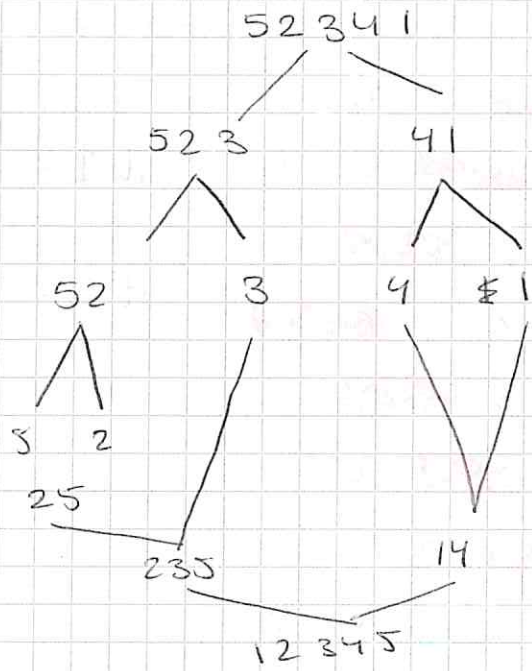
DDS Answers Tutorial 1

Question 1

insertion sort — Merge sort

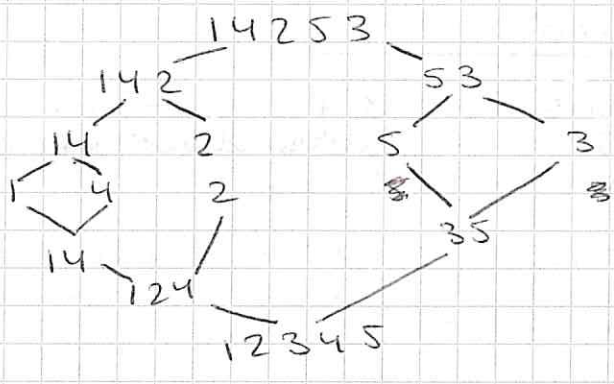
a)

5 2 3 4 1
2 5 3 4 1
2 3 5 4 1
2 3 4 5 1
2 3 4 1 5
2 3 1 4 5
2 1 3 4 5
1 2 3 4 5



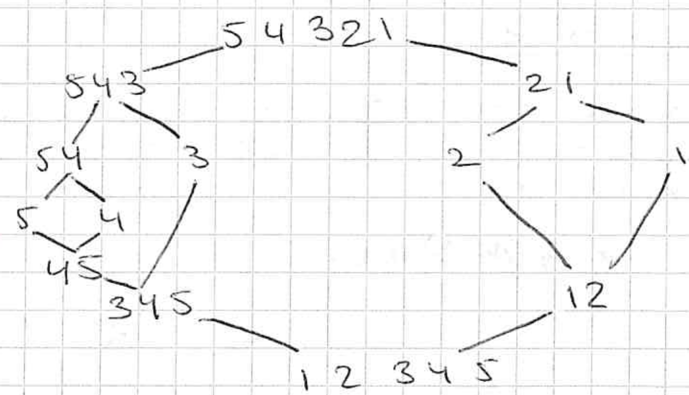
b)

1 4 2 5 3
1 2 4 5 3
1 2 4 3 5
1 2 3 4 5



c)

5 4 3 2 1
4 5 3 2 1
4 3 5 2 1
3 4 5 2 1
3 4 2 5 1
3 2 4 5 1
2 3 4 5 1
2 3 4 1 5
2 3 1 4 5
2 1 3 4 5
1 2 3 4 5



Question 2

a) In bubble sort, we go through the list $n-1$ times, and swap adjacent elements that are in the wrong order. As after i iterations, the i maximal elements are in the end of the array, we only care about the first $n-i$ elements in iteration i.

b)

iteration 1:

5 2 3 4 1

2 5 3 4 1

2 3 5 4 1

2 3 4 5 1

2 3 4 1 5

iteration 2:

2 3 1 4 5

it. 3:

2 1 3 4 5

it. 4:

1 2 3 4 5

it. 1:

1 4 2 5 3

1 2 4 5 3

it. 2:

1 2 4 3 5

1 2 3 4 5

Question 3

a) Comparisons : $1 + 2 + \dots + n-1 = \frac{1}{2}n(n-1)$

Swaps : $\frac{1}{2}n(n-1)$

$[n, n-1, \dots, 2, 1]$

b) Comparisons : $1 + 2 + \dots + n-1 = \frac{1}{2}n(n-1)$

Swaps : $1 + 2 + \dots + n-1 = \frac{1}{2}n(n-1)$

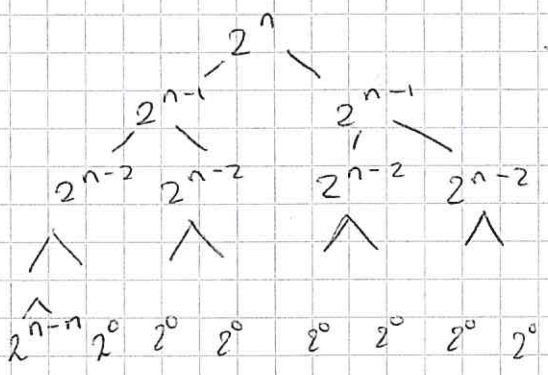
$[n, n-1, \dots, 2, 1]$

Question 4

a) At most $2n-1$. In adding the last ~~comparison~~ element, no comparison is needed.

