Conditional Probability $$P[A \mid B] = \frac{P[A \cap B]}{P[B]}$$

Multiplication Rule $$P[A \cap B] = P[A \mid B]\ P[B]$$

Bayes Theorem $$P[B \mid A] = \frac{P[A \mid B]\ P[B]}{P[A]}$$

Law of Total probability $$P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$$

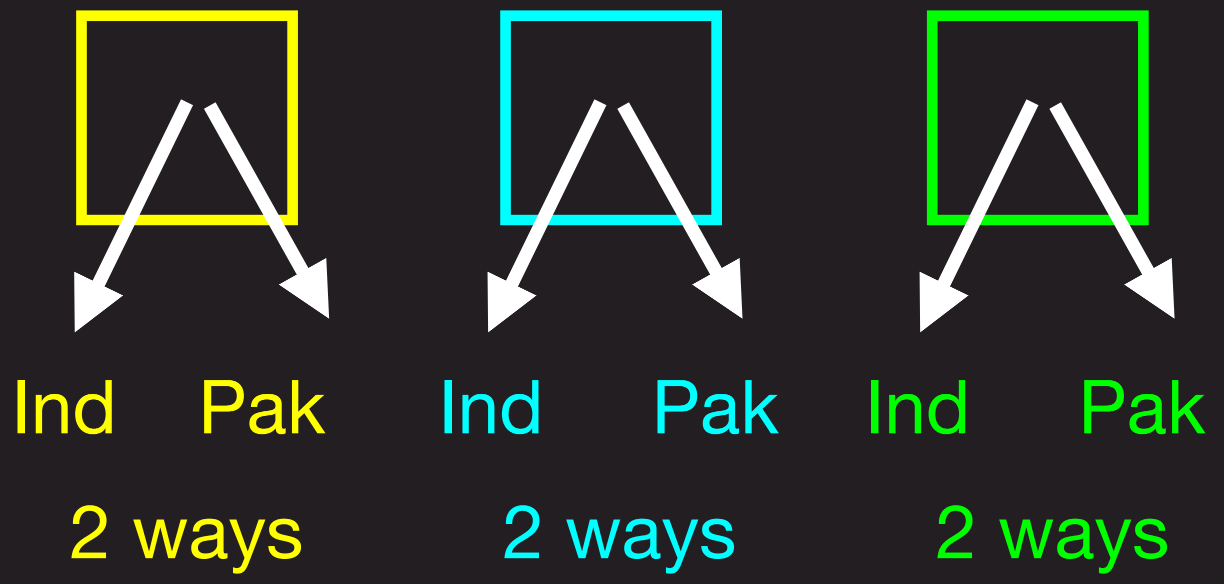
$$P[B] = P[B \cap A] + P[B \cap A^c]$$

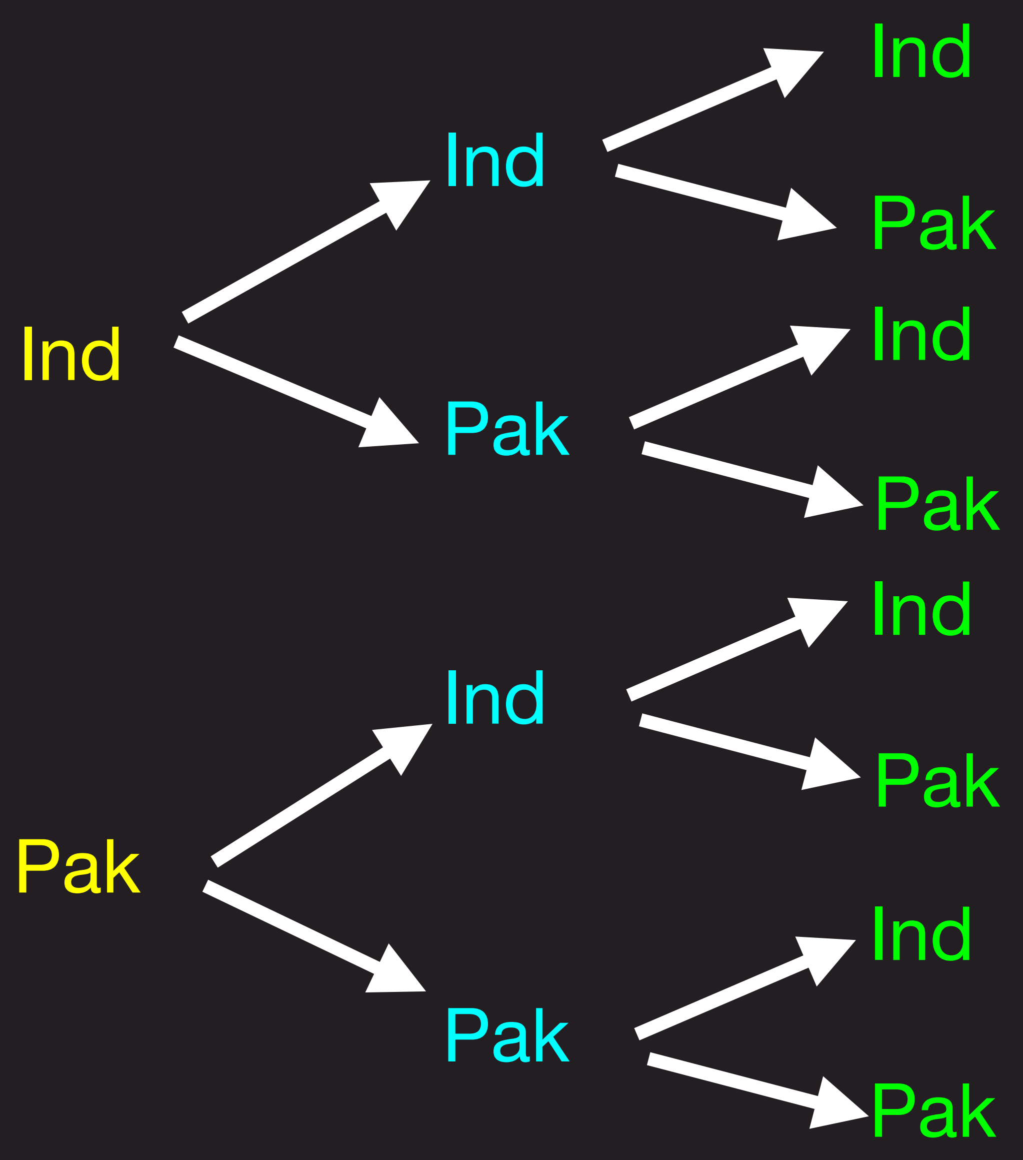
Independence $$P[A \mid B] = P[A]$$

$$P[A \cap B] = P[A]\ P[B]$$

India and Pakistan play a 3-match series. How many results are possible?

