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1(a) In convective zone:

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \rightarrow (1)$$

$$P = \frac{K_B \rho T}{\mu m_H} \quad (\text{IDG eqn}) \quad — (2)$$

$$\frac{dP}{dr} = -\frac{GM_\odot}{r^2}\rho \quad — (3)$$

$$\Rightarrow \frac{dT}{dr} = -\frac{2}{5}\frac{GM_\odot}{r^2}\frac{\mu m_H}{K_B}$$

$$\Rightarrow \int_{T_0}^{T} dT = \frac{-2}{5}\int_{r_0}^{r}\frac{GM_\odot}{r^2}\frac{\mu m_H}{K_B}\,dr \quad \left(\text{Here } r_0 = 0.7 R_\odot\right)$$

For density: From (1)

$$\frac{dT}{dR} = \frac{2}{5}\frac{T}{P}\frac{dP}{dr} \quad — (4)$$

From (2),

$$\frac{dP}{dr} = \frac{k_B}{\mu m_H}\left[T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right] \longrightarrow (5)$$

Substituting (5) in (4) we get:

$$\frac{dT}{dR} = \frac{2}{5}\frac{T}{P}\frac{k_B}{\mu m_H}\left[T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right]$$

$$\Rightarrow \frac{3}{2}\frac{dT}{T} = \frac{d\rho}{\rho}$$

and on integrating we get:

$$\frac{\ln T(r)}{T_0} = \ln\left[\left(\frac{\rho(r)}{\rho_0}\right)^{2/3}\right]$$

$$\Rightarrow \rho(r) = \rho_0\left[\frac{T(r)}{T_0}\right]^{3/2}$$

{ The boundary conditions used $T(r_0) = T_0$, $\rho(r_0) = \rho_0$ }

For pressure: $$\frac{dP}{dr} = -\frac{GM_\odot \rho(r)}{r}$$

$$P(r) = P_0 - GM_\odot \rho_0 \int_{r_0}^{r} \frac{\rho(r')}{r'^2}\,dr'$$

which can be found using integration of $\frac{\rho(r')}{r'^2}$ found before.

(b) $X = 0.7$, $Z = 0.02$.

No. of particles per unit volume.

$$n = (1.4 + 0.01 + 0.1 + 0.1725)\frac{\rho}{m_H} = 1.5825\frac{\rho}{m_H}$$

$$\Rightarrow \frac{m_H}{\rho} = 0.632$$

$\therefore$ Mean particle mass $= \mu m_H = 2.64 \times 10^{-27}$ Kg.

(c) $\frac{dM}{dr} = 4\pi r^2 \rho(r)$

$$\therefore M = \int_{r_0}^{R_0} 4\pi r^2 \rho(r)\, dr$$

$$= 4\pi \int_{r_0}^{R_0} r^2 \left(1 + 2.277\times10^7 \left(\frac{1}{r} - \frac{1}{0.7R_\odot}\right)\right)^{3/2}$$

Numerical integration of the above will yield the answer.

2.) HR diagram for a globular cluster will have apparent magnitude plotted against B−V, while HR diagram for stars not limited to a cluster will have absolute magnitude against B−V.

So, we pick a value of B−V on the main seq. (MS) of nearby stars (to globular cluster) and find what value of distance 'd' a star in the cluster of same temperature have same absolute magnitude.

From fig 3.5, we choose point (0.5, 5) and from fig. 3.8 point (0.5, 20)

Now, using $m - M = 5 \log \frac{d}{10}$; $m \rightarrow$ apparent magnitude, $M \rightarrow$ Absolute magnitude.

$$\Rightarrow 20 - 5 = 5 \log \frac{d}{10}$$

$$\Rightarrow \frac{d}{10} = 1000$$

$$\Rightarrow d = 10^4 pc = 32600 \, ly.$$

(b) The pre calculated value from internet is 33920 ly as distance to M_3.

This value differs from that obtained from calculations by $\frac{33920 - 32600}{33920} \times 100 \approx 4\%$.

3.) Proportionality relations derived in class:

$$L \propto M^3 \; ; \; L \propto T^6 \Rightarrow M \propto T^2 \text{ or } M = X T^2$$

X = Constant.

Wien's law: $\lambda_{max} T = 2.898 \times 10^{-3} \, mK$.

Finding X using $M_\odot$ and T_{eff} values of Sun.

$$X = \frac{M_\odot}{T_{eff}} = \frac{2 \times 10^{30}}{(6 \times 10^3)^2} = 5.56 \times 10^{22} \, Kg \, K^{-2}$$

* When $M = 9M_\odot$, $T_{eff}' = \sqrt{\frac{M T_{eff}^2}{M_\odot}} = 6000 \times \sqrt{\frac{9M_\odot}{M_\odot}}$

$= 18000\,K$.

