A very short anthology of some beautiful numbers, mathematical terms & expressions

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Armstrong Numbers: An n-digit number that is the sum of the nth powers of its digits is called an Armstrong number. It is also sometimes known as an n-narcissistic number or a perfect digital invariant (Madachy 1979), or a plus perfect number. The largest Armstrong number is 115132219018763992565095597973971522401 e.g., 153, 9474, 54748.
$153 = 1^3 + 5^3 + 3^3$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$
$54748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5$

Aronson's sequence: Aronson's sequence is an integer sequence defined by the English sentence "t is the first, fourth, eleventh, sixteenth, ... letter in this sentence." Spaces and punctuation are ignored. The first few numbers in the sequence are:
$1, 4, 11, 16, 24, 29, 33, 35, 39, 45, 47, 51, 56, 58, 62, 64, 69, 73, 78, 80, 84, 89, 94, 99, 104, 111, 116, 122, 126, 131, 136, 142, 147, 158, 164, 169, ...$

Astonishing numbers: Astonishing Number is a number N whose representation can be decomposed into two parts, a and b, such that N is equal to the sum of the integers from a to b and a + b = N where '+' denotes concatenation.Few Astonishing Numbers are: 15, 27, 429, 1353, 1863, 3388, 3591, 7119..
1+2+3+4+5=15
2+3+4+5+6+7=27
For every $a = \frac{2.10^n - 5}{15}$ and $b = \frac{8.10^n - 5}{15}$, N is an astonishing number.

Autobiographical numbers: These numbers are natural numbers with utmost 10 digits in which the first digit of the number (from left to right) tells you how many 0s there are in the number, the second digit tells you the number of 1's, the third digit tells you the number of 2's. . . and so on. Here is the full set of autobiographical numbers: $1210, 2020, 21200, 3211000, 42101000, 521001000, 6210001000$

Brun's constant: In number theory, Brun's theorem states that the sum of the reciprocals of the twin primes (pairs of prime numbers which differ by

2) converges to a finite value known as Brun's constant, usually denoted by $B_2 = 1.90216054\ldots$.

Chaitin's constant: In the computer science sub field of algorithmic information theory, a Chaitin constant (Chaitin omega number) or halting probability is a real number that, informally speaking, represents the probability that a randomly constructed program will halt. These numbers are formed from a construction due to Gregory Chaitin. Although there are infinitely many halting probabilities, one for each method of encoding programs, it is common to use the letter Ω to refer to them as if there were only one. Because Ω depends on the program encoding used, it is sometimes called Chaitin's construction instead of Chaitin's constant when not referring to any specific encoding.Each halting probability is a normal and transcendental real number that is not computable, which means that there is no algorithm to compute its digits. Each halting probability is Martin-Löf random, meaning there is not even any algorithm which can reliably guess its digits.

Eddington Number: In astrophysics, the Eddington number,N_{Edd} is the number of protons in the observable universe. The term is named for British astrophysicist Arthur Eddington, who in 1938 was the first to propose a value of N_{Edd} and to explain why this number might be important for physical cosmology and the foundations of physics.
N_{Edd}= 15 747 724 136 275 002 577 605 653 961 181 555 468 044 717 914 527 116 709 366 231 425 076 185 631 031 296

Cycling Numbers: Take the number, 1781. $17^2 + 81^2 = 6850, 68^2 + 50^2 = 7124, 71^2 + 24^2 = 5617, 56^2 + 17^2 = 3425, 34^2 + 25^2 = 1781$, the number we have taken. Such numbers are called cycling numbers. Other examples include 7141: $\{7141 \longrightarrow 6722 \longrightarrow 4973 \longrightarrow 7730 \longrightarrow 6829 \longrightarrow 5465 \longrightarrow 7141\}$.

Dissectible Numbers: A dissectible number is a three digit number abc with the property that when multiplied by any Wonderful Demlo number the product has the form $axx\ldots xbyy\ldots yc$, with the same number of x digits as y digits. Wonderful Demlo numbers are the palindrome numbers: {1, 121, 12321, 1234321, 123454321, 12345654321, 1234567654321,...}. The disssectible numbers are: {9, 18, 27, 36, 45, 54, 63, 72, 81, 108, 117, 126, 135, 144, 153, 162, 207, 216, 225, 234, 243, 306, 315, 324, 405} .Example: $1234321 \times 216 = \underline{2}666\underline{1}333\underline{6}$

Durable Palindromic Primes: If removal of the first and the last of a palindromic prime leaves another prime and repititions of the digit removal operation continue to produce primes until a single digit prime is reached, the largest prime in such a sequence is said to be parent durable palindromic prime. E.g., 11311. 11311, 131, 3 are primes. There are exactly ten parent durable palindromic primes. They are 11311, 37273, 71317, 937579, 1335331, 3315133, 9375739, 373929373 and 733929337. There are 27 durable palindromic primes.

Eporn: (Equal Products of Reversible Numbers) can be defined as the numbers which can be expressed as product of two reversible numbers in 2 different ways. E.g: $2520 = 120 \times 021 = 210 \times 012$
$63504 = 144 \times 441 = 252 \times 252$ .We know 6 eporns in four digit numbers: 2520,4030,5740,7360,7650,9760

Erdős Number: The Erdős number is the number of "hops" needed to connect the author of a paper with the prolific late mathematician Paul Erdős. An author's Erdős number is 1 if he has co-authored a paper with Erdős, 2 if he has co-authored a paper with someone who has co-authored a paper with Erdős, etc. 509 people have Erdős number 1. There is an entire list of people arranged by Erdős number. Terence Tao has an Erdos number=3.

