I. Introduction

In squash, the squash ball plays a vital role in the dynamics of the game. The squash ball is engineered in a manner that complements the intensity of the game. The squash ball is composed mainly of **isobutylene-isoprene rubber**, also known as raw Butyl rubber. Raw butyl rubber has a high thermal retention property, which enables the squash ball to retain its heat for longer periods in the game (Landman).

In a squash game, it is a standard practice to “warm the ball up” before a game is initiated due to the observed relationship between temperature and the bounce height of the squash ball. The warming of the squash ball is done by playing recursive shots on the wall in a short interval of time. The material of the ball is such that the ball is warmed quickly and it retains heat for successive play in the game. The observed relationship between the temperature and bounce height is such that an increase in the temperature of the ball will lead to a significant increase in the height of the bounce of the ball that can be noticeable from human perception. This is observed not only for the initial bounce but for subsequent bounces, wherein, the height of the second and third bounces are much higher for a warmer ball than a colder ball. Much of the literature published around thermodynamics of the squash ball, revolves around this direct proportional relationship of temperature vs bounce height, where the proportionality relationship was in a polynomial form.

On the contrary, I was interested in investigating the kinetic energy loss in the ball at different temperatures. When I started playing squash a year ago, I always struggled with the anticipating the bounce of the ball once it rebounded from the wall as the change (decrease) in bounce height of consecutive bounces seemed very drastic and irregular. As I entered high school, the knowledge of energy conversions provided greater clarity about what the relationship between loss in kinetic energy and temperature of the ball could possibly be, so I decided to investigate the relationship for this paper. An intriguing observation regarding squash balls is that balls with a higher temperature tend to have a lesser height change (decrease) in subsequent bounces, over time, than the height change in the comparatively cooler balls. This was a strange observation as even though warmer balls lose heat at a faster pace, they lost a relatively smaller height in subsequent bounces as compared to cooler balls. I planned to address this observation in my investigation. The loss in the bounce height could be attributed to the kinetic energy loss occurring in the ball at each interaction. A series of questions popped up in my head to address this phenomenon such as: Can you minimize the kinetic energy loss by increasing the temperature indefinitely? Is there a temperature value at which kinetic energy loss is completely minimized? Hence, the research question that I designed is **How does the loss in kinetic energy in a squash ball respond to change in temperature of the ball when dropped from a fixed height?**

Scope of Research:

By investigating this question, the general trend of temperature against kinetic energy loss may be identified. This may be advantageous to know, while initially warming the ball up, so that the player can heat the ball to an optimum temperature range, where the kinetic energy loss with each bounce will be reduced and the ball will be able to have more predictable bounces. By heating the ball to a temperature within this optimum range, the player can avoid the cumbersome process of reheating a ball during a game or even avoid losing points due to drastic bounce variations over the course of the game.

II. Background Information

The squash ball, during play, displays complex problems in rotational mechanics, thermal physics, Kinetic - Potential Energy conversions, and elasticity. Among these concepts, the ones that are going to be explored in this paper include Kinetic-Potential energy conversions and thermal physics.

The squash ball is hollow from the inside and has ambient pressure within the space in the ball. When the ball is subjected to higher temperature (than room temperature) the air pressure also experiences a proportional increase causing the ball to inflate slightly. The inflation occurs as the increased temperature raises the average kinetic energy of air molecules within the squash ball and the molecules collide with the walls of the squash ball with greater energy causing a microscopic expansion, unnoticeable by the human eye. Moreover, the molecules in the ball vibrate along their mean position due to the exposure of higher kinetic energy and the ball expands (Landman). Cooling the ball has the opposite effect as all the kinetic energy of the air within the ball is lost and the ball contracts as the molecules of air collide less frequently with the walls within the ball.

