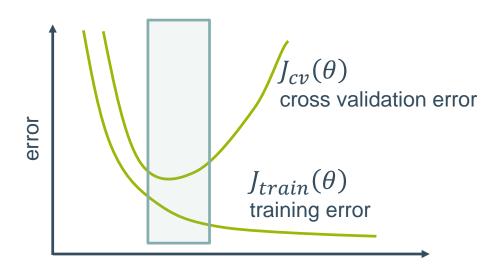
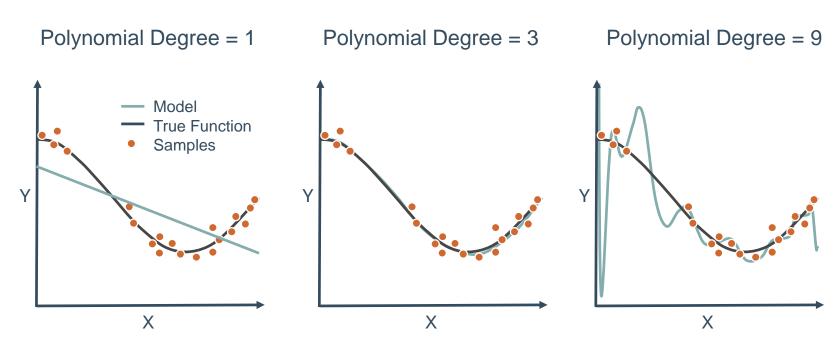


# REGULARIZATION AND FEATURE SELECTION

# MODEL COMPLEXITY VS ERROR



## PREVENTING UNDER—AND OVERFITTING



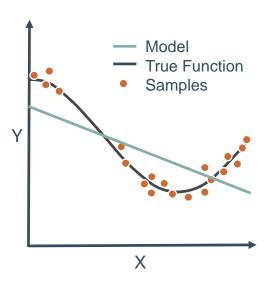
How to use a degree 9 polynomial and prevent overfitting?

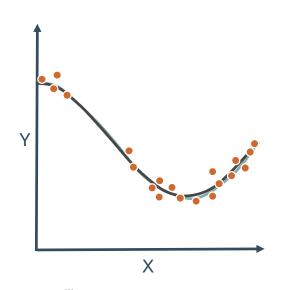
# PREVENTING UNDER—AND OVERFITTING

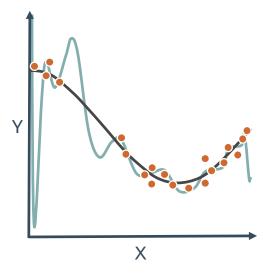


Polynomial Degree = 3

Polynomial Degree = 9

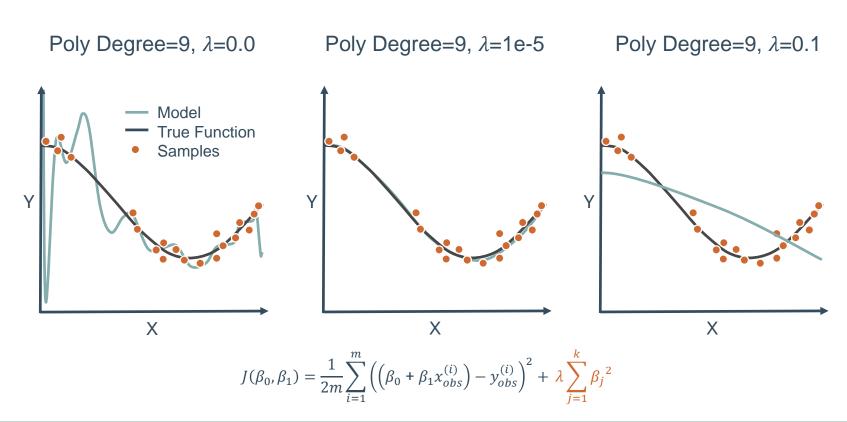






$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( \left( \beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2$$

