

TURING MACHINE

1) TM to accept the language $L = \{0^n 1^n \mid n \geq 1\}$. Write Instantaneous description for string 0011

Transitions made by TM are:

$$\delta(q_0, 0) = (q_1, x, R)$$

$$\delta(q_2, 0) = (q_2, 0, L)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

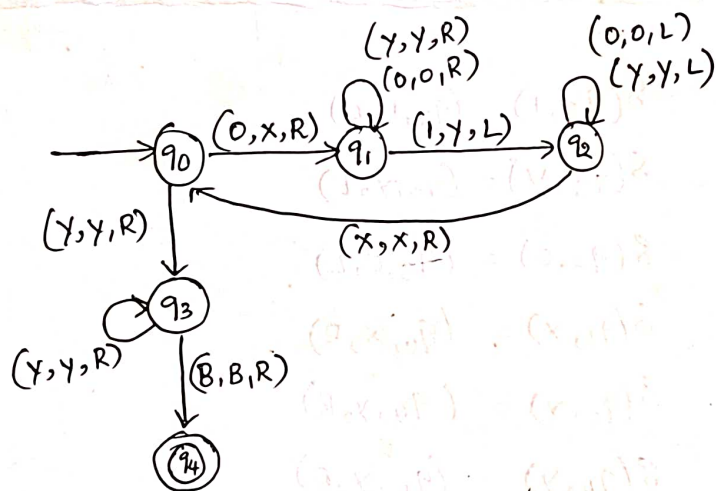
$$\delta(q_1, 1) = (q_2, y, L)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

Transition Diagram:



Note:

If question is $0^n 1^n: n \geq 0$, then you need to add transition from q_0 to final state (q_4) on a Blank Symbol i.e $\delta(q_0, B) = (q_4, B, R)$

Transition Table:

δ Present State	Tape Symbol (Γ)				
	0	1	x	y	B
$\rightarrow q_0$	xRq ₁			yRq ₃	
q ₁	0Rq ₁	yLq ₂		yRq ₁	
q ₂	0Lq ₂		xRq ₀	yLq ₂	
q ₃				yRq ₃	BRq ₄
(q ₄)					

TM is given by:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \text{ where}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Gamma = \{0, 1, x, y, B\}$$

$q_0 \in Q$ is the start state

$$\Sigma = \{0, 1\}$$

$F = q_4$ is the final state

Instantaneous Description (ID) for string 0011

(Initial ID) $q_0 0011 \vdash x q_1 011 \vdash x 0 q_1 11 \vdash x q_2 0 y 1 \vdash q_2 x 0 y 1 \vdash x q_0 0 y 1 \vdash x x q_1 y 1$
 $\vdash x x y q_1 1 \vdash x x q_2 y y \vdash x q_2 x y y \vdash x x q_0 y y \vdash x x y q_3 y \vdash x x y y q_3$
 $\vdash x x y y B q_4$ (Final ID)

2) TM to accept the language $L = \{0^n 1^n 2^n \mid n \geq 1\}$. Write ID for string 001122

Transitions made by TM are:

$$\delta(q_0, 0) = (q_1, x, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, 1) = (q_2, y, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, z) = (q_2, z, R)$$

$$\delta(q_2, 2) = (q_3, z, L)$$

$$\delta(q_3, z) = (q_3, z, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, y) = (q_3, y, L)$$

$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_4, y, R)$$

$$\delta(q_4, y) = (q_4, y, R)$$

$$\delta(q_4, z) = (q_5, z, R)$$

$$\delta(q_5, z) = (q_5, z, R)$$

$$\delta(q_5, B) = (q_6, B, R)$$

TM is given by:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \text{ where}$$

