Algorithms and Data Structures

Analyzing Algorithms: Runnning Time

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Analyzing algorithms: key points

Correctness:

Does the algorithm terminate for each input with a correct answer?

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Examples of simple data structure:

- arrays,
- stacks and queues,
- linked lists,
- hash tables,
- search trees, ...

$\longrightarrow$ defined and discussed later.

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Generally, we seek upper bounds on the running time of an algorithm.

$\longrightarrow$ Guarantees!

Different kinds of analysis

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- Requires information on statistical distribution of inputs.

Best-case: (bogus)

- $T(n) =$ minimal number of elementary steps the algorithm needs on an input of size n.
- Cheat with a slow algorithm that works fast on <u>some</u> input.

Insertion sort: best- and worst-case analysis

Best-case running time

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- Thus, $1+2+\ldots+n-1=\frac{n(n-1)}{2}$ comparisons in total.

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Remark

Insertion sort does not require any additional space.

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$\longrightarrow$ Asymptotic Analysis

Asymptotic analysis provides upper, lower, and tight bounds on the asymptotic growth of $T(n)$ using

- O-notation (”Big-Oh”),
- Ω-notation (”Big-Omega”), and
- Θ-notation (”Big-Omega”).

$O-, \Omega-,$ and Θ-notation are defined later.

Common running times

Actual running times corresponding to $T(n)$:

Input size n	Running time function $T(n)$:				
	$33n$	$46n\log(n)$	$13n^2$	$3,4n^3$	2^n
10	$0,00033s$	$0,0015s$	$0,0013s$	$0,0034s$	$0,001s$
10^2	$0,0033s$	$0,03s$	$0,13s$	$3,4s$	$4 \cdot 10^{16}y$
10^3	$0,033s$	$0,45s$	$13s$	$0,94h$	
10^4	$0,33s$	$6,1s$	$1300s$	$39d$	
10^5	$3,3s$	$1,3m$	$1,5d$	$108y$	

s	seconds
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Observation:

The impact of constant factors decreases with growing n.

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$T(n)$	New maximal input size
$\log(n)$	N^K
n	$K \cdot N$
n^2	$\sqrt{K} \cdot N$
2^n	$N + \log(K)$

Asymptotic Analysis

$\mathcal{O}$ ("big-Oh")-notation

Let $f, g : \mathbb{N} \to \mathbb{R}_{\geq 0}$ be two functions and $c > 0$.

Informal definition of $\mathcal{O}(f)$

$\mathcal{O}(f)$ is the set of all functions growing not faster than f.

- $g \in \mathcal{O}(f)$ means $c \cdot f(n)$ is an upper bound for $g(n)$.

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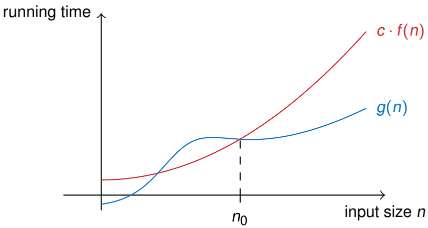
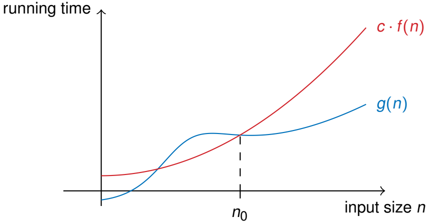


$\mathcal{O}$-notation

$\mathcal{O}$ provides an upper bound on the asymptotic growth of a functions.

Mathematical definition of $\mathcal{O}(f)$

$g \in \mathcal{O}(f)$ if and only if $\exists\ c > 0, n_0 \in \mathbb{N}$ with $\forall\ n \geq n_0 : 0 \leq g(n) \leq c \cdot f(n)$



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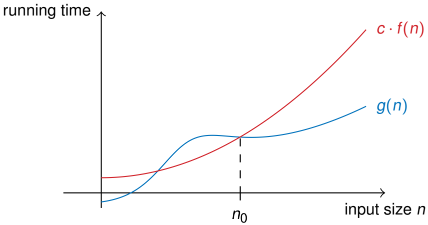

Certainly, the smaller upper bounds the better. For example,

$$g \in \mathcal{O}(n^2) \text{ provides more information than } g \in \mathcal{O}(n^3).$$

