leaf node based on node is less or greater than the leaf node. We note that a new node is always inserted as a leaf node.

Inserting a node

- A recursive algorithm for inserting a node into a BST is as follows. Assume we insert a node N to tree T. if the tree is empty, the we return new node N as the tree. Otherwise, the problem of inserting is reduced to inserting the node N to left of right sub trees of T, depending on N is less or greater than T. A definition is as follows.

Insert(N, T) = N if T is empty

$$= \text{insert}(N, T.\text{left}) \text{ if } N < T$$

$$= \text{insert}(N, T.\text{right}) \text{ if } N > T$$

Searching for a node

- Searching for a node is similar to inserting a node. We start from root, and then go left or right until we find (or not find the node). A recursive definition of search is as follows. If the node is equal to root, then we return true. If the root is null, then we return false. Otherwise we recursively solve the problem for T.left or T.right, depending on N < T or N > T. A recursive definition is as follows.

- Search should return a true or false, depending on the node is found or not.

Searching for a node

- Search(N, T) = false if T is empty Searching for a node is similar to inserting a node. We start from root, and then go left or right until we find (or not find the node).

- A recursive definition of search is as follows. If the node is equal to root, then we return true. If the root is null, then we return false. Otherwise we recursively solve the problem for T.left or T.right, depending on $N < T$ or $N > T$. A recursive definition is as follows.

- Search should return a true or false, depending on the node is found or not.

  Search(N, T) = false if T is empty

  $\qquad$ = true if $T = N$

  $\qquad$ = search(N, T.left) if $N < T$

  $\qquad$ = search(N, T.right) if $N > T$

Deleting a node

- A BST is a connected structure. That is, all nodes in a tree are connected to some other node. For example, each node has a parent, unless node is the root. Therefore deleting a node could affect all sub trees of that node. For example, deleting node 5 from the tree could result in losing sub trees that are rooted at 1 and 9.
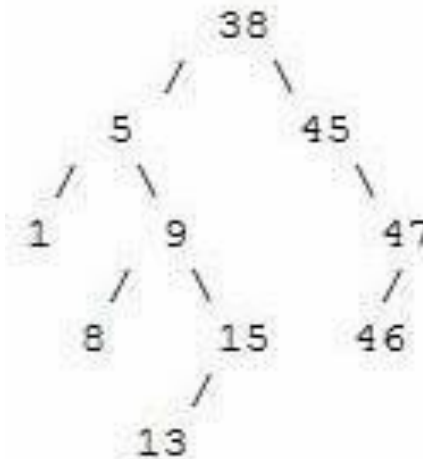
Balanced Search Trees

- A **self-balancing** (or **height-balanced**) **binary search tree** is any node-based binary search tree that automatically keeps its height (maximal number of levels below the root) small in the face of arbitrary item insertions and deletions.

- **AVL Trees:** An **AVL tree** is another balanced binary search tree. Named after their inventors, **A**delson-**V**elskii and **L**andis, they were the first dynamically balanced trees to be proposed. Like red-black trees, they are not perfectly balanced, but pairs of sub-trees differ in height by at most 1, maintaining an $O(\log n)$ search time. Addition and deletion operations also take $O(\log n)$ time.

AVL Tree - Definition

- **Definition of an AVL tree:** An AVL tree is a binary search tree which has the following properties:

  i. The sub-trees of every node differ in height by at most one.

  ii. Every sub-tree is an AVL tree.

- Balance requirement for an AVL tree: the left and right sub-trees differ by at most 1 in height.
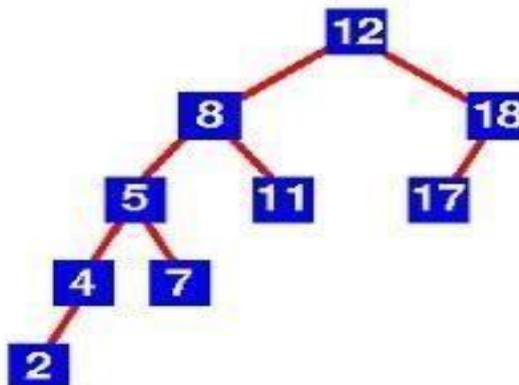
Balance requirement for an AVL tree: the left and right sub-trees differ by at most 1 in height.
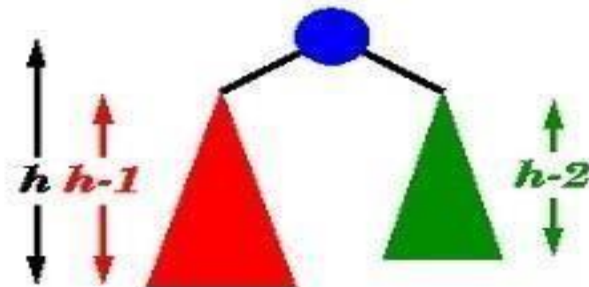
For example, here are some trees:
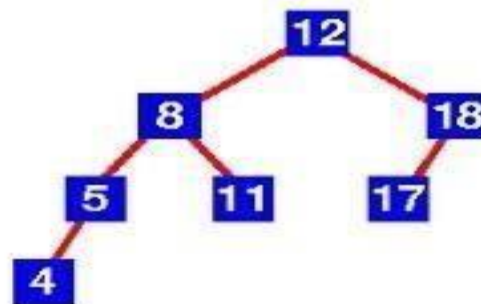
Yes this is an AVL tree. Examination shows that *each* left sub-tree has a height 1 greater than each right sub-tree.

No this is not an AVL tree. Sub-tree with root 8 has height 4 and sub-tree with root 18 has height 2.

