Mathematical Methods - Analysis
Assignment 4
Due: 13 November 2023

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1. For each of the following bivariate, real valued functions find all the critical points and then determine which of them are local extrema using the Hessian. Which of these extrema are global? Justify. [15 points]
   (a) $f(x, y) = e^{-(x^2+y^2)}, \quad (x, y) \in \mathbb{R}^2$.
   (b) $f(x, y) = \ln(1 + x^2 + y^2) \quad (x, y) \in \mathbb{R}^2$.
   (c) $f(x, y) = x^2y^2 \quad (x, y) \in \mathbb{R}^2$.
2. In each of the following determine the maximum and the minimum of the given function f on a given closed and bounded set S. [20 points]
   (a) $f(x, y) = xy - \sqrt{1 - x^2 - y^2}$ and $S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$.
   (b) $f(x, y) = x^3 + xy$ and $S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$.
3. Solve the following problems using Lagrange's multiplier. [15 points]
   (a) Minimize $x + y^2$ subject to the constraint $2x^2 + y^2 = 1$.
   (b) Maximize $x^2 + xy + y^2 + yz + z^2$ on the sphere of radius 1.
   (c) Maximize xyz subject to the constraints $x \geq 0, y \geq 0, z \geq 0$ and $xy + yz + xz = 2$.