Assignment -1

Compare raster scan and random scan display

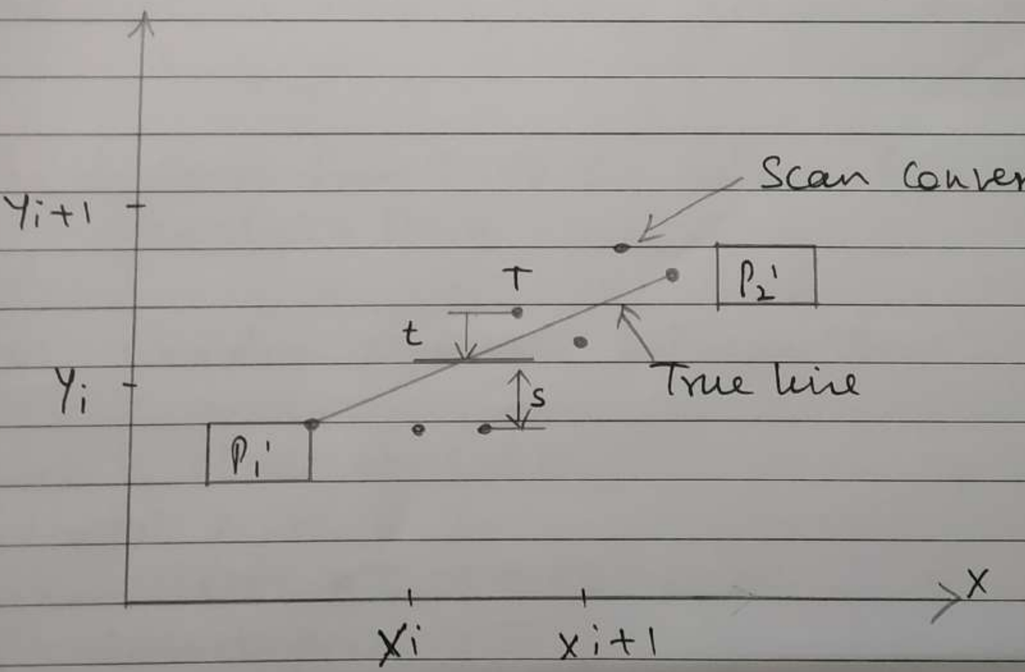
→ Random Scan	Raster Scan
The resolution of random scan is higher than raster scan	While the resolution of raster scan is lesser or lower than random scan
Costlier than raster scan	Cheaper than random scan
In random scan, any alteration is easy in comparision of raster scan	While in raster scan inter any alteration is not so easy
Interlacing is not used	Interlacing is used
Mathematical function is used for image or picture rendering It is suitable for application requiring polygon drawings	While in which, for image or picture rendering raster scan uses pixels. It is suitable for cooking realistic scenes
Electron Beam is directed to only that part of screen where picture is required to be drawn one line at a time.	Electron Beam is directed from Top to Bottom and one row at a time on screen. It is directed to whole screen.

Q2 Derive and write Bresenham's line drawing algorithm

Assume a pixel $P_1'(x_1', y_1')$, then select subsequent pixels as we work our way to the right, one pixel position at a time in the horizontal direction toward $P_2'(x_2', y_2')$

The next pixel is
i Either one to its right (lower-bound for line)
ii One top its right and up (upper-bound for line)

The line is best approximated by those pixels that fall the least distance from the path between P_1', P_2'



Scan Converting a line

To chooses the next one between the bottom pixel S and top pixel T

If S is chosen
we have $X_i+1 = X_i+1$ and $Y_i+1 = Y_i$
If T is chosen
we have $X_i+1 = X_i+1$ and $Y_i+1 = Y_i+1$