Balance Factor

- To implement our AVL tree we need to keep track of a **balance factor** for each node in the tree. We do this by looking at the heights of the left and right subtrees for each node. More formally, we define the balance factor for a node as the difference between the height of the left subtree and the height of the right subtree.

$$balanceFactor = height(leftSubTree) - height(rightSubTree)$$

- Using the definition for balance factor given above we say that a subtree is left-heavy if the balance factor is greater than zero. If the balance factor is less than zero then the subtree is right heavy. If the balance factor is zero then the tree is perfectly in balance.

Balance Factor
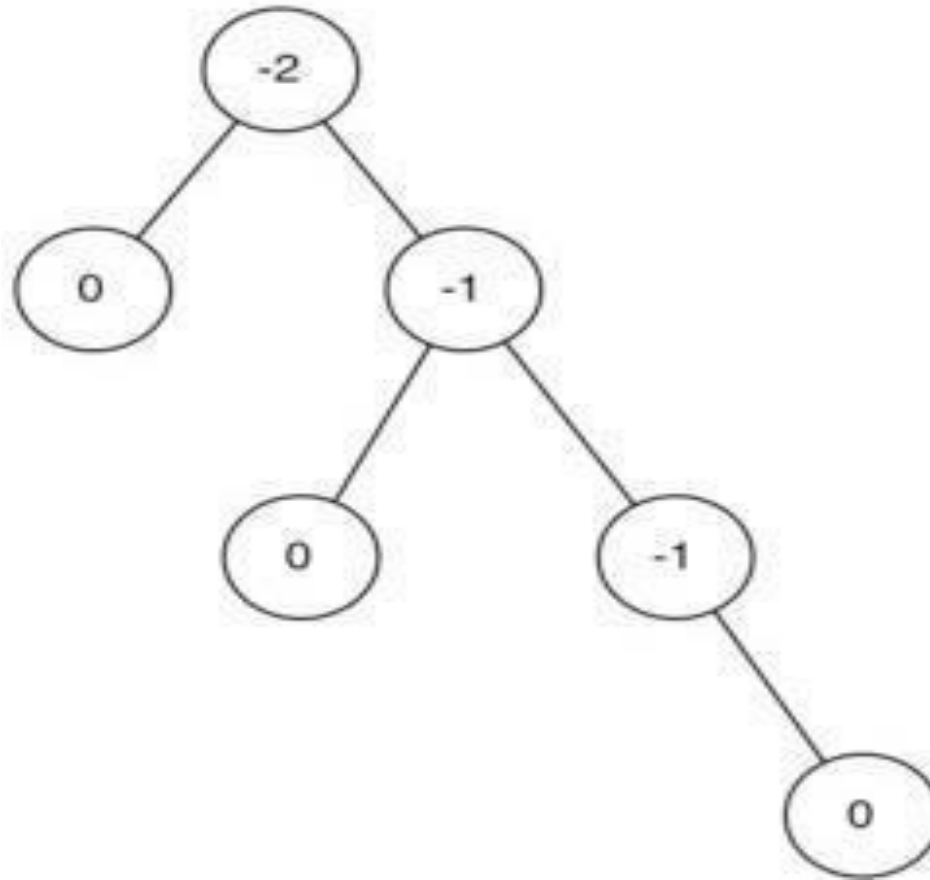
Introduction to M-Way Search Trees

- A **multiway tree** is a tree that can have more than two children. A **multiway tree of order m** (or an **m-way tree**) is one in which a tree can have m children.

- As with the other trees that have been studied, the nodes in an m-way tree will be made up of key fields, in this case m-1 key fields, and pointers to children.

- Multiday tree of order 5
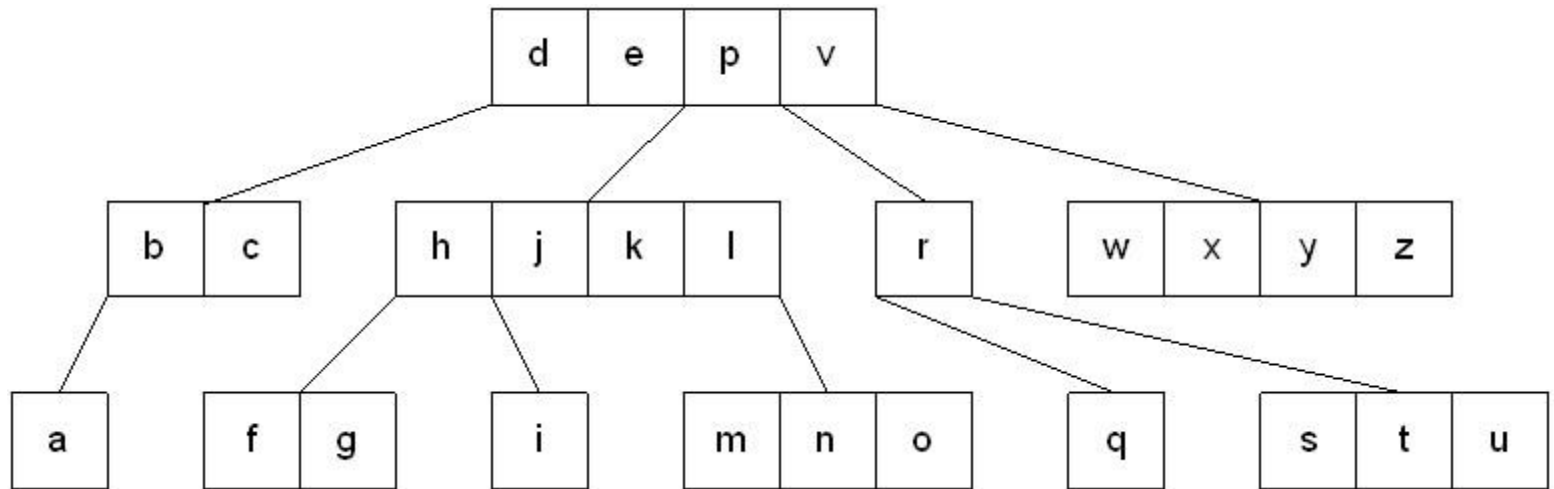
Properties of M-way Search Trees

