Summer Internship Project
Report

Galilean Transformations and the Galilean non-invariance of classical electromagnetism

Submitted by

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Certificate

This is to certify that this research work entitled on the Galilean non-invariance of classical electromagnetism was carried out by Gaurav Kanu, from National Institute of Science Education and Research, Bhubaneswar under my guidance.

Name of the guide
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Date:

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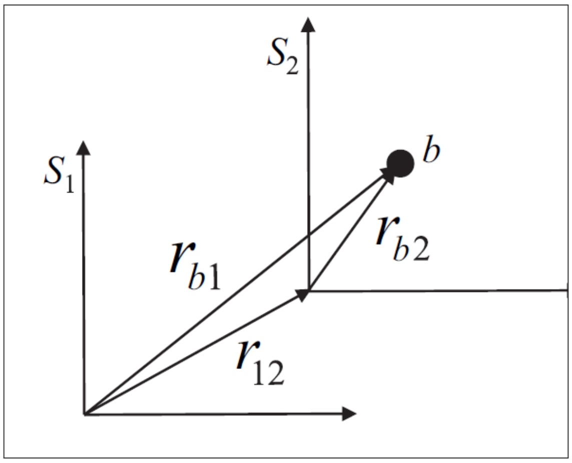
1 Abstract

To show Galilean non-invariance of classical electromagnetism in a pre-relativistic considerations, we need to avoid the use Lorentz transformations, indeed we have used Lorentz force to show it in a pre-relativistic manner. Maxwell's equation contains a parameter 'c' which leads us to doubt that non-invariance of the Galilean transformations is due to the parameter 'c' whereas I have analyzed the equations of Maxwell to the Galilean transformations and shown how they can be related pre-relativistically.

2 Introduction

Galilean transformations, also called Newtonian transformations, set of equations in classical physics that relate the space and time coordinates of two systems moving at a constant velocity relative to each other. Adequate to describe phenomena at speeds much smaller than the speed of light, Galilean transformations formally express the ideas that space and time are absolute; that length, time, and mass are independent of the relative motion of the observer; and that the speed of light depends upon the relative motion of the observer.

Let's consider two reference frames 'S_1' & 'S_2', with position vectors for the object 'b' :



$\vec{r}_{21}$= Position of S_2 w.r.t S_1
$\vec{r}_{\mathrm{b}1}$= Position of 'b' w.r.t S_1
$\vec{r}_{\mathrm{b}2}$ = Position of 'b' w.r.t S_2

The above position vectors are related by :

$$\vec{r}_{b1}(t) = \vec{r}_{21}(t) + \vec{r}_{b2}(t) \tag{1}$$

The velocity vectors are related by :

$$\vec{v}_{b1}(t) = \vec{v}_{21}(t) + \vec{v}_{b2}(t) \tag{2}$$

On diffrentiating the equation(2) w.r.t time we can see that acceleration relative to both the frames are the same :

$$\vec{a}_{b1} = \vec{a}_{b2}$$

From the above equations two facts can be deprived :
1) The displacement of object 'b' during the time interval(t_i to t_f) is not the same for S1 and S2.

$$\begin{aligned} \Delta\vec{r}_{b1} &= \vec{r}_{b1}(t_f) - \vec{r}_{b1}(t_i) \\ &= \vec{r}_{21}(t_f) + \vec{r}_{b2}(t_f) - \vec{r}_{b1}(t_i) - \vec{r}_{b2}(t_i) \\ &= \Delta\vec{r}_{b2} + \Delta\vec{r}_{21} \end{aligned} \tag{3}$$

2) If we consider two objects (say 'b'& 'c')relative position vector at particular time is the same for both S1 and S2 frames.

$$\begin{aligned} &\vec{r}_{cb1} = \vec{r}_{c1} - \vec{r}_{b1} \\ &= \vec{r}_{21} + \vec{r}_{c2} - (\vec{r}_{21} + \vec{r}_{b2}) \\ &= \vec{r}_{c2} - \vec{r}_{b2} = \vec{r}_{cb2} \end{aligned} \tag{4}$$