# **REGULARIZATION**

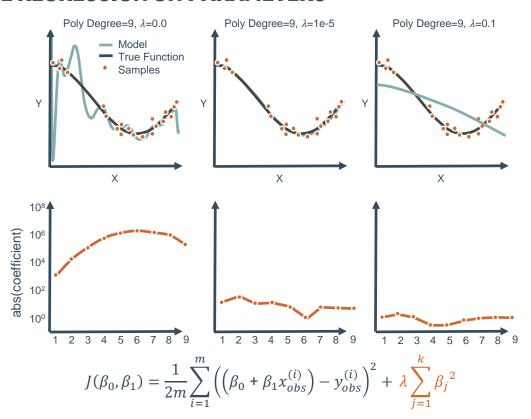


# **RIDGE REGRESSION (L2)**

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( \left( \beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

- Penalty shrinks magnitude of all coefficients
- Larger coefficients strongly penalized because of the squaring

# **EFFECT OF RIDGE REGRESSION ON PARAMETERS**

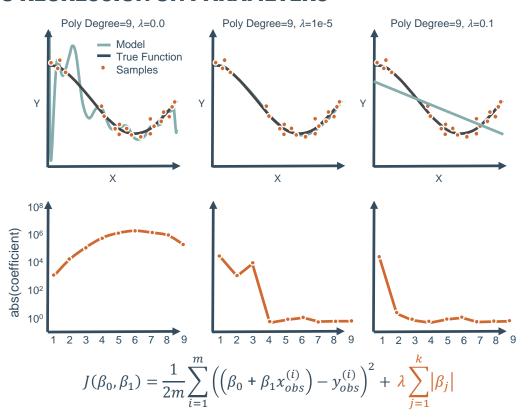


# LASSO REGRESSION (L1)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( \left( \beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} |\beta_j|$$

- Penalty selectively shrinks some coefficients
- Can be used for feature selection
- Slower to converge than Ridge regression

# **EFFECT OF LASSO REGRESSION ON PARAMETERS**

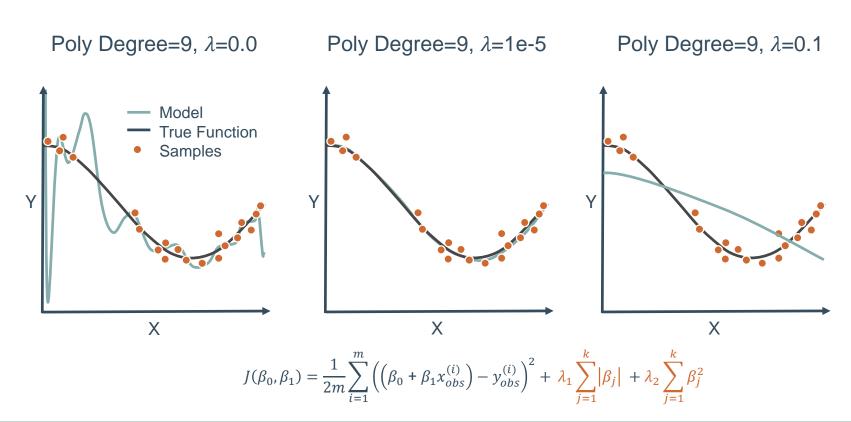


# **ELASTIC NET REGULARIZATION**

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( \left( \beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2 + \lambda_1 \sum_{j=1}^{k} |\beta_j| + \lambda_2 \sum_{j=1}^{k} |\beta_j|$$

- Compromise of both Ridge and Lasso regression
- Requires tuning of additional parameter that distributes regularization penalty between L1 and L2

# **ELASTIC NET REGULARIZATION**



# HYPERPARAMETERS AND THEIR OPTIMIZATION

• Regularization coefficients ( $\lambda_1$  and  $\lambda_2$ ) are empirically determined

