Homework 4

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Q1: Simulate the behavior of an enhanced Two-Channel Transparent Telerobotic Architecture ($C5 = -1,\ C_6 = 100,\ C4 = 0$) fo 60 seconds, considering the following parameters:

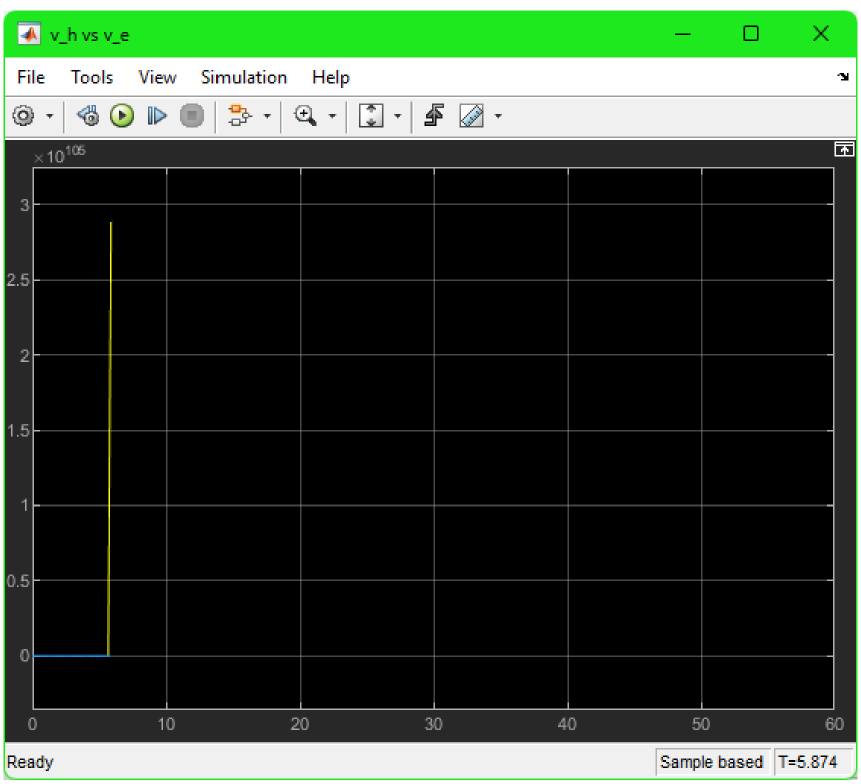
- **Two-way Communication Delay of 100ms**
- $Z_m = \frac{2s+13}{0.1s+1}$
- $Z_s = \frac{0.5s+3}{0.1s+1}$
- Z_e: a mass-damping model with a mass of 3 kg and damping of 3 N.s/m
- Z_h: a mass damping model with a mass of 0.1 kg and damping of 0.1 N.s/m
- $f_e^* = 0$
- $f_h^* = 20\sin(1t) + 20\cos(2t) + 20\sin(10t)$

Plot the Velocity of the leader robot versus the Velocity of the follower robot. Also, plot the interactive Force (F_h) at the leader robot felt by the user versus the Force at the environment side (F_e). Compare, evaluate, and discuss the transparency and performance of the system (Force tracking and Velocity tracking).

A) Mathematically explain if you believe the system should be stable or not.