2.1 Kinetic energy

We know that Kinetic energy is given by K.E $= \frac{1}{2}mv^2$.
Here we check during a time interval Δt how K.E. changes w.r.t both the frames.Here $\Delta K1$ is the change in K.E of object b from frame S_1.

$$\begin{aligned} &\Delta K_1 = \tfrac{1}{2}m_b v_{b1}^2(t_f) - \tfrac{1}{2}m_b v_{b1}^2(t_i) \\ &= \tfrac{1}{2}m_b(\vec{v}_{21} + \vec{v}_{b2})^2(t_f) - \tfrac{1}{2}m_b(\vec{v}_{21} + \vec{v}_{b2})^2(t_i) \\ &= \tfrac{1}{2}m_b(v_{21}^2 + 2\vec{v}_{21}\vec{v}_{b2} + v_{b2}^2)(t_f) - \tfrac{1}{2}m_b(v_{21}^2 + 2\vec{v}_{21}\vec{v}_{b2} + v_{b2}^2)(t_i) \\ &= \Delta K_{21} + \Delta K_2 + (m_b\vec{v}_{21}\vec{v}_{b2}(t_f)) - (m_b\vec{v}_{21}\vec{v}_{b2}(t_i)) \\ &= \Delta K_{21} + \Delta K_2 + m_b\vec{v}_{21}(\vec{v}_{b2}(t_f) - \vec{v}_{b2}(t_i)) \end{aligned} \tag{5}$$

Note: Here $\Delta K_{21} = 0$ because the relative velocity between the frames(S_1&S_2) is constant.

$$\therefore \Delta K_1 = \Delta K_2 + m_b v_{21}(v_{b2}(t_f) - v_{b2}(t_i)) \tag{6}$$

2.2 Work and the work-kinetic energy theorem

Similarly as kinetic energy work done can also be calculated w.r.t both the frames(S_1 & S_2) on object b. Here W_{b1} denotes work done on object b from frame S_1, during the time intervalΔt.

$$
\begin{aligned}
W_{b1} &= m_b a.[\vec{r}_{b1}(t_f) - \vec{r}_{b1}(t_i)] \\
&= m_b a.[\vec{r}_{21}(t_f) + \vec{r}_{b2}(t_f) - \vec{r}_{21}(t_f) - \vec{r}_{21}(t_i)]
\end{aligned}
$$

$$
W_{b1} = W_{b2} + m_b a.\Delta r_{21} \tag{7}
$$

The last term in the Eq.(6) and equation Eq.(7) is known as extra work and both the quantities are equal indeed ie; $m_b v_{21}(v_{b2}(t_f) - v_{b2}(t_i) = m_b a.\Delta r_{21}$ since from Work-KE theorem $\Delta KE = W$.

2.3 Potential Energy

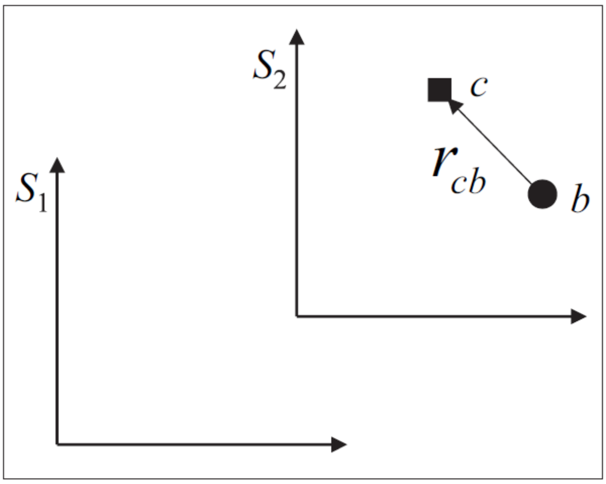


Fig. 2. A system with two objects *b* and *c*. The vector r_{cb} represents the relative position of *c* with respect to *b*. That vector is frame independent as shown in the text.

