25th November, 2022

DSML : CC Maths

Probability 3 : Problem Solving - 1

Recap :
* Experiment.
* Outcome.
* Sample Space
* Event.
* Conditional Probability.
* Multiplication Rule.
* Bayes' theorem.
* Law of total probability.

Today :
* Independence of events.
* Lots of problem solving.

Class starts @ 9:05 p.m.

Formulae seen so far:

① Conditional Probability: $P[A|B] = \frac{P[A \cap B]}{P[B]}$

② Multiplication Rule: $P[B] \cdot P[A|B] = P(A \cap B)$

③ Bayes' Theorem: $P[A|B] = \frac{P[B|A] \cdot P[A]}{P[B]}$

④ Law of total probability:

$$P[B] = P[B \cap A] + P[B \cap C]$$

↳ Works only when $B = (B \cap A) \cup (B \cap C)$

$$\therefore P[B] = P[A] \cdot P[B|A] + P[C] \cdot P[B|C].$$

Experiment: 1 coin toss + 1 dice throw.

Q] Let A : getting a heads in the coin toss.

B : getting a 3 on the dice roll.

Is event A independent of event B? → Yes.

$$S = \left\{ \begin{matrix} (H,1), (H,2), (H,3), (H,4), (H,5), (H,6) \\ (T,1), (T,2), (T,3), (T,4), (T,5), (T,6) \end{matrix} \right\}$$

$P[A] = \frac{6}{12} = \frac{1}{2}$.

$P[B] = \frac{2}{12} = \frac{1}{6}$.

$P[A \cap B] = \frac{1}{12}$.

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{1/12}{1/6} = \frac{1}{2}$$

$P[A \mid B] = P[A]$

Formulae seen so far:

① Conditional Probability: $P[A|B] = \dfrac{P[A \cap B]}{P[B]}$

② Multiplication Rule: $P[A \cap B] = P[A|B] \cdot P[B]$

③ Bayes' Theorem: $P[A|B] = \dfrac{P[B|A] \cdot P[A]}{P[B]}$

④ Law of total probability:

$P[B] = P[B \cap A] + P[B \cap C]$

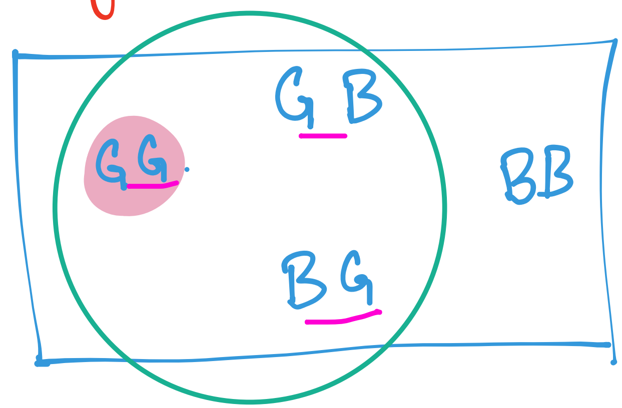
↳ Works only when $B = (B \cap A) \cup (B \cap C)$

$\therefore P[B] = P[A] \cdot P[B|A] + P[C] \cdot P[B|C]$.

⑤ Independence: A is independent of B if $P[A|B] = P[A]$.

Q] A family has 2 children, at least 1 is a girl. What is the probability that both are girls?

→ $S = \{ BB, BG, GB, GG \}$.



A : Event that both are girls.

B : At least one is a girl.

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{1/\cancel{4}}{3/\cancel{4}} = \frac{1}{3}$$

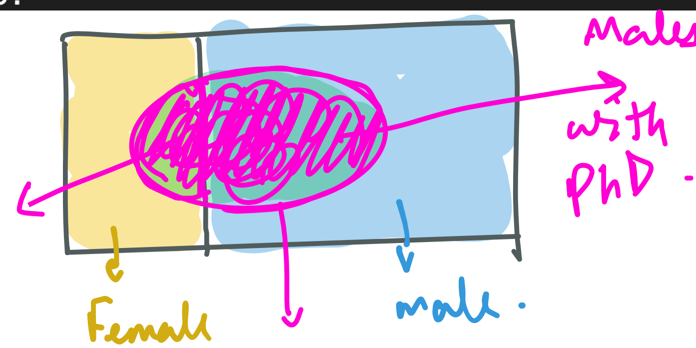
In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD.

