Quadratic Functions

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Objectives

- Quick revision of Quadratic Function.
- Factorising Quadratic Equations.
- Proving Basic Quadratic formulas of roots relationship.

Quick Revision

Forms of Quadratic Function

- $f(x) = ax^2 + bx + c$ is called the **standard form**.
- $f(x) = a(x - x_1)(x - x_2)$ is called the **factored form**, where x_1 and x_2 are the roots of the quadratic function.

Delta Δ

Δ determines tells us how many real solutions does the quadratic equation have:
$\Delta = \mathrm{b}^2 - 4ac$

$$\text{number of solutions} = \begin{cases} 2 & \text{when } \Delta > 0 \\ 1 & \text{when } \Delta = 0 \\ 0 & \text{when } \Delta < 0 \end{cases}$$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Graph of Quadratic Function

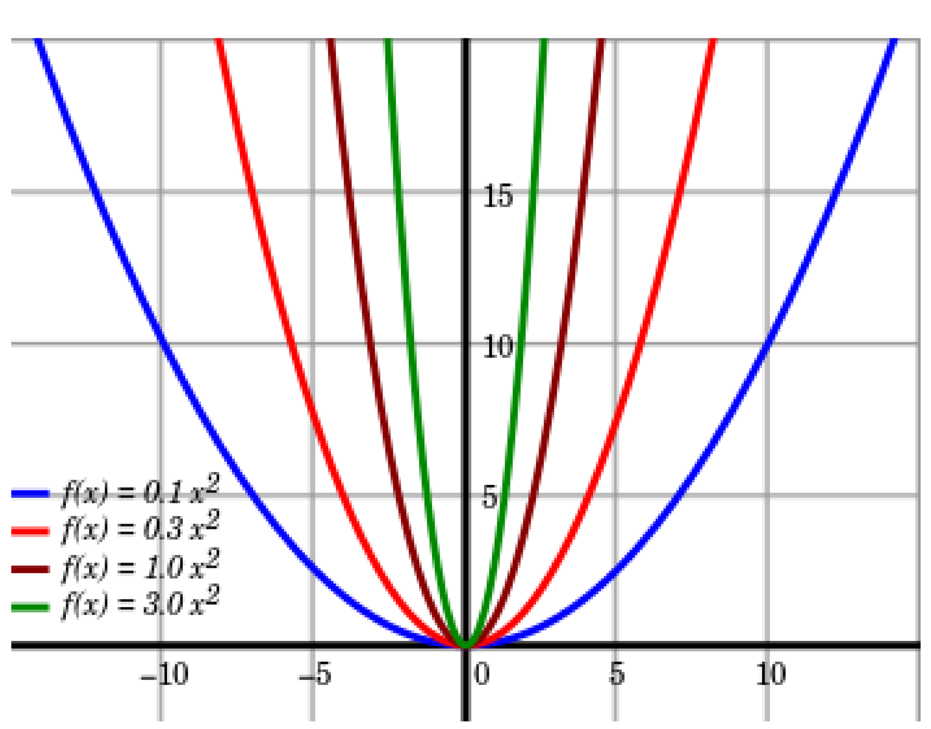

Figure 1: Graph of $f(x) = ax^2|_{\{0.1,0.3,1.0,3.0\}}$ Parabolic in nature

Factorising a Quadratic Equation

Factorising a quadratic equation means putting it into two brackets, and is useful if you're trying to draw its graph. Solving a quadratic equation is pretty easy if $a = 1$ (in $ax^2 + bx + c$ form), but can be a real pain otherwise.

In order to factorise a quadratic you should follow steps shown below:

1. Rearrange the equation into the standard $ax^2 + bx + c$ form.
2. Write down two brackets with an 'a': $a(x \quad)(x \quad)$
3. Find two numbers that multiply to give 'c' and add or subtract to give 'b' (ignoring the signs).
4. Put the numbers in brackets and choose their signs.

Factorising- Tasks

1. Factorise $x^2 - x - 12$.

By following the method shown before, we factorise $x^2 - x - 12$ as :
(x-4)(x+3)

2. Solve $x^2 - 8 = 2x$ by factorising.

By following the method shown before, we factorise $x^2 - 2x - 8$ as :
(x+2)(x-4)
Thus, we get $x = -2$ or $x = 4$

Myth of Delta Δ

It's commonly believed that in order to work out roots of a quadratic function you must count Δ and use other previously established formulas. However this is untrue since factorising in many cases is as good or even better than simply counting Δ. However Δ is useful if the coefficients are really huge.

Example of Factorisation

Solve $x^2 + 4x - 21 = 0$ by factorising.

$$x^2 + 4x - 21 = (x \quad)(x \quad)$$

1 and 21 multiply to give 21 - and add or subtract to give 22 and 20.
3 and 7 multiply to give 21 - and add or subtract to give 10 and 4.

$$x^2 + 4x + 21 = (x + 7)(x - 3)$$

And solving the equation:

$$(x + 7)(x - 3) = 0$$

we get

$$x = -7, \quad x = 3$$

Proof of Quadratic Formula- I

Let's prove that:

$$x_1 + x_2 = \frac{-b}{a}$$

When Δ is positive we have two roots:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}, \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Substituting for x_1 and x_2 respectively, we receive:

$$x_1 + x_2 = \frac{-b - \sqrt{\Delta}}{2a} + \frac{-b + \sqrt{\Delta}}{2a} =$$

$$= \frac{(-b - \sqrt{\Delta}) + (-b + \sqrt{\Delta})}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$$

The same we could do with another pattern, which state that $x_1 x_2 = \frac{c}{a}$, we would be proving this next.

Proof of Quadratic Formula- II

Let's prove that:

$$x_1 x_2 = \frac{c}{a}$$

When Δ is positive we have two roots:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}, \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Substituting for x_1 and x_2 respectively, we receive:

$$x_1 x_2 = \frac{-b - \sqrt{\Delta}}{2a} * \frac{-b + \sqrt{\Delta}}{2a} =$$

$$= \frac{(-b - \sqrt{\Delta}) * (-b + \sqrt{\Delta})}{2a} = \frac{b^2 - \Delta}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

Glossary

verb	noun	meaning
add	addition	$+$
subtract	subtraction	$-$
multiply	multiplication	$*$
divide	division	$\div$
solve	solution	getting answer
substitute	substitution	$t = x^2$

Table 1: Terms we come across

Some Necessary and Useful Vocabulary

- (n.) sign $\rightarrow + \text{ or } -$
- (n.) equation $\rightarrow$ *some* $f(x) = 0$
- (n.) factor $\rightarrow$ two multiplied factors give result
- (v.) factorise $\rightarrow$ putting into brackets
- (n.) coefficient $\rightarrow$ a constant number i.e. a, b, c in a pattern $ax^2 + bx + c$
- (n.) quadratic function $\rightarrow f(x) = ax^2 + bx + c$
- (n.) root $\rightarrow$ solution of quadratic equation
- (n.) formula = generalised solution technique