The actual y co-ordinate of line at $x = x_i+1$ is
$y = mx_i+1 + b$

$$\boxed{y = m(x_i+1) + b} \quad — ①$$

The distance from S to the actual line in y direction
$s = y - y_i$

The distance from T to the actual line in y direction
$t = (y_i+1) - y$

Now consider difference between these 2 distance
$s - t$
When $(s-t) < 0 \rightarrow s < t$
$\therefore$ closest pixel is S
When $(s-t) \geq 0 \rightarrow t < s$
$\therefore$ closest pixel is T
This difference is
$s - t = (y - y_i) - [(y_i+1) - y]$
$= 2y - 2y_i - 1$

$$\boxed{s - t = 2m(x_i+1) + 2b - 2y_i - 1}$$

Subsituting m by $\frac{\Delta y}{\Delta x}$, introducing decision variable

$$d_i = \Delta x \, (s-t)$$
$$d_i = \Delta x \left(2 \frac{\Delta y}{\Delta x} (x_i+1) + 2b - 2y_i - 1\right)$$
$$d_i = 2\Delta x y_i - 2\Delta y - 1 \, \Delta x \, 2b - 2y_i \, \Delta x - \Delta x$$
$$d_i = 2\Delta y x_i - 2\Delta x y + C$$

Where $C = 2\Delta y + \Delta x (2b-1)$

We can write decision variable d_{i+1} for the next step on

$$d_{i+1} = 2\Delta y . x_{i+1} - 2\Delta x y_{i+1} + C$$
$$d_{i+1} - d_i = 2\Delta y (x_{i+1} - x_i) - 2\Delta x (y_{i+1} - y_i)$$

Since $x(i+1) = x_i + 1$, we have

$$d_{i+1} + d_i = 2\Delta y (x_{i+1} - x_i) - 2\Delta x (y_{i+1} - y_i)$$

Special cases

If chosen pixel is at the top pixel T (ie $d_i \geq 0$)

$\rightarrow y_{i+1} = y_i + 1$

$$d_{i+1} = d_i + 2\Delta y - 2\Delta x$$

If chosen pixel is at the bottom pixel T (ie $d_i < 0$)

$\rightarrow y_{i+1} = y_i$

$$d_{i+1} = d_i + 2\Delta y$$

Finally we calculate d_i

$$d_i = \Delta x [2m(x_1+1)+2b-2y_1-1]$$

$$d_i = \Delta x [2(mx_1+b-y_1)+2m-1]$$

Since $mx_1+b-y_i=0$ and $m=\frac{\Delta y}{\Delta x}$, we have

$$d_i = 2\Delta y - \Delta x$$

Algorithm

1 Read the line end points (x_1, y_1) and (x_2, y_2) such that They are not equal [if equal than plot that point and end]

2 $\Delta x = |x_2 - x_1|$ and $\Delta y = |y_2 - y_1|$

3 [initialize starting point)

$x = x1$

$y = y_1$

4 $e = 2\Delta y - \Delta x$

5 $i = 1$ [initalize counter]

6 Plot (x, y)

7 While $(e \geq 0)$

{ $y = y+1$

$e = e - 2\Delta y$ }

$x = x+1$

$e = e + 2 * \Delta y$

8 $i = i+1$

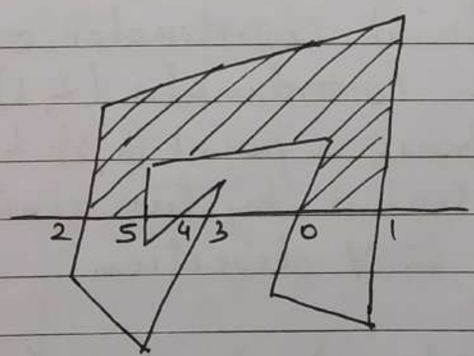
9 if $(i \leq \Delta x)$ then goto Step 6

10 Stop.

Q3 Write a short note scanline polygon fill algo

→ The scanline fill algorithm is an ingenious way of filling in irregular polygons. The algorithm begin with a set of points. Each point is connected to the next, and the line between them is considered to be an edge of an polygon