When the ball is dropped from the height h_1, it goes through a series of energy conversions due to changes at the molecular level. The squash ball is made of a highly elastic polymer structure that enables the ball to be conformed when a force is applied. This structure, gives the ball the property to store elastic potential energy when under stress, and also the property to effectively convert the elastic potential energy into kinetic or gravitational potential energy when the bonds inside the ball reinstate to form the original shape. Greater the elasticity of the ball, greater will be the energy stored within it to be converted into other forms in the next instant. The increased temperature loosens up the bonds between the molecules in the polymer, and allows the ball to store more elastic potential energy than before, which allows the ball to convert this increased elastic potential energy to other forms of energy such as gravitational potential and kinetic energy. This conversion in turn causes the higher bounce (Boorman). Cooling the ball has the opposite effect as the bonds between the molecules become stiffer and less elastic potential energy can be stored in the bonds, reducing compression and bounce back height. The degree to which an object can store energy when conformed is called resilience. The squash ball is identified as an object that has a fairly low resilience due to its material, butyl rubber; as it dissipates a high quantity of energy post its deformation when it collides. This energy is mainly lost in the form of heat and is consumed in heating the air trapped in the ball, which enables the ball to remain heated to higher temperatures (Eaton).
The collision of the ball in both a real game and with the ground is an inelastic collision, which means that kinetic energy is not conserved once the collision reaches completion, although, momentum can or cannot have been conserved (Fitzpatrick). This kinetic energy is lost in terms of heat and sound energy with the ground or the racquet.

III. Mathematical Model

The approach I took to solve this question was by considering the energy conversions that take place within the ball. To find the kinetic energy loss in the ball, velocities at different instances are needed to be known. By carefully selecting the instances to begin and end the measurement, velocity at different instances could be calculated. Here is the approach I took:
Consider a squash ball at room temperature at instance that is held at a fixed known height, h_1 (Figure 1 shows the ball's position at instance α). The total energy within the ball is gravitational potential energy and can be found by the equation

$$Gravitational\ potential\ energy = m \times g \times h_1$$

Where,
m is the mass of the ball in kg
g is the value of acceleration due to gravity, 9.807 ms^{-2}
h_1 is the height at which the ball is initially held in m.

Once the ball is dropped, the gravitational potential energy gradually gets converted into kinetic energy and at one instant β, the instant immediately before the ball contacts the ground, all the gravitational potential energy is converted to kinetic energy. The equation for total energy at this point is

$$Kinetic\ Energy = \frac{1}{2} \times m \times v_1^2$$

Where v_1 is the velocity just before impact in ms^{-1} (maximum velocity in the conversion). These two instances can be equated to get v_1 as at instant the total kinetic energy was 0J and at instant the total gravitational potential energy was 0J. Therefore each equation at instance and can represents the total energy at the respective instances and should give the same value. Note: Air resistance has been ignored in this case and may have an effect on reducing the total energy possessed by the ball at instance β. This is a fair assumption as the ball is dropped from a very small height, so that the effect of air resistance is minimal. Therefore the equation we get is

$$\frac{1}{2} \times m \times v_1^2 = m \times g \times h_1$$

By making v_1 the subject and simplifying we get

Equation 1

$$v_1 = \sqrt{2 \times g \times h_1}$$

Since h_1 is known v_1 can be calculated.

After the ball collides with the ground, all the kinetic energy stored within the ball is transformed into elastic potential energy. During this time period, the ball compresses slightly and loses energy as heat and sound due to the inelastic collision (collision where total kinetic energy is not conserved, however, momentum is). As the ball restores its natural shape, the elastic potential energy stored within the ball is converted into both kinetic energy and gravitational potential energy.

The instance δ, after the squash ball has completely decompressed, shows us the initial velocity after impact, where the gravitational potential energy is 0J due to the height being 0m and the kinetic energy is maximum as v_2 is the highest velocity post collision. The equation we get for the total energy at this instance is

$$Kinetic\ Energy = \frac{1}{2} \times m \times v_2^2$$

At instance γ the squash ball travels upwards and attains the maximum measured height h_2, where kinetic energy is 0J, as velocity is $0ms^{-1}$, and gravitational potential energy is maximum after collision. The total energy at this instance is given by the equation

$$Gravitational\ potential\ energy = m \times g \times h_1$$

Therefore we can equate these two instances, as both of these equations represent total energy post collision at different instances. Note: Air resistance has been ignored in this case and may have an effect on reducing the total energy possessed by the ball at instance γ. Hence, we get the following equation

$$\frac{1}{2} \times m \times v_2^2 = m \times g \times h_2$$

By making v_2 the subject and simplifying we get

Equation 2

$$v_2 = \sqrt{2 \times g \times h_2}$$

Since we have the two different instances of velocity the loss in kinetic energy can be given by the equation

$$Kinetic\ energy\ loss = \frac{1}{2} \times m \times v_1^2 - \frac{1}{2} \times m \times v_2^2$$