- m-way search tree is a m-way tree in which:
  - i.   Each node has m children and m-1 key fields
  - ii.   The keys in each node are in ascending order.
  - iii. The keys in the first i children are smaller than the ith key
  - iv.   The keys in the last m-i children are larger than the ith key
- 4-way search tree
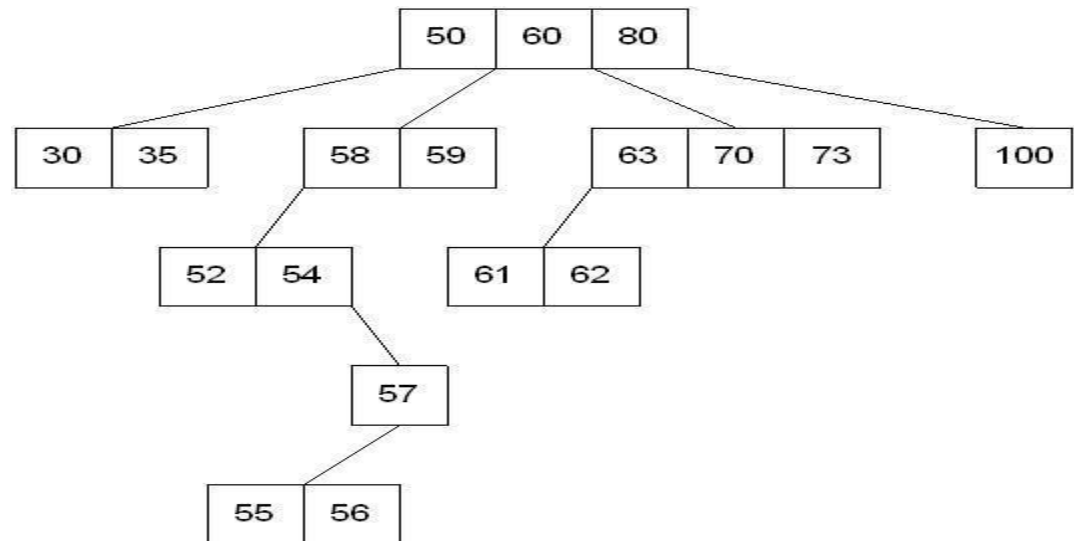
B -Trees

- An extension of a multiway search tree of order m is a **B-tree of order m**. This type of tree will be used when the data to be accessed/stored is located on secondary storage devices because they allow for large amounts of data to be stored in a node.

- A B-tree of order m is a multiway search tree in which:

  i.   The root has at least two subtrees unless it is the only node in the tree.

  ii.   Each nonroot and each nonleaf node have at most m nonempty children and at least m/2 nonempty children.

  iii. The number of keys in each nonroot and each nonleaf node is one less than the number of its nonempty children.

  iv.   All leaves are on the same level.

Searching a B -Tree

- Start at the root and determine which pointer to follow based on a comparison between the search value and key fields in the root node.

- Follow the appropriate pointer to a child node.

- Examine the key fields in the child node and continue to follow the appropriate pointers until the search value is found or a leaf node is reached that doesn't contain the desired search value.

Insertion into a B -Tree

- The condition that all leaves must be on the same level forces a characteristic behavior of B-trees, namely that B-trees are not allowed to grow at the their leaves; instead they are forced to grow at the root.

- When inserting into a B-tree, a value is inserted directly into a leaf. This leads to three common situations that can occur:

  i.   A key is placed into a leaf that still has room.

  ii. The leaf in which a key is to be placed is full.

  iii. The root of the B-tree is full.

Case 1: A key is placed into a leaf that still has room

This is the easiest of the cases to solve because the value is simply inserted into the correct sorted position in the leaf node.
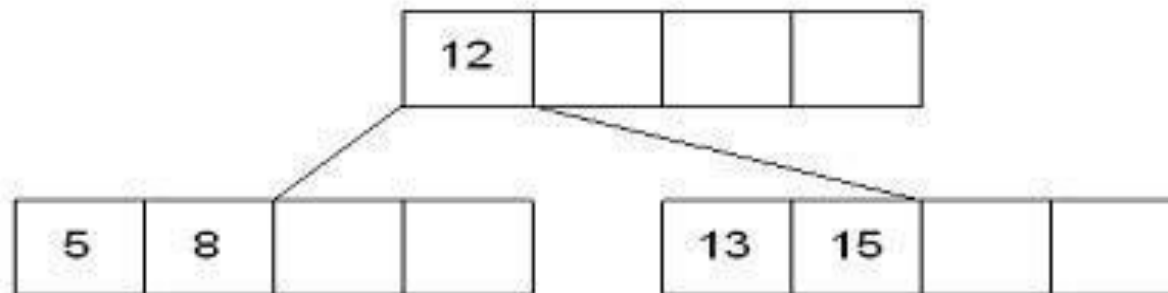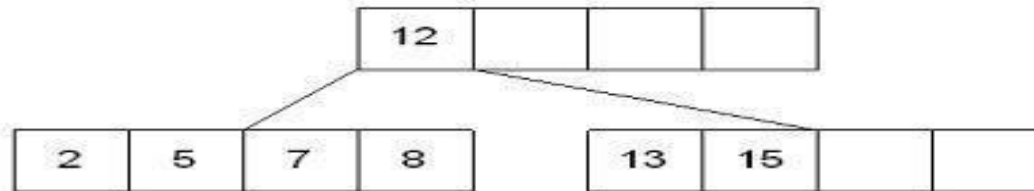

