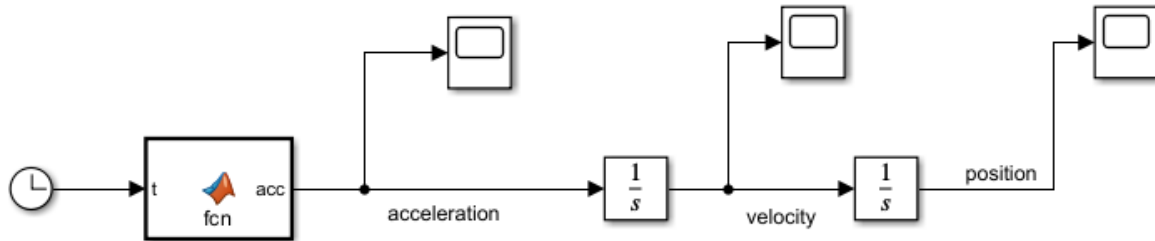
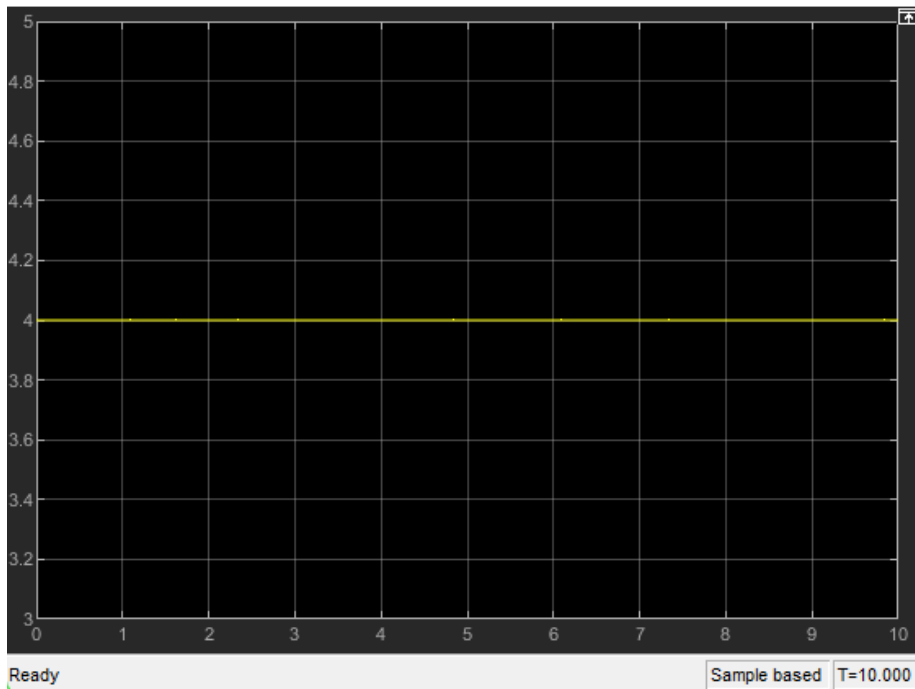


**Exercise 1:**

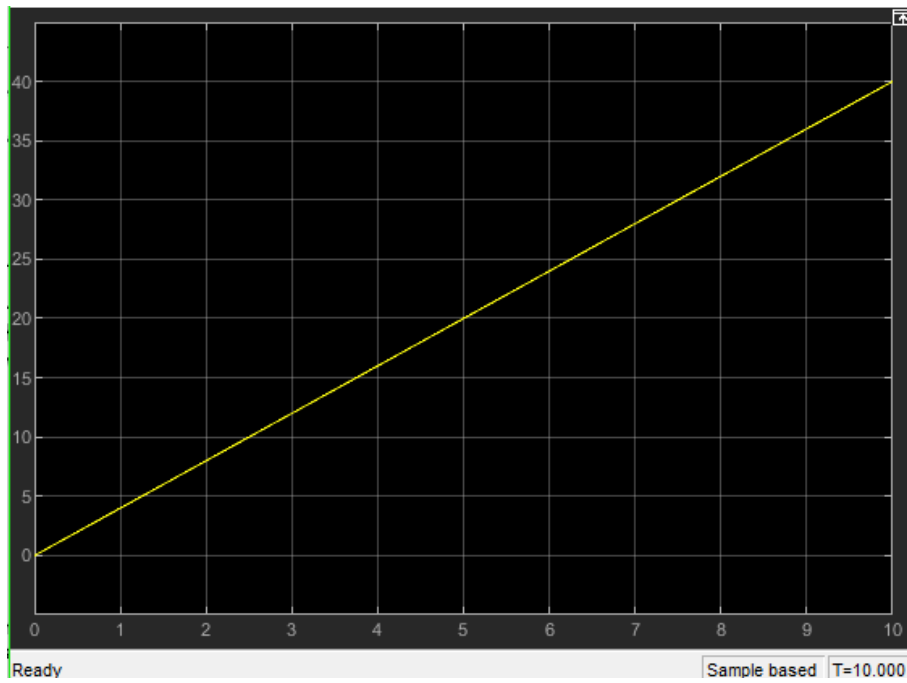
Exercises 1: Simulate the motion of a mass of 5kg when an external force of 20N is applied to it. Plot the position, velocity, and acceleration. (Hint: Use  $F=ma$ . You can also refer to Example 4 on ODE for Hint)



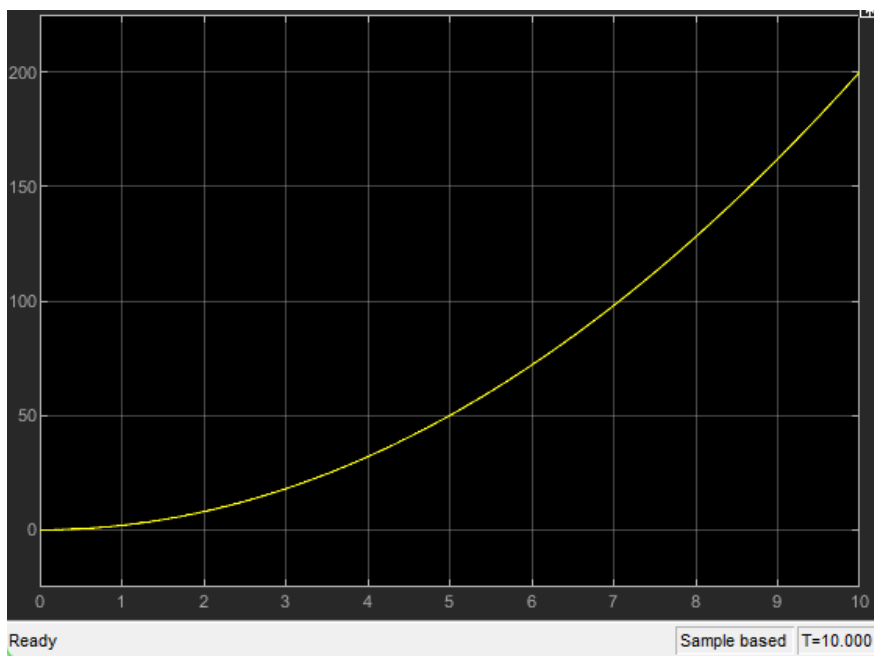
Acceleration vs t(s)



Velocity vs t(s)

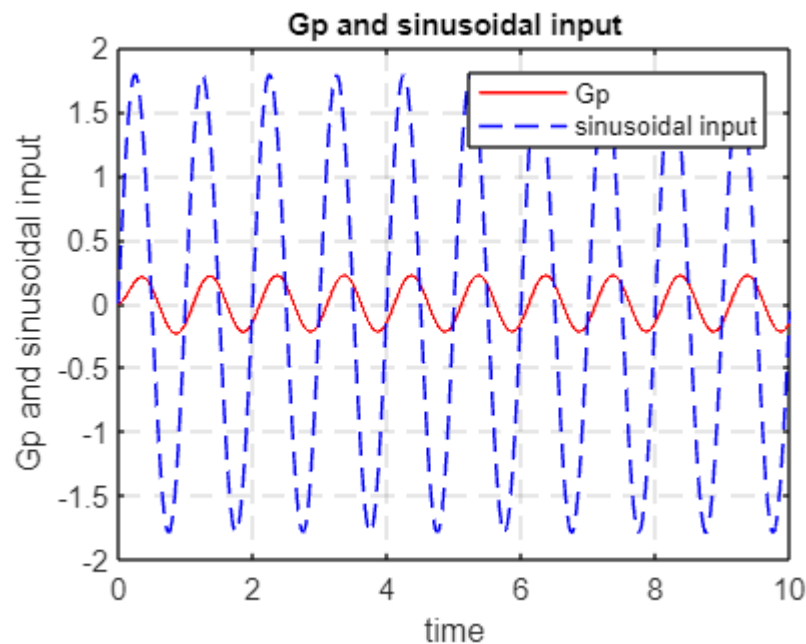
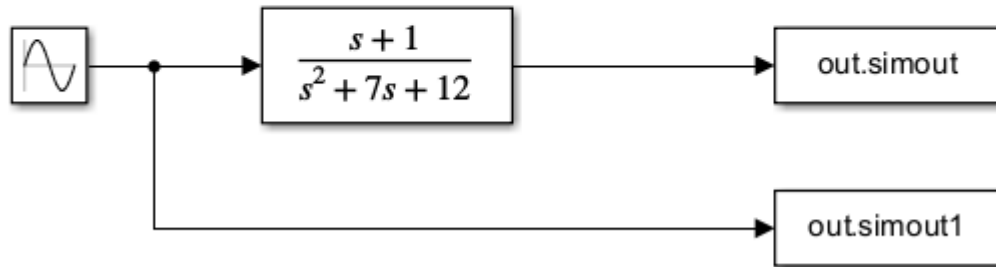


Position vs t(s)



**Exercise 2:**

Exercises 2: Consider a transfer function,  $G_p = \frac{s+1}{s^2+7s+12}$  with a sinusoidal input of amplitude 1.79 and frequency of 1 Hz. Output the sinusoidal input and system output to the MATLAB workspace using the 'To Workspace' block. Plot the input and output signal using MATLAB script on the same plot. Differentiate the two signals by changing their color or format. Show legend.

