

Part - B Asynchronous Sequential circuit

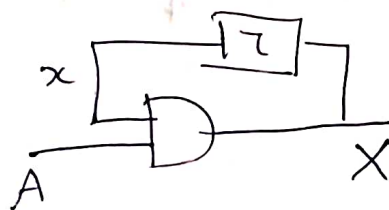
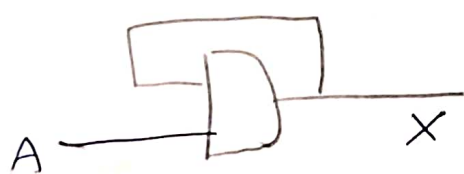
- Asynchronous sequential circuit also called event driven circuit does not have any clock to trigger change of state.
- state changes are triggered by change in input signal.

11-8 Analysis of asynchronous sequential circuit

- memory is the most important element in sequential circuit.
- In synchronous system we use clock driven flip flops which we cannot use here.
- This is done through feedback similar to basic latch portion of a flip flop.
- First we discuss how a two input AND gate and two input NAND gate behave with output fed back to one of the input.

AND Gate :

- The two input AND gate with output fed back as one input is shown below



X \ A	0	1
0	0	0
1	0	1

$$X = X \cdot A$$

→ The circuit can be redrawn that includes the effect of propagation delay of the gate (τ)

→ τ → Finite time after which a gate reacts to its input.

→ If x is current output obtained following logic relation & x is the feedback output we write

$$x = x(t - \tau)$$

→ The truth table is also called state table and each location in k map a state of asynchronous sequential ckt.

→ The encircled states indicate stable condition of the circuit.

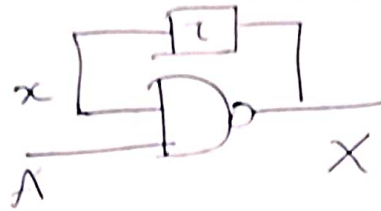
→ $A = 0$ $x = 0$ then $X = x \cdot A = 0 \cdot 0 = 0$

→ After time $t = \tau$ x takes the value X

→ $A = 0$, $x = 0$ represents a stable state and is encircled.

→ similarly $x = 0$, $A = 1$ / $x = 1$, $A = 1$ → stable state.

NAND Gate :



	A = 0	A = 1
x = 0	1	1
x = 1	0	0

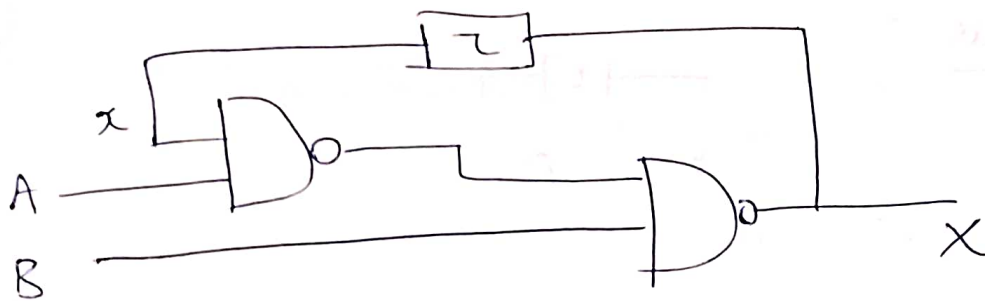
$$X = \overline{x \cdot A}$$

$$\begin{aligned} & \begin{cases} x=0 & A=0 & X=1 \\ & x=1 & A=0 \end{cases} \end{aligned}$$

- For $A = 1$ there is no stable state
 - $x = X'$ for both $x=0$ & $x=1$
 - Thus there is oscillation between $x=0$ & $x=1$ state
- $x=0 \quad A=1 \quad x=1$
 $x=1 \quad A=1 \quad x=0$
- $11 \rightarrow 0$
nand

Two input NAND Latch :

- In analysis of sequential circuit there is an important constraint to be followed.
- Though there can be more than one input feeding the circuit, at a time only one input variable can change.
- The other input can change only when the circuit is stabilized following the previous input change.
- If there are two or more output variables only one output variable can change.



x	AB			
	00	01	10	11
0	1	0	0	1
1	1	0	1	1

- If $X = x$ the circuit is stable.
- Arrows show the movements from transient states.
- let us see how input changes affect the output.

