AY 2022-23 - V SEM.

Theory of Computation - 21CS52.

Scheme and Solution.

1a. Assume L is regular language $L = \{a^n b^n \mid n \geq 0\}$

$w = xyz = a^n b^n$

Let $x = a^i$, $y = a^{n-i}$, $z = b^n$ — 1

$xy^kz = a^i \cdot a^{(n-i)k} \cdot b^n$ — ½

$k = 0$ $xy^0z = a^i b^0 b^n = a^i b^n$ since $i \neq n \notin L$.

Hence by contradiction $a^n b^n$ not regular using pumping lemma — ½

1b.

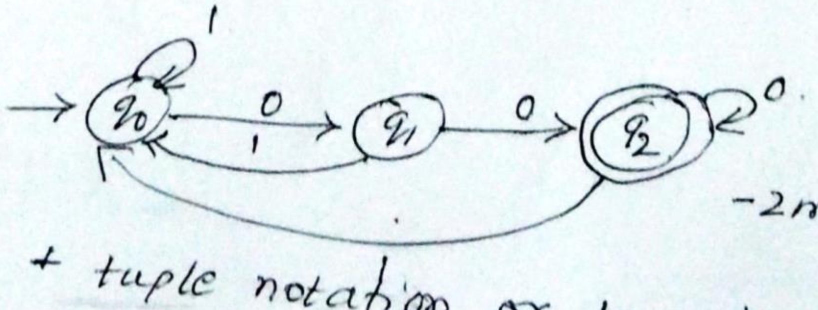
DFA	NFA	ε-NFA
$\delta: Q \times \Sigma \to Q$	$\delta: Q \times \Sigma \to 2^Q$	$\delta: Q \times \Sigma \cup \varepsilon \to 2^Q$ — 1M.
Deterministic	Nondeterministic	nondeterministic
Difficult to design	easy to design	flexible

1c. All strings having atleast 2 0s — 1m

$L = \{00, 100, 000, 001, 010, \ldots\}$ - 1M.

2a.

(i)

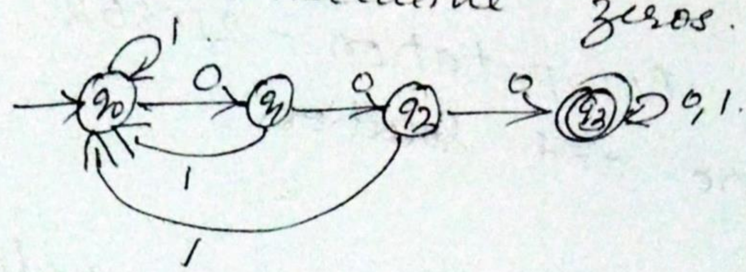


-2m

Page 1

+ tuple notation or transition table.

(ii) Three consecutive zeros.



– 2 marks

2b.

Transition table

δ_N	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$q_1 q_2$	q_1
$* q_2$	q_2	q_1, q_2

Soln

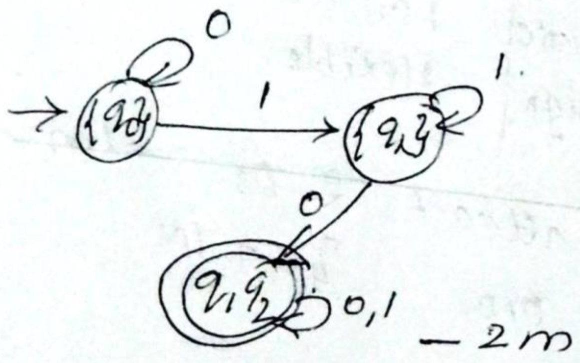
$\delta_D(q_0, 0) = \delta_N(q_0, 0) = \{q_0\}$
$\delta_D(q_0, 1) = \delta_N(q_0, 1) = \{q_1\}$ $\Big\}\frac{1}{2}$ m

$\delta_D(q_1, 0) = \delta_N(q_1, 0) = \{q_1 q_2\}$
$\delta_D(q_1, 1) = \delta_N(q_1, 1) = \{q_1\}$ $\Big\}\frac{1}{2}$ m