$\vec{r}_{cb}$= Relative position of 'c' w.r.t 'b'

Here for simplicity we have considered that the two objects 'b' and 'c' are interacting with each other with conservative forces and no external forces are acting on them. Work done from S_1 is given by :

$$
\begin{aligned}
W_1^{(S)} &= W_{b1} + W_{c1}, where \\
W_{b1} &= m_b \vec{a}_b.[\vec{r}_{b1}(t_f) - \vec{r}_{b1}(t_i)] \\
W_{c1} &= m_c \vec{a}_c.[\vec{r}_{c1}(t_f) - \vec{r}_{c1}(t_i)]
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&Using Newton's third law : m_b\vec{a}_b = -m_c\vec{a}_c \\
&W_1^{(S)} = m_c\vec{a}_c.[\vec{r}_{c1}(t_f) - \vec{r}_{c1}(t_i) - \vec{r}_{b1}(t_f) + \vec{r}_{b1}(t_i)] \\
&= m_c\vec{a}_c.[\vec{r}_{cb}(t_f) - \vec{r}_{cb}(t_i)] \\
&= W_2^{(S)}
\end{aligned} \tag{9}
$$

The net system work done can be defined by :

$$
\Delta U = -W^{(S)} = -m_c\vec{a}_c.[\vec{r}_{cb}(t_f) - \vec{r}_{cb}(t_i)] \tag{10}
$$

3 The Galilean non-invariance of classical electromagnetism

Galilean relativity predicts that an observer not at rest with respect to the ether measures a different speed of propagation for the electromagnetic waves. Michelson-Morley experiment provided the new invariance in the world of morden physics by Einstein known as special relativity.

Here so as to understand the Galilean non-invariance of classical electromagnetism we consider the value of c(speed of light) to be

$$c = \sqrt{\frac{1}{\mu_o \epsilon_o}} \tag{11}$$

3.1 Galilean relativity

Newtonian mechanics is invariant under the set of transformations, here for simplicity we are avoiding the rotating frames we are considering only parallel frames of reference.

$$t' = t + a, \qquad \vec{x'} = \vec{x} - \vec{v_o} t \tag{12}$$

From transformations,

$$\vec{v} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}, \qquad \vec{v'} = \frac{\mathrm{d}\vec{x'}}{\mathrm{d}t} \tag{13}$$

$$\vec{v'} = \vec{v} - \vec{v_o} \tag{14}$$

From the Galilean transformations of the velocities;

1)

$$\frac{\partial(t,x,y,z)}{\partial t'} = \frac{\partial}{\partial t}.\frac{\partial t}{\partial t'} + \frac{\partial}{\partial x}\frac{\partial x}{\partial t'} + \frac{\partial}{\partial y}\frac{\partial y}{\partial t'} + \frac{\partial}{\partial z}\frac{\partial z}{\partial t'}$$
$$= \frac{\partial t}{\partial t'}.(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial}{\partial z}\frac{\partial z}{\partial t})$$
$$= \frac{\partial t}{\partial t'}.(\frac{\partial}{\partial t} + \frac{\partial}{\partial x^i}.\frac{\partial x^i}{\partial t})$$
$$= \frac{\partial}{\partial t} + \frac{\partial}{\partial x^i}.\frac{\partial x^i}{\partial t}$$
$$\therefore \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{v}_o^i \frac{\partial}{\partial x^i} \tag{15}$$

2)

$$\frac{\partial(t,x,y,z)}{\partial x'^i} = \frac{\partial}{\partial t}.\frac{\partial t}{\partial x'^i} + \frac{\partial}{\partial x}\frac{\partial x}{\partial x'^i} + \frac{\partial}{\partial y}\frac{\partial y}{\partial x'^i} + \frac{\partial}{\partial z}\frac{\partial z}{\partial x'^i}$$

Differentiating w.r.t to x;

$$\frac{\partial(t,x,y,z)}{\partial x'} = \frac{\partial}{\partial t}.\frac{\partial t}{\partial x'} + \frac{\partial}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial}{\partial y}\frac{\partial y}{\partial x'} + \frac{\partial}{\partial z}\frac{\partial z}{\partial x'}$$
$$= \frac{\partial}{\partial x}.\frac{\partial x}{\partial x'}$$
$$= \frac{\partial}{\partial x}.\frac{\partial x'+v_o t}{\partial x'}$$
$$= \frac{\partial}{\partial x}.(\frac{\partial x'}{\partial x'} + v_o\frac{\partial t}{\partial x'})$$