Inserting the number 7 results in:
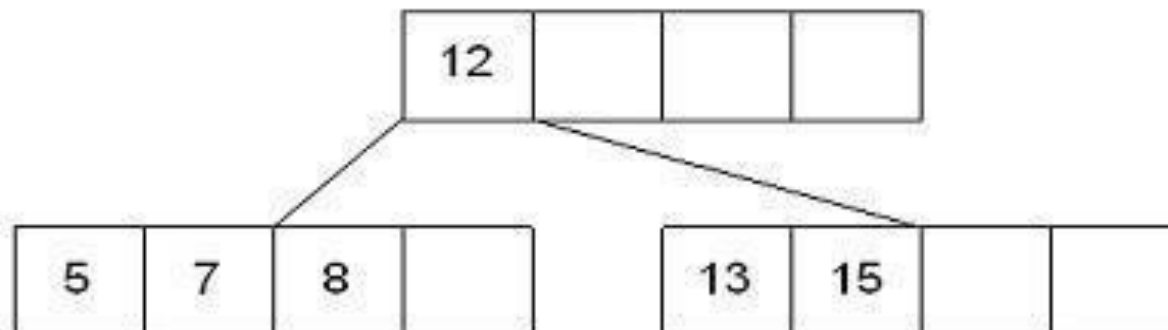
## Case 2: The leaf in which a key is to be placed is full

In this case, the leaf node where the value should be inserted is split in two, resulting in a new leaf node. Half of the keys will be moved from the full leaf to the new leaf. The new leaf is then incorporated into the B-tree.

The new leaf is incorporated by moving the middle value to the parent and a pointer to the new leaf is also added to the parent. This process is continues up the tree until all of the values have "found" a location.
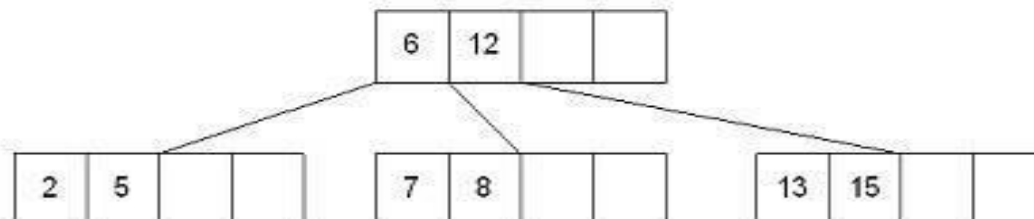
Insert 6 into the following B-tree:


results in a split of the first leaf node:
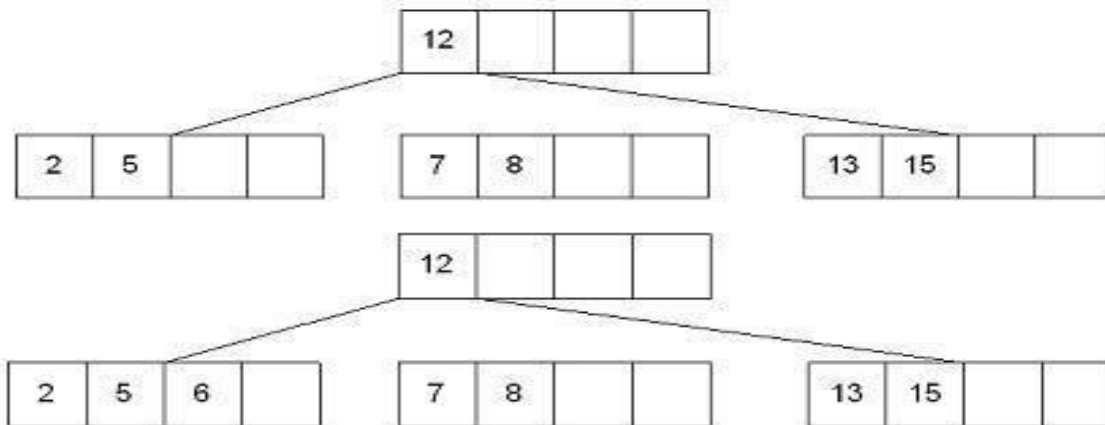
The new node needs to be incorporated into the tree – this is accomplished by taking the middle value and inserting it in the parent:

Case 3: The root of the B-tree is full

The upward movement of values from case 2 means that it's possible that a value could move up to the root of the B-tree. If the root is full, the same basic process from case 2 will be applied and a new root will be created. This type of split results in 2 new nodes being added to the B-tree.