Input AB change from 00 to 01 :

- The circuit moves from $xAB = 100$, a stable position to $xAB = 101$, which is unstable and then moves to $xAB = 001$.
- This is a stable state that has the output 0.
- Therefore $00 \rightarrow 01$ transition in AB has output X making $1 \rightarrow 0$ transition.

Input AB changes from 00 to 10:

→ The circuit moves from $xAB = 100$, a stable position to $xAB = 110$.

→ Therefore a $00 \rightarrow 10$ transition in AB does not alter the value of output $x = 1$

→ AB cannot change from 00 to 11 as there will be a finite delay.

→ Thus the transition path of AB is either $00 \rightarrow 01 \rightarrow 11$ or $00 \rightarrow 10 \rightarrow 11$

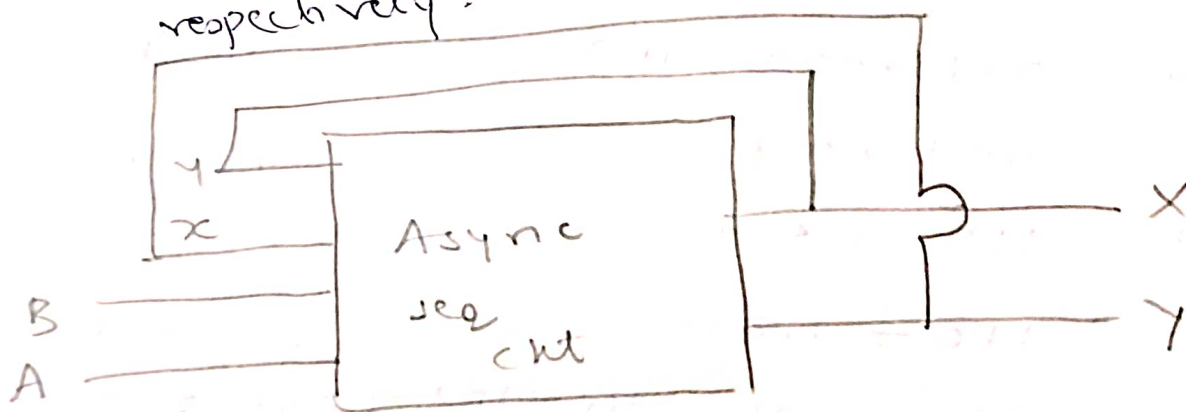
depending on which of A or B changes earlier.

<u>input AB</u>	state (xAB) transition	output x
$00 \rightarrow 01$	$100 \rightarrow 101 \rightarrow 001$	$1 \rightarrow 0 \rightarrow 0$
$00 \rightarrow 10$	$100 \rightarrow 110$	$1 \rightarrow 1$
$01 \rightarrow 00$	$001 \rightarrow 000 \rightarrow 100$	$0 \rightarrow 1 \rightarrow 1$
$01 \rightarrow 11$	$001 \rightarrow 011$	$0 \rightarrow 0$
$10 \rightarrow 00$	$110 \rightarrow 100$	$1 \rightarrow 1$
$10 \rightarrow 11$	$110 \rightarrow 111$	$1 \rightarrow 1$
$11 \rightarrow 01$	$011 \rightarrow 001$, $111 \rightarrow 101 \rightarrow 001$	$\left. \begin{array}{l} 0 \rightarrow 0, 1 \rightarrow 0 \\ 0 \rightarrow 1, 1 \rightarrow 1 \end{array} \right\}$
$11 \rightarrow 10$	$011 \rightarrow 010 \rightarrow 101$, $111 \rightarrow 110$	

Problems with Async seq ckt:

(34)

- Before we go for the design of asynchronous sequential circuit we will look at a few design related issues.
- There are no issues in sync ckt.
- Async ckt responds to all the transient values and problems like oscillation, critical race, hazards can cause major problem.
- EX :- ckt with two inputs A, B and two outputs x, y.
- Both the outputs are fed back to the input side in the form of x & y but with different prop delays.
- Thus x, y cannot change simultaneously
- They change with time delays τ_1 & τ_2 respectively.