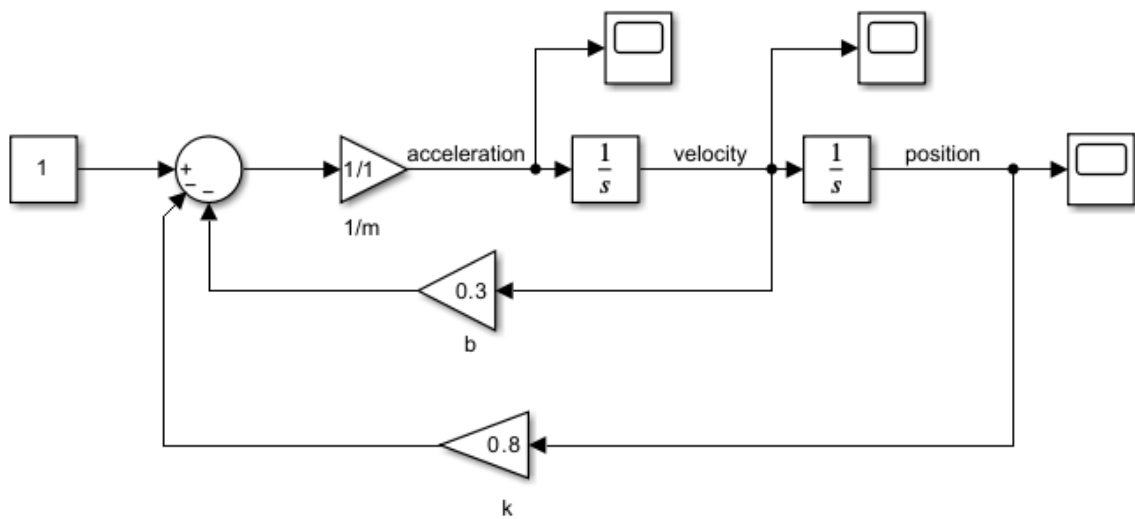


**Exercise 3:**

a)

Exercise 3:

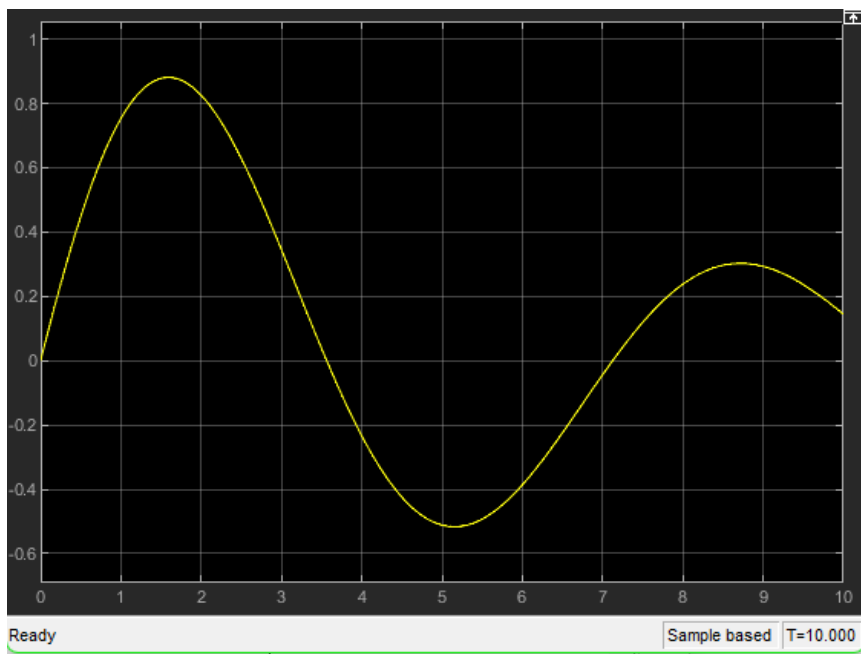
a) Simulate the system,  $F = m\ddot{x} + b\dot{x} + kx$  (called the mass-spring-damper model) with mass ( $m$ ) = 1kg, damping coefficient ( $b$ )= 0.3 Ns/m , and spring constant ( $k$ ) = 0.8 N/m. Consider an input force of 1N. Plot the position ( $x$ ), velocity( $\dot{x}$ ) and acceleration ( $\ddot{x}$ ).



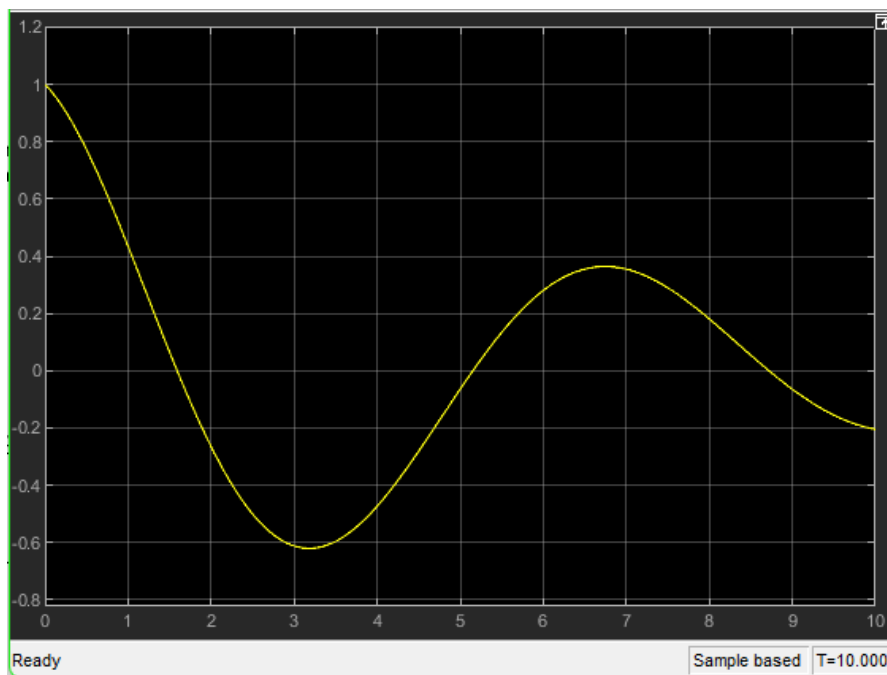
Position vs time



Velocity vs time

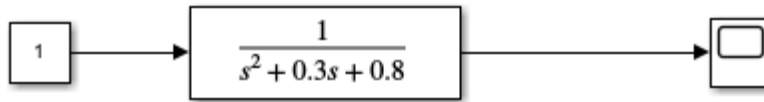


Acceleration vs time

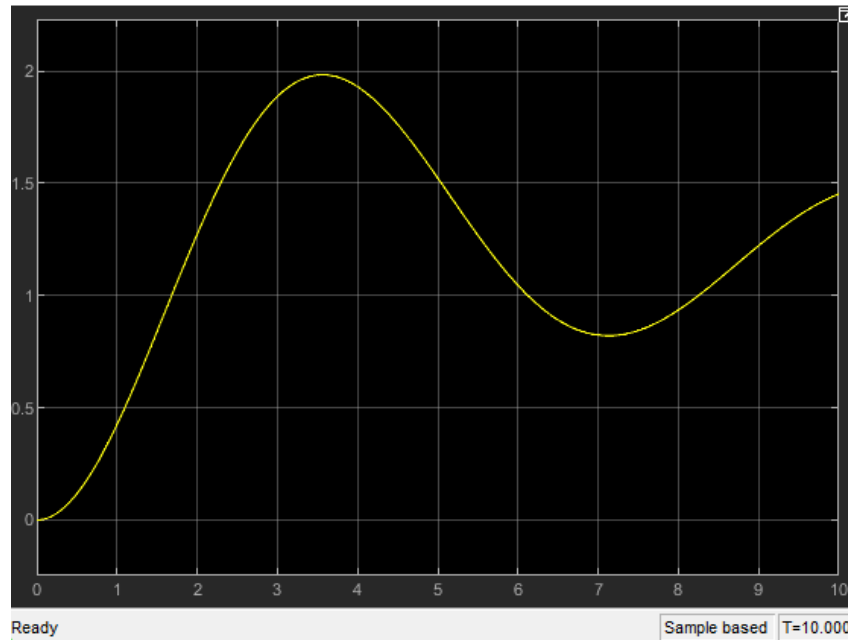


b)

b) Derive the transfer function of the system described in 'a)' with force (F) as the input and position (x) as the output. Simulate the system using the transfer function block in Simulink. Plot the output (position).

