GMM

→ EM algorithm (Expectation Maximation)

Kmeans

Assign the point to cluster having smallest dist

↳ Hard clustering

→ Normal mean

EM

→ In case of EM, we find the probabity of a point belong to each of cluster

↳ Soft clustering

→ Calculate weighted mean

K-means

→ Randomly select K centroid μ_K

→ Repeat

→ Assigned point to closest cluster

→ Update centroid μ_K

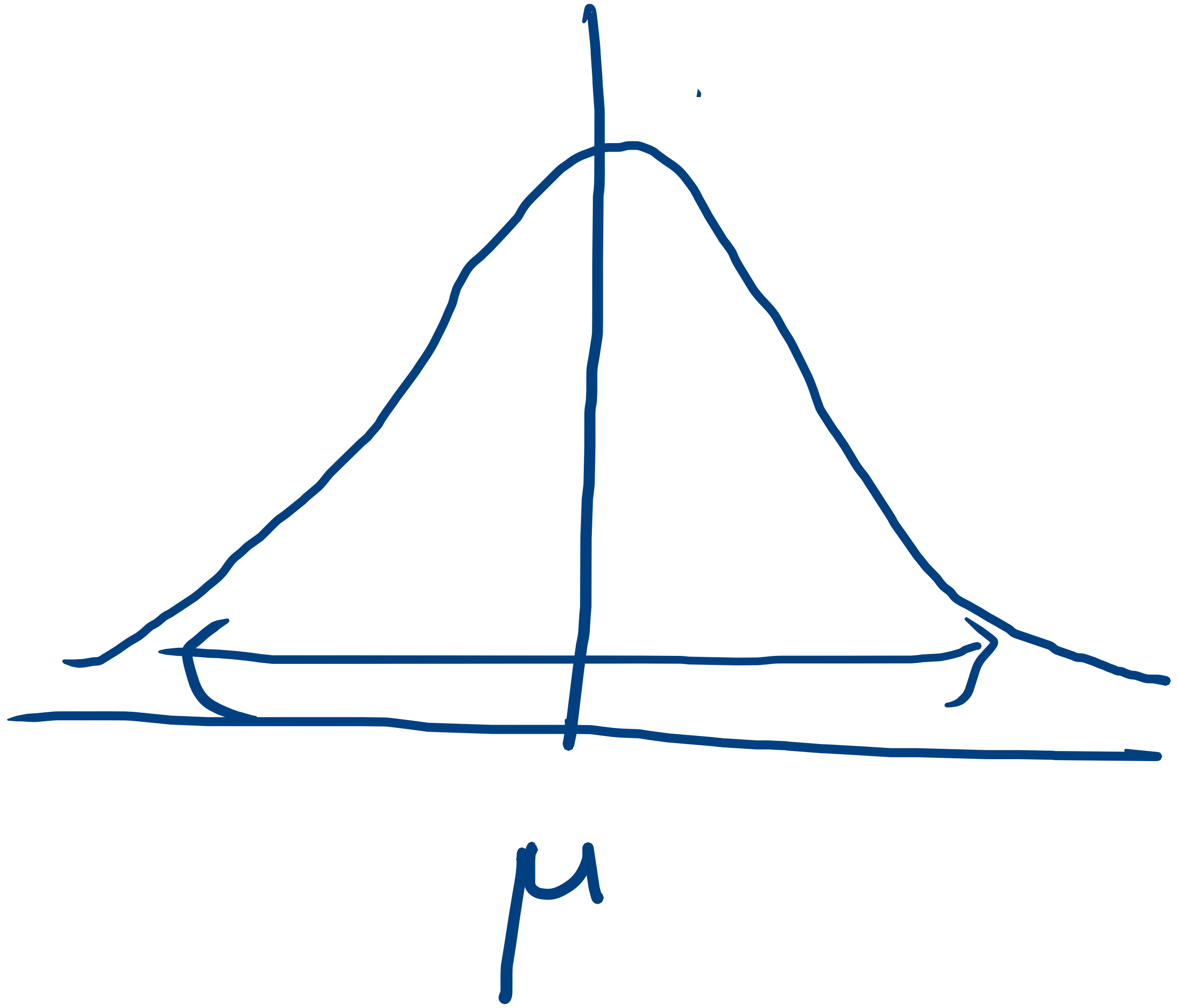
EM

→ Randomly place k Gaussian (μ_K, σ_K)