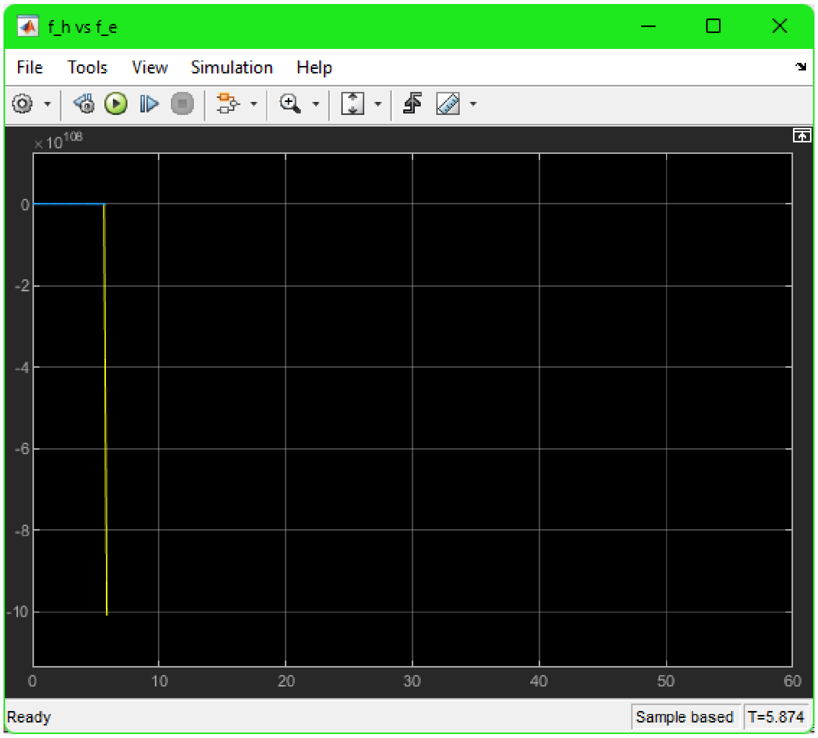
Velocity tracking

v_h vs v_e



Force tracking

f_h vs f_e



As seen in the graphs above, f_h is tracking f_e (same trajectory just shifted by a slight delay) as well as v_h is tracking v_e (same trajectory just shifted by a slight delay). This shows that the system has achieved transparency. This is expected to happen because we have configured the model to match the ideal scenario where you have kinematic correspondence ($v_h = v_e$) as well as ideal force response ($f_h = f_e$) with the exception of a delay in the curves. The system performs according to expectation because there is no echo on follower and leader side, which leads to acceptable force tracking as well as acceptable velocity tracking (ideal case for a scenario with a delay).

In order to prove stability, we can use Nyquist Stability Theory. The theory is given below

- Z_h *should not have RHP zeros (The admittance of human limb should be stable)*
- Z_e *should not have RHP zeros (The admittance of environment should be stable)*
- $|Zh| > |Ze|$ *This is called Small Gain Stability Condition of Interconnected systems*

This analysis is based on Nyquist Stability Theory

$$Z_h = 0.1s + 0.1$$
$$To\ find\ zeros\ of\ Z_h,$$

$$0 = 0.1s + 0.1$$
$$s = -1$$

Since the zeros of Z_h are not in the RHP, we can conclude that the first condition is satisfied.

$$Z_e = 3s + 3$$
$$To\ find\ zeros\ of\ Z_e,$$

$$0 = 3s + 3$$
$$s = -1$$

Since the zeros of Z_e are not in the RHP, we can conclude that the second condition is satisfied.

For the third condition,

$$Z_h = 2j\omega + 2\ where\ j = \sqrt{-1}$$
$$|Z_h| = \sqrt{(0.1\omega)^2 + 0.1^2}$$
$$|Z_h| = \sqrt{0.01\omega^2 + 0.01}$$