By simplifying it we get

Equation 3

$$Kinetic\ energy\ loss = \frac{1}{2} \times m \times (v_1^2 - v_2^2)$$

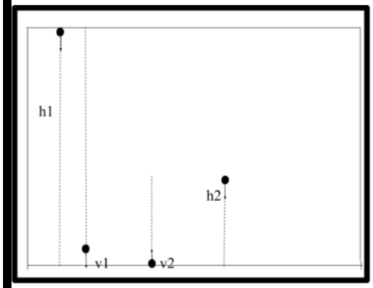

Figure 1: Position of the ball at different instances

In the equation v_1^2 is taken before v_2^2 as $v_1 > v_2$ so that the sign convention doesn't make energy values negative. Throughout the investigation, calculated values are taken up to 3 decimal places to obtain accurate answers, except in cases where measurements are conducted and the measurement limits and uncertainties of the apparatus is considered. The diagram shown in *Figure 1* illustrates the position of the ball at different instances.

IV. <u>Methodology</u>

Since a direct empirical relationship between kinetic energy loss and temperature cannot be obtained, I conducted an experiment where I could measure the desired quantities- height reached by the ball post the collision with the ground- that are needed to construct the relationship. The approach is as follows

- I setup an experiment to obtain v_2 for different values of temperature, so that the kinetic energy loss for each temperature value can be calculated.
- The experimental setup for obtaining v_2 involved fixing the temperature of the squash ball and then dropping a ball from a fixed height, h_1, to measure the rebound height, h_2, by clicking photos of the entire trajectory and using software analysis to find the maximum rebounded height. **Hence, the independent variable in the experiment is the temperature of the ball, and the dependent variable is the bounce height.**
- Using the value of h_2 from the experiment, the mathematical model can be utilized to calculate the velocities, v_1 and v_2. By using the values of the velocities, using *Equation 1 and 2*, kinetic energy loss can be calculated by *Equation 3*. In this manner, an indirect relationship between Kinetic energy loss and temperature can be obtained.
- By repeating the same practice for different temperatures, I plot the graph of Kinetic energy loss vs temperature. The equation of the graph can provide the relationship between kinetic energy loss against temperature and the obtained graph can showcase the general relationship between the variables.

V. Hypothesis

I predict that as **the temperature of the ball will increase, the amount of kinetic energy loss will decrease**. This will occur as I'm dropping the squash ball from the same height for every trial, so its velocity before impact will remain the same for all the trials, whereas, its velocity post the bounce will differ due to different heights it rebounds to. By referring to *Equations 1, 2, and 3*, it can be understood that with subsequent rise in temperature, v_1 will stay the same, whereas, v_2 will increase. This occurs because h_2 is known to increase with progressing rise in temperatures and h_1 remains constant throughout the investigation. The increase in h_2 with temperature is an observation that is made at a macroscopic level and will be evident with the data collected. Moreover, I predict that the relationship between temperature and kinetic energy loss will be in the form of a polynomial equation: $ax^n + bx^{n-1} + cx^{n-2}$ ....+d.
where the highest degree will be equal to the constant "n" and n could be defined within the domain of $3 \leq n \leq 6$. I propose that such a relationship takes place as during my research I encountered a research paper, which was written with the objective of finding the relationship between the ratio of velocities, v_1:v_2, also known as coefficient of restitution (Dependant variable), with the temperature (Independant variable) of the ball ("Influence of Temperature on Bouncing Balls."). The finding revealed that the equation of the graph resulted in a polynomial equation, with the highest degree being 3, with an R_2 value of 0.997. Since this investigation plots the temperature against Kinetic energy loss, where the only variable in *Equation 3* is velocity raised to a power, v_{22}, I predict that the Kinetic energy loss will be related to temperature with a polynomial equation in the domain defined above.

VI. Variables

1. Independent Variable

The temperature of the ball. The temperature will directly affect the elasticity of the ball and this will impact the height to which it is bounced. The temperature values used in the experiment are 27˚C, 35°C, 40°C, 45°C, 50°C, 55˚C, and 60˚C. The reason for choosing the room temperature, 27˚ C, was to establish the trend in the graph with all the possible values of temperature that may be encountered while warming the ball up, so that the obtained equation could have appropriate relevance in real life. The temperature range, 35˚C to 50˚C, is chosen because it is observed that 35˚C is the lowest temperature at which the ball has a significant bounce for play. On the other hand, the maximum temperature that a squash ball reaches in order to have thermal equilibrium with the surroundings of the court is around 45˚C, beyond this it is highly unlikely for a player to warm the ball up without mechanical assistance. The reason for choosing a higher temperature of 55˚C and 60˚C is to observe the relationship for higher temperatures and test whether a minimum kinetic energy loss value may exist.

