2021 Scheme

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NITTE EDUCATION TRUST

NITTE MEENAKSHI INSTITUTE OF TECHNOLOGY

An Autonomous Institution Approved by UGC/AICTE/Govt. of Karnataka
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Affiliated to Visveswaraya Technological University, Belagavi
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DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

MSE 2 SCHEME AND SOLUTION

Course Title with code	Theory of Computation(21CS52)	Maximum Marks	30 Marks
Date and Time	9/1/2024, 2.00-03.00pm	No. of Hours	1.0
Course Instructor(s)	Mrs. Shobha / Mrs. Shruthi Shetty J/Mrs. Sowmya P		

1. *Q1 (6 Marks) & Q6 (4 Marks) - Compulsory questions.*
2. *Q2 and Q5 Choice-based questions for 10 marks each.*
3. *Missing Data (if any) can be suitably assumed.*

Q. No	Question	MAX MARKS
1. a	$S \rightarrow aA$ $A \rightarrow aA \mid \epsilon$	2m
	Solution :- The grammar is Type2 and Type3.	[1M]
	Technical Justification :- Since the grammar consists of ϵ on the right hand of production, it can't be Type1, Also the grammar is a set of terminal strings that derive from the start symbol, and each production is left linear. Hence it can be considered as Type 2 and Type 3 grammar.	[1M]
b)	$S \rightarrow aSbS \mid aS \mid \epsilon$, The Parse Tree for the sentence "aab" is	

[1m]

Therefore since two parse trees are there for string aab, the [1m]

grammar is ambiguous.

c) <u>The Language of PDA</u>

1) <u>Acceptance by final state</u> [1M]

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA, then $L(M)$, the language accepted by final state is defined as

$L(M) = \{ w \mid (q_0, w, Z_0) \vdash^{*}_{P} (p, \epsilon, \alpha) \}$ for some state p in F, any stack string $\alpha \in \Gamma^*$.

2) <u>Acceptance by Empty Stack</u> [1M]

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, be a PDA, then $N(M)$ the language accepted by empty stack is defined as

$N(M) = \{ w \mid (q_0, w, Z_0) \vdash^{*} (p, \epsilon, \epsilon) \}$ for any state $p \in Q$.

<u>UNIT-3</u>

2. a) Grammar G generating set of all palindromes over $\Sigma = \{a, b\}$. is given by

$G = (V, T, P, S)$

where $V = \{S\}$

$T = \{a, b\}$ [2M]

$P = \{ S \rightarrow \epsilon \mid a \mid b \mid aSa \mid bSb \}$

S is the start symbol.

ii) Grammar G generating set of all non-palindromes over $\Sigma = \{a, b\}$ is defined as, $G = (V, T, P, S)$

where $V = \{S, A\}$

$T = \{a, b\}$ [2M]

$P = \{ S \rightarrow aSa \mid bSa \mid aAb \mid bAa$

$A \rightarrow aA \mid bA \mid \epsilon \}$

S is the start symbol.

2. b) Ambiguous Grammar

A context free grammar $G = (V, T, P, S)$ is ambiguous if there is atleast one string w in T^* for which we can find, 2 or more different parse trees, each with root labeled 'S' and yield w. [2M]

$S \rightarrow aB \mid bA$

$A \rightarrow aS \mid bAA \mid a$

$B \rightarrow bS \mid aBB \mid b$

Left most derivation for the given grammar. [2M]