→ Truth table → k-map.

xy \ AB				
	00	01	11	10
00	00	11	00	01
01	00	01	00	00
11	11	11	00	11
10	10	11	00	10

oscillation.

critical race

non critical race

oscillation :

→ $xyAB = 0000$

$x = x \quad y = y$

→ $AB \rightarrow 00 \text{ to } 10.$

$xyAB = 0010$

%r $x \neq y$

→ This is a transient state because $xy \neq xy$

→ After time t_2 the circuit goes to

$xyAB = 0010$

→ Thus the circuit oscillates between state 0010 & 0110.

→ output y oscillates between 0 & 1 with a time gap t_2 .

→ In asynchronous sequential circuits for any given input, transitions between two unstable states like these are to be avoided.

Critical race:

(36)

- This occurs when an input change tries to modify more than one output.
- consider the stable state $xyAB = 0000$.
- AB changes to 01 ckt moves to $xyAB = 0001$ where $xy = 11$.
- now depending which of τ_1 and τ_2 is lower. xy moves from 00 to either 01 or 10 .
- τ_1 - lower:
 x changes earlier ckt goes to $xyAB = 1001$.
 $xy = 11$.
- τ_2 - lower:
 y changes earlier ckt goes to $xyAB = 0101$.
 $xy = 01$.
- Thus depending on prop delays in feedback path, the circuit settles at two different states, generally two different outputs.
- such a situation is called critical race condition and is to be avoided in asyn seq ckt.

→ Race can be non critical, in which case its presence does not pose any problem for the circuit

→ Stable state $xyAB = 1110$

→ If AB changes to 11, the circuit goes to $xyAB = 1111$ where output $xy = 00$.

→ Again depending on propagation delays xy becomes either 01 or 10.

→ In both cases the final state is 0011 and output = 00. ~~since~~

→ since the race condition does not lead to two different state it is termed as non-critical race.

Hazards :-

→ Static & dynamic hazards causes mal functioning of asyn seq ckt.

→ In circuit with feedback even when these hazards are adequately covered there can be another problem called essential hazard.

→ This occurs when change in input does not reach one part of the circuit while from other part one output fed back to the input side

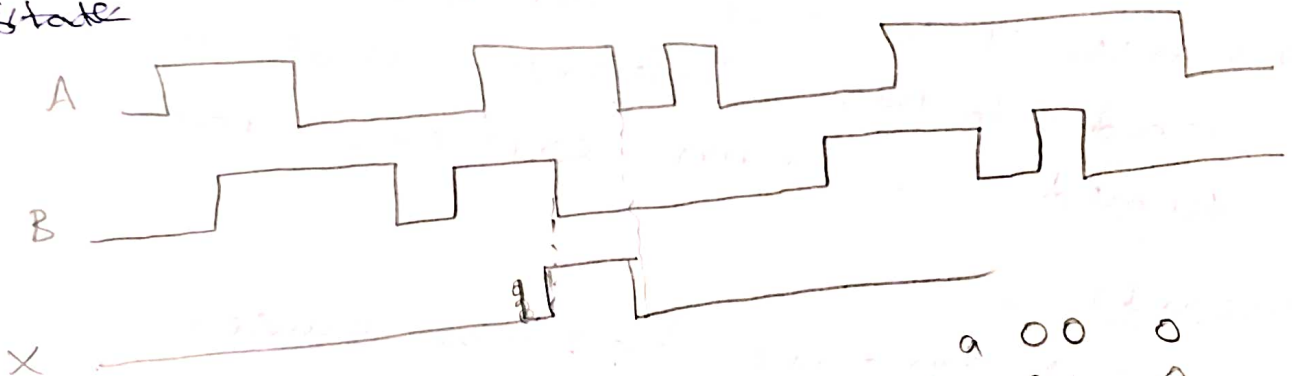
→ Essential hazard can be avoided. (38)
 by adding delay, may be in the
 form of additional gates that does
 not change the logic level.

Design of asynchronous sequential circuit:

The problem:

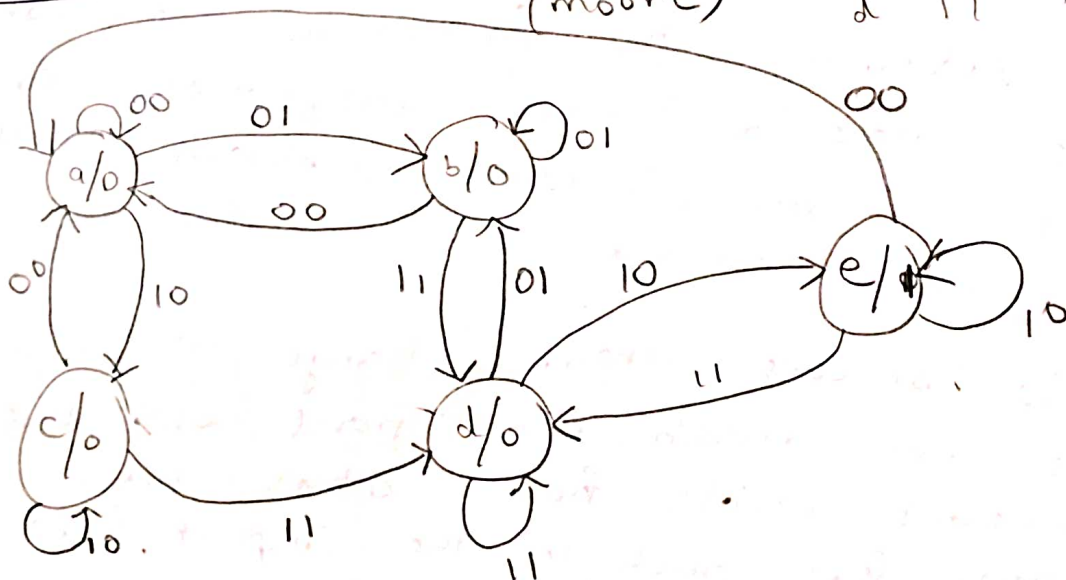
- Two inputs A, B , one output X
- X goes high if $A=1$ B makes transition $1 \rightarrow 0$.
- X remains high as long as this
 $A=1$ $B=0$ are maintained.

State



a	00	0
b	01	0
c	10	1
d	11	0

State transition diagram: (moore)



Primitive table :

- Next step is to form state table from state transition diagram.
- In this table if all the rows representing a state has only one stable state for all possible input combinations it is termed as primitive table or primitive flow table.
- Each row in this table has one dont care state.
- The dont care state in each row comes to a condition which asks both input to change at the same time.
- This condition is not allowed in an asynchronous sequential logic.
- The suffix like 1, 2 are given (not compulsory)
- This will be useful when we check state redundancy.

state redundancy. AB

	00	01	11	10	X
a	(a)	b	x1	c-	0
b	a	(b)	d	x2	0
c	a	x3	d	(c)	0
d	x4	b	(d)	e	0
e	a	x5	d	(e)	1

State reduction

b	d, x ₁			
	c, x ₂			
c	d, x ₁	b, x ₃		
	b, x ₃	c, x ₂		
d	d, x ₁	a, x ₄	a, x ₄	
	c, e	e, x ₂	b, x ₃	
e			c, e	
	a	b	c	d

→ c column → c, e condition is not satisfied
 ∴ e cannot be equal to a, b, c, d.

→ b column → b with d → not possible
 ∴ e, x₂ → not possible

→ b column → b, x₃ → possible if $\begin{matrix} x_3 = b \\ x_2 = c \end{matrix}$
 c, x₂

(bc) → Partition group.

→ a column → a with d → not possible
 ∴ c, e → not possible.

→ a column. → let $x_1 = d, x_2 = c, x_3 = b$

$\begin{matrix} d, x_1 \\ c, x_2 \end{matrix} \Rightarrow \begin{matrix} d, d \\ c, c \end{matrix} \Rightarrow (ab) \text{ can be grouped}$

→ a column → let $x_1 = d, x_2 = c, x_3 = b$

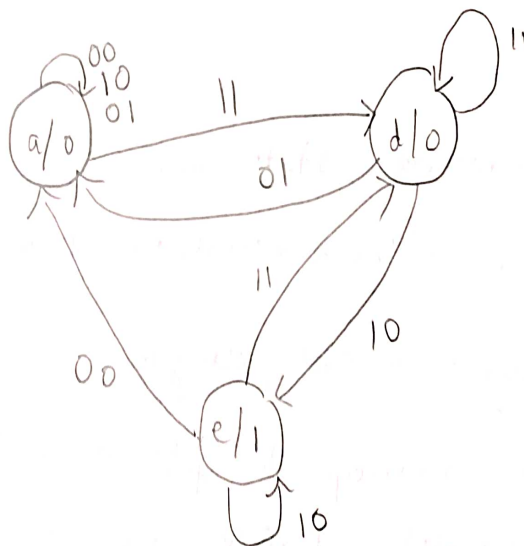
$\begin{matrix} d, x_1 \\ b, x_3 \end{matrix} \Rightarrow \begin{matrix} d, d \\ b, b \end{matrix} \Rightarrow (bac) \text{ can be grouped}$

