

## Multivariate Statistical Analysis Assignment 2

1. Three random variables  $X, Y, Z$  have a joint pdf

$$f(x, y, z) = \begin{cases} kxyz^2 & 0 < x, y < 1, 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find  $k$  and  $E(X)$
- Examine the independence of r.v.s.
- Find the marginal pdf of  $X$  and  $Z$ .
- Find the conditional distribution of  $X / Y = 1/2 \text{ and } Z = 1$
- Check the covariance of  $X$  and  $Z$

2. Suppose that  $X_1, X_2, X_3$  are independent r.v.s with unit variance. Let  $Y_1 = X_1 + X_2 + X_3$ ;  $Y_2 = X_1 - X_2$  and  $Y_3 = X_1 - X_3$ . Find the correlation and covariance matrices of  $Y^T = [Y_1, Y_2, Y_3]$

3. With  $\mu^T = [10 \ 4 \ 7]$  and i)  $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ .

and ii)  $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$  Find the distribution of

$X = \mu + BU$ . Verify the ranks of the resulting covariance matrices

4. Given  $X \sim N_3(\mu, \Sigma)$  where

$$i) \Sigma = \begin{bmatrix} 9 & -3 & -3 \\ -3 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$$

$$ii) \Sigma = \begin{bmatrix} 8 & -4 & -4 \\ -4 & 4 & 0 \\ -4 & 0 & 4 \end{bmatrix}$$

- a) Use an eigen value analysis to find  $B$ , where  $X = \mu + BV$ .
- b) Examine the structural relationship among  $X_i$  for (i)
5. For the distributions of  $X$  appearing in problems (3) and (4) above, write down the marginal distribution of  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  and the conditional distribution of  $x_1 \mid \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}$  and of  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_3$ .
- When do the distributions degenerate.