

**Hypothesis** is defined as an **Educated guess about Population** and a **A prediction of the relationship b/w or more variables**.

**Null Hypothesis**

A statement which states that there is no relationship b/w the variables.

**Alternate Hypothesis**

A statement which states that there is relationship b/w the variables.

## Statistical Tests :-

- They are conducted to test the hypothesis and to find the inferences about the ~~pop~~ population.
- For that, samples are ~~are~~ selected & various tests are performed on them to find the ~~to~~ inference about the population under study.

## Parametric Tests

- They are applied under the circumstances where the population is normally distributed or is assumed to be normally distributed.
- Parameter like mean, standard deviation etc are used.
- For example, T-test, Z-Test, F-Test, ANOVA, Pearson's Correlation Coefficient.
- These are applied where data is quantitative,  
Numerical.

## Non Parametric Test

- ~~At~~ They are applied under the circumstances where the population is not normally distributed (skewed distribution) or is not assumed to be normally distributed.

- Where parametric tests cannot be applied, then non-parametric tests ~~can~~ come into play. (Where we have no idea about the ~~pop~~ population)
- These tests are called as Distribution-free tests.  
Independent of Pop. distribution
- Parameters like mean, std etc are not used.
- For eg, Chi-square test, U-test (Mann-Whitney Test), H-Test (Kruskal-Wallis Test), Spearman's Rank correlation Test.
- These are applied where data is qualitative <sup>categorical</sup>.
- Applied where scale of measurement is either an ordinal or a nominal scale.

Parametric Test

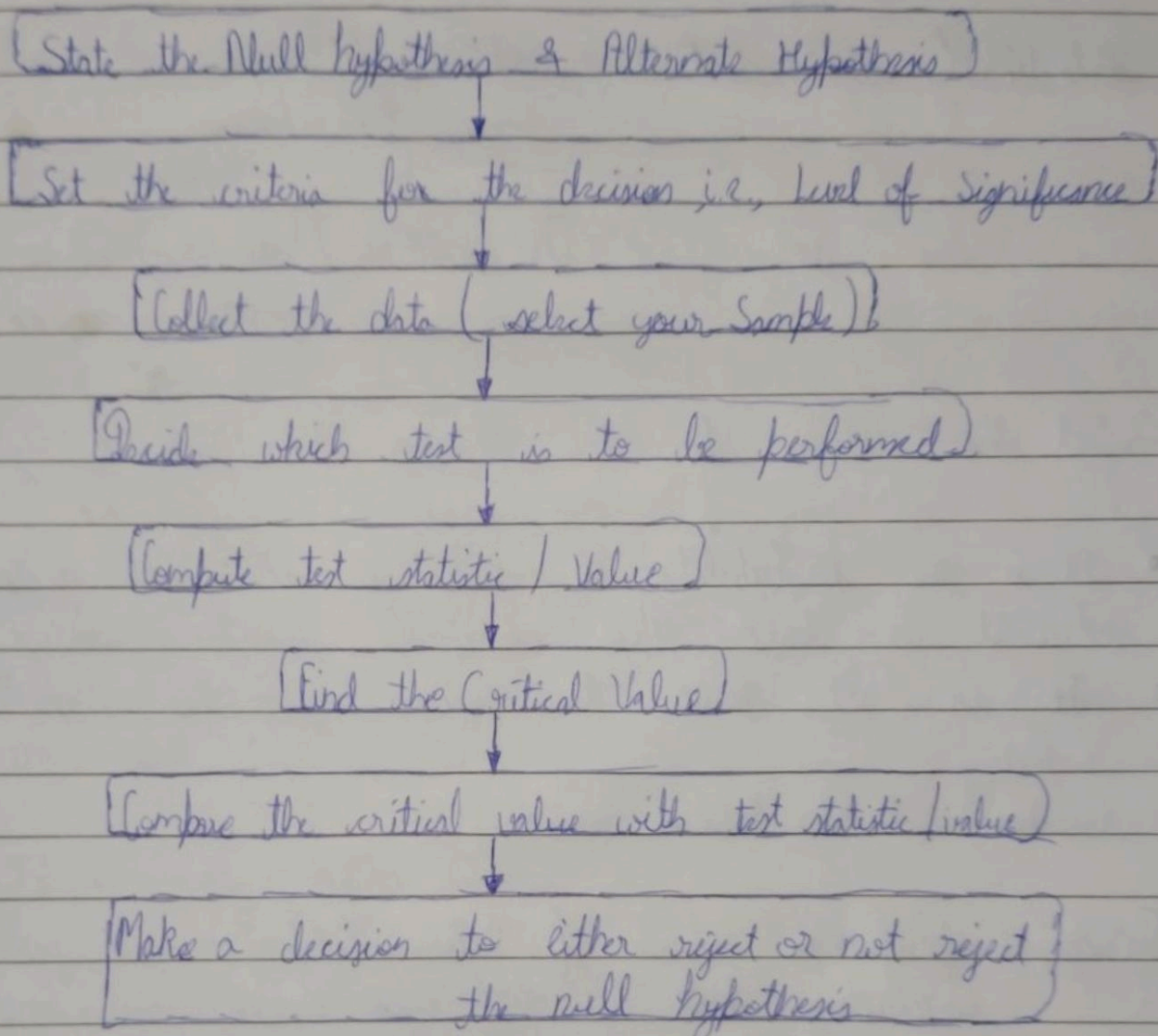
Non Parametric Test.