Inserting 13 into the following tree:
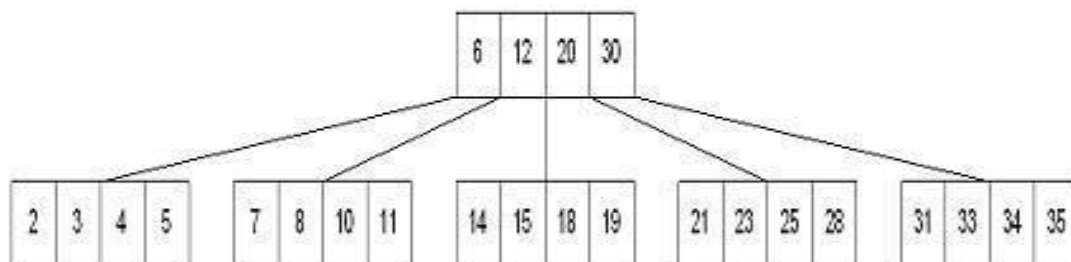
Results in:
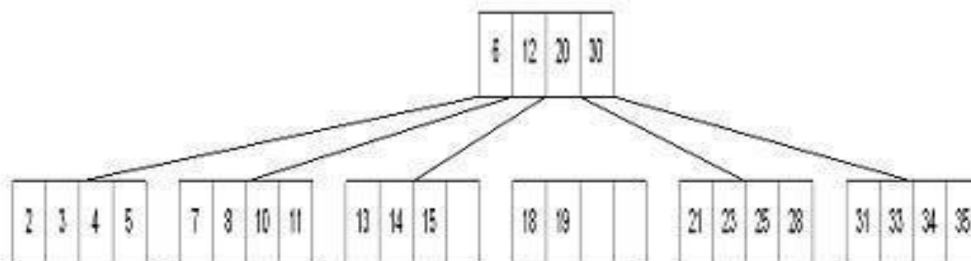
The 15 needs to be moved to the root node but it is full. This means that the root needs to be divided:
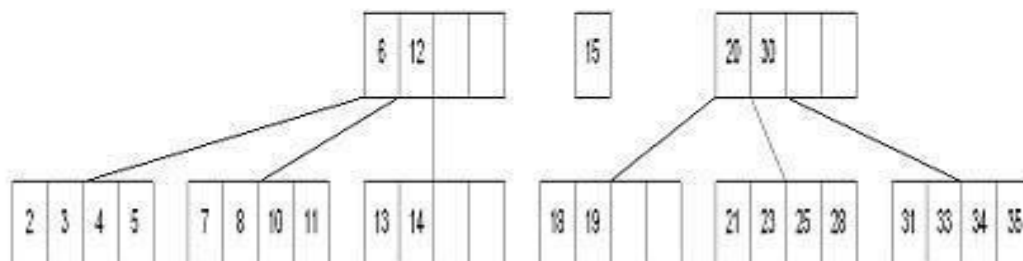
The 15 is inserted into the parent, which means that it becomes the new root node:

221

Deleting from a B -Tree

- The deletion process will basically be a reversal of the insertion process - rather than splitting nodes, it's possible that nodes will be merged so that B-tree properties, namely the requirement that a node must be at least half full, can be maintained.

- There are two main cases to be considered:
  - i. Deletion from a leaf
  - ii. Deletion from a non-leaf

Case 1: Deletion from a leaf

1a) If the leaf is at least half full after deleting the desired value, the remaining larger values are moved to "fill the gap".