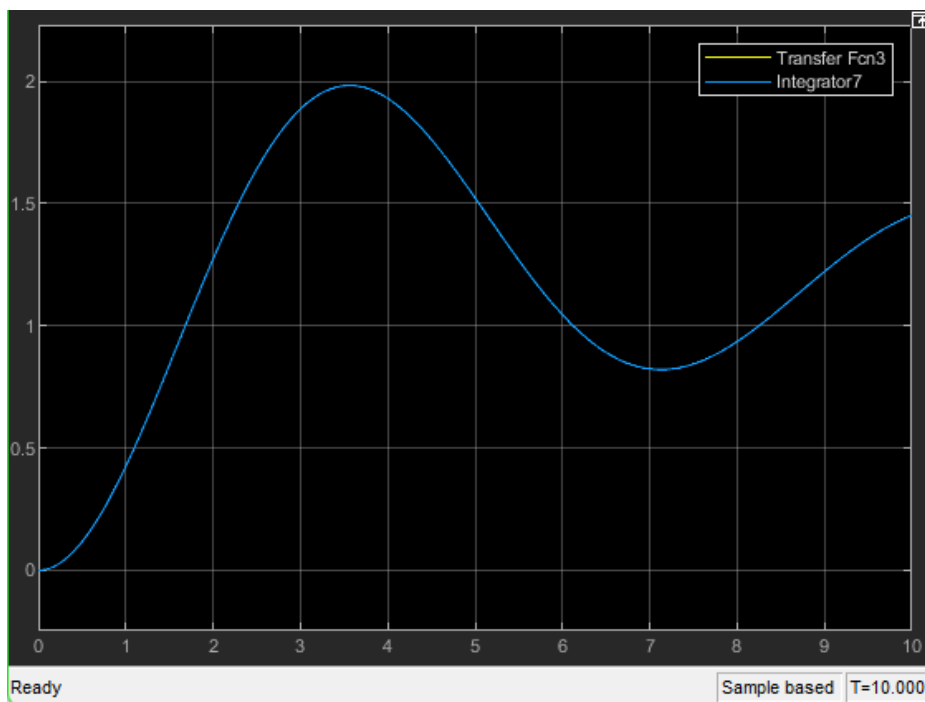
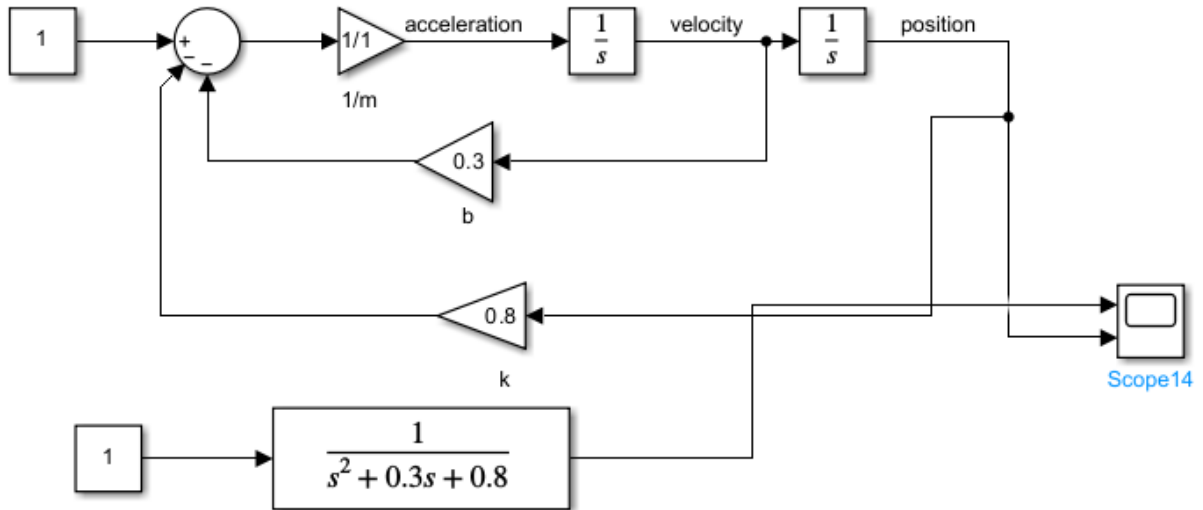


Position vs time



c)

c) Compare the position output from 'a)' and 'b)' by plotting them in the same figure. What do you observe? Do outputs of both the models give the same results?



The position outputs from both “a” and “b” overlap as seen in the graph above. The transfer function curve completely overlaps the mass-spring damp model curve. Therefore, it can be inferred that outputs of both the models do give the same results.

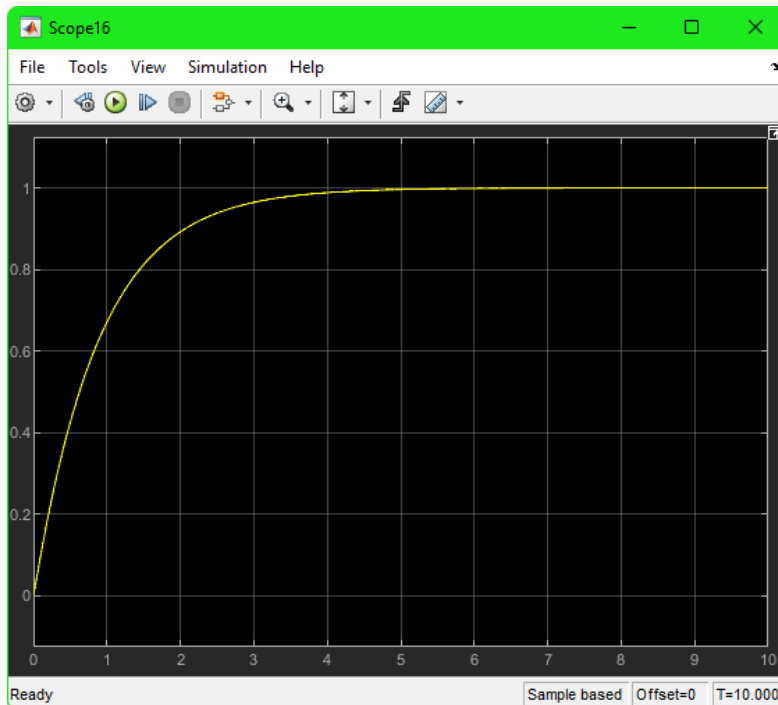
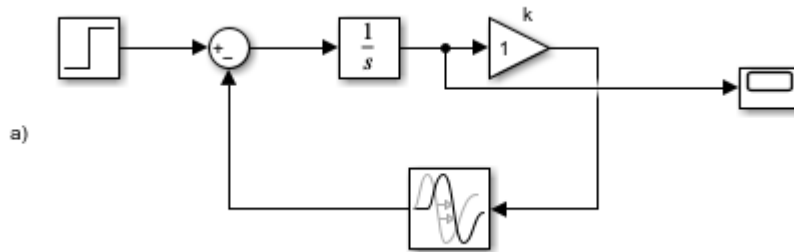
#### Exercise 4:

##### Exercise 4:

For a system given by  $y = \ddot{x}(t) + kx(t - T)$ , consider a delay of 100ms ( $'kx(t - T)'$  refers to a delayed signal  $'x'$ ) and a step input  $'y'$  applied at 0s. Simulate the system response  $'x'$  for the following values of  $k$ :