Advantage :- More Powerful  
Disadvantage :- Less Robust

More Robust  
Less Powerful.



## Steps in Hypothesis Testing?



## Analysis of each step:-

i) Set the null & alternate hypothesis:-

- Null Hypothesis:  $\Rightarrow$  A statement which assumes that there is no significant relationship b/w the variables.  
 $\Rightarrow$  It is to be tested for its verification and validity.

- Alternate Hypothesis: A statement which assumes that there is a significant relationship b/w the variables.

⇒ Also called as Research Hypothesis.

Statistically significant :- implies difference in result ~~test~~ <sup>doesn't</sup> occur by chance.

The relation b/w variables is not just by chance.

ii) Set the criteria for the decision, i.e., Level of Significance

- It is the probability of rejecting the null hypothesis when it is true. Also called as Type I error.
- It is set prior to conducting the hypothesis testing.
- Can be set at 5% or lower.

For eg, significance level of 5% indicates, a 5% risk of concluding that a difference exists when there is no actual difference.

- Lower significance level indicates that stronger evidence is required before rejecting the null hypothesis.
- It is denoted by ' $\alpha$ '.



iii) Select the sample. (Sampling methods)

iv) Decide the test and compute the value of test statistic.

v) Find critical value

- Critical value is the cut-off value which is to be compared with the test value to take a decision about the null hypothesis.
- It divides the graph into two sections: Rejection area and Acceptance area.
- If test value falls into the rejection area, then reject the null hypothesis.
- It is derived from the level of significance of the test.
- It is the table value of level of significance of the test.
- ~~It is the table value~~
- The table/critical value of 5% level of significance is 1.96.

vi) Compare the critical value with Test Value

If value of test statistic is greater than the critical value  $\Rightarrow$  Reject the null hypothesis

If the value of test statistic is less than ~~and~~ critical value  $\Rightarrow$  Fail to reject the null hypothesis

What is p-value?

- It is the probability of getting the value of our test statistic if the null hypothesis is true.
- The lower the p-value, the stronger is the evidence that the null hypothesis is false.
- Since, p-value is a probability value, therefore, it will always lie b/w 0 & 1.
- A high p-value indicates ~~that~~ the observed results are likely to occur by chance under the null hypothesis & that's why, in this case, null hypothesis would not be rejected.
- On the other hand, a low p-value indicates that the results are less likely to occur by chance under the null hypothesis & hence, in this case, null hypothesis would be rejected.

Relationship - b/w p-value, Critical Value & Test statistic

- The benefit of using p-value is that it calculates probability estimate, which can be tested at any desired level of significance by comparing this probability directly with significance level.
- For eg, assume z-value comes out to be 1.98 which is greater than ~~than~~ the critical value at 5% which is 1.96.



- Now, to check for a different significance level of 1%, a new critical value is to be calculated.
- But by calculating the p-value, no critical value is then to be calculated.
- We can compare p-values directly with significance level (5% or 1%).

## Parametric Tests

### T-Test

- It is a parametric test of Hypothesis testing based on Student's T distribution.
- It was developed by William Sealy Gosset.
- It is essentially, testing the significance of the difference of the mean values when the sample size is small (i.e., less than 30) & when population standard deviation is not available.
- It assumes:-
  - ⇒ Population distribution is normal.
  - ⇒ Samples are random & independent.
  - ⇒ Sample size is small.
  - ⇒ Population standard deviation is ~~small~~ not known.
- Mann-Whitney 'U' Test as a non-parametric counterpart of T-test



A T-Test can be a

One Sample T-test

To compare sample mean with that of population mean.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$\bar{x}$  is the sample mean

$s$  is sample std

Two Sample T-test

To compare means of two different samples.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\bar{x}_1 \rightarrow$  sample mean of 1st grp

$\bar{x}_2 \rightarrow$  " " " 2nd grp

$s_1 \rightarrow$  sample std of 1st group

$s_2 \rightarrow$  " " " 2nd group

If the value of test statistic is greater than the table value

$\Rightarrow$

Reject the null hypothesis

If the value of test statistic is less than the table value

$\Rightarrow$

Do not Reject the null hypothesis.