Deleting 6 from the following tree:
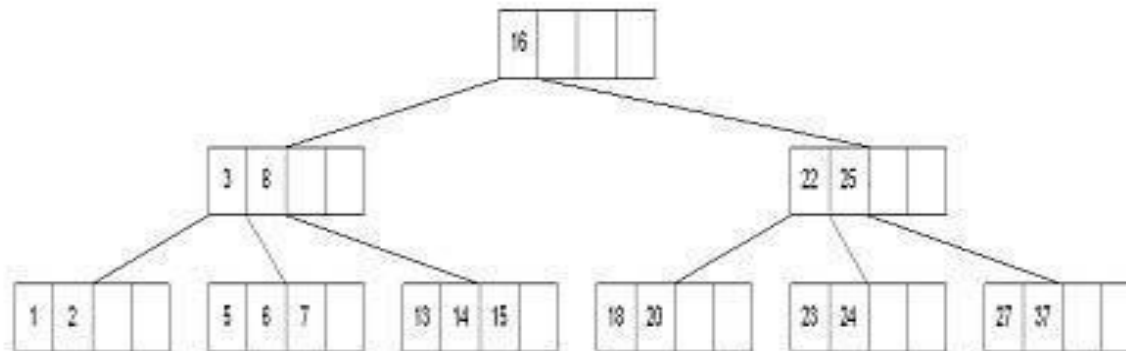
results in:
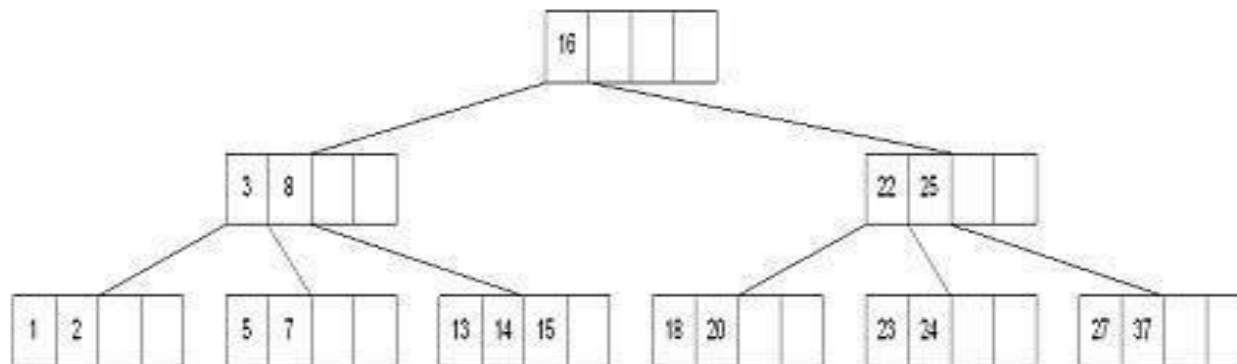
If there is a left or right sibling with the number of keys exceeding the minimum requirement, all of the keys from the leaf and sibling will be redistributed between them by moving the separator key from the parent to the leaf and moving the middle key from the node and the sibling combined to the parent.
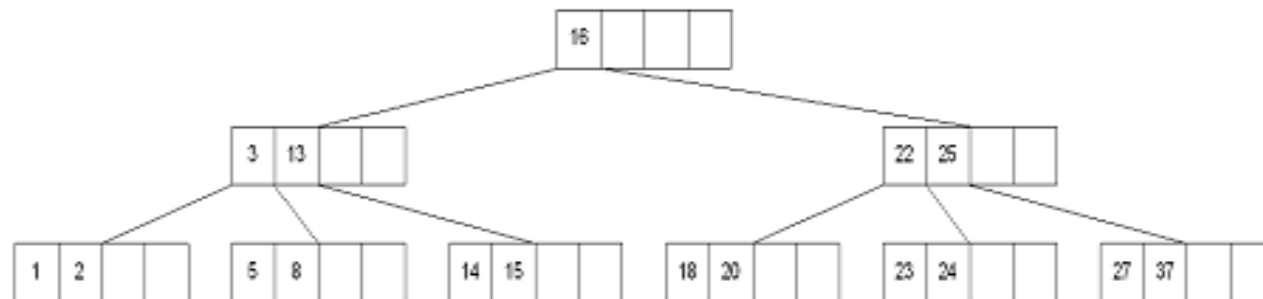
Now delete 8 from the tree:
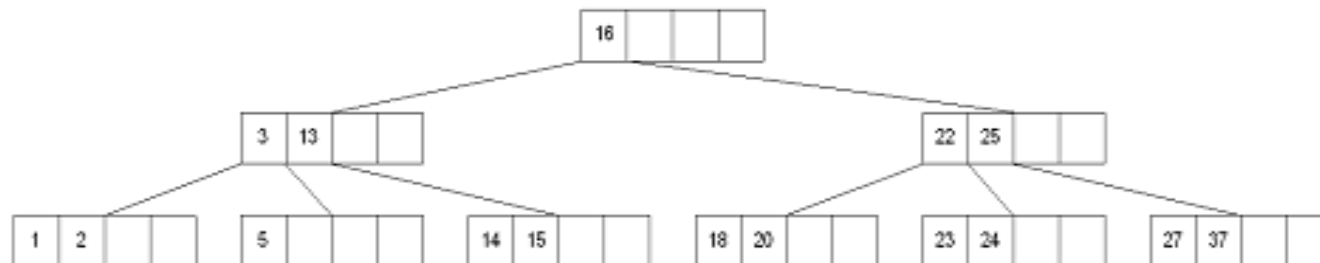
If the number of keys in the sibling does not exceed the minimum requirement, then the leaf and sibling are merged by putting the keys from the leaf, the sibling, and the separator from the parent into the leaf. The sibling node is discarded and the keys in the parent are moved to "fill the gap". It's possible that this will cause the parent to underflow. If that is the case, treat the parent as a leaf and continue repeating step 1b-2 until the minimum requirement is met or the root of the tree is reached.

Hashing

- Hashing is the technique used for performing almost constant time search in case of insertion, deletion and find operation.

- Taking a very simple example of it, an array with its index as key is the example of hash table. So each index (key) can be used for accessing the value in a constant search time. This mapping key must be simple to compute and must helping in identifying the associated value. Function which helps us in generating such kind of key- value mapping is known as Hash Function.

- In a hashing system the keys are stored in an array which is called the Hash Table. A perfectly implemented hash table would always promise an average insert/delete/retrieval time of O(1).

Hashing Function

- A function which employs some algorithm to computes the key *K* for all the data elements in the set *U,* such that the key *K* which is of a fixed size. The same key *K* can be used to map data to a hash table and all the operations like insertion, deletion and searching should be possible. The values returned by a **hash function** are also referred to as **hash** values, **hash** codes, **hash** sums, or **hashes**.
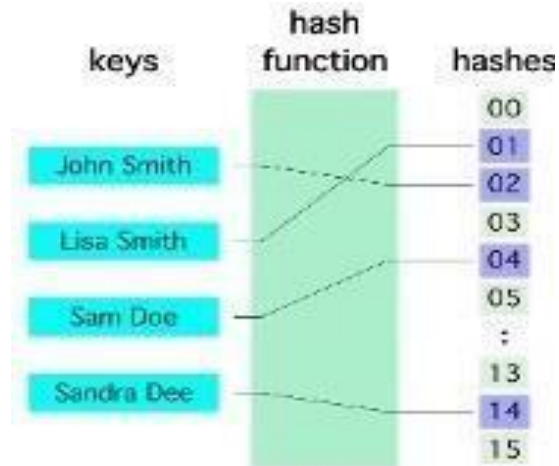
Hash Collision

- A situation when the resultant hashes for two or more data elements in the data set *U,* maps to the same location in the has table, is called a hash collision. In such a situation two or more data elements would qualify to be stored / mapped to the same location in the hash table.