- a)  $k = 1$
- b)  $k = 10$
- c)  $k = 15.74$
- d)  $k = 20$

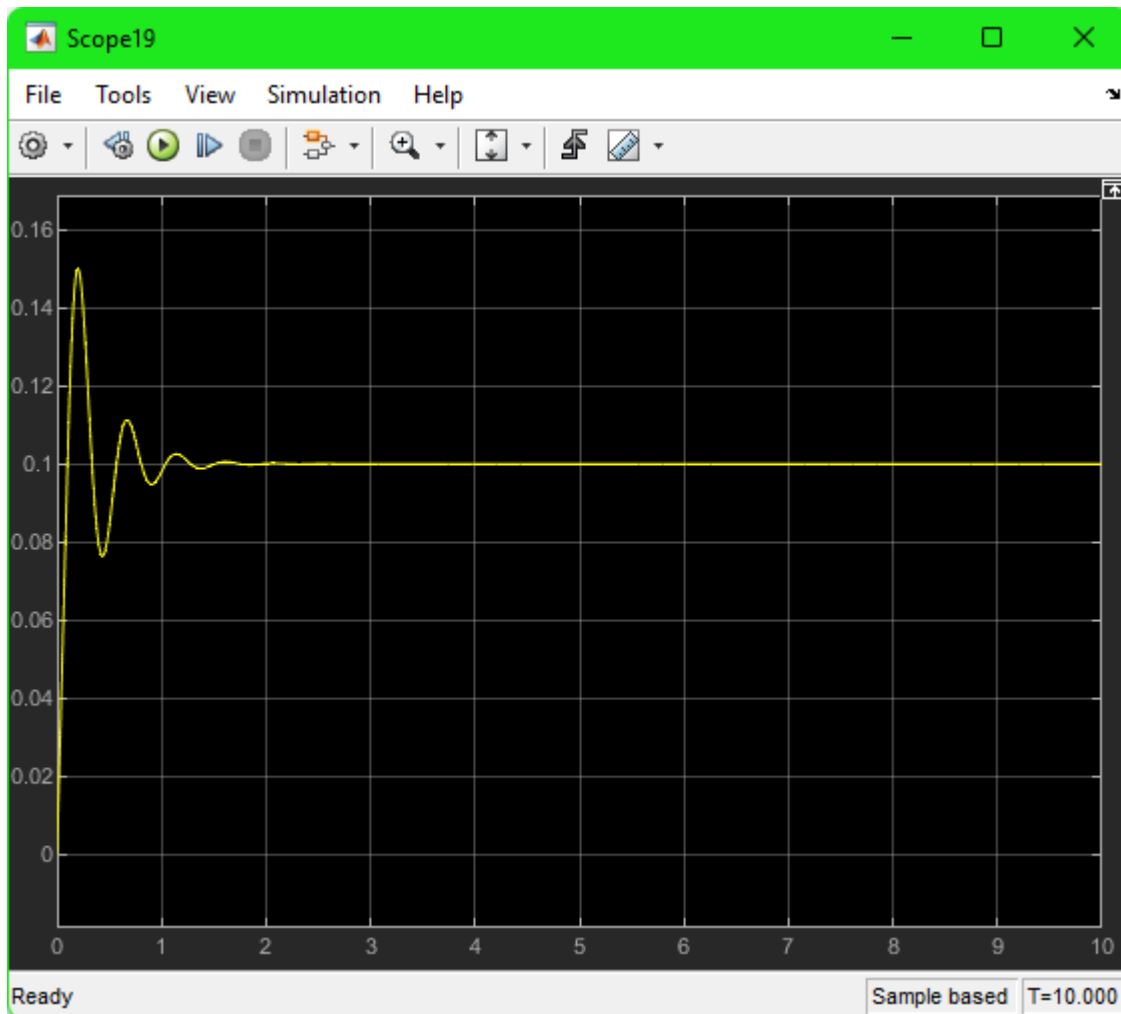
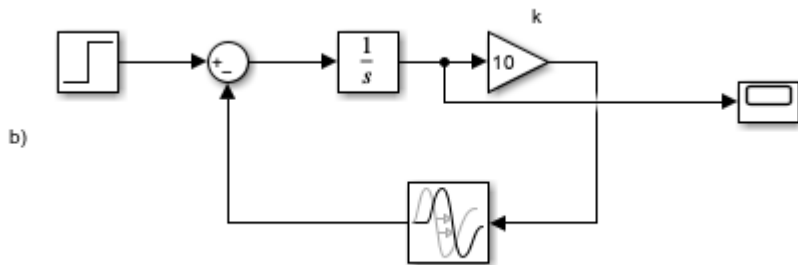
Plot the output for each of the cases and explain your observation. Is the output converging to a value (then the system is considered to be stable) or diverging (the system is unstable), or oscillating with constant amplitude (the system is asymptotically stable)? Is there an overshoot(oscillates and then converges)?





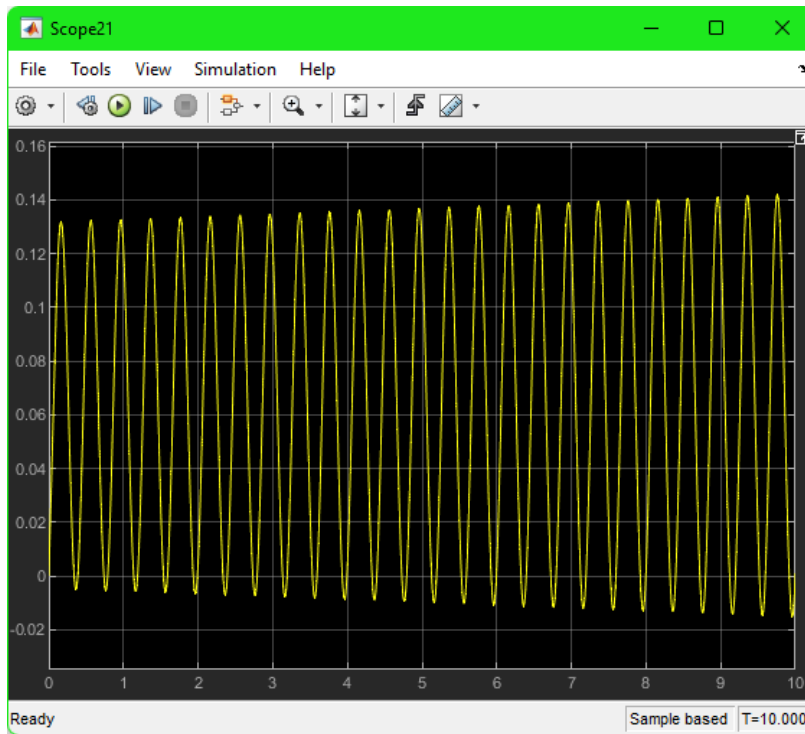
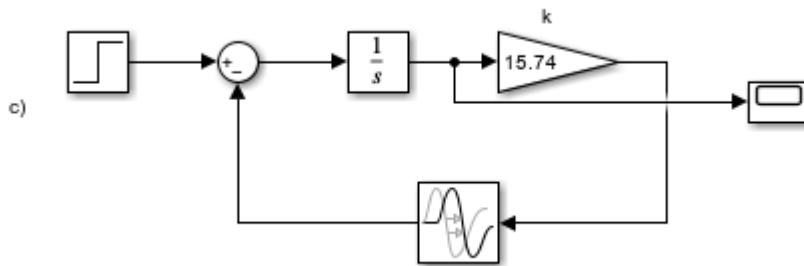
This plot approaches the value of 1 and therefore is converging and is stable.

b)



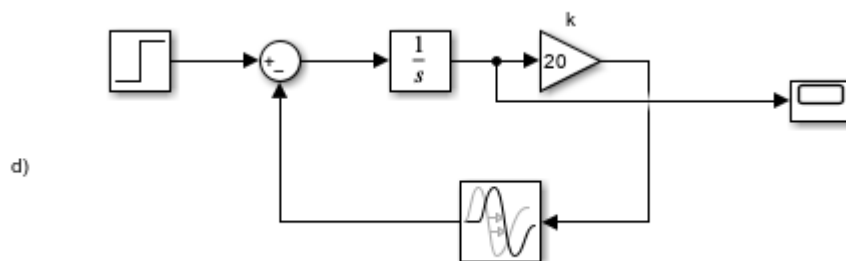
This plot overshoots 0.1 at first and then oscillates around the value 0.1 converging to it.

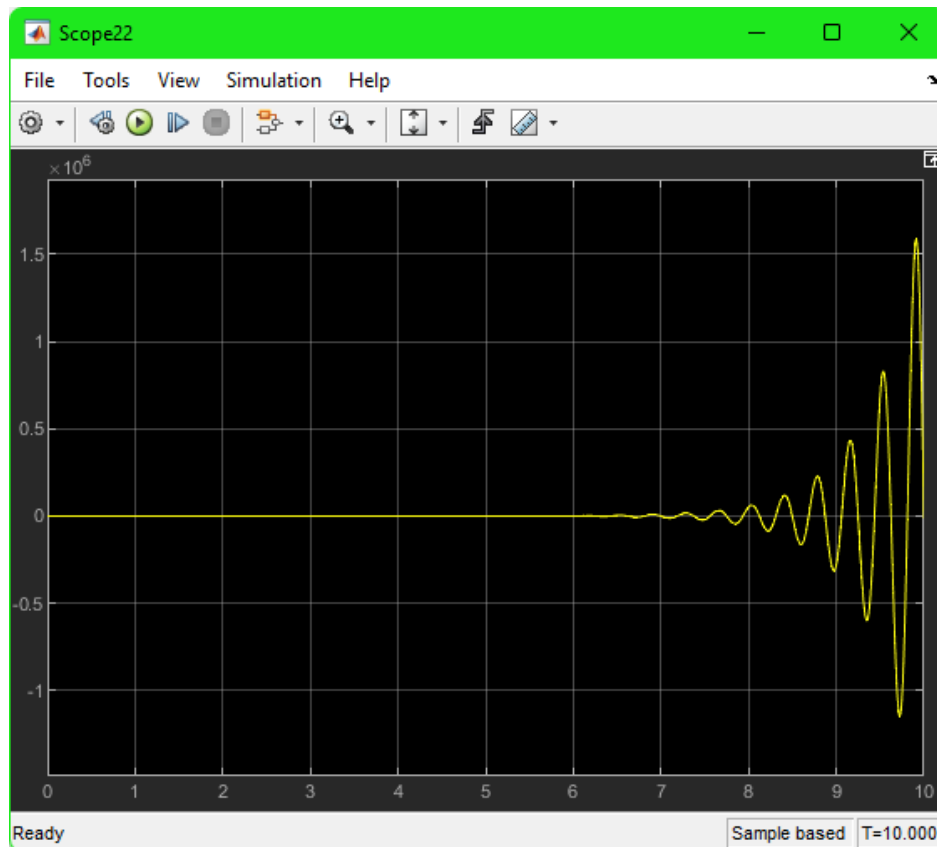
c)



