Linear Algebra and its Applications
Assignment 6
Due: 18 April 2024

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1. Prove that A^T has the same nonzero singular values as A. Is it true that $\|Ax\| = \|A^T x\|$, for all nonzero x? Justify your answer.

2. Find the 2-norm of the matrix $A - \sigma_1 u_1 v_1^T$. Here σ_1 is the first singular value and u_1, v_1 are the first columns of U and V respectively.

3. Find the full SVD of following matrices:

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

4. Find the singular values of AA^TA for any real matrix A.

5. Let A be an $m \times n$ real matrix of rank r. Consider the following $(m+n) \times (m+n)$ (block) matrix

$$S = \begin{bmatrix} \mathbf{0} & A \\ A^T & \mathbf{0} \end{bmatrix}.$$

   Find the (nonzero) eigenvalues of S along with the corresponding eigenvectors. Justify your claims.