Physics Extended Essay

Essay: 4000 words

Session: May 2020

Title: Investigating the restoring force on the maximum perpendicular dip of an intersection point on the string plane in a squash racquet.

Research Question:

What will be the restoring force at the point of intersection, which has the maximum perpendicular dip, on a string plane in a squash racquet when a uniform force is applied?

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I. Introduction:

Among the racquet sports, games played by sustaining a projectile in the air between 2 or more players with a racquet, squash has been steadily growing in popularity due to the dynamics of the game. This increased interest has gotten many to optimize both the technique and the equipment of the game.

Throughout the history of squash, enthusiasts have upgraded the frame, material of strings, and grip to enhance the performance of the player and make the game more entertaining and intense. In a fast-paced game of squash, an edge can be obtained over an opponent by ensuring that every shot is clean and well-timed, so that the ball is away from the reach of the adversary. To achieve this, it is important to understand the location on the string plane(gut), where the ball may receive the maximum impulse in countering with a fast-paced return or service. It has always been common knowledge in squash, even among the junior players, that the central region of the racquet head should be targeted in order to give a successful return. However, the *region* in the centre does not provide specificity, so the *specific spot* on the region is yet undetermined and can only be roughly distinguished by a regular player through well revised intuition. My squash coach's persistence on convincing me to target the centre of the racquet got me to question the viability of hitting the centre of the racquet and motivated me to investigate the ideal spot.

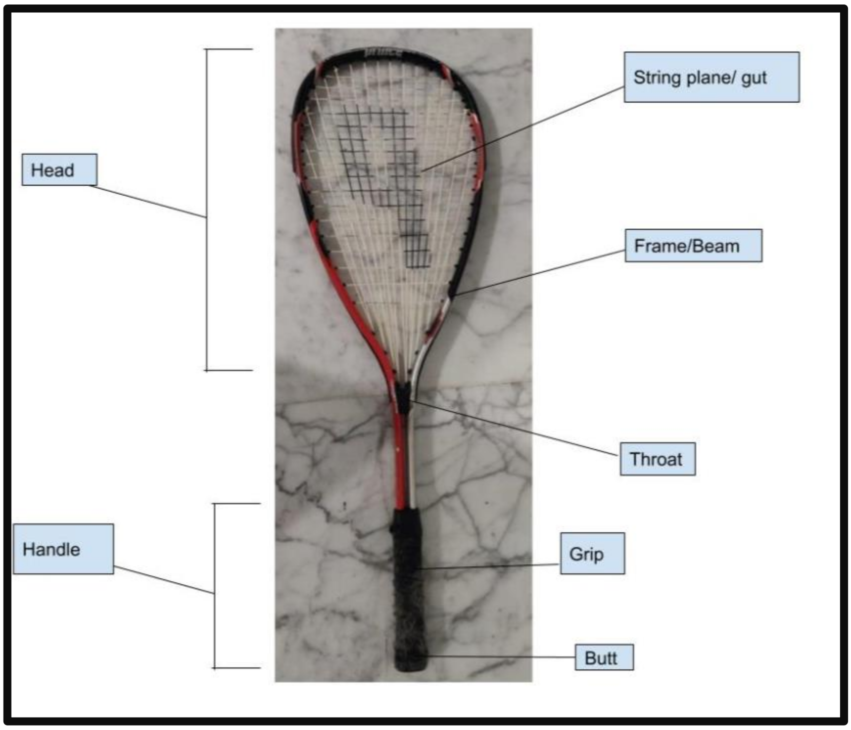


Figure 1: Parts of the racquet. Note: Throughout the investigation, the racquet will be viewed in the orientation that is shown in the figure.

Upon the impact of a ball and the gut**, a "dip" forms**, a temporary perpendicular deformation in the string plane, formed due to the elastic nature of the material of the strings, and impacts the impulse with which the ball leaves (*Figure 2* shows the image of a dip). Due to the formation of a dip**, a restoring force develops in the string**, which is the describing force that acts to retract a point to its initial position(Davecoulter). Restoring force is of 2 types in this case: restoring force exists in the strings that brings back the extended string to its original length, and a result of this is the second restoring force, which acts in the perpendicular direction opposite to the position of the dip; the latter is responsible for adding on to the ball's net impulse. Therefore, studying the relationship between the dip and the restoring force at different points provides insight into the optimum point.

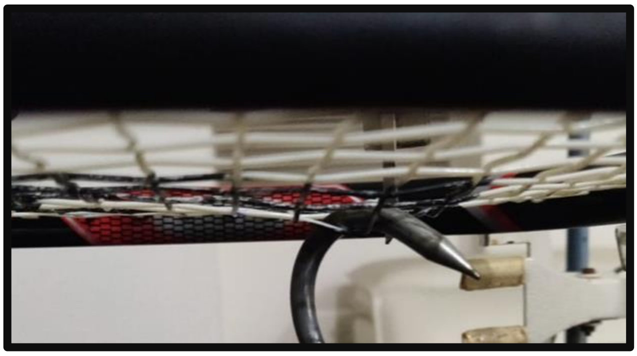


Figure 2: Image of a dip being formed

An interesting element of all racquet sports, including squash, are the intersection points of the horizontal and vertical strings. Different intersection points forms different sized dips and exert a different restoring force, based on the string lengths that form the intersection point and the location of the intersection point on the gut (*Figure 1* shows the image of different parts of the racquet).

Therefore, the objective of the essay is to experimentally find the point of intersection, where the maximum dip will exist, and then using the value of that dip to create a mathematical model to predict the restoring force that the point may exert. To fulfil this objective, the RQ designed is **What will be the restoring force at the point of intersection, which has the maximum perpendicular dip, on a string plane in a squash racquet when a uniform force is applied?**

II. Background Information

Among the literature that I surveyed, Mr. Howard Brody's paper, "Physics on the tennis racket", particularly stood out (Brody 482). The paper argued that the strings in the tennis racket could not "feed-back energy to the ball' during collision as the tennis ball compressed lesser than the strings itself resulting in a shorter dwell time, amount of time the ball and gut are at the point of maximum dip, on the gut (Brody). This meant that the restoring force had no effect whatsoever on the final impulse of the ball as the ball already rebounds before the force can exert itself. Although this may be true for the tennis racquet, it is not true for the squash racquet, where the ball compresses more than the strings, which leads to a prolonged dwell time causing the restoring force to have an effect on the ball's final momentum. (A proof of this is presented in the *Appendix 5*, where I proved this assumption to be true experimentally).

Another applicable research paper, authored by Mr. Howard Brody, is " Physics of the tennis racquet II: The sweet spot"(Brody 816). The paper described the several ideal spots, with different definitions, such as the point of **centre of percussion** and the point of **vibrational node**. He points differ in terms of the characteristics their characteristics. The "sweet spot" is the point on the racquet gut at which the player will feel most comfortable, least "jarring" in playing the shot. Since the definition I came up with is not matched by the sweet spot's, I can name the investigated point the **dominance spot**. Moreover, this paper also provided inspiration for the mathematical model I designed.

When a force is exerted at an intersection point, an extension takes place in both the horizontal and vertical strings. The perpendicular dip formed is a direct result of the extension of the individual strings. This leads to the exploration of key principles of Physics such as resolution of force vectors, restoring force in strings, and Hooke’s Law.

The intersection points, vary uniquely in terms of the distance from the frame both in the horizontal and vertical direction (*Figure 4* describes this unique characteristic for 2 points). This is the general reason why each point may project a different dip value.

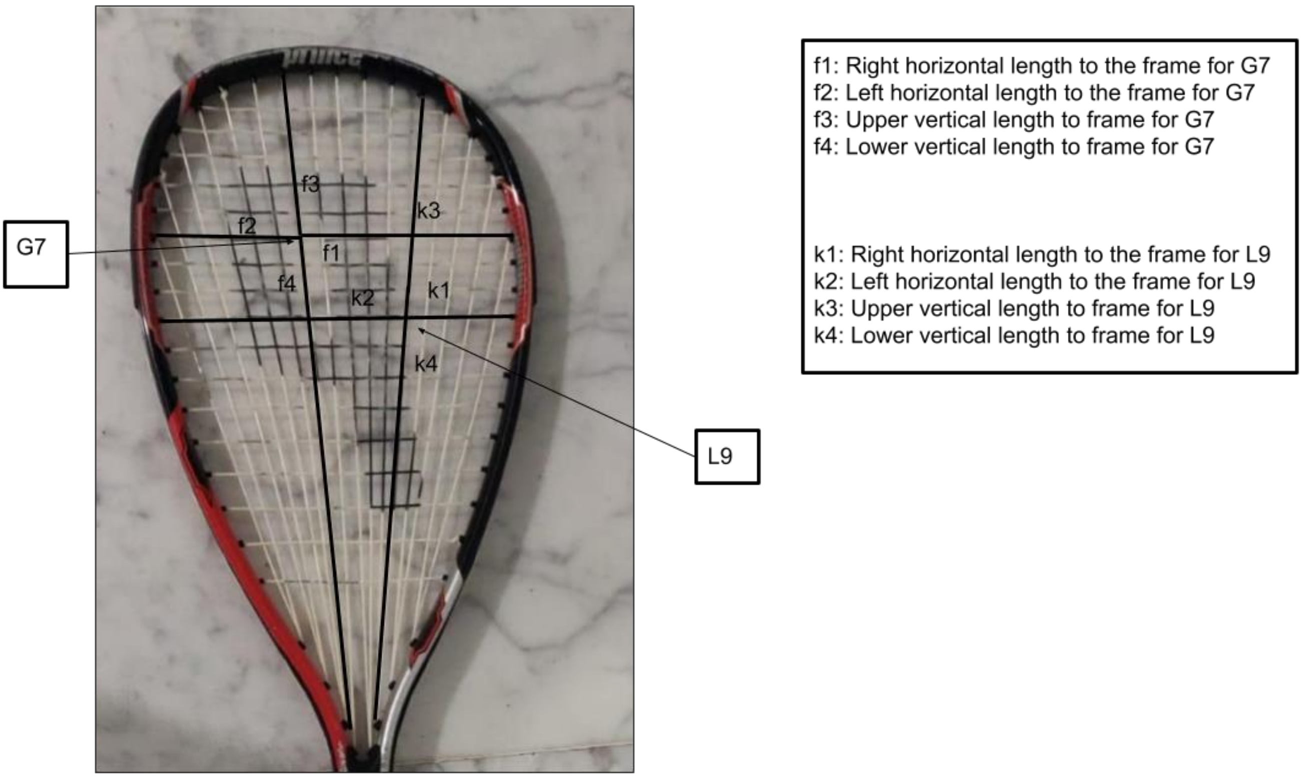
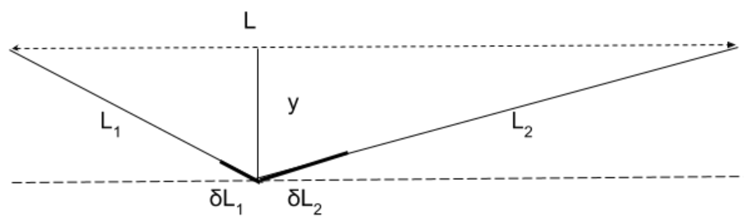