Ω (”big Omega”)- notation

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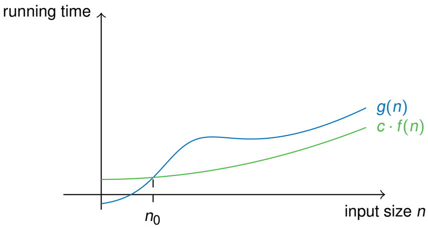

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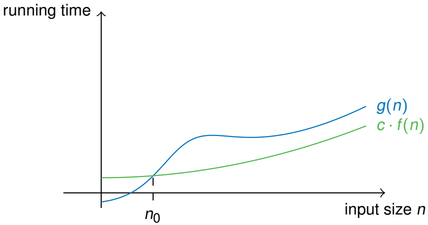

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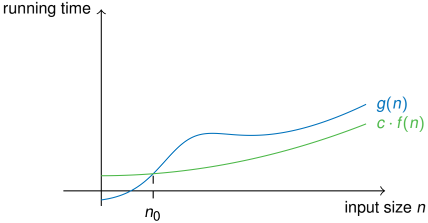

Certainly, the greater lower bounds the better. For example,

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$\Theta(f)$ denotes the set of all functions growing equally fast as f.

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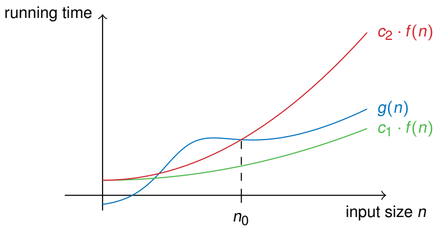

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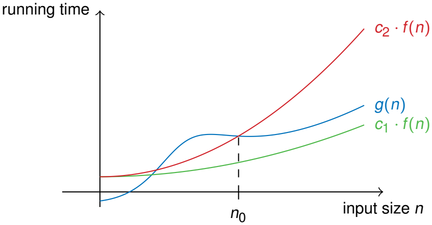
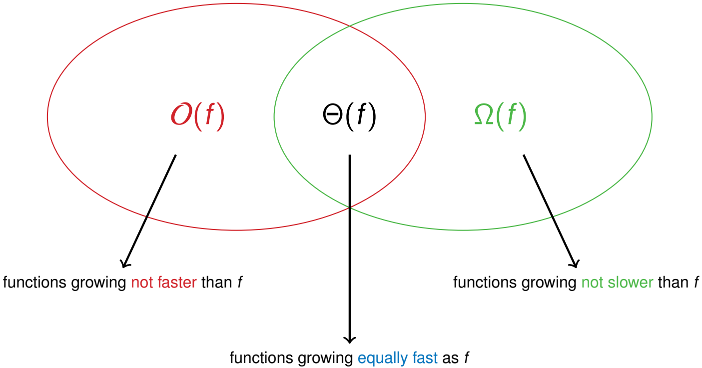


Relationship between the three sets



Running Time of Insertion Sort and Merge Sort

Running time of insertion sort (worst case)

Recall: Arithmetic series (cf. exercises)

$$\sum_{j=1}^{n} j = \frac{1}{2}n(n+1).$$

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Worst case: Input reverse sorted.

For some constant $c > 0$,

$$T(n) = \sum_{j=2}^{n} c(j-1) = c\sum_{j=1}^{n-1} j \in \Theta(n^2).$$

Running time of insertion sort (average case)

Suppose that we randomly choose n numbers and apply insertion sort.

Average case: all permutations equally likely.

For some constant $c > 0$,

$$T(n) = \sum_{j=2}^{n} \frac{c}{2}(j-1) = \frac{c}{2}\sum_{j=1}^{n-1} j = \frac{c}{4}n^2 - \frac{c}{4}n \in \Theta(n^2).$$

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Analog for $\mathcal{O}(f)$ and $\Omega(f)$.

Running time of merge sort (worst-case)

MergeSort $(A[1..n])$	$T(n)$	
IF $(n = 1)$	$\Theta(1)$	
RETURN A	$\Theta(1)$	
Sort $A[1\ ..\ \lfloor \frac{n}{2} \rfloor]$ und $A[\lfloor \frac{n}{2} \rfloor + 1\ ..\ n]$ recursively	$2T(\frac{n}{2})$	"sloppy"
Merge the two sorted subarrays	$\Theta(n)$	
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"sloppy" should be $T(\lfloor \frac{n}{2} \rfloor) + T(\lceil \frac{n}{2} \rceil)$, but it turns out not to matter asymptotically

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We derive the following recurrence for running time function $T(n)$:

$$T(n) = \begin{cases} \Theta(1) & \textbf{if } n = 1 \\ 2T(\frac{n}{2}) + \Theta(n) & \textbf{if } n > 1 \end{cases}$$

Recurrence for merge sort

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Next lecture: several ways to find upper bounds for recurrences of type

$$T(n) = aT(n/2) + f(n),$$

where a, b are some constants, and $f(n)$ is some function.

