CAP5415-Computer Vision
Lecture 8-Motion Models, Feature Tracking, and Alignment

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Readings

- Szeliski, R. Ch. 7
- Bergen et al. ECCV 92, pp. 237-252.
- **Shi, J. and Tomasi, C. CVPR 94, pp.593-600.**
- Baker, S. and Matthews, I. IJCV 2004, pp. 221-255.

- Slide Credits: Szeliski, Shah and B. Freeman

Recap: Estimating Optical Flow

- Assume the image intensity I is constant

Time = *t*

Time = *t+dt*

$$I(x, y, t) = I(x + dx, y + dy, t + dt)$$

First Assumption: Brightness Constraint

$$I(x,y,t) \simeq I(x+dx, y+dy, t+dt)$$

$$I(x(t)+u.\Delta t, y(t)+v.\Delta t) - I(x(t), y(t), t) \approx 0$$

Assuming I is differentiable function, and expand the first term using Taylor's series:

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Compact representation

$$I_x u + I_y v + I_t = 0$$

Brightness constancy constraint

Second Assumption: Gradient Constraint

Velocity vector is constant within a small neighborhood **(LUCAS AND KANADE)**

$$E(u,v) = \int_{x,y} (I_x u + I_y v + I_t)^2 dxdy$$

$$\frac{\partial E(u,v)}{\partial u} = \frac{\partial E(u,v)}{\partial v} = 0$$

$$2(I_x u + I_y v + I_t)I_x = 0$$
$$2(I_x u + I_y v + I_t)I_y = 0$$

Recap: Lucas-Kanade

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

Structural Tensor representation

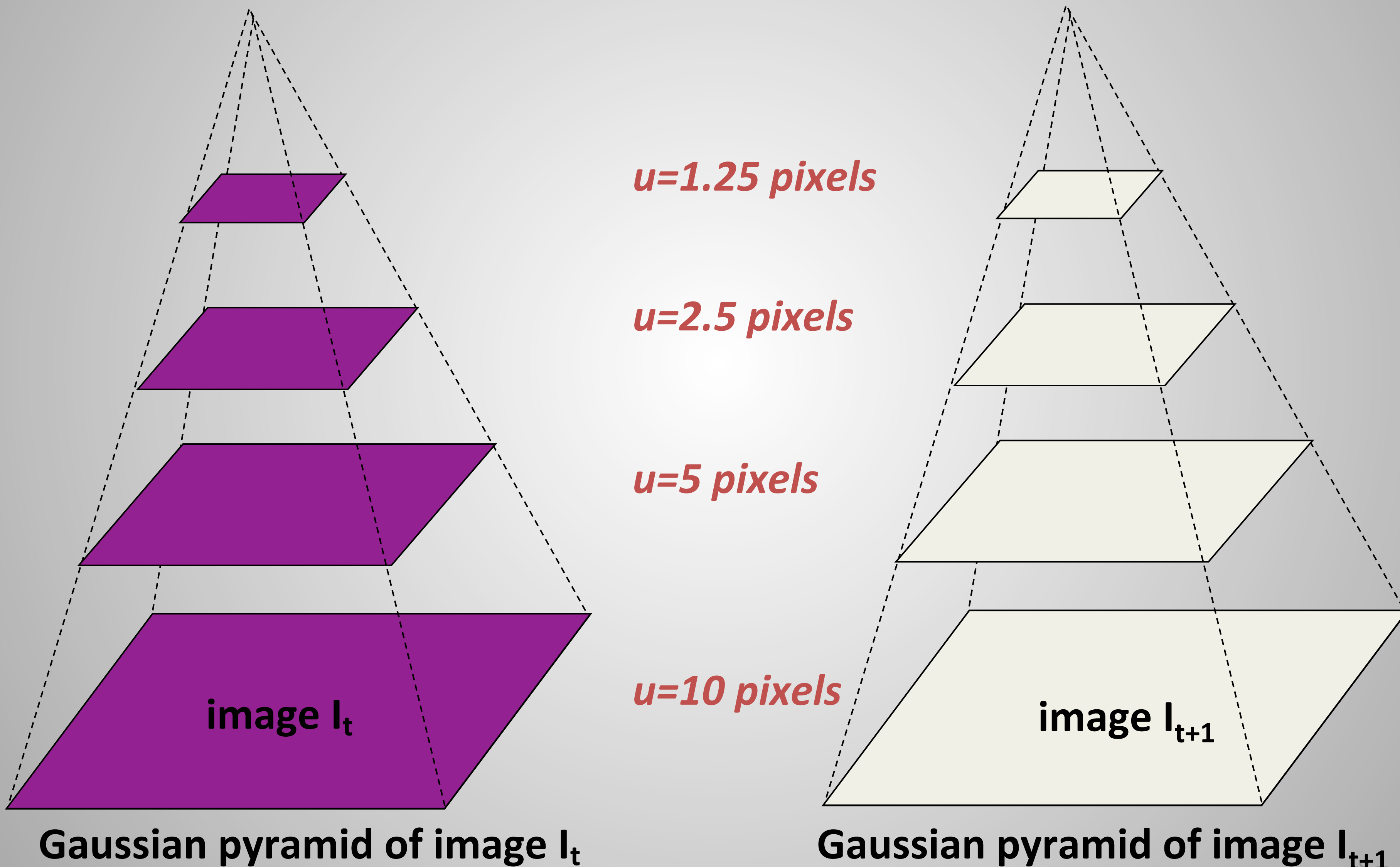
$$\begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} T_{xt} \\ T_{yt} \end{bmatrix}$$

$$u = \frac{T_{yt}T_{xy} - T_{xt}T_{yy}}{T_{xx}T_{yy} - T_{xy}^2} \text{ and } v = \frac{T_{xt}T_{xy} - T_{yt}T_{xx}}{T_{xx}T_{yy} - T_{xy}^2}$$

Pitfalls & Alternatives

- Brightness constancy is not satisfied
  - Correlation based method could be used
- A point may not move like its neighbors
  - Regularization based methods
- The motion may not be small (Taylor does not hold!)
  - Multi-scale estimation could be used

Multi-Scale Flow Estimation



Recap: Horn & Schunck

- Global method with smoothness constraint to solve aperture problem
- Minimize a global energy function

$$E(u,v) = \int_{x,y} [(I_x u + I_y v + I_t)^2 + \alpha^2(|\nabla u|^2 + |\nabla v|^2)]dxdy$$

- Take partial derivatives w.r.t. u and v:

$$(I_x u + I_y v + I_t)I_x - \alpha^2 \nabla u = 0$$
$$(I_x u + I_y v + I_t)I_y - \alpha^2 \nabla v = 0$$

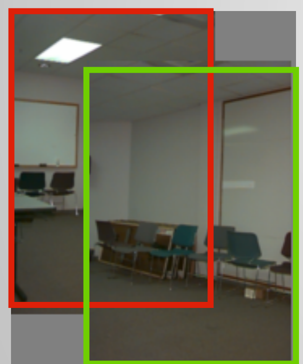
Global Motion Models (Parametric)

All pixels are considered to summarize global motion!

- 2D Models
  - Affine
  - Quadratic
  - Planar projective (homography)
- 3D Models
  - Inst. Camera motion models
  - Homography + epipole
  - Plane + parallax

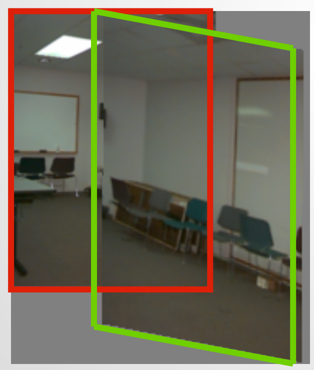
Motion Models

Translation



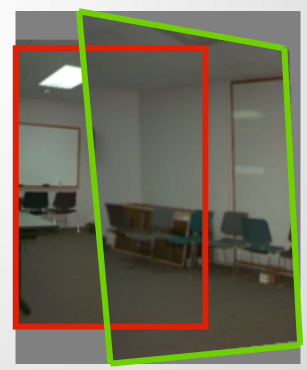
2 unknowns

Affine



6 unknowns

Perspective



8 unknowns

3D rotation

3 unknowns

Global Motion

Estimate motion using all pixels in the image

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Global Motion can be used to

- Remove camera motion
- Object-based segmentation
- generate mosaics

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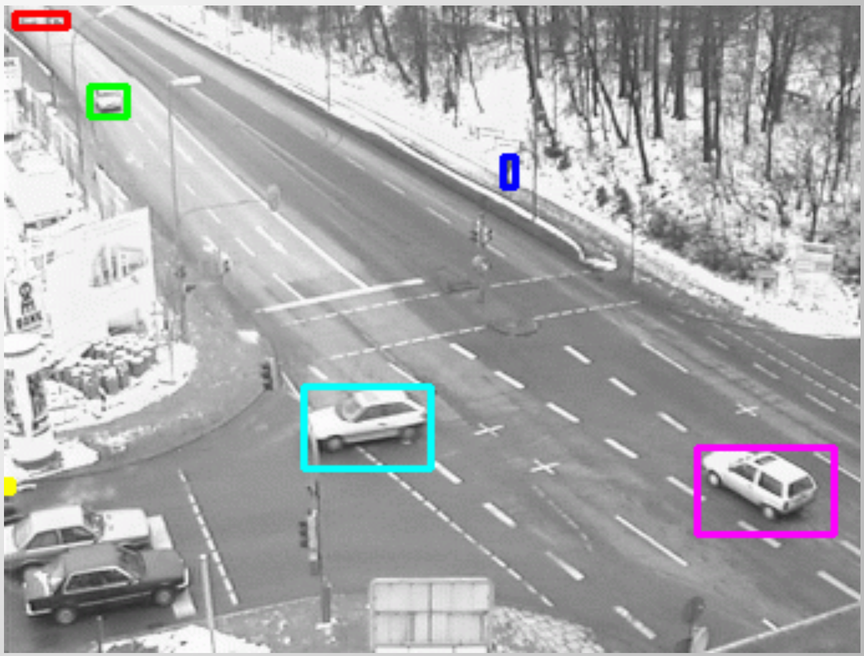
Recap: Object Tracking

- Track an object over a sequence of images



Challenges in Object Tracking

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- Which features to track?