$\delta_D(\{q_1 q_2\}, 0) = \delta_N(\{q_1 q_2\}, 0)$
$= \delta_N(q_1, 0) \cup \delta_N(q_2, 0)$ – $\frac{1}{2}$ m
$= q_1 q_2 \cup q_2 = \{q_1 q_2\}$ – 1m

$\delta_D(\{q_1 q_2\}, 1) = \delta_N(\{q_1 q_2\}, 1)$
$= \delta_N(q_1, 1) \cup \delta_N(q_2, 1)$ – $\frac{1}{2}$ m
$= \{q_1\} \cup \{q_1 q_2\}$
$= \{q_1 q_2\}$ – 1 m

DFA



– 2m

NFA to DFA using subset construction

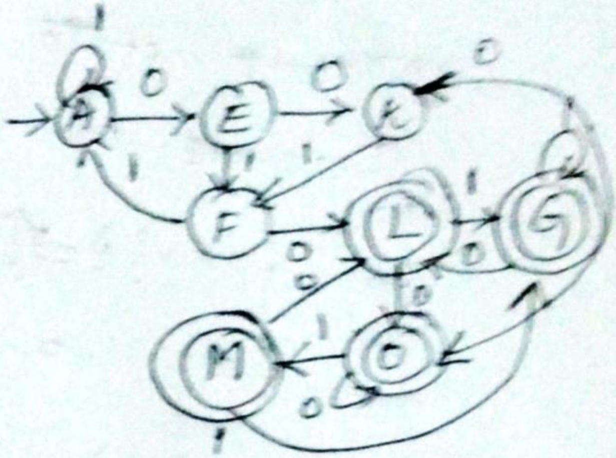
5a Soln

		0	1
A	→ p	{p,q}	{p}
B	q	{r}	{r}
C	r	{s}	ϕ
D	* s	{s}	{s}
E	pq	pqr	pr
F	pr	pqs	p
G	ps	pqs	ps
H	qr	rs	r
I	qs	rs	rs
J	rs	s	s
K	pqr	pqrs	pr
L	pqs	pqrs	ps
M	prs	pqs	ps
N	qrs	rs	rs
O	pqrs	pqrs	prs

Scheme

Table Construction: 4M

DFA transition diagram: 1M



5b

δ	ε	a	b	c
→ p	{q,r}	ϕ	{q}	{r}
q	ϕ	{p}	{r}	{p,q}
r	ϕ	ϕ	ϕ	ϕ

(i) ε-closure $(p) = \{p\ q\ r\}$.
ε-closure $\{q\} = \{q\}$.
ε-closure $\{r\} = \{r\}$ $\quad - \frac{1}{2} m$

(ii) Set of strings of length '3' or less accepted by the automaton. $- \frac{1}{2}$ m.

(iii) Start state of DFA

ε-closure of $p = \{p\ q\ r\}$.

$\delta_D(\{p\ q\ r\}, a) = E(\delta_\varepsilon(p, a) \cup \delta_\varepsilon(q, a) \cup \delta_\varepsilon(r, a))$

$= E\text{-closure}(p) = \{p\ q\ r\}$ $\quad - \frac{1}{2}$ m

$\delta_D(\{p\ q\ r\}, b) = E(\delta_\varepsilon(p, b) \cup \delta_\varepsilon(q, b) \cup \delta_\varepsilon(r, b)$

$= E\text{-closure}(\{q \cup r \cup \phi\})$

$= E\text{closure}(\{q, r\}) = \{E\text{closure}\{q\} \cup$

$E\text{closure}\{r\}\} = \{q\} \cup \{r\} = \underline{\{q, r\}}$ $\quad - \frac{1}{2}$ m

$\delta_D(\{p\ q\ r\}, c) = E(\delta_\varepsilon(p, c) \cup \delta_\varepsilon(q, c) \cup \delta_\varepsilon(r, c))$