#### Use Test Data to Tune $\lambda$ ?

	Date	Title	Budget	DomesticTotalGross	Director		Rating	Runtime
0	2013-11-22	The Hunger Games: Catching Fire	130000000	424668047	Francis Lawrence		PG-13	146
1	2013-05-03	Iron Man 3	200000000	409013994	Shane Black		PG-13	129
2	2013-11-22	Frozen	150000000	400738009	Chris BuckJennifer Lee		PG	108
3	2013-07-03	Despicable Me 2	76000000	368061265	Pierre CoffinChris Renaud		PG	98
4	2013-06-14	Man of Steel	225000000	291045518	Zack Snyder		PG-13	143
5	2013-10-04					Cuaron	PG-13	91
6	2013-06-21	Monsters University	анян	NG DAT <i>i</i>		anlon	G	107
7	2013-12-13	The Hobbit: The Deso		NU DAL		ckson	PG-13	161
8	2013-05-24	Fast & Furious 6				in	PG-13	130
9	2013-03-08	Oz The Great and Powerful	215000000	234911825	Sam Raimi		PG	127
10	2013-05-16	Star Trek Into Darkness	190000000	228778661 J.J. Abrams		PG-13	123	
11	2013-11-08	Thor: The Dark World	170000000	206362140	Alan Taylor		PG-13	120
12	2013-06-21	World War Z	190000000 202359711 Marc Forster		PG-13	116		
13	2013-03-22	The Croods	105000000	107100405	Wide Da	MiccoChris Sanders	PG	98
14	2013-06-28	The Heat	FOT	DATA		g	R	117
15	2013-08-07	We're the Millers	FSI	DATA		Marshall Thurber	R	110
16	2013-12-13	American Hustle				. Russell	R	138
17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luh	rmann	PG-13	143

## HYPERPARAMETERS AND THEIR OPTIMIZATION

- Regularization coefficients ( $\lambda$ \_1 and  $\lambda$ \_2) are empirically determined
- Want value that generalizes—do not use test data for tuning

#### Use Test Data to Tune $\lambda$ ?



## HYPERPARAMETERS AND THEIR OPTIMIZATION

- Regularization coefficients ( $\lambda$ \_1 and  $\lambda$ \_2) are empirically determined
- Want value that generalizes—do not use test data for tuning
- Create additional split of data to tune hyperparameters—validation set

#### Tune $\lambda$ with Cross Validation

	Date	Title	Budget	DomesticTotalGross	Directo		Dating	Runtime
	Date	Title	Buaget	Domestic i otal Gross	Director		Hating	Runtime
0	2013-11-22	The Hunger Games: Catching Fire	130000000	424668047	Francis Lawrence		PG-13	146
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3	2013-07-03	Despicable Me 2	AINII	NG DAT <i>i</i>	A	offinChris Renaud	PG	98
4	2013-06-14	Man of Steel				yder	PG-13	143
5	2013-10-04	Gravity				Cuaron	PG-13	91
6	2013-06-21	Monsters University	NaN	268492764	Dan Sc	Oan Scanlon		107
7	2013-12-13	The Hobbit: The Desolation of Smaug	NaN	258366855	Peter Jackson		PG-13	161
8	2013-05-24	Fast & Furious 6	400000000	000070050		in	PG-13	130
9	2013-03-08	Oz The Great and Pov	IDAT	TOUDAT		imi	PG	127
10	2013-05-16	Star Trek Into Darknes	ШАІ	'ION DAT <i>a</i>		ams	PG-13	123
11	2013-11-08	Thor: The Dark World			•	/lor	PG-13	120
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Import the class containing the regression method

from sklearn.linear\_model import Ridge

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#### Create an instance of the class

RR = Ridge(alpha=1.0)

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#### Import the class containing the regression method

from sklearn.linear\_model import Ridge

#### Create an instance of the class

RR = Ridge(alpha=1.0)

#### Fit the instance on the data and then predict the expected value

```
RR = RR.fit(X_train, y_train)
y_predict = RR.predict(X_test)
```

#### Import the class containing the regression method

```
from sklearn.linear_model import Ridge
```

#### Create an instance of the class

```
RR = Ridge(alpha=1.0)
```

#### Fit the instance on the data and then predict the expected value

```
RR = RR.fit(X_train, y_train)
y_predict = RR.predict(X_test)
```

The RidgeCV class will perform cross validation on a set of values for alpha.

## LASSO REGRESSION: THE SYNTAX

#### Import the class containing the regression method

```
from sklearn.linear_model import Lasso
```

#### Create an instance of the class

```
LR = Lasso(alpha=1.0)
```

#### Fit the instance on the data and then predict the expected value

```
LR = LR.fit(X_train, y_train)
y_predict = LR.predict(X_test)
```

The LassoCV class will perform cross validation on a set of values for alpha.