This plot oscillates with an increasing amplitude and is divergent. It is also unstable.

d)





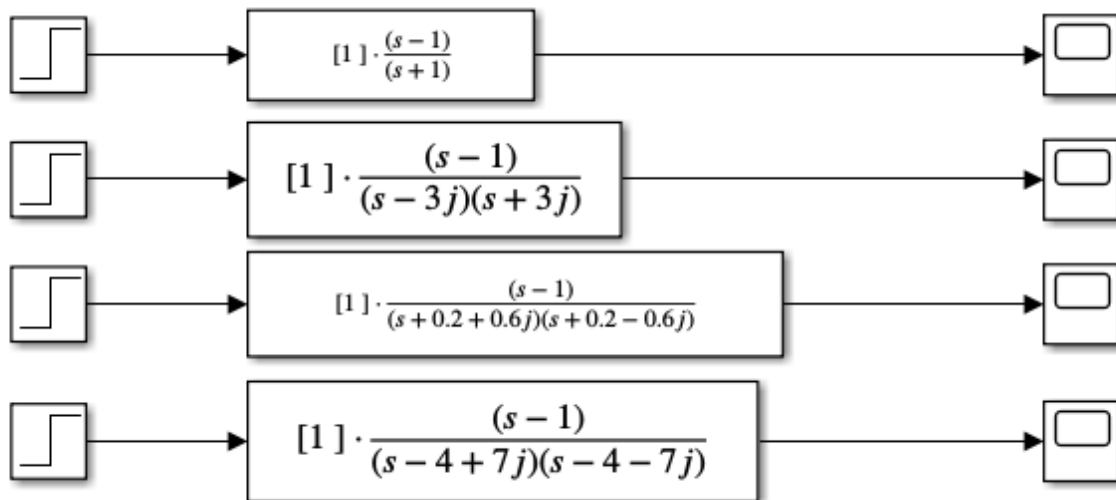
This plot oscillates with an increasing amplitude and is divergent. It is also unstable.

### Exercise 5:

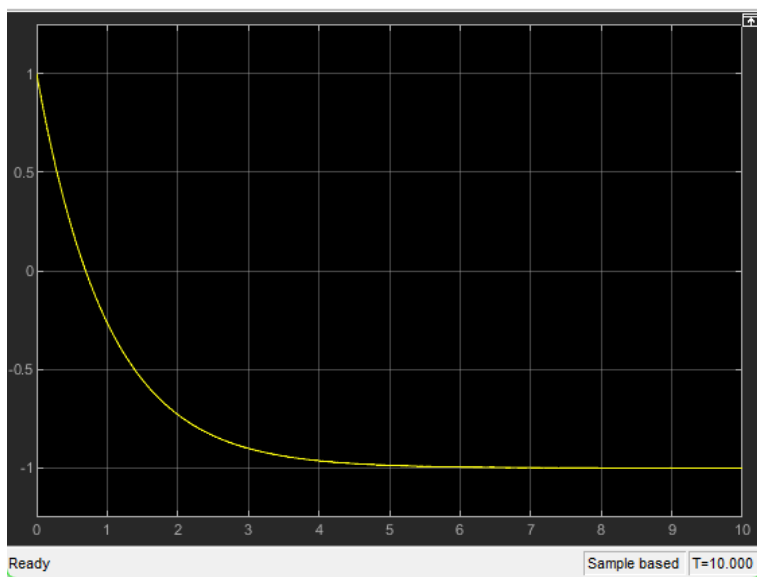
## Exercise 5:

Consider a step input with step time of 0. Use the Zero-Pole block to simulate the system for the following values of poles and explain your observations (also plot them). In all cases, there is one zero ( $z = 1$ )

- a)  $p_i = -1$
- b)  $p_i = \pm 3j$  (i.e.  $+3j$  and  $-3j$ )
- c)  $p_i = -0.2 \pm 0.6j$
- d)  $p_i = 4 \pm 7j$



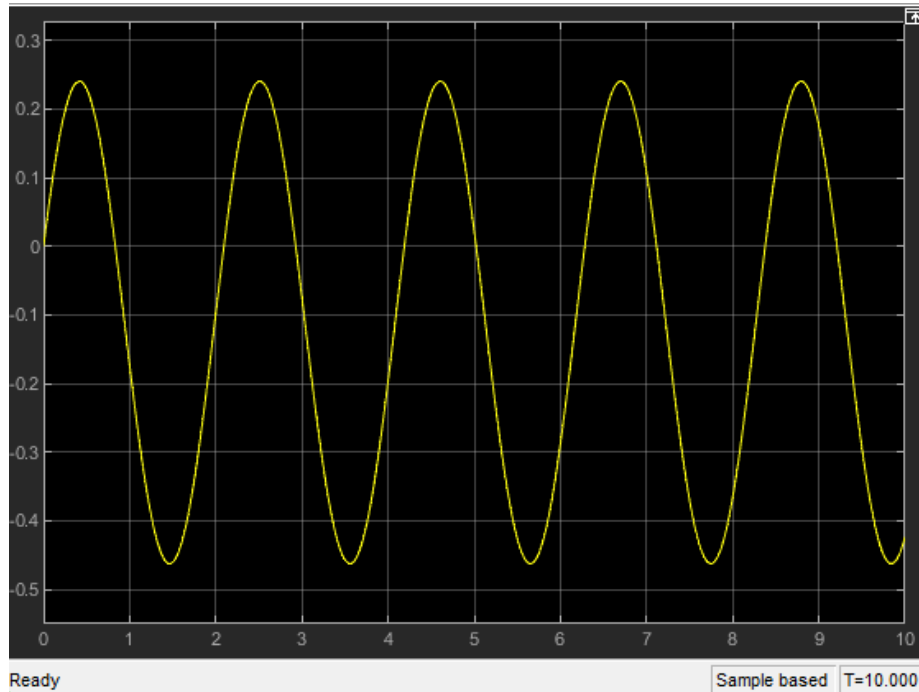
a)



The pole of this curve is  $s=-1$ . When solved in the denominator, this graph has “s” as a negative real value which means that they are on the left half-plane which results in an exponentially

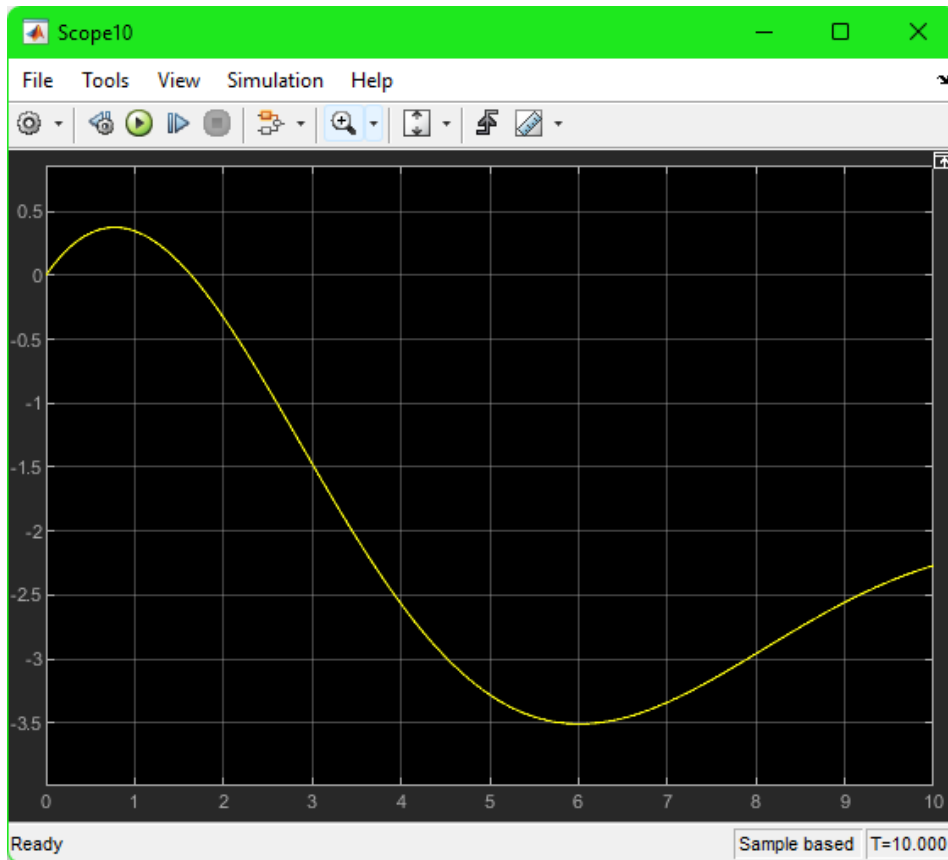
decaying component in the homogenous response. This system is stable as it converges. The zero of this graph occurs at  $s=1$ .