At least n.

b)



$\longrightarrow 2^0 \cdot (2 \cdot (2^{n-1}) - 1) \qquad 2^n - 2^0$

$\longrightarrow 2^1 \cdot (2 \cdot (2^{n-2}) - 1) \qquad 2^n - 2^1$

$\longrightarrow 2^2 \cdot (2 \cdot (2^{n-3}) - 1) \qquad 2^n - 2^2$

$\longrightarrow 2^{n-1} \cdot (2 \cdot (2^0) - 1) \qquad 2^n - 2^{n-1}$

$+$

$n \cdot (2^n) - (2^n - 1)$

$(n-1)2^n + 1$

Homework Tutorial 1.

Question 1

a)

Insertion Sort

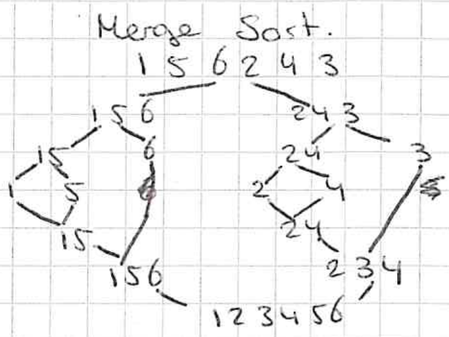
1 5 6 2 4 3
1 5 2 6 4 3
1 2 5 6 4 3
1 2 5 4 6 3
1 2 4 5 6 3
1 2 4 5 3 6
1 2 4 3 5 6
1 2 3 4 5 6

don't look at the colors! sorry :)

Bubble Sort

1 5 6 2 4 3
1 5 2 6 4 3
1 5 2 4 6 3
1 5 2 4 3 6
1 2 5 4 3 6
1 2 4 5 3 6
1 2 4 3 5 6
1 2 3 4 5 6

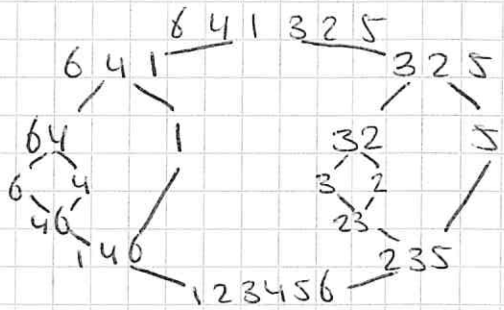
Merge Sort.



b)

6 4 1 3 2 5
4 6 1 3 2 5
4 1 6 3 2 5
1 4 6 3 2 5
1 4 3 6 2 5
1 3 4 6 2 5
1 3 4 2 6 5
1 3 2 4 6 5
1 2 3 4 6 5
1 2 3 4 5 6

6 4 1 3 2 5
4 6 1 3 2 5
4 1 6 3 2 5
4 1 3 6 2 5
4 1 3 2 6 5
4 1 3 2 5 6
1 4 3 2 5 6
1 3 4 2 5 6
1 3 2 4 5 6
1 2 3 4 5 6



Question 2

a) Selection Sort starts sorting from the left and aims to find the minimum element of all unsorted elements. Then, it swaps this element with the first 'unsorted' position in the array.

b) 1.

2 6 3 4 1 5
1 6 3 4 2 5
1 2 3 4 6 5
1 2 3 4 6 5
1 2 3 4 6 5
1 2 3 4 5 6

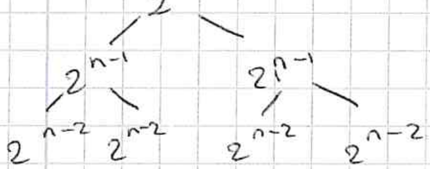
2.

1 4 2 5 6 3
1 4 2 5 6 3
1 2 4 5 6 3
1 2 3 5 6 4
1 2 3 4 6 5
1 2 3 4 5 6

Question 3.

a) Best case: 1, 2, ..., n-1, n.

Then ~~swaps~~



$2^{n-1} = 2^{n-1}$

$2^1 \cdot 2^{n-2} = 2^{n-1}$

$\vdots$

$2^{n-1} \cdot 2^0 = 2^{n-1}$

$n \cdot 2^{n-1}$

b) Selection Sort.

best case : $[1, \ldots, n]$

\# swaps : 0

\# comp. : $\frac{1}{2} n(n-1)$