16th December, 2022

DSML : CC Maths

Probability II - Problem Solving.

Recap: (a) Probability theory.
(b) Bayes' theorem.
(c) Combinatorics.
(d) Descriptive statistics.
(e) Binomial Distribution.
(f) Gaussian Distribution
(g) Poisson Distribution.
(h) Geometric Distribution.
(i) Exponential Distribution.
(j) Confidence Intervals.

OTP: 2023

Class starts @ 9:05 p.m.

Today: (a) Log-normal Distribution.
(b) Problem solving.

Log. normal Distribution:

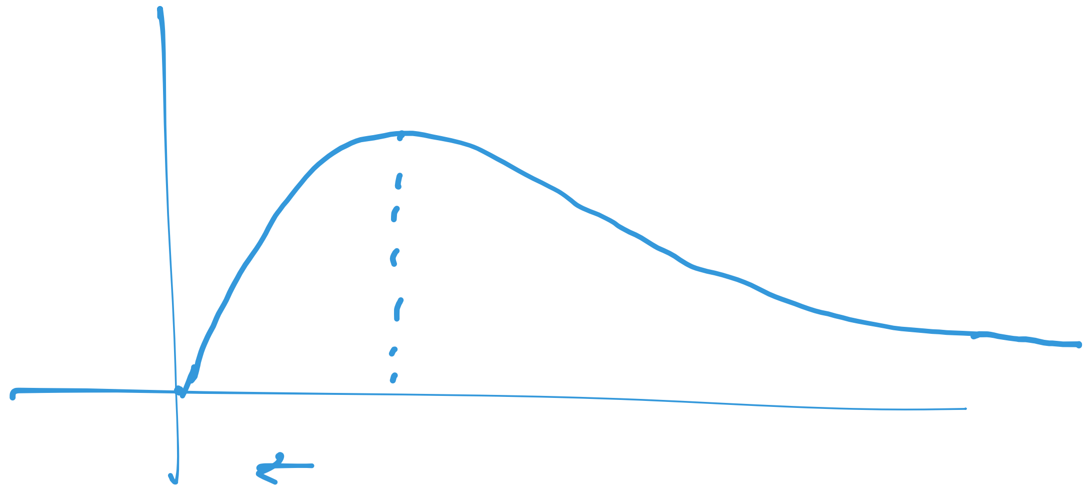
X : count of no. of days of hospitalization.

3-5.

1- 2.

20-30

>300



Left-skewed

di

If $\log(\text{data}) \to$ is Gaussian,

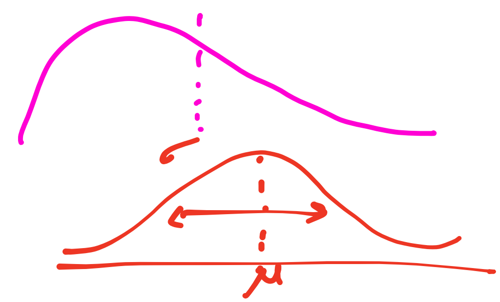
then data is log-normal.

Distributions	Parameters
1 binom.	n, k, p.
2 poisson.	$\lambda \to$ rate. $k \to$
3 Gaussian	$\mu \to$ mean, $\sigma \to$ std.
4 exponential	λ.
5 Geom	$p, \; k \to$.
6 Log-normal	μ, σ.

X

L.N. :

Gauss :