## LASSO REGRESSION: THE SYNTAX

#### Import the class containing the regression method

```
from sklearn.linear model import Lasso
```

#### Create an instance of the class

```
LR = Lasso(alpha=1.0)
```



regularization parameter

Fit the instance on the data and then predict the expected value

```
LR = LR.fit(X_train, y_train)
y predict = LR.predict(X test)
```

The LassoCV class will perform cross validation on a set of values for alpha.

## **ELASTIC NET REGRESSION: THE SYNTAX**

#### Import the class containing the regression method

```
from sklearn.linear model import ElasticNet
```

#### Create an instance of the class

```
EN = ElasticNet(alpha=1.0,11 ratio=0.5)
```

Fit the instance on the data and then predict the expected value

```
EN = EN.fit(X_train, y_train)
y_predict = EN.predict(X_test)
```

The **ElasticNetCV** class will perform cross validation on a set of values for l1\_ratio and alpha.

## **ELASTIC NET REGRESSION: THE SYNTAX**

#### Import the class containing the regression method

```
from sklearn.linear model import ElasticNet
```

#### Create an instance of the class

```
EN = ElasticNet(alpha=1.0,11 ratio=0.5)
```



alpha is the regularization parameter

Fit the instance on the data and then predict the expected value

```
EN = EN.fit(X_train, y_train)
y_predict = EN.predict(X_test)
```

The ElasticNetCV class will perform cross validation on a set of values for l1\_ratio and alpha.

## **ELASTIC NET REGRESSION: THE SYNTAX**

#### Import the class containing the regression method

```
from sklearn.linear model import ElasticNet
```

#### Create an instance of the class

```
EN = ElasticNet(alpha=1.0,11 ratio=0.5)
```



l1\_ratio distributes alpha to L1/L2

Fit the instance on the data and then predict the expected value

```
EN = EN.fit(X_train, y_train)
y_predict = EN.predict(X_test)
```

The ElasticNetCV class will perform cross validation on a set of values for l1\_ratio and alpha.

# FEATURE SELECTION

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- For L1-regularization, this is accomplished by driving some coefficients to zero

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- Regularization performs feature selection by shrinking the contribution of features
- For L1-regularization, this is accomplished by driving some coefficients to zero
- Feature selection can also be performed by removing features

# WHY IS FEATURE SELECTION IMPORTANT?

 Reducing the number of features is another way to prevent overfitting (similar to regularization)

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- For some models, fewer features can improve fitting time and/or results

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- Reducing the number of features is another way to prevent overfitting (similar to regularization)
- For some models, fewer features can improve fitting time and/or results
- Identifying most critical features can improve model interpretability

## **RECURSIVE FEATURE ELIMINATION: THE SYNTAX**

#### Import the class containing the feature selection method

```
from sklearn.feature selection import RFE
```

#### Create an instance of the class

```
rfeMod = RFE(est, n_features_to_select=5)
```

#### Fit the instance on the data and then predict the expected value

```
rfeMod = rfeMod.fit(X_train, y_train)
y_predict = rfeMod.predict(X_test)
```

The RFECV class will perform feature elimination using cross validation.

# **RECURSIVE FEATURE ELIMINATION: THE SYNTAX**

#### Import the class containing the feature selection method

```
from sklearn.feature selection import RFE
```

#### Create an instance of the class

```
rfeMod = RFE(est, n features to select=5)
```



est is an instance of the model to use

Fit the instance on the data and then predict the expected value

```
rfeMod = rfeMod.fit(X_train, y_train)
y_predict = rfeMod.predict(X_test)
```

The RFECV class will perform feature elimination using cross validation.