Partition groups are

$$P = (abc)(d)(e)$$

Reduced state table

	AB				
	00	01	11	10	x
a	(a)	(a)	d	(a)	0
d	x4	a	(d)	e	0
e	a	x5	d	(e)	1



State assignment:

→ suppose $\{a, d, e\}$ is represented by $\{00, 01, 10\}$
it occurs twice for $d \rightarrow e$ $e \rightarrow d$ transition.

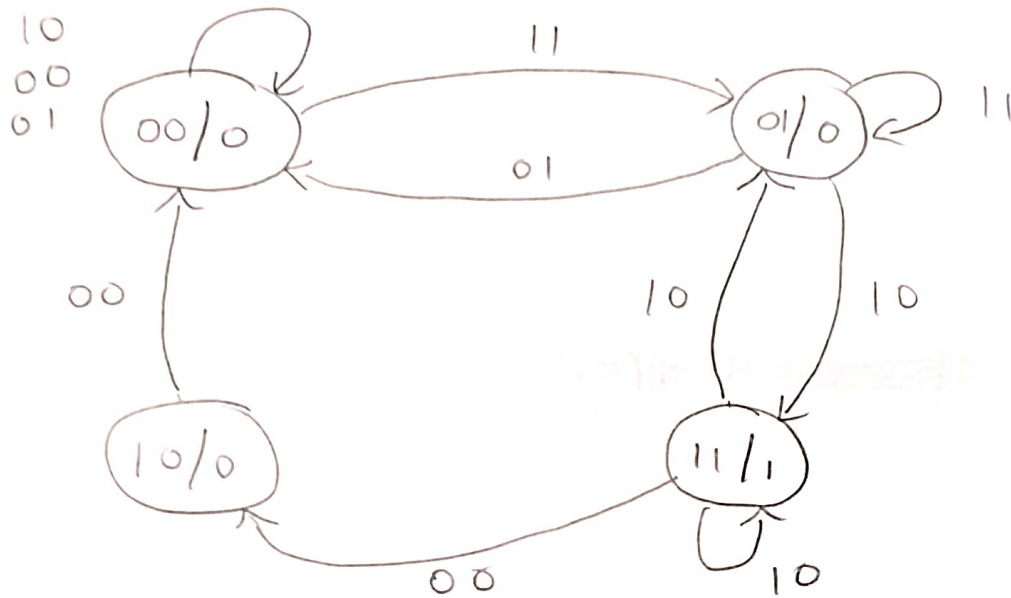
→ There arises a critical race problem
for 01 to 10 transition

→ we can solve this by changing the
representation $\{00, 01, 11\}$

→ Here $e \rightarrow a$ transition cause problem

→ Hence we introduce a dummy variable ϕ between e & a . (4.2)

$a : 00$ $d : 01$ $e : 11$ $\phi : 10$



→ we represent state variables by P & Q , feedback variables p & q .

Design eqns & ckt diagram :

→ we use k-map to get expression of state variables P & Q as function of input A, B & p & q .

→ $A, B \rightarrow$ input variables $\rightarrow 00, 01, 11, 10$

→ $p, q \rightarrow$ states, $a, d, e, \phi \rightarrow 00, 01, 11, 10$

→ Inside kmap fill up the next states.
 PQ

→ Prepare two kmaps to derive equation for P & equation for Q .

AB

PQ	00	01	11	10
a ← 00	00	00	01	00
d ← 01	X	00	01	11
e ← 11	10	X	01	11
f ← 10	00	X	X	X

↓ ↓
PQ

eqn for P

AB

PQ	00	01	11	10
00	0	0	0	0
01	X	0	0	1
11	1	X	0	1
10	0	X	X	X

$$P = \bar{Q} \bar{B}$$

eqn for Q

AB

PQ	00	01	11	10
00	0	0	1	0
01	X	0	1	1
11	0	X	1	1
10	0	X	X	X

$$Q = \bar{Q}A + AB$$

PQ	0/1
00 →	0
01 →	0
10 →	0
11 →	1

PQ

	0	1
0	0	0
1	0	1

$$X = PQ$$