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- Efficient tracking

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- Which features to track?
- Efficient tracking
- Appearance constraint violation
- …

Challenges in Object Tracking

- **Which features to track?**
- Efficient tracking
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- …

Shi-Tomasi Feature Tracker

- Good Features to Track

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  - Find good features using **eigenvalues of Hessian** matrix (threshold on the smallest eigenvalue when computing Harris corner detection)

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  - Track from frame to frame with LK

Shi-Tomasi Feature Tracker

- Good Features to Track
  - Find good features using eigenvalues of Hessian matrix (threshold on the smallest eigenvalue when computing Harris corner detection)
  - Track from frame to frame with LK
  - Check consistency of tracks by “affine registration” to the first observed instance of the feature

Shi-Tomasi Feature Tracker

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

Shi and Tomasi
CVPR 1994
Good Features
To Track.

KLT Tracking

- KLT: Kanade-Lucas-Tomasi

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- Tracking in single/multiple camera(s)

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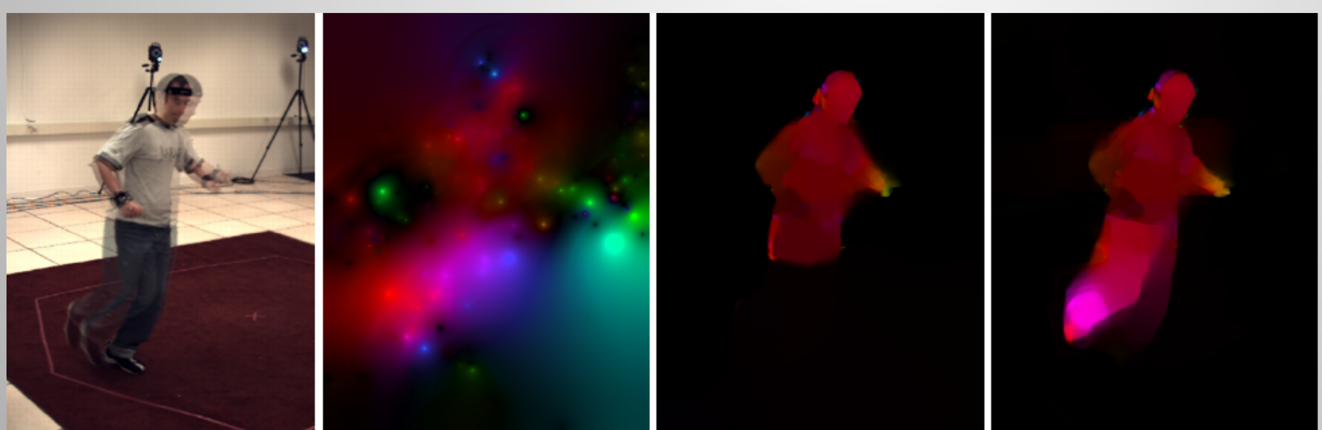
- Multiple object tracking
- Tracking in single/multiple camera(s)
- Tracking in fixed/moving camera

KLT Tracking Algorithm

- Find ***GoodFeaturesToTrack***
  - Harris Corners (thresholded on smallest eigenvalues)
- Use LK algorithm to find optical flows
- Use Coarse-to-Fine strategy to deal with large movements
- When creating long tracks, check appearance of registered patch against appearance of initial patch to find points that have drifted

Recent Developments at Optical Flow

- Start with LK or similar methods
  - +Gradient consistency
  - +Energy minimization with smoothing term
  - +Region matching
  - +KeyPoint matching

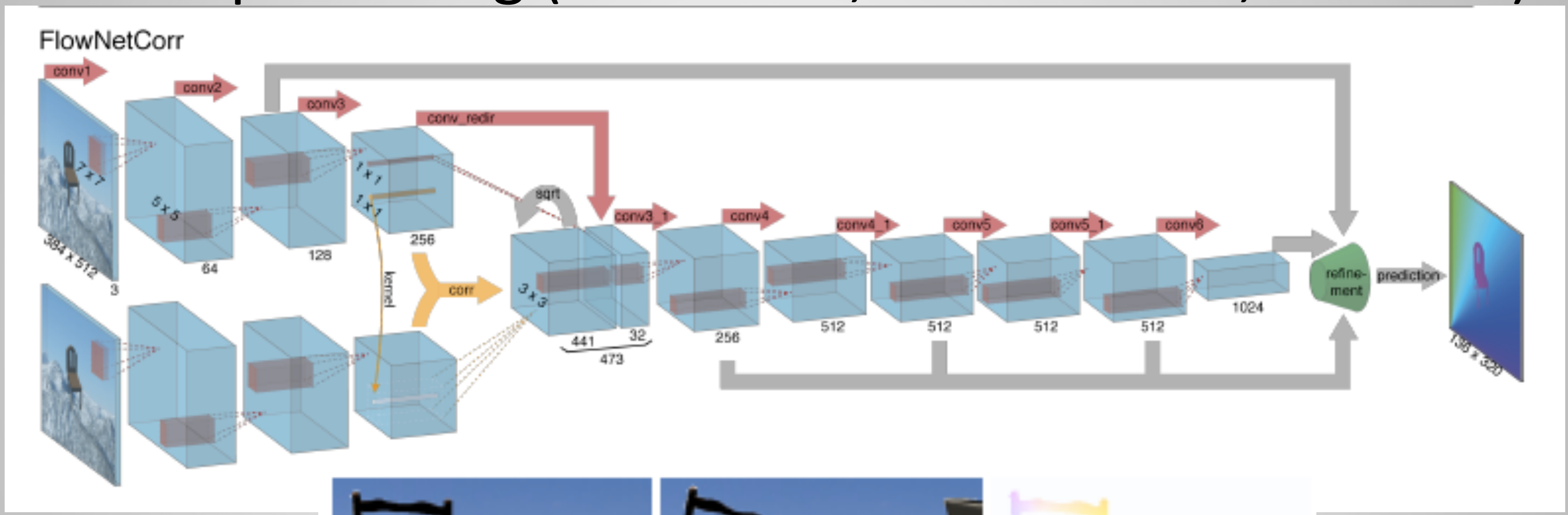


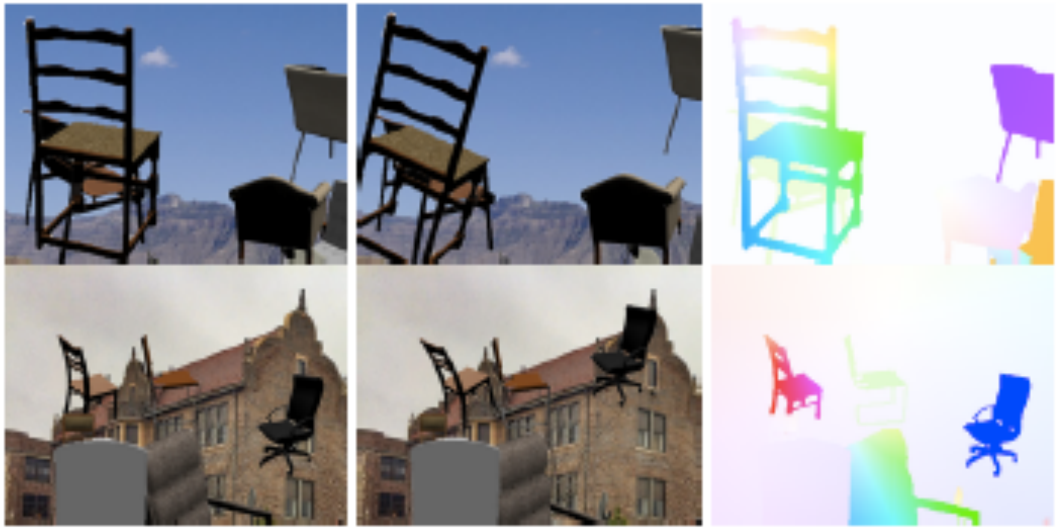
Region-based +Pixel-based +Keypoint-based