2. Dependent Variable

The height of the squash ball measured post the collision with the ground. Once the ball is dropped from a fixed height, the height to which it will bounce, h_2, will be measured by software and will vary for different values of temperature of the ball.

3. Controlled Variable

Table 1: Controlled variables

Variable	***Why and how the variable is to be controlled***
The height from which the ball is dropped.	The height from which the ball is dropped needed to be controlled otherwise the relationship between bounce height and temperature cannot be investigated as change in drop height will directly affect post collision bounce height. By fixing a point on the wall, behind the ball dropping position, as a reference point, this variable is fixed. For every trial, the ball was dropped from that same point and this height is referred to as the constant, h_1, in the entire procedure.
The volume of water used in the electric heater and water bath.	The change in the volume of water used in the electric heater for different trials may influence the distribution of heat within the squash ball as different volumes of water will lead to unequal distribution of heat within the ball. The volume of water that was fixed was 1000cm3.
The position of the ball within the electric heater.	The position of ball within the water bath matters to ensure that the entire ball is heated uniformly. Using tongs, the ball was suspended at a fixed position during the heating period for each of the trials to ensure that the ball receives equivalent heat throughout and every point is heated to the same desired temperature.
External factors	Factors such as wind, surface of ground, room temperature were controlled to ensure minimal impact on the experiment. Blowing wind can change the movement of the ball while being dropped, so the effect of wind was minimised by conducting the experiment in a room with closed windows. Moreover, the surface of ground may affect the bounce back of the ball depending on the texture of the surface; keeping the surface constant by performing the investigation in only one location controlled this. The room temperature affects the air resistance as at different temperatures there will be different air densities; the temperature was controlled by performing the experiment in an air conditioned room fixed at 27℃.

VII. Apparatus

Table 2: Apparatus

Equipment	**Properties**	**Quantity**
Squash Ball		
450 W electric heater	-	1
Measuring tape	Uncertainty: ±0.001m	1
Object of reference for the bounce to be compared to the bounce height.	Rectangular red suitcase used in this case. Height: 0.725m	1
Infra-red thermometer- To measure the temperature of the ball once heated	Uncertainty ±0.1°C	1
Water	Volume of water used: 1000cm3	-
Image processing software- Fiji software used in this case(Schindelin).	The software should enable scale setting for the objects in the image and allow the user to measure the dimensions of objects in the picture.	-
Laptop	-	1

Camera- Mobile phone camera used (Oneplus6 phone camera used).	High Resolution along with high storage capacity	1
Multiple image capturing software (also known as burst photography)- GonnyCam application for mobile phone used in this case("GonnyCam HD Burst Camera").	This was the software that enabled the camera to capture 100 images in 3-4 seconds, so that the instant at which the ball reaches the maximum rebound height can be measured. This burst photography gives images at a rate that is similar to a video with 30 frames per second.	-
Tongs	-	1
Water absorbent handkerchief	Water absorption property	1
Thermometer	Uncertainty: 0.1˚C	1

VIII. Experimental Setup

Set your camera at an appropriate distance from the place of dropping the ball, so that the camera captures the entire scene of the fall of the ball. Set the camera to take 100 shots in a very short time interval (3s-4s) to capture the entire trajectory of the ball post collision, so that the highest height that the ball reached can be identified out of the pictures clicked.

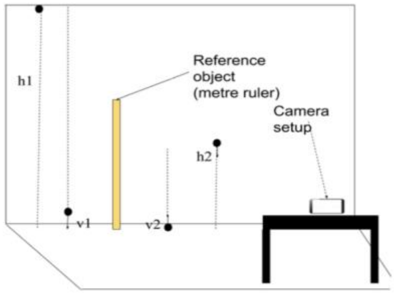

Figure 2: Diagram of the setup

IX. Procedure

Part 1: Varying the temperature of the ball. Procedure described for experiment with 35˚C, however the same procedure applies to all temperatures: 27 ˚C, 35 ˚C, 40 ˚C, 45 ˚C, 50 ˚C, 55˚C, and 60˚C.

a) Using the electric heater, warm the water in an insulated container up then detach the insulated container from the heater. This is done to create a water bath inside which the ball can be kept to make it reach a certain temperature. The water within the bath has been intentionally heated to a higher temperature as the specific heat capacity of the ball and the bath differ: butyl rubber has a specific heat capacity of 1966 J/kgK, whereas, water has 4190 kJ/kgK (Hume) . Although the ball has a lower specific heat capacity, it takes time to heat up to the same temperature as the water due to it having structural differences along with the material (the ball is hollow). This step is done to make the ball reach the desired temperature quicker . The water in the heater, being insulated, acts as a water bath so that the ball within it can be controlled at a temperature.