The point of each edge are adjusted to ensure that point with smaller value appears first. Next, a data structure is created that contain a list of edges that begin on each scanline image. The program progresses from the first scanline upward. for each line, any pixels that contain an intersection between this scanline and an edge of the polygon are filled in. Then, the algorithm progresses along the scanline Turning on when it reaches another one, all the way across scanline



Q4 Transformation matrices for translations, rotation scaling, reflection and shearing in 2D co-ordinates and homogenous system

→ Scaling $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

Rotation (clockwise) : $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Translation : $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$

Rotation (Anti-clock : $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Shearing (x-direction) $\begin{bmatrix} 1 & 0 \\ Sh_x & 1 \end{bmatrix}$

Reflection (x-axis) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Shearing (Y-direction) $\begin{bmatrix} 1 & Sh_y \\ 0 & 1 \end{bmatrix}$

Reflection : (Y-axis) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Shearing (both) $\begin{bmatrix} 1 & Sh_y \\ Sh_x & 1 \end{bmatrix}$

Reflection : (origin) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

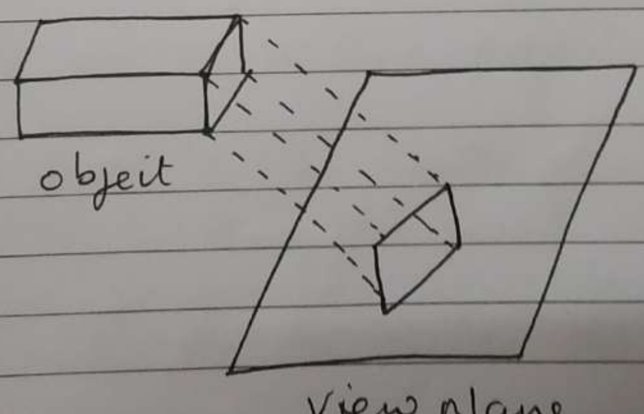
Reflection (Y = X) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection (Y = -X) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Q8 Explain parallel and perspective projections and derive matrix for oblique projection

→ Parallel Projection

i In parallel projection, Z co-ordinate is discarded and parallel lines from each vertex on the object are extended until they intersect view plane.
ii Point of intersection is the projection of vertex
iii We connect the projected vertices by line segment which correspond to connections on the original object
iv Parallel projection perserves relative proportions of object
v Accurate views of the various sides of an object are obtained with a parallel projection But not a realistic representation
vi Parallel projection is shown below



Perspective projection:

i In perspective projection, lines of projection are not parallel

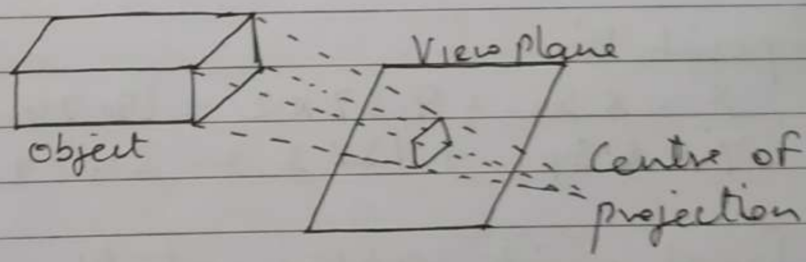
ii Perspective projection transform object position to the view plane while converging to a center point of projection

iii All projection are converged at single point called center of projection or projection reference point

iv Perspective projection produces realistic views but does not perserve relative proportions

v Projection of distant objects are smaller than the projection of object of the same size that are closer to the projection plane