Large displacement optical flow, Brox et al., CVPR 2009

Recent Developments at Optical Flow

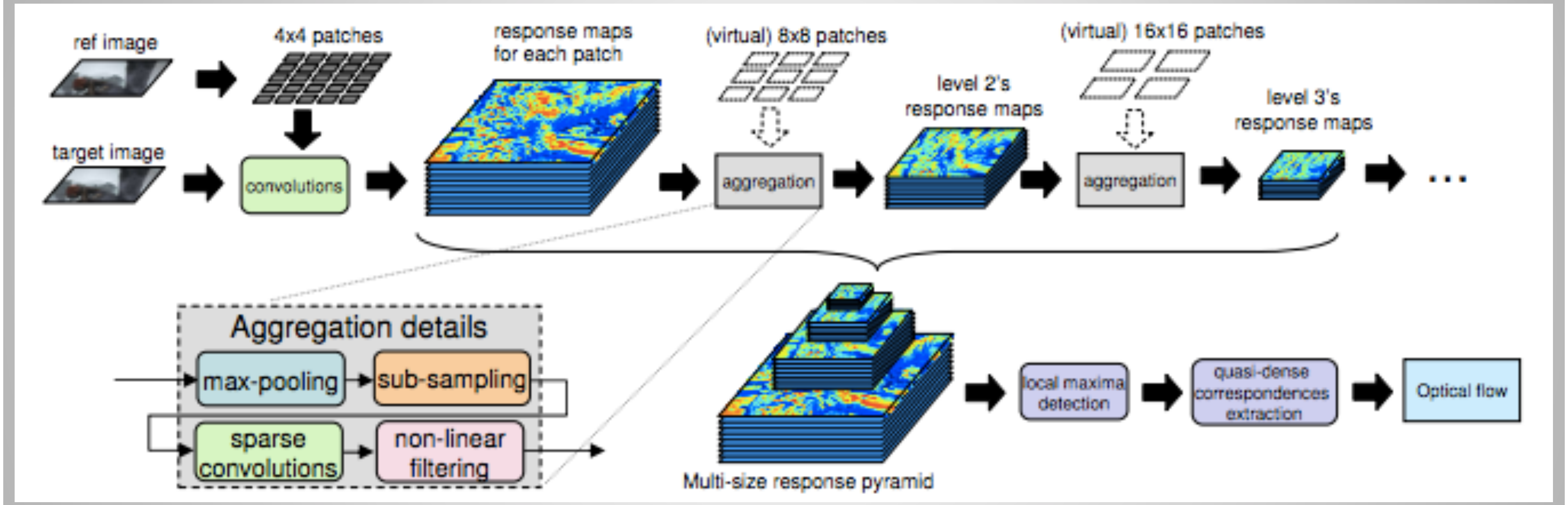
- Use of Machine Learning
  - Deep Learning (ICCV 2015, Fischer et al., FlowNet)





DeepFlow (Large Displacement Optical Flow)

- Basically it is a matching algorithm with variational approach [Weinzaepfel et al., ICCV 2013].



- Dense correspondence (matching)
- Self-smooth matching
- Large displacement optical flow
  - https://www.youtube.com/watch?v=k_wkDLJ8lJE

- Can we use SIFT features for tracking?

Ex: SIFT Tracking

Frame 0 → Frame 100

How to evaluate correctness of optical flows?

Optical Flow - Quantitative Evaluation

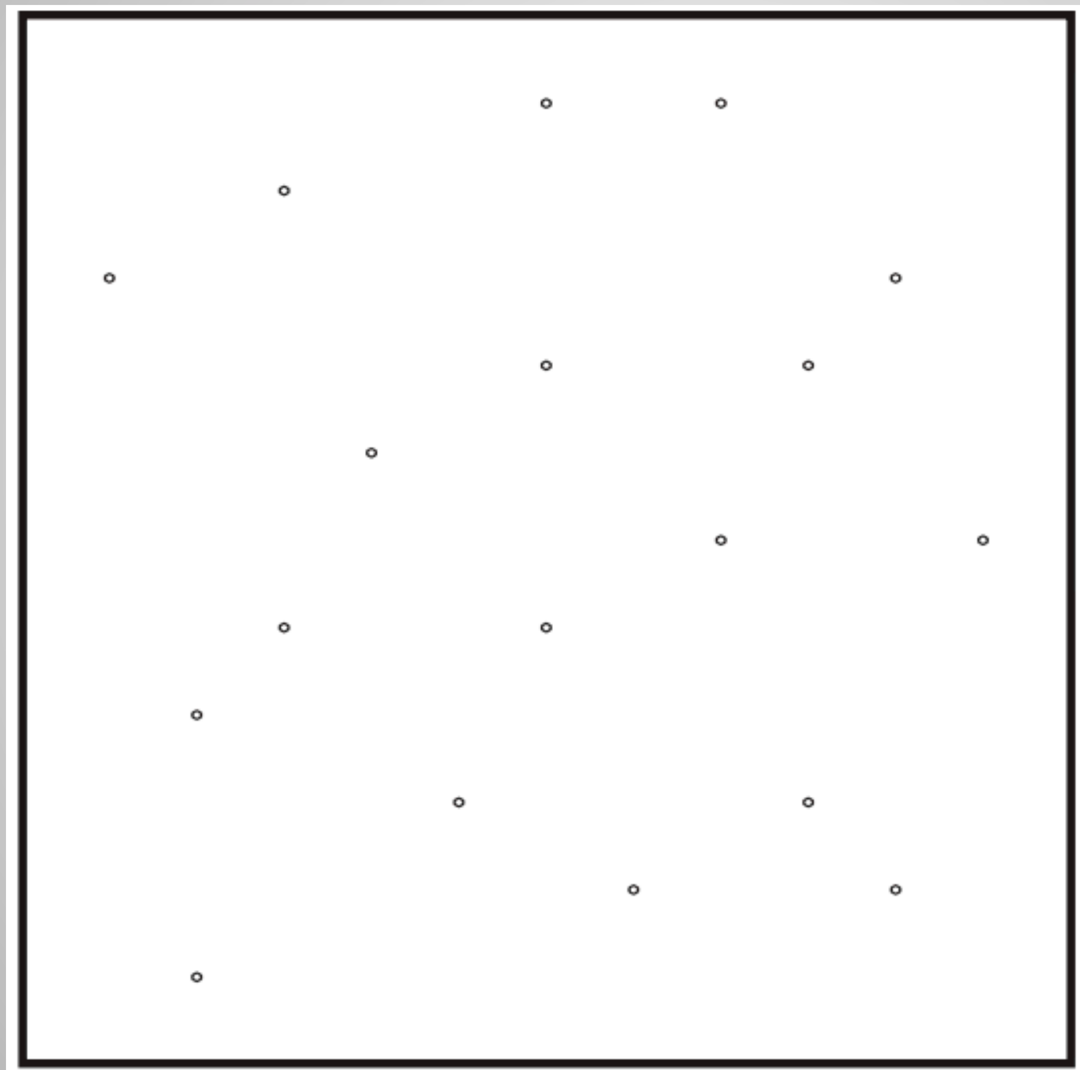
$$E_{ep2} = \sqrt{(u - u^*)^2 + (v - v^*)^2}$$

$$E_{ep1} = |u - u^*| + |v - v^*|$$

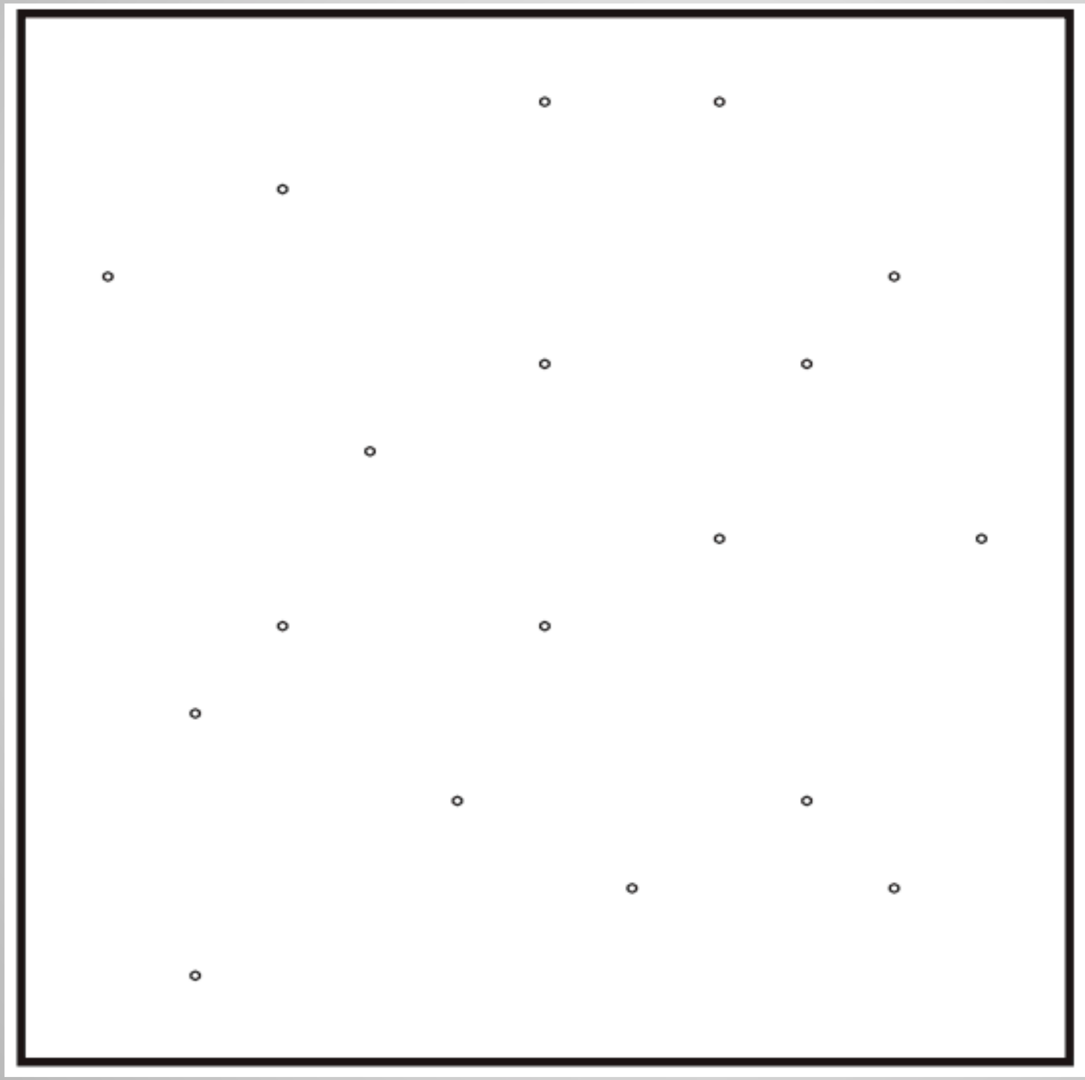
- Where **u**=*(u,v)* is computed, **u=***(u*,v*)* ground truth velocity vectors.