$$= \frac{\partial}{\partial x} \tag{16}$$

Similarly on differentiating w.r.t 'y' and 'z' we get;

$$\frac{\partial(t,x,y,z)}{\partial y'} = \frac{\partial}{\partial y} \tag{17}$$

$$\frac{\partial(t,x,y,z)}{\partial z'} = \frac{\partial}{\partial z} \tag{18}$$

From equations 14, 15, 16, we can conclude;

$$\frac{\partial(t,x,y,z)}{\partial x'^i} = \frac{\partial}{\partial x^i} \tag{19}$$

Since small change in the coordinates of S_1 frame is equal to small change in S_2 frame and by using equation(4) we can write

$$\vec{\nabla}' = \vec{\nabla} \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{v}_o.\nabla \tag{20}$$

Spatial volume(V) is a Galilean invariant;

$$V = V' \tag{21}$$

3.2 Electric charge invariance

In classical electromagnetism charge(q) is conserved and from(19) volume is also Galilean invariant, therefore the charge density$\rho = \rho(t, x(t))$ is also Galilean invariant.

$$\rho' = \rho \tag{22}$$

We can define charge density as

$$\vec{j} = \rho\vec{v} \tag{23}$$

under Galilean transformations current density can be defined as(w.r.t S1 and S2)

$$\vec{j}' = \vec{j} - \rho\vec{v}_o \tag{24}$$

3.3 Maxwell's equations

In SI units, the four differential Maxwell's equations of classical electromagnetism in vacuo read

$$\vec{\nabla}.\vec{B} = 0 \tag{25}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{26}$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_o} \tag{27}$$

$$\vec{\nabla} \times \vec{B} = \mu_o\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \tag{28}$$

Where $\vec{E}$ and $\vec{B}$ are electric and magnetic fields and Maxwell's constant c from (11).

3.4 Galilean transformations for the fields and invariance of Maxwell's equations

To prove Galilean non-invariance in electromagnetism in a pre-relativistic manner we avoid the use of Lorentz transformations instead we are using Lorentz force and compare it with the Maxwell's equations and Galilean relativity.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{29}$$

according to Newton's second law, mass and acceleration both are Galilean invariant itself and from charge invariance, we have

$$\vec{F} = \vec{F}' \implies \vec{E}' + \vec{v}' \times \vec{B}' = \vec{E} + \vec{v}_o \times \vec{B} \tag{30}$$

recalling Eq.14, we get

$$\vec{E}' + \vec{v} \times (\vec{B}' - \vec{B}) = \vec{E} + \vec{v}_o \times \vec{B}' \tag{31}$$

The only possible solution of (31) not involving velocities in a specific reference frame, and without restrictions on the choice of v, is

$$\vec{B}'(x', t) = \vec{B}(x, t) \tag{32}$$

for Electric field;

$$\vec{E}'(x', t') = \vec{E}(x, t) + \vec{v}_o \times \vec{B}(x, t) \tag{33}$$

3.5 Maxwell's equations and Galilean invariance

3.5.1 Magnetic Gauss law

From equations (25) ,(20), (32) we can immediately find

$$\vec{\nabla}'.\vec{B}' = \vec{\nabla}.\vec{B} = 0 \tag{34}$$

Therefore, magnetic Gauss law is Galilean invariant.

3.5.2 Faraday's law

From equations (20), (26), (32), (33) we can write as;

$$\vec{\nabla}' \times \vec{E}' + \tfrac{\partial \vec{B}'}{\partial t'} = \vec{\nabla} \times (\vec{E} + \vec{v}_o \times \vec{B}) + (\tfrac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B}$$
$$= \vec{\nabla} \times \vec{E} + \vec{\nabla} \times (\vec{v}_o \times \vec{B}) + \tfrac{\partial \vec{B}}{\partial t} + (\vec{v}_o.\vec{\nabla})\vec{B}$$

$$\vec{\nabla}' \times \vec{E}' + \frac{\partial \vec{B}'}{\partial t'} = \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \tag{35}$$

