IB Mathematics HL
Internal Assessment

Using parametric equations to determine the ideal lob shot in squash

Research Question: What is the ideal approach that a player should take to achieve the lob service in squash?

I. Aim

The aim of this paper is to investigate the ideal approach a player must take to deliver an effective lob shot service that should land in a specific location in a court.

II. Introduction

In a fast paced game like squash, it is always beneficial to gain the upper hand on your opponent by making them face a difficult service. However, the front wall is huge and the location to aim for an effective service is always confusing. On top of that, experimentation can often lead to faults and loss in points. In these conditions, if the ideal target location is known then it can be much easier to control power and perform an effective service to have an advantageous start to the game. This is the reason why I decided to investigate the ideal location on the wall to perform an excellent service. I investigated the lob shot, where the ball lands in the corner (*Figure 1* shows the trajectory)and the ball goes 'dead'- no rebound from the walls to play the ball back (Strachan). The investigation can be conducted as during the service, the player is restricted to the service box (shown in *Figure 1*) and a fixed destination can be chosen in the court, in this case the back court left corner. I've been playing squash for almost a year now and the ideal spot on the wall for the lob shot always intrigued me. I always wanted to apply my academics to sports to give me an edge and improve my game.

The dimensions, of the squash court, considered in this investigation are of the standard squash court(Shown in *Figures 1*, *2*, *3*). The ball used real life empirical investigation is a standard one dot Dunlop squash ball, and the squash racquet used is the Prince O3 silver squash racquet (140.0000g). Throughout the report, 4 decimal places are going to be used for all numeric values (except measured values in trials due to limitations of the instruments'' least count), and the original and derived quantities are going to be in terms of standard units.

III. Theory

To analyse the ideal approach, I have to take certain quantities into account so that I can either vary or keep them constant to make the ball reach the destination. The quantities I take into account are-

- Position of the player within the service box.
- Location of the ball on the wall.
- Height at which the service is played
- Average velocity with which the ball travels

According to squash rules, the server is supposed to begin the service from the service box(shown in *Figure 1*)(Strachan). We can make this point fixed in this case, so that a position to commence the shot is identified.

The shot's trajectory can be modelled to understand the location on the wall where the ball must be aimed.

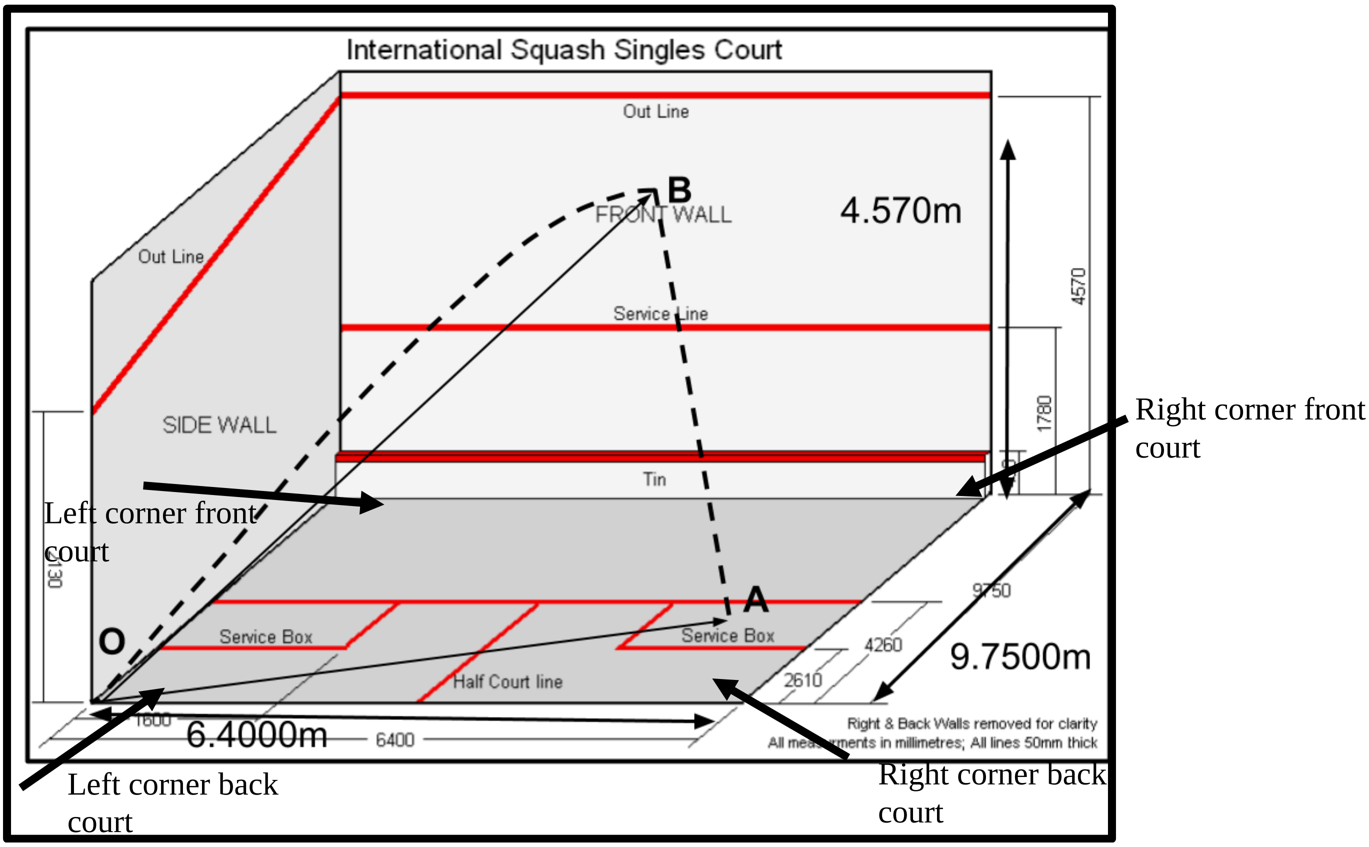

Figure 1: The lob service's approximate trajectory (“Squash Court Lines for Beginners.”)

In *Figure 1*, the dotted line represents a sample trajectory. The direction of the shot is from the service box to the front wall, followed by the rebound of the ball from the front wall to the left corner in the back court, which is the target location. $\overrightarrow{AB}$ represents the straight line motion and $\overrightarrow{BO}$ represents the projectile motion. **Projectile motion** is the type of motion where an object travels in a parabolic trajectory and is subject to acceleration, due to gravity, in only one direction; the path that the object follows is called its trajectory(“Projectile Motion”). Due to differences in the type of motion, vectors $\overrightarrow{AB}$ and $\overrightarrow{BO}$ can be broken into two parts.

I used vector analysis for this investigation, as the ball travels in three dimensions. The vector analysis consists of two parts: position vector and velocity vectors. The position vectors define the location of points of interest, and the velocity vectors determine the rate at which the ball reaches those locations. To account for the motion in all 3 directions, parametric equations are used. The equations are helpful to model how the ball can land in the desired location as they show the different positions of the ball at different instances. Moreover, integration is also used in this investigation through the principles of kinematics.

The first step is to determine the origin and the dimensions of the court, so that the coordinates system in the investigation can be determined.