- **Hash collision resolution techniques:**

- **Open Hashing (Separate chaining):**Open Hashing, is a technique in which the data is not directly stored at the hash key index (k) of the Hash table. Rather the data at the key index (k) in the hash table is a pointer to the head of the data structure where the data is actually stored. In the most simple and common implementations the data structure adopted for storing the element is a linked-list.
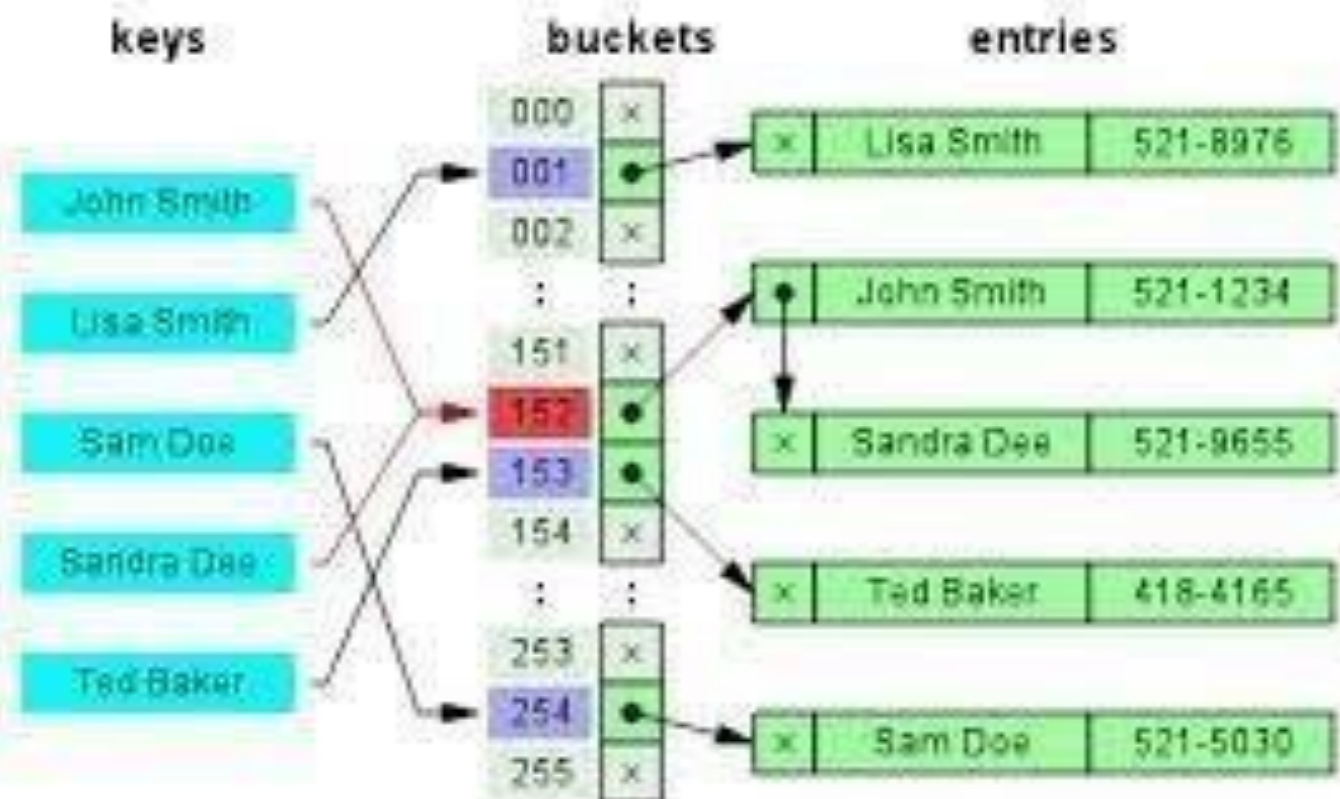
Closed Hashing (Open Addressing)

- In this technique a hash table with pre-identified size is considered. All items are stored in the hash table itself. In addition to the data, each hash bucket also maintains the three states: EMPTY, OCCUPIED, DELETED. While inserting, if a collision occurs, alternative cells are tried until an empty bucket is found. For which one of the following technique is adopted.

- Liner Probing

- Quadratic probing

- Double hashing

A Comparative Analysis of Closed Hashing vs Open Hashing

Open Addressing	Closed Addressing
All elements would be stored in the Hash table itself. No additional data structure is needed.	Additional Data structure needs to be used to accommodate collision data.
In cases of collisions, a unique hash key must be obtained.	Simple and effective approach to collision resolution. Key may or may not be unique.
Determining size of the hash table, adequate enough for storing all the data is difficult.	Performance deterioration of closed addressing much slower as compared to Open addressing.
State needs be maintained for the data (additional work)	No state data needs to be maintained (easier to maintain)
Uses space efficiently	Expensive on space

Applications of Hashing

- A **hash function** maps a variable length input string to fixed length output string -- its **hash value**, or **hash** for short. If the input is longer than the output, then some inputs must map to the same output -- a **hash collision**.

- Comparing the hash values for two inputs can give us one of two answers: the inputs are definitely not the same, or there is a possibility that they are the same. Hashing as we know it is used for performance improvement, error checking, and authentication.

- In error checking, hashes (checksums, message digests, etc.) are used to detect errors caused by either hardware or software. Examples are TCP checksums, ECC memory, and MD5 checksums on downloaded files.

Applications of Hashing

- Construct a *message authentication code* (MAC)
- Digital signature
- Make commitments, but reveal message later
- Timestamping
- Key updating: key is hashed at specific intervals resulting in new key

8. **Sorting and Searching**

- Sorting refers to arranging data in a particular format.

- Sorting algorithm specifies the way to arrange data in a particular order

- Sorting is nothing but arranging the data in ascending or descending order.

- Most common orders are in numerical or lexicographical order.

- The importance of sorting lies in the fact that data searching can be optimized to a very high level, if data is stored in a sorted manner.

- Sorting is also used to represent data in more readable formats.

8. **Sorting and Searching**

- Sorting refers to the operation or technique of arranging and rearranging sets of data in some specific order.

- A collection of records called a list where every record has one or more fields.

- The fields which contain a unique value for each record is termed as the *key* field.

