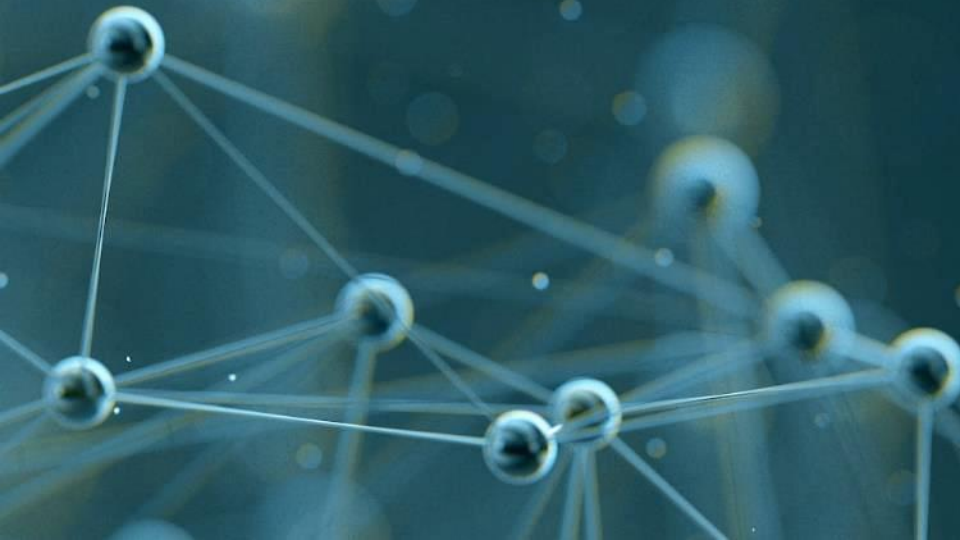
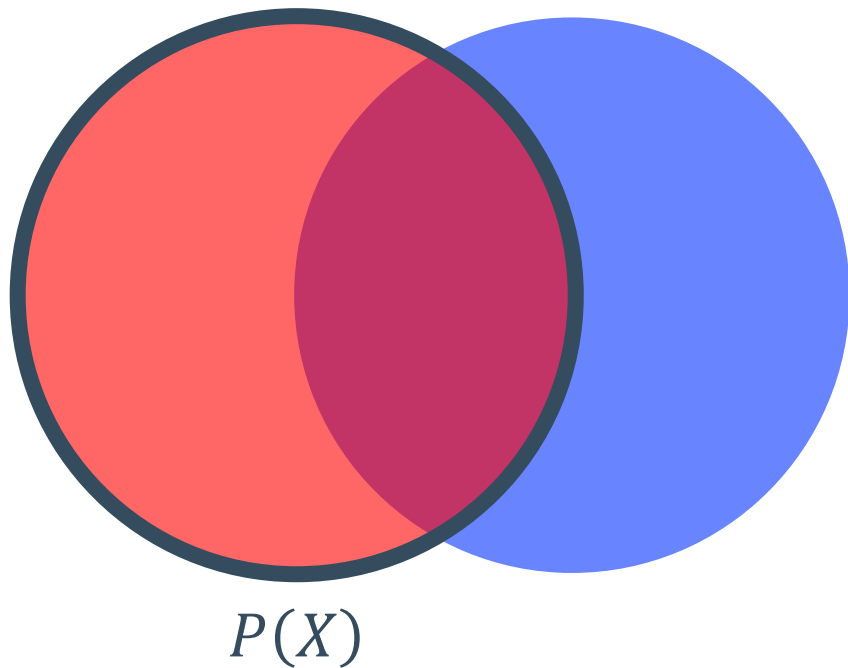