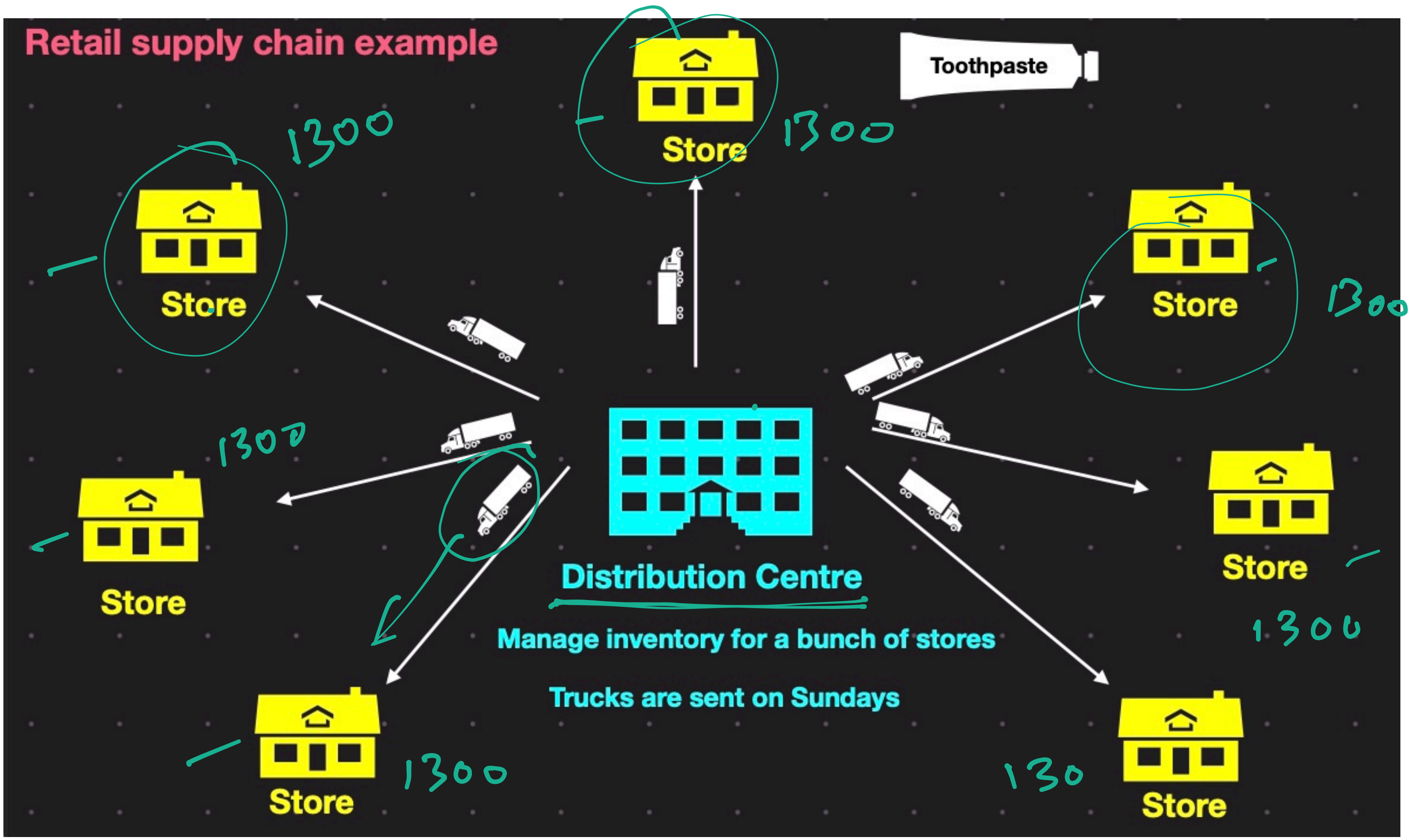


$X \longrightarrow$ lognormal with parameters μ, σ.

$$E[X] = \underline{e^{\mu + \sigma^2/2}}$$

$$\mathrm{Var}[X] = \underline{\left(e^{\sigma^2} - 1\right)\left(e^{2\mu + \sigma^2}\right)}.$$

wikipedia.


Retail supply chain example
Toothpaste
Store
1300
Store
1300
Store
1300
Store
1300
Distribution Centre
Manage inventory for a bunch of stores
Trucks are sent on Sundays
Store
1300
Store
1300
Store
130
Store

Airline overbooking.

5% of the people making reservations don't show up. Suppose 52 tickets are sold and we have 50 seats on the flight. Prob. that every passenger gets a seat?

$X \rightarrow$ No. of people who ~~don't~~ show up.

$p \rightarrow$ prob. that 1 person doesn't show.

$P[X \le 50]$

$\hookrightarrow 0.05 \rightarrow 0.95$

$1 - P[X \le 1]$.

$P[X \ge 2.]$

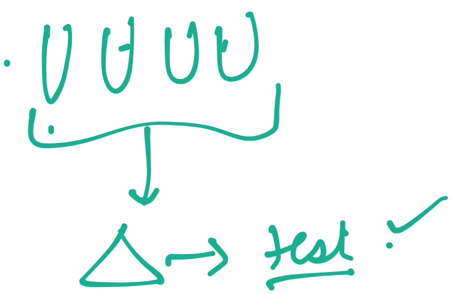
$n = 52.$

binom

$1 - \text{binom.cdf}(n = 52, p = 0.05, k = 1)$

Pooled Blood test.

