NITTE MEENAKSHI INSTITUTE OF TECHNOLOGY
(AN AUTONOMOUS INSTITUTE AFFILIATED TO VTU, BELAGAVI)

5th Semester Mid-Semester Examination BE Degree (MSE-III Scheme)
Academic year 2023-2024
Department of Computer Science and Engineering
Theory of Computation (21CS52)

1a) i) Recursively Enumerable languages —— (1M)

A language L is recursively enumerable, if it is accepted by a TM, ie., given a string w which is input to TM, the machine halts and outputs Yes if it belongs to the language. If w does not belong to language L, the TM halts and outputs No.

ii) Decidable and Undecidable problems —— (1M)

Any instance of a problem for which the TM halts whether input is accepted or rejected is called decidable problem. And those problems for which TM will not halt are called undecidable problems.

1b) The difference between nondeterministic and deterministic TM lies only in the definition of δ.

For nondeterministic TM, δ is given by

$$Q \times \Gamma = 2^{Q \times \Gamma \times \{L,R\}} \qquad \text{—— (1M)}$$

For each state q & tape symbol X, $\delta(q, X)$ is a set of tuples —— (1M)

$$\{(q_1, X_1, D), (q_2, X_2, D), (q_3, X_3, D) \dots (q_i, X_i, D)\}$$

The machine can choose any of the tuples as next move

1c) Formal definition of TM is given by ~~(2M)~~

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where → (1M)

Q is set of finite states ~~Page 1 of 1.~~

Σ is set of input alphabets

Γ is set of tape symbols
δ is transition function from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\}$
q_0 is start state
B is special symbol indicating blank character
$F \subseteq Q$ is set of final states. (1M)

2a) TM for $L = \{0^n 1^n \mid n >= 1\}$

$W = 000111$

Steps:- (1M)

i) Replace leftmost 0 by X and change the state to q_1 and then move the read-write head towards right. After zero is replaced, replace corresponding 1 so that number of zeroes matches with number of 1's.

ii) Search for leftmost 1 & replace it by the symbol Y and move towards left so as to obtain the leftmost 0. Steps (i) and (ii) can be repeated.

δ transition is as follows. (3M)

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

$\delta(q_2, Y) = (q_2, Y, L)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, X) = (q_0, X, R)$

$\delta(q_0, Y) = (q_3, Y, R)$

$\delta(q_3, Y) = (q_3, Y, R)$

$\delta(q_3, B) = (q_4, B, R)$

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ (1M)

where,

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

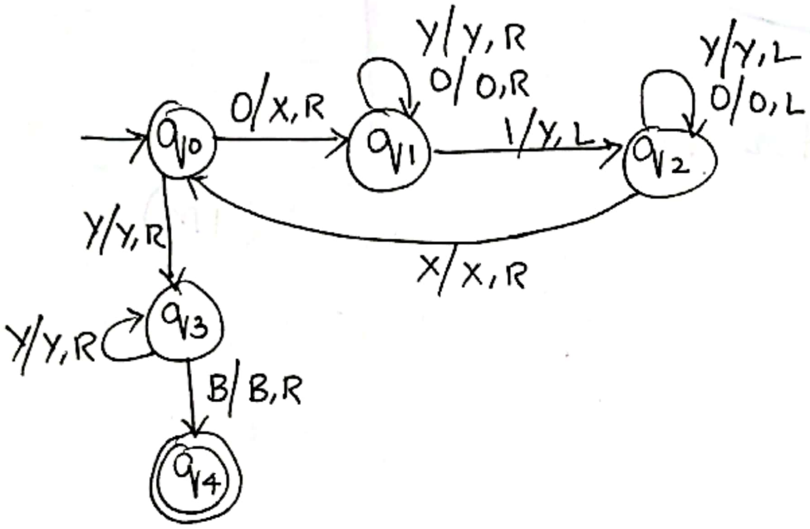
$\Gamma = \{0, 1, X, Y, B\}$

q_0 is start state

$B \in \Gamma$ is blank symbol

q_4 is final state

δ is as shown above



(1M)

2b) The Halting problem can be stated as 'Given a Turing Machine M and an input string w with the initial configuration q_0, after some or all computations do the machine M halts?'. Given the description of any arbitrary TM M and input string w, there should be a single TM that will predict whether or not the computation of M applied to w will halt. (3M)

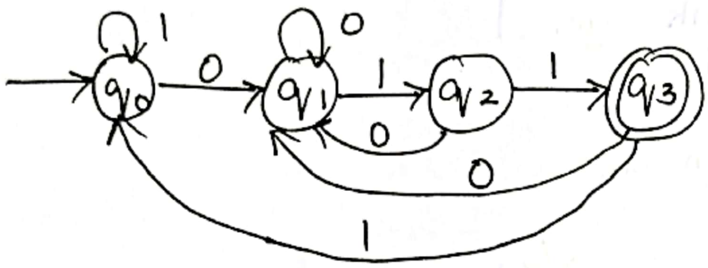
If M enters into a infinite loop, then no matter how long we wait, we can never be sure that M is in fact in a loop. The machine may be in loop because of very long computation.