# NAÏVE BAYES

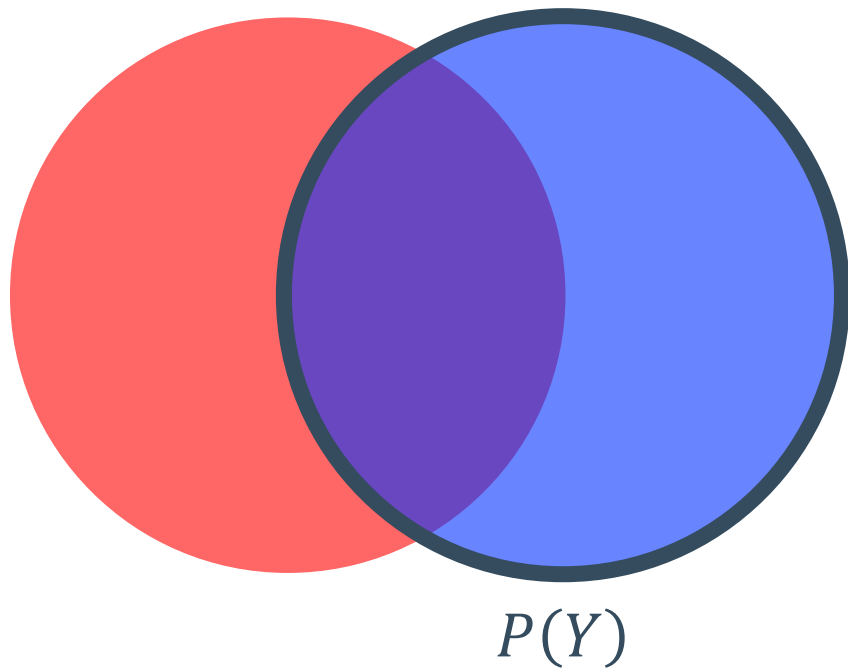


# PROBABILITY BASICS



LinSingle event probability:  
 $P(X)$

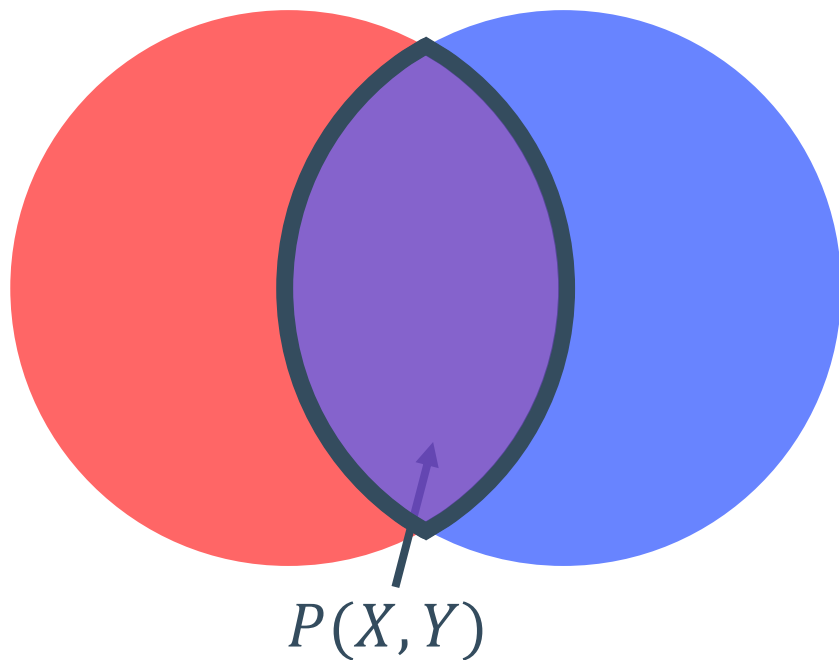
# PROBABILITY BASICS



LinSingle event probability:

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# PROBABILITY BASICS



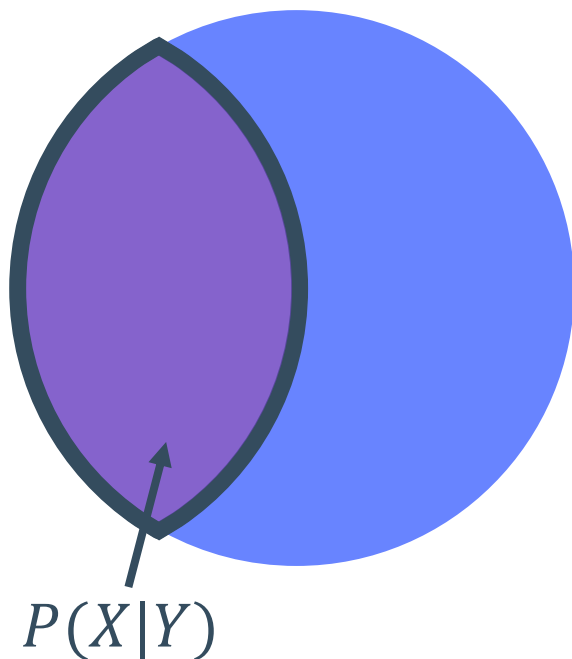
Single event probability:

$$P(X), P(Y)$$

Joint event probability:

