TERMINAL VELOCITY FOR FREE FALL

Ravneet Kaur (2020PHY1064) Gaurav Chandra (2020PHY1122) Khushi Sethi (2020PHY1155)

S.G.T.B. Khalsa College, University of Delhi, Delhi-110007, India.

Project Report Submitted to

Dr. Mamta and Dr. H. C. Ramo

as part of internal assessment for the course

"32223902 - Computational Physics Skills"

July 19, 2023

PLAN

1. Study of free-fall motion.
2. Effect of air resistance on the motion.
3. Model of air drag studied is of the form:

$$F_d = a \times v + b \times v^2$$

4. Investigation of different stages for parachute deployment.
5. Solving the parachute problem using **Python Programming Language**.
6. Detection of Terminal Velocity from the data.
7. Calculation of total time of parachuting.

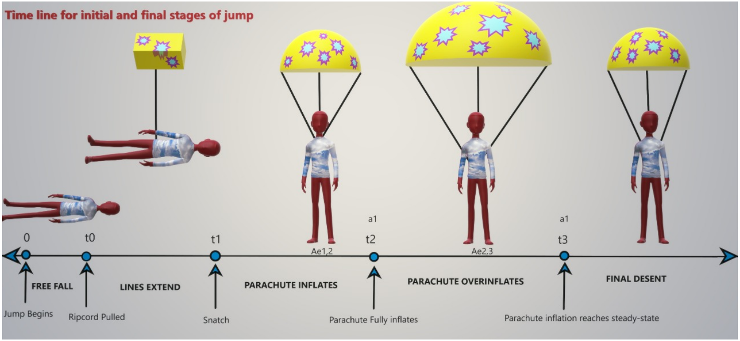
THEORY

In our Parachute Problem, we aim to study the free-fall Motion in the presence of air resistance which includes all the stages of parachute deployment.

- Form of air drag is decided by the Reynolds number Re.
- Reynolds-number[1] is a dimensionless quantity that is used to determine the type of flow pattern as laminar or turbulent. $\mathrm{Re} = \frac{inertialforce}{viscousForce} = \frac{\rho \times d \times v}{\mu}$
- For Re $<<$1, viscous forces dominates in and the drag force is approximately linear in the velocity.
- When Re $> 10^5$, the inertial forces dominates and the drag force is approximately quadratic in the velocity.
- In our case,density $\rho \approx 1kgm^3$ and viscosity$\mu \approx 1.5 \times 10^{-5} kgms^{-1}$.So the value of Reynolds number must be $> 10^5$,so drag force is of the form : $F_d = b \times v^2$

[1] a

In our Parachute Problem, we have a Parachutist who jumps from an aircraft with initial velocity, v = 0 at a height[2], y = 17982 ft (5480.9136 m). This can be modelled in five distinct stages as follows :



- The first stage is free fall, which will occurs till $t_0 = 20$ sec, ripcord is pulled at t_0.

[2]Data taken from [b]

- The second stage is where snatch force is applied, this will change the shape of parachutist from spread eagle position to up-right position. Snatch force will occurs at t_1-t_0 = 0.5 sec.
- The third stage is when the parachute is fully inflated, at time t = t_2 parachute will be at steady state area a_1.
- The fourth stage is when the parachute will over-inflate due to surrounding air pressure, at time t = t_3 parachute will again be at its steady state area a_1.
- The fifth stage is when parachutist will achieve terminal velocity and reaches the ground safely.

THEORY

Parachute Problem is a second order differential equation.

$$\boxed{\frac{d^2y}{dt^2} = -g + \frac{k}{m} \times v^2}$$

Reduced the ODE into a system of two first-order differential equations for the velocity and position.

$$\boxed{\frac{dy}{dt} = v}$$

$$\boxed{\frac{dv}{dt} = -g + \frac{k}{m} \times v^2}$$

THEORY

Converted the Physics Problem into a Mathematical Problem by introducing new variables:

Let

$$\gamma = \frac{y}{y_1} \quad where, \quad y_1 = H$$

$$\nu = \frac{v}{v_1} \quad where, \quad v_1 = \sqrt{gH}$$

$$T = \frac{t}{t_1} \quad where, \quad t_1 = \frac{\sqrt{H}}{\sqrt{g}}$$

$$\kappa = \frac{k}{k_1} \quad where, \quad k_1 = \frac{m}{H}$$

$$\boxed{\frac{d\nu}{dT} = \kappa \times \nu^2 - 1}$$

$$\boxed{\frac{d\gamma}{dT} = \nu}$$

k is the coefficient of drag[3])

$$k = \frac{1}{2}\rho \begin{cases} 1.95 b_0, & t \leq t_0 \\ 1.95 b_0 + 0.35 b_1 \frac{l(t-t_0}{t_1 - t_0}), & t_0 < t \leq t_1 \\ 0.35 b_1 h + 1.33 A^e_{1,2}(t), & t_1 < t \leq t_2 \\ 0.35 b_1 h + 1.33 A^e_{2,3}(t), & t_2 < t \leq t_3 \\ 0.35 b_1 h + 1.33 a_1, & t > t_3 \end{cases}$$

a_1	b_0	b_1	h	l	m	t_0	t_1	t_2	t_3	C_d^H	C_d^F	C_d^C
43.8 m^2	0.5 m^2	0.1 m^2	1.78 m	8.96 m	97.2 kg	20.0 sec	20.5 sec	21.5 sec	23.2 sec	1.33	1.95	0.35