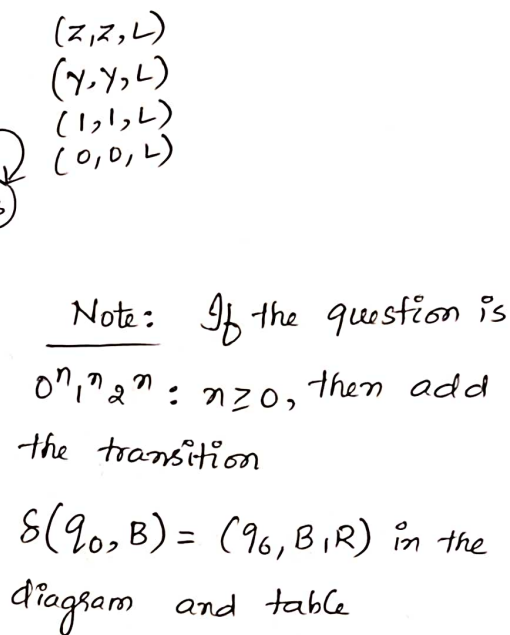
$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{0, 1, 2\}$$

$$\Gamma = \{0, 1, 2, x, y, z, B\}$$

$q_0 \in Q$ is the start state, $F = q_6$ is the final state

states



Transition Table:

8 States	Tape Symbol (τ)						
	0	1	2	x	y	z	B
$\rightarrow q_0$	XR q_1				YR q_4		B
q_1	OR q_1	YR q_2	,		YR q_1		
q_2		IR q_2	ZL q_3			ZR q_2	
q_3	OL q_3	IL q_3		XR q_0	YL q_3	ZL q_3	
q_4					YR q_4	ZR q_5	
q_5						ZR q_5	BR q_6
q_6							

Instantaneous Description (ID) for string 001122:

$q_{0001122} \vdash x q_{01122} \vdash x o q_{11122} \vdash x o y q_{2122} \vdash x o y i q_{222} \vdash x o y q_{3122} \vdash$
 $x o q_{3y122} \vdash x q_{30y122} \vdash q_3 x o y i z z \vdash x q_{00} y i z z \vdash x x q_{1y122} \vdash x x y q_{1122} \vdash$
 $x x y y q_{22} \vdash x x y y z q_{22} \vdash x x y y q_{322} \vdash x x y q_{3y22} \vdash x x q_{3y y z z} \vdash x q_{3x y y z z}$
 $\vdash x x q_{0y y z z} \vdash x x y q_{4y z z} \vdash x x y y q_{422} \vdash x x y y z q_{52} \vdash x x y y z z q_{5} \vdash$
 $x x y y z z B q_6 \text{ (Final ID)}$

3) Obtain TM to accept set of all palindromes over $\{0,1\}^*$. Write ID function $N = W^R$

Transitions made by TM are:

$$\delta(q_0, 0) = (q_1, x, R)$$

$$\delta(q_0, 1) = (q_2, y, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_3, B, L)$$

$$\delta(q_1, x) = (q_3, x, L)$$

$$\delta(q_1, y) = (q_3, y, L)$$

$$\delta(q_2, 0) = (q_2, 0, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_4, B, L)$$

$$\delta(q_2, x) = (q_4, x, L)$$

$$\delta(q_2, y) = (q_4, y, L)$$

$$\delta(q_3, 0) = (q_5, x, L)$$

$$\delta(q_4, 1) = (q_5, y, L)$$

$$\delta(q_5, 0) = (q_5, 0, L) \quad \delta(q_5, 1) = (q_5, 1, L) \quad \delta(q_5, x) = (q_0, x, R) \quad \delta(q_5, y) = (q_0, y, R)$$

For odd length palindrome:

$$\delta(q_3, x) = (q_6, x, R)$$

$$\delta(q_4, y) = (q_6, y, R)$$

For even length palindrome:

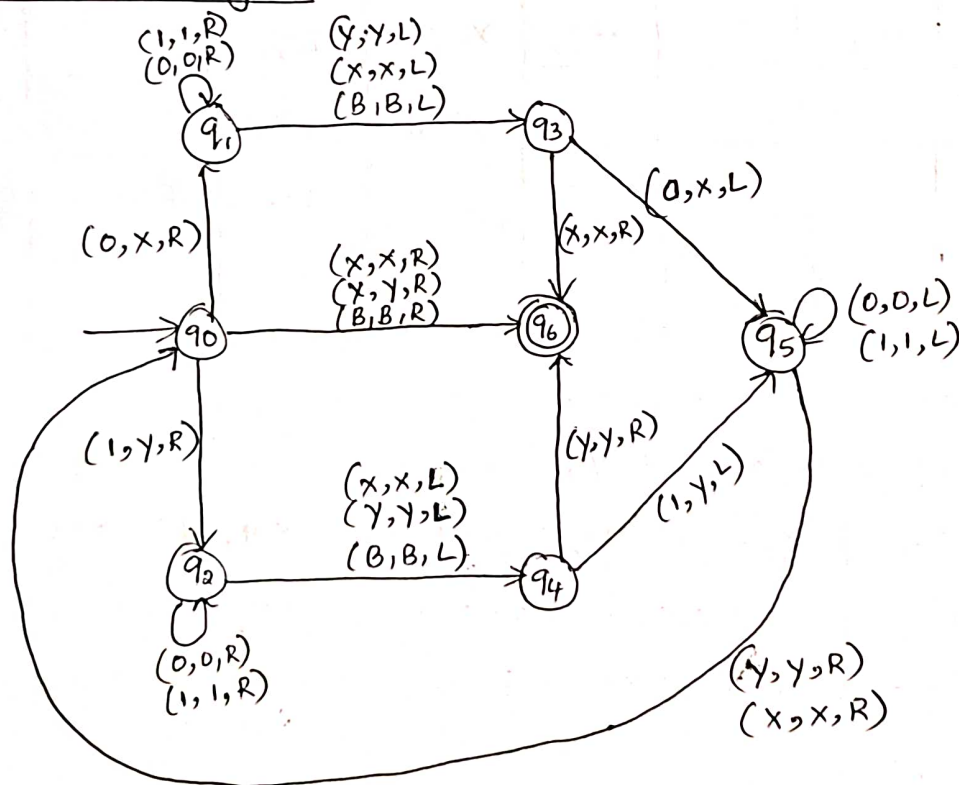
$$\delta(q_0, x) = (q_6, x, R)$$

$$\delta(q_0, y) = (q_6, y, R)$$

$$\delta(q_0, B) = (q_6, B, R)$$

Since ϵ is accepted

Transition Diagram:



Note: 1) If question is $\{0,1\}^+$, then you need to remove the transition $\delta(q_0, B) = (q_6, B, R)$ from the diagram

2) If the question is $WW^R = \text{Set of palindromes of even length}$ then remove two transitions from diagram i.e $\delta(q_3, x) = (q_6, x, R)$ and $\delta(q_4, y) = (q_6, y, R)$

Transition Table:

States	Tape Symbol (Γ)				
	0	1	x	y	B
$\rightarrow q_0$	XRq_1	YRq_2	XRq_6	YRq_6	BRq_6
q_1	ORq_1	IRq_1	XLq_3	YLq_3	BLq_3
q_2	ORq_2	IRq_2	XLq_4	YLq_4	BLq_4
q_3	XLq_5		XRq_6		
q_4		YLq_5		YRq_6	
q_5	OLq_5	ILq_5	XRq_6	YRq_6	
(q_6)					

T_M is given by:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \text{ where}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\} \quad \Sigma = \{a, b\} \quad \Gamma = \{a, b, x, y, B\} \quad F = q_6$$

ID for string 1001:

$$\begin{aligned} & q_0 1001 \vdash Yq_2 001 \vdash Y0q_2 01 \vdash Y00q_2 1 \vdash Y001q_2 \vdash Y00q_4 1 \vdash Y0q_5 0Y \vdash \\ & Yq_5 00Y \vdash q_5 Y00Y \vdash Yq_0 00Y \vdash Yxq_1 0Y \vdash Yx0q_1 Y \vdash Yxq_3 0Y \vdash \\ & Yq_5 xxY \vdash Yxq_0 xy \vdash Yxxq_6 Y \text{ (Final ID)} \end{aligned}$$

4) T_M to accept strings of a's and b's such that $N_a(w)$ is equal to $N_b(w)$

Write Instantaneous description for string bbabaa

Transitions made by T_M is given as follows:

$$\delta(q_0, B) = (q_5, B, R) \rightarrow (\text{Since } \epsilon \text{ is accepted})$$