$$P(X, Y)$$

# PROBABILITY BASICS



**Single event probability:**

$$P(X), P(Y)$$

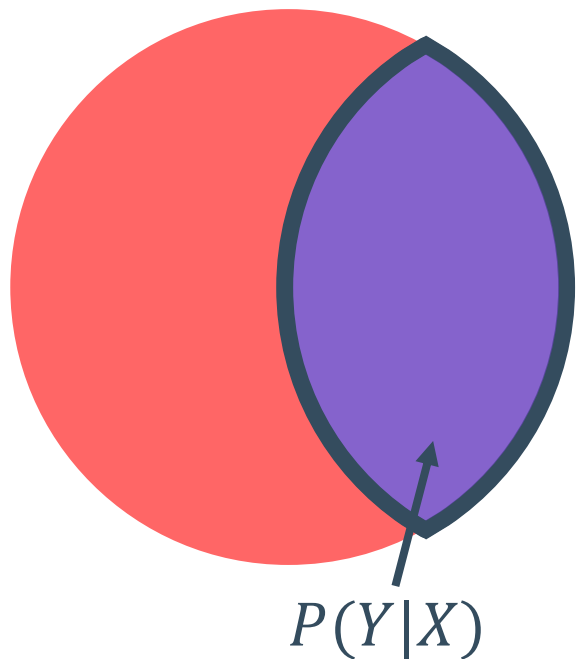
**Joint event probability:**

$$P(X, Y)$$

**Conditional probability:**

$$P(X|Y)$$

# PROBABILITY BASICS



**Single event probability:**

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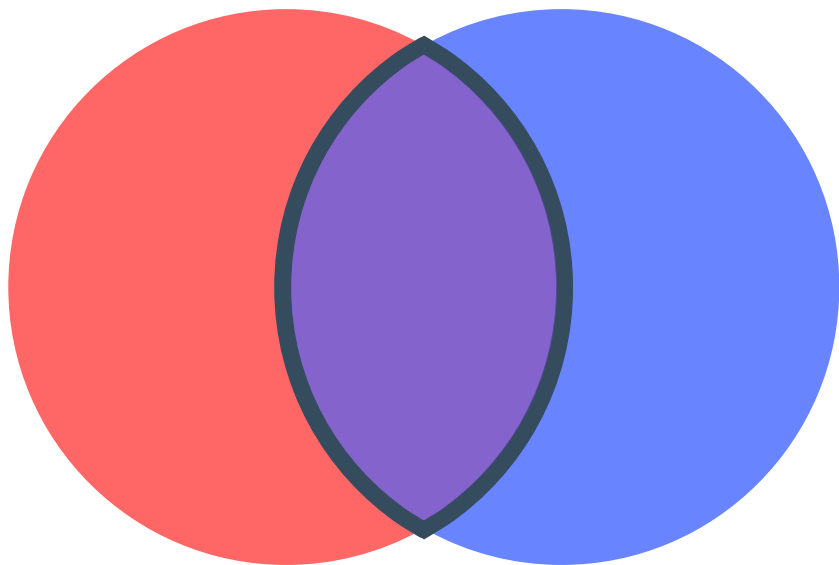
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# PROBABILITY BASICS



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**Joint event probability:**

$$P(X, Y)$$

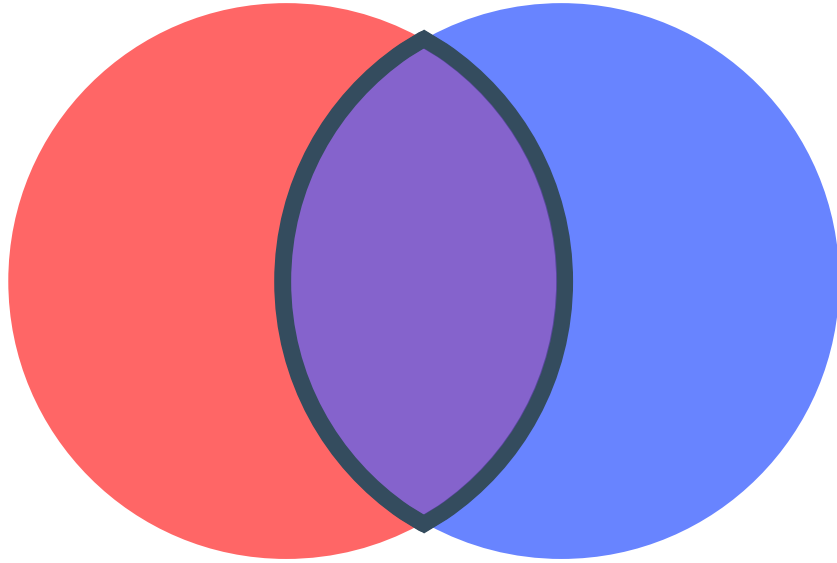
**Conditional probability:**

$$P(X|Y), P(Y|X)$$

**Joint and conditional relationship:**

$$P(X, Y) = P(Y|X) * P(X) = P(X|Y) * P(Y)$$

# BAYES THEOREM DERIVATION

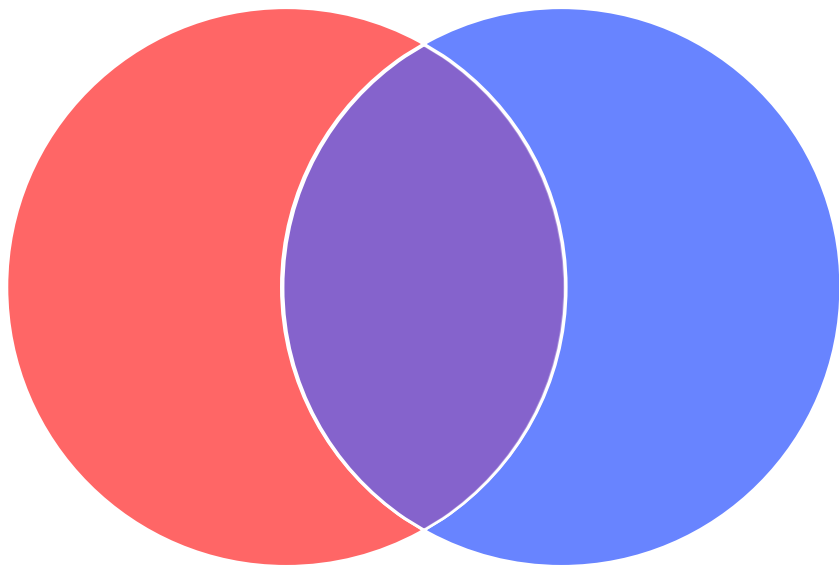


By conditional and joint relationship:

$$P(Y|X) * P(X) = P(X|Y) * P(Y)$$



# BAYES THEOREM DERIVATION



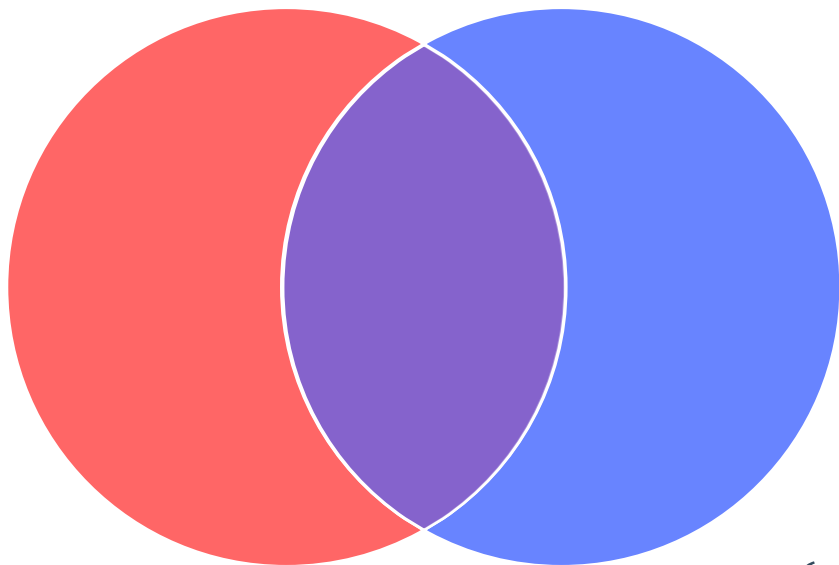
**By conditional and joint relationship:**

$$P(Y|X) * P(X) = P(X|Y) * P(Y)$$

**To invert conditional probability:**

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

# BAYES THEOREM DERIVATION



By conditional and joint relationship:

$$P(Y|X) * P(X) = P(X|Y) * P(Y)$$

To invert conditional probability:

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

$$P(X) = \sum_Z P(X, Z) = \sum_Z P(X|Z) * P(Z)$$

# BAYES THEOREM

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

# BAYES THEOREM

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

$$\textit{posterior} = \frac{\textit{likelihood} * \textit{prior}}{\textit{evidence}}$$

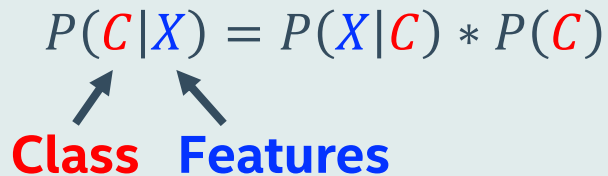
# NAÏVE BAYES CLASSIFICATION

$$P(Y|X) = \frac{P(X|Y) * P(Y)}{P(X)}$$

$$\textit{posterior} = \frac{\textit{likelihood} * \textit{prior}}{\textit{evidence}}$$

# TRAINING NAÏVE BAYES

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

$$P(\text{Class} | \text{Features}) = P(\text{Features} | \text{Class}) * P(\text{Class})$$


# TRAINING NAÏVE BAYES: THE NAÏVE ASSUMPTION

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

Difficult to calculate  
joint probabilities  
produced by expanding  
for all features

$$P(C|X) = P(X|C) * P(C)$$

$$\begin{aligned} P(C|X) &= P(X_1, X_2, \dots, X_n|C) * P(C) \\ &= P(X_1|X_2, \dots, X_n, C) * P(X_2, \dots, X_n|C) * P(C) \\ &\dots \end{aligned}$$

# TRAINING NAÏVE BAYES: THE NAÏVE ASSUMPTION

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

**Solution:** assume all  
features independent  
of each other

$$P(C|X) = P(X|C) * P(C)$$

$$P(C|X) = P(X_1|C) * P(X_2|C) * P(X_n|C) * P(C)$$



# TRAINING NAÏVE BAYES: THE NAÏVE ASSUMPTION

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

**Solution:** assume all  
features independent  
of each other

This is the “naïve”  
assumption

$$P(C|X) = P(X|C) * P(C)$$

$$P(C|X) = P(X_1|C) * P(X_2|C) * P(X_n|C) * P(C)$$

$$P(C|X) = P(C) \prod_{i=1}^n P(X_i|C)$$

# TRAINING NAÏVE BAYES

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

Class assignment is  
selected based on  
*maximum a posteriori*  
(MAP) rule

$$P(C|X) = P(X|C) * P(C)$$

$$\underset{k \in \{1, \dots, K\}}{\operatorname{argmax}} P(C_k) \prod_{i=1}^n P(X_i|C_k)$$

# TRAINING NAÏVE BAYES

For each class ( $C$ ),  
calculate probability  
given features ( $X$ )

Class assignment is  
selected based on  
*maximum a posteriori*  
(MAP) rule



Means select potential  
class with largest value

$$P(C|X) = P(X|C) * P(C)$$

$$\underset{k \in \{1, \dots, K\}}{\operatorname{argmax}} P(C_k) \prod_{i=1}^n P(X_i|C_k)$$

# THE LOG TRICK

Multiplying many values  
together causes  
computational instability  
(underflows)

$$\underset{k \in \{1, \dots, K\}}{\operatorname{argmax}} P(\textcolor{red}{C}_k) \prod_{i=1}^n P(\textcolor{blue}{X}_i | \textcolor{red}{C}_k)$$

# THE LOG TRICK

Multiplying many values together causes computational instability (underflows)