# **RECURSIVE FEATURE ELIMINATION: THE SYNTAX**

#### Import the class containing the feature selection method

```
from sklearn.feature selection import RFE
```

#### Create an instance of the class

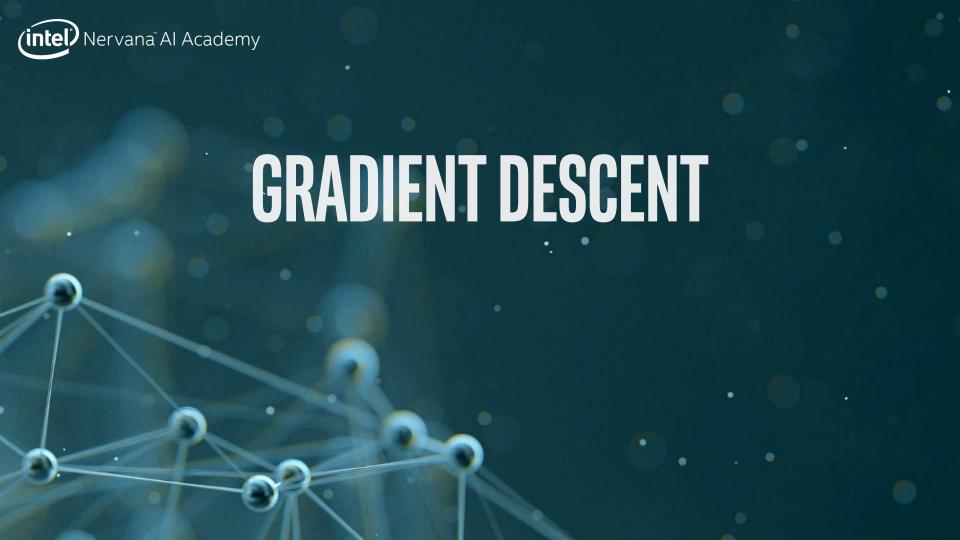
```
rfeMod = RFE(est, n_features_to_select=5)
```



Fit the instance on the data and then predict the expected value

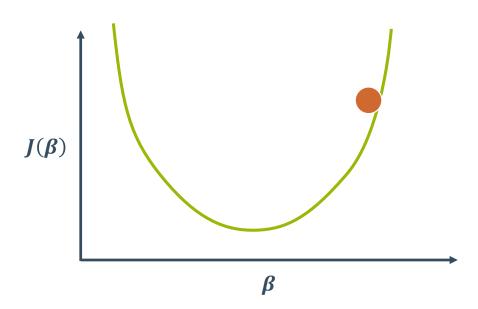
```
rfeMod = rfeMod.fit(X_train, y_train)
y_predict = rfeMod.predict(X_test)
```

The RFECV class will perform feature elimination using cross validation.



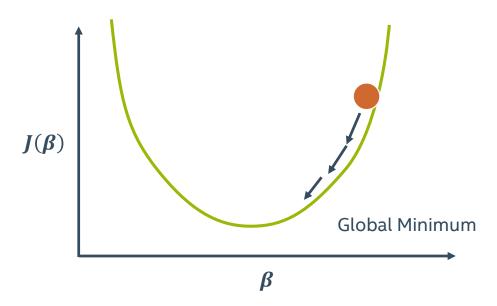
# **GRADIENT DESCENT**

Start with a cost function  $J(\beta)$ :



# **GRADIENT DESCENT**

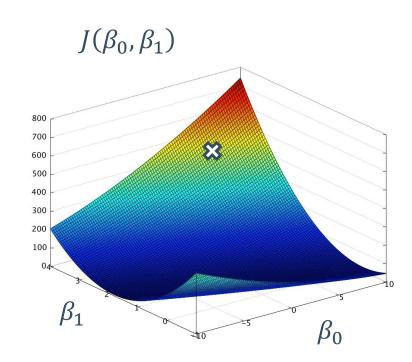
Start with a cost function  $J(\beta)$ :



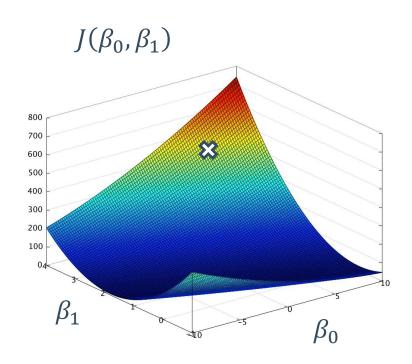
Then gradually move towards the minimum.