Figure 4: Diagram showing the different distances to the frame of 2 points G7 and L9. Note: Please refer to Appendix 1 for the naming of the intersection points.

Hooke's Law states that, under the elastic limit, stress is directly proportional to strain along the axis of an elastic material(Fitzpatrick). Stress is defined as the amount of restoring force exerted per unit area of cross section, when a body is subject to an extension (Vanderbuilt University). Strain is the ratio of the extension to the length of the body when the body experiences a force(Vanderbuilt University).



L - total length of the unstretched string
y - length of the dip
L_1- length of the first segment
L_2- length of the second segment
δL_1 - length of the extension in the first segment
δL_2- length of the extension in the second segment
F_R- The restoring force along the string
A- Area of cross section of the string

Figure 3: Change in dip as a result of change in extension

For a horizontal string experiencing a force at the intersection point, the Hooke's Law can be invoked for one segment of the string. A segment of a string, is a portion on either side of the intersection point.

$$Stress \propto Strain$$

Using *Figure 3*, the proportionality between stress and strain can be written as follows

Equation 1

$$\frac{F_R}{A} \propto \frac{\delta L_1}{L_1}$$

Since the area of cross section is constant, it is observed that there exists a proportional relationship between restoring force and strain:

Equation 2

$$F_R \propto \frac{\delta L_1}{L_1}$$

When the total length of the string, L, will increase, the length of each segment, L_1 and L_2 will also increase. Hence, the extensions in the segments of the string strings, δL_1 and δL_2, will also proportionately increase. Therefore we can write

$$F_R \propto \delta L_1$$

By referring to *Figure 3*, it is known that

Equation 3

$$y \propto \delta L_1$$

The perpendicular dip is the result of the extension in the string, so with an increase in the extension, a larger dip will be produced.

Using *Equation 2 and 3*, it can be inferred that

$$F_R \propto y$$

However, it is important to note that this proportionality relationship with the dip will involve the vertical component of the restoring force in the strings; this is because restoring force in the strings is at an angle to the horizontal. Since the definition of Hooke's law allows the restoring force to be projected along the string's axis, the effect of the restoring force on the dip is measured by the resolution of vectors, where the vertical component has an effect on the dip and it is assumed that the horizontal vectors get cancelled(Further elaboration occurs in the subheading Mathematics Model*)*.

Each intersection point differs in terms of its location on the gut, this leads to different sized dips being formed. The size of the maximum dip is to be experimentally determined and the magnitude of the restoring force, using the value of that dip, is theoretically investigated.

III. <u>Methodology</u>

(Please refer to *Appendix 1* for the naming of the intersection points)

The investigation is approached by the following steps:

⇒ I empirically determine the maximum dip corresponding to a few intersection points that are chosen based on them having traits to likely be the points with the maximum dip.

⇒ The experiment I devised to measure these dips consists of applying a uniform force individually on the selected intersection points, the independent variable, and then measuring the maximum dip generated, the dependent variable, by a Vernier calliper.

⇒ Once I have identified the point with the maximum dip, I plan to use the characteristics of the point and the dip to find the restoring force using the model that I have created.

⇒ The model redefines certain variable quantities in terms of experimentally measurable quantities, such as horizontal and vertical distance of the point from the frame.

⇒ Finally, I use the model to find the restoring force of all the points I have considered to plot bar graphs for each intersection point to establish a general trend across the racquet.

IV. Hypothesis

I predict that the maximum dip will be within the vertical strings G to K and between the horizontal strings 7-11. I hypothesize this because the strings covered in the prediction are the longest strings on the racquet in both the vertical and horizontal direction. In a squash racquet, the strings are fixed on either side of the frame, so when a force is applied perpendicular to a set of strings with variable length, the maximum dip will appear on the longest string for the same intersection point. (Further elaboration in *Appendix 6*)

Moreover, I also predict that the readings around central regions will tend to have very similar readings and there will be high discrepancy in dips between the points near the endpoints of the string and the central region.

V. Variables

Independent Variables

⇒ The intersection point on the string plane where the force is applied.

Dependant Variables

⇒ The length of the dip formed at the intersection point where the force is applied.

Controlled Variables

⇒ The racquet used in the investigation

⇒ Type of point where the force is applied- The force is only applied to the intersection of two strings in the vertical and horizontal direction. (Please refer to *Appendix 4* for different types of ball racquet collisions).

⇒ The force applied on the intersections was kept constant by hanging 12 kg mass at every intersection investigated.

VI. Apparatus

Table 1: Apparatus used along with quantity and properties

Equipment	Properties	Quantity
Squash Racquet- Prince TF Nitro	The squash racquet should be sturdy and the frame should be able to sustain heavy weights.	1
G-Clamps	-	2
12kg of weights	-	1
Hook	-	1
Stands with clamps	-	3 stands, 5 clamps
Analog Vernier Calliper	Uncertainty ± 0.1cm	1
Measuring Tape	Uncertainty ± 0.1cm	1
Newton-meter	Uncertainty ± 0.5kg	1
Tables	2 tables placed at a height of 0.5-0.7m and 1 at a height of 0.9-1.1m	3

VII. Safety Precautions

Table 2: Safety precautions that are to be taken in the experiment

Safety hazard	Precaution
Hazard due to heavy apparatus (12kg bag)	Place the bag at an elevation that is at level with the hook, so that the bag does not need to be lifted from a large height. Since the bag is heavy, care should be taken when lifting it to ensure it doesn’t fall and cause damage
Sharp object hazard	While handling the Vernier calliper, sharp edges may cause a hurt hazard.

VIII. Experimental Setup

The following procedure is for the measurement of dips of the selected points, and point G6 is taken as an example. (Photos of the position of the equipment and experimental setup are available in *Appendix 2*)

1. Using 2 G-Clamps, clamp a squash racquet by the handle near the corner of a table and allow the head of the racquet to be suspended beyond the table edge. Ensure that one of the clamps is at the base of the handle and the other is at the neck of the racquet at a distance of 10.00 cm. Ensure that the racquet is clamped at a height of at least 90.00 cm.
2. Place 2 tables on either side of the racquet and place 2 stands, with fixed clamps, on the tables. The attached clamps are placed under the frame of the hanging racquet head to support the racquet from bending when the force is applied.

IX. Procedure

Part i: Placing the Vernier caliper on the gut

1. Using 2 clamps, that are attached to a stand, clamp a Vernier caliper over the gut of the racquet at a height that is nominal to the string plane. Determine this height by judgement and ensure it is about 0.01 to 0.05cm above the string. Horizontally translate the stand around the gut to ensure that the Vernier caliper is not getting stuck in the gut of the racquet. Position the Vernier caliper above the designated point. Ensure that the Vernier caliper is perpendicular to the squash racquet strings.

2. Hang the Newton-meter on the intersection point, ensuring that the hook is a diagonal to the "+" shape that the intersection is forming. Hang 12kg mass on the hook and note the dip that develops at that point. Ensure the racquet remains horizontal by placing the stands with clamps below the frame in such a way that any unbalanced force is cancelled out (*Figure 5* shows the placement of the hook).

Figure 5: The manner in which the hook is to be attached to the intersection point. The hook will be diagonal to the "+" pattern.

Part ii: Collecting the data- Using the Vernier depth gauge

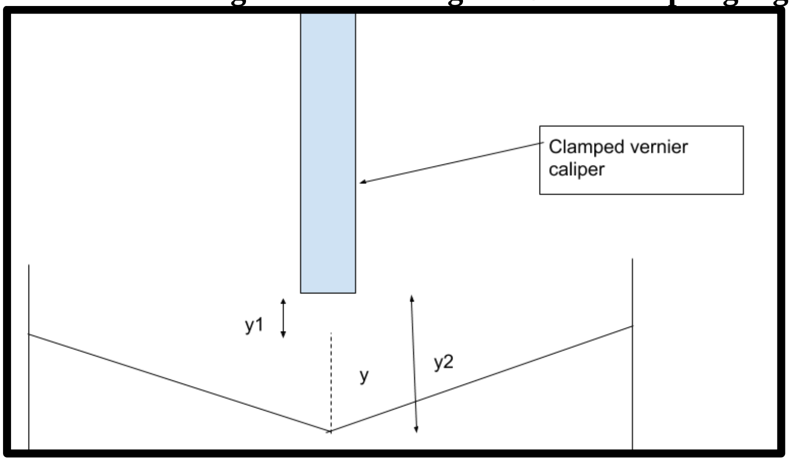


Figure 6: Data collection with the Vernier calliper.

