Mathematical Methods - Analysis
Assignment 2
Due: 4 September 2023

(**Note**: Question 1 is worth 10 marks and the remaining are worth 5 marks each.)

1. Find the derivatives of functions given below. Clearly specify those subsets of the domain on which either the function or the derivative is undefined.

   (a) $f(x) = \sqrt{e^{2x} + 3}$.

   (b) $f(x) = \frac{(2x^2 + x - 1)^{\frac{5}{2}}}{(3x+2)^9}$.

   (c) $f(x) = \sin(\log(2x + 1))$.

   (d) $g(x) = \frac{\sin(5x+2)}{\cos(x^2-1)}$.

   (e) $g(x) = \frac{\log(x^2+2)}{e^{-x}}$.

2. Find the equation of the tangent line to the graph of the function

$$g(t) = \frac{t}{t+5},$$

   when $t = 2$.

3. Suppose a gas is pumped into a spherical balloon at a constant rate of 50 cm^3/sec. Assume that the gas pressure remains constant, and that the balloon always has a spherical shape. How fast is the radius increasing when it is 5 cm?

4. Sand is falling on a pile, always having the shape of a cone, at the rate 3 cm^3/sec. Assuming that the diameter at the base of the pile is always three times the height. At what rate is the height increasing when it is 4 cm?

5. A swimming pool is 25 feet wide, 40 ft long, and 3 ft deep on one end and 9 ft deep at the other, the bottom being an inclined plane. If the water is pumped in to the pool at the rate of 10 ft^3/ min, how fast is the water level rising when it is 4 ft deep at the deep end?