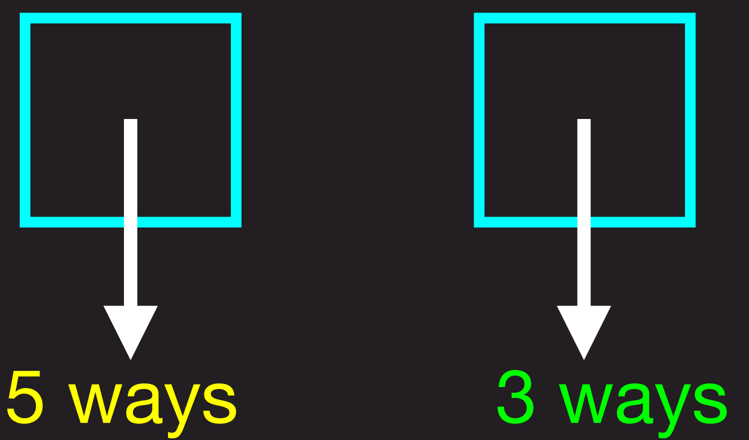
Note that we consider (Ind, Ind, Pak) different from (Ind, Pak, Ind) etc.





Total number of ways = 2 * 2 * 2 = 8

In a bowl-out, for a specific ball you have to choose a bowler and a wicket keeper. Suppose you have 5 bowlers and 3 wicket keepers. How many ways can you select for a ball?



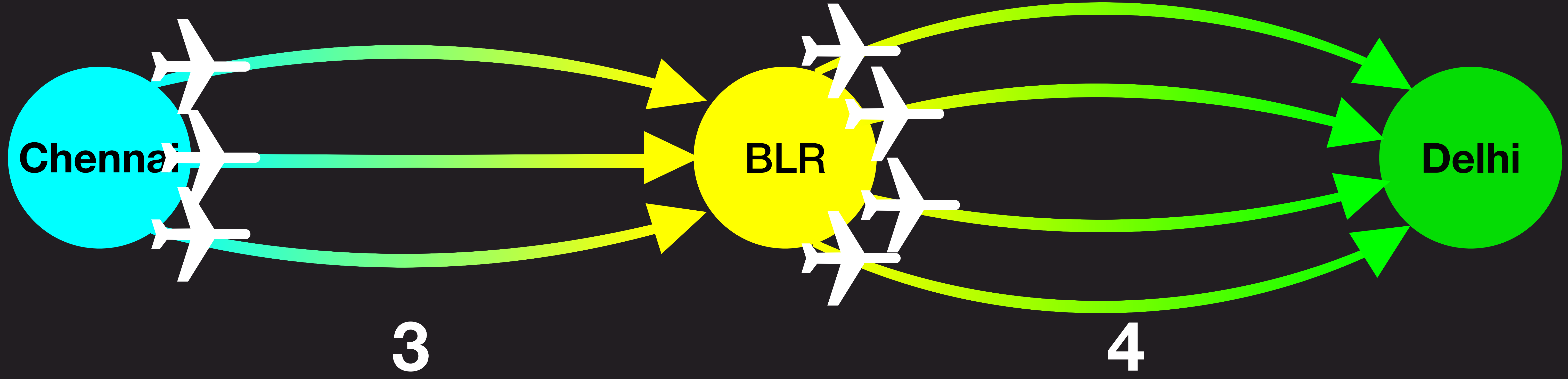
Total number of ways = 5 * 3 = 15

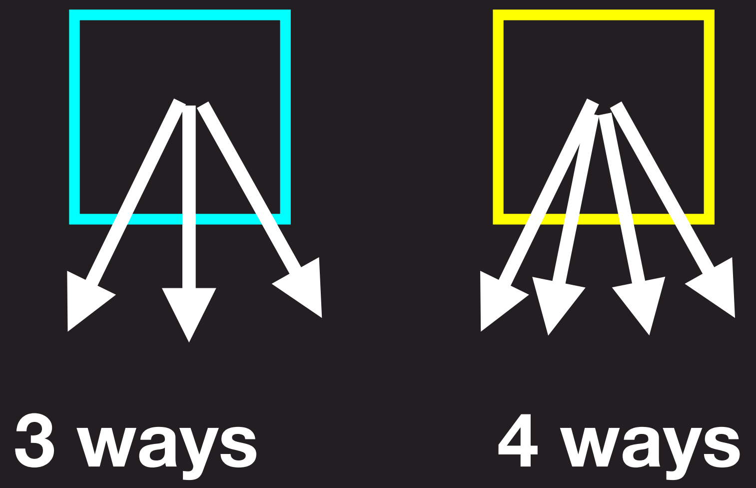
(B1, W1), (B2, W1), (B3, W1), (B4, W1), (B5, W1)

