Chennai Mathematical Institute

Multivariate Statistical Analysis

Mid Term Test

Time : 60 Minutes Max marks: 20

Choose the Best possible answer

A 1. An advantage of using an experimental multivariate design over separate univariate designs is that using the multivariate analysis is:

A. Allows you to look at more complex relationships than does univariate strategy
B. Provides a more powerful test of hypotheses

a) Both A and B holds
b) A holds but not B
c) B holds but not A
d) Both A and B do not hold

A 2. If X has a multivariate Normal distribution, then

A. Linear combinations of X are normally distributed
B. All subsets of the components of **X** have a MVN distribution
C. All the conditional distributions of the components are not MVN

a) A and B are true
b) A and C are true
c) B and C are true
d) A,B, and C are true

c 3. Let X and Y be independent standard normal random variables. Define the random variable Z by $Z = X$ if $XY > 0$; $Z = -X$ if $XY < 0$

A. Z is a standard normal random variable
B. Random Vector (Y, Z) is not jointly normal

a) A is true but not B
b) B is true but not A
c) Both A and B are true
d) Both A and B are not true

d 4. Examine the following statements

A. If the components of MVN are not independent, then the vector may not be jointly Normal
B. Uncorrelated jointly normal random variables are independent

a) A holds but not B
b) B holds but not A
c) A and B do not hold
d) A and B holds

5. If Y follows MVN with (μ,Σ), then $Y'AY \sim \chi^2(r,\gamma)$ where $\gamma = \mu' A\mu$ and $r(A) = r$ iff

A. $A\Sigma A = \Sigma$ and $A\Sigma$ is idempotent

A

B. $A\Sigma A = \Sigma$ and $A^{-1}\Sigma$ is idempotent

a) A holds
b) B holds
c) A and B holds
d) A and B does not hold

6. Examine the following properties on p variate MVN

A. Invariant under linear transformation

A

B. Uncorrelated sub-vectors are independent

a) A and B holds
b) A holds but not B
c) B holds but not A
d) A and B hold under certain other conditions

7. The MD between Y and μ in the metric of Σ is $D(Y,\mu) = [(Y-\mu)\ \Sigma^{-1}\ \ \ (Y-\mu)]^{1/2}$

A. Invariant under appropriate linear transformations

c

B. Adjusted Euclidean distance in the metric of reciprocal variance

a) A holds and not B
b) B holds and not A
c) A and B holds together
d) A and B do not hold

8. Q-Q plot can be used to

A. order Mahalanobis distances versus estimated quantiles for a sample of size *n* from a chi-squared distribution with *p* degrees of freedom for evaluating multivariate normality

c

B. points on the right side of the plot for which the Mahalanobis distance is notably greater than the chi-square quantile value exhibit the presence of outliers.

a) A is true
b) B is true
c) A and B are true
d) A and B are not true

9. Examine the following statements in PCA

A. The construction of principal components does not require that the variables in Y have a normal multivariate distribution.

D

B. Construction of PCA can be carried out through SVD of the data matrix

C. PCA minimizes the perpendicular distance between a data point and the principal component

a) A and B are true
b) A and C are true
c) B and C are true
d) A, B, and C are true

10. Examine the following statements in PC

D

A. PC's are invariant to linear transformation to the original variables F

B. PC's are not sensitive to the presence of outliers F

a) Both A and B are true
b) A alone is true
c) B alone is true
d) A and B are untrue

11. Examine the following statements on Mahalanobis distance

B

A. MD is effectively a multivariate equivalent of Euclidean distance T

B. MD could be used for classification and Outlier detection T

C. MD is not possible for highly correlated variables F

a) A,B and C are true
b) A,B is true but not C
c) A,C is true but not B
d) B,C is true but not A