(1M)

Formally, Halting problem is stated as 'Given an arbitrary TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ and input $w \in \Sigma^*$, does M halt on input w?'

3a) TM to accept language ending with 001

DFA for accepting language ending with 001



(2M)

Transition table for DFA

(1M)

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
$* q_3$	q_1	q_0

Using the above transition table we can write the transition function for TM as follows (2M)

δ	0	1	B
q_0	$q_1, 0, R$	$q_0, 1, R$	—
q_1	$q_1, 0, R$	$q_2, 1, R$	—
q_2	$q_1, 0, R$	$q_3, 1, R$	—
q_3	$q_1, 0, R$	$q_0, 1, R$	q_4, B, R
q_4	—	—	—

3b) A Multi-tape TM is a standard TM with more number of tapes. Each tape is controlled independently with independent read-write head. (2M)

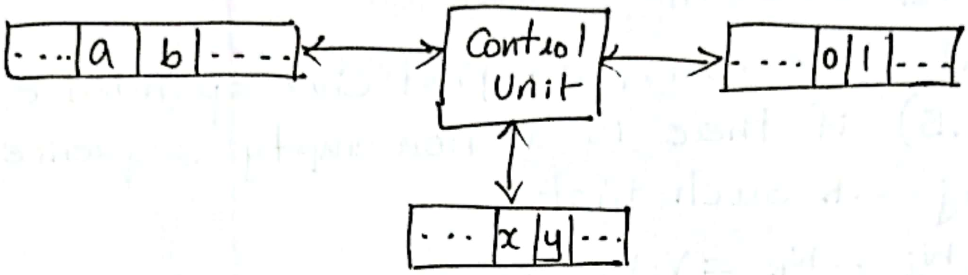


Fig: Multi-tape TM

The formal definition of Multi-tape TM is (1M)

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$Q \rightarrow$ set of finite states

$\Sigma \rightarrow$ set of input symbols

$\Gamma \rightarrow$ set of tape symbols

$\delta \rightarrow Q \times \Gamma^n = Q \times \Gamma^n \times \{L, R\}^n$

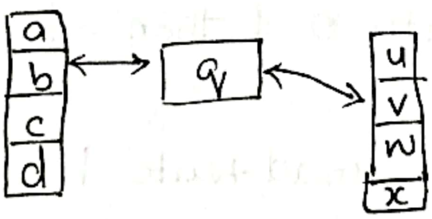
$q_0 \rightarrow$ start state

$B \rightarrow$ Blank character

$F \rightarrow$ final state.

The move of multi-tape TM depends on current state & scanned symbol by each of tape heads.

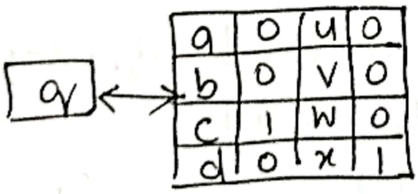
(1M)



Multitape

The above 2-tape TM can be simulated using single-tape TM which has 4 tracks as shown below.

(1M)



The machine in state q and when first read/write head points to symbol c & second read/write head points to symbol x, then machine enters into state p, if & only if this transition is defined for TM with multi-tapes.

(2M)

4a) The Post Correspondence problem can be stated as.
Given two sequences of n strings on some alphabet Σ

$A = w_1, w_2, w_3 \dots w_n$

$B = v_1, v_2, v_3 \dots v_n$

We say that there exists a Post Correspondence solution for pair (A,B) if there is a non empty sequence of integers $i, j \dots k$ such that

$w_i w_j \dots w_k = v_i v_j \dots v_k$.

Given $w_1 = 10, w_2 = 011, w_3 = 101$ (1M)

$x_1 = 101, x_2 = 11, x_3 = 011$

~~For any~~ Here we don't get any PC-solution such that $w_i w_j \dots w_k = x_i x_j \dots x_k$ (1M)

Any string composed of elements of A will be longer than the corresponding string from B. So we can tell that given instance of PCP has no solution. (2M)

4b) Let $x = 4$ and $y = 3$

Represent it using unary notation

$x = 1111$ $y = 111$

Store x on tape, end with 0 & then store y as below

1111 0 111 B

Replace 1 by 1 and move read-write head towards right & stay in q_0

$\delta(q_0, 1) = (q_0, 1, R)$ (1M)

On encountering a 0, change state to q_1, replace 0 by 1 & move pointer towards right.