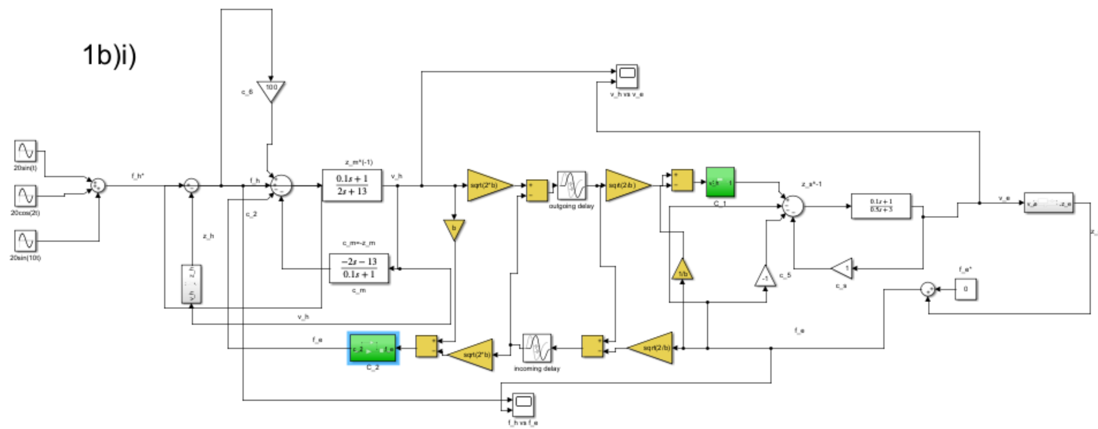
$$Z_e = 3j\omega + 3\ where\ j = \sqrt{-1}$$
$$|Z_e| = \sqrt{(3\omega)^2 + 3^2}$$
$$|Z_e| = \sqrt{9\omega^2 + 9}$$

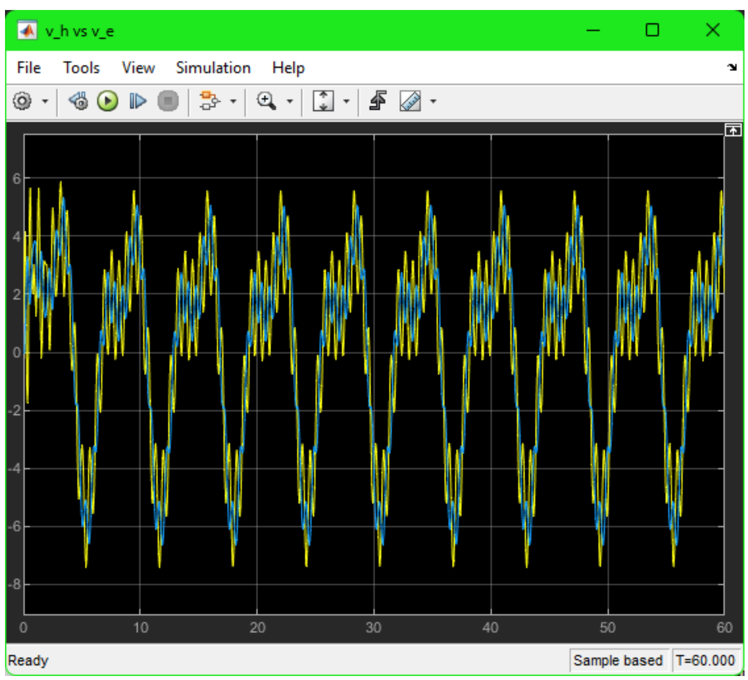
$$\sqrt{0.01\omega^2 + 0.01} < \sqrt{9\omega^2 + 9} = |Z_h| < |Z_e|$$

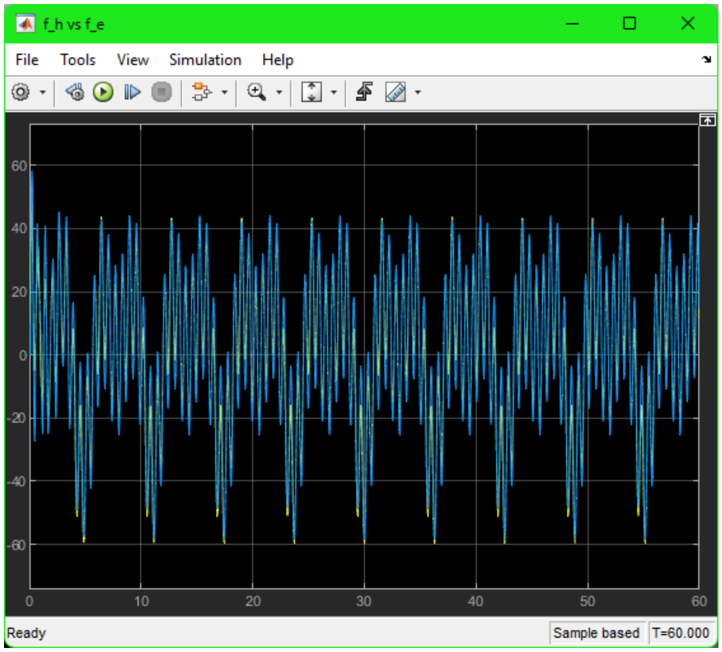
Therefore, the third condition has shown to have failed the satisfaction criteria. **The system is unstable.**