1. What is the probability that a randomly chosen faculty member is a female and has PHD?
2. What is the probability that a randomly chosen faculty member is a male and has PHD?
3. What is the probability that a randomly chosen faculty member has a PHD?
4. What is the probability that a randomly chosen PHD holder is female?

- $P[phd \cap F] = \underbrace{P[phd \mid F]} \cdot \underbrace{P[F]}$

$= 0.6 \times 0.3$

$= 0.18.$



- $P[phD \cap M] = P[phd \mid M] \cdot P[M].$

$= 0.4 \times 0.7.$

$= 0.28.$

$\boxed{PhD} = (M \cap Phd) \cup (F \cap Phd)$

$\cdot P[phd] = P[phd \cap M] + P[phd \cap F]$

$= 0.18 + 0.28 = 0.46$

$P[F \mid phd] = \dfrac{P[F \cap phd]}{P[Phd]}. = \dfrac{0.18.}{0.46} = 0.39.$

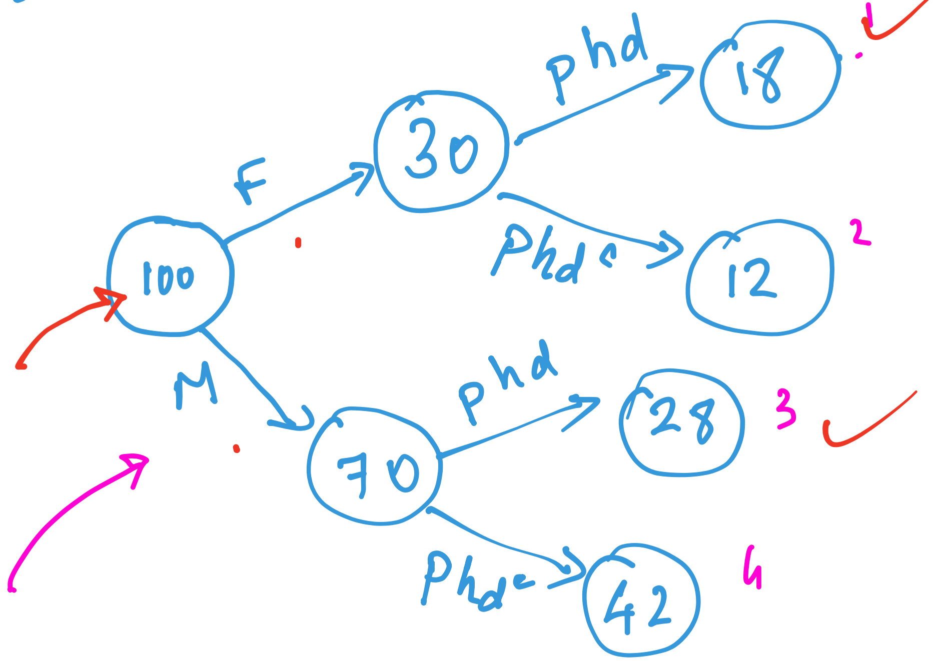
Alternate method.

In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD.

What is the probability that a randomly chosen PHD holder is female?