• Now imagine there are two parameters  $(\beta_0, \beta_1)$ 

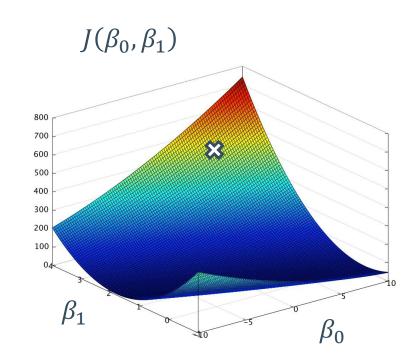
- Now imagine there are two parameters  $(\beta_0, \beta_1)$
- This is a more complicated surface on which the minimum must be found



- Now imagine there are two parameters  $(\beta_0, \beta_1)$
- This is a more complicated surface on which the minimum must be found
- How can we do this without knowing what  $J(\beta_0, \beta_1)$  looks like?

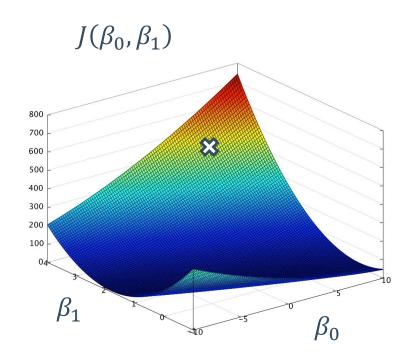


- Compute the gradient,  $\nabla J(\beta_0, \beta_1)$ , which points in the direction of the biggest increase!
- $-\nabla J(\beta_0, \beta_1)$  (negative gradient) points to the biggest decrease at that point!



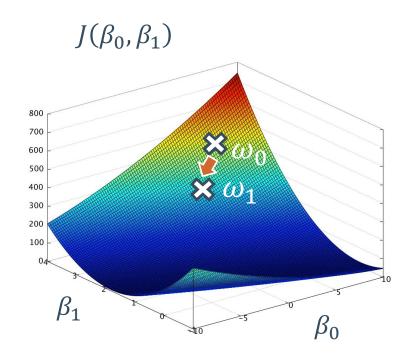
 The gradient is the a vector whose coordinates consist of the partial derivatives of the parameters

$$\nabla J(\beta_0, \dots, \beta_n) = \langle \frac{\partial J}{\partial \beta_0}, \dots, \frac{\partial J}{\partial \beta_n} \rangle$$



■ Then use the gradient ( $\nabla$ ) and the cost function to calculate the next point ( $\omega_1$ ) from the current one ( $\omega_0$ ):

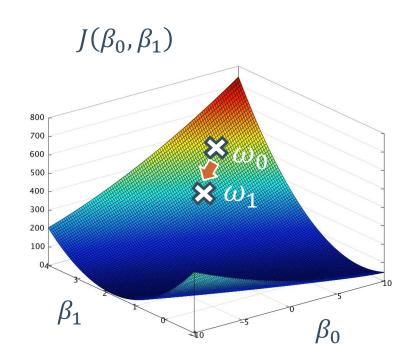
$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$



■ Then use the gradient ( $\nabla$ ) and the cost function to calculate the next point ( $\omega_1$ ) from the current one ( $\omega_0$ ):

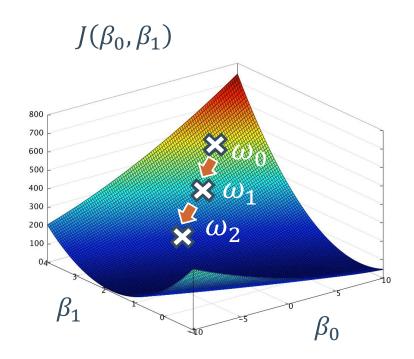
$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$

• The learning rate  $(\alpha)$  is a tunable parameter that determines step size



 Each point can be iteratively calculated from the previous one

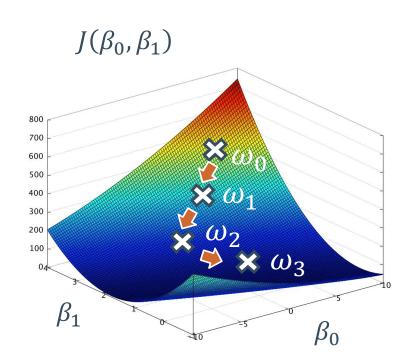
$$\omega_{2} = \omega_{1} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$