$S \Rightarrow aB$

$\Rightarrow aaBB$

$\Rightarrow aaaBBB$

$\Rightarrow aaabBB$

$\Rightarrow aaabbB$

$\Rightarrow aaabbaBB$

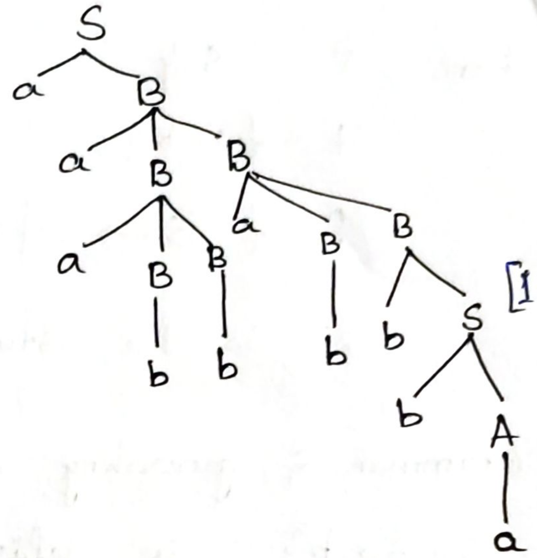
$\Rightarrow aaabbabB$

$\Rightarrow aaabbabbS$

$\Rightarrow aaabbabbbA$

$\Rightarrow aaabbabbba$

Parse Tree



[1M]

The left most derivation by applying some other productions to obtain string 'aaabbabbba'

$S \Rightarrow aB$

$S \Rightarrow aaBB$

$S \Rightarrow aaaBBB$

$S \Rightarrow aaabSBB$

~~S~~ $\Rightarrow aaabbABB$

$\Rightarrow aaabbaBB$

$\Rightarrow aaabbabB$

$\Rightarrow aaabbabbS$

$\Rightarrow aaabbabbbA$

$\Rightarrow aaabbabbba$

[2M]

3 a) $L = L_1 \cup L_2$, grammar for this can be obtained as $S \to S_1 | S_2$

$L_1 = \{ a^n b^m \mid n \geq 0, \ m > n \}$ [2M]

$L_2 = \{ 0^n 1^{2n} \mid n \geq 0 \}$ [2M]

$P = \{ S \to S_1 | S_2$ [1M]

$S_1 \to AB$

$S_2 \to 0S_2 11 | \epsilon$

$A \to aAb | \epsilon$

$B \to bB | b \}$

where $V = \{ S_1, S_2, A, B \}$

$\Sigma = \{ a, b, 0, 1 \}$

S is the start symbol.

3 b) <u>Left Most Derivation</u> , " w = '+*-xyxy' "

$G = E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$

<u>LMD</u> - [2M]

$E \Rightarrow +EE$

$\xRightarrow{lm} +*EEE$

$\xRightarrow{lm} +*-EEEE$

$\xRightarrow{lm} +*-xEEE$

$\xRightarrow{lm} +*-xyEE$

$\xRightarrow{lm} +*-xyxy$

<u>RMD</u> - [2M]

$E \xRightarrow{rm} +EE$

$\xRightarrow{rm} +Ey$

$\xRightarrow{rm} +*EEy$

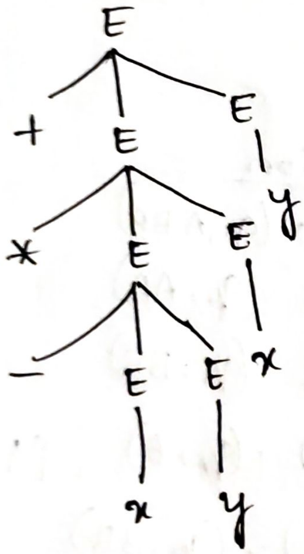
$\xRightarrow{rm} +*Exy$

$\xRightarrow{rm} +*-EExy$

$\xRightarrow{rm} +*-Eyxy$

$\xRightarrow{rm} +*-xyxy$.

<u>Parse Tree</u> [1M]



<u>Unit - 4</u>

4. a) The grammar,

$S \rightarrow aABB \mid aAA$

$A \rightarrow aBB \mid a$

$B \rightarrow bBB \mid A$

$C \rightarrow a$,

For the production,

$B \to A$, since it is not in GNF, convert it to GNF by replacing A by $aBB | a$.