→ Repeat

E-step → Find the probability that a x_i came from cluster c_K

M-step → Update μ_K, σ_K, $P(K)$

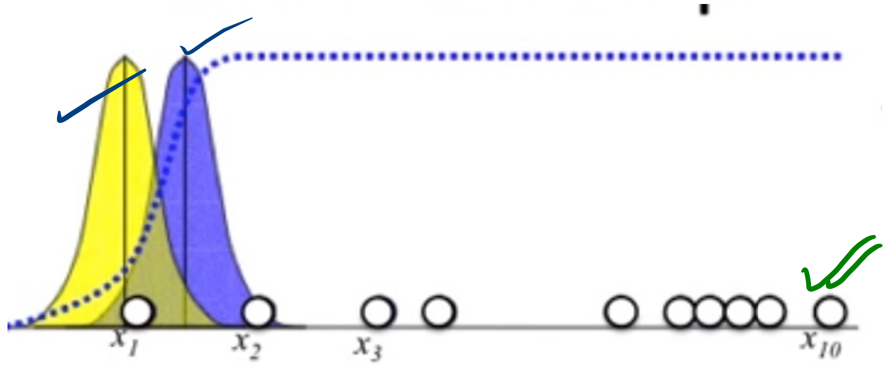


$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$\sigma^2 =$ Variance

μ

EM algo using 1D



y	0.8	0.3
B	0.2	0.7

$$P(x_i/y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(x_i-\mu_y)^2}{2\sigma_y^2}\right)$$

$$P(x_i/b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left(-\frac{(x_i-\mu_b)^2}{2\sigma_b^2}\right)$$

→ Given a point x_i, Prob that it belong to blue cluster

$$b_i = P(b/x_i) = \frac{P(x_i/b)\ P(b)}{P(x_i/b)\ P(b) + P(x_i/y)\ P(y)}$$

Posterior

Prior

$$y_i = P(y/x_i) = 1 - b_i$$

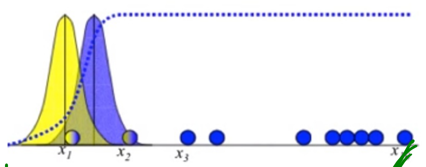
$P(b)$
$P(y) = 0.5$

EM: 1-d example

$$P(x_i \mid b) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp\left\{-\frac{(x_i-\mu_b)^2}{2\sigma_b^2}\right\}$$

$$b_i = P(b \mid x_i) = \frac{P(x_i \mid b)P(b)}{P(x_i \mid b)P(b) + P(x_i \mid a)P(a)}$$

$$a_i = P(a \mid x_i) = 1 - b_i$$

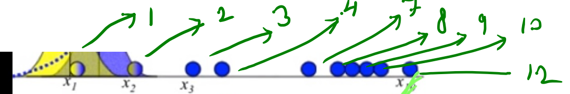


y_i: .080 0.60 0.02 0.01 0
B_i: .20 0.40 0.98 0.99 1

$$b_i = \frac{P(x_i/b)\,P(b)}{P(x_i/b)\,P(b) + P(x_i/y)P(y)} = \frac{P(x_i/b)}{P(x_i/b) + P(x_i/y)} = \frac{10^{-3}}{10^{-3}+10^{-4}} = 1$$

$$y_i = 1 - b_i = 0$$

$$\mu_b = \frac{\sum b_i x_i}{\sum b_i}$$



$$\mu_b = \frac{0.8 \times 1 + 0.6 \times 2 + 0.02 \times 3 + 0.01 \times 4}{0.8 + 0.6 + 0.02 + 0.01 + 0 \ldots 0}$$

