Tutorial - 4.

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Q1) $T(n) = aT(n/b) + f(n^2)$
$a \geq 1, b \geq 1$
On compairing
$a = 3, b = 2, f(n) = n^2$
Now, $c = \log_b a = \log_2 3 = 1.584$
$n^c = n^{1.584} < n^2$.
$\therefore f(n) > n^c$
$\therefore T(n) = O(n^2)$.

Q2). $a \geq 1, b > 1$
$a = 4, b = 2, f(n) n^2$
$c = \log_2 4 = 2$.
$n^c = n^2 = f(n) = n^2$
$\therefore T(n) = \theta(n^2 \log_2 n)$

Q3) $a = 1$
$b = 2$.
$f(n) = 2^n$
$c = \log_b a = \log_2 c = 0$
$n^c = n^0 = 1$.
$f(n) > n^c$
$T(n) = \theta(2^n)$.

Q4). $a = 2^n$
$b = 2, f(n) = n^2$
$c = \log_b a = \log_2 2^n$
$n^c \Rightarrow n^n$.
$f(n) = n^a$
$f(n) = \theta(n^2 \log_2 n)$

Q5) $a = 16, b = 4$
$f(n) = n$
$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 4$
$= 2$
$n^c \Rightarrow n^2$ & c
$f(n)$ & $< n^c$
$\therefore T(n) = \theta(n^2)$

Q6) $a = 2, b = 2$
$f(n) = n \log n$
$c = \log_2 n$
$n^c = n^1 = n$
$n \log n > n$.
$f(n) > n^c$
$T(n) = \theta(n \log n)$.

Q7) $a=2,\ b=2,\ f(n)=n/\log n$

$c=\log_2 2=1$

$n^c=n^1=1$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n)=\theta(n)$.

Q8) $a=2,\ b=4,\ f(n)=n^{0.51}$

$c=\log_b a=\log_4 2=0.5$

$n^c=n^{0.5}$

$\because n^{0.5}<n^{0.51}$

$f(n)>n^c$.

$\therefore T(n)=\theta(n^{0.51})$

Q9) $a=0.5,\ b=2$

$a\geq 1$ but here a is 0.5

so we cannot apply master's theorem.

Q10) $a=16,\ b=4,\ f(n)=n!$

$\therefore c=\log_b a=\log_4 16=2$.

$n^c=n^2$

As $n!>n^2$

$\therefore T(n)=\theta(n!)$

Q11) $a=4,\ b=2,\ f(n)=\log n$.

$c=\log_b a=\log_2 4=2$.

$n^c=n^2$

$f(n)=\log n$

$\because \log n<n^2$.

$f(n)<n^c$

$T(n)=\theta(n^c)$

$=\theta(n^2)$.

Q12) $a=\sqrt{n},\ b=2$

$c=\log_b a=\log_2\sqrt{n}=\frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n<\log(n)$.

$\therefore f(n)>n^c$

$T(n)=\theta(f(n))$

$=\theta(\log(n))$.

Q13) $a=3;\ b=2;\ f(n)=n$

$c=\log_b a=\log_2 3=1.5849$

$n^c=n^{1.5489}$.

$\Rightarrow f(n)<n^c$

$T(n)=\theta(n^{1.5849})$

Q14) $a=3,\ b=3$

$c=\log_b a=\log_3 3=1$.

$n^c=n^1=n$

As $\text{sqrt}(n)<n$

$f(n)<n^c$

$T(n)=\theta(n)$.

Q15) $a = 4, b = 2$.

$c = \log_b a = \log_2 4 = 2$.

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$T(n) = \theta(n^2)$.

Q16) $a = 3, b = 4, f(n) = n \log n$.

$c = \log_b a = \log_4 3 = 0.792$.

$n^c = n^{0.792}$

$n^{0.792} < n \log n$.

$T(n) = \theta(n \log n)$.

Q17) $T(n) = 3T(n/3) + n/2$.

$a = 3; b = 3$.

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$.

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$.

$\therefore T(n) = \theta(n)$

Q18) $a = 6; b = 3$.

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$.

As $n^{1.6309} < n^2 \log n$.

$\therefore T(n) = \theta(n^2 \log n)$.

Q19) $a = 4, b = 2, f(n) = \frac{n}{\log n}$.

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$.

$\frac{n}{\log n} < n^2$.

$T(n) = \theta(n^2)$.

Q20) $a = 64$ $b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$\therefore n^2 \log n > n^2$.

$T(n) = \theta(n^2 \log n)$

Q21) $a = 7; b = 3; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \theta(n^2)$

Q22) $a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$.

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \theta(n(2 - \cos n))$.