Hence the production will be

$B \to bBB | aBB | a$

GNF conversion - [1M]

$\therefore$ The converted grammar is

$S \to aABB | aAA$

$A \to aBB | a$

$B \to bBB | aBB | a$

$C \to a$

<u>Step 2</u>: Push S on to the stack,

[1M]

$\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

<u>Step 3</u>: Transitions are:

Production	Transition
$S \to aABB$	$\delta(q_1, a, S) = (q_1, ABB)$
$S \to aAA$	$\delta(q_1, a, S) = (q_1, AA)$
$A \to aBB$	$\delta(q_1, a, A) = (q_1, BB)$
$A \to a$	$\delta(q_1, a, A) = (q_1, \epsilon)$ [1M]
$B \to bBB$	$\delta(q_1, b, B) = (q_1, BB)$
$B \to aBB$	$\delta(q_1, a, B) = (q_1, BB)$
$B \to a$	$\delta(q_1, a, B) = (q_1, \epsilon)$
$C \to a$	$\delta(q_1, a, C) = (q_1, \epsilon)$

<u>Step 4</u>: Finally in state q_1, the transition defined is [1M]

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

4. b) PDA for the language $L = \{ww^R \mid w \in (a+b)^*\}$ by a final state.

Solnⁿ: Step 1: Let q_0 be the initial state and Z_0 be the initial symbol on the stack. In state q_0, when the top of the stack is Z_0, whether the input is a or b, push the symbols on to the stack. Remain in state q_0.

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$
$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

Transitions – [2M]

The string w can be ϵ, hence

$\delta(q_0, \epsilon, Z_0) = (q_1, Z_0)$

Once the first scanned symbol, pushed on to the stack, next, push the symbols, until it reaches the mid point.

$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, b, a) = (q_0, ba)$

Since the end of w, and start w^R will be the same character, if both the symbol are same, i.e the input string and the top of stack then the middle part is reached. But this can not be always. Hence considering both case. Therefore following are the transition when mid is not reached.

$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, b) = (q_0, bb)$

The following transition when mid is reached.

$\delta(q_0, a, a) = (q_1, \epsilon)$
$\delta(q_0, b, b) = (q_1, \epsilon)$

Reading the strings of w^R

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

Finally in state q_1, if the string is ww^R, then no input symbol to scanned and the stack should be empty, i.e the stack should contain Z_0. Hence change the state to q_2 and do not alter the contents of the stack.

The transition for this can be,

$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$

As per the condition to be deterministic $\delta(q, a, Z)$ should have only one component.

But for the language, there are 2 transitions each having 2 components, i.e

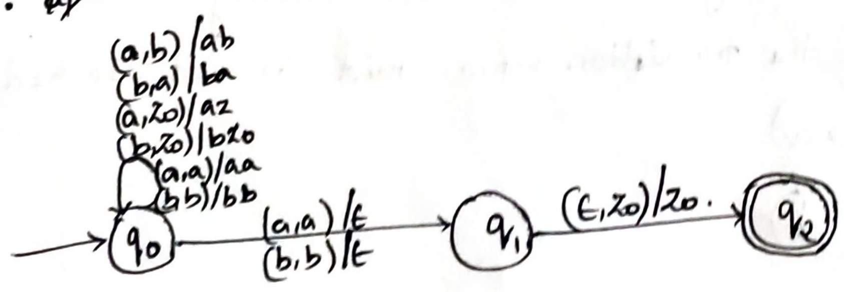
$\delta(q_0, a, a) = (q_0, aa)\ (q_1, \epsilon)$ [2M]

$\delta(q_0, b, b) = (q_0, bb)\ (q_1, \epsilon)$ "Non-deterministic

Hence the first condition fails. Therefore the given PDA is non-deterministic.

[1M]

5. a) Transition Diagram



5 b) $S \to aAa \mid bBb \mid \epsilon$

5 b) $A \to C \mid a$

$B \to C \mid b$

$C \to CDE \mid \epsilon$

$D \to A \mid B \mid ab$

i) eliminate ϵ - productions. [2M]

S, C is Nullable variable, where as A, B, D is Nullable through C.

