① Given three T/F questions, count total ways to answer the questions?

____ ____ ____

T F

T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

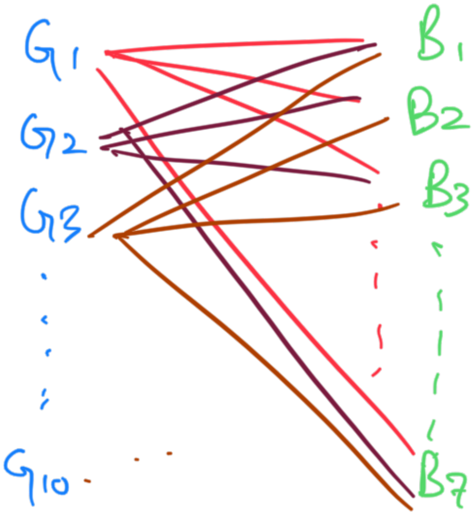
total 8 possibilities

$\underset{T/F}{2}$ $\underset{T/F}{2}$ $\underset{T/F}{2}$ → $2 * 2 * 2 = \boxed{8}$

Q1 and Q2 and Q3

multiplication

② 10 girls & 7 boys.
Count total ways to make a boy + girl pair?



each girl can form a pair with either of 7 boys.

1 girl → 7 boys

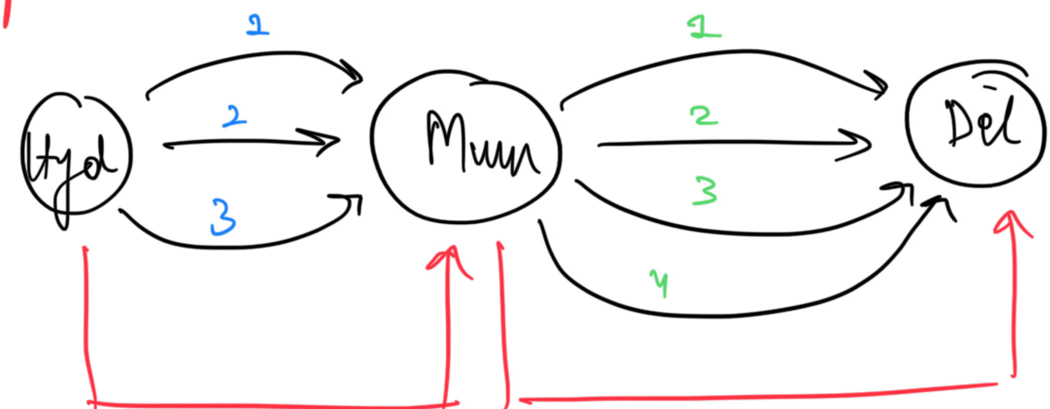
10 girls → $\boxed{70 \text{ boys}}$.