Refer to *Figure 6* for visual representation of the calculation

1. Measure the height between the clamped Vernier calliper and the unbent strings to get the quantity y_1. This can be done by scrolling down the thumb screw so that the lowermost point on the depth rod is at the same level as the lower string (whether it is vertical or horizontal). For example, the measurement taken for point G6 trial 1 is as follows

$$y_1 = Main\ scale\ reading + Vernier\ scale\ reading \times Least\ count$$

$$y_1 = 0.4 + 3 \times 0.01$$

$$y_1 = 0.43\ cm$$

("How To Read A Vernier Caliper")

2. Apply the 12kg mass on the intersection point. The dip, y_2, formed in this case is measured by using the Vernier depth gauge. By extending the depth rod to the lowermost point of the intersection, the dip was calculated. The measurement done for trial 1 of point G6 is as follows

$$y_2 = Main\ scale\ reading + Vernier\ scale\ reading \times Least\ count$$

$$y_2 = 2.2 + 2 \times 0.01$$

$$y_2 = 2.22\ cm$$

3. Hence the actual depth, y, due to the force acting is given by the formula

$$y = y_2 - y_1$$

$$y = 2.22 - 0.43$$

$$y = 1.79cm$$

4. Take 3 trials for each point of intersection and take an average of the value of y to reduce random errors.
5. Repeat the experiment for the intersection points chosen(Please refer to *Appendix 2* to determine the selection of points).

X. Data Analysis

Table 3: Dip values with uncertainty

Intersection point	y/cm (Systematic uncertainty: ±0.01cm)				Intersection point	y/cm (Systematic uncertainty: ±0.01cm)			
	Trial 1	Trial 2	Trial 3	Average		Trial 1	Trial 2	Trial 3	Average
F4	1.73	1.63	1.72	1.69±0.06	G4	1.75	1.86	1.94	1.85±0.11
F5	1.75	1.91	1.86	1.84±0.09	G5	1.96	1.74	1.88	1.86±0.12
F6	1.44	1.39	1.69	1.51±0.14	G6	1.79	1.81	1.74	1.78±0.05
F7	1.95	1.94	1.99	1.96±0.04	G7	1.85	1.89	1.72	1.82±0.10
F8	1.48	1.42	1.57	1.49±0.09	G8	1.62	1.90	1.76	1.86±0.15
H4	1.87	1.96	1.74	1.93±0.12	I4	1.81	1.85	1.74	1.80±0.07
H5	2.12	2.10	1.99	2.07±0.08	I5	2.16	2.21	1.78	2.05±0.23
H6	1.81	1.90	1.61	1.77±0.16	I6	1.99	1.95	1.99	1.98±0.03
H7	1.83	2.36	1.77	1.99±0.28	I7	1.95	1.92	1.86	1.91±0.04
H8	1.55	1.48	1.32	1.45±0.13	I8	1.75	1.74	1.66	1.72±0.06
J4	1.88	1.85	1.52	1.75±0.19	K4	1.77	1.68	1.66	1.70±0.07
J5	1.81	1.76	1.77	1.85±0.03	K5	1.85	1.82	1.66	1.78±0.11
J6	1.88	1.82	1.89	1.86±0.05	K6	1.82	1.89	1.87	1.85±0.05
J7	2.12	2.11	2.06	2.10±0.04	K7	1.83	1.81	1.68	1.77±0.09
J8	1.91	1.82	1.72	1.82±0.11	K8	1.74	1.81	1.82	1.79±0.05

Example 1: Calculating y_{avg} for intersection point G6. y_{avg} stands for average value of y

$$y_{avg} = \frac{Trial\ 1 + Trial\ 2 + Trial\ 3}{3}$$

$$y_{avg} = \frac{1.79 + 1.81 + 1.74}{3}$$

$$y_{avg} = 1.78\ cm$$

Example 2: Calculating uncertainty in y_{avg} for intersection point G6.

$$\Delta y_{avg} = Systematic\ uncertainty + Random\ Uncertainty$$

$$Systematic\ uncertainty\ in\ \Delta y_{avg} = 0.01\ cm$$

$$Random\ uncertainty\ in\ \Delta y_{avg=}\ \frac{Maximum\ value - Minimum\ Value}{2}$$

$$\Delta y_{avg} = 0.01 + \frac{1.81 - 1.74}{2}$$

$$\Delta y_{avg} = 0.05\ cm$$

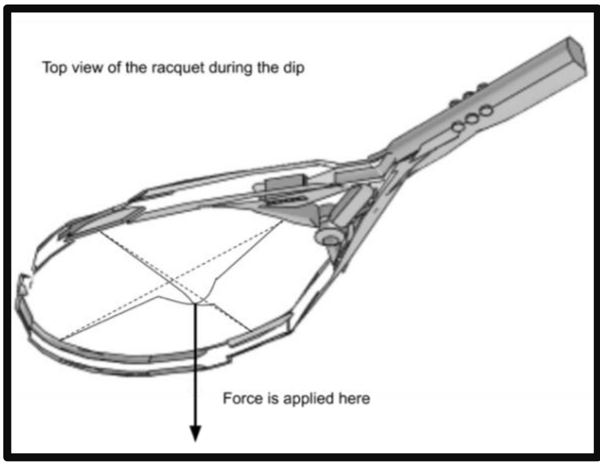
By referring to *Table 3*, the point at which the maximum dip formed, J7, is identified.

The value of the dip at J7 2.10$\pm$0.04cm.

XI. Mathematical Model

I created a mathematical model, which can give the restoring force at the maximum dip formed, when a force is applied. The model is specifically **created for the points of intersection as these points have been investigated in the experiment**.

Approximation in the model



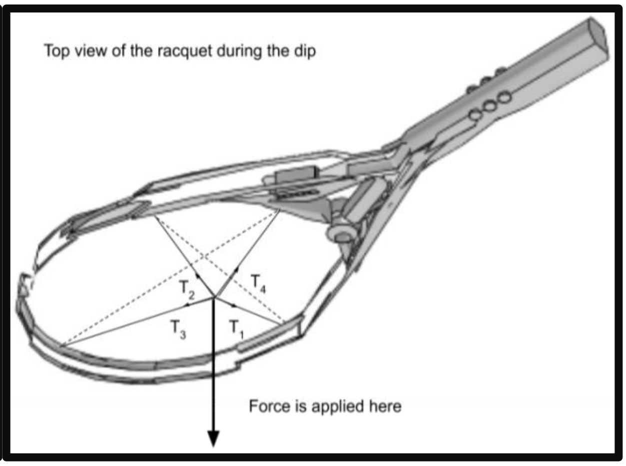


Figure 7: Actual dip that takes place

Figure 8: Approximation being made for the dip

Figure 7 shows how the strings bend in a racquet during experimentation. However, for this model, the bend in the string is assumed to be in the manner shown in *Figure 8*. The purpose of choosing this **approximation is that it forms a right angled triangle** in the position where the dip occurs and assists in forming relationships between several quantities.

Moreover, the approximation can be justified by the fact that the strings are majorly composed of polyamide, which has a very high Young's modulus value, measure of an object's resistance to deformation when experiencing a force, 2.95×10^9 Nm^{-2}("Young's Modulus - Tensile and Yield Strength for common Materials").

This high value lets the string extend by a minimal amount in either consideration of dips.

Another assumption in the model is that the area of cross section of the strings is that of a circle and that the strings are cylindrical, however, in reality the strings do not necessarily have to have this shape as the strings deform over time.

Figure 8 shows the tensions acting in the individual segments of the strings, due to the force applied at the intersection. Restoring force in context of the strings is the **tension in the strings,** which will differ on either side of the point due to difference in the length of the segments**.** Hence, the variables for restoring force in strings are in terms of T, where T represents the tension in the string for various segments. The two segments on the horizontal string experience tensions T_1 and T_2 and the two segments on the vertical string experience tensions T_3 and T_4.

Derivation of the model

The objective of this model is to get the expression of restoring force in terms of the length of the strings and the dip of the intersection, as these quantities can be measured.

Till now, the restoring force in the string, F_R, considered, was along the axis of extension of the string. However, in the model, the restoring force is along the perpendicular axis(vertical component) and therefore must be defined by a new name, F_{RD}, where F_{RD} stands for restoring force in the dip.

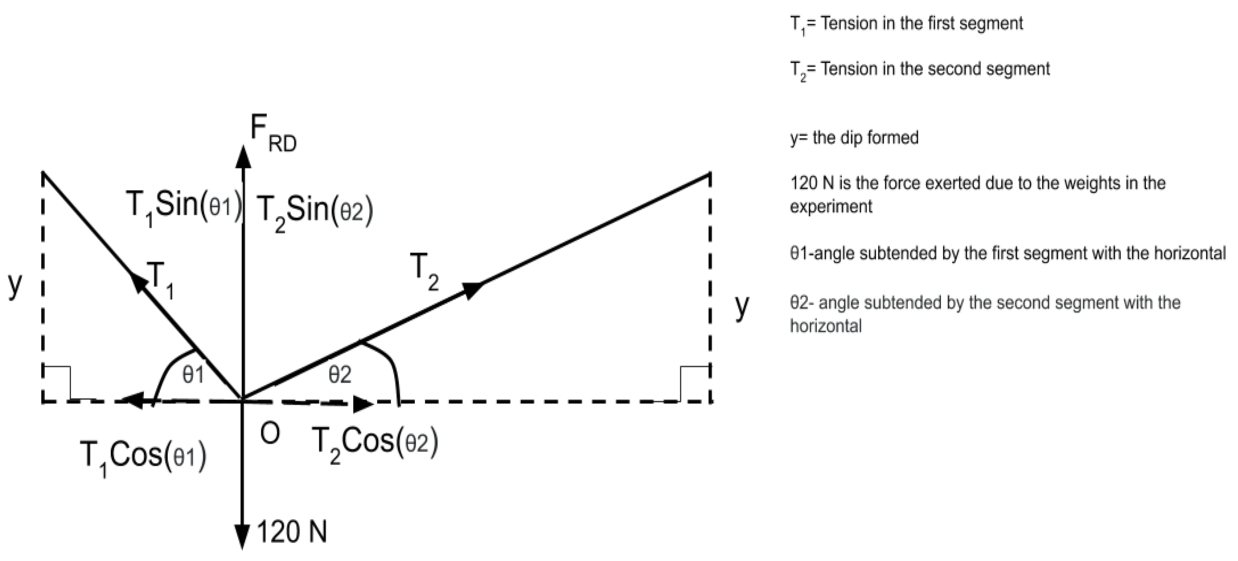


Figure 9: The vector plane showing forces acting on the string during the period of depression on the horizontal strings. Note: Figure not to scale.

