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Tutorial - 2

Ans 1

```
Void func (int n)
{
   int j = 1, i = 0;
   while (i < n)
   {
     i = i + j
     j ++;
   }
}
```

$j=1$, $i = 0+1$
$j = 2$, $i = 0+1+2$.
$j=3$, $i = 0+1+2+3$.

Loop ends when $i \geq n$

$0+1+2+3 \ldots n > n$.

$\frac{K(K+1)}{2} > n$.

$K^2 > n$

$K > \sqrt{n}$

$O(\sqrt{n})$.

Ans 2. Recurrence Relation F.S Fibonacci Series

$$T(n) = T(n-1) + T(n-2)$$

$T(0) = T(1) = 1$.

- if $T(n-1) \approx T(n-2)$.

(Lower Bound)

$$T(n) = 2T(n-2).$$
$$= 2\{2T(n-4)\} = 4T(n-4)$$
$$= 4(2T(n-6))$$
$$= 8T(n-6).$$
$$= 8(2T(n-8))$$
$$= 16T(n-8).$$

$$T(n) = 2^K T(n-2K).$$

$n - 2^K = 0$

$n = 2^K$.

$K = \frac{n}{2}$

$T(n) = 2^{n/2} T(0)$
$= 2^{n/2}$.

$$T(n) = \Omega(2^{n/2}).$$

- if $T(n-2) \approx T(n-1)$

$$T(n) = 2T(n-1)$$
$$= 2(2T(n-2)) = 4T(n-2).$$
$$= 4(2T(n-3)) = 8T(n-3).$$
$$= 2^K T(n-K)$$

$n - K = 0$

$K = n.$

$T(n) = 2^K \times T(0) = 2^n.$

$= T(n) = O(2^n)$ (upper bound).

Ans 3 • $O(n(\log n)) \Rightarrow$

```
for (int i=0; i<n; i++)
{
   for (int i = 1; j<n; j=j*2)
   {
      // some O(1).
   }
}
```

• $O(n^3) \Rightarrow$

```
for (int i = 0; i<n; i++).
{
 for (int j=0; j<n; j++).
 {
   for (int K= ; K<n; K++).
   {
     // some O(1).
   }
}
```

Ans ~~3~~ 4 ~~int~~. $T(n) = T(n/4) + T(n/2) + cn^2$

• Lets assume $T(n/2) >= T(n/4)$.

So, $T(n) = 2T(n/2) + cn^2$

applying master's Theorem $(T(n) = aT(\frac{n}{b}) + f(n))$

$a = 2, b = 2$, $f(n) = n^2$.

$c = \log b^a = \log_2 2 = 1$

Compare n^c and $f(n) = n^2$. ($n^c = n$)

$f(n) > n^2$ so, $T(n) = \Theta(n^2)$

Ans 5. for

i	j
1	1
2	1 + 3 + 5
3	1 + 4 + 7
1	1 + 5 + 9
⋮	⋮
n	

$j = (n-1)/i$ times

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-2)}{3} + \dots + \frac{(n-1)}{n}$$

$$T(n) = n\left[1 + \tfrac{1}{2} + \tfrac{1}{3} + \dots + \tfrac{1}{n}\right] - 1 \times \left[1 + \tfrac{1}{2} + \tfrac{1}{3} + \dots + \tfrac{1}{n}\right]$$

$$n \log n - \log n$$

$$T(n) = O(n \log n).$$

Ans 6 :- for i

1
2
2^{k}
2^{k^2}
2^{k^3}
⋮
2^{k^m}

~~too~~ where

$2^{k^m} <= n$

$k^m = \log_2 n.$

$m = \log k \log_2 n.$

$$\therefore \sum_{i=1}^{m} 1$$

$1 + 1 + 1 \dots m$ times

$T(n) = O(\log_k \log n)$

Ans 7. Given algorithm divides array in 99% and 1% part

$$\therefore T(n) = T(n-1) + O(1)$$

$$T(n) = (T(n-1) + T(n-2) + \dots + T(1) + O(1)) \times n$$

$$= n \times n$$

$$\therefore T(n) = O(n^2)$$

Lowest height = 2

highest height = n.

$\therefore$ difference $= n-2 \quad n > 2$

The given algorithm produces linear result

Ans 8. (a) $n, n!, \log n, \log\log n$, rest $(n), \log(n!), n\log n, \log^2(n), 2^n, 2^{2^n}, 4^n, n^2, 100$

$100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

(b) $2^{(2^n)}, 4n, 2n, 1, \log(n), \log(\log(n)), \sqrt{\log(n)}, \log 2n, 2\log(n), n, \log(n!), n!, n2, n\log(n)$.

$1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$.

(c) $8^{2n}, \log_2(n), n\log_6(n), n\log_2(n), \log(n!), n!, \log_2(n), 96, 8n^2, 7n^3, 5n$.

$96 < \log_3 n < \log 2n < 5n < n\log_6(n) < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$.