Recursion tree (rough idea)

Question: How can we solve a recurrence of type $T(n) = 2T(\frac{n}{2}) + cn$ where $c > 0$ is some constant?

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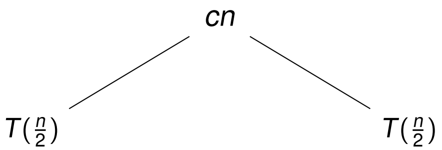

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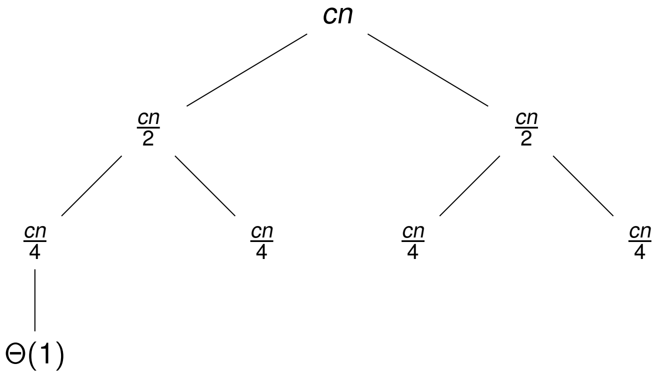

Recursion tree (rough idea)

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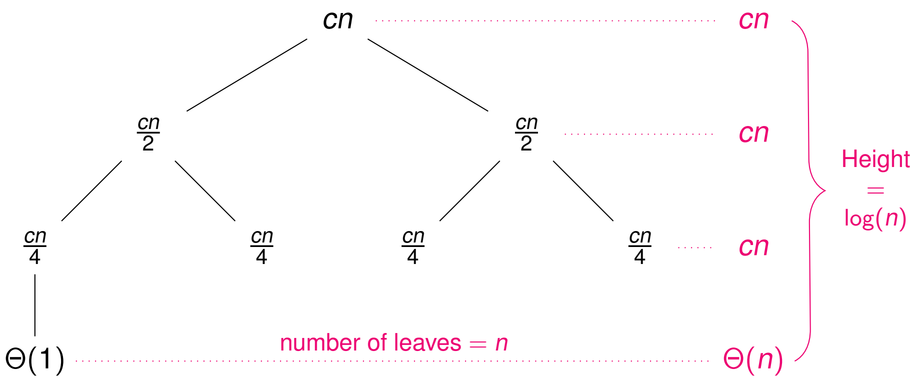

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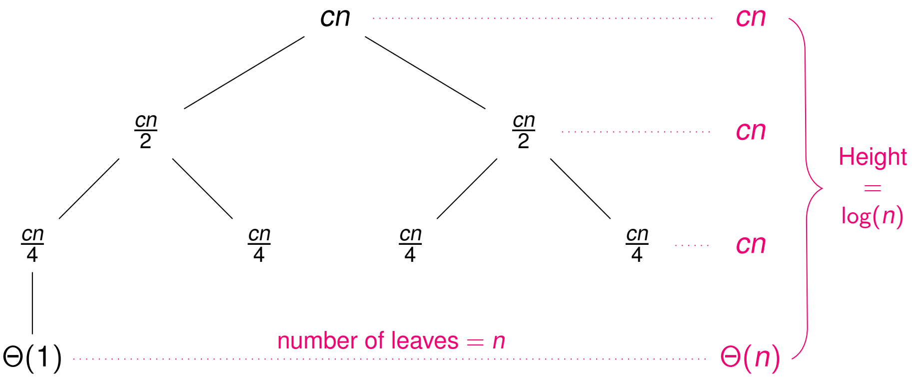

$$\Rightarrow T(n) = \Theta(n \cdot \log(n))$$

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But: without any additional information on the input instance it is not possible to beast the worst case running time of $\Theta(n \cdot \log(n))$.