

Assignment 1

AI1110: Probability and Random Variables

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Question : A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.5 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution.

The Sample space of for tosses is
 $S = \{TTTT, TTTH, TTHT, THTT, HTTT, TTHH, THTH, THHT, HTHT, HHTT, HTTH, THHH, HTHH, HHHT, HHHH\}$

After 4 tosses he can have 5 different amounts

- 1) 4 heads & 0 tails - $1*4 - 1.5*0 = 4$
- 2) 3 heads & 1 tails - $1*3 - 1.5*1 = 1.5$
- 3) 2 heads & 2 tails - $1*2 - 1.5*2 = -1$
- 4) 1 head & 3 tails - $1*1 - 1.5*3 = -3.5$
- 5) 0 heads & 4 tails - $1*0 - 1.5*4 = -6$

Let us solve this by binomial random variable. here, random variable is a function assigning values of number of heads in 4 tosses to amount won after 4 tosses

we know, for binomial random variable, probability is given by $\Pr(x, n, P)$

x = number of successes (here number of heads)

n = number of trials (here 4)

P = probability of success (probability of head = 0.5)

$\Pr(x, n, P) = {}^nC_x * P^x * (1 - P)^{n-x}$

$$\begin{aligned} 1) \text{ Probability of having 4 heads and amount Rs=4 is } & \Pr(X = 4, 4, 0.5) \\ & = {}^4C_4 * 0.5^4 * (1 - 0.5)^{4-4} \\ & = \frac{1}{16} \end{aligned}$$

$$\begin{aligned} 2) \text{ Probability of having 3 heads and amount Rs=1.5 is } & \Pr(X = 3, 4, 0.5) \\ & = {}^4C_3 * 0.5^3 * (1 - 0.5)^{4-3} \\ & = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 3) \text{ Probability of having 2 heads and amount Rs=-1 is } & \Pr(X = 2, 4, 0.5) = {}^4C_2 * 0.5^2 * (1 - 0.5)^{4-2} \\ & = \frac{3}{8} \end{aligned}$$

$$\begin{aligned} 4) \text{ Probability of having 1 head and amount Rs=-3.5 is } & \Pr(X = 1, 4, 0.5) = {}^4C_1 * 0.5^1 * (1 - 0.5)^{4-1} \\ & = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 5) \text{ Probability of having 0 head and amount Rs=-6 is } & \Pr(X = 0, 4, 0.5) = {}^4C_0 * 0.5^0 * (1 - 0.5)^{4-0} \\ & = \frac{1}{16} \end{aligned}$$