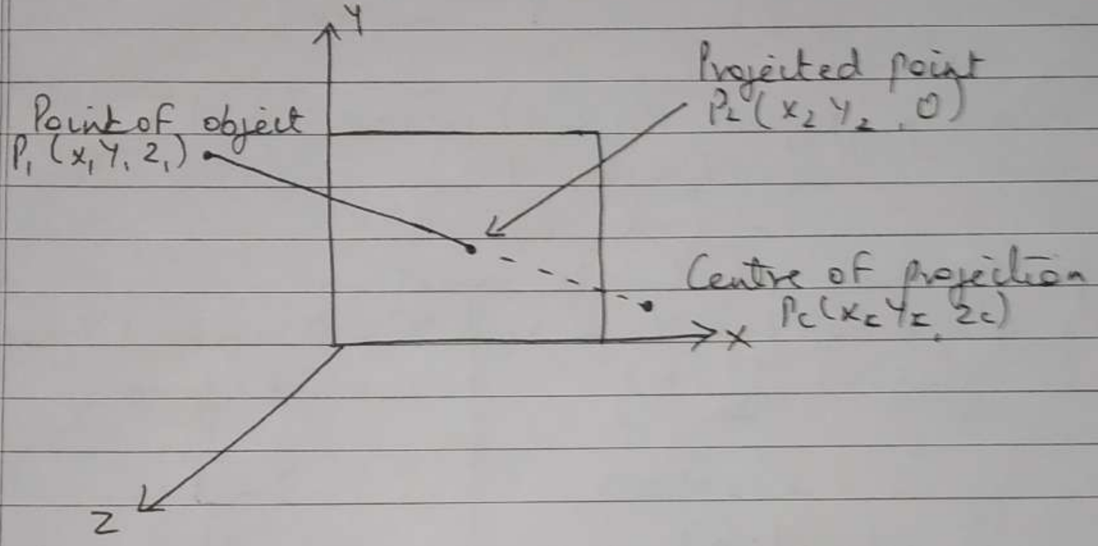
vi Perspective projection is shown below



Matrix of perspective projection

Let us consider the centre of projection is at $P_c(x_c, y_c, z_c)$ and the point on object is $P_1(x_1, y_1, z_1)$ then the parametric equation for line containing these points can be given as

$X_2 = X_c + (X_1 - X_c)\,U$
$Y_2 = Y_c + (Y_1 - Y_c)\,U$



for projected point Z_2 is 0.

$0 = Z_c + (Z_1 - Z_c)\,U$

$U = -Z_c / Z_1 - Z_c$

Substituting value of U is first two equation

$X_2 = (X_c - Z_c) * (X_1 - X_c) / (Z_1 - Z_c)$

$= X_c Z_1 - X_c Z_c - X_1 Z_c + X_c Z_c / Z_1 - Z_c$

$= X_c Z_1 - X_1 Z_c / Z_c - Z_c$

$Y_2 = (Y_c - Z_c) * (Y_1 - Y_c) / (Z_1 - Z_c)$

$= Y_c - Z_1 - Y_c Z_c - Y_1 Z_c + Y_c Z_c / Z_1 - Z_c$

$= Y_c Z_1 - Y_1 Z_c / Z_1 - Z_c$

Above equation can be represented in the Homogenous

$$[x_2\ y_2\ z_c\ 1] = [x_1\ y_1\ z_1\ 1]\begin{bmatrix} -Z_c & 0 & 0 & 0 \\ 0 & -Z_c & 0 & 0 \\ X_c & Y_c & 0 & 1 \\ 0 & 0 & 0 & -Z_c \end{bmatrix} \quad \begin{bmatrix} -Z_c & 0 & 0 & 0 \\ 0 & -Z_c & 0 & 0 \\ X_c & X_c & 0 & 1 \\ 0 & 0 & 0 & Z_c \end{bmatrix}$$

Here, we have taken center of projection $P_c(x_c\ y_c\ z_c)$
If we take center of projection on $-v_c z$ axis
Such that $x = 0$

$y = 0$

$z = -z_c$

Q9 Write a short note on Animation Techniques

→ frame by frame

Earlier in traditional method, animation was done by hand because of the absence of the Computer aided drawing facilities And these traditional method required a lot of effort for even making a Short vedio because of the fact that every second of animation requires 24 frames to process

Procedural:

In this method, set of rules are used to animate the objects. Animator defines or specify the initial rules and procedure to process and later runs simulation. Many of the times rules of procedure are based on real worlds physical rule which are shown by Mathematical equations

Behavioral:

According to this technique, to a certain extent the character or object determines its own action which allows the character to improve later, and in turn, it frees the animator in determining each and every details of the characters motion

Motion Capture:

It uses live action footage of a living being character which is recorded to computer via video and later that action is used to animate the character which gives the real feel to the viewer as if real human character has been animated.