On Encountering a in state q_0 :

$$\delta(q_0, a) = (q_1, x, R) \quad \delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_1, a) = (q_1, a, R) \quad \delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_1, y) = (q_1, y, R) \quad \delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

On Encountering symbol b in state q_0

$$\delta(q_0, b) = (q_3, x, R)$$

$$\delta(q_4, b) = (q_4, b, L)$$

$$\delta(q_3, b) = (q_3, b, R)$$

$$\delta(q_4, y) = (q_4, y, L)$$

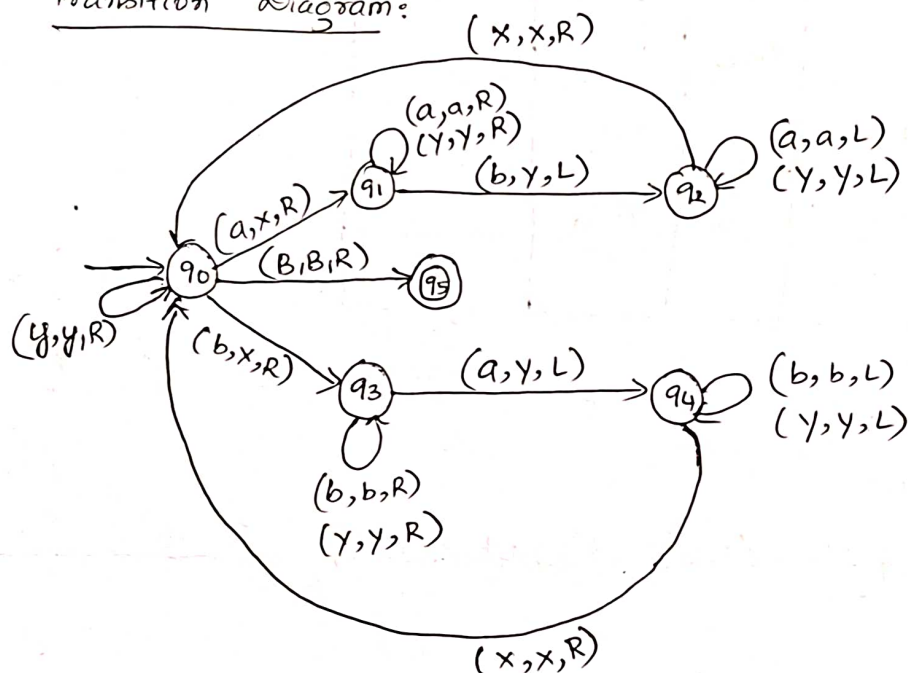
$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_4, x) = (q_0, x, R)$$

$$\delta(q_3, a) = (q_4, y, L)$$

$$\delta(q_0, y) = (q_0, y, R)$$

Transition Diagram:



Transition Table:

S states	Tape Symbol (T)				
	a	b	x	y	B
$\rightarrow q_0$	XRq ₁	XRq ₃		YRq ₀	BRq ₅
q ₁	aRq ₁	YLq ₂		YRq ₁	
q ₂	aLq ₂		XRq ₀	YLq ₂	
q ₃	YLq ₄	bRq ₃		YRq ₃	
q ₄		bLq ₄	XRq ₀	YLq ₄	
(q ₅)					

ID for string bbabaa is

- (Initial ID) $q_0 b b a b a a \vdash x q_3 b a b a a \vdash x b q_3 a b a a \vdash x q_4 b y b a a \vdash q_4 x b y b a a \vdash$
 $x q_0 b y b a a \vdash x x q_3 y b a a \vdash x x y q_3 b a a \vdash x x y b q_3 a a \vdash x x y q_4 b y a \vdash x x q_4 y b y a$
 $\vdash x q_4 x y b y a \vdash x x q_0 y b y a \vdash x x y q_0 b y a \vdash x x y x q_3 y a \vdash x x y x y q_3 a \vdash x x y x q_4 y y$
 $\vdash x x y q_4 x y y \vdash x x y x q_0 y y \vdash x x y x y q_0 y \vdash x x y x y y q_0 \vdash x x y x y y B q_5$ (Final ID)