Work with log values and sum the results

$$\underset{k \in \{1, \dots, K\}}{\operatorname{argmax}} P(\mathbf{C}_k) \prod_{i=1}^n P(X_i | \mathbf{C}_k)$$

$$\log(P(\mathbf{C}_k)) \sum_{i=1}^n \log(P(X_i | \mathbf{C}_k))$$

# EXAMPLE: PREDICTING TENNIS WITH NAÏVE BAYES

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

# EXAMPLE: TRAINING NAÏVE BAYES TENNIS MODEL

$$P(\text{Play}=\text{Yes}) = 9/14$$

$$P(\text{Play}=\text{No}) = 5/14$$

# EXAMPLE: TRAINING NAÏVE BAYES TENNIS MODEL

$$P(\text{Play=Yes}) = 9/14$$

Outlook	Play=Yes	Play=No
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

$$P(\text{Play=No}) = 5/14$$

Temperature	Play=Yes	Play=No
Hot	2/9	2/5
Mild	4/9	2/5
Cool	3/9	1/5



# EXAMPLE: TRAINING NAÏVE BAYES TENNIS MODEL

$$P(\text{Play}=\text{Yes}) = 9/14$$

Outlook	Play=Yes	Play=No
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

Humidity	Play=Yes	Play=No
High	3/9	4/5
Normal	6/9	1/5

$$P(\text{Play}=\text{No}) = 5/14$$

Temperature	Play=Yes	Play=No
Hot	2/9	2/5
Mild	4/9	2/5
Cool	3/9	1/5

Wind	Play=Yes	Play=No
Strong	3/9	3/5
Weak	6/9	2/5

Create probability lookup tables based on training data

# EXAMPLE: PREDICTING TENNIS WITH NAÏVE BAYES

Predict outcome for the following:

$x' = (\text{Outlook}=\text{Sunny}, \text{Temperature}=\text{Cool}, \text{Humidity}=\text{High}, \text{Wind}=\text{Strong})$

$$P(\text{yes}|\text{sunny, cool, high, strong}) = P(\text{sunny}|\text{yes}) * P(\text{cool}|\text{yes}) * \\ P(\text{high}|\text{yes}) * P(\text{strong}|\text{yes}) * P(\text{yes})$$

$$P(\text{no}|\text{sunny, cool, high, strong}) = P(\text{sunny}|\text{no}) * P(\text{cool}|\text{no}) * \\ P(\text{high}|\text{no}) * P(\text{strong}|\text{no}) * P(\text{no})$$

# EXAMPLE: PREDICTING TENNIS WITH NAÏVE BAYES

**Predict outcome for the following:**

$x' = (\text{Outlook}=\text{Sunny}, \text{Temperature}=\text{Cool}, \text{Humidity}=\text{High}, \text{Wind}=\text{Strong})$

Feature	Play=Yes	Play=No
Outlook=Sunny	2/9	3/5

# EXAMPLE: PREDICTING TENNIS WITH NAÏVE BAYES

**Predict outcome for the following:**

$x' = (\text{Outlook}=\text{Sunny}, \text{Temperature}=\text{Cool}, \text{Humidity}=\text{High}, \text{Wind}=\text{Strong})$

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Outlook=Sunny	2/9	3/5
Temperature=Cool	3/9	1/5
Humidity=High	3/9	4/5
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<b>Overall Label</b>	<b>9/14</b>	<b>5/14</b>

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# LAPLACE SMOOTHING

**Problem:** categories with no entries result in a value of "0" for conditional probability

$$P(C|X) = P(X_1|C) * P(X_2|C) * P(C)$$

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# LAPLACE SMOOTHING

**Problem:** categories with no entries result in a value of "0" for conditional probability

**Solution:** add "1" to numerator and denominator of empty categories

0

$$P(C|X) = P(X_1|C) * P(X_2|C) * P(C)$$

$$P(X_1|C) = \frac{1}{\text{Count}(C) + n}$$

$$P(X_2|C) = \frac{\text{Count}(X_2 \& C) + 1}{\text{Count}(C) + m}$$

# TYPES OF NAÏVE BAYES

**Naïve Bayes Model**

**Bernoulli**

**Data Type**

**Binary (T/F)**

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**Naïve Bayes Model**

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**Discrete (e.g. count)**

# TYPES OF NAÏVE BAYES

## Naïve Bayes Model

**Bernoulli**

**Multinomial**

**Gaussian**

## Data Type

**Binary (T/F)**

**Discrete (e.g. count)**

**Continuous**

# COMBINING FEATURE TYPES

## Problem

Model features contain different data types (continuous and categorical)

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## Problem

Model features contain different data types (continuous and categorical)

## Solution

- **Option 1:** Bin continuous features to create categorical ones and fit multinomial model
- **Option 2:** Fit Gaussian model on continuous features and multinomial on categorical features; combine to create "meta model" (week 10)

# DISTRIBUTED COMPUTING WITH NAÏVE BAYES

- Well-suited for large data and distributed computing—limited parameters and log probabilities are a summation
- Scikit-Learn implementations contain a "partial\_fit" method designed for out-of-core calculations



# NAÏVE BAYES: THE SYNTAX

Import the class containing the classification method

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from sklearn.naive_bayes import BernoulliNB
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BNB = BernoulliNB(alpha=1.0)
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Laplace smoothing  
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Laplace smoothing  
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Fit the instance on the data and then predict the expected value