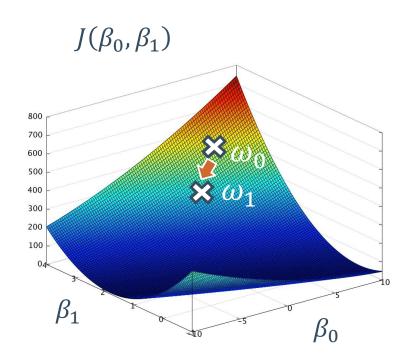
 Each point can be iteratively calculated from the previous one

$$\omega_2 = \omega_1 - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2$$

$$\omega_{3} = \omega_{2} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$

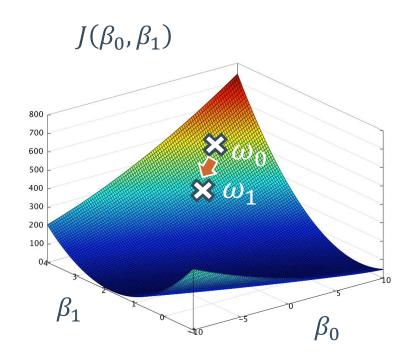


 Use a single data point to determine the gradient and cost function instead of all the data



 Use a single data point to determine the gradient and cost function instead of all the data

$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$

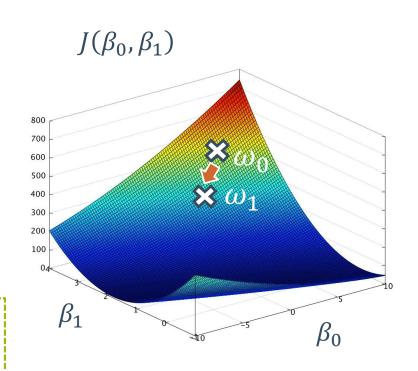


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$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{m} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$



$$\omega_1 = \omega_0 - \alpha \nabla \frac{1}{2} \left( \left( \beta_0 + \beta_1 x_{obs}^{(0)} \right) - y_{obs}^{(0)} \right)^2$$

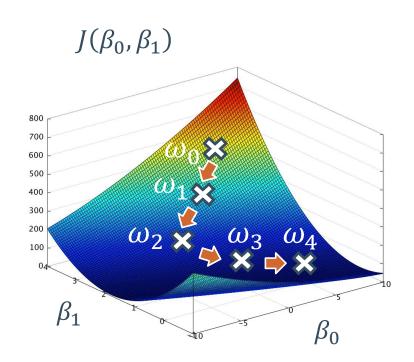


 Use a single data point to determine the gradient and cost function instead of all the data

$$\omega_1 = \omega_0 - \alpha \nabla \frac{1}{2} \left( \left( \beta_0 + \beta_1 x_{obs}^{(0)} \right) - y_{obs}^{(0)} \right)^2$$

. . .

$$\omega_4 = \omega_3 - \alpha \nabla \frac{1}{2} \left( \left( \beta_0 + \beta_1 x_{obs}^{(3)} \right) - y_{obs}^{(3)} \right)^2$$

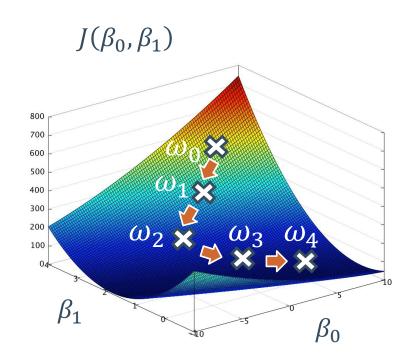


 Use a single data point to determine the gradient and cost function instead of all the data

$$\omega_1 = \omega_0 - \alpha \nabla \frac{1}{2} \left( \left( \beta_0 + \beta_1 x_{obs}^{(0)} \right) - y_{obs}^{(0)} \right)^2$$