$P[F] = 0.3.$ $\quad P[phd \mid F] = 0.6.$

$P[M] = 0.7$ $\quad P[phd \mid M] = 0.4.$



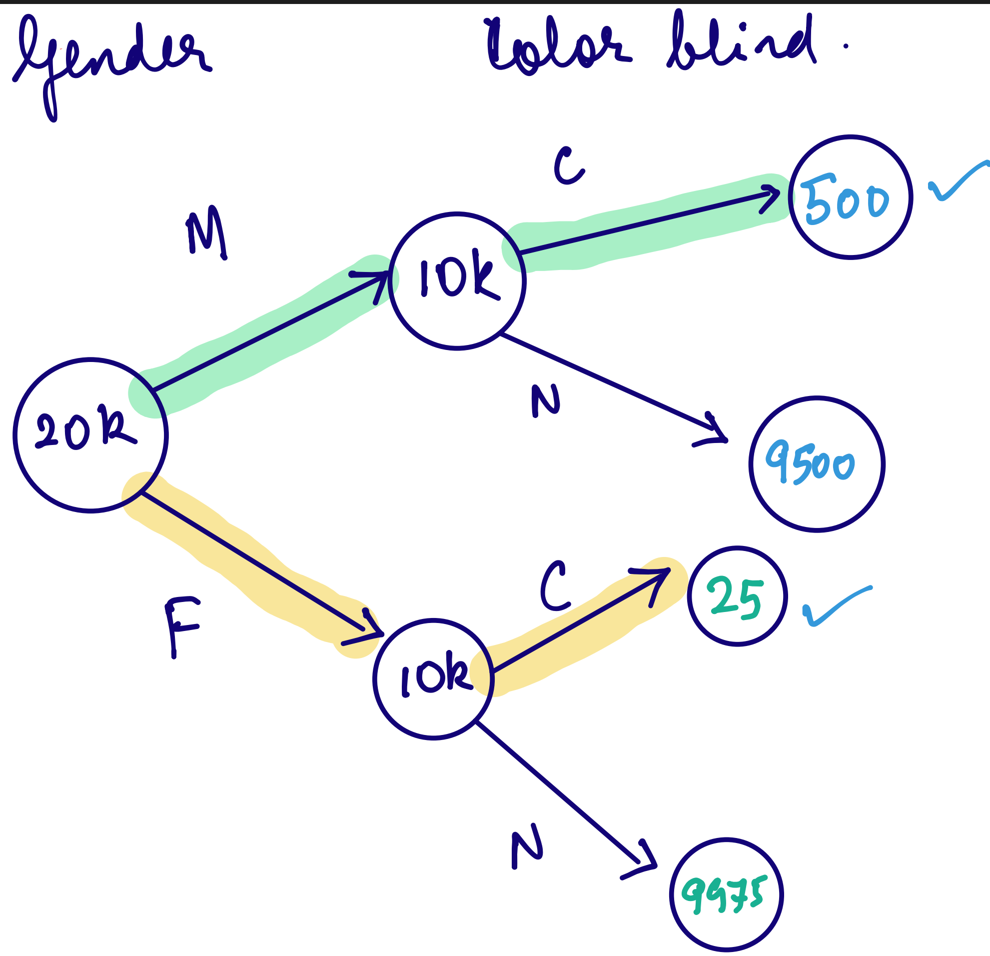
numerator: Females with phd.

$$\frac{18}{18+28} = \frac{18}{46}$$

denominator: total members with a Phd.

$= \underline{0.39}$

Suppose 5 percent of men and 0.25 percent of the women are color-blind. A random color-blind person is chosen. What is the probability of this person being male? Assume there are equal number of men and women overall.



$10000 \times 0.0025.$

$\rightarrow 25$

$$\frac{500}{500 + 25}$$

$= 0.95$

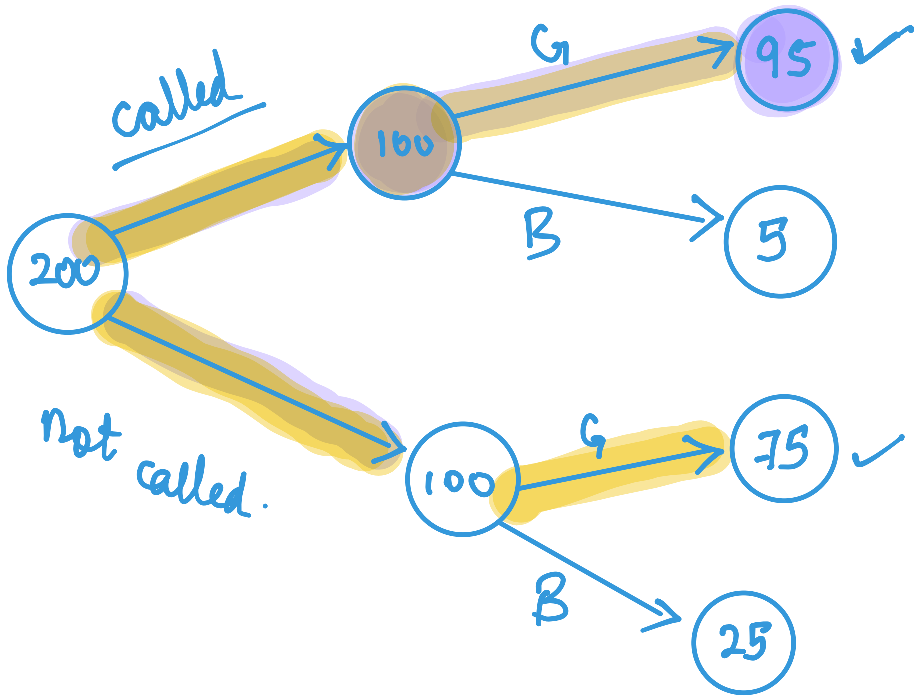
Amazon Interview question.