$\delta(q_0, 0) = (q_1, 1, R)$ (1M)

Replace 1 by 1, move read-write head towards right & stay in q_1 only

$\delta(q_1, 1) = (q_1, 1, R)$ (1M)

On encounteing B, change state to q_2, replace B by B & move pointer towards left.

$$\delta(q_1, B) = (q_2, B, L) \quad (1M)$$

change 1 to 0, change state to q_3 & move head towards left.

$$\delta(q_2, 1) = (q_3, 0, L) \quad (1M)$$

5a) $L = \{a^n b^n c^n \mid n >= 1\}$

$w = aabbcc$ (½M)

Replace a by X, b by Y and c by Z as follows

In state q_0, if scanned symbol is a, replace by X & change state to q_1

$$\delta(q_0, a) = (q_1, X, R) \quad (½ M)$$

In state q_1, if encounter a or Y, replace a by a & Y & Y & remain in q_1

$$\delta(q_1, a) = (q_1, a, R) \quad (½ M)$$
$$\delta(q_1, Y) = (q_1, Y, R)$$

In state q_1 on encountering b, change state to q_2 & replace b by Y

$$\delta(q_1, b) = (q_2, Y, R)$$

In state q_2, replace b by b, Z by Z & remain in q_2

$$\delta(q_2, 1) = (q_2, 1, R) \quad (½ M)$$
$$\delta(q_2, Z) = (q_2, Z, R)$$

In state q_2, on encountering c, change state to q_3 & replace c by Z & move left.

$$\delta(q_2, c) = (q_3, Z, L) \quad (½ M)$$

In state q_3, stay in q_3, move left & replace Z by Z, b by b, Y by Y, a by a.

(1M)

$$\delta(q_3, Z) = (q_3, Z, L)$$
$$\delta(q_3, b) = (q_3, b, L)$$
$$\delta(q_3, Y) = (q_3, Y, L)$$
$$\delta(q_3, a) = (q_3, a, L)$$

On encountering X, replace X by X, change state to q_0, move towards right

(½ M)

$$\delta(q_3, X) = (q_0, X, R)$$

In state q_0, if input symbol is Y, change state to q_4 & replace Y by Y

(½ M)

$$\delta(q_0, Y) = (q_4, Y, R)$$

In state q_4, replace Y by Y & stay in state q_4

$$\delta(q_4, Y) = (q_4, Y, R)$$

(½ M)

In state q_4, replace Z by Z & change state to q_5

$$\delta(q_4, Z) = (q_5, Z, R)$$

(½ M)

In state q_5, replace Z by Z. On encountering B, change state to q_6 & replace B by B

$$\delta(q_5, Z) = (q_5, Z, R)$$
$$\delta(q_5, B) = (q_6, B, R)$$

(½ M)

5b) $w_1 = 1$, $w_2 = 10111$, $w_3 = 10$

$v_1 = 111$, $v_2 = 10$, $v_3 = 0$

$\overbrace{1\,0\,1\,1\,1}^{w_2}\ \overbrace{1}^{w_1}\ \overbrace{1}^{w_1}\ \overbrace{1\,0}^{w_3}$ —— (1½M)

$\underbrace{1\,0}_{v_2}\ \underbrace{1\,1\,1}_{v_1}\ \underbrace{1\,1\,1}_{v_1}\ \underbrace{0}_{v_3}$ —— (1½M)

For the given instance, ~~we hav~~ there exists a PC-solution as shown above

So the PC-solution for the given instance is $(2, 1, 1, 3)$ —— (1M)

6a) ~~Repla~~ $a = 1111$, $b = 111$ $w = 11110111$

~~Replace~~ Let q_0 be start state & let read-write head point to first digit of a. (½M)

(i) ~~Replace~~ Replace leftmost digit of a by X, the move till we get leftmost digit of b

(ii) Replace it by X & move left till leftmost 1 of a is obtained.

(iii) Repeat through step (i) till More 1's in a which results in no 1's in b

~~step 1:~~ XXX X10XXX B where $a > b$

In state q_0, if 1 is encountered, change state to q_1, replace 1 by X & move towards right

$\delta(q_0, 1) = (q_1, X, R)$ (½M)

In state q_1, on encountering 0, change state to q_2

$\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_1, 0) = (q_2, 0, R)$ (½M)

In state q_2, if encounter 1, change state to q_3, replace 1 by X, move towards left

$$\delta(q_2, 1) = (q_3, X, L)$$ (½M)

In state q_3, if encounter 0, change state to q_4 & replace 0 by 0

$$\delta(q_3, 0) = (q_4, 0, L)$$ (½ M)

In state q_4, if encounter 1, replace 1 by 1, move towards left. Replace X by X & change state to q_0.

$$\delta(q_4, 1) = (q_4, 1, L)$$

$$\delta(q_4, X) = (q_0, X, R)$$ (1M)

In state q_2 if B is encountered, it means a>b & the machine should enter into final state q_6

$$\delta(q_2, B) = (q_6, B, L)$$ (½ M)

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