Consider *Figure 9*, which shows the vector plane with the forces in the string and the dip. The assumption made in this case is that the horizontal components of the tension formed in string segments are equal and get cancelled. This assumption is made to account for the fact that the intersection point does not have horizontal movement when the 12 kg mass is relieved. So the horizontal components can be written as

$$T_1 Cos(\theta 1) = T_2 Cos(\theta 2)$$

The net force that causes the strings to come back to their default position can be given by

$$T_{horizontal} = T_1 Sin(\theta 1) + T_2 Sin(\theta 2)$$

Where $T_{horizontal}$ represents the total restoring force acting on the intersection point due to the horizontal string.

Now to investigate the value of $Sin(\theta 1)$, $Sin(\theta 2)$, T_1, T_2 we can consider the length and displacement of the string in a vector plane

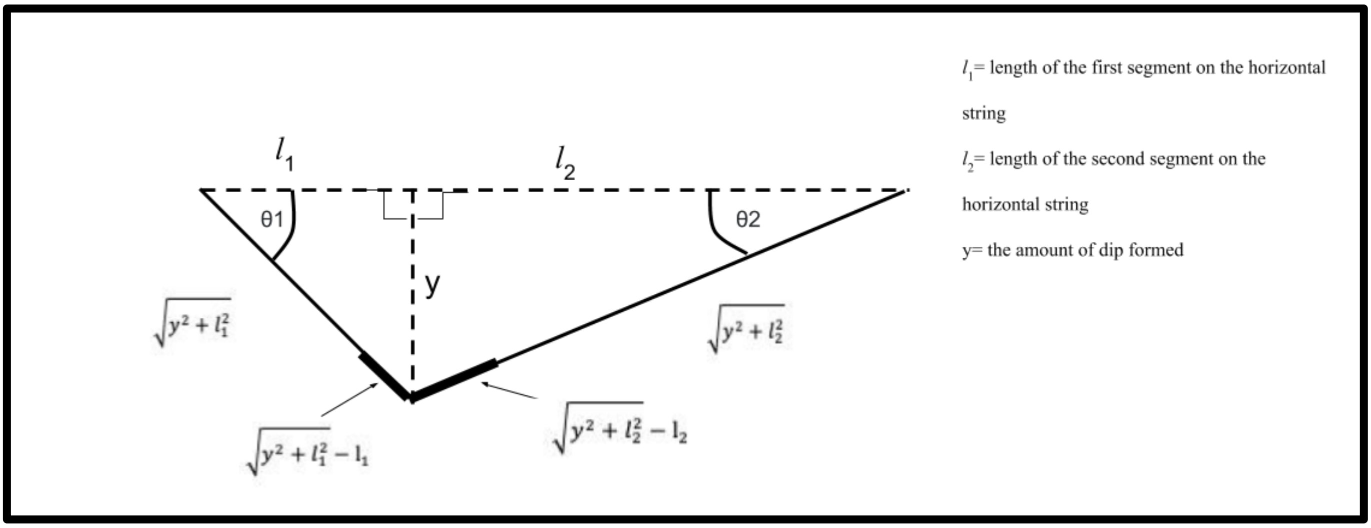


Figure 10: The displacement and extension in the vector plane showing the lengths and extensions that occur in the horizontal string.

Figure 10 shows the angles formed in the string, due to the interior alternate property. Therefore, values of $Sin(\theta 1)$ and $Sin(\theta 2)$ can be obtained from the positions shown in the diagram.

$$Sin(\theta 1) = \frac{y}{\sqrt{y^2 + l_1^2}}$$

$$Sin(\theta 2) = \frac{y}{\sqrt{y^2 + l_2^2}}$$

To find the values of T_1 and T_2, the extensions of the strings will need to be considered; in *Figure 10*, the emboldened lines represent the extension. The extension in line segment, l_1, is given by the quantity δl_1, and the extension in l_2 is given by the quantity δl_2.

$$\delta l_1 = \sqrt{y^2 + l_1^2} - l_1$$

$$\delta l_2 = \sqrt{y^2 + l_2^2} - l_2$$

To find the expressions of T_1 and T_2, the concept of Hooke's Law will be used. For the Tension T_1 in the string-

$$Stress \propto Strain$$

$$\frac{F_R}{A} = k \times \frac{\delta l_1}{l_1}$$

A is the area of cross section of the strings

k represents the Young's Modulus, E, of the material of the string

F_R is the restoring force along the axis experiencing the extension, which is T_1 in this case

$$\frac{T_1}{A} = E \times \frac{\delta l_1}{l_1}$$

$$T_1 = \frac{E \times \delta l_1 \times A}{l_1}$$

$$T_1 = \frac{E \times (\sqrt{y^2 + l_1^2} - l_1) \times A}{l_1}$$

Similarly for T_2

$$T_2 = \frac{E \times (\sqrt{y^2 + l_2^2} - l_2) \times A}{l_2}$$

Therefore, the final expression for $T_{horizontal}$ is

$$T_{horizontal} = T_1 Sin(\theta 1) + T_2 Sin(\theta 2)$$

$$T_{horizontal} = \frac{E \times (\sqrt{y^2 + l_1^2} - l_1) \times A}{l_1} \times \frac{y}{\sqrt{y^2 + l_1^2}} + \frac{E \times (\sqrt{y^2 + l_2^2} - l_2) \times A}{l_2} \times \frac{y}{\sqrt{y^2 + l_2^2}}$$

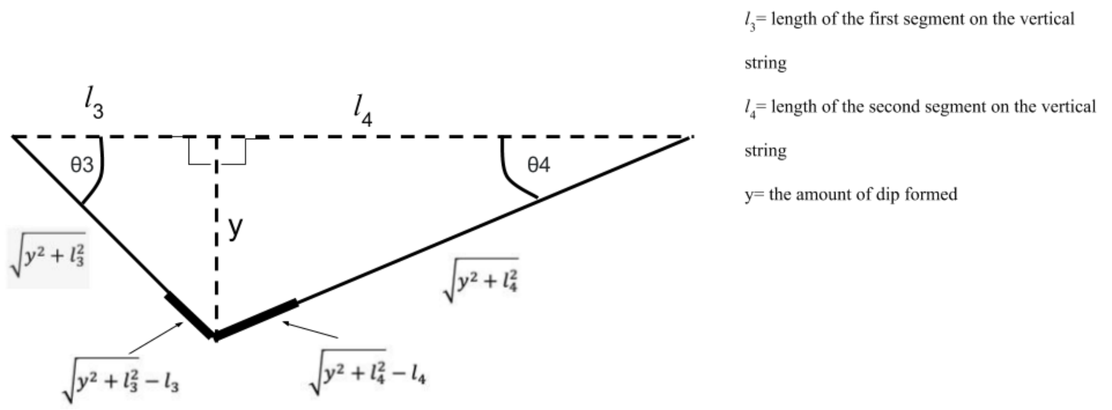


Figure 11: The displacement and extension in the vector plane showing the lengths and extensions that occur in the vertical string.

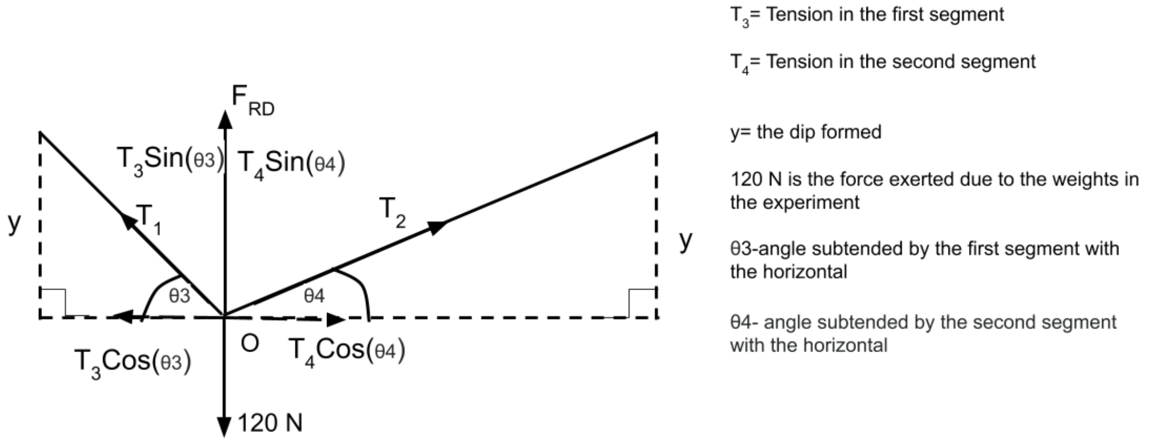


Figure 12: The vector plane showing forces acting on the string during the period of depression on the vertical string. Note: Figure not to scale.

