Linear Algebra and its Applications
Assignment 2
Due: 12 February 2024

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1. Let A be the following 3×3 matrix, all of whose principal submatrices are invertible:

$$A = \begin{bmatrix} b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 \\ 0 & a_3 & b_3 \end{bmatrix}$$

   Find LU factorization of A, i.e., express the entries of L, U in terms of a_i, b_i, c_i's. In general, a matrix is called tridiagonal if the non-zero entries, if any, occur on the main diagonal, the diagonal upon the main diagonal and the diagonal below the main diagonal. Suppose A is an $n \times n$ tridiagonal matrix whose LU factorization existsts, using the above exercise, directly write down a formula for the entries of L and U in terms of those of A (without proof).

2. Write down the pseudocode of an algorithm that computes the LU factorization of a tridiagonal matrix. Make sure to use the special structure. Find the exact operation count of your algorithm.

3. Find the LU factorization of the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 7 \\ 2 & 4 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

   Make sure that your pivot is the largest absolute nonzero number among the available entries in that column.

4. Consider the following 4×4 matrix A:

$$\begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

   Find four conditions of a, b, c, d to get $A = LU$ with four nonzero pivots.