(B1, W2), (B2, W2), (B3, W2), (B4, W2), (B5, W2)

(B1, W3), (B2, W3), (B3, W3), (B4, W3), (B5, W3)

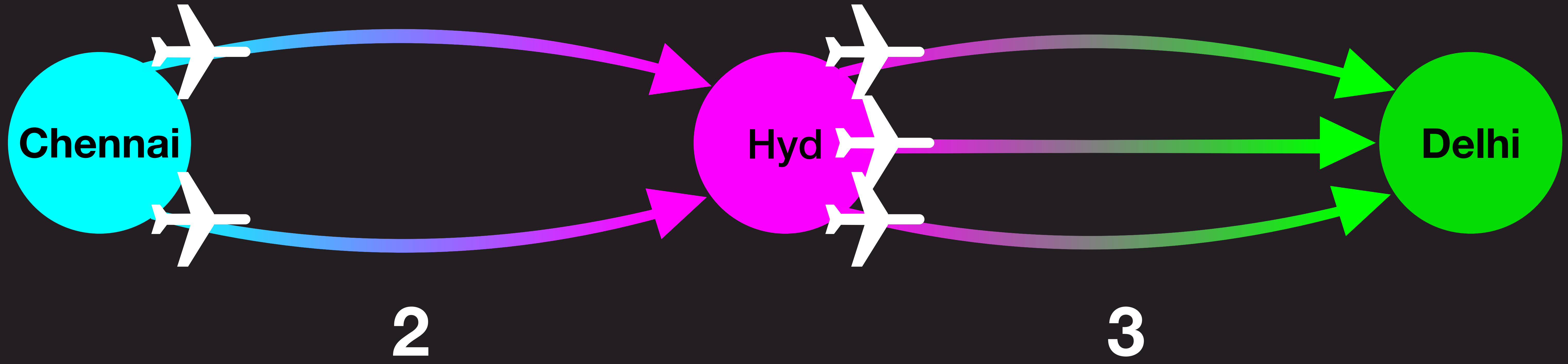
There are 3 ways to move from Chennai to Bangalore.
There are 4 ways to move from Bangalore to Delhi.
What are the total ways of moving from Chennai to Delhi?



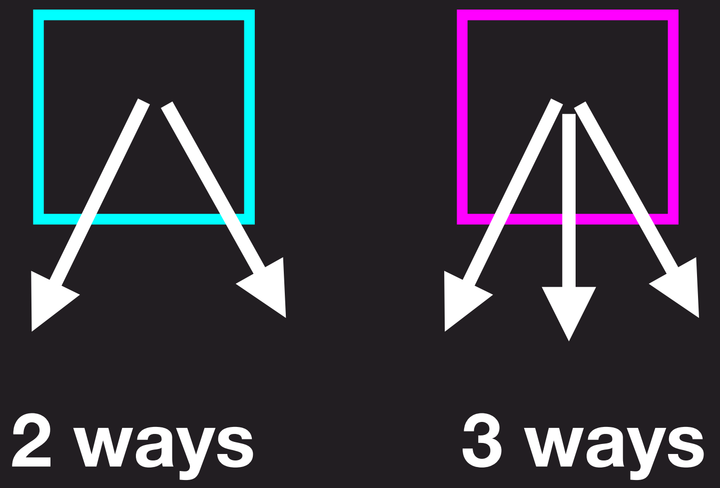


Total number of ways = 3 * 4 = 12

There are 2 ways to move from Chennai to Hyderabad.
There are 3 ways to move from Hyderabad to Delhi.
What are the total ways of moving from Chennai to Delhi?