B) Apply Wave-Variable control approach. Is the system stable? Explain why. Choose a value of “b” of your choice that reduces the force tracking error (submit new Simulink file). Explain your choice. Plot the Velocity of the leader robot versus the Velocity of the follower robot. Also, plot the interactive Force (Fh) at the leader robot felt by the user versus the Force at the environment side (Fe). Now choose a value of "b" of your choice that reduces the velocity tracking error (submit new Simulink file). Explain your choice. Now, compare, evaluate, and discuss the transparency and performance of the system for both values of "b" and Explain your observations.

Apply Wave-Variable control approach. Is the system stable? Explain why.







Yes, the system is stable. As seen in the graphs above, both the force and velocity graphs show oscillatory behavior and do not diverge. Therefore, the system is passive and dissipates energy over a period of time. This makes the system stable.

Choose a value of "b" of your choice that reduces the force tracking error (submit new Simulink file). Explain your choice.

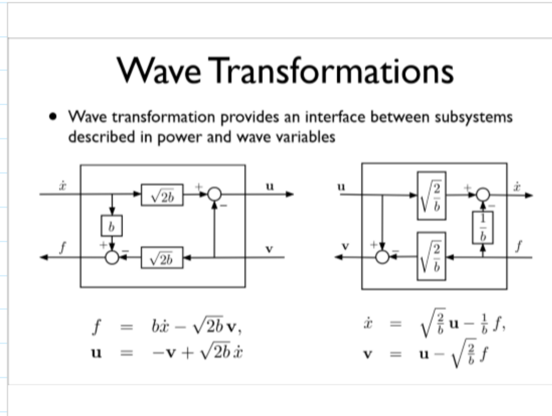
The value of b chosen is 0.01. By experimenting with different values of b some higher some lower, it was discovered that lower values of b reduced force tracking error and improved its transparency, however, increased velocity tracking error and worsened its transparency. An informal proof below shows how force is independent of b and makes the delayed force from the environment match the force from the environment.

$$f = b\dot{x} - \sqrt{2b}\left(u - \sqrt{\frac{2}{b}}\,f\right)$$

$$f = b\dot{x} - \sqrt{2b}\,u + 2f$$

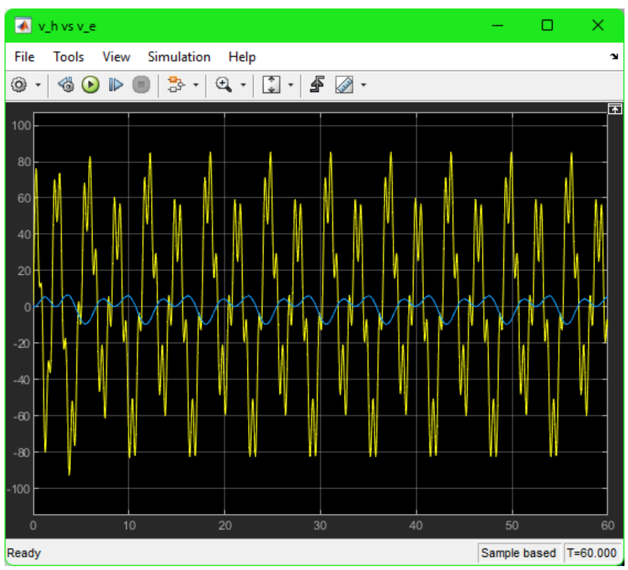
$$f \approx \lim_{b \to 0}\left(\underset{\downarrow\, 0}{b\dot{x}} - \underset{\downarrow\, 0}{\sqrt{2b}\,u} + 2f\right)$$