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BNB = BNB.fit(X_train, y_train)  
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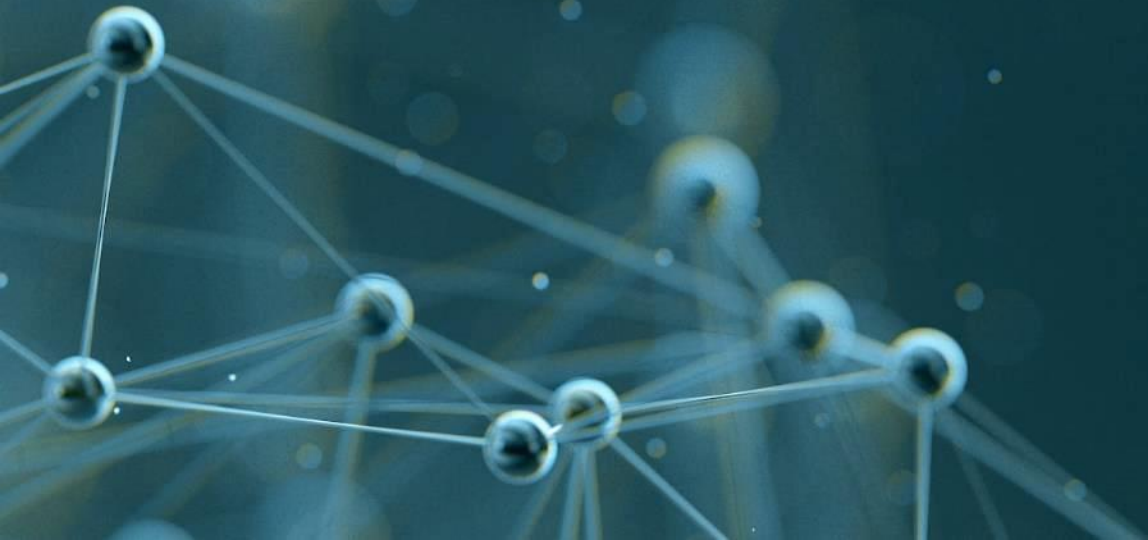
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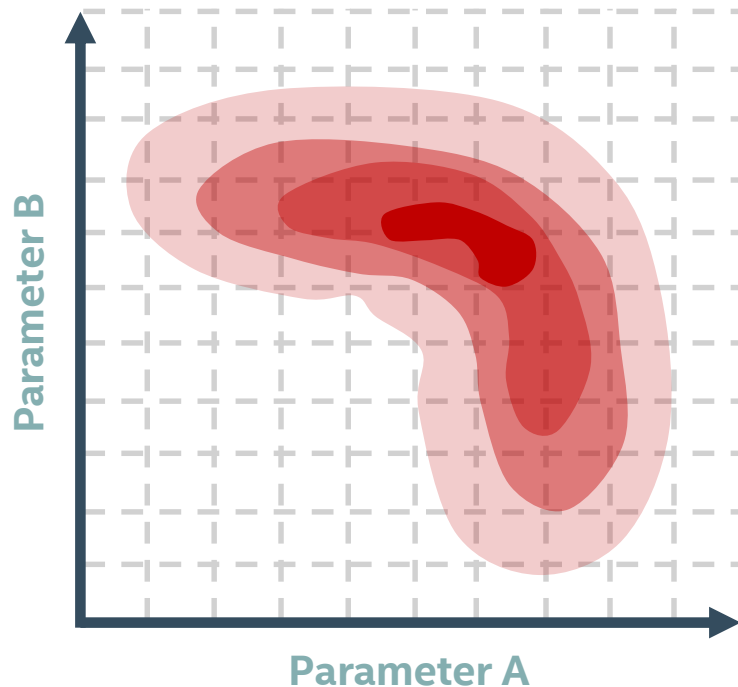
Other naïve Bayes models: `MultinomialNB`, `GaussianNB`.

# GRID SEARCH AND PIPELINES



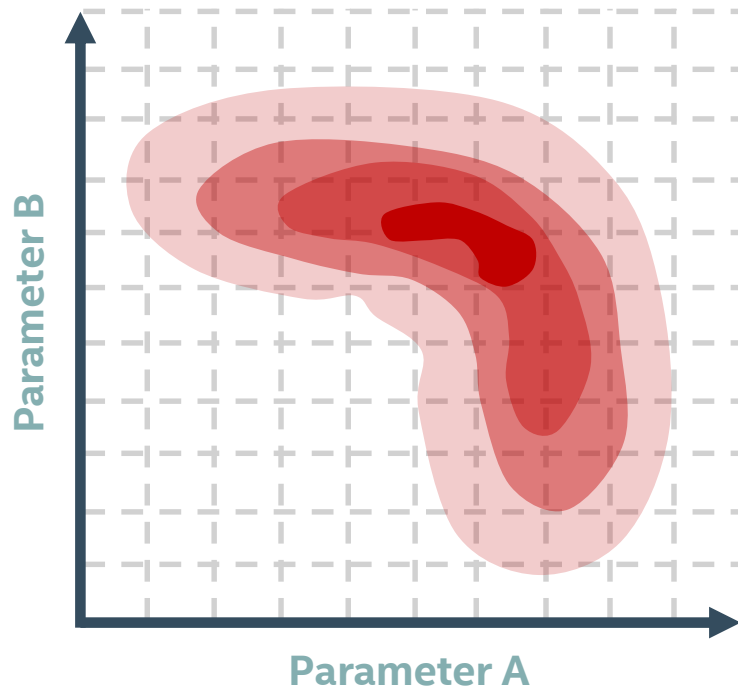
# GENERALIZED HYPERPARAMETER GRID SEARCH

- Hyperparameter selection for regularization / better models requires cross validation on training data
- Linear and logistic regression methods have classes devoted to grid search (e.g. LassoCV)