Exotic Numbers: Exotic numbers are numbers that can be expressed using each of its own digits in any order only once using any mathematical symbols. e.g:
$715 = (7 - 1)! - 5$
$120 = ((1 + 2)! - 0!)!$
$2592 = 2^5 \times 9^2$
$343 = (3 + 4)^3$

Filzian number: A Filzian number is a positive integer that is equal to sum of its digits times the product of its digits.
$1 \times 1 = 1$
$(1 + 3 + 5) \times 1 \times 3 \times 5 = 135$
$(1 + 4 + 4) \times 1 \times 4 \times 4 = 144$
(Try to prove that they are the only such numbers!)

Fortuitous numbers: The numbers that are equal to the product of the lengths of the words in its name! E.g:
84,672 is sounded and read "EIGHTY FOUR THOUSAND SIX HUNDRED SEVENTY TWO". Count the letters in each of those words, multiply the counts, and you get $6 \times 4 \times 8 \times 3 \times 7 \times 7 \times 3 = 84,672$.
333,396,000 (THREE HUNDRED AND THIRTY THREE MILLION, THREE HUNDRED AND NINETY SIX THOUSAND) and $5 \times 7 \times 3 \times \ldots \times 6 \times 3 = 333,396,000$. The next few Fortuitous numbers are23,337,720,000, 19,516,557,312,000, 56,458,612,224,000, and 98,802,571,392,000.

Hilbert number: The number, $2^{\sqrt{2}}$ is called the hilbert number.Gelfond–Schneider theorem (1934):If a and b are algebraic numbers with $a \neq 0, 1$, and b irrational, then any value of a^b is a transcendental number. Therefore $2^{\sqrt{2}}$ is a transcendental number.

The Hofstadter–Conway 10,000 dollar sequence: It is defined as follows: a(1)=a(2)=1,

a(n)=a(a(n-1))+a(n-a(n-1)), $n \geq 3$.
The first few terms of this sequence are 1, 1, 2, 2, 3, 4, 4, 4, 5, 6, 7, 7, 8, 8, 8, 8, 9, 10, 11, 12, ... This sequence acquired its name because John Horton Conway offered a prize of 10,000 dollars to anyone who could demonstrate a particular result about its asymptotic behavior. The prize, since reduced to 1,000 dollars, was claimed by Collin Mallows.In private communication with Klaus Pinn, Hofstadter later claimed he had found the sequence and its structure some 10–15 years before Conway posed his challenge.

Kaprekar's Constant : 6174 is known as Kaprekar's constant after the Indian mathematician D. R. Kaprekar. This number is notable for the following rule:
1)Take any four-digit number, using at least two different digits (leading zeros are allowed).
2)Arrange the digits in descending and then in ascending order to get two four-digit numbers, adding leading zeros if necessary.
3)Subtract the smaller number from the bigger number.
Go back to step 2 and repeat.
The above process, known as Kaprekar's routine, will always reach its fixed point, 6174, in at most 7 iterations. Once 6174 is reached, the process will continue yielding 7641 – 1467 = 6174. For example, choose 1495:
9541 – 1459 = 8082
8820 – 0288 = 8532
8532 – 2358 = 6174
7641 – 1467 = 6174
The only four-digit numbers for which Kaprekar's routine does not reach 6174 are repdigits such as 1111, which give the result 0000 after a single iteration. All other four-digit numbers eventually reach 6174 if leading zeros are used to keep the number of digits at 4.

Mill's constant : In number theory, Mills' constant is defined as the smallest positive real number A such that the floor function of the double exponential function $\lfloor A^{3^n} \rfloor$ is a prime number for all natural numbers n. This constant is named after William H. Mills who proved in 1947 the existence of A based on results of Guido Hoheisel and Albert Ingham on the prime gaps. Its value is unknown, but if the Riemann hypothesis is true, it is approximately 1.3063778838630806904686144926... First 4 Mill's primes are: 2, 11, 1361, 2521008887

Palindromic Numbers:A palindromic number (also known as a numeral palindrome or a numeric palindrome) is a number (such as 16461) that remains the same when its digits are reversed. Interestingly, one can calculate the no of such numbers:
The number of palindromes $\leq 10^n = 2(10^{n/2} - 1)$ if n is even and
$2 \times |(10^{(n-1)/2} - 1)| + 9 \times (10^{(n-1)/2})$ if n is odd. For verification, check it yourselves!

Ramsay Number: The Ramsey number R(m,n) gives the solution to the party problem, which asks the minimum number of guests R(m,n) that must be invited so that at least m will know each other or at least n will not know each other.By symmetry, $R(m,n) = R(n,m).R(3,9) = 36$. If you invite 36 guests to a party, there will exist at-least 3 of them who will know each other!

Self Descriptive Number:A 10-digit number satisfying the following property: Number the digits 0 to 9, and let digit n be the number of 'n's in the number. There is exactly one such number: 6210001000.

n	0	1	2	3	4	5	6	7	8	9
No of 'n's	6	2	1	0	0	0	1	0	0	0

Truncatable primes: A left-truncatable prime is a prime number which, in a given base, contains no 0, and if the leading ("left") digit is successively removed, then all resulting numbers are prime. For example, 9137, since 9137, 137, 37 and 7 are all prime. Another such number is 357686312646216567629137. A right-truncatable prime is a prime which remains prime when the last ("right") digit is successively removed. 7393 is an example of a right-truncatable prime, since 7393, 739, 73, and 7 are all prime. A left-and-right-truncatable prime is a prime which remains prime if the leading ("left") and last ("right") digits are simultaneously successively removed down to a one or two digit prime. 1825711 is an example of a left-and-right-truncatable prime, since 1825711, 82571, 257, and 5 are all prime.In base 10, there are exactly 4260 left-truncatable primes, 83 right-truncatable primes, and 920,720,315 left-and-right-truncatable primes.