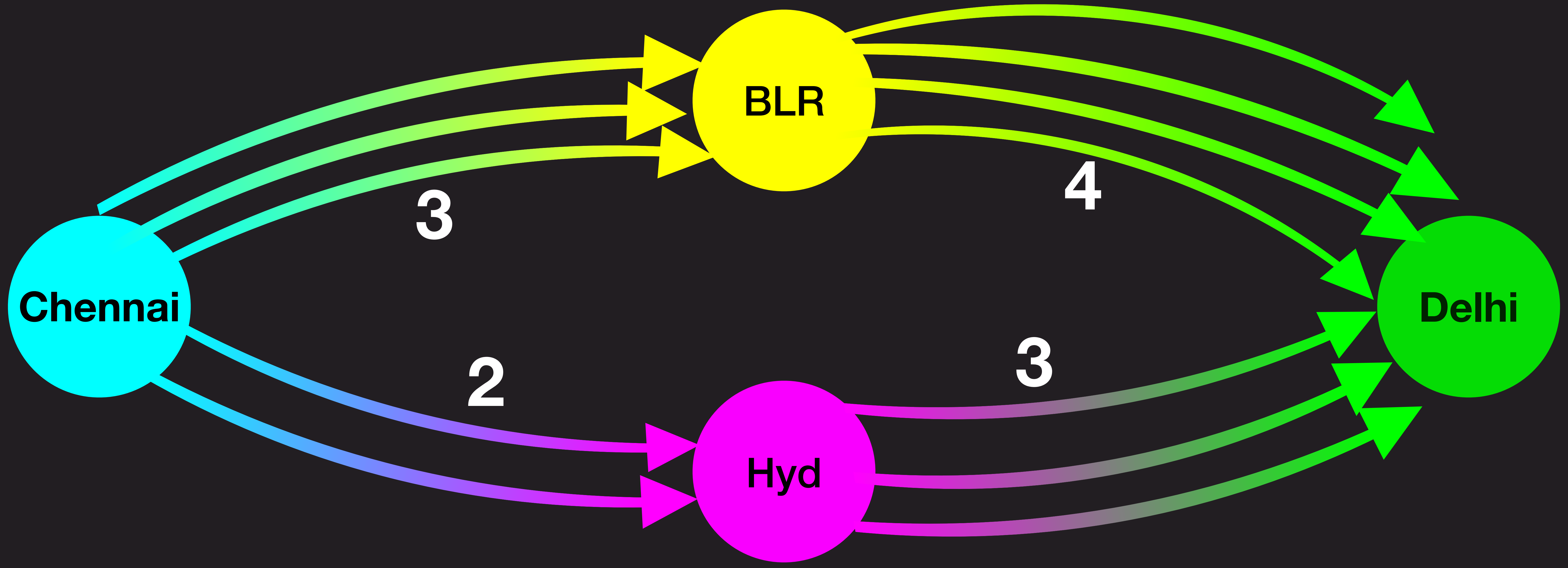
Total number of ways = 2 * 3 = 6



There are 3 ways to move from Chennai to Bangalore, and 4 ways to move from Bangalore to Delhi.

There are 2 ways to move from Chennai to Hyderabad, and 3 ways to move from Hyderabad to Delhi.

In how many ways can we move from Chennai to Delhi?

