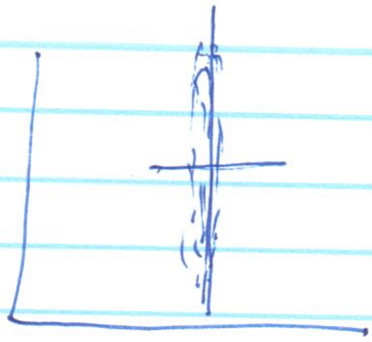
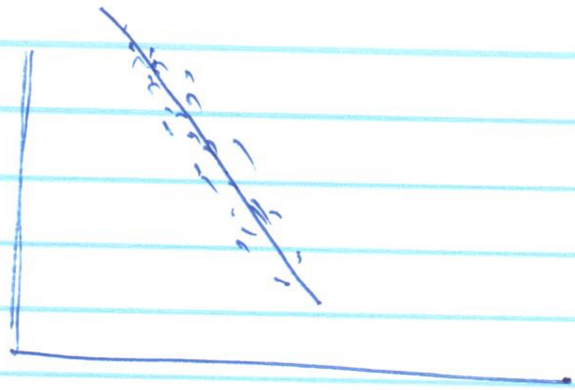
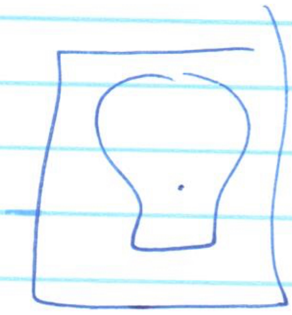
 



77760 pixels

X

1×77760

$L \rightarrow 77760 * 4$

Z $\underline{1 \times 4} = \boxed{0 \quad 0 \quad 0 \quad 0}$

X

X

$(N \times D)$

Symmetric $X = X^T$

Square.

Diagonal

$X = \cdot X \cdot x \cdot \cdot$

Matrix Decomposition.

Singular Value Decomposition (SVD)

$$\underset{N\times D}{X} = \underset{N\times N}{U}\ \underset{N\times D}{S}\ \underset{D\times D}{V^T}$$

Orthonormal matrices
$U \rightarrow U^T U = I$
$V \rightarrow V^T V = I$

Each column of $U \rightarrow$ left singular vector

Each column of $V \rightarrow$ right singular vector

$$S \rightarrow \begin{array}{c c} & D \\ \begin{array}{c} D \\ N \end{array} & \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots \sigma_2 & \\ & & & \sigma_D \\ \hline & 0 & & \end{bmatrix} \end{array}$$

If $N > D$

singular values

$$\boxed{U, S, V = \text{np.linalg.svd}(X)}$$

① Equivalence to PCA ② Approximating a matrix ③ Recommender system

$X = USV^T$ let X be our data matrix

let us assume that X is mean centered

$$X^T X = (USV^T)^T (USV^T)$$

$$= V(US)^T USV^T$$

$$= VS^T U^T U S V^T$$

$$= VS^T S V^T$$

$$= VDV^T$$

$(AB)^T = B^T A^T$

If S is diagonal

$S^T S = \begin{bmatrix} \sigma_1^2 & & 0 \\ 0 & \sigma_2^2 & \\ & & \ddots \end{bmatrix}$

let $D = S^T S$

$$\boxed{Ax = \lambda x}$$

$$X^T X V = V D \underbrace{V^T V}$$

$$\boxed{X^T X V = V D}$$

Any column of V

$$\boxed{(X^T X) V_i = V_i \sigma_i^2}$$

scalar

i^{th} column vector of V

V_i will be the i^{th} eigen vector of $X^T X$

$\Rightarrow$ V_i is the i^{th} PC of X

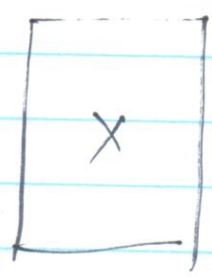
and σ_i^2 is the ~~square of the~~ i^{th} eigen value of $X^T X$

Two ways to do PCA

Method #1	Method #2
$X^T X \longrightarrow$ find eigen values of $X^T X$.	Perform SVD on X and choose the first right singular vector as the first PC.

Choose first L r. singular vectors to get the L principal components.