Similarly, by referring to *Figure 12 and Figure 11* the equations for the vertical string can be written as

$$T_3Cos(\theta 3) = T_4Cos(\theta 4)$$

$$T_{vertical} = T_3Sin(\theta 3) + T_4Sin(\theta 4)$$

Where $T_{vertical}$ represents the total restoring force acting on the intersection point due to the vertical string.

$$\delta l_3 = \sqrt{y^2 + l_3^2} - l_3$$

$$\delta l_4 = \sqrt{y^2 + l_4^2} - l_4$$

$$Sin(\theta 3) = \frac{y}{\sqrt{y^2 + l_3^2}}$$

$$Sin(\theta 4) = \frac{y}{\sqrt{y^2 + l_4^2}}$$

$$T_3 = \frac{E \times (\sqrt{y^2 + l_3^2} - l_3) \times A}{l_3}$$

$$T_4 = \frac{E \times (\sqrt{y^2 + l_4^2} - l_4) \times A}{l_4}$$

$$T_{vertical} = \frac{E \times (\sqrt{y^2 + l_3^2} - l_3) \times A}{l_3} \times \frac{y}{\sqrt{y^2 + l_3^2}} + \frac{E \times (\sqrt{y^2 + l_4^2} - l_4) \times A}{l_4} \times \frac{y}{\sqrt{y^2 + l_4^2}}$$

Therefore, the total restoring force will be the sum of the force exerted by the horizontal string and the vertical string. (Refer to *Appendix 6* for the proof of this statement)

$$F_{RD} = T_{horizontal} + T_{vertical}$$

Calculations for point J7

The Young's Modulus, E, for polyamide, is known. Area of cross section is given by the formula= $\pi \times r^2$ where r stands for radius of the string. The diameter of the string is 1.2mm, so **r= 0.6mm** (Crandall). All the strings on the squash racquet are assumed to be identical in terms of width and material, so area and Young's modulus, E, are constants. (Note: 4 decimal places are used for all the calculations, except for quantities that were measured)

For point J7, the values of the variables are as follows

y= 0.0210 $\pm$0.0004m

l_1 = 0.1160 $\pm$0.001m

$l_2 = 0.0870 \pm 0.001$m

$l_3 = 0.1010 \pm 0.001$m

$l_4 = 0.2550 \pm 0.001$m

The lengths of the strings were measured by a measuring tape.

For the horizontal string, the calculations are as follows (Note the answers given may slightly differ from the answers if calculated with the numbers presented as these numbers have been rounded off when being carried forward to another equation)

$$Sin(\theta 1) = \frac{0.0210}{\sqrt{(0.0210)^2 + (0.1160)^2}}$$

$$Sin(\theta 1) = 0.1781$$

$$Sin(\theta 2) = \frac{0.0210}{\sqrt{(0.0210)^2 + (0.0870)^2}}$$

$$Sin(\theta 2) = 0.2346$$

$$T_1 = \frac{2.95 \times 10^9(\sqrt{(0.0210)^2 + (0.1160)^2} - 0.1160) \times \pi \times (0.0006)^2}{0.1160}$$

$$T_1 = 54.2315\ N$$

$$T_2 = \frac{2.95 \times 10^9(\sqrt{(0.0210)^2 + (0.0870)^2} - 0.0870) \times \pi \times (0.0006)^2}{0.0870}$$

$$T_2 = 95.1892\ N$$

$$T_{horizontal} = 54.2315 \times 0.1781 + 95.1892 \times 0.2346$$

$$T_{horizontal} = 32.1438\ N$$

For the vertical string the calculations are as follows

$$Sin(\theta 3) = \frac{0.0210}{\sqrt{(0.0210)^2 + (0.1010)^2}}$$

$$Sin(\theta 3) = 0.2036$$

$$Sin(\theta 4) = \frac{0.0210}{\sqrt{(0.0210)^2 + (0.2550)^2}}$$

$$Sin(\theta 4) = 0.0821$$

$$T_3 = \frac{2.95 \times 10^9(\sqrt{(0.0210)^2 + (0.1010)^2} - 0.1010) \times \pi \times (0.0006)^2}{0.1010}$$

$$T_3 = 71.3544\ N$$

$$T_4 = \frac{2.95 \times 10^9(\sqrt{(0.0210)^2 + (0.2550)^2} - 0.2550) \times \pi \times (0.0006)^2}{0.2550}$$

$$T_4 = 11.2945\ N$$

$$T_{vertical} = 71.3544 \times 0.2036 + 11.2945 \times 0.0821$$

$$T_{vertical} = 15.4524\ N$$

$$F_{RD} = 32.1438 + 15.4524$$

$$F_{RD} = 47.5962\ N$$

The calculation of uncertainty for F_{RD} in this formula is as follows

$$\frac{\Delta T_{horizontal}}{T_{horizontal}} = \left(\frac{\Delta l_1}{l_1} + \frac{\Delta y}{y} + \frac{\Delta l_1}{l_1} + \frac{\Delta l_1}{l_1} + \frac{\Delta y}{y} + \frac{\Delta y}{y} + \frac{\Delta l_1}{l_1}\right) + \left(\frac{\Delta l_2}{l_2} + \frac{\Delta y}{y} + \frac{\Delta l_2}{l_2} + \frac{\Delta l_2}{l_2} + \frac{\Delta y}{y} + \frac{\Delta y}{y} + \frac{\Delta l_2}{l_2}\right)$$

$$\Delta T_{horizontal} = \left(4 \times \frac{\Delta l_1}{l_1} + 6 \times \frac{\Delta y}{y} + 4 \times \frac{\Delta l_2}{l_2}\right) \times T_{horizontal}$$

$$\Delta T_{horizontal} = \left(4 \times \frac{0.001}{0.1160} + 6 \times \frac{0.0004}{0.0210} + 4 \times \frac{0.001}{0.0870}\right) \times 32.1438$$

$$\Delta T_{horizontal} = \pm 6.2599\ N$$

$$\frac{\Delta T_{vertical}}{T_{vertical}} = \left(\frac{\Delta l_3}{l_3} + \frac{\Delta y}{y} + \frac{\Delta l_3}{l_3} + \frac{\Delta l_3}{l_3} + \frac{\Delta y}{y} + \frac{\Delta y}{y} + \frac{\Delta l_3}{l_3}\right) + \left(\frac{\Delta l_4}{l_4} + \frac{\Delta y}{y} + \frac{\Delta l_4}{l_4} + \frac{\Delta l_4}{l_4} + \frac{\Delta y}{y} + \frac{\Delta y}{y} + \frac{\Delta l_4}{l_4}\right)$$

$$\Delta T_{vertical} = \left(4 \times \frac{\Delta l_3}{l_3} + 6 \times \frac{\Delta y}{y} + 4 \times \frac{\Delta l_4}{l_4}\right) \times T_{vertical}$$

$$\Delta T_{vertical} = \left(4 \times \frac{0.001}{0.1010} + 6 \times \frac{0.0004}{0.0210} + 4 \times \frac{0.001}{0.2550}\right) \times 15.4524$$

$$\Delta T_{vertical} = \pm 2.6204\ N$$

$$\Delta F_{RD} = \Delta T_{vertical} + \Delta T_{horizontal}$$

$$\Delta F_{RD} = 6.2599 + 2.6204$$

$$\Delta F_{RD} = \pm 8.8802 N$$

XII. <u>Graphical Analysis</u>

To reveal a general trend, I found the restoring force, F_{RD}, for each intersection point considered in the experiment. An accurate representation of the restoring force vs the intersection points could be through a bar graph. The trend has been considered row-wise (according to the numbers rather than the letters).

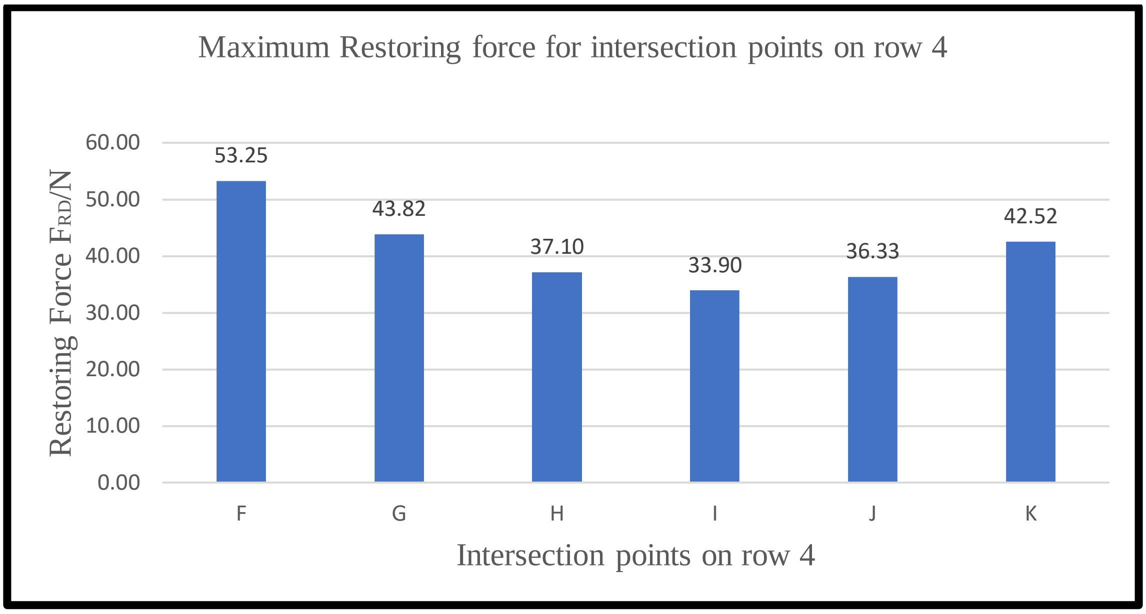


Figure 13: Bar graph representation of Restoring Force F_{RD} vs Intersection points on row 4