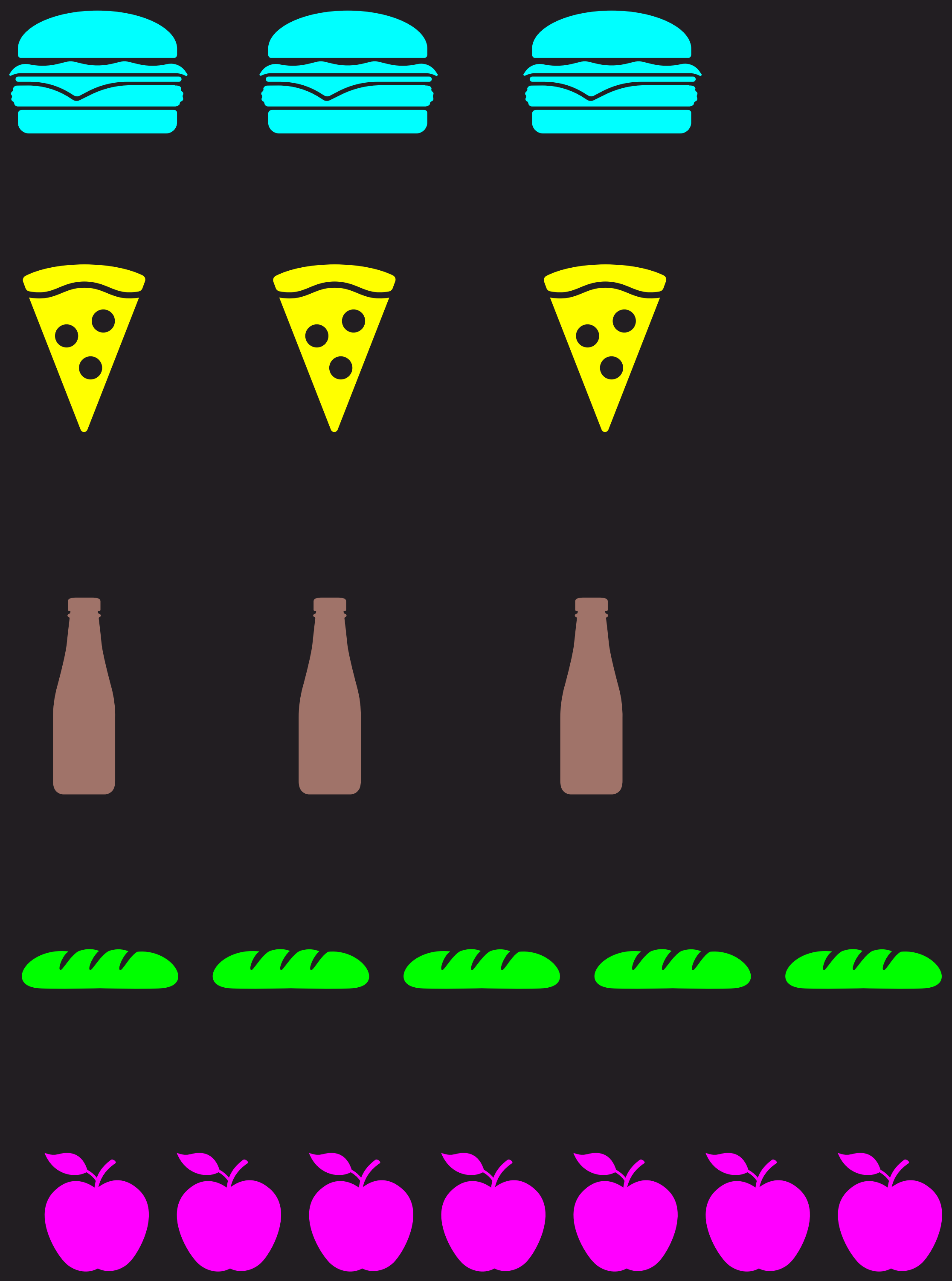


Via Mumbai 3 * 4 = 12

Via Nagpur 2 * 3 = 6

Total = 12 + 6 = 18

A fast food outlet has the following types of items in their menu



3 types of Burgers

3 types of Pizza

3 types of Drinks

5 types of Sandwiches

7 types of Fruits

You can choose one of the following combos:

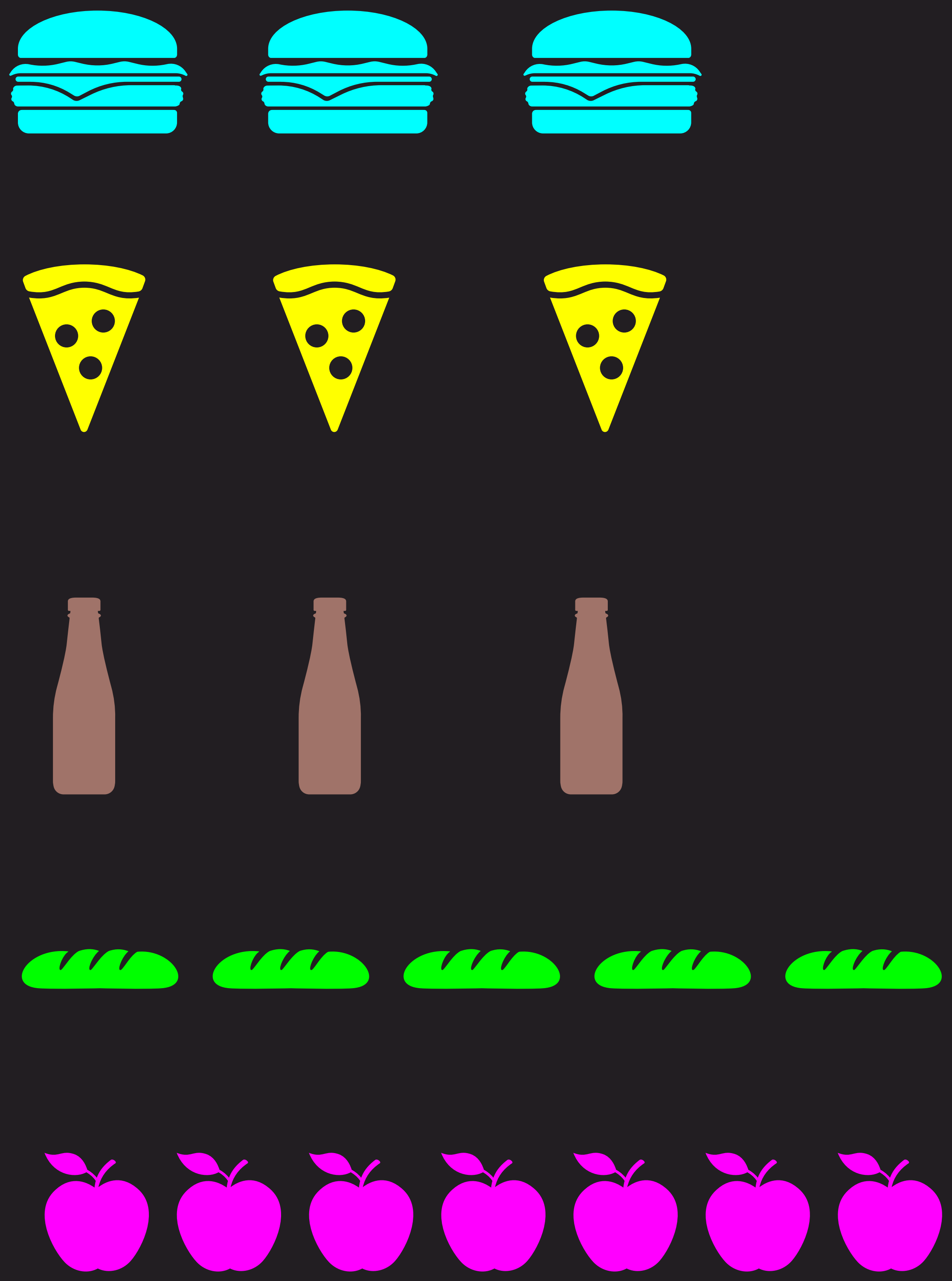
1 Burger and 1 Sandwich

1 Fruit and 1 Drink

1 Pizza

How many such combos can you make?

A fast food outlet has the following types of items in their menu



3 types of Burgers

3 types of Pizza

3 types of Drinks

5 types of Sandwiches

7 types of Fruits

You can choose one of the following combos:

1 Burger and 1 Sandwich

1 Fruit and 1 Drink

1 Pizza

How many such combos can you make?