$$E_{ang} = \arccos\left(\frac{\mathbf{u}^T\mathbf{u}^*}{|\mathbf{u}||\mathbf{u}^*|}\right)$$

Interpretation of Optical Flow Fields

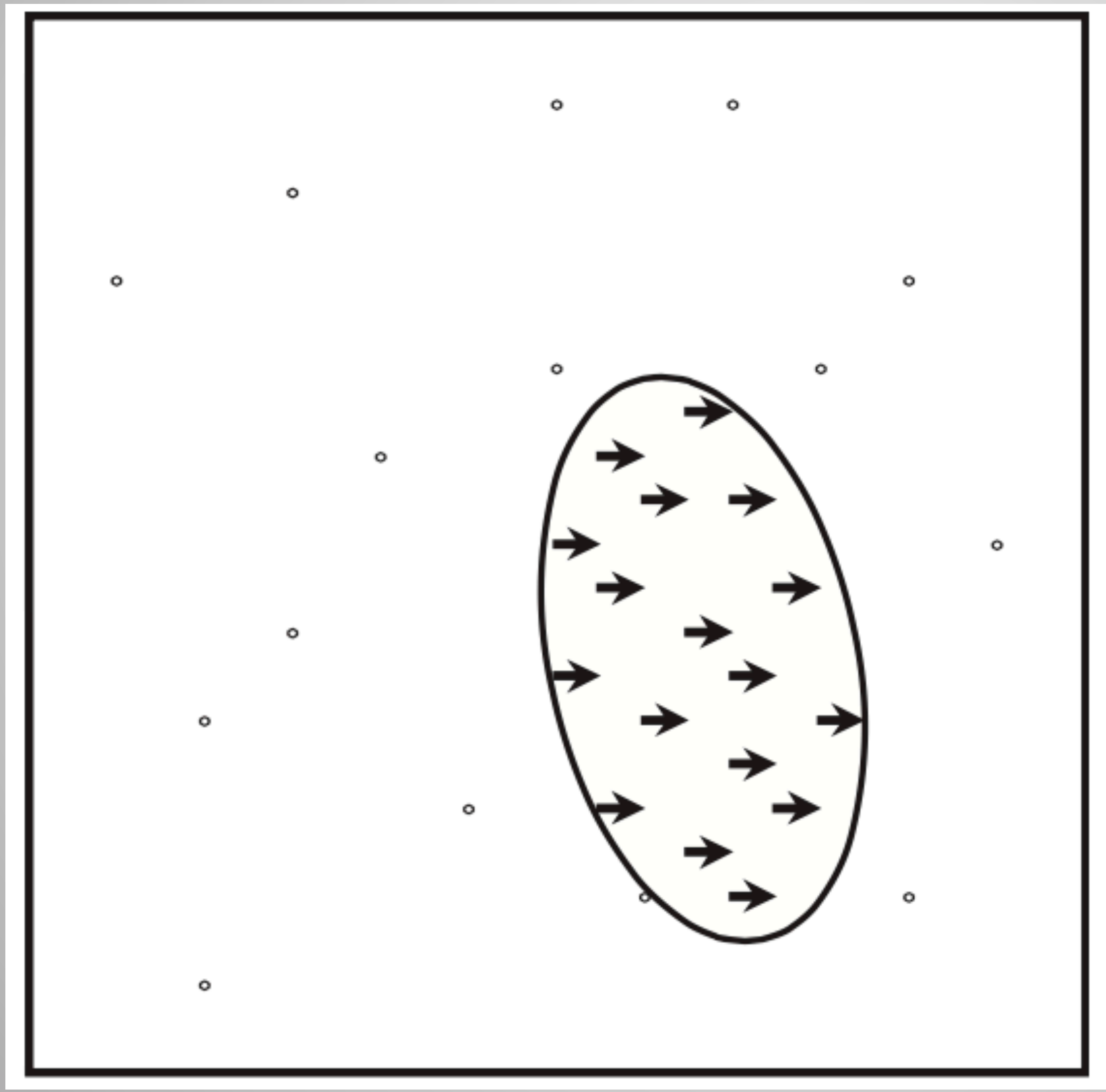


Interpretation of Optical Flow Fields

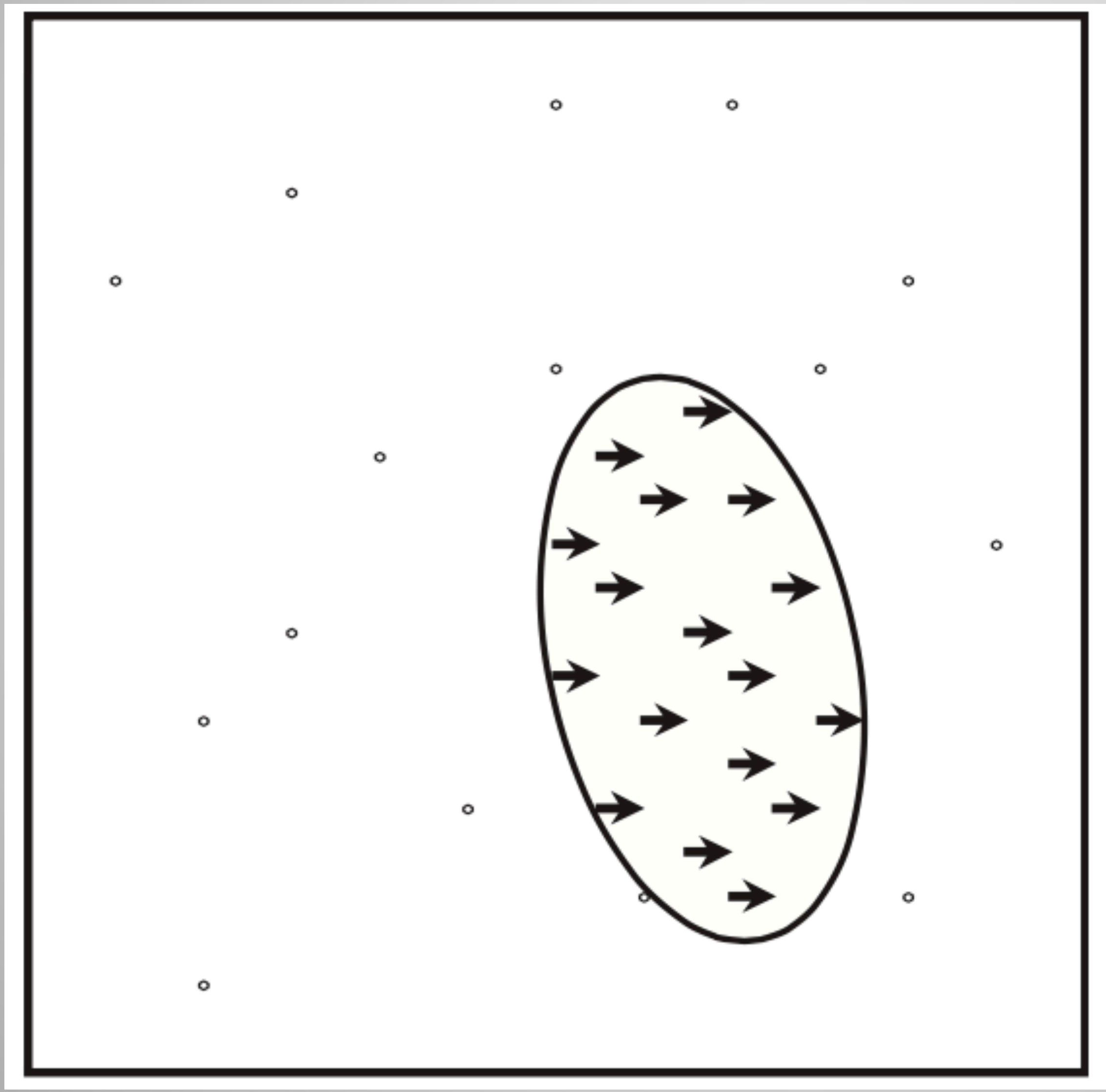


Object features all have Zero velocity.