b) Dip the squash ball, by holding it with tongs the predetermined fixed position. Check the temperature of the ball at regular intervals of 30-45s.

c) To check the temperature of the ball, remove the ball by the use of tongs and then gently wrap it in a highly water absorbent handkerchief to make sure it has dried completely (this is done to ensure that the infra-red thermometer does not pick up the temperature of water that is there on the surface instead of the ball). Then using the infra-red thermometer measure the temperature of the ball. Strategically heat the ball to a higher temperature of 38˚C, so that the trend in the gradual decrease in temperature can be identified and the ball can be dropped at the instant it reaches 35˚C.

d) Carry the ball to the dropping point, using tongs, from the heater when the temperature reaches close to 35˚C and then drop it gently from the predetermined height, h_1. Ensure that the camera captures the entire fall to determine the maximum height reached by the ball can be identified clearly.

Part 2: The image analysis of the squash ball

a) Once the images of the entire trajectory of the ball are found, identify the image in which the squash ball reaches the maximum height. This will have to be done manually to minimize the significance of an error. After identifying the image, upload it on a computer to analyze it using a software. In this case, Fiji software was used as the image processor.

b) Using the reference object to set the scale in the image, measure the height of the bounce back using appropriate tools in the image processor.
c) Repeat the experiment with different temperatures for the same ball.
d) Take 3 trials for each temperature.

Safety Precautions

Table 3: Safety Precautions

Safety Hazard	Precautions
Warm water and ball	When working with the electric heater, special care should be taken while handling the ball within the water bath. Lab coat must be worn in order to be safeguarded from burns in case spillage occurs while handling the electric heater. Use of tongs and gloves is also necessary when transporting the ball to the drop location safety due to the burning hazard from the high temperature of the ball.
Infra-red thermometer	The Infra-red thermometer should be used with care and should be switched on only when necessary to measure the temperature of the ball as it can blind someone if directly pointed at the eye (Nair). Safety goggles are needed to be worn when using the device.
Height related Hazards	Since the ball is being dropped from a height of 2.113m, which had to be reached by standing on a platform, a falling hazard develops. Special care should be taken in order to ensure that there is no water, due to spillage, on the platform that can cause a person to slip and fall from the height. Moreover, care should be taken so that the person doesn't lose their balance and fall.

X. Data Analysis and processing

As per the hypothesis, velocity, v_1 remained the same for all trials because height, h_1 was kept constant (refer to *Equation 1*). The values for those quantities are as follows: h_1= 2.113m and v_1= 6.438ms^{-1}. Mass of the yellow single dot squash ball is $23.829 \pm 0.001g$.

Table 4: Data analysis

	Yellow Single dot					
Temperature/ ˚C	h_2/m					
	Trial1	Trial2	Trial3	Average h_2	v_2/ms^{-1}	Kinetic Energy loss/J
27.0	0.493	0.477	0.458	0.476 ± 0.019	3.056 ± 0.059	0.383 ± 0.009
35.0	0.544	0.613	0.591	0.583 ± 0.036	3.381 ± 0.103	0.358 ± 0.016
40.0	0.645	0.616	0.641	0.634 ± 0.016	3.526 ± 0.043	0.346 ± 0.028
45.0	0.699	0.720	0.725	0.715 ± 0.014	3.744 ± 0.023	0.327 ± 0.023
50.0	0.840	0.804	0.739	0.794 ± 0.052	3.947 ± 0.128	0.308 ± 0.069
55.0	0.874	0.864	0.842	0.860 ± 0.017	4.107 ± 0.041	0.293 ± 0.024
60.0	0.920	0.932	0.905	0.919 ± 0.015	4.246 ± 0.033	0.279 ± 0.020

XI. Calculations with example and uncertainty

Example 1: The average h_2 calculation at 35°C

The average value of h_2 is calculated by the formula:

$$Average\ h_2 = \frac{a+b+c}{3}$$

Where a,b,c are values of the trials of h_2

$$= \frac{0.544+0.613+0.591}{3}$$
$$= 0.583m$$

Uncertainty in h_2 is given by the formula

$$\Delta h_2 = \Delta RE + \Delta SE$$

Where RE is random error and SE is systematic error.