1 Burger and 1 Sandwich	3 * 5 = 15
1 Fruit and 1 Drink	7 * 3 = 21
1 Pizza	3

Total = 15 + 21 + 3 = 39

Permutations

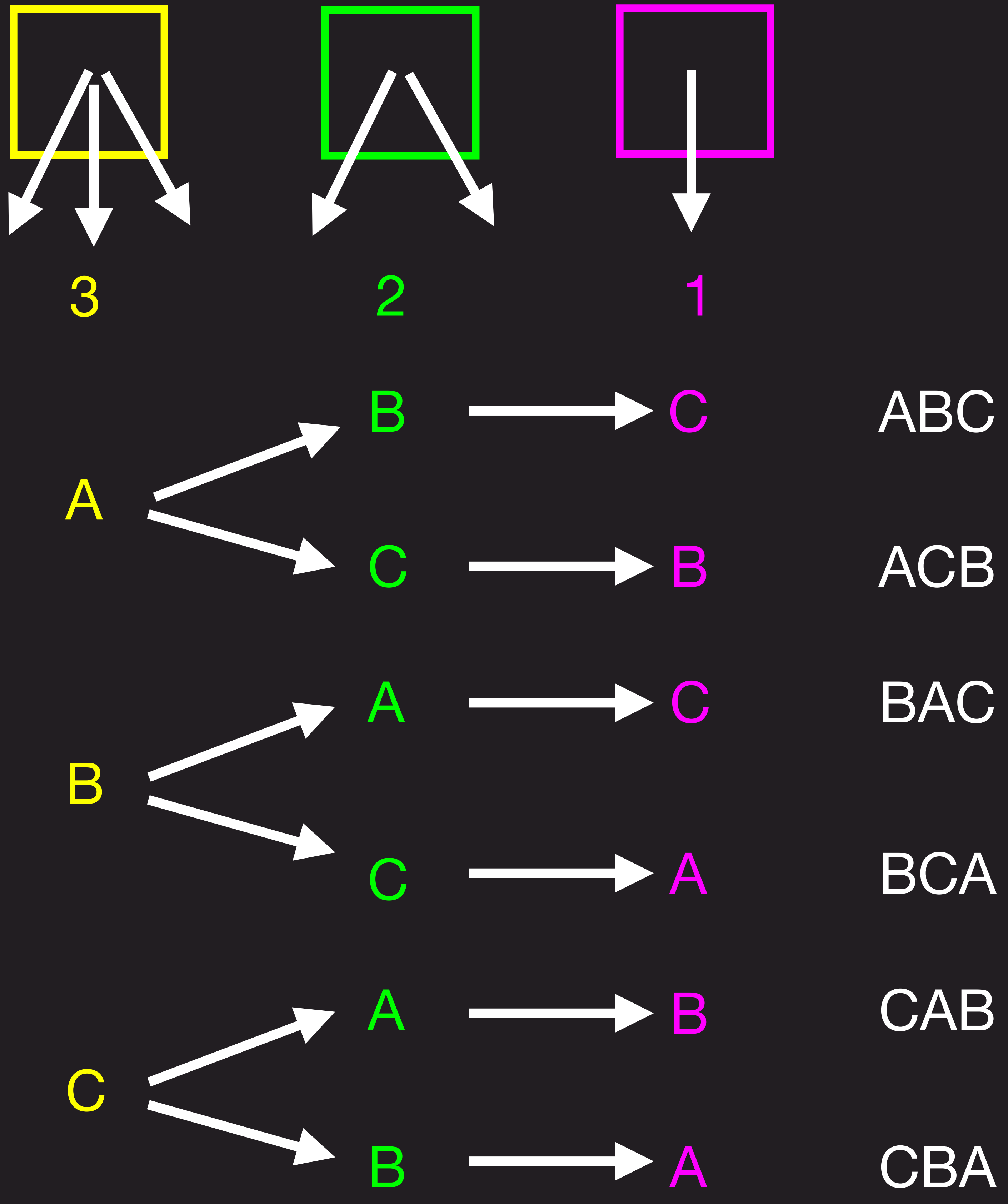
Arrangement of objects

Order matters!

$$(i, j) \neq (j, i)$$

$$a\ b \neq b\ a$$

What is the number of ways of arranging 3 characters: A, B, C?



Total number of ways = 3 * 2 * 1 = 6

This number is called “3 factorial”

$3! = 3 * 2 * 1$

What is the number of ways of arranging 4 characters: A, B, C, D?

4 3 2 1

Total number of ways $= 4! = 4 * 3 * 2 * 1 = 24$

4 factorial

What is the number of ways of arranging N distinct objects?