# GENERALIZED HYPERPARAMETER GRID SEARCH

- Grid search can be useful for other methods too, so a generalized method is desirable
- Scikit-learn contains `GridSearchCV`, which performs a grid search with parameters using cross validation





# GRID SEARCH WITH CROSS VALIDATION: THE SYNTAX

Import the class containing the grid search method

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from sklearn.linear_model import LogisticRegression  
from sklearn.model_selection import GridSearchCV
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Create an instance of the estimator and grid search class

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LR = LogisticRegression(penalty='l2')
GS = GridSearchCV(LR, param_grid={'c':[0.001, 0.01, 0.1]},
                  scoring='accuracy', cv=4)
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logistic  
regression  
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```

Fit the instance on the data to find the best model and then predict

```
GS = GS.fit(X_train, y_train)
y_train = GS.predict(X_test)
```

# OPTIMIZING THE REST OF THE PIPELINE

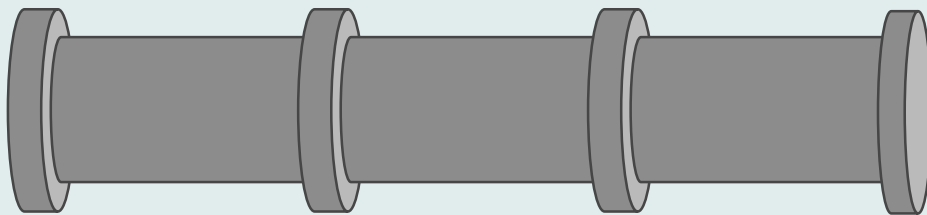
- Grid searches enable model parameters to be optimized

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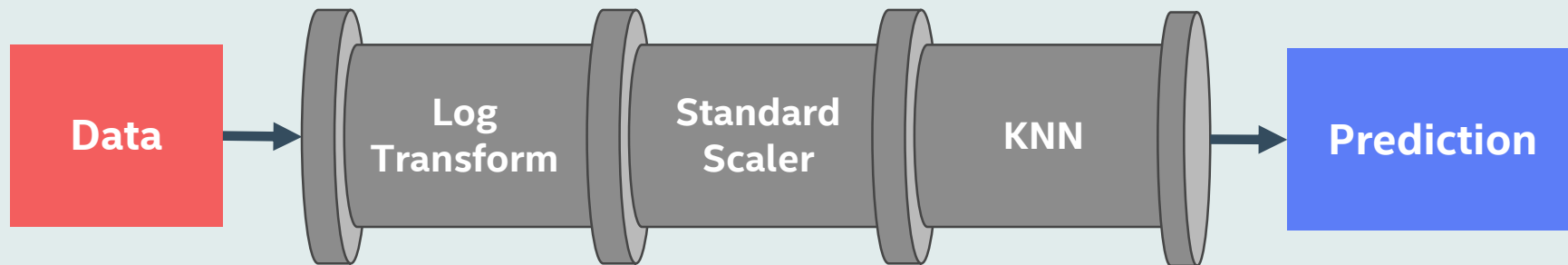
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**Pipelines!**

# AUTOMATING MACHINE LEARNING WITH PIPELINES

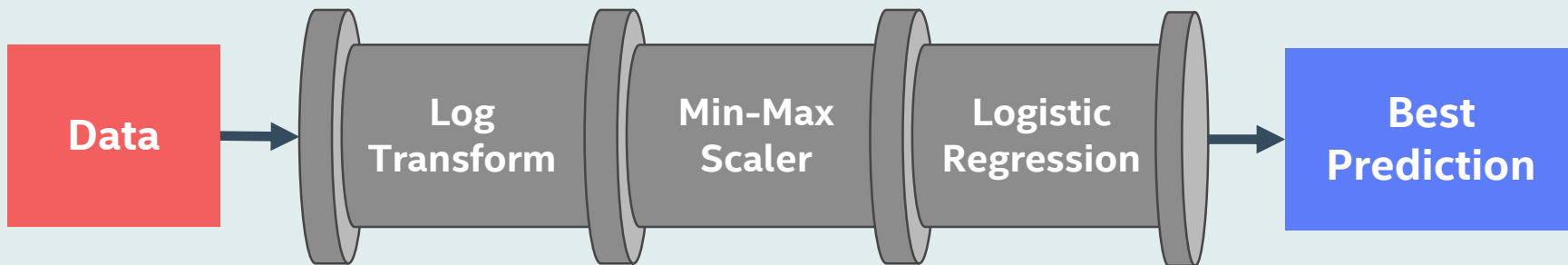
- Machine learning models often selected empirically





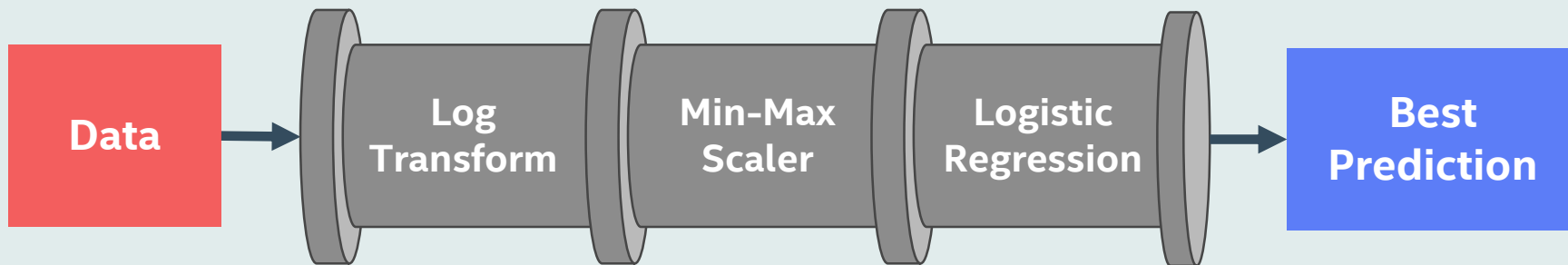
# AUTOMATING MACHINE LEARNING WITH PIPELINES