$= E(r \cup \{p\ q\}) = E(\{p\ q\ r\})$

$= \{p\ q\ r\} \cup \{q\} \cup \{r\} = \underline{\{p\ q\ r\}}$ $\quad - \frac{1}{2}$

$\delta_D(\{q, r\}, a) = E(\delta_\varepsilon(\{q, r\}, a) = E(p) = \underline{\{p\ q\ r\}}$ $\quad - \frac{1}{2}$

$\delta_D(\{q, r\}, b) = E(\delta_\varepsilon(q) \cup \delta_\varepsilon(r, b)) = E(r) = \underline{\{r\}}$ $\quad - \frac{1}{2}$

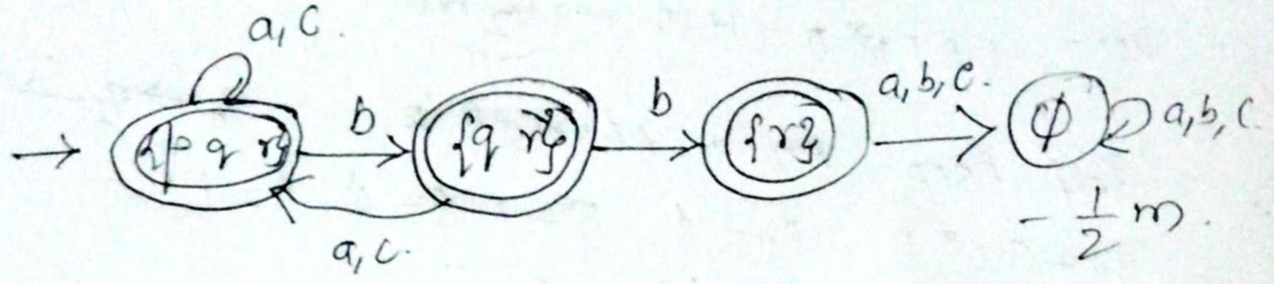
$\delta_D(\{q, r\}, c) = E(\delta_\varepsilon(q, c) \cup \delta_\varepsilon(r, c)) = E(p, q) = \underline{p\ q\ r}$ $\quad - \frac{1}{2}$