b)



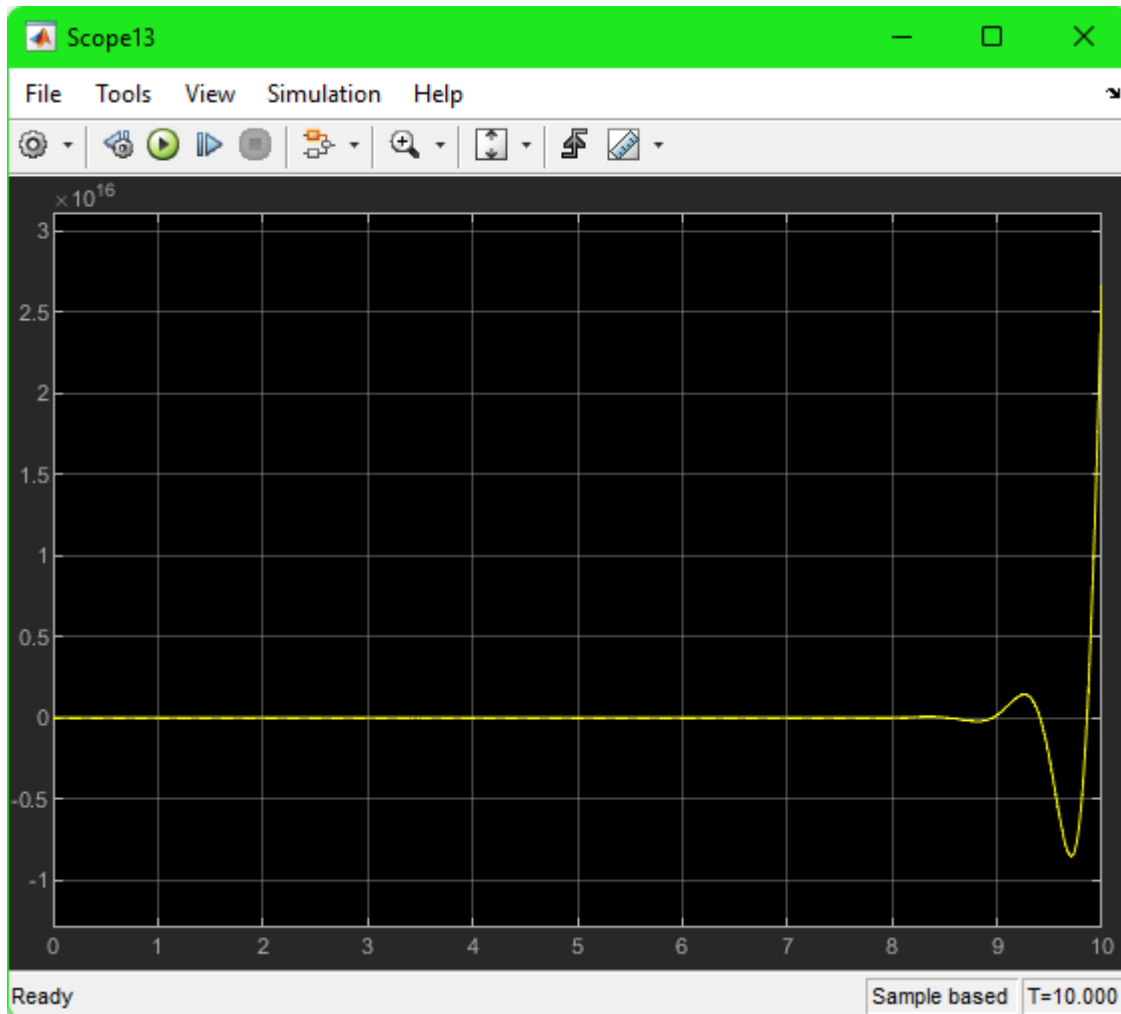
The poles of this curve is  $s=3j$  and  $s=-3j$ . When solved in the denominator, this graph has “s” as a pole pair on the imaginary axis ( $\pm j\omega$ ) which means that they generate a response that is oscillatory with a constant amplitude determined by initial conditions. The zero of this graph occurs at  $s=1$ .

c)



The poles of this curve is  $s=-0.2+0.6j$  and  $s=-0.2-0.6j$ . When solved in the denominator, this graph has “s” as a complex conjugate pole pair ( $-\sigma \pm j\omega$ ) which means that they are on the left hand plane and generate a response with a decaying sinusoidal component. The zero of this graph occurs at  $s=1$ .

d)



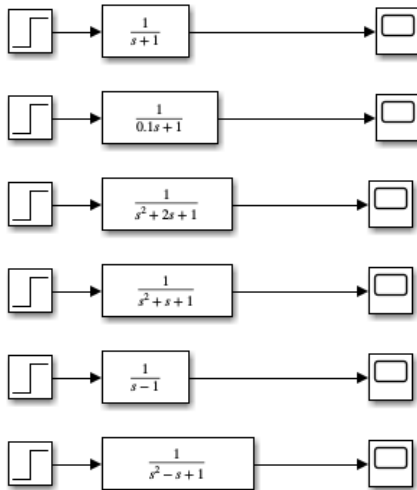
The poles of this curve is  $s=4 + 7j$  and  $s=4 - 7j$ . When solved in the denominator, this graph has “s” as a complex conjugate pole pair ( $\sigma \pm j\omega$ ) which means that they are on the right hand plane and generate a response with an increasing sinusoidal component. The zero of this graph occurs at  $s=1$ .

### Exercise 6:

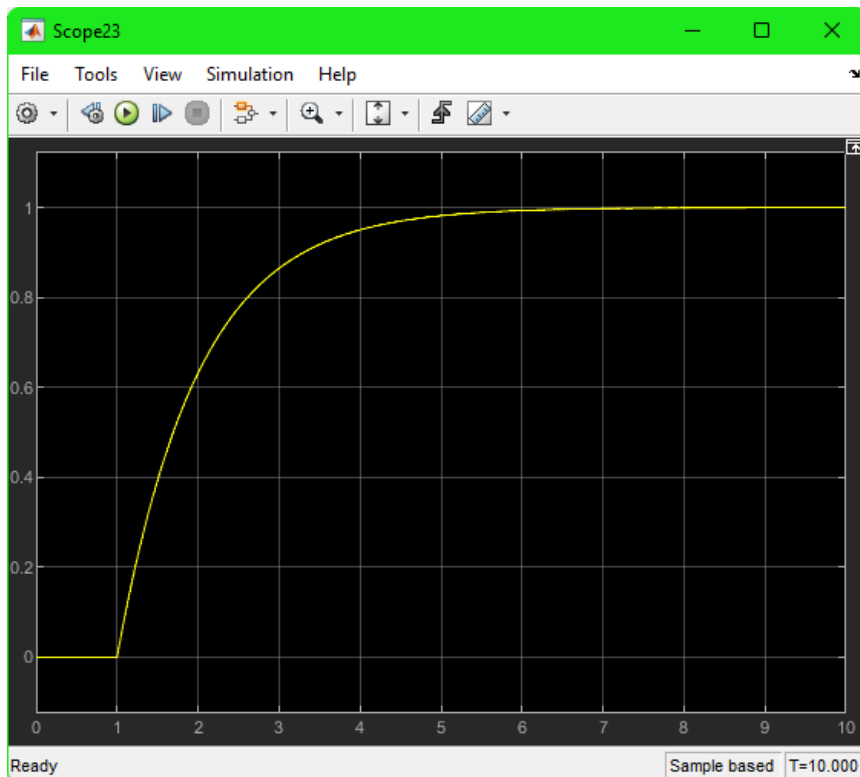
## Exercise 6:

Plot the output. For all transfer function, separately explain (for the following items, all plots be provided):