Interpretation of Optical Flow Fields

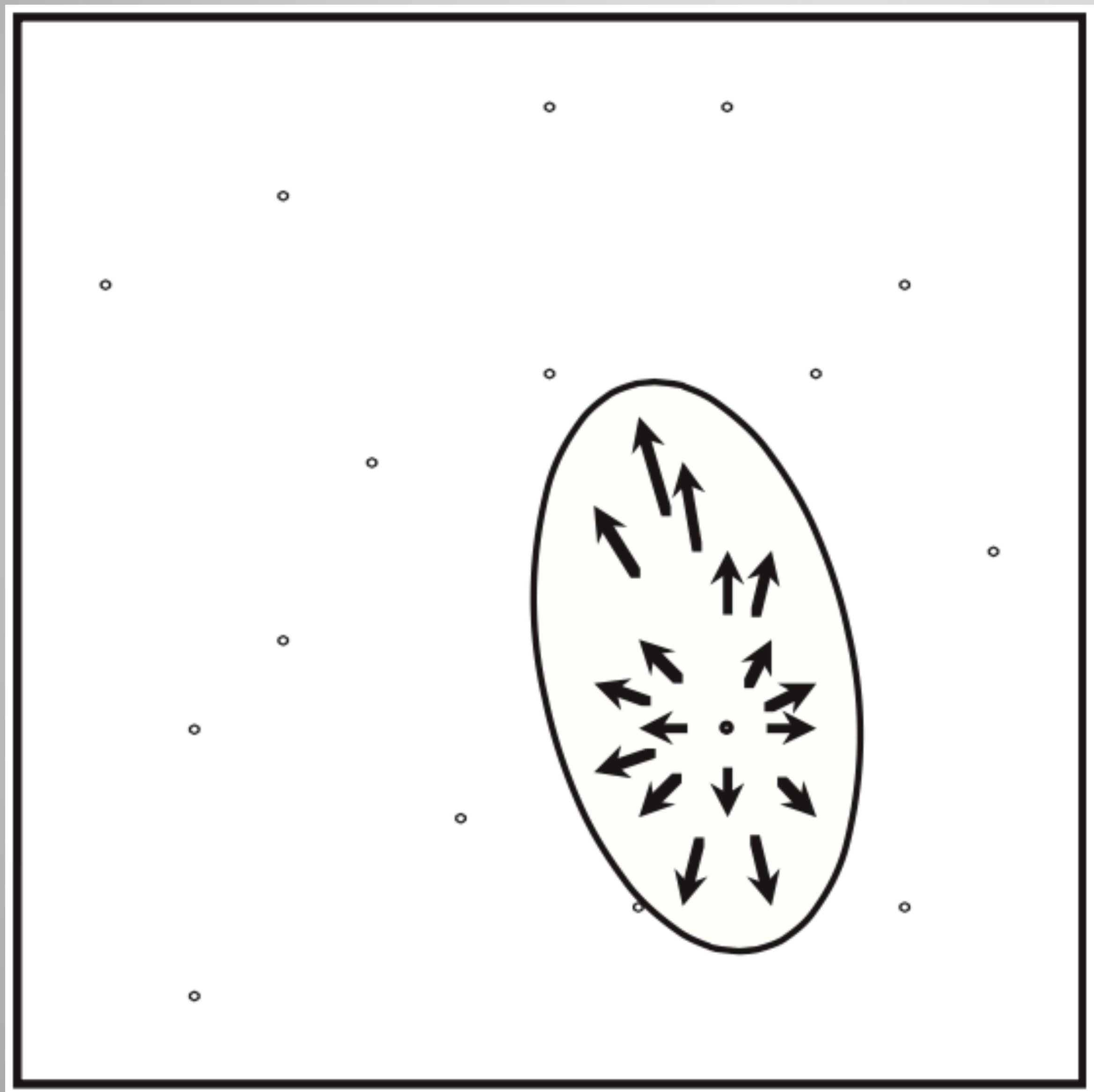


Interpretation of Optical Flow Fields

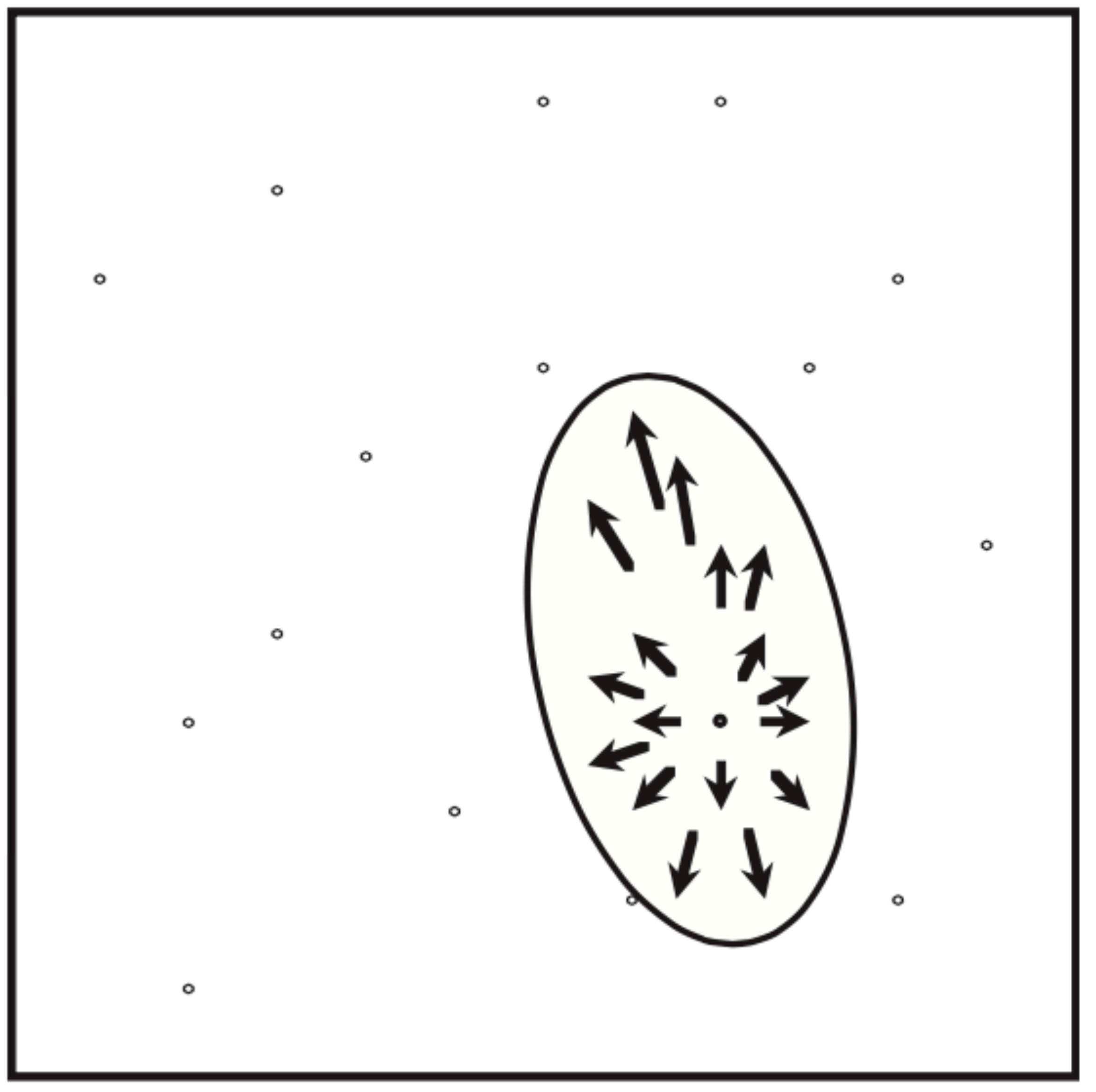


Object is moving to the Right.