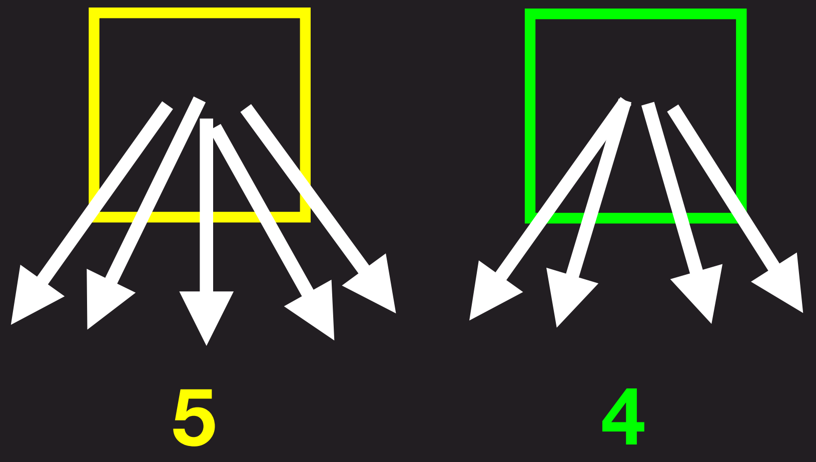
$$N * (N-1) * (N-2) * \cdots * 3 * 2 * 1 = N!$$

N factorial

In how many ways can we arrange 0 distinct characters?

$$0! = 1$$

Given 5 different characters, in how many ways can we arrange them in 2 places?



Total number of ways = $5 * 4 = 20$

$$^5P_2 = 5 * 4 = \frac{5 * 4 * 3 * 2 * 1}{3 * 2 * 1} = \frac{5!}{3!}$$

Given “N” distinct objects, count the number of ways in which can we arrange them in 3 places.

$$^{N}P_3 = N * (N-1) * (N-2)$$

Given “N” distinct objects, count the number of ways in which can we arrange them in 4 places.

$$^{N}P_4 = N * (N-1) * (N-2) * (N-3)$$

Given “N” distinct objects, count the number of ways in which can we arrange them in “k” places.

$$^{N}P_k = N * (N-1) * (N-2) * (N-3) * \cdots * (N-k+1)$$

$$^{N}P_k = \frac{N * (N-1) * (N-2) * (N-3) * \cdots * (N-k+1) * (N-k) * (N-k-1) * \cdots 3 * 2 * 1}{(N-k) * (N-k-1) * \cdots 3 * 2 * 1}$$

$$^{N}P_k = \frac{N!}{(N-k)!}$$

Let us see the same for 5 objects in 2 places

$$^{5}P_2 = 5 * 4 \quad = \frac{5 * 4 * 3 * 2 * 1}{3 * 2 * 1} = \frac{5!}{3!}$$

$$N = 5 \qquad k = 2 \qquad N - k = 3$$

$$\frac{N!}{(N-k)!} = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

There are 4 players P1, P2, P3, and P4 who can play in the top-order positions of 1, 2, and 3.

How many arrangements of top-order can we make from 3 of these 4 players?

Sachin, Sehwag, Kohli, Rohit

Examples of top-order

Sachin, Sehwag, Kohli

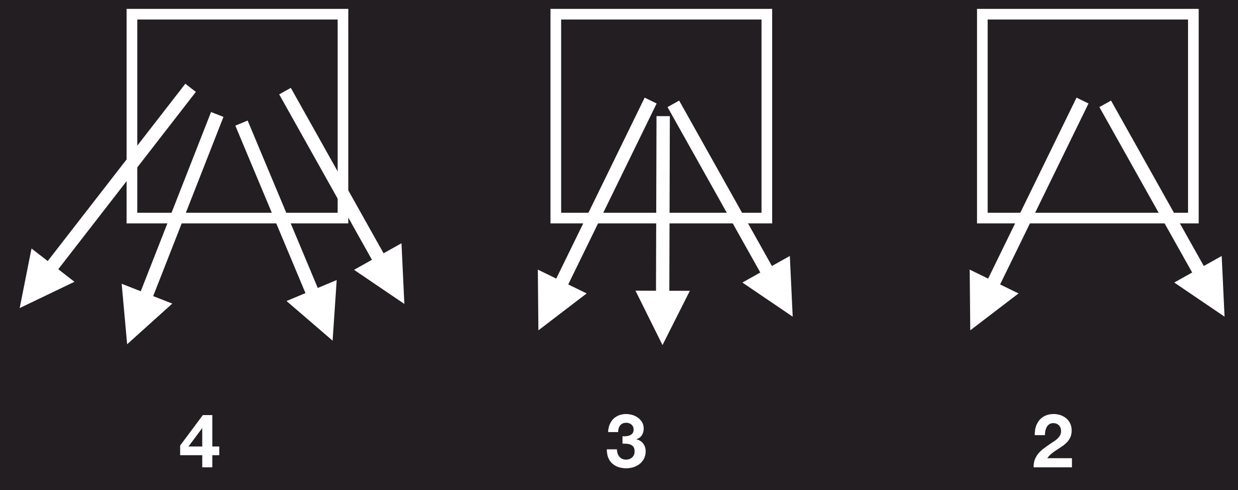
Sehwag, Sachin, Kohli

Sachin, Rohit, Kohli

Sachin, Kohli, Sehwag

How many more like this?

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2



$$24 = 4 * 3 * 2$$

$${}^4P_3$$

Sachin, Sehwag, Kohli, Rohit

Suppose we have to select 3 players out of 4 players in our team.

In how many ways can we do this?