$\underset{G}{\underline{10}}$ & $\underset{B}{\underline{7}}$ → $\boxed{70}$

10 options

7 options

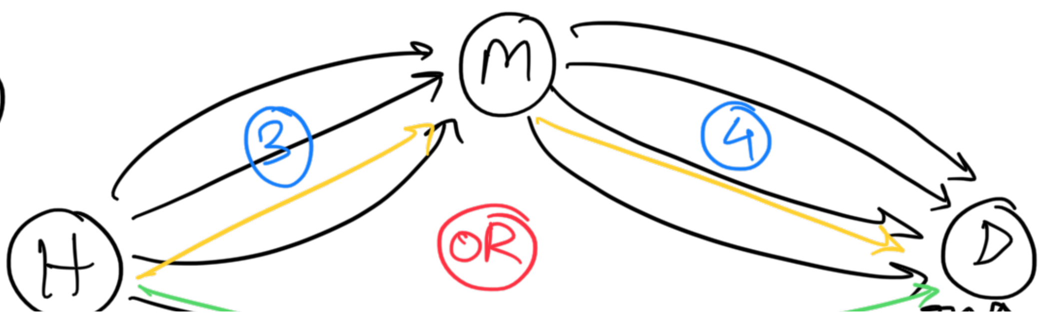
3

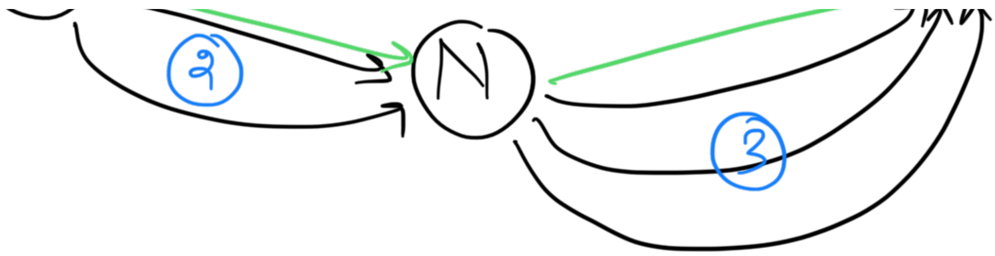


3 options and 4 options

$3 \times 4 = \boxed{12}$

5





→ H → M → D or H → N → D

$$\to \underbrace{(H \,\&\, M \,\&\, D)}_{3\times 4} \quad \text{or} \quad \underbrace{(H \,\&\, N \,\&\, D)}_{2\times 3}$$

$$\boxed{12 \quad + \quad 6} \to \boxed{18}$$

(6) You can gift one of the following combos:

Pen - 3
Book - 5
Flower - 7
Chocolate - 3
Ring - 3

→ 1 pen & 1 book or
1 Flower & 1 chocolate or
1 ring

$$\boxed{3\times 5 + 7\times 3 + 3} \to 39.$$

Permutations → arrangement of objects.

not the same

$i,j \neq j,i$

order matters

(7) Count no. of ways to arrange $\boxed{a, b, c}$

slightly different

a b c

a c b

a b c ✓

a c b ✓

$\boxed{\begin{matrix} aab \\ abb \end{matrix}}$ ✗

we will miss a character

repetition is not allowed!

a/b/c ___ ___ ___

3 options

a ___ ___

b a c

2 options

a b ___

c

1 option

3

and ✗

2

and ✗

1

3!

???

6

→ a, b, c, d → how many ways to arrange?

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ → AND $= 4!$

Result →

No. of ways to arrange n distinct objects at n places

$= \boxed{n!}$

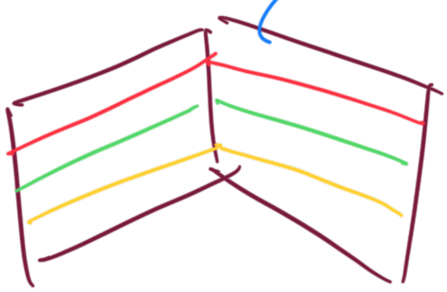
Important

(9) No. of ways to arrange 0 objects?

mathematically → $0! = 1$

Intuition →

3 100 Rs notes



→ ways to arrange $= 3!$

how many ways can I keep

a wallet with
0 notes ?

$\boxed{1 \text{ way}}$ → empty state

(10) 5 different characters.
ways to arrange in 2 slots?

5 4 | ~~3 2 1~~ → 5!

these slots do not exist

5×4 → $\boxed{20}$

for 3 slots →

5 4 3 | ~~2 1~~

$5 \times 4 \times 3 = \boxed{60}$

No. of ways to arrange n distinct objects at r positions. →

if $r = 3$ →

$n(n-1)(n-2)$

$r = 4$

$n(n-1)(n-2)(n-3)$

$r = k$

$n(n-1)(n-2)\ldots\ldots[n-(k-1)]$

$(n-k+1)$

$n(n-1)(n-2)\ldots(n-k+1)(n-k)(n-k-1)\ldots\ldots(1)$

$(n-k)(n-k-1)\ldots\ldots 1$

$n!$

$(n-k)!$

$$\frac{n!}{(n-k)!}$$

n distinct objects $\longrightarrow$ r slots.

$$\boxed{\frac{n!}{(n-r)!} = {}^nP_r.}$$

Combinations $\longrightarrow$ selection of objects!

$\longrightarrow$

O O   O O $\longrightarrow$ $i, j = j, i$

are the same

order does not matter!

① → Dhoni, Kohli, Bumrah, Rohit, Chahal

② → K, R, D, C, B

same choice !!

(11) 3 batsmen from 4 players. How many ways to select?

P_1 P_2 P_3 P_4

A:
$P_1 P_2 P_3$
$P_1 P_3 P_2$
$P_2 P_1 P_3$
$P_2 P_3 P_1$
$P_3 P_2 P_1$
$P_3 P_1 P_2$

B:
$P_1 P_3 P_4$
$P_1 P_4 P_3$
$P_3 P_4 P_1$
$P_3 P_1 P_4$
$P_4 P_3 P_1$
$P_4 P_1 P_3$

C:
$P_1 P_2 P_4$
$P_1 P_4 P_2$
$P_2 P_4 P_1$
$P_2 P_1 P_4$
$P_4 P_1 P_2$
$P_4 P_2 P_1$

D:
$P_2 P_3 P_4$
$P_2 P_4 P_3$
$P_3 P_2 P_4$
$P_3 P_4 P_2$
$P_4 P_2 P_3$
$P_4 P_3 P_2$

$6 \times 4 = 24$ arrangements.