$$\Delta h_2 \Delta RE = \frac{(Range)}{2}$$
$$\Delta h_2 \Delta SE = 0.001$$
$$\Delta h_2 = \frac{0.613-0.544}{2} + 0.001$$
$$\Delta h_2 = \pm 0.036m$$

Example 2: The calculation of v_2 for 35°C

The velocity after the bounce is given by the formula

$$v_2 = \sqrt{2 \times g \times h_2}$$
$$v_2 = \sqrt{2 \times 9.807 \times 0.583}$$
$$v_2 = 3.381ms^{-1}$$

The uncertainty in velocity after bounce is given by the formula.

$$\frac{\Delta v_2}{v_2} = \frac{1}{2} \times \frac{\Delta h_2}{h_2}$$
$$\Delta v_2 = \frac{1}{2} \times \frac{0.036}{0.583} \times 3.381$$
$$\Delta v_2 = \pm 0.103\ ms^{-1}$$

Example 3: Calculation of Kinetic Energy loss for 35°C and its uncertainty

$$KE\ loss = \frac{1}{2} \times m \times (v_1^2 - v_2^2)$$
$$KE\ loss = \frac{1}{2} \times 0.0238 \times (6.438^2 - 3.381^2)$$
$$KE\ loss = 0.358J$$

For uncertainty in Kinetic Energy Loss:

$$k = (v_1^2 - v_2^2)$$
$$k = (v_1 + v_2)(v_1 - v_2)$$
$$a = (v_1 + v_2)$$
$$b = (v_1 - v_2)$$
$$\Delta b = \Delta v_1 + \Delta v_2$$
$$\frac{\Delta k}{k} = \frac{\Delta v_1 + \Delta v_2}{v_1 + v_2} + \frac{\Delta v_1 + \Delta v_2}{v_1 - v_2}$$

$$\frac{\Delta KE\ loss}{KE\ loss} = \frac{\Delta m}{m} + \frac{\Delta v_1 + \Delta v_2}{v_1 + v_2} + \frac{\Delta v_1 + \Delta v_2}{v_1 - v_2}$$
$$\Delta KE\ loss = \left(\frac{0.001}{23.829} + \frac{0.001+0.103}{6.438+3.381} + \frac{0.001+0.103}{6.438-3.381}\right) \times 0.358$$
$$\Delta KE\ loss = \pm 0.016\ J$$

XII. Graphical Analysis

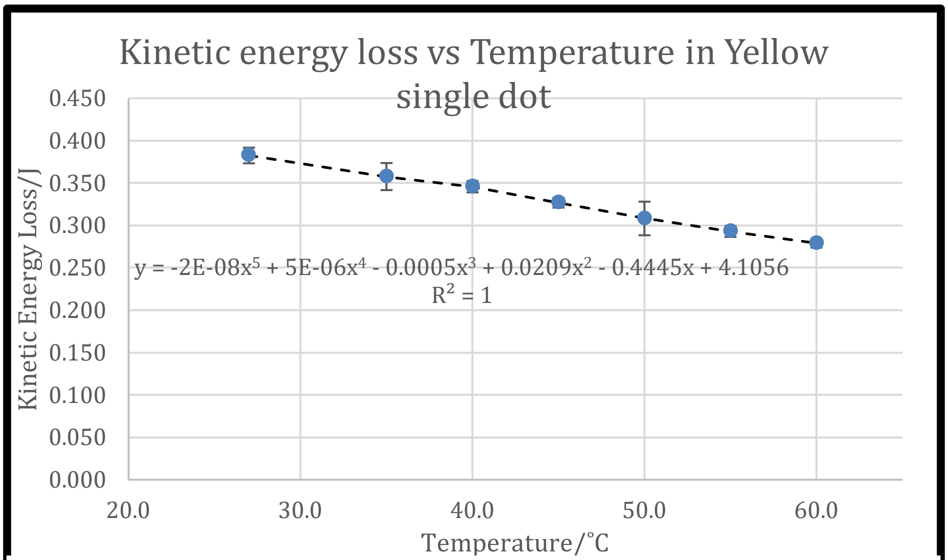

Figure 3: The graph of kinetic energy loss vs temperature

(Note Horizontal error bars for temperature are plotted for this graphs, but aren't visible due to their constant small value of ±0.1℃ for all the temperatures)

x = Temperature(℃). The graph of Temperature vs kinetic energy loss for the squash ball, used in the investigation, shows an exact match for the equation of the graph, as the software generated equations gave the R_2 value to be 1. As predicted by the hypothesis, the highest power of the polynomial relationship fell within the forecasted range with the specific value being n=5.