Interpretation of Optical Flow Fields

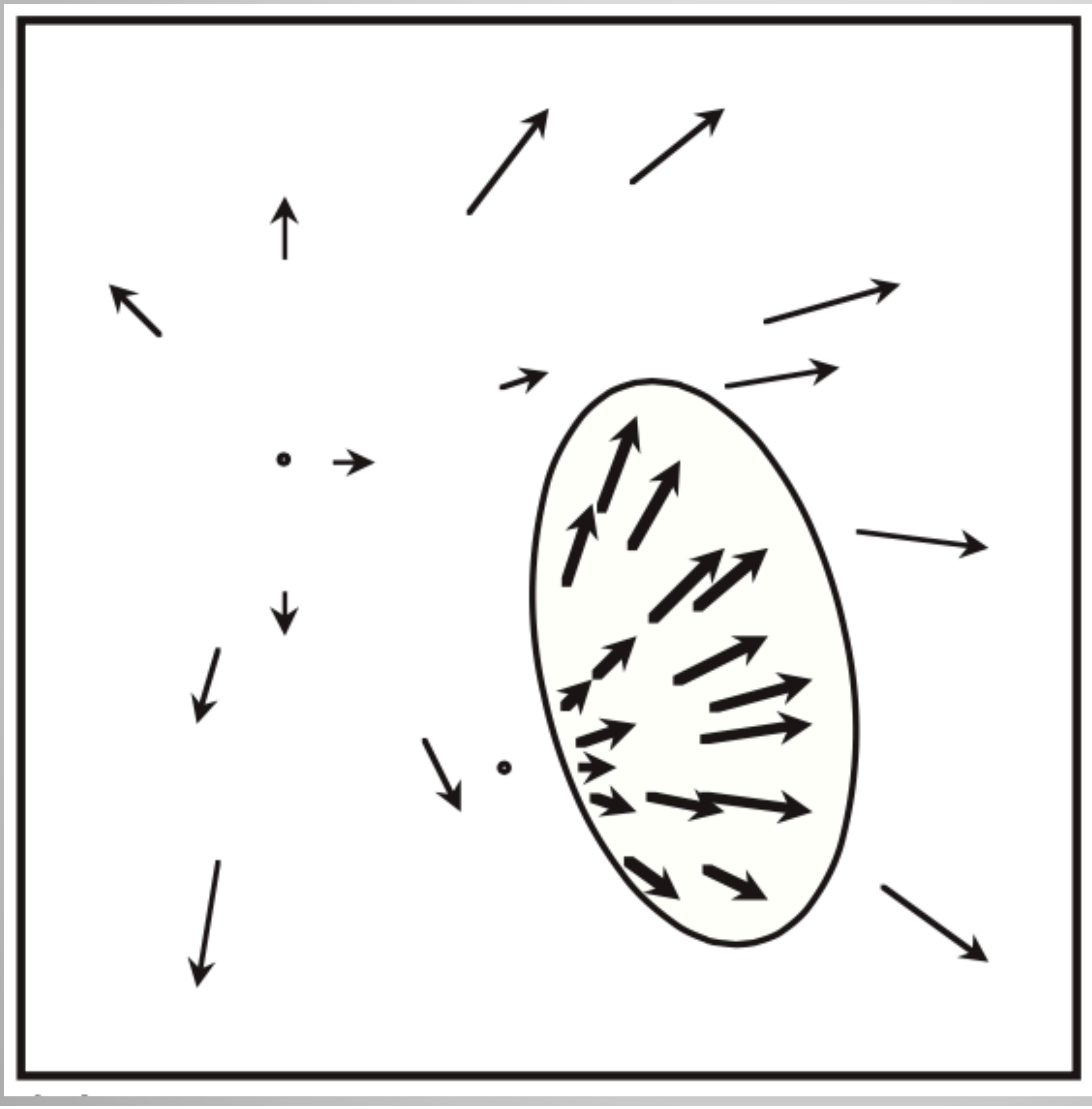


Interpretation of Optical Flow Fields

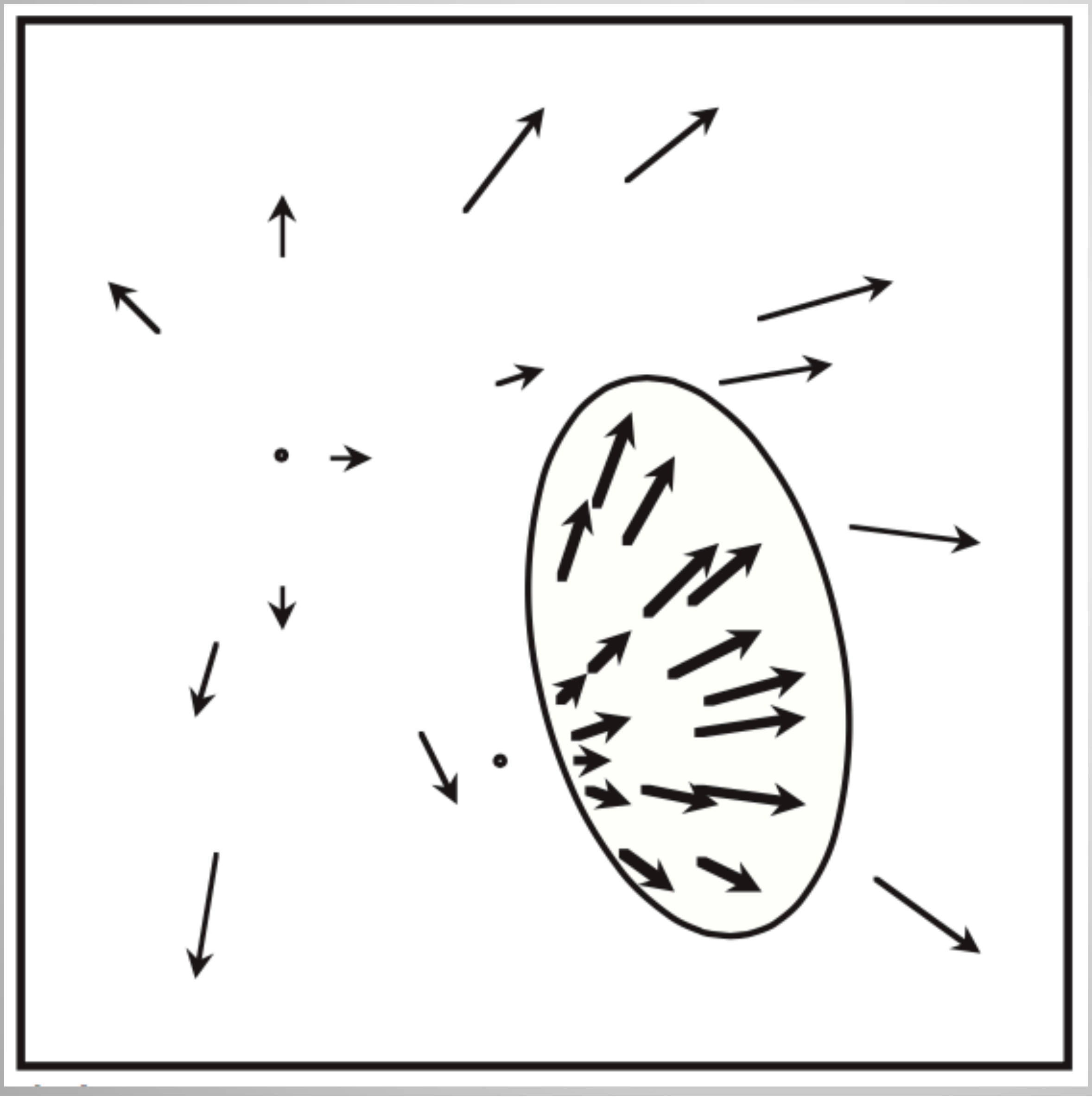


Object is moving
Directly toward the camera
that is stationary

Interpretation of Optical Flow Fields

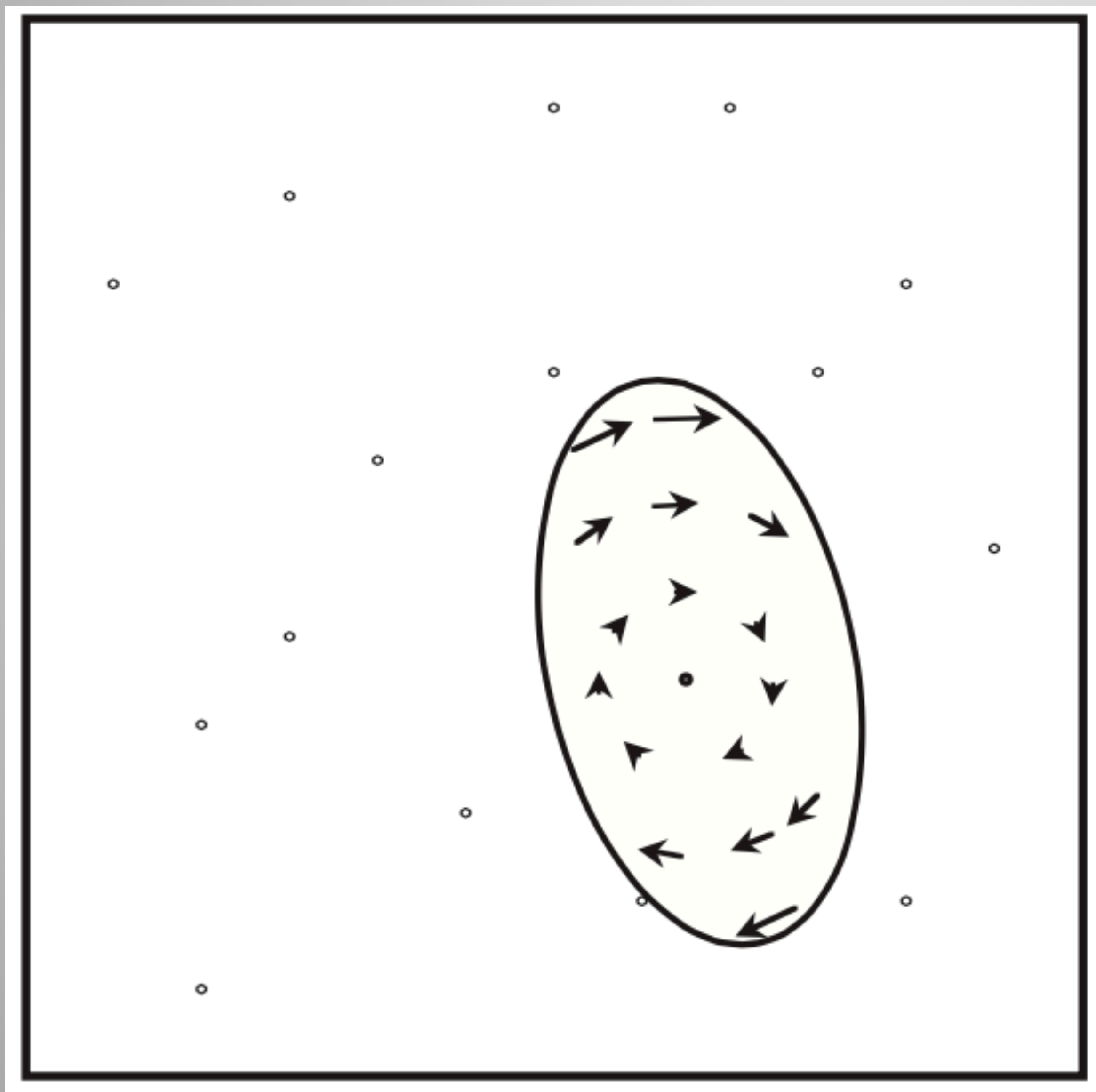


Interpretation of Optical Flow Fields

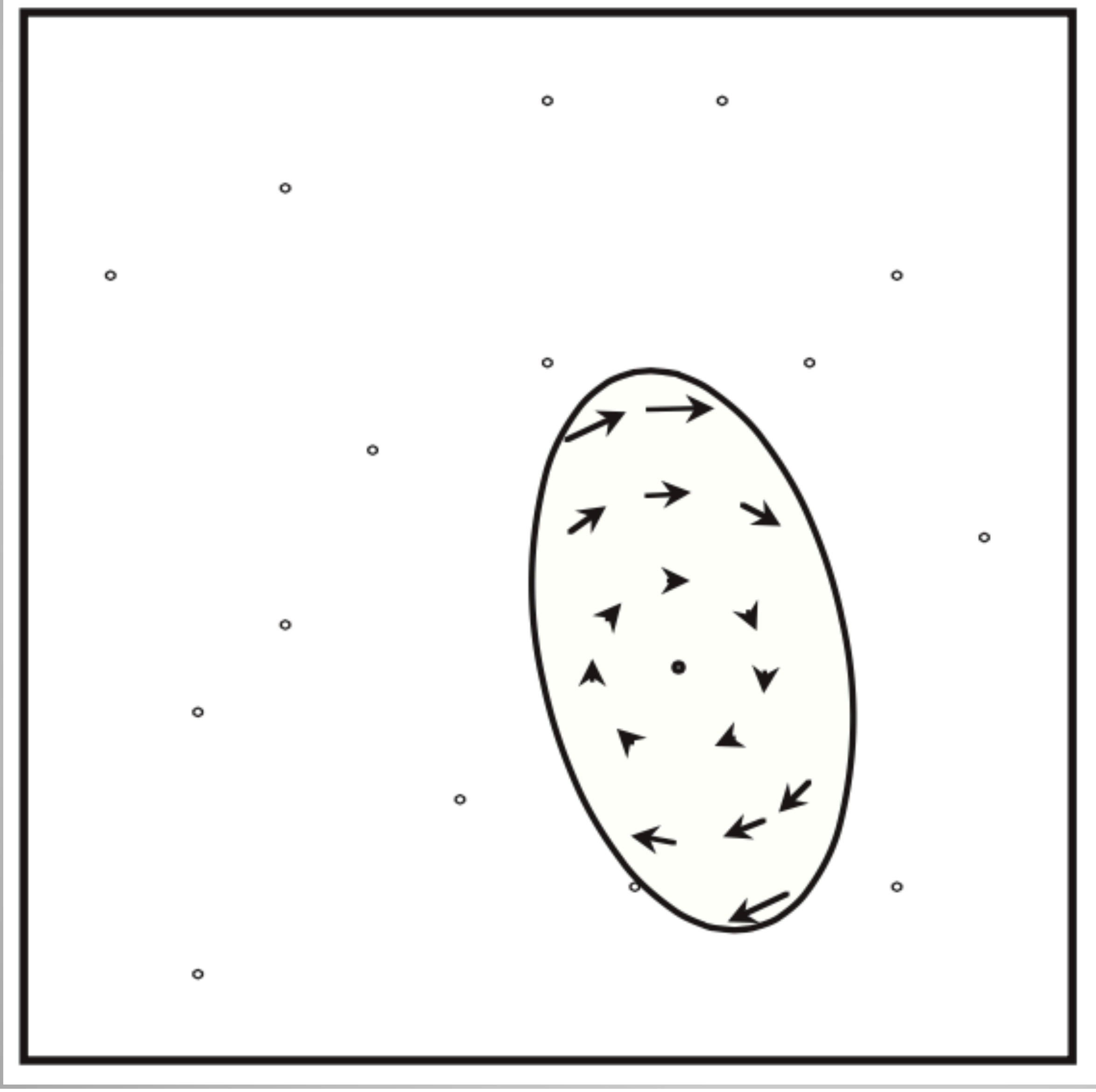


Camera is moving into
the scene, and an object
moving passed the camera

Interpretation of Optical Flow Fields

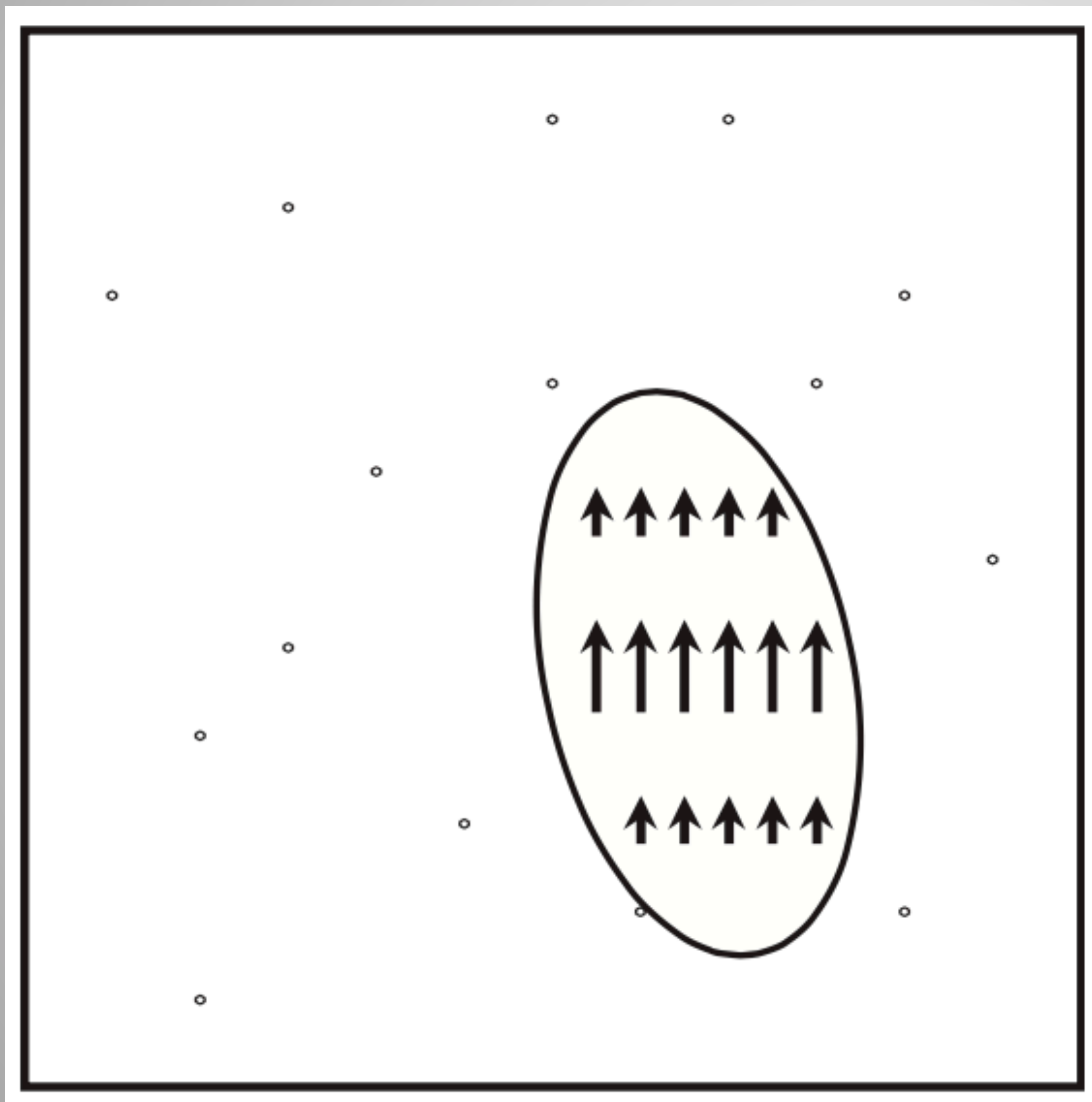


Interpretation of Optical Flow Fields



Object is rotating about
the line of sight to the camera

Interpretation of Optical Flow Fields



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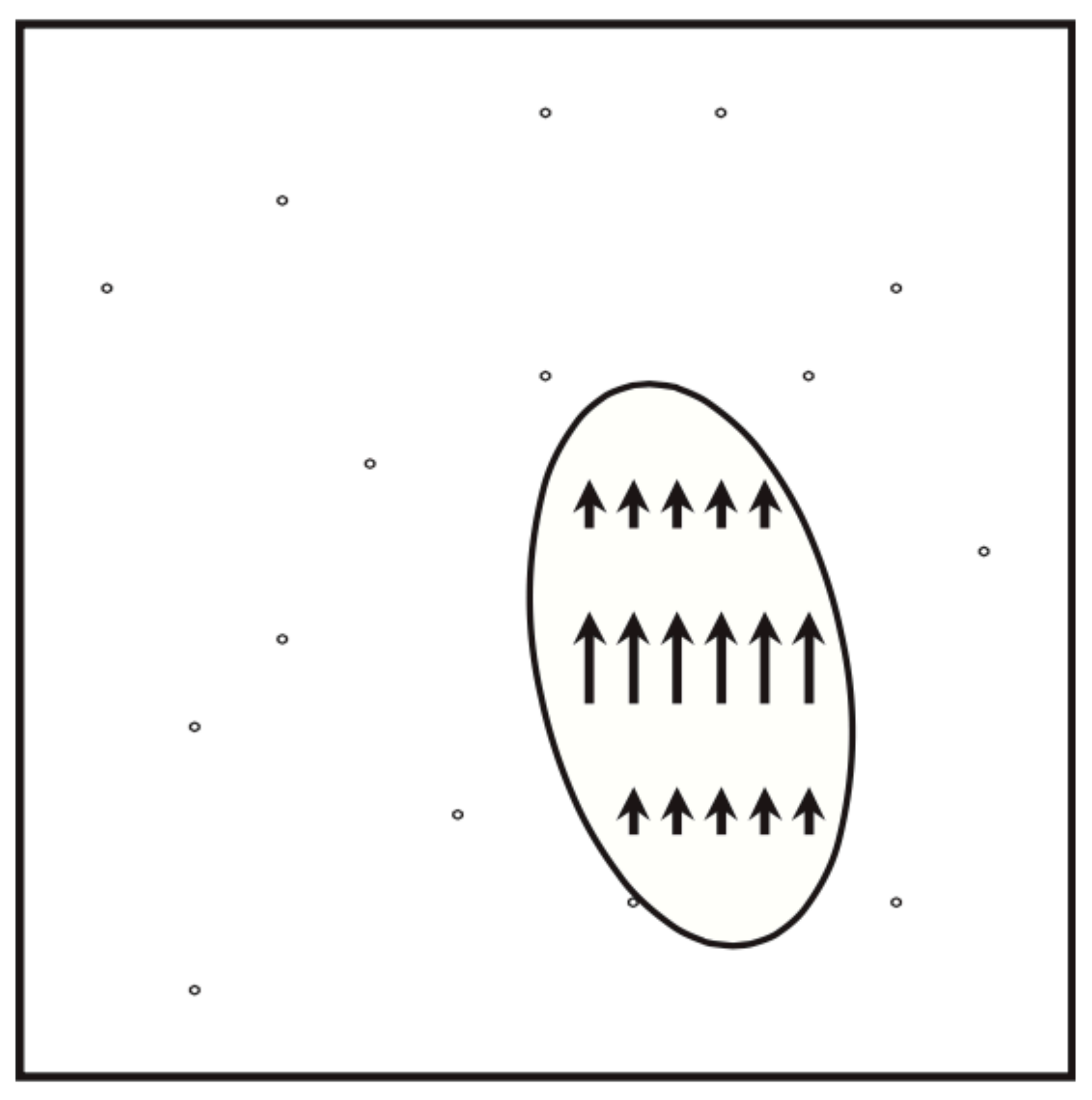