4 selections

3!

$\frac{24}{3!}$ → internal arrangements

permutations

permutations within a selection.

→ ways to arrange n objects in r places $= {}^nP_r$

$$\boxed{\frac{n!}{(n-r)!}}$$

→ ways to arrange r objects in r places $= r!$

$$\boxed{\frac{{}^nP_r}{r!}} \rightarrow \boxed{\frac{n!}{(n-r)!\,r!}} \rightarrow {}^nC_r$$

Properties →

(A) ${}^nC_1 \rightarrow \frac{n!}{(n-1)!\,1!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = \boxed{n}$

(B) ${}^nC_0 \rightarrow \frac{n!}{(n-0)!\,0!} = \frac{n!}{n!} = \boxed{1}$

(12) what is ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$

$\hookrightarrow \{1, 2, 3\} \rightarrow n = 3$

$\rightarrow {}^3C_0 = 1 \rightarrow \{\}$

$\rightarrow {}^3C_1 = 3 \rightarrow \{1\}, \{2\}, \{3\}$

$\rightarrow {}^3C_2 = 3 \rightarrow \{1, 2\}, \{2, 3\}, \{1, 3\}$

$\rightarrow {}^3C_3 = 1 \rightarrow \{1, 2, 3\}$

all possible subsets of a set

(2^n)

$\{1, 2, 3\}$

T F

$\hookrightarrow 2 * 2 * 2 \rightarrow 2^n$

F	F	F	$\rightarrow \{\}$
T	F	F	$\rightarrow \{1\}$
F	T	F	$\rightarrow \{2\}$
T	T	F	$\rightarrow \{1, 2\}$
T	T	T	$\rightarrow \{1, 2, 3\}$

Q Given 5 players, count the number of ways to select 2 players.

$\hookrightarrow {}^5C_2$

$P_1 P_2$ $P_3 P_4 P_5$
$P_1 P_3$ $P_2 P_4 P_5$
$P_1 P_4$ $P_2 P_3 P_5$
$P_1 P_5$ $P_2 P_3 P_4$

$P_2 P_3$ $P_1 P_4 P_5$
$P_2 P_4$ $P_1 P_3 P_5$
$P_2 P_5$ $P_1 P_3 P_4$

$P_3 P_4$ $P_1 P_2 P_5$
$P_3 P_5$ $P_1 P_2 P_4$

$P_4 P_5$ $P_1 P_2 P_3$

Yellow → selections
Green → rejections

$$\boxed{{}^5C_2 = {}^5C_3}$$

$$\boxed{{}^nC_r = {}^nC_{n-r}.}$$

Q Given n items → select r items from them

$$\boxed{{}^nC_r}$$

→ a_1, a_2, a_3 . . . . a_n

2 possible options →

include this in selection — OR — not include this in selection

total elements left after this → $(n-1)$ $(n-1)$

more elements to be selected → $(r-1)$ (r)

$$\boxed{{}^nC_r} = \boxed{{}^{n-1}C_{r-1} + {}^{n-1}C_r}$$

Mathematical Proof

$${}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

$$\frac{(n-1)!}{(r-1)!\,[(n-\cancel{1})-(r-\cancel{1})]!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$\frac{(n-1)!}{\boxed{(r-1)!}\,(n-r)!} + \frac{(n-1)!}{\boxed{r!}\,(n-r-1)!}$$

$$\frac{(n-1)!}{(r-1)!\,\boxed{(n-r)!}} + \frac{(n-1)!}{r(r-1)!\,\boxed{(n-r-1)!}}$$

$$\frac{(n-1)!}{(r-1)!\,(n-r)(n-r-1)!} + \frac{(n-1)!}{r(r-1)!\,(n-r-1)!}$$

$$\frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{1}{r} + \frac{1}{n-r}\right]$$

$$\downarrow$$

$$\left[\frac{n-\cancel{r}+\cancel{r}}{(n-r)r}\right]$$

$$\frac{(n-1)!\,n}{(n-r-1)!\,(n-r)\;(r-1)!\,r}$$

$$\downarrow$$

$$\frac{n!}{(n-r)!\;r!} \Rightarrow {}^{n}C_{r}$$

since $\boxed{LHS = RHS}$

→ hence, proved ←

—— x —————— x ——————