[3]DE in New Millennium:Douglas B. Meade and Allan A.Struthers [c]

PACKAGES AND METHODS USED

THE PACKAGES WE USED IN OUR PROJECT ARE :

- **NUMPY PACKAGE** : Made numpy Ndarrays .
- **CSV PACKAGE** : Stored the data in csv files .

THE METHODS WE USED ARE **EULER METHOD** AND **RUNGE-KUTTA**[4] **4 METHOD**

- Solved the system of first order differential equations.
- Compared the results from both the methods.

We have made PYTHON programs on both of these methods,and made modules for both , to use them in the main program.

[4] Advanced Engineering Mathematics 10th Edition By ERWIN KREYSZIG

RESULTS

TIME TAKEN TO REACH TERMINAL VELOCITY

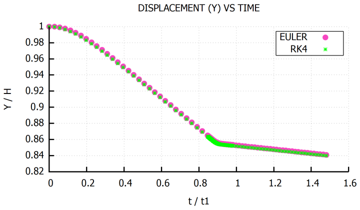
At the time 25.3657 seconds, Parachutist achieves Terminal Velocity, $V_t = 5.7188\ ms^{-1}$ in the negative Y-axis.

THE HEIGHT AND DRAG COEFFICIENT

The height at which Terminal Velocity is achieved is $h = 4658.77$ metres and the Drag coefficient at this point is $K = 29.1018$ kgm^{-1}

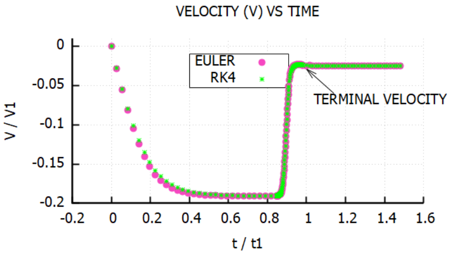
THE TOTAL TIME OF PARACHUTING

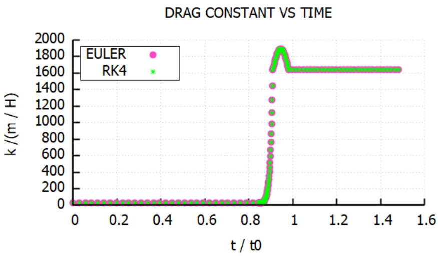
It takes the Parachutist 814.64 sec to reach the ground after reaching Terminal Velocity,So the Total Time of Parachuting is 14 minutes.



- Kink at 0.9 signifies that the 4^{th} stage has ended.
- Initially, Distance covered at higher rate and then reduces after kink which signifies that terminal velocity has been attained.

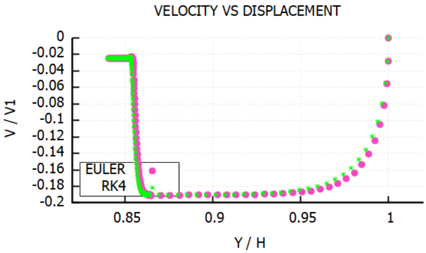
- Initially, velocity increases up-till the free fall.
- It starts decreasing after 0.8 on x axis where the parachute is opened.
- Constant value is achieved i.e., the **TERMINAL VELOCITY**.





- Initially Drag is constant.
- After 0.8 on x axis, it starts increasing **exponentially** up-to the steady state area.
- Then, follows the **sin**π function, it reaches the steady state area and then its magnitude becomes constant.

- As the distance decreases, velocity increases.
- After 0.86 on x axis, where free-fall is ending, velocity starts decreasing.
- Achieves a constant value which is the **TERMINAL VELOCITY**.



- We have solved the parachute problem in the presence of air resistance. We have modelled the problem into five different stages.
- We have solved the second order differential equation of parachute problem by reducing it into first order ODE's. We used Euler and RK4 methods to solve the ODE's and compared them. We have calculated terminal velocity,and time,height ,drag coefficient when terminal velocity is reached.
- We have studied the non-dimensional model and plotted various plots between different parameters using Gnuplots.

CONCLUSION

- We have studied how various factors affect the parachutist's safe landing.
- We have learnt the importance of Reynolds's number in parachuting and how it decides the drag constant.
- We had a great experience on using PYTHON PROGRAMMING LANGUAGE and GNUPLOTS.


Thank you