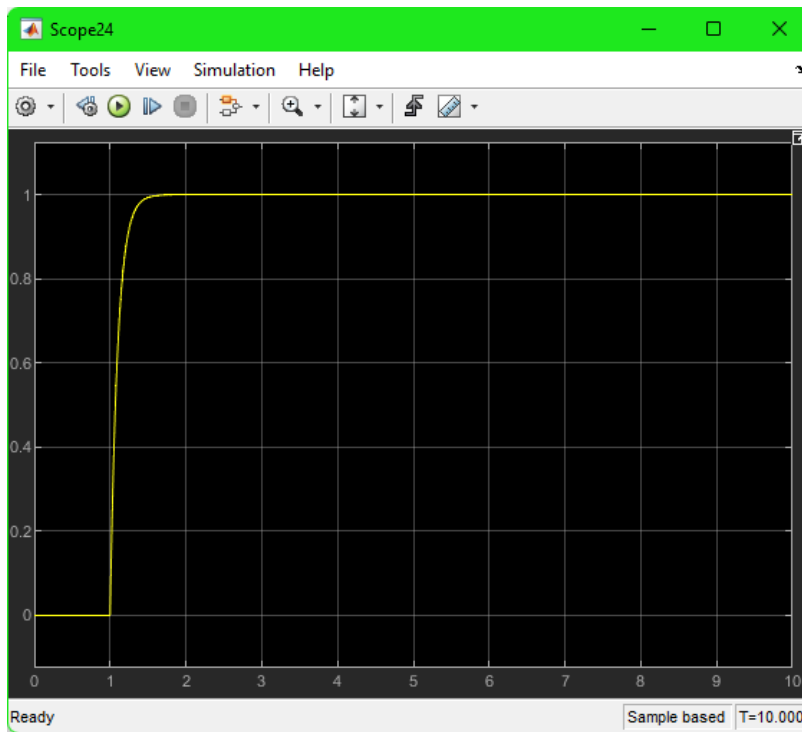
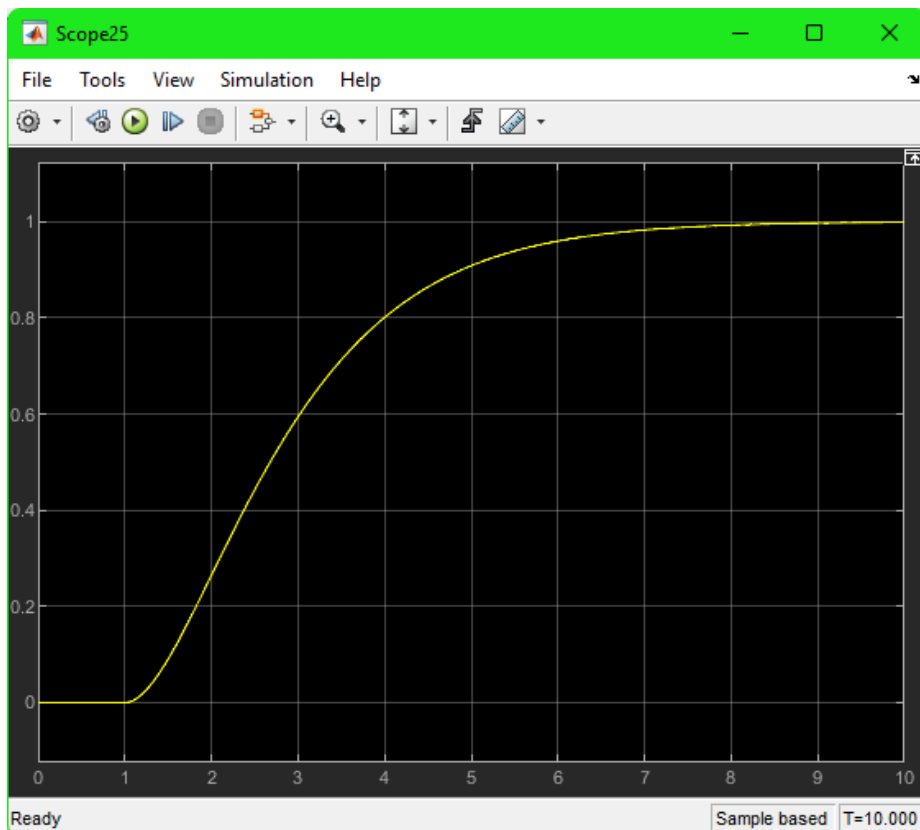
- 1) If they converge and if yes, why they converge
- 2) If they diverge and if yes, why they diverge
- 3) If they oscillate and if yes, why they oscillate
- 4) Also, compare G1 and G2 and tell which one is faster?
- 5) Compare G3 and G4 and tell what do you observe as the difference.
- 6) Compare G1 and G2 and explain which one is faster and why.
- 7) Change the input and consider two new inputs separately: a)  $\sin(0.1 t)$  and b)  $\sin(20t)$ . Simulate G1 for these two inputs for 100 seconds. Compare the input and output. Which one of the inputs has a larger drop in magnitude? Can you explain why G1 (and similarly G2, G3, G4) are named low pass filters? Why G3 is a better low pass filter than G4?

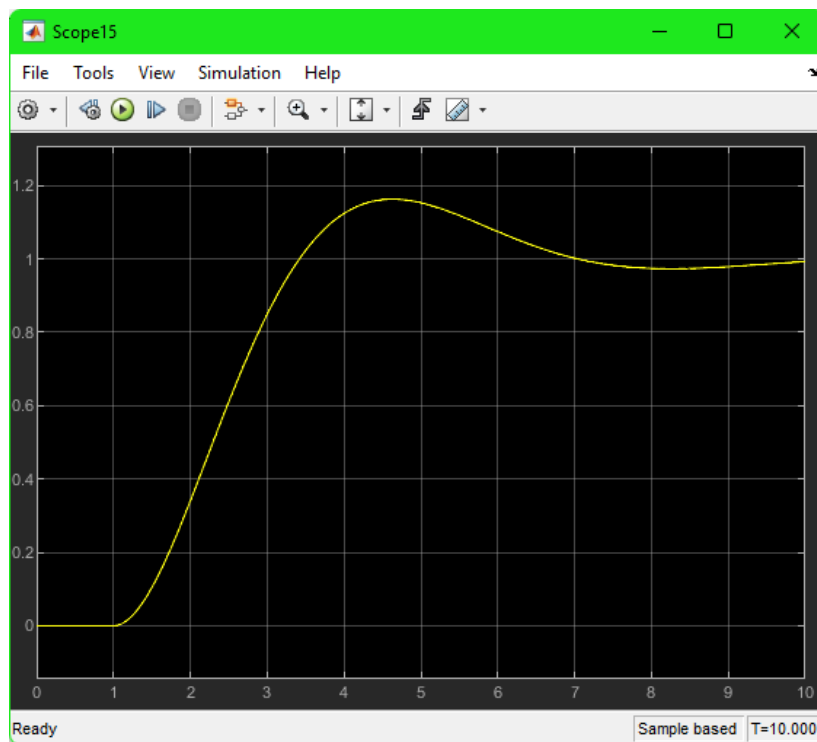
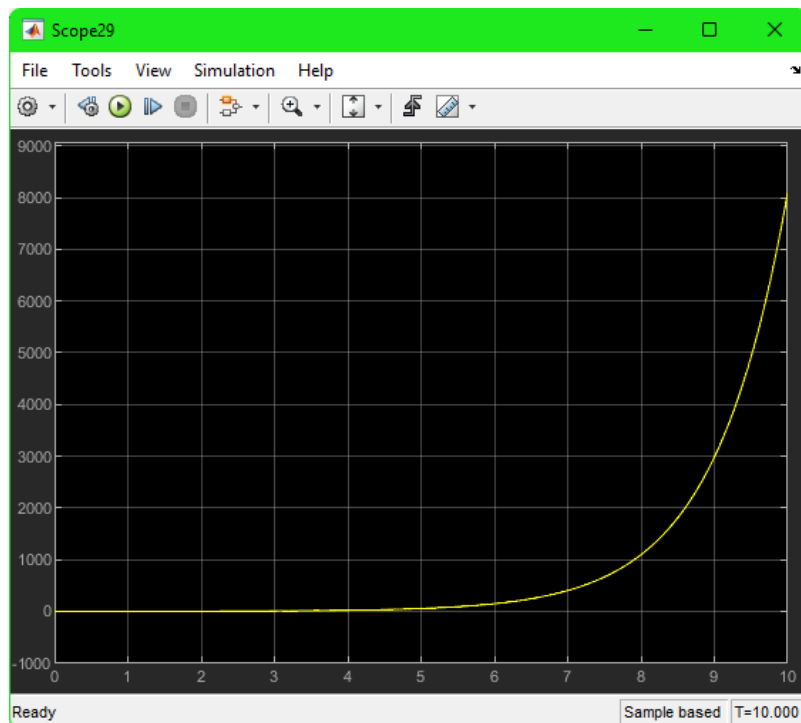