$$\delta_D(r,a) = E(\delta_\varepsilon(r,a)) = E(\phi) = \phi$$

$$\delta_D(r,b) = E(\delta_\varepsilon(r,b)) = E(\phi) = \phi$$

$$\delta_D(r,c) = E(\delta_\varepsilon c) = E(\phi) = \phi$$

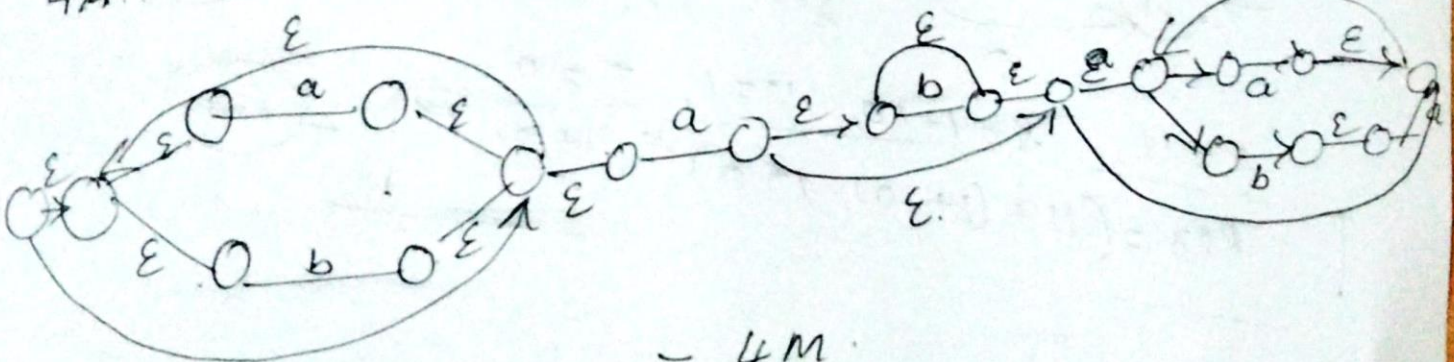
$\frac{1}{2}m$



$-\frac{1}{2}m.$

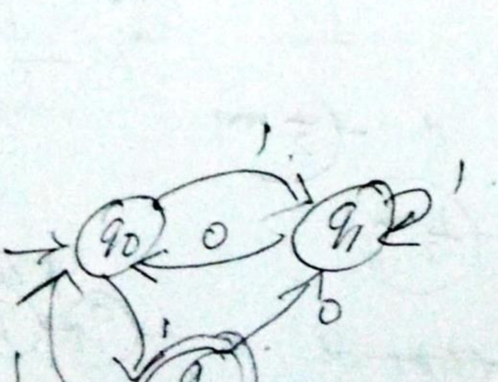
$(a+b)^* ab^*a\ (a+ba)^*$ to εNFA.

4A.



$-\ 4M.$

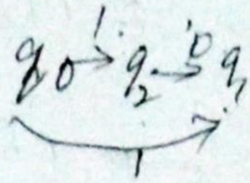
4b.



Soln:

Eliminate q_2

0 - 2 - 1



$RE_1 = R_{11} + QS^*P$

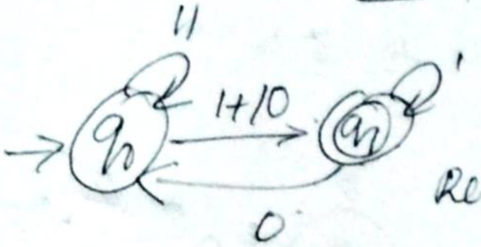
$R_{11} = 1 \quad Q = 1 \quad S = \phi, \ P = 0$

$Re_1 = 1 + 1\phi^*0 = 1+10$ — ½ m

Self loop on '0' through '2' $\quad q_0 \xrightarrow{1} q_2 \xrightarrow{1} q_0$.

$Re_2 = R_{11} + QS^*P$, $R_{11} = \phi$, $Q = 1$, $S = \phi$, $P = 1$.

$Re_2 = \phi + 1 \cdot \phi^* 1 = \underline{\underline{11}}$ — $\frac{1}{2}$ m



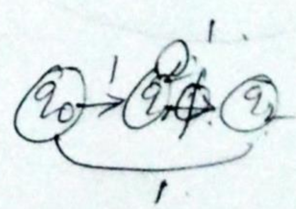
Use stopping condition

$Re_3 = (R + SU^*T)^* SU^*$ — $\frac{1}{2}$ m

$R = 11, \quad S = 1+10, \quad U = 1, \quad T = 0.$

$$\boxed{Re_3 = (11 + (1+10) \cdot 1^* 0)^* \cdot (1+10) \cdot 1^*}$$ — 1 m

Eliminate q_1

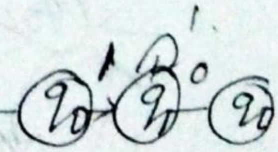


Consider 0 - 1 - 2

$R_{11} = 1, \ S = 1, \ Q = 1, \ P = \phi$ — $\frac{1}{2}$ m

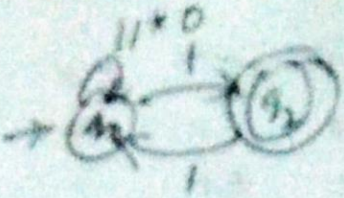
$Re_4 = 1 + 11^*\phi = \underline{\underline{1}}$. — $\frac{1}{2}$ m

Consider loop on q_0 through q_1



$R_{11} = \phi, \ Q = 1, \ S = 1, \ P = 0$ — $\frac{1}{2}$ m

$Re_5 = \phi + 11^*0 = 11^*0$.