-   Various sorting techniques are analyzed in various cases and named these cases as follows:
  - Best case
  - Worst case
  - Average case

Example

•**Telephone Directory** − The telephone directory stores the telephone numbers of people sorted by their names, so that the names can be searched easily.
•**Dictionary** − The dictionary stores words in an alphabetical order so that searching of any word becomes easy.

Searching

- Searching in data-strucutre refers to the process of finding a desired element in set of items. The desired element is called "target".

- The set of items to be searched in, can be any data-structure like − list, array, linked-list, tree or graph.

- Search refers to locating a desired element of specified properties in a collection of items.

•Searching Algorithms are designed to check for an element or retrieve an element from any data structure where it is stored.

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Searching

- **Linear Search**

 Linear search searches all items and its worst execution time is n where n is the number of items. Also khown as **Sequential Search**

- **Binary Search**

 Binary search requires items to be in sorted order but its worst execution time is constant and is much faster than linear search. Also known as **Interval Search**

- **Interpolation Search**

 Interpolation search requires items to be in sorted order but its worst execution time is O(n) where n is the number of items and it is much faster than linear search.

Searching

- **Linear Search**

Linear search searches all items and its worst execution time is n where n is the number of items. Also khown as **Sequential Search**

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Interpolation search requires items to be in sorted order but its worst execution time is O(n) where n is the number of items and it is much faster than linear search.

Sequential search

Sequential search is also called as Linear Search.

•Sequential search starts at the beginning of the list and checks every element of the list.

•It is a basic and simple search algorithm.

•Sequential search compares the element with all the other elements given in the list. If the element is matched, it returns the value index, else it returns -1.

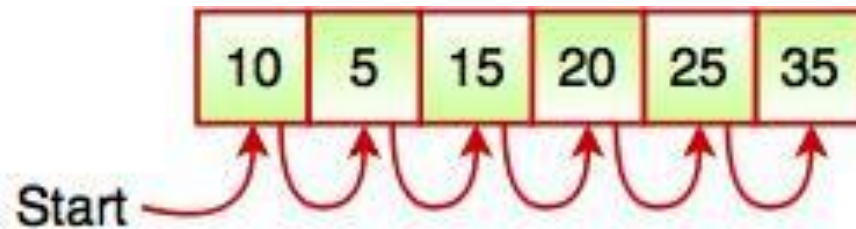Fig. Sequential Search

Sequential search ALgorithm

Linear Search (Array A, Value x)
Step 1: Set i to 1
Step 2: if i > n then go to step 7
Step 3: if A[i] = x then go to step 6
Step 4: Set i to i + 1
Step 5: Go to Step 2
Step 6: Print Element x Found at index i and go to step 8
Step 7: Print element not found
Step 8: Exit

Linear Search

| 10 | 14 | 19 | 26 | 27 | 31 | 33 | 35 | 42 | 44 |

=
33

Binary search

- Binary Search is used for searching an element in a sorted array.
- It is a fast search algorithm with run-time complexity of O(log n).
- Binary search works on the principle of divide and conquer.
- This searching technique looks for a particular element by comparing the middle most element of the collection.
- It is useful when there are large number of elements in an array.

Binary Searching

Fig. Working Structure of Binary Search

Binary search Algorithm

1.Start with the middle element:

•If the **target** value is equal to the middle element of the array, then return the index of the middle element.

•If not, then compare the middle element with the target value,

•If the target value is greater than the number in the middle index, then pick the elements to the right of the middle index, and start with Step 1.

•If the target value is less than the number in the middle index, then pick the elements to the left of the middle index, and start with Step 1.

2.When a match is found, return the index of the element matched.

3.If no match is found, then return -1

Binary search Example

list

0	1	2	3	4	5	6	7	8
10	12	20	32	50	55	65	80	99

search element   12

Step 1:

search element (12) is compared with middle element (50)

list

0	1	2	3	4	5	6	7	8
10	12	20	32	50	55	65	80	99

12

Both are not matching. And 12 is smaller than 50. So we search only in the left sublist (i.e. 10, 12, 20 & 32).

list

0	1	2	3	4	5	6	7	8
10	12	20	32	50	55	65	80	99

Step 2:

search element (12) is compared with middle element (12)

list

0	1	2	3	4	5	6	7	8
10	12	20	32	50	55	65	80	99

12

Both are matching. So the result is "Element found at index 1"

Binary search Example

search element   **80**

Step 1:

search element (80) is compared with middle element (50)

0	1	2	3	4	5	6	7	8
list | 10 | 12 | 20 | 32 | **50** | 55 | 65 | 80 | 99 |

80

Both are not matching. And 80 is larger than 50. So we search only in the right sublist (i.e. 55, 65, 80 & 99).

0	1	2	3	4	5	6	7	8
list | 10 | 12 | 20 | 32 | 50 | **55** | **65** | **80** | **99** |

Step 2:

search element (80) is compared with middle element (65)

0	1	2	3	4	5	6	7	8
list | 10 | 12 | 20 | 32 | 50 | **55** | **65** | **80** | **99** |

80

Both are not matching. And 80 is larger than 65. So we search only in the right sublist (i.e. 80 & 99).

0	1	2	3	4	5	6	7	8
list | 10 | 12 | 20 | 32 | 50 | 55 | 65 | **80** | **99** |

Step 3:

search element (80) is compared with middle element (80)

0	1	2	3	4	5	6	7	8
list | 10 | 12 | 20 | 32 | 50 | 55 | 65 | **80** | **99** |

80

Both are not matching. So the result is "Element found at index 7"

Binary search Class Work

Searching for 14 in 12 elements Array

| 1 | 3 | 5 | 6 | 7 | 8 | 9 | 11 | 14 | 15 | 19 | 27 |

THANK YOU