A blood bank conducts tests on pooled samples of size 4.



If clean, the bank stores all 4.
If dirty, the bank separately tests all 4.
The probability that any sample is contaminated is $p = 0.1$.
Find the expected number of tests.

*binom

$n = 4$.
$p = 0.1$.

5

X count of contaminated sample. $P[X=0]$

$Y \rightarrow$ total tests conducted.

1 - If 1st test was healthy

5 - If 1st test was contaminated.

1

$P[Y=1] = P[X=0]$

binom.pmf(n=4, p=0.1, k=0).

$$E[Y] = P[Y=1]\cdot 1 + P[Y=5]\times 5$$

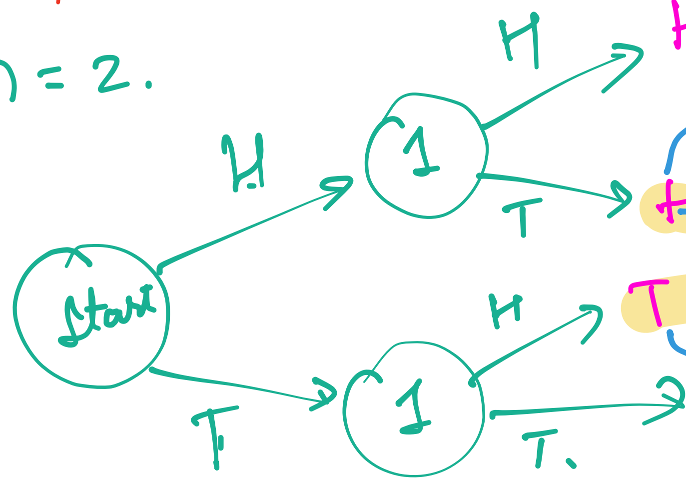
Simulate a fair coin from a biased coin.

$P[H] = 0.7$

* How can we now design an experiment so that the coin lands heads 50% of the time?

condition.