Q6 Explain Liang-Barsky Line clipping Algo. Also clip line from (30, 60) to (60, 20) against window (XW_{min}, YW_{min}) → (10, 10) (XW_{max}, YW_{max}) → (50, 50)

→ Liang-Barsky line clipping algorithm is faster line clipping algorithm based on analysis of the parametric equation of a line segment

$$X = X_1 + U \Delta X$$
$$Y = Y_1 + U \Delta Y$$

where, $\Delta x = X_2 - X_1$ & $\Delta Y = Y_2 - Y_1$

2 Using these equations Cyrus & Beck developed an algorithm that is generated more efficient than the cohen Sutherland algorithm

3 Later Liang-Barsky independently devised an even faster parametric line clipping algorithm

4 In this approach we first the point clipping condition parametric form

$$X_{min} \le X_1 + U \Delta X \le X_{max}$$
$$Y_{min} \le Y_1 + U \Delta Y \le Y_{max}$$

5 Each of these four equalities can be expressed as
$u p_k \leq q_k$ for $k = 1, 2, 3, 4$

6 Parameters P & q are defined as
$p1 = -\Delta x$ and $q1 = x_1 - x_{min}$ (Left Boundary)
$p2 = \Delta x$ and $q2 = x_{max} - x_1$ (Right Boundary)
$p3 = -\Delta y$ and $q3 = y_1 - y_{min}$ (Bottom Boundary)
$p4 = \Delta y$ and $q4 = y_{max} - y_1$ (Top Boundary)

7 If a line is parallel to a view boundary, the p value for that boundary is zero

8 If line is parallel to x axis for example then P_1 & P_2 must be zero
- Given $pk = 0$, if $qk < 0$ then line is trivially invisible because it is outside view window
- Given $pk = 0$, if $qk > 0$ then line is inside the corresponding window boundary

9 When $pk < 0$, as U increase line goes from the outside to inside ie entering

10 When $pk > 0$, line goes from inside to outside ie exiting

Problem
Given $(x_{min}, y_{min}) = (10, 10)$ $(x_{max}, y_{max}) = (50, 50)$
$P_1 (30, 60)$ $P_2 (60, 25)$

→ set $U_{min} = 0$ and $U_{max} = 1$

$U_{left} = q_1/p_1$ | $U_{right} = q_2/p_2$

$= x_1 - x_{min} / -\Delta x$ | $x_{max} - x_1 / \Delta x$

$30 - 10 / -(60 - 30)$ | $50 - 30 / (60 - 30)$

$20/30$ | $20/30$

-0.67 | 0.67

U_{bottom} q_3/p_3 | $U_{top} = q_4/p_4$

$y_1 - y_{min} / -\Delta y$ | $y_{max} - y_1 / \Delta y$

$60 - 10 / -(25 - 60)$ | $50 - 60 / (25 - 60)$

$50/35$ | $-10/35$

1.43 | 0.29

Since U_{left} is less than U_{min} and U_{bottom} is greater than U_{max}, so we ignore it

$U_{right} = U_{min} = 0.67$ (entering) $U_{top} = U_{max} = 0.29$

$Q - P \ (\Delta x, \Delta y) = (30, -35)$

Since $U_{min} > U_{max}$, there is no line segment draw

Q5 find out the final coordinates of a figure bounded by the co-ordinates (2,3) (10,3) (2,7) (10,7) when rotated about a point (8,8) by 30° in clockwise direction and scaled by two units in x direction and three units in y direction

→ Given A(2,3) B(10,3) C(2,7) D(10,7)

rotation unit (8,8)

$\theta = 30°$

$S_x = 2$ $S_y = 3$

Solution:

rotating point A(2,3) w.r.t (5,8)