Use stopping condition

$R = 11^*0, \; S = 1, \; U = \phi, \; T = 1$

$(11^*0 + 1\phi^*1)^*1\phi^*$ — $\frac{1}{2}$m

$Re_b = (11^*0 + 1\phi^*1)^*1 \cdot \phi^*$

$\boxed{Re_b = (11^*0 + 11)^*1}$ — 1m

Final $RE = Re_5 + Re_6$

$= (11 + (1+10)1^*0)^*(1+10)1^* +$

$(11^*0 + 11)^*1$ — $\frac{1}{2}$m

$(a+b)^*ab + (a+b)^*ba$

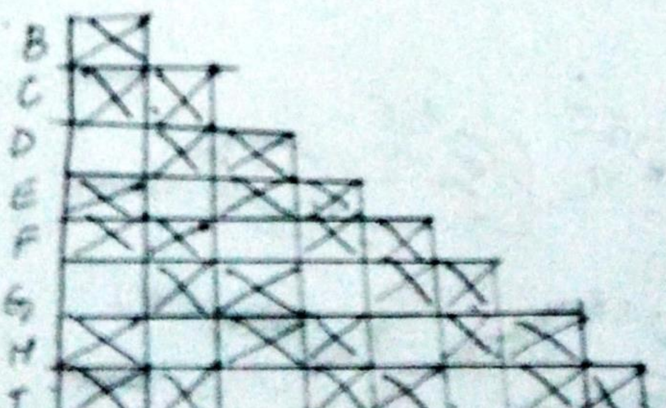
5a

(i) $RE = (a+b)^*ab + (a+b)^*ba$ — 2m

$= (a+b)^*(ab + ba)$.

(ii) $RE = (0+1)^*0(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$ — 2m

5b



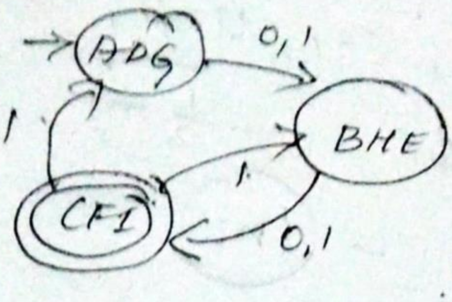
Scheme:

Table filling : 6 mar

→ ADG BHE BHE

BHE CFI CFI

* CFI ADG BHE

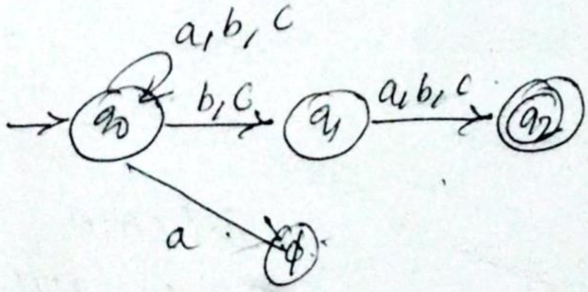


— 2 marks

AB	BC	EF
AD	BE	EH
AE	BF	FI
AG	BH	EB
AH	BI	EC
BD	CE	EH
BE	CF	FI
BG	CH	FB
BH	CI	CC
CF	DG	HI
CI	DA	HE
DE	EF	HI
DG	EH	HB
DH	EI	HC
EG	FH	IB
EH	FI	IC
FI	GA	BE
GH	HI	BC

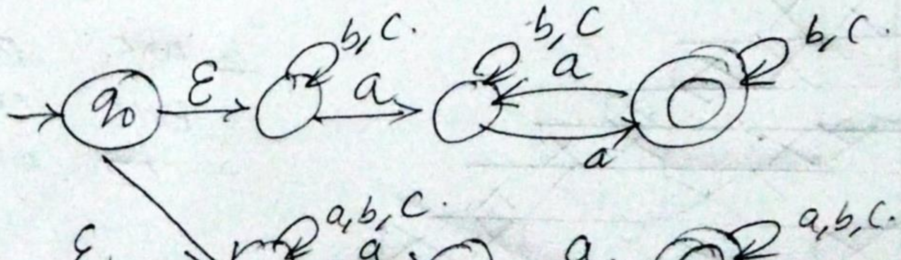
6a:

(i)



- 2m

(ii)



-2m