Rank of a matrix



e.g

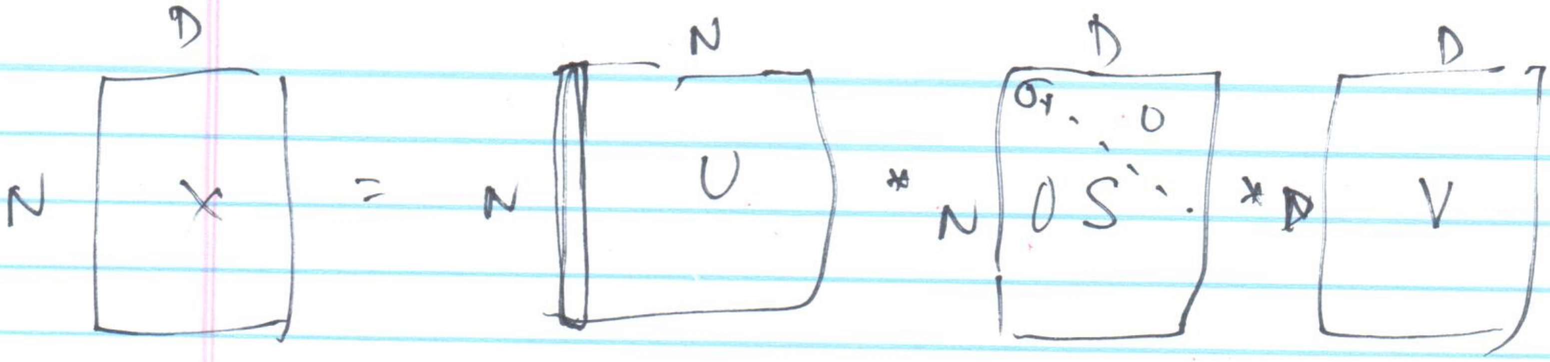
$$X = \begin{bmatrix} 1 & 1 & 2 \\ 7 & 9 & 16 \\ 3 & 1 & 4 \\ 4 & 8 & 12 \end{bmatrix}$$

rank of X is 2

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 7 & 7 & 7 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

rank$(X) = 1$

$$\underline{X = U S V^T} \longrightarrow \text{SVD}$$



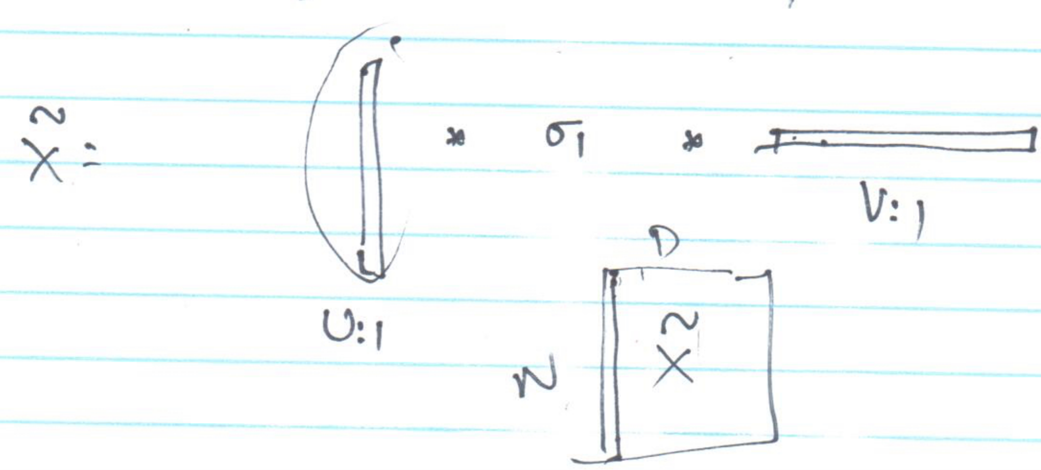
$$\underset{(N \times 1)}{U_{:1}} * \underset{(1 \times 1)}{\sigma_1} * \underset{(1 \times D)}{V_{:1}^T} = \underset{(N \times D)}{\tilde{X}}$$

$$U_{:1,2} * \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} * V_{:1,2}^T = \tilde{X}$$

Choosing first "few" (L) singular vectors & values → we get an approximation of X.

$$\boxed{\tilde{X} = U_{:1} * \sigma_1 * V_{:1}^T}$$

→ rank of ($\tilde{X}$) will be 1



ECM theorem

Among all rank-1 approximations of X

SVD approximation is the best