

Answer Any THREE Questions.

1. Given

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 6 \\ 4 & 0 & 4 \end{bmatrix}$$

Consider the two linear combinations  $b'X = 2X_1 + 2X_2 - X_3$

And  $c'X = X_1 - X_2 + 3X_3$

Find the sample means, variances, and sample covariance of the two linear combinations.

2.

Let  $\mathbf{X}_{(3 \times 1)}$  be  $N_3(\mu, \Sigma)$  with

$$\Sigma = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Are  $X_1$  and  $X_2$  independent? What about  $(X_1, X_2)$  and  $X_3$ ?

3. Let  $X_1, X_2, X_3$  and  $X_4$  be independently and identically distributed  $3 \times 1$  random vectors with

$$\mu = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Now consider two linear combinations of random vectors

$$\frac{1}{2}X_1 + \frac{1}{2}X_2 + \frac{1}{2}X_3 + \frac{1}{2}X_4$$

and

$$X_1 + X_2 + X_3 - 3X_4$$

Find the mean vector and covariance matrix for each linear combination of vectors and also the covariance between them.

4.

Let  $\mathbf{X}$  be distributed as  $N_p(\mu, \Sigma)$  with  $|\Sigma| > 0$ . Then

$(\mathbf{X} - \mu)' \Sigma^{-1} (\mathbf{X} - \mu)$  is distributed as  $\chi_p^2$ , where  $\chi_p^2$  denotes the chi-square distribution with  $p$  degrees of freedom.