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2
P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4

$$\frac{24}{6} = 4$$

$$6 \rightarrow 3!$$

$$^4C_3 = \frac{^4P_3}{3!}$$

$$24 = 4 * 3 * 2$$

$$^4P_3$$

Combinations

$$(i, j) = (j, i)$$

$$^{n}C_{k} = \frac{^{n}P_{k}}{k!}$$

Suppose we have to select 2 players out of 5 players in our team.
In how many ways can we do this?

$$^5C_2 = \frac{5 * 4}{2 * 1} = 10$$

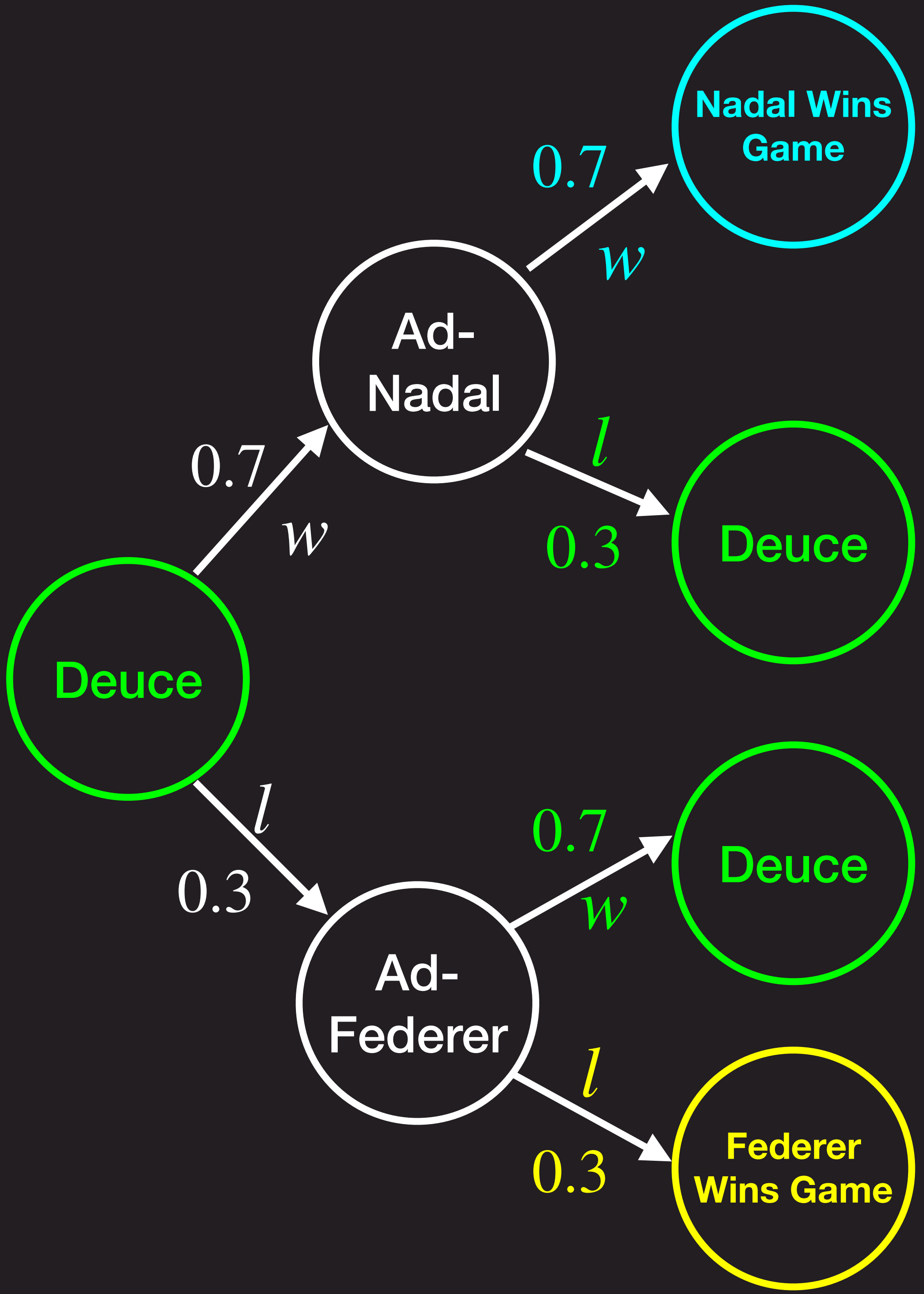
Three students are randomly chosen from across cohorts at Scaler.
Each student is equally likely to belong to any of the 12 cohorts starting in each month: January, Feb, … , December

What is the probability that no two students belong to the same cohort?

$$\frac{{}^{12}P_3}{12^3} = 0.76$$

Nadal Vs Federer on Clay

Nadal wins 70% of the points he plays against Federer on clay
Suppose a tennis game between Nadal and Federer is on Deuce.
What is the probability of Nadal winning the game?



Let $P[N]$ denote the probability that Nadal won the game

$$P[N] = P[N|ww]P[ww] + P[N|wl]P[wl] + P[N|lw]P[lw] + P[N|ll]P[ll]$$

$$P[N|ww] = 1 \qquad P[ww] = 0.7 * 0.7$$

$$P[N|ll] = 0 \qquad P[ll] = 0.3 * 0.3$$

$$P[N|wl] = P[N] \qquad P[wl] = 0.7 * 0.3$$

$$P[N|lw] = P[N] \qquad P[lw] = 0.3 * 0.7$$

$$P[N] = (1)(0.7 * 0.7) + P[N](0.7 * 0.3) + P[N](0.3 * 0.7) + (0)(0.3 * 0.3)$$

$$P[N](1 - 2 * 0.7 * 0.3) = 0.7 * 0.7$$

$$P[N] = \frac{0.7 * 0.7}{1 - 2 * 0.7 * 0.3} = 0.84$$