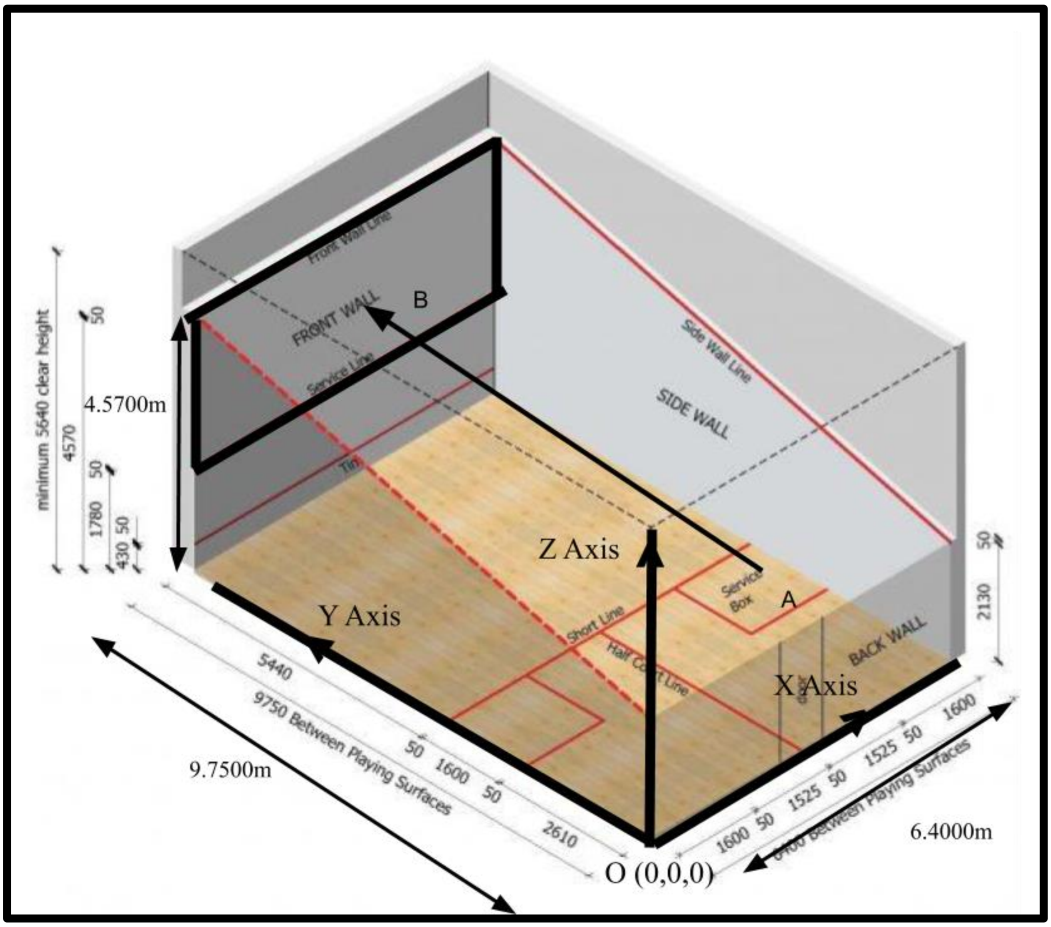

Figure 2: Dimensions of a standard squash court and axes. Note the dimensions are in mm the conversion from m to mm is 1m is 10^5mm. ("Court Specifications.")

As shown *Figure 2*, the origin is set at the back court left corner. This makes the target coordinates of the ball to be:

$$\text{Target location in the investigation} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix}$$

The x, y, and z axes are set according to the following logic: When facing the front wall, the horizontal translation along the court is on the x axis, vertical translation along the court is on the y axis, and the perpendicular translation to both the x and y axis in the court is on the z axis.

IV. <u>The Approach</u>

The aim of the approach is to find a set of equations that show the position of the ball at any instant desired. In order to do this, both the position and velocity vectors of the ball are needed to be found and written in the following formula

Formula 1

$$\vec{r} = (initial\ position\ vector) + (velocity\ vector) \times t$$

t- stands for time since the shot is played from the service box.

⇒ A ball is assumed to follow a hypothetical trajectory and, as an example, a point on the wall is decided to be the target location. First, I define the position vector, $\overrightarrow{OA}$, that gives the position of the beginning of the service. Then, I find the position vector $\overrightarrow{OB}$, which gives the sample location on the wall that I am targeting. Following that, I find $\overrightarrow{AB}$ as the position vector for the ball in the first part of the shot.

- ⇒ Using this position vector, I find the velocity vector, by dividing the components of the position vector by the average time taken for the shot to reach the wall in the first part. This time was found experimentally. The velocity vector will give the position of the ball with respect to the time. Using the position and velocity vector, the position of the ball at any instant is found out using *Equation 1*.
- ⇒ This vector is written in the parametric equation form, to examine the position of the ball in the x, y, and z axes at every instant.
- ⇒ A second set of parametric equations are then found out to model the trajectory of the ball post the collision, as the ball undergoes projectile motion.
- ⇒ The second set of parametric equations are inputted into spreadsheet software, where the time when all the three equations reach 0 simultaneously, is found.
- ⇒ If the sample location on the wall taken does not lead to the ball reaching the desired coordinates in time, the vector $\overrightarrow{OB}$'s components, first, and second parametric equations will be written as variables. By manually entering coordinates of different locations on the wall, on software, a set of coordinates on the wall will be found which lead to a specific instant when the ball reaches its final intended destination.
- ⇒ Finally, the total time for the theoretical shot will be found by the sum of the time taken for $\overrightarrow{AB}$ and $\overrightarrow{OB}$. This time will be compared to the real life trials where the lob shot was played to assess the accuracy of the model created.

V. Straight Line Motion

Refer to *Figure 2* for the illustrations of the dimensions and positions considered.
The straight line motion segment is analysed by determining position vector $\overrightarrow{OA}$, the starting point.
For this trial, the service is commenced from the centre of the service box. The service box is a perfect 2 dimension square(shown in *Figure 2*), of dimensions 1.6000m×1.6000m; due to this, the x component will be $6.4000m - 0.8000m = 5.6000m$. Moreover, the y component will be $2.6100m + 0.5000m + 0.8000\ m = 3.4600($. In squash, it is generally advised to strike the ball from the player's waist height so the height taken in this case is 0.9000 m, my waist height, which will be the z component.

So the position vector, $\overrightarrow{OA}$, will be $\begin{pmatrix} 5.6000 \\ 3.4600 \\ 0.9000 \end{pmatrix}$

For this trial, the location of position vector, $\overrightarrow{OB}$, is from the centre of the front wall (shown in *Figure 2*). The total length of the front wall is 6.4000m, in the x axis. Therefore, the x component is $\frac{6.4000}{2} = 3.2000m$("Court Specifications."). The y component is 9.7500m as the front wall covers the entire vertical length of the court. For the z component, the centre of the middle segment (highlighted in *Figure 2*) of the front wall is taken; this is done as, conventionally, a lob service is targeted on the middle segment. The centre of the middle segment is taken for this trial which gives us $\frac{1.8300+4.5700}{2} = 3.2000$m.

So the position vector $\overrightarrow{OB}$ is $\begin{pmatrix} 3.2000 \\ 9.7500 \\ 3.2000 \end{pmatrix}$

To find the vector, $\overrightarrow{AB}$, which defines the change in position of the ball, we use the following formula
Formula 2

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

Therefore, position vector $\overrightarrow{AB} = \begin{pmatrix} -2.4000 \\ 6.2900 \\ 2.3000 \end{pmatrix}$

To find the total distance travelled by the ball during the straight line motion, the magnitude of $\overrightarrow{AB}$ can be found. The magnitude of the position vector $\overrightarrow{AB}$ is found by using the following formula
("*Magnitude of a Vector")(Fabio 915)*

Formula 3

$$\left|\overrightarrow{AB}\right| = \sqrt{(x)^2 + (y)^2 + (z)^2}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(-2.4000)^2 + (6.2900)^2 + (2.3000)^2}$$

Therefore, $\left|\overrightarrow{AB}\right| = 7.1143\text{m}$

VI. Calculating velocity in straight line motion

To obtain the average velocity of the ball, the displacement of the ball per unit time needs to be found. Since the position of the ball in each of the axis is found, the time taken for the ball to undergo the change in the position is needed to find the velocity of the ball in each of the axes. This time can be found experimentally. Upon empirically performing this service in the court, where the ball arrived at the destination corner and the targeted spot on the wall was hit, I got the results shown in *Table 1* for the time taken to hit the wall from the service box.

Table 1: Timings for $\overrightarrow{AB}$ motion

Trial No	Time/s
1	0.45
2	0.55
3	0.51
4	0.33
5	0.41
6	0.41

The time taken is measured using a Winner Digital stopwatch W-999. The average time taken is 0.44s, which was found over 6 trials.