50% of the people who gave the first round were called for the second round

95% of the people who got invited for the second round felt that they had a good first round

75% of the people who did not get invited for the second round also felt that they had a good first round

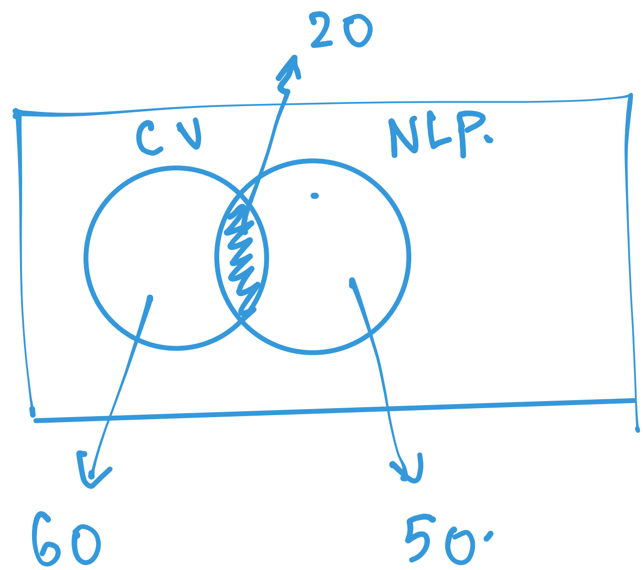
Given that a person felt good about the first round, what is the probability that he cleared the first round?



$$\frac{95}{95+75} = \frac{95}{170}.$$

0.5588.

Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language process (NLP). Also, it is seen that 20 have taken both CV and NLP. Given that a person has taken NLP, what is the probability that he has also taken CV?



$P[CV \cap NLP \mid NLP]$

A: No. of students who have taken CV.

B: Students who have taken NLP.

$P[A] = \frac{60}{100}$

$P[B] = \frac{50}{100}$

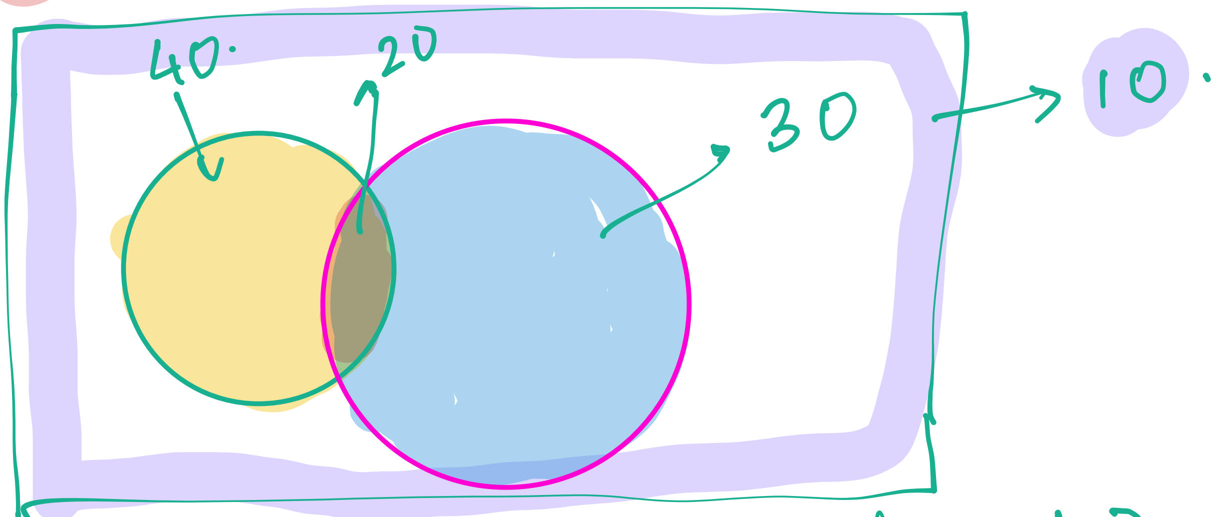
$$P[A \mid B] = \frac{P[A \cap B]}{P[B]}$$

$$= \frac{20/100}{50/100} = \frac{2}{5}$$

60 students → CV.

50 students → NLP.