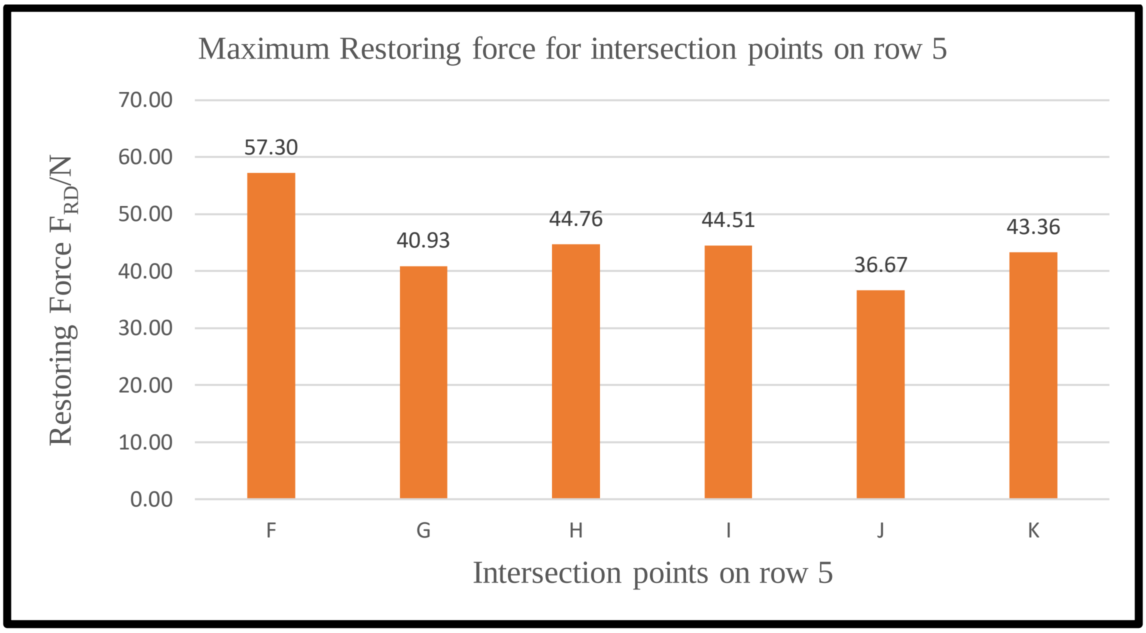


Figure 14: Bar graph representation of Restoring Force F_{RD} vs Intersection points on row 5.

Figure 15: Bar graph representation of Restoring Force F_{RD} vs Intersection points on row 6.

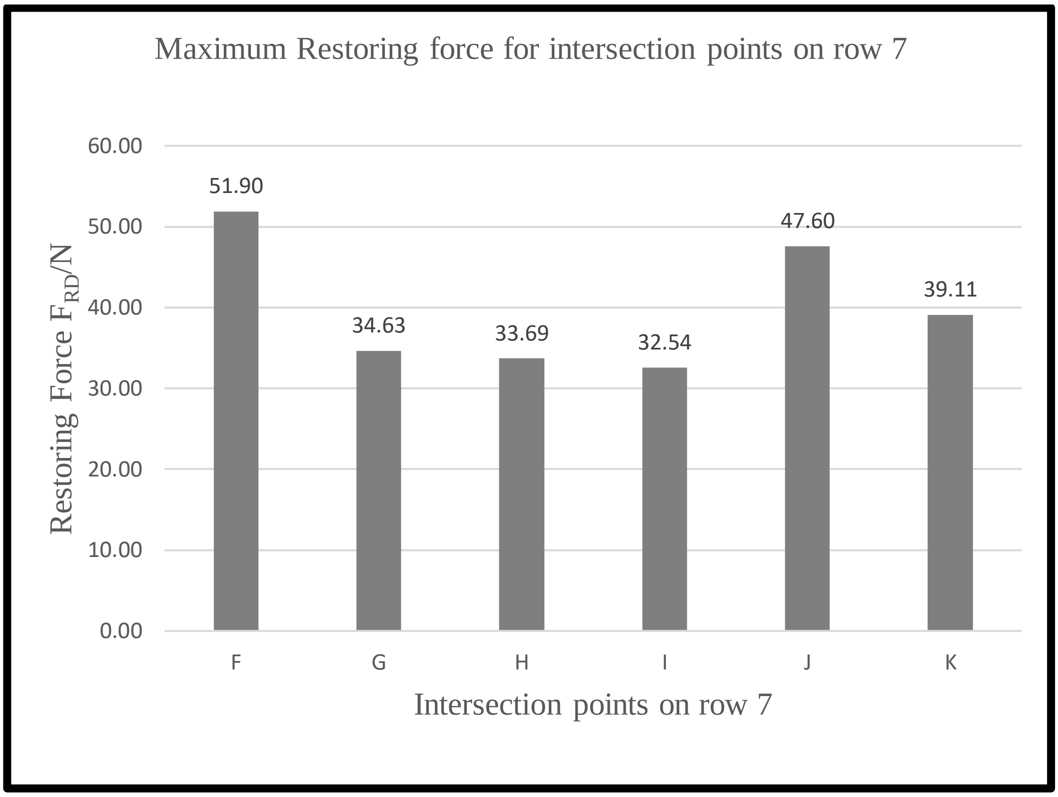


Figure 16: Bar graph representation of Restoring Force F_{RD} vs Intersection points on row 7.

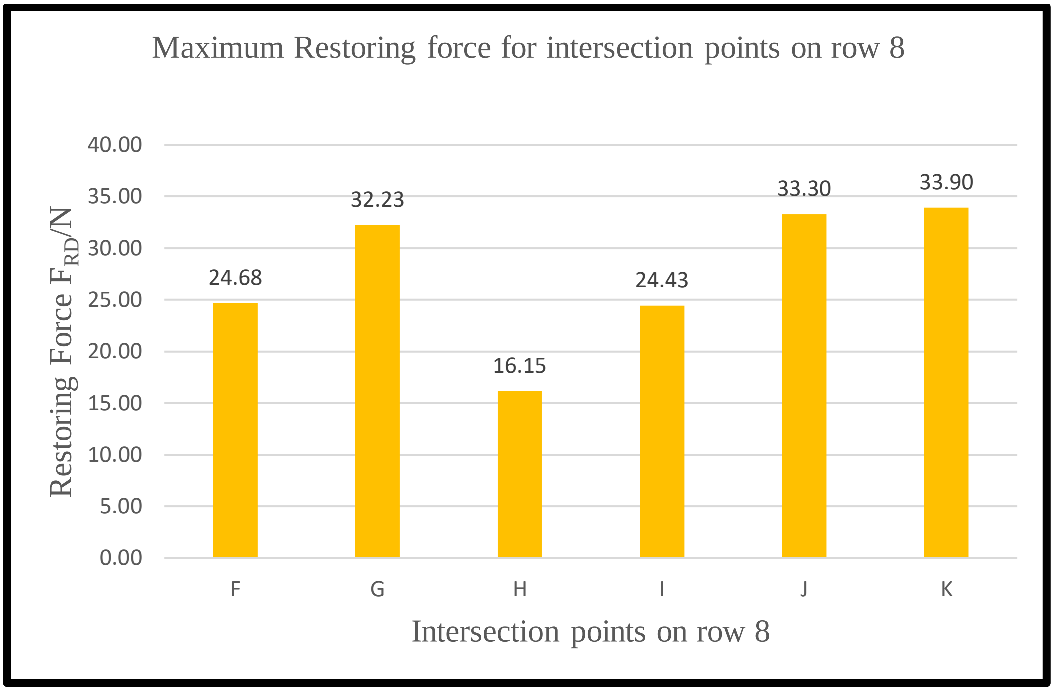


Figure 17: Bar graph representation of Restoring Force F_{RD} vs Intersection points on row 8.

By comparing the graphs, shown in *Figures 13, 14, 15, 16, 17, and Table 3*, the relationship between the extent of the dip and restoring force can be obtained. It can be inferred from the graphs that the points closer to the endpoints exhibited a higher restoring force. It is also generally noticed that these points had relatively lower dip values as compared to the values near the central region. This trend contrasts the hypothesis, wherein, it was predicted that the central region will have higher restoring forces due to greater dips. It can also be noticed that up to 3 points (F7, F5, and F4) have higher restoring force values when compared to the predicted highest restoring force value for point J7. This is unusual as although the three points have lower dip values than J7, they produce higher restoring force values. The obtained results undermine the predictions of the study and suggest that the unaccounted factors play a larger role.

Another relation that can be inferred is that the maximum on average restoring force for points on a particular row, occurred on row 5, and the rows following row 5 reduced in on average restoring force. A similar observation was made for the dips where row 5 and 7 have greater dips on average as compared to other rows.

Graphical analysis reveals that the proportionality between the dip and restoring force is obeyed on average, but there tend to be some anomalies in some points. The anomalies can be explained by considering the effect of assumptions in the model that may have had a larger effect.

XIII. Evaluation

Overall, the experiment was an effective way to measure the dip of the points on the racquet. By hanging heavy mass of 12kg, on the intersection points, substantial difference was observed between dips of most points.

The uncertainty values in the experiment proved to be within an acceptable range with the highest percentage uncertainty being 14.07% and the lowest being 1.52%.

The value of restoring force, F_{RD}, given by the mathematical model is justifiable in the real world as F_{RD} in each case was less than the average tension that the gut experiences, 27 pounds of force, or 120N (Haneburry). This force is the overall tension experienced in each string when unstretched due to the framework of the racquet. None of the values obtained by the mathematical model exceeded this value of tension as in such a case the string would snap or become permanently damaged/extended due to excessive force along the string. This verifies the realistic nature of the model.