Since I know the time taken for the ball to cover $\left|\overrightarrow{AB}\right|$, I can calculate the magnitude of the average velocity vector. This can be done by dividing $\left|\overrightarrow{AB}\right|$ by the average time taken.

$$\left|\overrightarrow{AB}\right| = \frac{7.1143}{0.44} = 16.1689\text{m}s^{-1}$$

By using this value of time, the velocity vector can be calculated by dividing the position vector by the time taken. Therefore, the velocity vector, in unit vector notation, is

$$\overrightarrow{AB} = \frac{-2.4000}{0.44}\vec{\imath} + \frac{6.2900}{0.44}\vec{\jmath} + \frac{2.3000}{0.44}\vec{k}$$

(Fabio 923)

The velocity vector $\overrightarrow{AB}$ can be written in the vector form as $\begin{pmatrix} -5.4545 \\ 14.2954 \\ 5.2272 \end{pmatrix}$

To find the position of the ball at any instant, the position vector and velocity vector can be written in the vector form (Fabio 944)

Equation 1

$$\vec{r} = \begin{pmatrix} 5.6000 \\ 3.4600 \\ 0.9000 \end{pmatrix} + \begin{pmatrix} -5.4545 \\ 14.2954 \\ 5.2272 \end{pmatrix} \times t$$

(where t stands for time since the service began)

Hence we can write the parametric equations for the vector $\overrightarrow{AB}$

$$0 \le t \le 0.44s \begin{cases} \mathrm{x(t)} = 5.6000 - 5.4545 \times \mathrm{t} \\ y(t) = 3.4600 + 14.2954 \times t \\ z(t) = 0.9000 + 5.2272 \times t \end{cases}$$

The equations are restricted in the domain of 0≤t≤0.44s. The vector is restricted to the domain as 0.44s is the average time taken for the straight line motion, AB. Post this time domain, the will ball collide with the sample location on the wall and the parametric equations change in different components.

VII. <u>Projectile Motion</u>

Projectile motion will begin once the ball rebounds from the wall. The change in parametric equations will occur differently for each component. To maintain simplicity, the domain of the parametric equations, post the rebound from the wall, have been defined within the domain t≥0, although, the equation shows the time from t≥0.44s, where t is the time since the service is started. This is done to simplify the readings from the software while measuring the time. The time periods for the two motions will be added later to understand the total time taken for the service.

An assumption is made in these equations: the ball and wall have an inelastic collision. Inelastic collisions are defined as collisions where kinetic energy before collision is equal to kinetic energy after the collision. Therefore, the ball does not lose any energy when it collides with the wall. This also means that the velocity of the ball before the collision is equal to the velocity of the ball after collision.

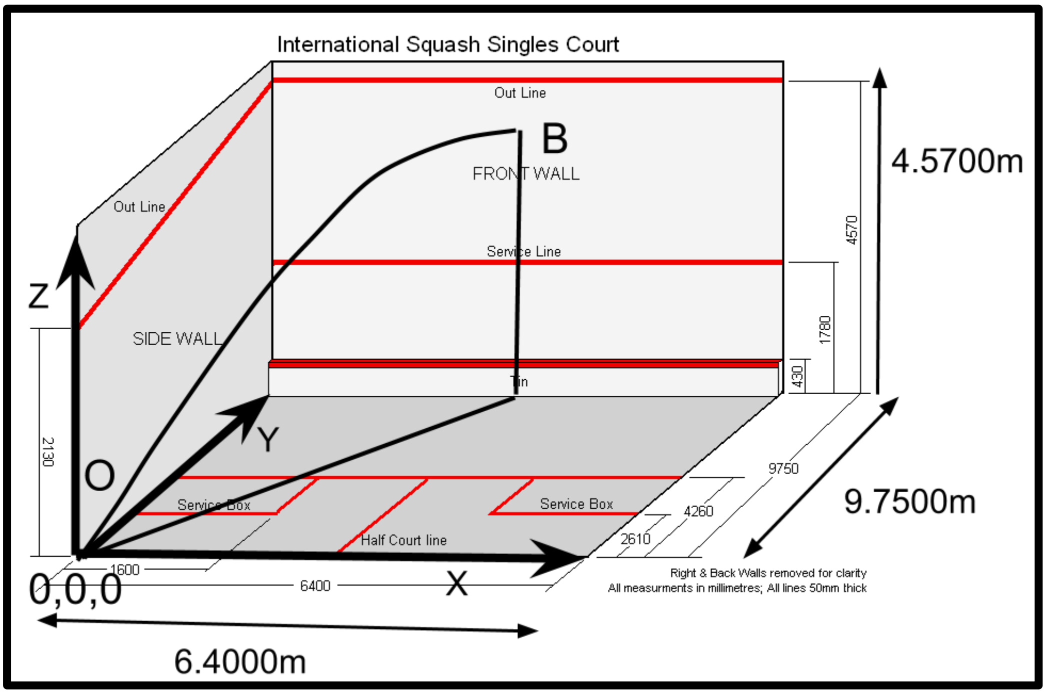

Figure 3: The trajectory of the ball post the collision with the ball ("Squash Court Lines for Beginners.")

It can be inferred from *Figure 3* that during the projectile motion part, x, y, and z component will decrease.

Since the ball begins at the same position on the wall, $\overrightarrow{OB}$, from which the ball rebounded during the straight line motion, the position vectors in the parametric equations remain unchanged post the collision. Therefore, the constant terms in the pre collision equations and post collision equations remain the same.

Hence, position vector(constant term) in the x-axis for the new equation is $\frac{6.4000}{2} = 3.2000\text{m}$

This constant comes from the position vector of $\overrightarrow{OB}$. It is assumed that the rate at which the ball will travel after the collision with the wall will remain the same, so the parametric equation that forms in the x direction is

$$x(t) = 3.2000 - 5.4545 \times t$$

The parametric equation in the y axis will have the fixed point to be 9.7500m as it will be the entire length of the court in the y axis, same as position vector of $\overrightarrow{OB}$. It is assumed that the rate at which y will decrease will have the same magnitude but the sign will be negative as the distance between the wall and the origin is decreasing, so the equation is

$$y(t) = 9.75000 - 14.2954 \times t$$

VIII. Finding the parametric equation in the z axis

The parametric equation in the z axis will have the fixed point(constant) to be 3.2000m, same as position vector $\overrightarrow{OB}$. However, a linear equation won't apply in this case. Unlike the velocity vectors in the x and y axis, the z axis is affected by the acceleration due to gravity, which causes the ball to go up, reach a maximum height and then fall down. This can be modelled by a quadratic equation, since the motion is parabolic with the coordinates of the maximum height being the coordinates of the vertex. Finding the quadratic equation in the z axis is the next part.

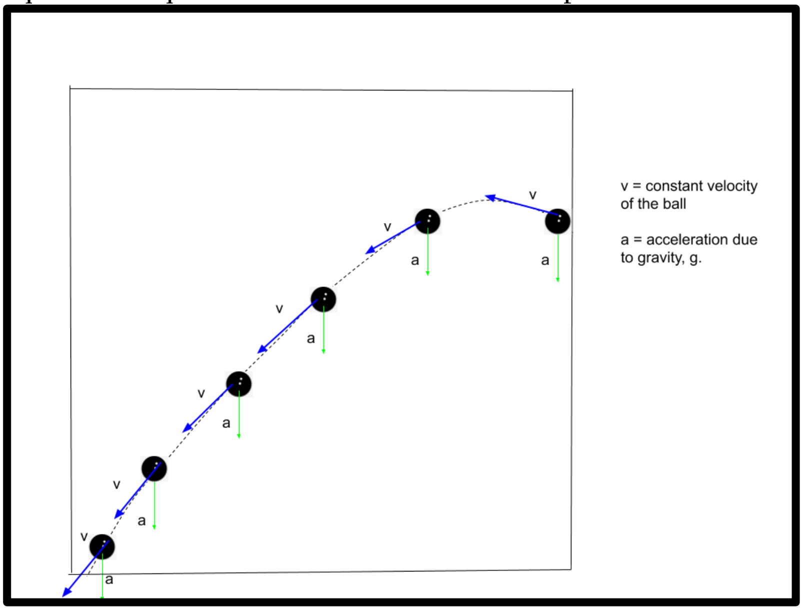

Figure 4: Kinematic representation of the projectile motion of the ball along with the assumption of the constant velocity

To find the position vector at any instance for the z component, post the collision, the known information can be considered:

We know that

$$z(0) = 3.2000m$$

Velocity of the ball post the collision with the ball will remain constant(an assumption). Since velocity is the first derivative of displacement, we can write

(Elert)
(Fabio, 715)

$$\frac{dz}{dt} = 5.2272\text{m}s^{-1}$$

where $\frac{dz}{dt}$ is velocity as it shows change in position in z axis with respect to time.

The ball also faces constant acceleration due to gravity in the downward direction. Since acceleration is rate of change of velocity, we can write

$$\frac{d^2z}{dt^2} - 9.8067\text{m}s^{-2}$$

Where $\frac{d^2z}{dt^2}$ is acceleration as it shows change in velocity with respect to time.