$n = 2.$



$HH \to p^2 \to 0.49$

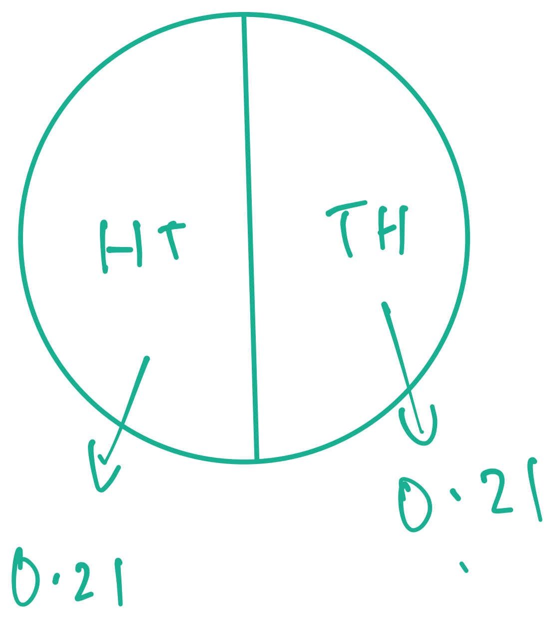
$HT \to p(1-p) \to 0.21$

$TH \to p(1-p) \to 0.21$

$TT \to (1-p)^2 \to 0.09.$

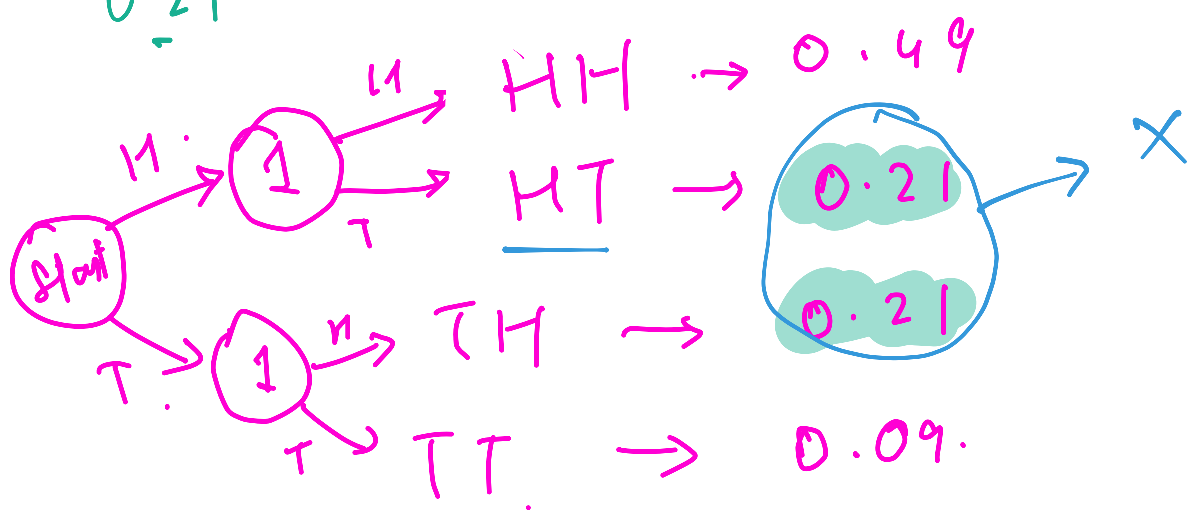
$X : \{HT, TH\}$ $\qquad Y = X^c = \{TT, HH\}.$

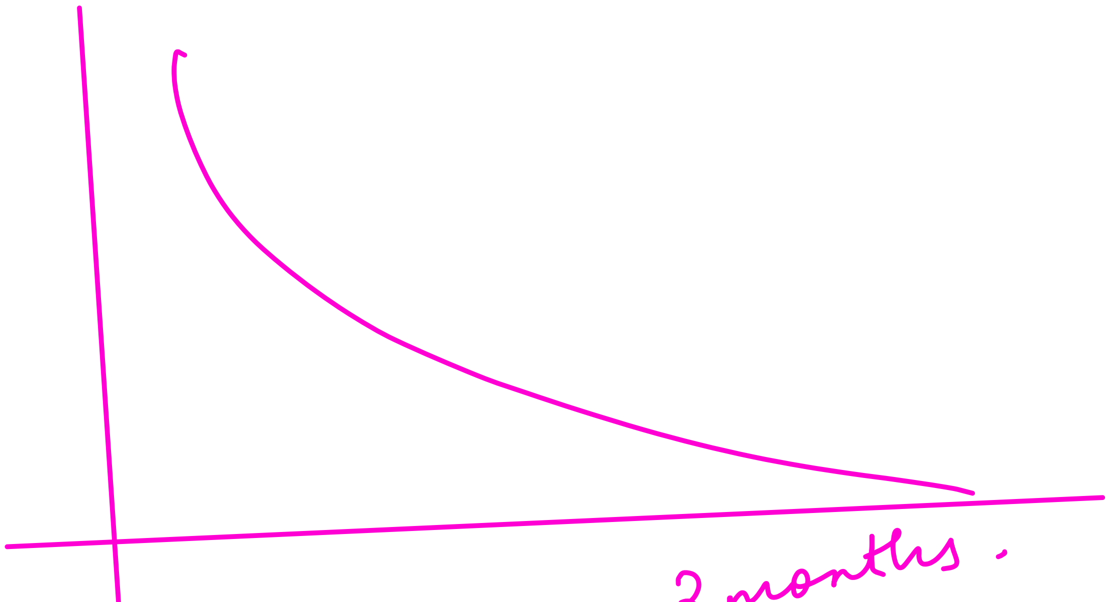
$$P(HT \mid X) = \frac{P(HT \cap X)}{P(X)} = \frac{0.21}{0.42} = \frac{1}{2}$$



$P(X) = 0.42$

$X = TH$ or HT





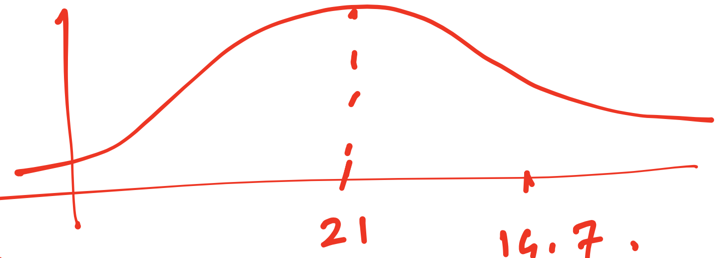
mu = 2 months.