$$f \approx 2f$$



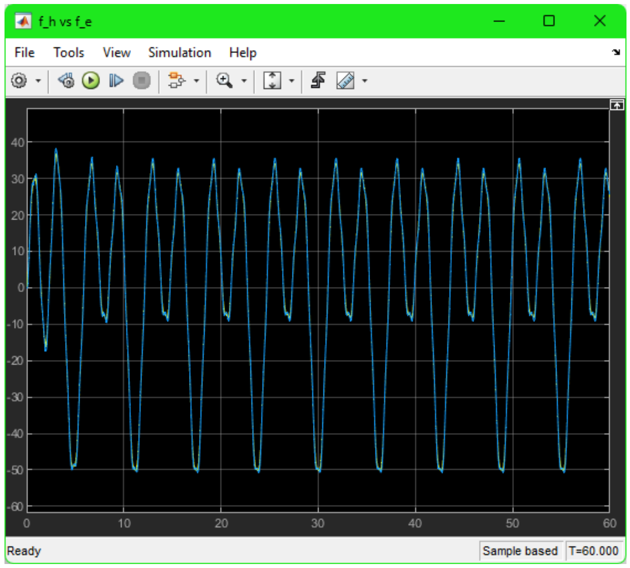
Velocity tracking

v_h vs v_e



Force tracking

f_h vs f_e



Now choose a value of "b" of your choice that reduces the velocity tracking error (submit new Simulink file). Explain your choice.

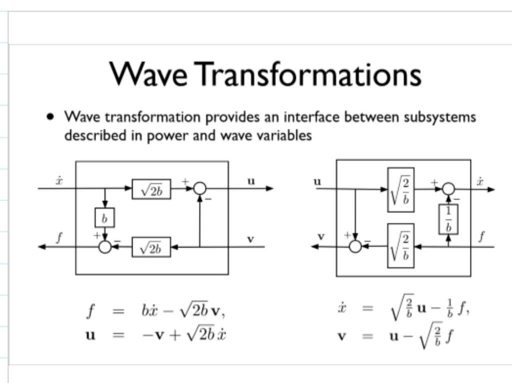
The value of b chosen is 10000. By experimenting with different values of b some higher some lower, it was discovered that higher values of b reduced velocity tracking error and improved its transparency, however, increased force tracking error and worsened its transparency. An informal proof below shows how velocity is independent of the inverse of b and makes the delayed force from the environment match the force from the environment.

$$\dot{x} = \sqrt{\frac{2}{b}}\left(-v+\sqrt{2b}\,\dot{x}\right) - \frac{1}{b}f$$

$$\dot{x} = -\sqrt{\frac{2}{b}}\,v + 2\dot{x} - \frac{1}{b}f$$

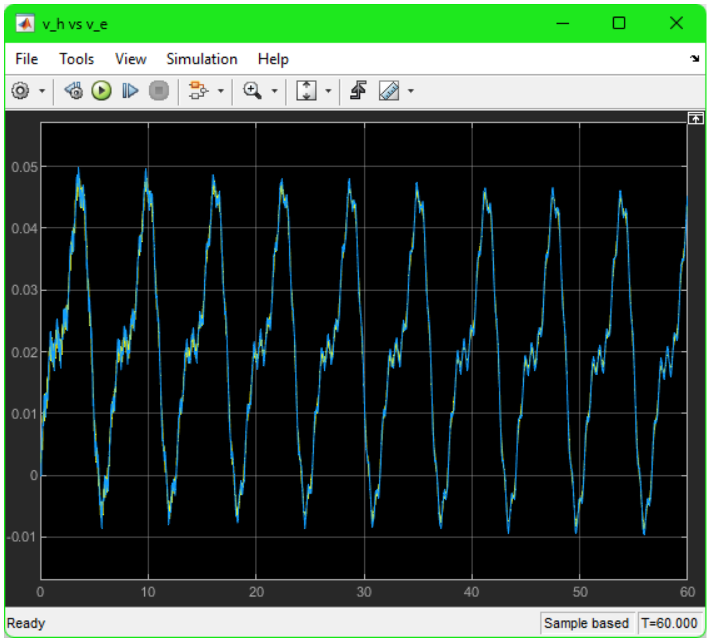
$$\dot{x} \approx \lim_{b\to\infty}\left(-\underset{\downarrow\,0}{\sqrt{\frac{2}{b}}} - \underset{\downarrow\,0}{\frac{1}{b}}f + 2\dot{x}\right)$$