The equation of the yellow single dot squash ball is

$$f(x) = -2.000 \times 10^{-8}x^5 + 5.000 \times 10^{-6}x^4 - 0.001x^3 + 0.021x^2 - 0.445x + 4.106$$

Where f(x)= Kinetic energy loss(J). The graph passes through all the error bars along with the points. The investigation involved considering 7 points so that the graph can be drawn to an accurate measure. Moreover, the hypothesis of declining Kinetic energy loss with subsequent increase in temperature turned out to be evident in the negative slope of the graph. There is a decreasing trend in the kinetic energy loss as the temperature increased and same can be said vice versa. Therefore it can be inferred that there is an inverse relationship between the kinetic energy lost by the ball and the temperature of the ball in the temperature range from $27 \leq x \leq 60$ where x is the temperature in ℃.

The size of the horizontal error bars proves that uncertainties in temperature are constant and negligible. The size of the vertical error bars varied for different temperatures. Although the error bars are significant for 2 points (35℃ and 50℃), the overall error bars justify the quintic (n=5) relationship that has been derived through the graph.

XIII. Evaluation

Both the procedure and the mathematics model, managed to function appropriately and deliver values with acceptable errors. The experiment produced values for h_2 that were precise and by repeating the experiments, the effect of anomalies was minimized. The model and procedure combined, validated the prediction in the hypothesis about the proportional relationship between bounce height, h_2, and velocity v_2.

My observation about the ball undergoing decreasing loss in height at subsequent bounces at higher temperatures can be validated by this experiment. This occurrence is seen as the ball at higher temperatures, has a smaller kinetic energy loss. An explanation for this observation can be that at higher temperatures, the bonds between the molecules in the polymer may have expanded and may retain this expansion even after the ball cools down. This might explain why a ball at room temperature and a ball that has dropped to room temperature after it has been heated at a higher temperature might differ in their bounce heights where the latter might experience a higher bounce.

It can be inferred from Equation 3 that there may exist a minimum kinetic energy loss value in the context of varying the temperature as the value of $v_1 > v_2$, due to thermal and sound energy losses. To approach this minimum kinetic energy loss value, higher temperature ranges could be explored.

The optimum temperature range could be between 45-50℃, since this is the temperature range that can be achieved by the strokes of the player. A higher temperature range would result in lower kinetic energy loss, however, other mechanical means may be required to heat the ball beyond 50℃. The percentage uncertainty discrepancy can be evaluated for the following variables

Quantity	Maximum percentage uncertainty	Minimum percentage uncertainty
h_2	$\pm 6.483\%$	$\pm 1.578\%$
v_2	$\pm 3.242\%$	$\pm 0.789\%$
Kinetic energy loss	$\pm 4.465\%$	$\pm 1.788\%$

The range of percentage uncertainty for v_2 and h_2 and Kinetic energy loss was within an acceptable range. The experiment was acceptably accurate and precise. However there were some limitations that can be worked on to provide better results in terms of reduced systematic and random errors if care is taken.

Table 5: Random and systematic errors

Source of error	Significance and evidence	Improvements
Systematic errors affecting accuracy		
The limitation of the camera of clicking 100 pictures of the ball trajectory,	This has a low significance. The camera was able to capture the 100 shots in an average 3.388s, which allows each picture to show the instantaneous position of the ball at a rate of 0.034s per	Install applications that may take more number of pictures in lesser time. For example 200 shots in 4 seconds, or a video recording where frame by frame analysis can take place. However, there may be a limitation of technology to identify the maximum height instantaneously. A frame by frame video analysis could have been conducted instead, but for that, a 60 frames per second camera would be required,