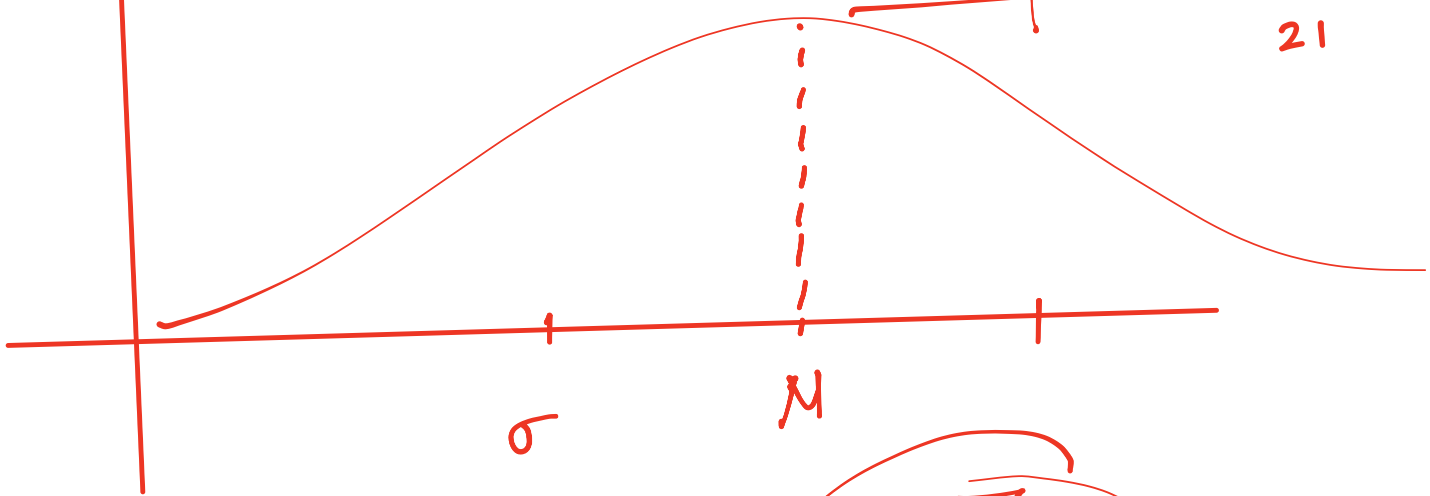
$\lambda = \frac{1}{mu} = 0.5$.

$P(X \leq 6)$?

expon.cdf(6).

$$Z = \frac{28-21}{14.7}$$





$P(E) = 0.7.$

$n = 70$

$p = 0.3.$

norm.cdf.