To find velocity, acceleration can be integrated

$$\frac{dz}{dt} = \int (-9.8067)\, dt$$

$$\frac{dz}{dt} = -9.8067 \times t + c_1$$

Here, c_1, is the constant of integration.
At time t=0, the ball had an initial velocity of 5.2272ms^{-1}. At this instance, the acceleration faced by the ball was 0. Hence, the velocity of the ball post the impact remains the same. By substituting t as 0, the result can be seen

$$5.2272 = 0 + c_1$$

$$c_1 = 5.2272$$

The final expression for velocity obtained is

$$\frac{dz}{dt} = -9.8067 \times t + 5.2272$$

Therefore, this expression of velocity can be integrated to find the equation of the position of the ball in the z-axis.

$$z(t) = \int(-9.8067 \times t + 5.2272)\, dt$$

$$z(t) = -9.8067 \times \frac{t^2}{2} + 5.2272 \times t + c_2$$

Here, c_2, is the constant of integration.
At time t=0, the position of the ball was on the wall, so its position in the z-axis was at $3.2000m$. By substituting t as 0, the position is evident

$$3.2000 = 0 + 0 + c_2$$
$$c_2 = 3.2000$$

Therefore, the final expression for position of the ball is

$$z(t) = -4.9034 \times t^2 + 5.2272 \times t + 3.2000$$

Finally, the parametric equations for the ball post the collision are

$$t \geq 0 \begin{cases} x(t) = 3.2000 - 5.4545 \times t \\ y(t) = 9.7500 - 14.2954 \times t \\ z(t) = -4.9034 \times t^2 + 5.2272 \times t + 3.2000 \end{cases}$$

IX. Finding the time at which origin is reached for 1st trial

	A	B	C	D
1	t	x	y	z
2	0.0000	3.2000	9.7500	3.2000
3	0.1000	2.6546	8.3205	3.6737
4	0.2000	2.1091	6.8909	4.0493
5	0.3000	1.5637	5.4614	4.3269
6	0.4000	1.0182	4.0318	4.5063
7	0.5000	0.4728	2.6023	4.5878
8	0.6000	-0.0727	1.1728	4.5711
9	0.7000	-0.6182	-0.2568	4.4564
10	0.8000	-1.1636	-1.6863	4.2436
11	0.9000	-1.7091	-3.1159	3.9328
12	1.0000	-2.2545	-4.5454	3.5239
13	1.1000	-2.8000	-5.9749	3.0169
14	1.2000	-3.3454	-7.4045	2.4118
15	1.3000	-3.8909	-8.8340	1.7087
16	1.4000	-4.4363	-10.2636	0.9075
17	1.5000	-4.9818	-11.6931	0.0083
18	1.6000	-5.5272	-13.1226	-0.9891
19	1.7000	-6.0727	-14.5522	-2.0844
20	1.8000	-6.6181	-15.9817	-3.2779
21	1.9000	-7.1636	-17.4113	-4.5694
22	2.0000	-7.7090	-18.8408	-5.9590

For the ball to reach origin, all three functions have to simultaneously become 0 at a specific instant. This can be done by plotting them on excel against possible time instances that may give the result. Time until 2.0000 seconds is considered to cover all possible positions in the trajectory until the components become negative. At greater time values, all the values in all 3 components become negative, which doesn't make sense logically as the ball has escaped the court in that case. *Microsoft Excel* was used to find the outcome by entering appropriate formulae(shown in *Figure 10* in *Appendix 1*) in the spreadsheet to give the results in *Figure 5.*

Figure 5: Results for the 1st trial with the target location of the centre of the middle segment on the front wall

As seen in *Figure 5*, targeting this spot on the wall does not get the ball to reach the origin for any of the values of time taken. To find the spot where the ball will reach the target location, the spot on the wall where the ball is being targeted must change($\overrightarrow{OB}$ must change). Since there are numerous possible spots on the wall, which if targeted, will lead to the ball to reach the origin, I considered the components of $\overrightarrow{OB}$ as variables and found the pre collision and post collision equations in terms of those variables. To find the spot, I can input various value of coordinates in these equations to get the resulting positions at different instances.

X. Rewriting $\overrightarrow{OB}$ and x, y, and z equations in terms of variables to find the spot on the wall

To rewrite the equations in terms of variables, the initial vectors will be reconsidered. The position vector, $\overrightarrow{OA}$ will remain the same as the serving position is fixed. For $\overrightarrow{OB}$, some of the components maintain the same distance that they did in the earlier part. The distance from the corner to the wall is fixed, so the y component will remain 9.7500m. However, the x and z components can change in $\overrightarrow{OB}$ to make the ball land in the corner. To determine the possible combinations, I took x and z coefficients of $\overrightarrow{OB}$ as variables α and β, where $0m \leq \alpha \leq 6.4000m$ and $0m \leq \beta \leq 4.5700m$.

$$\text{Position vector } \overrightarrow{OB} = \begin{pmatrix} \alpha \\ 9.7500 \\ \beta \end{pmatrix}$$

$$\text{Position Vector } \overrightarrow{AB} = \begin{pmatrix} \alpha - 5.6000 \\ 6.2900 \\ \beta - 0.9000 \end{pmatrix}$$

To find the distance travelled by the ball, the magnitude of $\overrightarrow{AB}$ is found

$$\left|\overrightarrow{AB}\right| = \sqrt{(\alpha - 5.6000)^2 + 6.2900^2 + (\beta - 0.9000)^2}$$

Earlier, to find the velocity vector, $\overrightarrow{AB}$, the time taken for the straight line motion was considered. Alternatively, the velocity vector can be found by using the following formula

$$\vec{v} = \frac{Component\ of\ \overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} \times |\vec{v}|$$

Since $|\vec{v}|$ is already known to be $16.1689ms^{-1}$, the following equations show position of the ball at any instance for the straight line motion $\overrightarrow{AB}$ by using *Formula 1*

$$0 \leq t \leq 0.44s \begin{cases} x(t) = 5.6000 - \dfrac{(\alpha - 5.6000)}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t \\ y(t) = 3.4600 + \dfrac{6.2900}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t \\ z(t) = 0.9000 + \dfrac{\beta - 0.9000}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t \end{cases}$$

The equations post collision change accordingly

$$t \geq 0 \begin{cases} x(t) = \alpha + \dfrac{(\alpha - 5.6000)}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t \\ y(t) = 9.7500 - \dfrac{6.2900}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t \\ z(t) = -4.9034 \times t^2 + \dfrac{\beta - 0.9000}{\left|\overrightarrow{AB}\right|} \times 16.1689 \times t + \beta \end{cases}$$

XI. Finding the time at which origin is reached with x and z components as variables

I test all the time values in the equation till 2.0000s. By entering this formulae in a spreadsheet software, I can vary α and β, which will change the spot on the wall. By manually changing α and β, I can ensure that for one time value, the ball reaches the origin. By trying different combinations with the variables and formulae on the excel sheet, used earlier, I got the results shown in *Figure 6.*

	A	B	C	D
1	α	4.0000	β	2.0000
2	AB	6.5829		
3				
4	t	x	y	z
5	0.1000	3.6070	8.2050	2.2211
6	0.2000	3.2140	6.6601	2.3442
7	0.3000	2.8210	5.1151	2.3692
8	0.4000	2.4280	3.5702	2.2962
9	0.5000	2.0350	2.0252	2.1251
10	0.6000	1.6420	0.4803	1.8559
11	0.7000	1.2490	-1.0647	1.4886
12	0.8000	0.8561	-2.6097	1.0233
13	0.9000	0.4631	-4.1546	0.4599
14	1.0000	0.0701	-5.6996	-0.2015
15	1.1000	-0.3229	-7.2445	-0.9610
16	1.2000	-0.7159	-8.7895	-1.8186
17	1.3000	-1.1089	-10.3344	-2.7743
18	1.4000	-1.5019	-11.8794	-3.8280
19	1.5000	-1.8949	-13.4243	-4.9798
20	1.6000	-2.2879	-14.9693	-6.2296
21	1.7000	-2.6809	-16.5143	-7.5776
22	1.8000	-3.0739	-18.0592	-9.0236
23	1.9000	-3.4669	-19.6042	-10.5676
24	2.0000	-3.8599	-21.1491	-12.2097

Figure 6: Results of the trial with variables in x and z components

Figure 6 shows that changing the variables does not lead to the exact origin for all 3 components. Through trial and error, I got the best case scenario, where the x and z coefficients are closest and nearly 0 at time 0.9000 seconds (shown in *Figure 6* 13th row): $\begin{pmatrix} 0.4631 \\ -4.1546 \\ 0.4599 \end{pmatrix}$. However, the y component is negative in this case, making this an incorrect answer.