- Machine learning models often selected empirically
- By trying different processing methods and tuning multiple models



# AUTOMATING MACHINE LEARNING WITH PIPELINES

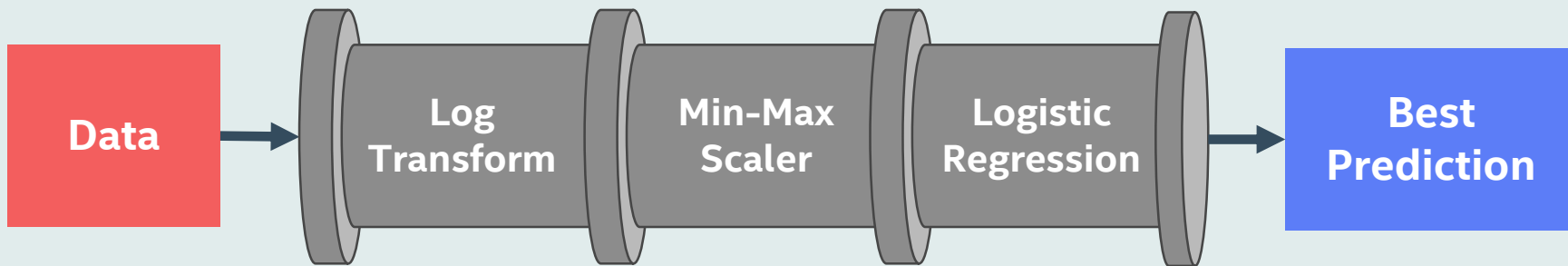
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**How to automate this process?**

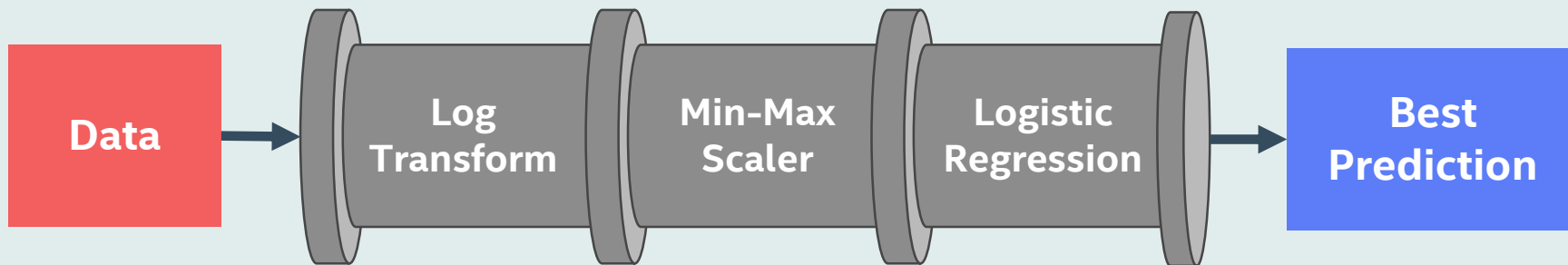
# AUTOMATING MACHINE LEARNING WITH PIPELINES

- Pipelines in Scikit-Learn allow feature transformation steps and models to be chained together



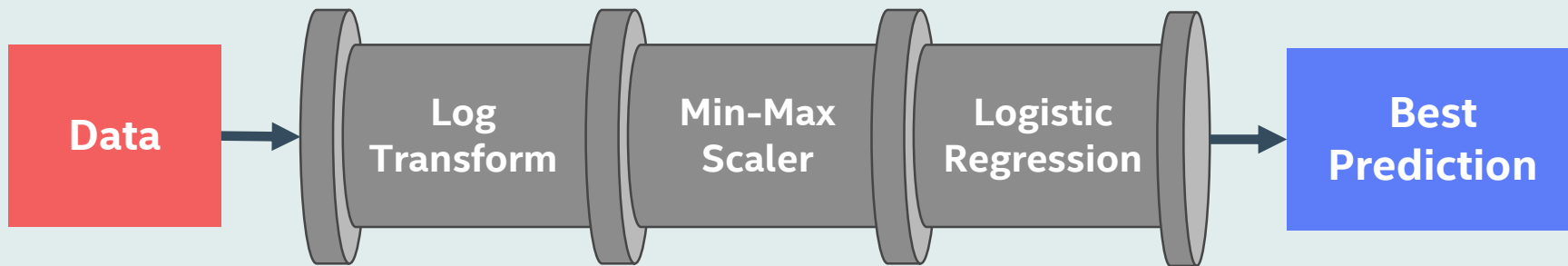
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**Pipelines make automation and reproducibility easier!**

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feature  
scaler class





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Features can be combined from different transform method using  
`FeatureUnion`