3.5.3 Gauss law

From equations (20), (27), (32), (33), and the vector formula $\nabla.(A \times B) = B.(\nabla \times A) - A.(\nabla \times B)$ we get;

$$\vec{\nabla}'.\vec{E}' - \tfrac{\rho'}{\epsilon_o} = (\vec{\nabla}.\vec{E} - \tfrac{\rho}{\epsilon_o}) - \vec{v}_o.(\vec{\nabla} \times \vec{B})$$
$$= \vec{\nabla}.\vec{E} - \tfrac{\rho}{\epsilon_o} + \vec{\nabla}.(\vec{v}_o \times \vec{B})$$

$$\vec{\nabla}'.\vec{E}' - \frac{\rho'}{\epsilon_o} = \vec{B}.(\vec{\nabla} \times \vec{v}_o) - \vec{v}_o.(\vec{\nabla} \times \vec{B}) \tag{36}$$

Hence, we see that the Galilean invariance of Gauss' law is assured if

$$\vec{v}_o.(\vec{\nabla} \times \vec{B}) = 0 \tag{37}$$

which in general is not satisfied.

3.5.4 Ampere's law

From equations (20), (28), (32), (33) and (11) we can write

$$\vec{\nabla}' \times \vec{B}' - \mu_o \vec{j}' - \frac{1}{c^2}\frac{\partial \vec{E}'}{\partial t'} = \nabla \times \vec{B} - \mu_o(\vec{j} - \rho\vec{v}_o) - \frac{1}{c^2}(\frac{\partial}{\partial t} + \vec{v}_o.\nabla)(\vec{E} + \vec{v}_o \times \vec{B})$$
$$= \vec{\nabla} \times \vec{B} - \mu_o\vec{j} + \mu_o\rho\vec{v}_o - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2}(\vec{v}_o.\vec{\nabla})\vec{E} - \frac{1}{c^2} \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})(\vec{v}_o \times \vec{B}))$$
$$= (\vec{\nabla} \times \vec{B} - \mu_o\vec{j} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}) + \mu_o\rho\vec{v}_o - \frac{1}{c^2}(\vec{v}_o.\vec{\nabla})\vec{E} - \frac{\vec{v}_o}{c^2} \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B})$$
$$= (\vec{\nabla} \times \vec{B} - \mu_o\vec{j} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}) + \mu_o\rho\vec{v}_o - \mu_o\epsilon_o(\vec{v}_o.\vec{\nabla})\vec{E} - \vec{v}_o(\mu_o\epsilon_o) \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B})$$
$$= (\vec{\nabla} \times \vec{B} - \mu_o\vec{j} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}) + \mu_o\rho\vec{v}_o - \mu_o\epsilon_o((\vec{v}_o.\vec{\nabla})\vec{E} + \vec{v}_o \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B}))$$

$$\vec{\nabla}' \times \vec{B}' - \mu_o\vec{j}' - \frac{1}{c^2}\frac{\partial \vec{E}'}{\partial t'} = (\vec{\nabla} \times \vec{B} - \mu_o\vec{j} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}) + \mu_o\rho\vec{v}_o - \mu_o\epsilon_o((\vec{v}_o.\vec{\nabla})\vec{E} + \vec{v}_o \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B})) \tag{38}$$

the above equation will be null if

$$\rho\vec{v}_o = \epsilon_o((\vec{v}_o.\vec{\nabla})\vec{E} + \vec{v}_o \times ((\frac{\partial}{\partial t} + \vec{v}_o.\vec{\nabla})\vec{B})) \tag{39}$$

which in general is not satisfied.

4 Conclusion

In this project I have shown, how the Galilean non-invariance of classical electromagnetism can be proved without recourse to the historically posterior knowledge of the Lorentz transformations.
Indeed, the definition of Maxwell's parameter c in terms of observer-independent quantities implies its scalar invariant character under frame transformations; hence it can't be determined that the Galilean non-invariance actually comes from the presence of this parameter c in Maxwell's equations.
During this project I came across many Galilean invariant and non-invariant terms not only from the Newtonian mechanics but some from classical electromagnetism.
I also came across that the theory of special relativity do not hold to Galilean invariance, because Galilean transformations are not applicable for high velocity objects.

Acknowledgment

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