Although the mathematics model used gives realistic values that the ball may exert during play, it has its limitations. For instance, the net horizontal movement, at the maximum dip for each point was supposed to cancel out as

$$T_1 Cos(\theta 1) = T_2 Cos(\theta 2)$$

However, with the values calculated the result for the horizontal string is

$$54.2315 \times 0.9840 = 95.1892 \times 0.9720$$

$$53.3637 \neq 92.5230$$

This happens for the vertical string as well

$$T_3 Cos(\theta 3) = T_4 Cos(\theta 4)$$

$$71.3544 \times 0.9790 = 11.2945 \times 0.9966$$

$$70.9334 \neq 12.5421$$

The assumption that horizontal forces get cancelled may not hold true for the model, however, this can explain the non-vertical rebound of the squash ball, when the ball is dropped on the gut. It can be observed that the ball acquires a slight sideways motion instead of a straight perpendicular rebound, which may be the result of the horizontal net force by the strings on the ball.

The strings are composed of mixtures different materials other than polyamide and have different filament structures, this alters the Young's Modulus value in the model by a small degree, which may have an effect on the values obtained(Crandall).

Table 4: Weakness, random, and systematic errors along with significance and improvements

Source of error/ weakness	Significance and evidence	Improvements
Weakness		
The varying sizes of gaps in the gut	The vertical strings converged down the racquet, leading to the formation of uneven shapes in the gaps on the gut; this leads to an unfair comparison as points on 9-11 may be on stiffer strings due to the proximity to other strings and may cause them to have lower dips.	Create a physical model with the same tension and lengths of the strings as the racquet, but also ensure that the gap space is kept constant and then measure the values of the dip for intersection points.
Overlapping or underlapping strings	Both horizontal and vertical strings alternately underlap and overlap. Consequently, the points where the horizontal string is underlapping, the racquet could have higher resistance to dipping as the horizontal string is shorter. Therefore unfair comparisons can be made between intersection points with either string underlapping or overlapping.	Compare the values of the dips of intersection points that have the horizontal string underlapping. Same for the vertical strings. Then take the highest dip separately for both the cases and find restoring force separately.
Random errors affecting precision		

Parallax error	While measuring the dip, the judgement of whether the depth rod and the lowest point of the dip were aligned, depended on human eyesight and this leads to a parallax error.	To avoid surrounding colour from interfering with alignment, hold a black paper behind the calliper while measuring the dip. Another solution can be by ensuring that the depth rod and the dip are viewed at a 90˚ angle.
Systematic errors affecting accuracy		
Permanent Deformation in the strings.	Due to prolonged exertion of the 12 kg mass on the gut, there could be permanent deformation in the strings. Evidence for this may be observed in the long term as the strings may become looser.	To reduce this deformation, a lower mass can be hung instead of 12 kg. By reducing the exposure time of the mass on the strings, deformation could be reduced.

Further scope for research

Interesting research ideas on similar lines may include: Investigating the dip and creating a mathematical model for restoring force

⇒ For different cases of collision of the ball and the gut in a squash racquet. (*Appendix 3* shows pictures of different types of collisions).

⇒ For collision of the ball and the gut in different types of racquets for different sports (racquetball, tennis).

⇒ For collision of the ball and the gut in a squash racquet for different stringing patterns.

XIV. <u>Conclusion</u>

The experiment was done with the objective of finding the intersection point with the lowest dip on the squash racquet gut when a uniform force was used to test each point.

This was successfully achieved with the point being J7, whose value for the dip was $2.10 \pm 0.04cm$. The restoring force obtained for this point through the mathematics model is $47.5962 \pm 8.8802N$. Hence, the RQ is successfully answered.

The hypothesis correctly predicted the trend in the dips, however, the prediction in the trend of restoring force did not match. Although the hypothesized relationship between dip and restoring force was inaccurate, it explained the general trend observed in the points.

The investigation refined my experiment skills and led me to design my own setup to find the dip. I got a better understanding of the mechanics behind strings in the racquet and was able to accurately point out the dominance spot based on the results. Moreover, I familiarised myself with the operation of a Vernier calliper and improved my ability to handle common apparatus within the laboratory.

Conceptually, the investigation led me to revise my understanding of resolution of vectors and Hooke's law. I was also introduced to a new perspective of the Hooke's law(namely the stress and strain relationship) along with a new concept: Young's Modulus. By referring to research papers on the same topic, I was able to generate a deep understanding of the ideas behind this investigation and contribute to the scope, which is to gain a better insight into the complexity of the string arrangements.

Through the theoretical learnings from this investigation, I hope to improve my ability in the court and inspire others to investigate a unique spot in terms of different definitions and perspectives.

Prior to the investigation, I used to believe that there could exist an optimum point on the racquet, however, I now understand that the gut could provide various points where high values of restoring force may be achieved and there may not be a specific spot necessarily, instead, there may exist a variety of spots that maybe useful to target.

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XVI. Appendix 1: The coordinate system on the racquet

Figure 18: The co-ordinates in a squash racquet

Since the scope of the investigation is to investigate different intersection points on the racquet, it is useful to establish a system where each point can be identified by a name that informs its location on the racquet. *Figure 18* shows the key to understanding the coordinate system that I developed for this investigation.

Moving horizontally across the gut, the vertical strings have been given tags in the alphabetical order with each subsequent vertical string being named an alphabet that follows the previous.

Moving vertically down the gut, the horizontal strings have been given tags in the numerical order with each subsequent horizontal string being named a number that is greater than the previous by 1 number.

The way that the identification of the intersection can take place is by understanding the following formula to define the location.

$$\text{Intersection point} = \text{the alphabet in the vertical string} \times \text{the number on the horizontal string}$$

For example: The intersection point that lies on the vertical string C and on the horizontal string 11 can be called “C11”.

It is important to maintain the order by first putting the alphabet and then following it up by the number to maintain coherence in understanding the name and location.

This system can help accurately point out the intersection point that is being spoken about using *Figure 18* that shows all the intersection points on the squash racquet being defined.

XVII. Appendix 2: Design of the experiment

The reason for taking both dips, y_1 and y_2, was because the strings had a width that were significant enough to be accounted for, so by taking the reading y_1 this string width was measured and subtracted from the dip. Moreover, the Vernier calliper had to be removed from the clamps post the reading of the dip, y_2, although, the calliper was clamped to the same marked position that was determined, a minor change in positioning took place when it was reattached. Therefore, by subtracting the length, y_1, every time only the value of the dip was obtained.

This squash racquet gut has a complex structure with about 272 intersections. For the purpose of this paper, the points of intersections were short listed by the following logic:

The vertical strings from 'A' to 'E' are at a high proximity to their horizontal nodes, which helps in eliminating them as near the nodes the dip would be lesser compared to the rest of the string. With the same logic strings 'L' to 'P' is also eliminated. Hence, the vertical strings considered include strings 'F' to 'K'

Moreover, the horizontal strings were filtered based on similar lines. The strings from '1' to '4' were eliminated due to their proximity with the nodes, however, in the lower segment strings '17' to '11' were eliminated as besides the reason of the node, the strings were different as the vertical strings of these intersection points were sticking closer to each other and increasing the resistance to dip. Hence, the horizontal strings considered include strings '4' to '11'.

The G clamps are fixed in such a way that any moment about the centre of mass of the racquet was cancelled by the racquet being clamped on the table.

The racquet used in the experiment may be severely damaged due to prolonged exposure to the force.

