Assignment 1

AI1110: Probability and Random Variables
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Question : A fair coin is tossed four times, and a person win Rs 1 for each head and loseRs 1.5 for each tail that turns up.From the sample space calculate how miany different amounts of money you canhave after four tosses and the probability of having each of these amounts.

Solution.

The Sample space of for tosses is S = {TTTT,TTTH,TTHT,THTT,HTTT, TTHH,THTH,THHT,HTHT,HHTT,HTTH,THHH, HTHH,HHTH,HHHT,HHHH}

After 4 tosses he can have 5 different amounts

1) 4 heads & 0 tails - 1*4-1.5*0=4
2) 3 heads & 1 tails - 1*3-1.5*1=1.5
3) 2 heads & 2 tails - 1*2-1.5*2=-1
4) 1 head & 3 tails - 1*1-1.5*3=-3.5
5) 0 heads & 4 tails - 1*0-1.5*4=-6

Let us solve this by binomial random variable. here,random variable is a function assigning values of number of heads in 4 tosses to amount won after 4 tosses

we know,for binomial random variable,probability is given by $\Pr(x, n, P)$
x=number of successes(here number of heads
n=number of trials(here 4)
P=probability of success(probability of head=0.5)
$\Pr(x, n, P) = {}^{n}C_{x} * P^{x} * (1-p)^{n-x}$

1) Probability of having 4 heads and amount Rs=4 is $\Pr(X = 4, 4, 0.5) = {}^{4}C_{4} * 0.5^{4} * (1-0.5)^{4-4}$ = $\frac{1}{16}$
2) Probability of having 3 heads and amount Rs=1.5 is $\Pr(X = 3, 4, 0.5) = {}^{4}C_{3} * 0.5^{3} * (1-0.5)^{4-3}$ = $\frac{1}{4}$
3) Probablity of having 2 heads and amount Rs=-1 is $\Pr(X = 2, 4, 0.5) = {}^{4}C_{2} * 0.5^{2} * (1-0.5)^{4-2}$ = $\frac{3}{8}$
4) Probability of having 1 head and amount Rs=-3.5 is $\Pr(X = 1, 4, 0.5) = {}^{4}C_{1} * 0.5^{1} * (1-0.5)^{4-1}$ = $\frac{1}{4}$
5) Probability of having 0 head and amount Rs=-6 is $\Pr(X = 0, 4, 0.5) = {}^{4}C_{0} * 0.5^{4} * (1-0.5)^{4-0}$ = $\frac{1}{16}$

sr.no:	parameter	value	description
1	H_0	Rs 4	0 heads
2	H_1	Rs 1.5	1 heads
3	H_2	Rs -1	2 heads
4	H_3	Rs -3.5	3 heads
5	H_4	Rs -6	4 heads