$A' = (2-8,\ 3-8)$

$(-6,\ -5)\ \therefore x = -6 \quad y = -5$

performing clockwise rotation

$x' = x \cdot \cos 30 - y \cdot \sin 30$

$= -6 \cdot \frac{\sqrt{3}}{2} - (-5) \cdot \frac{1}{2}$

$= -3\sqrt{3} + 2.5$

$x' = -2.69$

$y' = x \cdot \sin 30 + y \cdot \cos 30$

$= -6 \cdot \frac{1}{2} + (-5)\ \frac{\sqrt{3}}{2}$

$= -3 - 4.33$

$y' = -7.33$

$x' = x' + 8 \qquad y' = y' + 8$

$-2.69 + 8 \qquad -7.33 + 8$

$= 5.31 \qquad = 0.67$

rotating point B(10,3) w.r.t (8,8)

$B' = (10-8,\ 3-8)$

$= (2,\ -5)$

performing clockwise rotation

$x' = x \cos 30 - y \sin 30$	$y' = x \sin 30 + y \cos 30$
$= 2 \cdot \frac{\sqrt{3}}{2} + 5 \cdot \frac{1}{2}$	$= 2 \cdot \frac{1}{2} - 5 \cdot \frac{\sqrt{3}}{2}$
$= \sqrt{3} + 2.5$	$= 1 - 4.33$
$x' = 4.23$	$y' = -3.33$

$\therefore x' = x' + 8$
$= 4.23 + 8$
$= 12.23$

$\therefore y' = y' + 8$
$= -3.33 + 8$
$= 4.67$

rotating point $C(2,7)$ w.r.t $(8,8)$
$C' = (2-8,\ 7-8)$
$= (-6, -1)$

performing clockwise rotation

$\therefore y' = x \cdot \cos 30 - y \sin 30$
$= -6 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}$
$= -3\sqrt{3} + 0.5$
$= -4.69$

$\therefore x' = x' + 8$
$= -4.69 + 8$
$= 3.31$

$y' = x \cdot \sin 30 + y \cdot \cos 30$
$= -6 \cdot \frac{1}{2} - \frac{\sqrt{3}}{2}$
$-3 - \sqrt{3}/2$
-3.86

$\therefore y' = y' + 8$
$= -3.86 + 8$
$= 4.14$

rotating point $D(10,7)$ w.r.t $(8,8)$
$D' = (10-8,\ 7-8)$
$= (2, -1)$

performing clockwise rotation

$\therefore x' = x \cdot \cos 30 - y \sin 30$
$= 2 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}$
$\sqrt{3} + 0.2$
$= 1.87$

$\therefore x' = x' + 8$
$= 1.87 + 8$
$= 9.87$

$y' = x \cdot \sin 30 + y \cdot \cos 30$
$= 2 \cdot \frac{1}{2} - \frac{\sqrt{3}}{2}$
$= 1 - 0.86$
$= 0.13$

$y' = y' + 8$
$= 0.13 + 8$
$= 8.13$

$\therefore$ $A'(5.31, 0.67)$ $B'(12.23, 4.67)$ $C'(3.31, 4.14)$ $D'(9.87, 8.13)$

Scaling above points

for point A'

$x' = x \cdot S_x = 5.31 \times 2 = 10.62$

$y' = y \cdot S_y = 0.67 \times 3 = 2.01$

for point B'

$x' = x \cdot S = 12.23 \times 2 = 24.46$

$y' = y \cdot S_y = 4.67 \times 3 = 14.01$

for point C'

$x' = x \cdot S_x = 3.31 \times 2 = 6.62$

$y' = y \cdot S_y = 4.14 \times 3 = 12.42$

for point D'

$x' = x \cdot S_x = 9.87 \times 2 = 19.74$

$y' = 8 \cdot S_y = 8.13 \times 3 = 24.39$

$\therefore$ final points after rotation and scaling are

$(10.62, 2.01)$ $(24.46, 14.01)$ $(6.62, 12.42)$ $(19.74, 24.39)$