XVIII. Appendix 3: Photos of the experimental setup

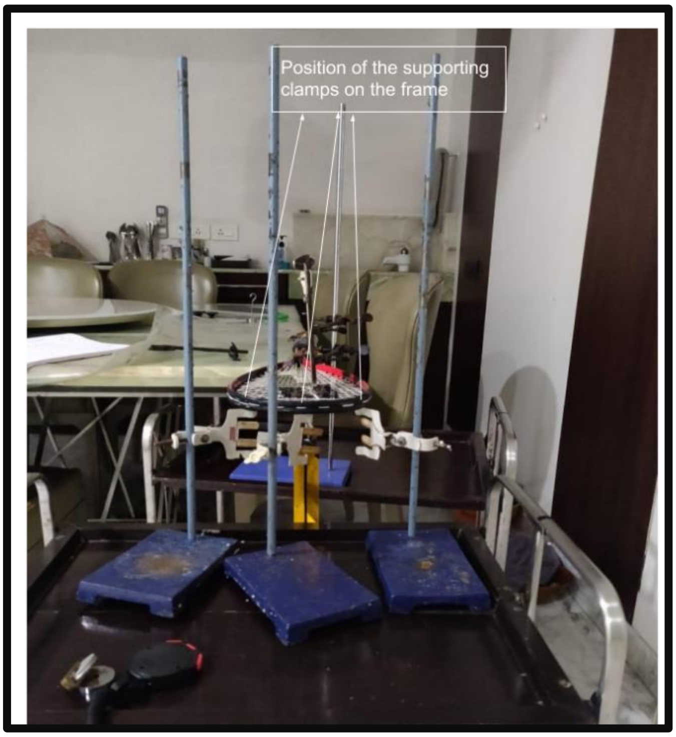

Figure 19: Position of the stands when supporting the racquet

Figure 20: Position of the G-Clamps

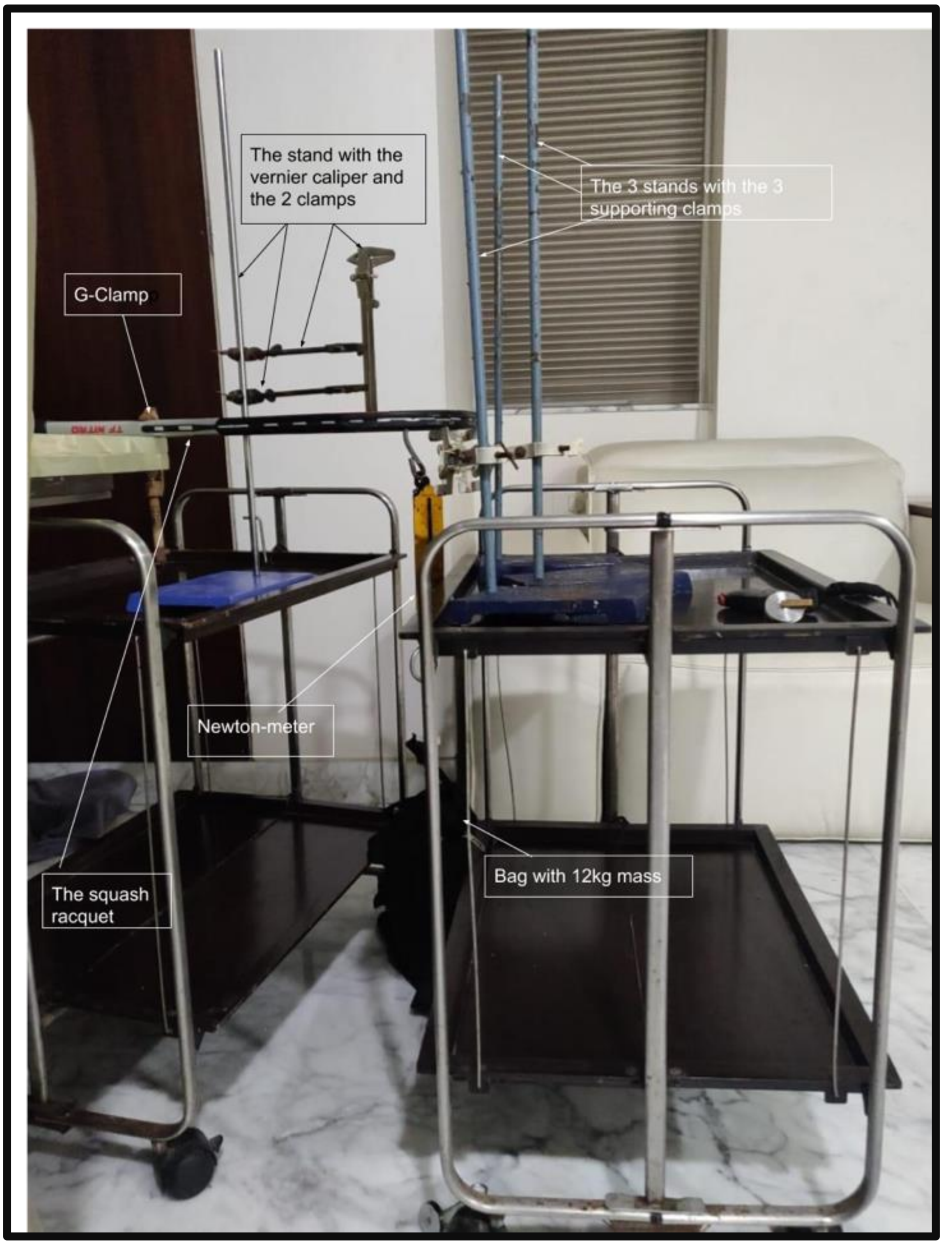

Figure 21: All the apparatus and procedure

Figure 22: Image of the dip being formed in the experiment

XIX. Appendix 4: Different types of ball-gut collisions

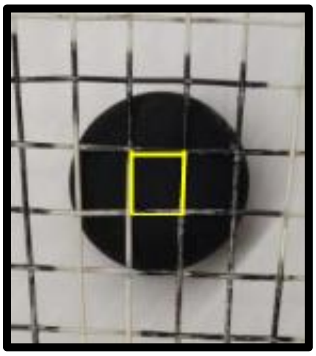

Figure 23: Ball impacting 1 square

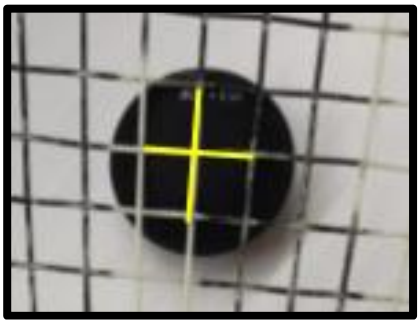

Figure 24: Ball impacting intersection point

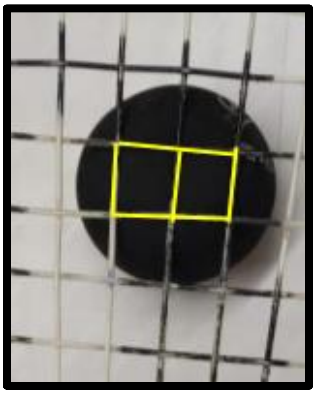

Figure 25: Ball impacting 2 squares

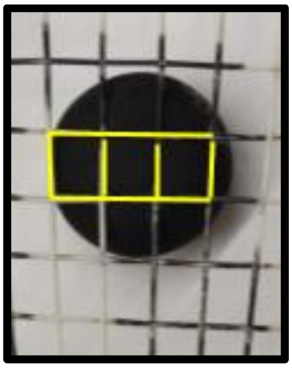

Figure 26: Ball impacting 3 square

XX. Appendix 5: Proving that the dip formed is greater than the compression in the ball

I conducted a series of preliminary experiments to check the viability of the experiment as a key component was in proving that the squash ball compressed less than the strings in the gut. This was done to assess whether the ball really gets impulse from the string or that it rebounds from the racquet before any assistance can be provided by the gut. By following the same procedure as described under the subheading procedure, I managed to find the dip values for the intersection points when a mass of 12kg was applied to the intersection point.

and then compared it to the compression values of the squash ball for force values that were similar to the ones used in the experiment.

$$Force\ applied\ on\ the\ intersection\ point\ =\ m \times g$$

$$Force\ applied\ on\ the\ intersection\ point\ =\ 12.0 \times 9.8067$$

$$Force\ applied\ on\ the\ intersection\ point = 117.6804$$

To compare the values I referred to the experiment conducted by WSF, World Squash Federation(the body that currently governs the proceedings of squash), which provided the specifications for the Standard Yellow Dot Championship Squash Ball. The experiment showed that for the squash ball compressed to about half its diameter, about 16mm, with a force of 44.8N to 57.6N("Squash balls"), whereas the preliminary experiments that I conducted on the intersections of the strings showed that a force of about 120N is required to have a perpendicular displacement of the same amount in the strings on average across all the intersection points I experimented on.

Since the ball compressed more than the strings, the ball will have a longer dwell time and the strings will be able to affect the ball's impulse successfully. This enables the investigation conducted to have a useful purpose.

XXI. Appendix 6: Interesting Physics aspects on the Squash racquet

After researching on the depression formed when a force is applied on strings, I found that a force applied at **the center of the string** will lead to the greatest depression formed. Therefore, for the overall racquet, the maximum dip in the string plane will be formed where the longest string in the y axis intersects the longest string in the x axis at each of its respective centers. This would be the case in a racquet where the frame is a perfect circle, which enables the longest string in the y plane and the longest string in the x plane to intersect at both their centers. Since the squash racquet head frame is in an irregular shape, this logic may not apply.

Figure 27: Approximate intersection of the longest strings in the x and y plane for a squash racquet. Note: the intersection does not occur at the centre of both the strings

Figure 28: Approximate intersection of the longest strings in the x and y plane in circular frame racquet. Note: It is at the centre of both the strings

As seen in *Figure 27 and Figure 28,* in a squash racquet, the intersection of the longest strings in the x and y plane does not occur at the center of both the strings, whereas in a circular frame racquet the intersection of longest strings is approximately in the center of both the strings. This unique shape of the squash racquet leads to the maximum dip to exist at any point on the string plane, which provoked my curiosity to find it.

The squash racquet can be described as a stiff body with a string membrane, where each vertical string is alternately overlapped or underlapped with the horizontal strings in a system of entanglement. Each vertical string, at the point of intersection, remains in contact with the horizontal string and exerts a reaction force, due to Newton's third law of motion: To every action there is an equal but opposite reaction, which is counteracted by the reaction force of the horizontal string on the vertical string intersection, resulting in the net force to be 0N.

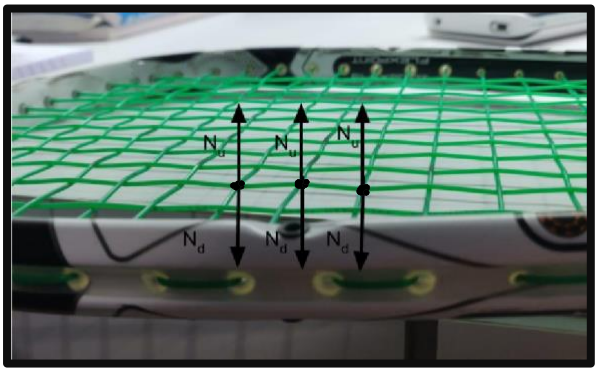

Figure 29: Reaction force in the strings cancelling out. Note: Please notice the alternately overlapping and underlapping pattern of the vertical or horizontal strings.

Figure 29 illustrates Newton's third law of motion, on the intersection points, as it shows the upward reaction force, N_u, acting on the string that is overlapping by the underlapping string, and the downward reaction force, N_d, acting on the string that is underlapping by the overlapping string. The forces N_u and N_d are equal and in opposite directions so they cancel each other out. This also explains how the net force acting in the perpendicular direction to the strings is always the vector sum of the forces exerted by horizontal and the vertical strings. It can then reasonably be inferred that during the course of collision of the ball and the gut, the reaction forces due to the strings are negligible and will have no effect on the impulse with which the ball leaves. Therefore, the restoring force that the racquet experiences is the only force acting on the intersection point at the very instant.