XII. **Finding the time at which origin is reached with x,z components, and height as variables**

I reconsidered the lob shot and the parts of the shot that I could vary to get a reliable answer. To answer the RQ, the time at which all 3 components simultaneously reach closest to 0 (and be positive)for a particular instant is needed to be found. In a lob shot, the height at which the shot is played may also vary to a small degree. So I decided to make the height at which the shot was taken as another variable in $\overrightarrow{OA}$. Since this step led to minor changes in the formulae, I changed the formulae on spreadsheet and got the results shown in *Figure 7*

A	B	C	D	E	F	G
α	4.0000	β	2.0000		height	0.7
AB	6.6192					
t	x	y	z			
0.1000	3.6092	8.2135	2.2685			
0.2000	3.2183	6.6771	2.4390			
0.3000	2.8275	5.1406	2.5114			
0.4000	2.4367	3.6041	2.4857			
0.5000	2.0458	2.0676	2.3619			
0.6000	1.6550	0.5312	2.1401			
0.7000	1.2642	-1.0053	1.8202			
0.8000	0.8733	-2.5418	1.4023			
0.9000	0.4825	-4.0782	0.8863			
1.0000	0.0916	-5.6147	0.2722			
1.1000	-0.2992	-7.1512	-0.4400			
1.2000	-0.6900	-8.6876	-1.2502			
1.3000	-1.0809	-10.2241	-2.1585			
1.4000	-1.4717	-11.7606	-3.1648			
1.5000	-1.8625	-13.2971	-4.2692			
1.6000	-2.2534	-14.8335	-5.4717			
1.7000	-2.6442	-16.3700	-6.7723			
1.8000	-3.0350	-17.9065	-8.1709			
1.9000	-3.4259	-19.4429	-9.6676			
2.0000	-3.8167	-20.9794	-11.2623			

Figure 7: Results of the trial with variables in x, z components and height

Figure 7 just shows a specific case where I take the height to be 0.70000m.

Varying the height doesn't affect the results by a great margin and the disparity between the value of y component and the x, z components made the result obtained incorrect again.

XIII. **Finding the time at which origin is reached with x, z components, height and velocity of the ball as variables**

Since varying the height didn't produce an accurate answer, I thought of varying a quantity which will affect the y component significantly. I decided to take the velocity with which the ball was hit, at $\overrightarrow{OA}$, as a variable. The value of velocity is strategically reduced in this case as, traditionally, lob shots tend to be much slower than regular shots, which gives the lob shot their parabolic trajectory.

By taking velocity, height, and location on the wall as variables, I made changes to the equations to get the results shown in *Figure 8*

	A	B	C	D	E	F	G	H	I
1	α	3.5000	β	3.5000		height	0.9	velocity	9
2	AB	7.1228							
3									
4	t	x	y	z					
5	0.1000	3.2347	8.9552	3.7795					
6	0.2000	2.9693	8.1605	3.9609					
7	0.3000	2.7040	7.3657	4.0443					
8	0.4000	2.4386	6.5709	4.0296					
9	0.5000	2.1733	5.7761	3.9168					
10	0.6000	1.9079	4.9814	3.7059					
11	0.7000	1.6426	4.1866	3.3970					
12	0.8000	1.3772	3.3918	2.9900					
13	0.9000	1.1119	2.5970	2.4850					
14	1.0000	0.8465	1.8023	1.8819					
15	1.1000	0.5812	1.0075	1.1807					
16	1.2000	0.3159	0.2127	0.3815					
17	1.3000	0.0505	-0.5821	-0.5159					
18	1.4000	-0.2148	-1.3768	-1.5112					
19	1.5000	-0.4802	-2.1716	-2.6047					
20	1.6000	-0.7455	-2.9664	-3.7962					
21	1.7000	-1.0109	-3.7611	-5.0858					
22	1.8000	-1.2762	-4.5559	-6.4734					
23	1.9000	-1.5416	-5.3507	-7.9592					
24	2.0000	-1.8069	-6.1455	-9.5429					
25	1.2267	0.2450	0.0005	0.1515					

Figure 8: The results of the trial with variables in x, z components, height, and velocity

Through trial and error, I changed each variable until I found the perfect combination so that all 3 coefficients are closest to 0. The result is shown in the above image where the t is 1.2267s (25th row in the *Figure 8*) and the x, y, and z components $\begin{pmatrix} 0.2450 \\ 0.0005 \\ 0.1514 \end{pmatrix}$ are at the final destination. This the first result, where all 3 components are below 0.5000m. The distance of the landing spot from the origin is given by

$$\sqrt{(0.2450)^2 + (0.0005)^2 + (0.1514)^2} = 0.2880m$$

Although this may seem like a significant distance numerically, the obtained position can lead to a highly effective return practically, due to the low value of each component.

In this case, the height at which the ball was served remained the same but the initial velocity deviated by a big margin when compared to the case where we took centre of the wall as the location. For the best case scenario, the quantities obtained are as follows:

$$\text{Target location on the wall, } \overrightarrow{OB}: \begin{pmatrix} 3.5000 \\ 9.7500 \\ 3.5000 \end{pmatrix}$$

Height at which the shot is played: 0.9000m

Velocity of the ball throughout the shot: $9.0000ms^{-1}$

XIV. <u>Time taken for the shot</u>

The total time taken for the shot should be calculated by taking the sum of the time taken for straight line motion and the time taken for projectile motion-

Time taken for projectile motion is 1.2267s. Time taken for the straight line motion can be determined by the average velocity taken into consideration

The average velocity at time 1.2267s is 9.0000m/s and the magnitude of position vector $\overrightarrow{AB}$ is 7.1228m. The time can then be found by

$$\frac{7.1228}{9.0000} = 0.7914s$$

Therefore, the total time taken is $0.7914 + 1.2267 \; = \; 2.0181s.$

The fall in the pace of the ball during the shot led to the long duration of the shot. This is also often seen in a squash game where lob shots are slower and more confusing to play. This is because the opponent anticipates the ball to rebound from the back wall, but the ball gets goes 'dead; in the corner upon direct impact without any opportunity for the opponent to return the service. Nevertheless, the values of the ideal shot can be compared to the experimental values that I found by measuring the time for the entire shot in real life. These are the same trials in which I measured the time for the ball to reach the wall from the service box.

Table 2: Time taken for entire shot to get over

Trial No	Time/s
1	1.48
2	1.45
3	1.52
4	1.39
5	1.45
6	1.62

Table 2 shows that the average time taken for the entire lobs shot is 1.5267s. This value comparable to the theoretical value that we got: 2.0181s. The difference between the two values show are due various ways that the shot can be played and the possible combinations that can be made with height, velocity, and spot on the wall.

XV. Evaluation

The investigation was conducted to answer the proposed research question: **What is the ideal approach that a player should take to achieve the lob service in squash?**

After taking all the variables into consideration, the traits of the shot that I found are reminiscent to those in real games. The investigation started with finding ideal location on the wall to target to perform a lob service, however, improvisations made the investigation to be the ideal approach to perform the lob service. By using the equations found in the investigation, there is more scope to discover other combinations of velocity, height, and location on the wall that will result in a lob shot in the target location. The equations may also help personalize the shot for an individual, based on their waist height. This difference will lead to unique locations that must be targeted on the wall based on the waist height of the player.