while measuring h_2 in the experiment.	shot, which is reasonable enough to determine the maximum height as there will be momentary stalling that can be captured within the time/shot value of the camera.	because a 30 frames per second camera would give the same result as the camera used. However, due to the fact that recordings of about 21 trials would be needed to be taken, the memory capacity of the device would be insufficient. Costs would also increase as a result. Therefore, video method was not preferred.
Non-uniform heating of the ball as the ball was manually held within the heater, by tongs, to maintain its position.	This has low significance. During the heating period of the squash ball within the water bath, the ball was held at a position by judgement. This was done for 2 reasons: 1) Speed- It was done to allow the ball to be quickly removed and checked for the temperature change without having a significant loss in temperature as compared to other means of keeping the position constant 2)The squash ball is hollow inside and the air within it caused it to float in the bath **which is why manually holding the ball down is necessary**. The evidence for this error was that the position at which the tongs held the ball had a temperature difference of about ± 0.2 ℃ as compared to the rest of the ball.	Use a stand and thread, with strengthened support to keep the ball at a fixed position within the water bath, so that uniform heating takes place within the ball, as the thread is too thin to affect the uniform heating. The support is necessary to keep the less dense ball from floating to the surface.
Random errors affecting precision		
Sideways bounce of the ball post the drop in the experiment causing variation in h_2.	This has a low significance. During the segment of the experiment when h_2 is being found, the uneven release of the ball by the both sides of the tongs causes the ball to have a spin as it is falling. This spin leads to the ball having a variation in the bounce back height, h_2. The evidence of this is found in the photos, where there is no perfect vertical bounce back in any photo. Most photos show that the ball has had a net horizontal movement as a result of an imperfect drop.	Ensure tongs release the ball properly. Discard trials where ball travelled sideways significantly. Ensure both the tongs and ball are dry when the ball is being released.
Non-uniform heating of the ball	This had Medium Significance. Even though the position tended to be the same within the water bath, there was a slight variation, in the temperature of the ball, of ± 0.3 ℃. Evidence is that the upper segment of the ball tended to have a higher temperature as compared to the lower segment.	Use a larger water bath, with the ball kept at a fixed position, to ensure that the ball doesn't face different temperature throughout the bath.

Table 6: Weakness and improvements

Weakness in the experiment	Effect and reason
When measuring the temperature of the ball, after it was removed from the water bath, the ball had a gradual temperature fall. Because of this, the ball may not have been at the desired temperature when it was dropped.	The ball was intentionally heated at a slightly higher temperature (about 1-1.5℃ higher) so this fall could not affect the values obtained. This was observed when the infra-red thermometer was used to measure the exact temperature of the ball. The temperature of the ball fell in a predictable trend, which was identified to ensure that the ball was as close to the desired temperature as possible when the ball was being dropped. The reason for this is that the room temperature was much lower than the temperature of the ball most of the trials, so the ball, in order to reach an equilibrium temperature, lost heat as time went on.
The squash ball was freshly	Over time, as the ball gets worn out in a game, the relationship between

bought from the store and wasn't subject to the conditions of the game prior to the experiment.	temperature and kinetic energy loss might not hold with these equations as the ball's elasticity may be affected by the collisions with the walls during a game and the frequent expansion and contraction of the ball may cause the relationship to alter. This factor might cause the obtained relationship to modify over time and usage.

XIV. Further research suggestions

There are many different types of squash balls available for junior and senior players. The balls differ in terms of the height of the rebound bounce and the ability to retain the heat in the ball once warmed up. By using the same procedure for different balls, the ball with the lowest kinetic energy loss can be identified. Graphs of the different balls can be compared and the equation between kinetic energy loss and temperature may be analyzed for different balls.

XV. Conclusion

This experiment was done to establish the relationship between kinetic energy loss(J) and the temperature(℃) in a squash ball. Post collection and processing of data, the association can be determined by the equations discovered by graphical analysis. The findings fit flawlessly with the hypothesis as the negative correlation was correctly predicted. The hypothesis forecasted, "the relationship between temperature(℃) and kinetic energy loss(J) will be in the form of a **polynomial equation**: $ax^n + bx^{n-1} + cx^{n-2} .. + d$ where the highest degree will be equal to the constant "n" and n could be defined within the domain of 3≤n≤6, which was successfully met as given by the following equation

$$f(x) = -2.000 \times 10^{-8}x^5 + 5.000 \times 10^{-6}x^4 - 0.001x^3 + 0.021x^2 - 0.445x + 4.106$$

Therefore, the hypothesis correctly predicted the trend. The research question "**How does the loss in kinetic energy in a squash ball respond to change in temperature of the ball when dropped from a fixed height?"** has been successfully investigated and answered, and the established trend was identified in the graph, shown in *Figure 3*. The methodology followed refined my knowledge of energy conversions and I learnt new content related to thermal retention property of materials. Moreover, the data collection process improved my image processing skills and enabled me to learn new features related to image editing. My earlier view, that warming the ball beyond a certain threshold may have negative implications, has been revised by the results of this investigation, which showed that rising temperatures of the ball may end up reducing the need to warm the ball up repeatedly. Through scientific analysis, I have better understood the dynamics of the squash ball and I intend to incorporate my findings in my daily play to improve my game.

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