## Z-Test

- It is a parametric test of hypothesis testing.
- It is used to determine whether the means are different when the population variance is known & sample size is large (i.e., greater than 30).
- It assumes:
  - ⇒ Population distribution is normal
  - ⇒ Samples are random & independent.
  - ⇒ Sample size is large.
  - ⇒ Population standard deviation is known.

(A Z-Test can be a)

One Sample Z-Test

To compare sample mean with that of population mean.

Two Sample Z-Test

To compare means of two different samples

$$Z\text{-test} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$



T-test is used when



Sample size is small and the population variance is not known

Z-test is used when



Sample size is large and population variance is known

Sample size is large and the population variance is not known.



Z-test is used.

Sample size is small and population variance is known.



Z-test is used.

### F-Test

- It is a parametric test of hypothesis testing based on Snedecor F-distribution.
- F-test is named after its test statistic,  $F$ .
- It is a test for the null hypothesis that two normal populations have the same variance.
- An F-test is regarded as a comparison of equality of sample variances.

- F-statistic is simply a ratio of two variances.
- It is calculated as:-

$$F = \frac{s_1^2}{s_2^2}, \text{ where } s^2 = \frac{\sum_{i=1}^n (x - \bar{x})^2}{n-1}$$

- By changing the variances in the ratio, F-test becomes a very flexible test. It can ~~be~~ then be used to:-
  - ⇒ Test the overall significance for a regression model.
  - ⇒ To compare the fits of different model.
  - ⇒ To test equality of means.
- It assumes:-
  - ⇒ Population distribution is normal.
  - ⇒ Samples are drawn randomly & Independently.

## ANOVA

- Also called as (Analysis of Variance), it is parametric test of hypothesis testing.
- It was developed by Ronald Fisher, also referred to as Fisher's ANOVA.
- It is an extension of T-test & Z-test.
- It is used to test the significance of the differences of the mean values among more than two sample groups.



- It uses F-test to statistically test the equality of means & the relative variance b/w them.

- It assumes:

- ⇒ Population distribution is normal.

- ⇒ Samples are random & independent.

- ⇒ Homogeneity of sample variances.

- One-way ANOVA & Two-ways ANOVA are its types.

$$F\text{-statistic} = \frac{\text{Variance b/w the sample means}}{\text{Variance within samples}}$$

### Chi-Square Test

- It is a non parametric test of hypothesis testing.

- As a non parametric test, chi square can be used i.

- i) As a goodness of fit

- ii) As a test of independence of two variables.

- It helps in assessing the goodness of fit b/w a set of observed values and those expected theoretically.

- It makes comparison b/w the expected & observed frequencies.

- Greater the difference, greater is the value of chi square.

- If there is no diff. b/w expected & observed frequencies, then the value of Chi-Square is 0.

- It is also called as the 'Goodness of Fit Test' which determines whether a particular distribution fits the observed data or not.

- It is calculated as:-

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$\swarrow$  Observed freq.       $\searrow$  Expected freq.

- Chi-Square is also used to test the independence of two variables.

- Conditions of Chi-Square Test:

⇒ Observations recorded & used are collected on a random basis.

⇒ All the items in sample must be independent.

⇒ No grp should contain very few items, say less than 5.

⇒ The overall no. of items must also be reasonably large.

It should normally be at least 50, however small the no. of grps may be.

- Chi Square can be used as a parametric test to test for a population variance on the basis of sample variance.



- If we take each one of a collected of sample variances, divide them by the known population variance and multiply the quotient by  $(n-1)$ , where  $n$  means the no. of items in the sample, we get the value of chi square.

- It is calculated as:

$$\chi^2 = \frac{\sigma_s^2}{\sigma_p^2} (n-1)$$

### Mann-Whitney U-Test

- It is a non-parametric test of Hypothesis testing.
- This test is used to investigate whether 2 independent samples were selected from population having same distribution.
- It is a true non-parametric ~~counter~~ counterpart of T-test & gives the accurate estimates of significance especially when sample sizes are small & population is not normally distributed.
- It is based on the comparison of every observation in the first sample ~~sets~~ with every observation in the other sample.
- The test statistic used here is 'U'.

- Max. value of 'U' is ' $n_1 * n_2$ ' and min. value is 0.

### "Kruskal - Wallis H-Test"

- It is a non parametric test of hypothesis testing.
- This test is used for comparing two or more independent samples of equal or different sample sizes.
- It extends the Mann Whitney U-Test which is used for comparing only 2 groups.
- One way ANOVA is the parametric equivalent of this test. And that's why, it's also called as 'One way ANOVA on ranks'.
- It uses ranks instead of actual data.
- ~~It uses ranks on~~
- It doesn't assume the pop<sup>n</sup> to be normally distributed.
- The test statistic used here is 'H'.