Weaknesses

In the initial part of the service, $\overrightarrow{AB}$, the motion of the ball is assumed to be straight line, however, this might not be the case as a certain curvature will always form due to the acceleration of gravity acting on the ball in the duration. This is a reasonable assumption to make as the ball will have a very high velocity once it leaves the racquet and since the distance to the wall is short the effect of the acceleration of gravity will be minimal. Therefore, even though the acceleration due to gravity acted on the ball initially, it is assume that the ball wasn't significantly affected by it. Another noteworthy objection is that the average velocity is used in the investigation, whereas, in a shot, velocity may constantly change. The ball may decelerate after leaving the racquet as the initial impact may lead to it having the greatest velocity. Moreover, the effect of the acceleration due to gravity and air resistance may slow the ball down over time. However, it can again be argued that due to the small distance between the player and the wall, the magnitude of variation may be minimal and may not have a very major effect, so that the average velocity consideration is justified. The collision of the ball with the wall is also assumed to elastic. Elastic collisions are interactions where no change in both kinetic energy and velocity takes place post collision of an object. It is unreasonable to assume this since the collision will lead to the ball slowing down as the ball loses kinetic energy in terms of sound. The friction of the ball with the wall will also dissipate some heat energy and lead the collision to become inelastic. Moreover, the investigation also doesn't consider the rotational dynamics of the ball. The ball, after contacting the racquet, may not retain a fixed face and may have a spin throughout the shot. The spin may significantly affect the landing location and cause a disparity between theoretical prediction of the landing location of the ball and the practical result if attempted.

In the investigation, the trajectory of the ball is assumed to be parabolic after contacting the wall, hence, the equation of the z component is in form of a quadratic. However, this might not be the case as the position of the ball in the z-axis with respect to time, post collision, may follow any random polynomial function, due to an indefinite trajectory. The warmth of the ball is also not taken into consideration. The squash ball is made of the polymer of raw butyl rubber, which gives the ball the property of higher bounce with a higher temperature. This may impact the investigation as the collision of the ball with the wall may be affected as the ball will travel a greater height or smaller height before dropping to the origin, which may in turn explain the difference in time in the experiment and the theory. The squash ball retains heat to a high degree, which causes it to have a steady increase in temperature between consecutive trials; therefore, during consecutive trials (conducted to measure the empirical time of the shot), the ball will have a higher bounce with a higher temperature. This property will make the trials unfair and introduce an error to the measurement of the time. Due to symmetry, the lob shot may have identical results if attempted from the left service box to the right corner target. Furthermore, the newness of the ball is also a factor that may affect the result as newer balls have a higher bounce than worn out balls due to material properties of the ball. Moreover, this analysis can be used to target any location during service and traits for lob shots with different target locations can be investigated using this analysis.

Further scope for improvement and alternative method.

Had varying velocity and height not given me an answer close to the origin, I would have changed the position vector $\overrightarrow{OA}$, in the service box, to ensure the service landed closest to the origin. This would have been achieved by taking the three components of $\overrightarrow{OA}$ as variables within the service box and limiting them to a certain domain to take the service box into consideration. Moreover, the x component would have to be defined to a domain which will take the length of racquet into account, while playing the lob shot in the service box, for the full swing to take place without crashing the racquet with the wall.

Aim analysis
The aim of the investigation was successfully answered, due to the close proximity of the answer to the origin. Practically, the answer found in this investigation will, if not exactly at the origin, lead to a successful service that will be difficult to counter in a game. The requirements for the service found can be practically attempted by a player and by continual practice and judgement the service investigated can be mastered!

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XVI. Appendix: The formulae used in the spreadsheet software to get the results

E5

	A	B	C	D
1	t	x	y	z
2	0	=3.2-5.4545*A2	=9.75-14.2954*A2	=-9.8067/2*A2^2+5.2272*A2+3.2
3	0.1	=3.2-5.4545*A3	=9.75-14.2954*A3	=-9.8067/2*A3^2+5.2272*A3+3.2
4	0.2	=3.2-5.4545*A4	=9.75-14.2954*A4	=-9.8067/2*A4^2+5.2272*A4+3.2
5	0.3	=3.2-5.4545*A5	=9.75-14.2954*A5	=-9.8067/2*A5^2+5.2272*A5+3.2
6	0.4	=3.2-5.4545*A6	=9.75-14.2954*A6	=-9.8067/2*A6^2+5.2272*A6+3.2
7	0.5	=3.2-5.4545*A7	=9.75-14.2954*A7	=-9.8067/2*A7^2+5.2272*A7+3.2
8	0.6	=3.2-5.4545*A8	=9.75-14.2954*A8	=-9.8067/2*A8^2+5.2272*A8+3.2
9	0.7	=3.2-5.4545*A9	=9.75-14.2954*A9	=-9.8067/2*A9^2+5.2272*A9+3.2
10	0.8	=3.2-5.4545*A10	=9.75-14.2954*A10	=-9.8067/2*A10^2+5.2272*A10+3.2
11	0.9	=3.2-5.4545*A11	=9.75-14.2954*A11	=-9.8067/2*A11^2+5.2272*A11+3.2
12	1	=3.2-5.4545*A12	=9.75-14.2954*A12	=-9.8067/2*A12^2+5.2272*A12+3.2
13	1.1	=3.2-5.4545*A13	=9.75-14.2954*A13	=-9.8067/2*A13^2+5.2272*A13+3.2
14	1.2	=3.2-5.4545*A14	=9.75-14.2954*A14	=-9.8067/2*A14^2+5.2272*A14+3.2
15	1.3	=3.2-5.4545*A15	=9.75-14.2954*A15	=-9.8067/2*A15^2+5.2272*A15+3.2
16	1.4	=3.2-5.4545*A16	=9.75-14.2954*A16	=-9.8067/2*A16^2+5.2272*A16+3.2
17	1.5	=3.2-5.4545*A17	=9.75-14.2954*A17	=-9.8067/2*A17^2+5.2272*A17+3.2
18	1.6	=3.2-5.4545*A18	=9.75-14.2954*A18	=-9.8067/2*A18^2+5.2272*A18+3.2
19	1.7	=3.2-5.4545*A19	=9.75-14.2954*A19	=-9.8067/2*A19^2+5.2272*A19+3.2
20	1.8	=3.2-5.4545*A20	=9.75-14.2954*A20	=-9.8067/2*A20^2+5.2272*A20+3.2
21	1.9	=3.2-5.4545*A21	=9.75-14.2954*A21	=-9.8067/2*A21^2+5.2272*A21+3.2
22	2	=3.2-5.4545*A22	=9.75-14.2954*A22	=-9.8067/2*A22^2+5.2272*A22+3.2

$$z(t) = -4.9034 \times t^2 + 5.2272 \times t + 3.2000$$

Figure 10: Formulae used for the results for the 1st trial. Note: The results are shown only for the projectile motion part($\overrightarrow{BO}$). The calculations for $\overrightarrow{AB}$ are to be added onto this result.

$$x(t) = 3.2000 - 5.4545 \times t$$

$$y(t) = 9.75000 - 14.2954 \times t$$

$$z(t) = -4.9034 \times t^2 + \frac{\beta - 0.9000}{|\overrightarrow{AB}|} \times 16.1689 \times t + \beta$$