20 students → Both.



only CV = 40

only NLP = 30.

40 + 20 + 30 = 90.

* Two events A and B are :

* Independent if : $\underline{P[A] = P[A \mid B]}$.

* Mutually exclive if : ① $\underline{A \cap B} = \phi$

② $P(A \cap B) = 0$.

Dice: $\underline{3}$ 6.

$P(3) = 1/6.$

$P(6) = 1/6.$

$P(3) + P(6) = \frac{1}{6} + 1/6$

$= \frac{1}{3}$

$P(A \mid B) = \frac{\overbrace{P(A \cap B)}^{\to 0}}{P(B)} = 0. = P(B \mid A)$

$P(T) = 0.03.$ $\quad P(B \mid T) = P(B).$

$P(B) = 0.07.$

(i) $P(T \cap B)$

(ii) $P(T \cup B) = P(T) + P(B) - P(T \cap B)$

$\underline{P(T \cap B)} = P(T) \cdot P(B \mid T)$

$= P(T) \cdot P(B).$

$= 0.03 \times 0.07.$

Customers = 100.

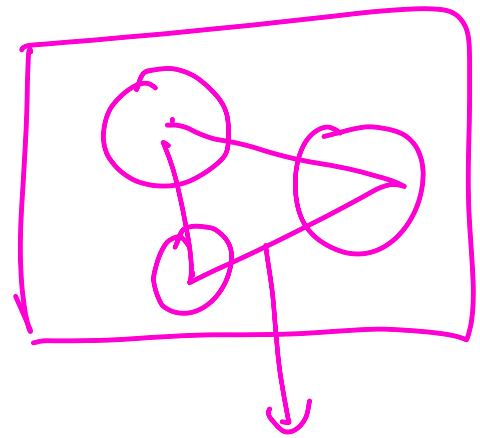
Large 20.

Medium 18.

Small 12.

→ 50 → didn't buy.

50 → bought.



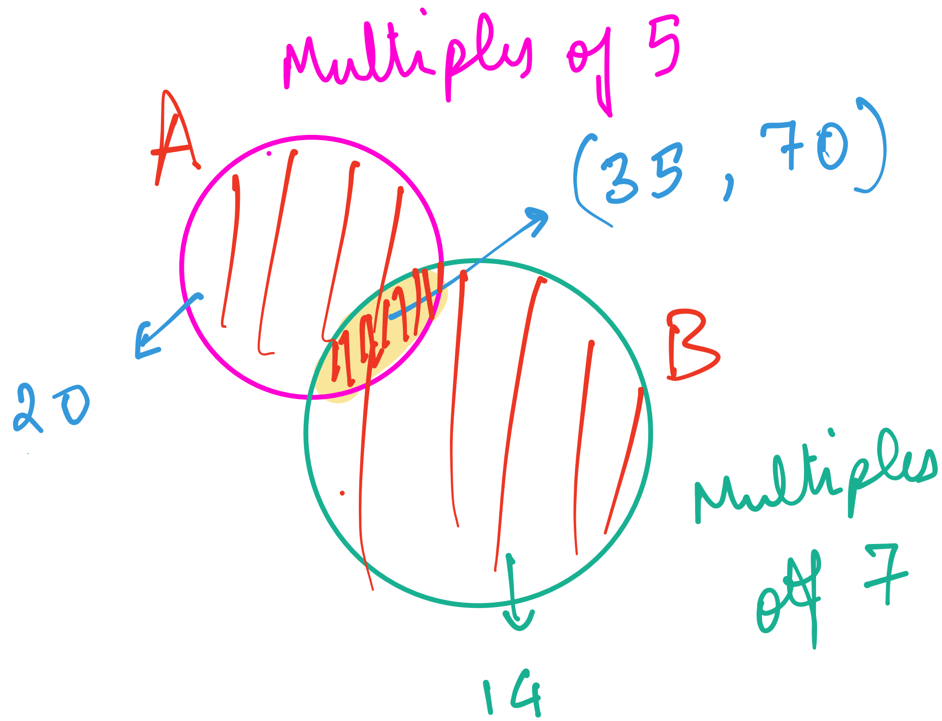
Customers who bought something.

$$B = (L \cap B) \cup (M \cap B) \cup (S \cap B)$$

100

1 – 100.

5 or 7.



$14 + 20 - 2 =$

A and B $\rightarrow$ $A \cap B$.

A or B $\rightarrow$ $A \cup B$.