12. Suppose X1, . . . , Xn are i.i.d. $N_p(\mu, \Sigma)$; then the sample mean and sample variance are independent, with

A. $\sqrt{n}(\bar{X} - \mu) \sim N_p(0, \Sigma)$,

B. $(n - 1)S \sim W_p(n - 1, \Sigma)$.

C

a) A is true
b) B is true
c) A and B are true
d) A and B are not true

13. let $X \sim N_p(\mu, \Sigma)$, $mS \sim W_p(m, \Sigma)$. then $T^2{}_p(m) = (\bar{X} - \mu)' S^{-1} (\bar{X} - \mu)$ and $[(m - p + 1)/(mp)]\, T^2{}_p(m)$ follows $F_{p,m-p+1}$:

A. If X and S are independent

B. If $\bar{X}$ and S are independent

B

a) A is true
b) B is true
c) A and B are true
d) A and B are not true

14. If $M \sim W_p(n; \Sigma)$ and $a \in R^p$ is such that $a'\Sigma a \neq 0$ and $n > p-1$ then

A. $a'Ma/a'\Sigma a$ follows χ^2_n

B. $a', \Sigma^{-1}a/a'M^{-1} a$ follows χ^2_{n-p+1}

C

a) A is true
b) B is true
c) A and B are true
d) A and B are not true

$$\frac{a^T M a}{a^T \Sigma a}$$

$$\frac{a^T \Sigma^{-1} a}{a^T M^{-1} a}$$

15. If A is idempotent of rank r then
   A. If y is $N_p(O, I)$, then $y'Ay$ is $\chi 2(r)$
   B. y is $N_p(\mu, \sigma^2 I)$, then $y'Ay/\sigma^2$ is $\chi 2(r, \mu' A\mu/\sigma^2)$

(a)

   a) A is true
   b) B is true
   c) A and B are true
   d) A and B are not true

16. Examine the following
   A. Let B is a k × p matrix of constants, A is a p × p symmetric matrix of constants, and y is distributed as $N_p(\mu, \Sigma)$. Then By and y'Ay are independent if and only if $B\Sigma A = O$
   B. Let A and B be symmetric matrices of constants. If y is $N_p(\mu, \Sigma)$, then y'Ay and y'By are independent if and only if $A \Sigma B = O$.

(c)

   a) A is true
   b) B is true
   c) A and B are true
   d) A and B are not true

17. If y us $N_p(\mu, \Sigma)$, then
   A. $E(Y'AY) = tr(A\Sigma) + \mu' A\mu$
   B. $var(y'Ay) = 2tr[(A\Sigma)^2] + 4\mu' A\Sigma A\mu$.
   C. $cov(y, y'Ay) = 2\Sigma A\mu$.

(D)

   a) A and B are true
   b) A and C are true
   c) B and C are true
   d) A, B and C are true

18. Examine the following for Mahalanobis distance:

   A. a measure that quantifies the dissimilarity between two data points in a multidimensional space, considering the covariance structure of the data. T
   B. It takes into account the correlations between variables, making it suitable for datasets where variables are interrelated. T
   C. It is not scale-invariant, meaning it is affected by the scaling of variables F

(A)

   a) A and B are true
   b) A and C are true
   c) B and C are true
   d) A, B, and C are true

19. Let X follows $N_p(\mu, \Sigma)$, and $Y = \Sigma^{-1/2} (X - \mu)$ then
   A. Y follows $N_p(0, I_p)$ T
   B. Y follows $N_p(\mu, \Sigma)$ F
   a) A is true
   b) B is true

(A)

   c) A and B are true
   d) A and B are not true

20. If $X \sim N_3(\mu, \Sigma)$ with $\mu = [\,-2\ 3\ 1\,]'$ and

$$\Sigma = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & -3 \\ 0 & -3 & 4 \end{pmatrix}.$$

Then marginal distribution of X_1 and X_3 and $[X_1 \quad X_3]'$ are

A. $X_1 \sim N(2, 2)$ F

B. $X_2 \sim N(1, 4)$ T

B C. $\cancel{X_3}$ $[X_1, X_3] \sim$

$$N_2\left(\begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}\right).$$ T

a) A and B are but not C
b) B and C are true but not A
c) A and C are true but not B
d) A,B, and C are true