Therefore the new productions, eliminating 'ϵ'

$S \to aAa \mid bBb \mid aa \mid bb$

$A \to C \mid a$

$B \to C \mid b$

$C \to CDE \mid CE \mid DE \mid E$

$D \to A \mid B \mid ab.$

ii) Eliminating unit Productions [2M]

Following unit productions, $A \to C$, $B \to C$, $C \to E$,

$D \to A$, $D \to B$,

Considering the dependency, the unit pairs obtained are (A, A), (A, C) (A, E), (B, B), (B, C), (C, C), (D, D), (D, A), (D, C), (D, B), therefore the production after eliminating unit productions are

$S \to aAa \mid bB \mid b \mid aa \mid bb$

$A \to a \mid CDE \mid CE \mid DE$

$B \to b \mid CDE \mid CE \mid DE$

C → CDE | CE | DE

D → ab | a | b | CDE | CE | DE

iii) <u>Eliminating useless productions</u> [2M]

1) Identifying Generating variable – S, A, B, D.

Hence the production, by eliminating 'C' will be

S → aAa | bBb | aa | bb

A → a | ~~CDE~~ | ~~CE~~ | ~~DE~~

B → b | ~~CDE~~ | ~~CE~~ | ~~DE~~

D → ab | a | b | ~~CDE~~ | ~~CE~~ | ~~DE~~

2) Identifying the Reachability variable

Here, the variable S is reachable to variable A and B. D is not reachable by S. Hence eliminating the productions with 'D'

The production is

S → aAa | aa | bBb | bb

A → a

B → b

5. a) PDA to accept the language $L = \{a^n b^{2n} \mid n \geq 1\}$ is given by $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

where $Q = \{q_0, q_1, q_2\}$ [02M]

$\Sigma = \{a, b\}$

$\Gamma = \{a, Z_0\}$

$\delta =:$ $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

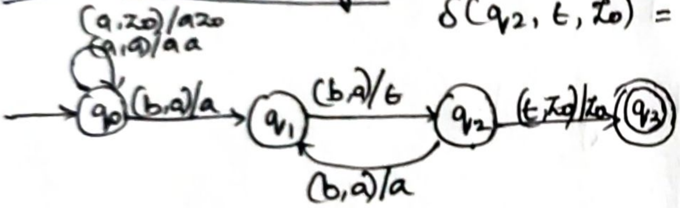
$\delta(q_1, b, a) = (q_2, \epsilon)$

$\delta(q_2, b, a) = (q_1, a)$

$\delta(q_2, \epsilon, Z_0) = (q_3, Z_0)$

$q_0 \in Q$ is start state, $Z_0 \in \Gamma$ is the symbol on the stack. $F = \{q_3\}$ is the final state.

PDA transition diagram



[01M]

Intantaneous description

$(q_0, aaabbbbbb, Z_0) \vdash (q_0, aabbbbbb, aaZ_0)$

$\vdash (q_0, abbbbbb, aa)$

$\vdash (q_0, bbbbbb, aa)$

$\vdash (q_1, bbbbb, aa)$ [1M]

$\vdash (q_1, bbbb, \epsilon)$

$\vdash (q_1, bbb, aa)$

$\vdash (q_1, bb, \epsilon)$

$\vdash (q_1, b, \frac{a}{Z_0})$

$\vdash (q_1, \epsilon, Z_0)$

$\vdash (q_2, \epsilon, Z_0)$

2021 Scheme

6. a) Grammar to generate the Integers.

The production for sign $S \rightarrow + | - | \epsilon$

Decimal digits, $D \rightarrow 0|1|2| \ldots 9$

$G = (V, T, P, S)$, where

for grammar. [2M]

$V = \{D, S, N, I\}$

$T = \{+, -, 0, 1, 2, \ldots 9\}$

$P = \{$

$I \rightarrow N | SN$

$N \rightarrow D | ND | DN$

$S \rightarrow + | - | \epsilon$

$D \rightarrow 0 | 1 | 2 | \ldots | 9$

$\}$

$S = I$, which is the start symbol.

Deriving, 2024

$I \Rightarrow N$

$\Rightarrow ND$

$\Rightarrow N4$

$\Rightarrow ND4$

$\Rightarrow N24$

$\Rightarrow ND24$

$\Rightarrow N024$

$\Rightarrow D024$

$\Rightarrow 2024$

for ID [2M]

Faculty Signature	Course Co-Ordinator/Mentor Signature	HoD Signature
8/01/2024	08/01/2024.	9/1/24