$$\dot{x} \approx 2\dot{x}$$



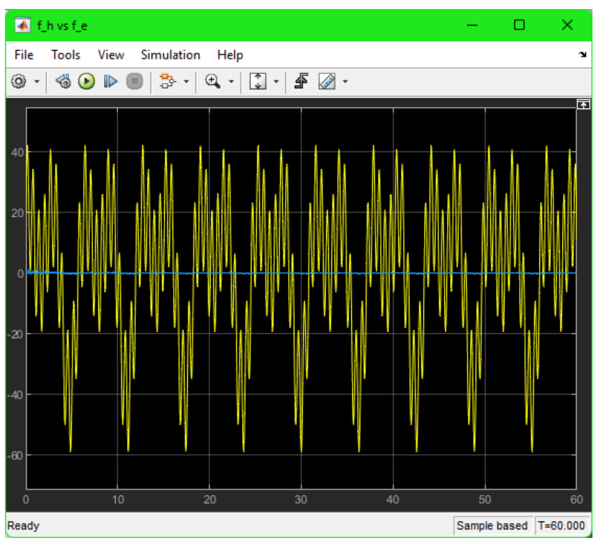
Velocity tracking

v_h vs v_e



Force tracking

f_h vs f_e



Now, compare, evaluate, and discuss the transparency and performance of the system for both values of "b" and Explain your observations.

For low values of b (b=0.01), it is seen in the above graphs that there is better force tracking, f_h is tracking f_e (same trajectory just shifted by a slight delay) but very poor velocity tracking v_h is not tracking v_e. This shows that the system has achieved ideal force response ($f_h = f_e$) with the exception of a delay in the curves. This is expected to happen because we have configured the model to have a small b which boosts the transparency of force but reduces the transparency of velocity.

For high values of b (b=10000), it is seen in the above graphs that there is better velocity tracking, v_h is tracking v_e (same trajectory just shifted by a slight delay) but very poor force tracking, f_h is not tracking f_e. This shows that the system has achieved kinematic correspondence (v_h = v_e) with the exception of a delay in the curve. This is expected to happen because we have configured the model to have a large b which boosts the transparency of velocity but reduces the transparency of force.

All things considered, we see that there is a tradeoff to be made between force tracking and velocity tracking by selecting a value for b.

C) If the delay was changing by time, can we still say that the system would remain stable? If yes, why? If no, Why? If any change would be needed to guarantee stability of a time-varying delay, please explain.

When communication delay is a function of time (varying), passivity can be lost even in wave variables domain. Stability for constant time delay does not imply stability for time-varying delay. The distortion of wave signals due to time-varying communication delay may introduce energy into the communication block, which makes it non-passive and, therefore, unstable. One possible way to dissipate the excess energy is by appropriate scaling of wave variables. This method, called Passivation by Scaling the Wave Variables, was devised by (Lozano et. al., 2002).

Passivation by Scaling the Wave Variables involves distinguishing between outgoing delay by multiplying it by a gain, g_1, and incoming delay by multiplying it by a gain, g_2.

There's an assumption to consider in this scenario:

1. The value of rate of change of delay

$$\dot{T}_i^2 < 1 = \frac{dT_i}{dt} < 1;\ i = 1, 2$$

We restrict the value of square of incoming gain and square of outgoing gain

$$g_i^2 \leq 1 - \dot{T}_i^2;\ i = 1, 2$$