Mathematical Methods - Analysis
Assignment 1
Due: 22 August 2023

(**Note**: Each question is worth 10 marks)

1. Recall that for a convergent sequence $\{a_n\}_{n=1}^{\infty}$ converging to a, given any $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that for all $n > N$ we have $\mid a_n - a \mid < \epsilon$. In each of the following sub-question you are given a convergent sequence (by specifying its nth term), its limit and a value of ϵ. Find the value of N (as an integer). Make sure to show your work.
   (a) $a_n = \frac{2n}{n+3}, a = 2, \epsilon = \frac{1}{5}$.
   (b) $a_n = \frac{1}{\sqrt{n+1}}, a = 0, \epsilon = \frac{3}{10}$.
   (c) $a_n = \frac{10^7}{n}, a = 0, \epsilon = \frac{1}{20}$.
   (d) $a_n = \frac{1}{\sqrt{n+1}+\sqrt{n}}, a = 0, \epsilon = \frac{1}{10}$.
   (e) $a_n = \frac{1}{n} + \frac{\sin n}{n+1}, a = 0, \epsilon = \frac{1}{100}$.
2. Recall that a sequence $\{a_n\}$ is divergent if for a given large number $M > 0$ there is an index N after which all terms of the sequence are bigger (or smaller) than M. All the sequences below are divergent, for the given M find N (as an integer).
   (a) $a_n = \log n, M = 100$.
   (b) $a_n = \log(\frac{1}{n}), M = -100$.
   (c) $a_n = \sqrt{n}, M = 500$.
   (d) $a_n = \frac{n^2}{n+5}, M = 1000$.
   (e) $a_n = \frac{n}{10^9}, M = 10000$
3. Determine if the given sequence converges or diverges or oscillates. Find the limit if the sequence converges or provide a brief justification why it doesn't converge.
   (a) $a_n = \sin(\frac{n\pi}{2})$.
   (b) $a_n = 2^{\frac{1}{n}}$.
   (c) $a_n = \frac{n \sin(n!)}{n^2+5}$.
   (d) $a_n = \sqrt{n+7} - \sqrt{n+4}$.
   (e) $a_n = b^n$, where b is a real number.