	A	B	C	D
1	α	4	β	2
2	AB	`=SQRT((B1-5.6)^2+6.29^2+(D1-0.9)^2)`		
3				
4	t	x	y	z
5	0.1	`=$B$1+($B$1-5.6)/$B$2*16.1689*A5`	`=9.75-(6.29/$B$2)*16.1689*A5`	`=(-9.8067/2)*A5^2+($D$1-0.9)/$B$2*16.1689*A5+$D$1`
6	0.2	`=$B$1+($B$1-5.6)/$B$2*16.1689*A6`	`=9.75-(6.29/$B$2)*16.1689*A6`	`=(-9.8067/2)*A6^2+($D$1-0.9)/$B$2*16.1689*A6+$D$1`
7	0.3	`=$B$1+($B$1-5.6)/$B$2*16.1689*A7`	`=9.75-(6.29/$B$2)*16.1689*A7`	`=(-9.8067/2)*A7^2+($D$1-0.9)/$B$2*16.1689*A7+$D$1`
8	0.4	`=$B$1+($B$1-5.6)/$B$2*16.1689*A8`	`=9.75-(6.29/$B$2)*16.1689*A8`	`=(-9.8067/2)*A8^2+($D$1-0.9)/$B$2*16.1689*A8+$D$1`
9	0.5	`=$B$1+($B$1-5.6)/$B$2*16.1689*A9`	`=9.75-(6.29/$B$2)*16.1689*A9`	`=(-9.8067/2)*A9^2+($D$1-0.9)/$B$2*16.1689*A9+$D$1`
10	0.6	`=$B$1+($B$1-5.6)/$B$2*16.1689*A10`	`=9.75-(6.29/$B$2)*16.1689*A10`	`=(-9.8067/2)*A10^2+($D$1-0.9)/$B$2*16.1689*A10+$D$1`
11	0.7	`=$B$1+($B$1-5.6)/$B$2*16.1689*A11`	`=9.75-(6.29/$B$2)*16.1689*A11`	`=(-9.8067/2)*A11^2+($D$1-0.9)/$B$2*16.1689*A11+$D$1`
12	0.8	`=$B$1+($B$1-5.6)/$B$2*16.1689*A12`	`=9.75-(6.29/$B$2)*16.1689*A12`	`=(-9.8067/2)*A12^2+($D$1-0.9)/$B$2*16.1689*A12+$D$1`
13	0.9	`=$B$1+($B$1-5.6)/$B$2*16.1689*A13`	`=9.75-(6.29/$B$2)*16.1689*A13`	`=(-9.8067/2)*A13^2+($D$1-0.9)/$B$2*16.1689*A13+$D$1`
14	1	`=$B$1+($B$1-5.6)/$B$2*16.1689*A14`	`=9.75-(6.29/$B$2)*16.1689*A14`	`=(-9.8067/2)*A14^2+($D$1-0.9)/$B$2*16.1689*A14+$D$1`
15	1.1	`=$B$1+($B$1-5.6)/$B$2*16.1689*A15`	`=9.75-(6.29/$B$2)*16.1689*A15`	`=(-9.8067/2)*A15^2+($D$1-0.9)/$B$2*16.1689*A15+$D$1`
16	1.2	`=$B$1+($B$1-5.6)/$B$2*16.1689*A16`	`=9.75-(6.29/$B$2)*16.1689*A16`	`=(-9.8067/2)*A16^2+($D$1-0.9)/$B$2*16.1689*A16+$D$1`
17	1.3	`=$B$1+($B$1-5.6)/$B$2*16.1689*A17`	`=9.75-(6.29/$B$2)*16.1689*A17`	`=(-9.8067/2)*A17^2+($D$1-0.9)/$B$2*16.1689*A17+$D$1`
18	1.4	`=$B$1+($B$1-5.6)/$B$2*16.1689*A18`	`=9.75-(6.29/$B$2)*16.1689*A18`	`=(-9.8067/2)*A18^2+($D$1-0.9)/$B$2*16.1689*A18+$D$1`
19	1.5	`=$B$1+($B$1-5.6)/$B$2*16.1689*A19`	`=9.75-(6.29/$B$2)*16.1689*A19`	`=(-9.8067/2)*A19^2+($D$1-0.9)/$B$2*16.1689*A19+$D$1`
20	1.6	`=$B$1+($B$1-5.6)/$B$2*16.1689*A20`	`=9.75-(6.29/$B$2)*16.1689*A20`	`=(-9.8067/2)*A20^2+($D$1-0.9)/$B$2*16.1689*A20+$D$1`
21	1.7	`=$B$1+($B$1-5.6)/$B$2*16.1689*A21`	`=9.75-(6.29/$B$2)*16.1689*A21`	`=(-9.8067/2)*A21^2+($D$1-0.9)/$B$2*16.1689*A21+$D$1`
22	1.8	`=$B$1+($B$1-5.6)/$B$2*16.1689*A22`	`=9.75-(6.29/$B$2)*16.1689*A22`	`=(-9.8067/2)*A22^2+($D$1-0.9)/$B$2*16.1689*A22+$D$1`
23	1.9	`=$B$1+($B$1-5.6)/$B$2*16.1689*A23`	`=9.75-(6.29/$B$2)*16.1689*A23`	`=(-9.8067/2)*A23^2+($D$1-0.9)/$B$2*16.1689*A23+$D$1`
24	2	`=$B$1+($B$1-5.6)/$B$2*16.1689*A24`	`=9.75-(6.29/$B$2)*16.1689*A24`	`=(-9.8067/2)*A24^2+($D$1-0.9)/$B$2*16.1689*A24+$D$1`

Figure 11: Formulae of the results of the trial with variables in x and z components Note: The results are shown only for the projectile motion part($\overrightarrow{BO}$). The calculations for $\overrightarrow{AB}$ are to be added onto this result.

$$\mathrm{x(t)} = \alpha + \frac{(\alpha - 5.6000)}{|\overrightarrow{AB}|} \times 16.1689 \times t$$

$$y(t) = 9.7500 - \frac{6.2900}{|\overrightarrow{AB}|} \times 16.1689 \times t$$

$$z(t) = -4.9034 \times t^2 + \frac{\beta - 0.9000}{|\overrightarrow{AB}|} \times 16.1689 \times t + \beta$$

	A	B	C	D	E	F	G
1	α	4	β	2		height	0.9
2	AB	=SQRT((B1-5.6)^2+6.29^2+(D1-G1)^2)					
3							
4	t	x	y	z			
5	0.1	=$B$1+($B$1-5.6)/$B$2*16.1689*A5	=9.75-(6.29/$B$2)*16.1689*A5	=(-9.8067/2)*A5^2+($D$1-$G$1)/$B$2*16.1689*A5+$D$1			
6	0.2	=$B$1+($B$1-5.6)/$B$2*16.1689*A6	=9.75-(6.29/$B$2)*16.1689*A6	=(-9.8067/2)*A6^2+($D$1-$G$1)/$B$2*16.1689*A6+$D$1			
7	0.3	=$B$1+($B$1-5.6)/$B$2*16.1689*A7	=9.75-(6.29/$B$2)*16.1689*A7	=(-9.8067/2)*A7^2+($D$1-$G$1)/$B$2*16.1689*A7+$D$1			
8	0.4	=$B$1+($B$1-5.6)/$B$2*16.1689*A8	=9.75-(6.29/$B$2)*16.1689*A8	=(-9.8067/2)*A8^2+($D$1-$G$1)/$B$2*16.1689*A8+$D$1			
9	0.5	=$B$1+($B$1-5.6)/$B$2*16.1689*A9	=9.75-(6.29/$B$2)*16.1689*A9	=(-9.8067/2)*A9^2+($D$1-$G$1)/$B$2*16.1689*A9+$D$1			
10	0.6	=$B$1+($B$1-5.6)/$B$2*16.1689*A10	=9.75-(6.29/$B$2)*16.1689*A10	=(-9.8067/2)*A10^2+($D$1-$G$1)/$B$2*16.1689*A10+$D$1			
11	0.7	=$B$1+($B$1-5.6)/$B$2*16.1689*A11	=9.75-(6.29/$B$2)*16.1689*A11	=(-9.8067/2)*A11^2+($D$1-$G$1)/$B$2*16.1689*A11+$D$1			
12	0.8	=$B$1+($B$1-5.6)/$B$2*16.1689*A12	=9.75-(6.29/$B$2)*16.1689*A12	=(-9.8067/2)*A12^2+($D$1-$G$1)/$B$2*16.1689*A12+$D$1			
13	0.9	=$B$1+($B$1-5.6)/$B$2*16.1689*A13	=9.75-(6.29/$B$2)*16.1689*A13	=(-9.8067/2)*A13^2+($D$1-$G$1)/$B$2*16.1689*A13+$D$1			
14	1	=$B$1+($B$1-5.6)/$B$2*16.1689*A14	=9.75-(6.29/$B$2)*16.1689*A14	=(-9.8067/2)*A14^2+($D$1-$G$1)/$B$2*16.1689*A14+$D$1			
15	1.1	=$B$1+($B$1-5.6)/$B$2*16.1689*A15	=9.75-(6.29/$B$2)*16.1689*A15	=(-9.8067/2)*A15^2+($D$1-$G$1)/$B$2*16.1689*A15+$D$1			
16	1.2	=$B$1+($B$1-5.6)/$B$2*16.1689*A16	=9.75-(6.29/$B$2)*16.1689*A16	=(-9.8067/2)*A16^2+($D$1-$G$1)/$B$2*16.1689*A16+$D$1			
17	1.3	=$B$1+($B$1-5.6)/$B$2*16.1689*A17	=9.75-(6.29/$B$2)*16.1689*A17	=(-9.8067/2)*A17^2+($D$1-$G$1)/$B$2*16.1689*A17+$D$1			
18	1.4	=$B$1+($B$1-5.6)/$B$2*16.1689*A18	=9.75-(6.29/$B$2)*16.1689*A18	=(-9.8067/2)*A18^2+($D$1-$G$1)/$B$2*16.1689*A18+$D$1			
19	1.5	=$B$1+($B$1-5.6)/$B$2*16.1689*A19	=9.75-(6.29/$B$2)*16.1689*A19	=(-9.8067/2)*A19^2+($D$1-$G$1)/$B$2*16.1689*A19+$D$1			
20	1.6	=$B$1+($B$1-5.6)/$B$2*16.1689*A20	=9.75-(6.29/$B$2)*16.1689*A20	=(-9.8067/2)*A20^2+($D$1-$G$1)/$B$2*16.1689*A20+$D$1			
21	1.7	=$B$1+($B$1-5.6)/$B$2*16.1689*A21	=9.75-(6.29/$B$2)*16.1689*A21	=(-9.8067/2)*A21^2+($D$1-$G$1)/$B$2*16.1689*A21+$D$1			
22	1.8	=$B$1+($B$1-5.6)/$B$2*16.1689*A22	=9.75-(6.29/$B$2)*16.1689*A22	=(-9.8067/2)*A22^2+($D$1-$G$1)/$B$2*16.1689*A22+$D$1			
23	1.9	=$B$1+($B$1-5.6)/$B$2*16.1689*A23	=9.75-(6.29/$B$2)*16.1689*A23	=(-9.8067/2)*A23^2+($D$1-$G$1)/$B$2*16.1689*A23+$D$1			
24	2	=$B$1+($B$1-5.6)/$B$2*16.1689*A24	=9.75-(6.29/$B$2)*16.1689*A24	=(-9.8067/2)*A24^2+($D$1-$G$1)/$B$2*16.1689*A24+$D$1			
25							