$\therefore \lambda_{max} = \frac{2.898 \times 10^{-3}}{18000\,K} = 0.161 \times 10^{-6}$

$= 161\,nm$. (UV)

* When $M = 0.25\,M_\odot$,

$T_{eff} = \sqrt{\frac{M\,T_{eff}^2}{M_\odot}} = 6000\sqrt{\frac{0.25\,M_\odot}{M_\odot}}$

$= 3000\,K$

$\therefore \lambda_{max} = \frac{2.898 \times 10^{-3}}{3000} = 0.966 \times 10^{-6}$

$= 966\,nm$ (IR).

5.) Energy generation rates per unit volume.

$r\Delta E = \rho E$

where $E_{pp} = 0.24\,\rho X^2 \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right]$ W/Kg

$\Rightarrow r\Delta E = 0.24\,\rho^2 X^2 \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right]$ W/m² here substitute $\rho = 1.48 \times 10^5 \ kg \ m^{-3}$

$T = 1.56 \times 10^7 K$

$X \doteq X_H = 0.64$ to obtain.

Energy generation per unit volume:

$$= 0.24 (1.48 \times 10^5)^2 (0.64)^2 \left(\frac{10^6}{1.56 \times 10^7}\right)^{2/3}$$

$$\exp\left[-33.8 \left(\frac{10^6}{1.56 \times 10^7}\right)^{1/3}\right] W/m^2$$

$$= 1078.9 \ W/m^2.$$

→ For CNO cycle, the corresponding eqn. for E becomes:

$$E_{CNO} = 8.7 \times 10^{20} \rho X_{CNO} X_H \left(\frac{10^6}{T}\right)^{2/3} \exp\left(-152.3 \left(\frac{10^6}{T}\right)^{1/3}\right)$$

$$\Rightarrow r \Delta E = 8.7 \times 10^{20} \rho^2 X_{CNO} X_H \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-152.3 \left(\frac{10^6}{T}\right)^{1/3}\right] W/kg$$

here, use $X_{CNO} = 0.015$ and others values as mentioned before:

$$\Rightarrow r \Delta E = 8.7 \times 10^{20} \times (1.48 \times 10^5)^2 \times 0.015 \times 0.064 \left(\frac{10^6}{1.56 \times 10^7}\right)^{2/3}$$

$$\exp\left(-152.3 \left(\frac{10^6}{1.56 \times 10^7}\right)^{1/3}\right]$$

$$= 151.7 \ W/m^2.$$

1/ (a) According to Virial Theorem:

$$E_T = -\frac{1}{2} E_G = -\frac{GM\Delta M}{2R} \quad ; \Delta M = \text{Mass accreted.}$$

Luminosity: $L = \frac{\Delta E_T}{\Delta t}$

$$= -\frac{GM}{2R}\frac{\Delta M}{\Delta t}$$

$\Rightarrow$ Luminosity due to release of energy following accretion is:

$$L_{acc} = -\frac{GM\dot{M}}{2R}.$$

(b) Equation for Eddington luminosity limit given in text:

$$L < \frac{4\pi cGM}{\underbrace{\chi}_{\text{opacity}}}$$

We consider an equality to get maximum accretion rate.

$$L_e = \frac{4\pi cGMm_H}{\sigma_T} \quad \left(\because \chi = \frac{\sigma_T}{m_H} \text{ for Thompson scattering of fully ionised H}\right)$$

Using condition $L_{acc} = L_e$ we get:

$$\frac{4\pi cGMm_H}{\sigma_T} = \frac{1}{2}\frac{GM\dot{M}}{R}.$$

$$\Rightarrow \dot{M}_{edd} = \frac{8\pi cm_H R}{\sigma_T M_\odot} M_\odot yr^{-1}$$

(c) Now we find $\dot{M}_{edd}$ for 3 cases.

(i) $R = R_\odot$

$$\dot{M}_{edd} = \frac{8\pi \times 3\times10^{8}\times 1.67\times10^{-27}\times 6.317\times10^{6}}{1.99\times10^{30}}$$

$$= 19.37\times10^{6}\times10^{-12}$$

$$= 1.93\times10^{-5}\ M_\odot\, yr^{-1}.$$

(ii) $R = 10^{-2} R_\odot$

$$\dot{M}_{edd} = 1.93\times10^{-7}\ M_\odot\, yr^{-1}$$

(iii) $R = 2\times10^{-5} R_\odot$

$$\dot{M}_{edd} = 2\times10^{-5}\times 1.93\times10^{-5}$$

$$= 3.86\times10^{-10}\ M_\odot\, yr^{-1}.$$

(d) Eddington limit can be exceeded in the below cases:

→ Fall back of debris onto a Black hole (BH) after a star is tidally disrupted by the BH.

→ Nova can shine better than L_{edd} for a long period of time.