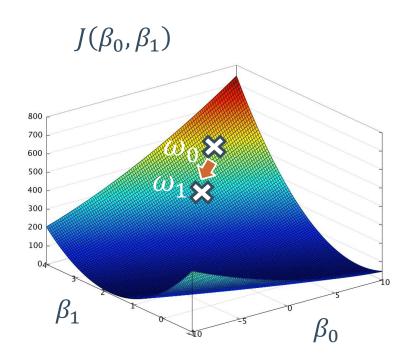
 $\omega_4 = \omega_3 - \alpha \nabla \frac{1}{2} \left( \left( \beta_0 + \beta_1 x_{obs}^{(3)} \right) - y_{obs}^{(3)} \right)^2$ 

 Path is less direct due to noise in single data point—"stochastic"



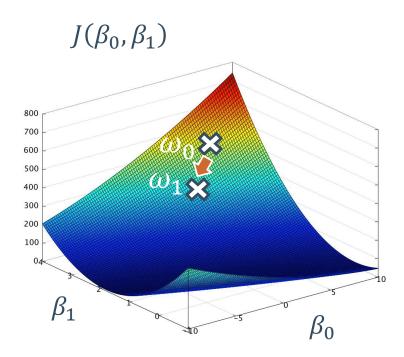
 Perform an update for every n training examples

$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{n} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$



Perform an update for every n training examples

$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{n} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$

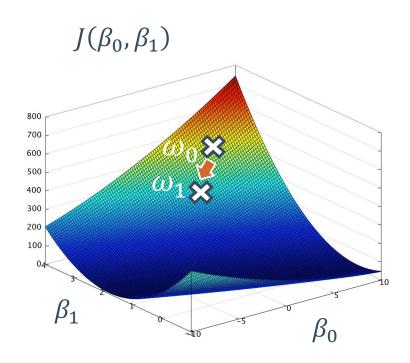


 Perform an update for every n training examples

$$\omega_{1} = \omega_{0} - \alpha \nabla \frac{1}{2} \sum_{i=1}^{n} \left( \left( \beta_{0} + \beta_{1} x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^{2}$$

### **Best of both worlds:**

- Reduced memory relative to "vanilla" gradient descent
- Less noisy than stochastic gradient descent



Mini batch implementation typically used for neural nets

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- Batch sizes range from 50–256 points

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- Trade off between batch size and learning rate ( $\alpha$ )

- Mini batch implementation typically used for neural nets
- Batch sizes range from 50–256 points
- Trade off between batch size and learning rate  $(\alpha)$
- Tailor learning rate schedule: gradually reduce learning rate during a given epoch

Import the class containing the regression model

from sklearn.linear\_model import SGDRegressor

### Import the class containing the regression model

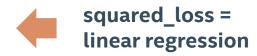
```
from sklearn.linear_model import SGDRegressor
```

### Create an instance of the class

### Import the class containing the regression model

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```

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### Import the class containing the regression model

```
from sklearn.linear_model import SGDRegressor
```

#### Create an instance of the class



regularization parameters

### Import the class containing the regression model

```
from sklearn.linear_model import SGDRegressor
```

### Create an instance of the class

### Fit the instance on the data and then transform the data

```
SGDreg = SGDreg.fit(X_train, y_train)
y_pred = SGDreg.predict(X_test)
```

### Import the class containing the regression model

```
from sklearn.linear_model import SGDRegressor
```

### Create an instance of the class

#### Fit the instance on the data and then transform the data

```
SGDreg = SGDreg.partial_fit(X_train, y_train)
y_pred = SGDreg.predict(X_test)
```



Mini-batch

version

### Import the class containing the regression model

```
from sklearn.linear_model import SGDRegressor
```

#### Create an instance of the class

### Fit the instance on the data and then transform the data

```
SGDreg = SGDreg.fit(X_train, y_train)
y_pred = SGDreg.predict(X_test)
```

Other loss methods exist: epsilon\_insensitive, huber, etc.

Import the class containing the classification model

from sklearn.linear\_model import SGDClassifier

### Import the class containing the classification model

```
from sklearn.linear_model import SGDClassifier
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### Create an instance of the class

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### Fit the instance on the data and then transform the data

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y_pred = SGDclass.predict(X_test)
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SGDclass = SGDclass.partial_fit(X_train, y_train)
y_pred = SGDclass.predict(X_test)

mini-batch
version
```



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See SVM lecture (week 7)