Figure 12: Formulae of the results of the trial with variables in x, z components, height. Note: The results are shown only for the projectile motion part($\overrightarrow{BO}$). The calculations for $\overrightarrow{AB}$ are to be added onto this result.

$$\mathrm{x(t)} = \alpha + \frac{(\alpha - 5.6000)}{|\overrightarrow{AB}|} \times 16.1689 \times t$$

$$y(t) = 9.7500 - \frac{6.2900}{|\overrightarrow{AB}|} \times 16.1689 \times t$$

$$z(t) = -4.9034 \times t^2 + \frac{\beta - \theta}{|\overrightarrow{AB}|} \times \delta \times t + \beta,$$
$$\theta \textit{ represents the variable for height}, \delta \textit{ represents the variable for velocity}$$

A	B	C	D	F	G	H	I
Name Box		β	3.5	height	0.9	velocity	9
AB	=SQRT((B1-5.6)^2+6.29^2+(D1-G1)^2)						
t	x	y	z				
0.1	=$B$1+($B$1-5.6)/$B$2*$I$1*A5	=9.75-(6.29/$B$2)*$I$1*A5	=(-9.8067/2)*A5^2+($D$1-$G$1)/$B$2*$I$1*A5+$D$1				
0.2	=$B$1+($B$1-5.6)/$B$2*$I$1*A6	=9.75-(6.29/$B$2)*$I$1*A6	=(-9.8067/2)*A6^2+($D$1-$G$1)/$B$2*$I$1*A6+$D$1				
0.3	=$B$1+($B$1-5.6)/$B$2*$I$1*A7	=9.75-(6.29/$B$2)*$I$1*A7	=(-9.8067/2)*A7^2+($D$1-$G$1)/$B$2*$I$1*A7+$D$1				
0.4	=$B$1+($B$1-5.6)/$B$2*$I$1*A8	=9.75-(6.29/$B$2)*$I$1*A8	=(-9.8067/2)*A8^2+($D$1-$G$1)/$B$2*$I$1*A8+$D$1				
0.5	=$B$1+($B$1-5.6)/$B$2*$I$1*A9	=9.75-(6.29/$B$2)*$I$1*A9	=(-9.8067/2)*A9^2+($D$1-$G$1)/$B$2*$I$1*A9+$D$1				
0.6	=$B$1+($B$1-5.6)/$B$2*$I$1*A10	=9.75-(6.29/$B$2)*$I$1*A10	=(-9.8067/2)*A10^2+($D$1-$G$1)/$B$2*$I$1*A10+$D$1				
0.7	=$B$1+($B$1-5.6)/$B$2*$I$1*A11	=9.75-(6.29/$B$2)*$I$1*A11	=(-9.8067/2)*A11^2+($D$1-$G$1)/$B$2*$I$1*A11+$D$1				
0.8	=$B$1+($B$1-5.6)/$B$2*$I$1*A12	=9.75-(6.29/$B$2)*$I$1*A12	=(-9.8067/2)*A12^2+($D$1-$G$1)/$B$2*$I$1*A12+$D$1				
0.9	=$B$1+($B$1-5.6)/$B$2*$I$1*A13	=9.75-(6.29/$B$2)*$I$1*A13	=(-9.8067/2)*A13^2+($D$1-$G$1)/$B$2*$I$1*A13+$D$1				
1	=$B$1+($B$1-5.6)/$B$2*$I$1*A14	=9.75-(6.29/$B$2)*$I$1*A14	=(-9.8067/2)*A14^2+($D$1-$G$1)/$B$2*$I$1*A14+$D$1				
1.1	=$B$1+($B$1-5.6)/$B$2*$I$1*A15	=9.75-(6.29/$B$2)*$I$1*A15	=(-9.8067/2)*A15^2+($D$1-$G$1)/$B$2*$I$1*A15+$D$1				
1.2	=$B$1+($B$1-5.6)/$B$2*$I$1*A16	=9.75-(6.29/$B$2)*$I$1*A16	=(-9.8067/2)*A16^2+($D$1-$G$1)/$B$2*$I$1*A16+$D$1				
1.3	=$B$1+($B$1-5.6)/$B$2*$I$1*A17	=9.75-(6.29/$B$2)*$I$1*A17	=(-9.8067/2)*A17^2+($D$1-$G$1)/$B$2*$I$1*A17+$D$1				
1.4	=$B$1+($B$1-5.6)/$B$2*$I$1*A18	=9.75-(6.29/$B$2)*$I$1*A18	=(-9.8067/2)*A18^2+($D$1-$G$1)/$B$2*$I$1*A18+$D$1				
1.5	=$B$1+($B$1-5.6)/$B$2*$I$1*A19	=9.75-(6.29/$B$2)*$I$1*A19	=(-9.8067/2)*A19^2+($D$1-$G$1)/$B$2*$I$1*A19+$D$1				
1.6	=$B$1+($B$1-5.6)/$B$2*$I$1*A20	=9.75-(6.29/$B$2)*$I$1*A20	=(-9.8067/2)*A20^2+($D$1-$G$1)/$B$2*$I$1*A20+$D$1				
1.7	=$B$1+($B$1-5.6)/$B$2*$I$1*A21	=9.75-(6.29/$B$2)*$I$1*A21	=(-9.8067/2)*A21^2+($D$1-$G$1)/$B$2*$I$1*A21+$D$1				
1.8	=$B$1+($B$1-5.6)/$B$2*$I$1*A22	=9.75-(6.29/$B$2)*$I$1*A22	=(-9.8067/2)*A22^2+($D$1-$G$1)/$B$2*$I$1*A22+$D$1				
1.9	=$B$1+($B$1-5.6)/$B$2*$I$1*A23	=9.75-(6.29/$B$2)*$I$1*A23	=(-9.8067/2)*A23^2+($D$1-$G$1)/$B$2*$I$1*A23+$D$1				
2	=$B$1+($B$1-5.6)/$B$2*$I$1*A24	=9.75-(6.29/$B$2)*$I$1*A24	=(-9.8067/2)*A24^2+($D$1-$G$1)/$B$2*$I$1*A24+$D$1				
1.2267	=$B$1+($B$1-5.6)/$B$2*$I$1*A25	=9.75-(6.29/$B$2)*$I$1*A25	=(-9.8067/2)*A25^2+($D$1-$G$1)/$B$2*$I$1*A25+$D$1				

Figure 13: Results of the trial with variables in x, z components, height, and velocity. Note: The results are shown only for the projectile motion part ($\overrightarrow{BO}$). The calculations for $\overrightarrow{AB}$ are to be added onto this result.

$$\mathrm{x(t)} = \alpha + \frac{(\alpha - 5.6000)}{|\overrightarrow{AB}|} \times \delta \times t, \delta \textit{ represents the variable for velocity}$$

$$y(t) = 9.7500 - \frac{6.2900}{|\overrightarrow{AB}|} \times \delta \times t, \delta \textit{ represents the variable for velocity}$$