$\mu = n * p. = 70 \times 0.3 = 21$

$\sigma^2 = n * p * q. = 70 \times 0.3 \times 0.7$

21×0.7

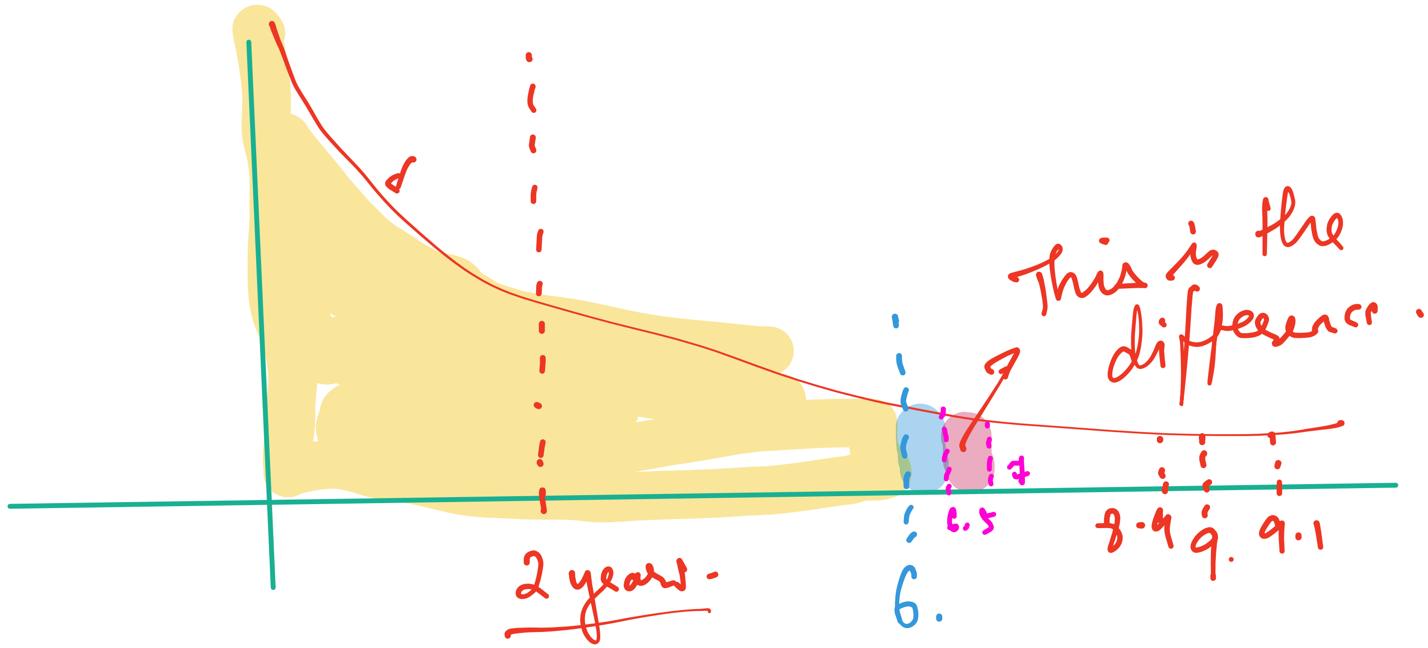
14.7

$\sigma = \sqrt{npq}.$

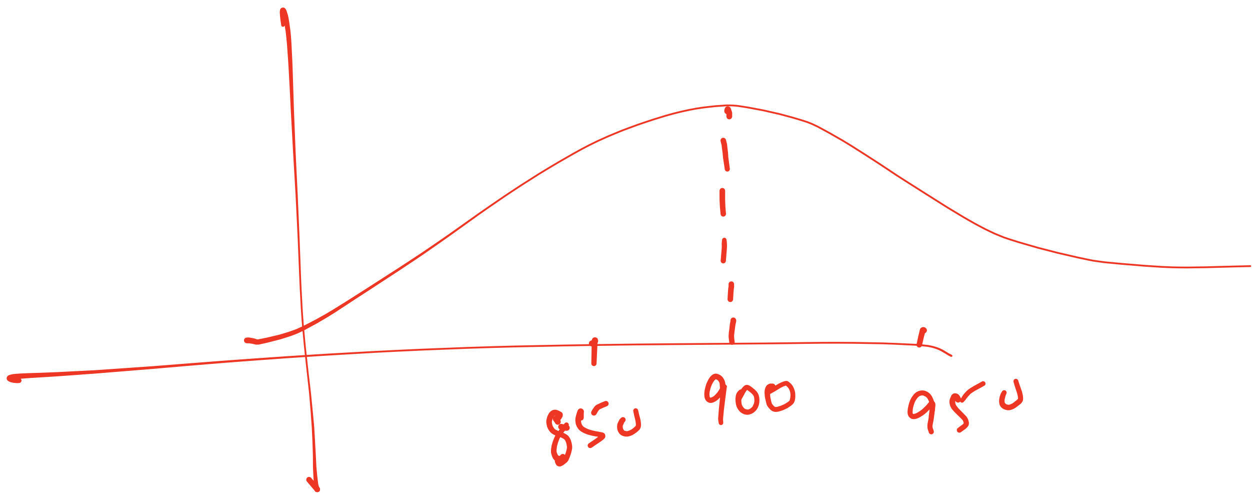
At least.

Electronic gadget:

$\frac{1}{7}$



$P(X \leq 6.5) \neq P(X \leq 7)$



$\sigma : 50$

$\mu = 900$

$n = 20$

S

$P(S \leq 875)$

$\mu = 900$

$\sigma = \dfrac{50}{\sqrt{20}}$

$Z = \left(\dfrac{875 - 900}{50} \times \sqrt{20} \right)$