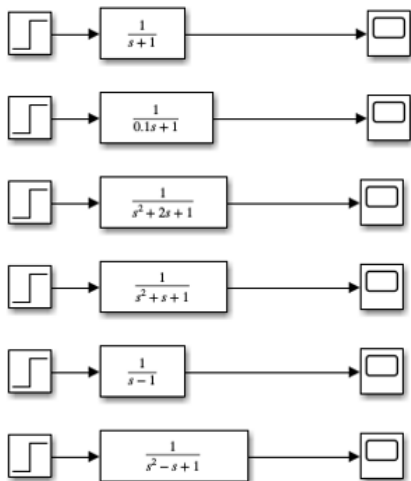
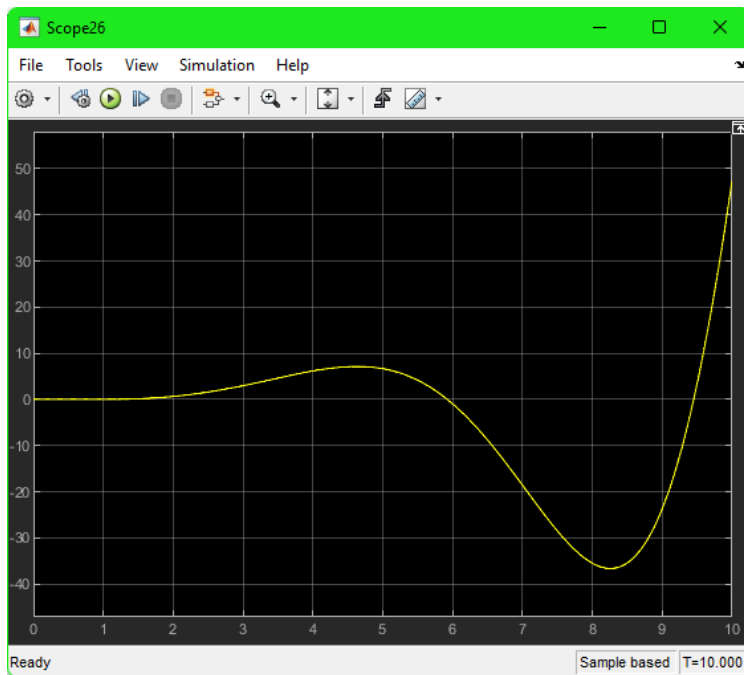
## G1





**G2****G3**

**G4****G5**

**G6****1**

1) If they converge and if yes, why they converge

G1, G2, and G3 converge. Each of these graphs when solved in the denominator, have “s” as a negative real value which means that they are on the left half-plane which results in an exponentially decaying component in the homogenous response.

2) If they diverge and if yes, why they diverge

G5 diverges. G5 when solved in the denominator, has “s” as a positive real value which means that it is on the right half-plane which results in an exponentially increasing component in the homogenous response.

3) If they oscillate and if yes, why they oscillate

G4 and G6 oscillate. Each of these graphs when solved in the denominator, have “s” as a complex conjugate pole pair value which means that they have a sinusoid oscillatory response.

4) Also, compare G1 and G2 and tell which one is faster?

G2 is faster.

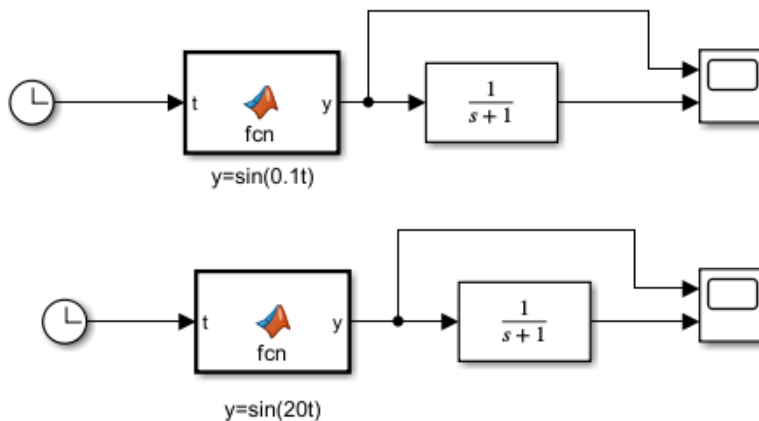
5) Compare G3 and G4 and tell what do you observe as the difference.

G3 directly approaches 1. G4 first overshoots 1 and then oscillates about 1. Solving the denominators of both the transfer functions, we get that s has a negative real solution for G3 which means it converges and s has a negative complex solution for G2, which means it has a decaying sinusoid component (it oscillates).

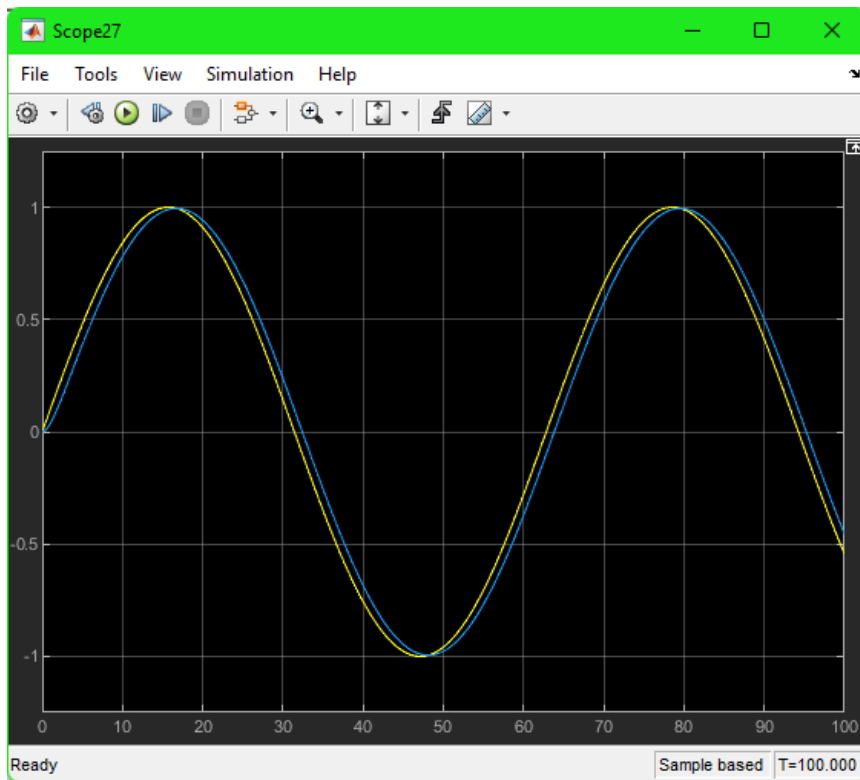
6) Compare G1 and G2 and explain which one is faster and why.

G2 is faster. Solving the denominators of both the transfer functions, we get  $s=-1$  for G1 and  $s=-10$  for G2. This means that G2 converges at a faster rate than G1 and is shown by the steeper slope in the G2 graph.

7) Change the input and consider two new inputs separately. Simulate G1 for these two inputs for 100 seconds. Compare the input and output. Which one of the inputs has a larger drop in magnitude? Can you explain why G1 (and similarly G2, G3, G4) are named low pass filters? Why G3 is a better low pass filter than G4?

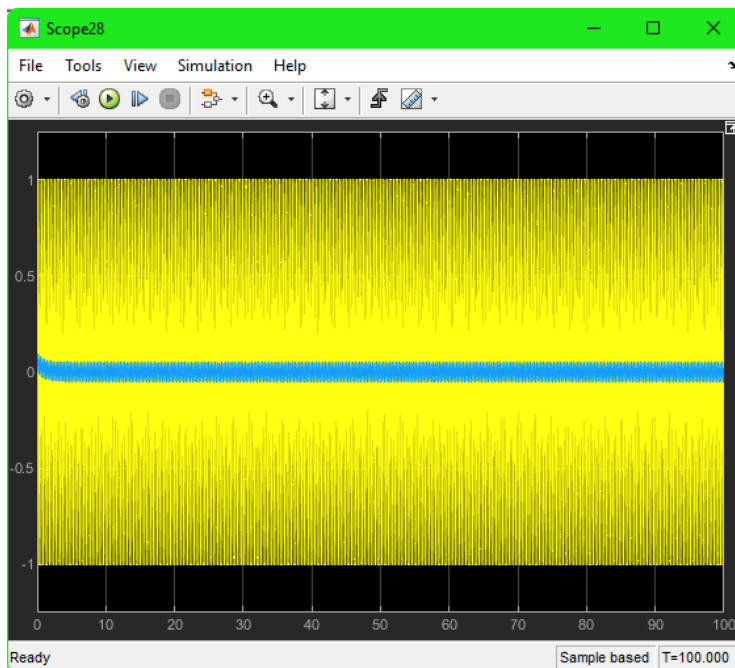


a)  $\sin(0.1 t)$



We see a phase shift between the input curve and the G1 curve but no noticeable drop in magnitude.

b)  $\sin(20t)$ .



We see a major drop in magnitude (amplitude) when comparing the input curve and the G1 curve.

Input “b” has a larger drop in magnitude (amplitude). This is most likely due to the low pass filter effect of G1 on the input. Since the input curve in b has a larger frequency, the low-pass filter effect was larger in that case.

G1, G2, G3, and G4 are named low pass filters mainly because they converge to a specific number. If there is a high-frequency input to these transfer functions, we see that the output has lower amplitude. G3 than is a better low pass filter because G3 exponentially decays and converges whereas G4 oscillates and eventually converges to a number.