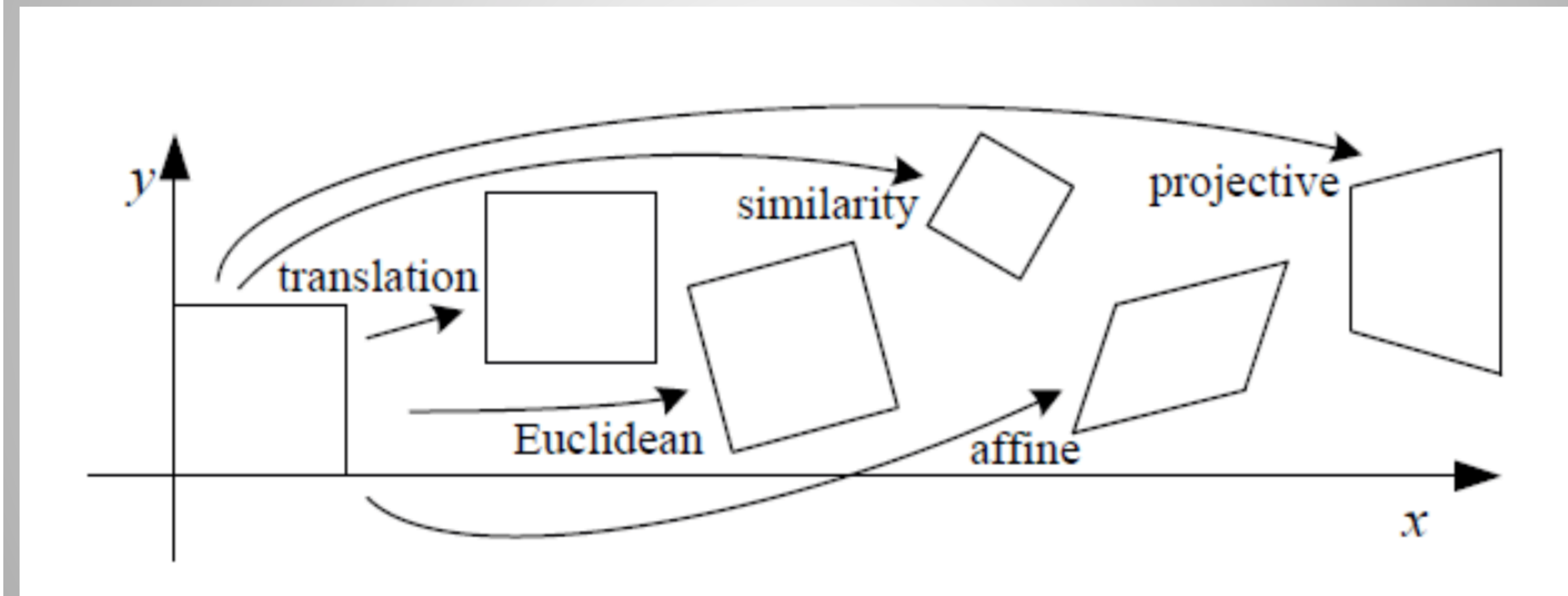


Object is rotating about an axis perpendicular to the line of sight.

Application in Image Alignment

- Motion can be used for image alignment



Pixel locations at time t+1 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ Pixel locations at time t

Practice: Homogenous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

Practice: Homogenous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

$$\textbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Practice: Homogenous Coordinates

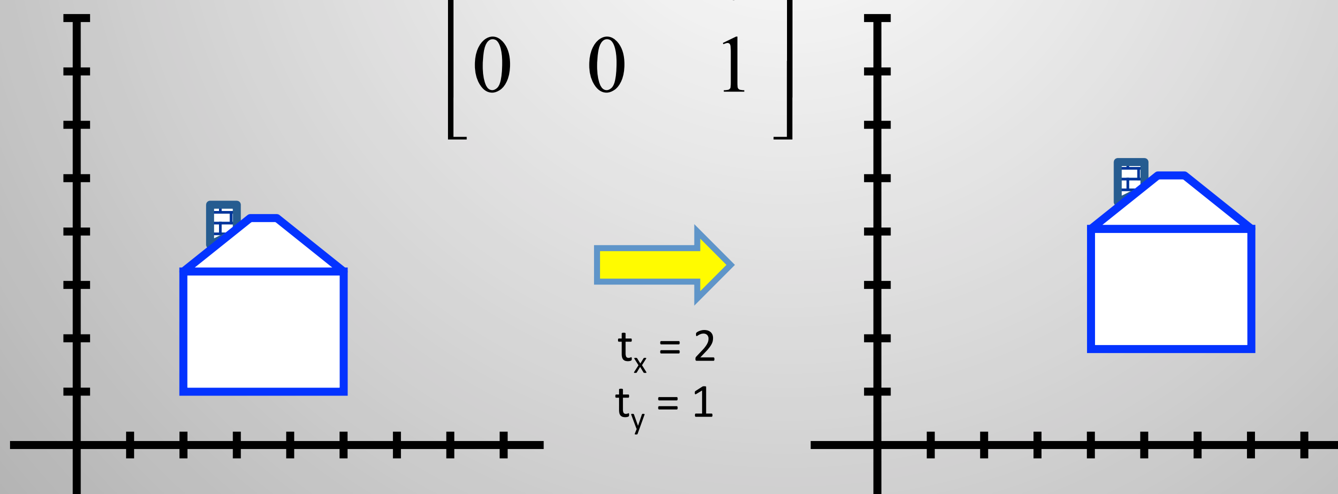
Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

$$\boldsymbol{Translation} = \begin{bmatrix} 1 & 0 & \boldsymbol{t_x} \\ 0 & 1 & \boldsymbol{t_y} \\ 0 & 0 & 1 \end{bmatrix}$$



Practice: Basic 2D Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh}_x & 0 \\ \boldsymbol{sh}_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

Affine Transformation

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations
- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Models change of basis

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

projective

Affine Transformation

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine matrix decomposition

Translation+rotation+scaling

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

p' = $T(t_x,t_y)$ $R(\Theta)$ $S(s_x,s_y)$ **p**

Questions?

- PA2 will be including optical flow estimations.