$$\sigma_b^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

$P(b/x_i)$

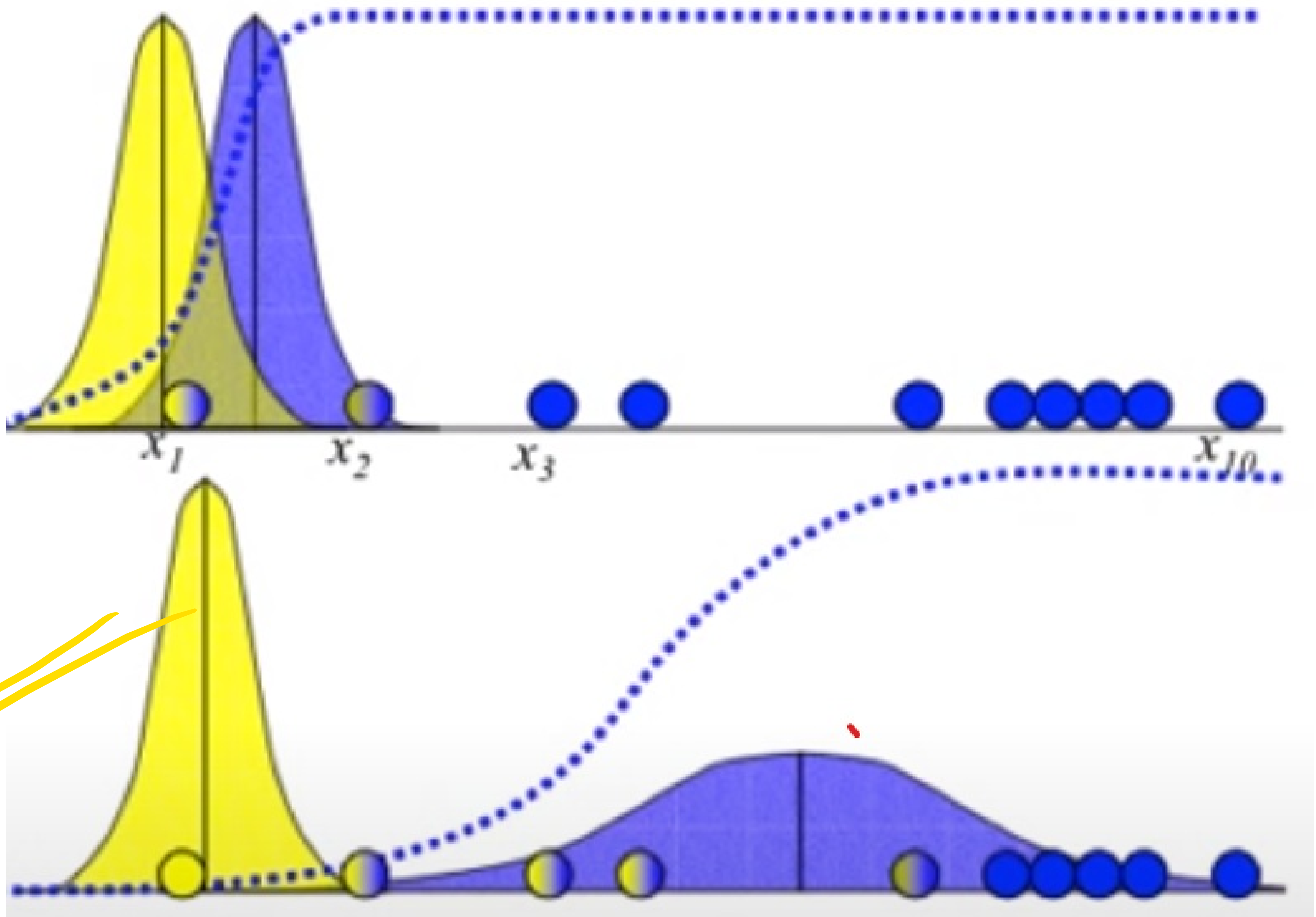
$$= \frac{\sum b_i (x_i - \mu_b)^2}{\sum b_i} = \frac{b_1(x_1-\mu_b)^2 + b_2(x_2-\mu_b)^2 \ldots}{b_1 + b_2 \ldots b_n}$$

$P(y/x_i)$

$$\mu_y = \frac{\sum y_i x_i}{\sum y_i} \qquad \sigma_y^2 = \frac{\sum y_i (x_i - \mu_y)^2}{\sum y_i}$$

$$P(b) = \frac{b_1 + b_2 \ldots b_n}{n} = \frac{\sum b_i}{n}$$

$$P(y) = 1 - P(b)$$



μ_y, σ_y

μ_b, σ_b

b_i 0.05 0.2 0.5 0.6 0.95 1 1 1 1

y_i 0.95 0.8 0.5 0.4 0.05 0 0 0 0

$\rightarrow$ Bayes

Random

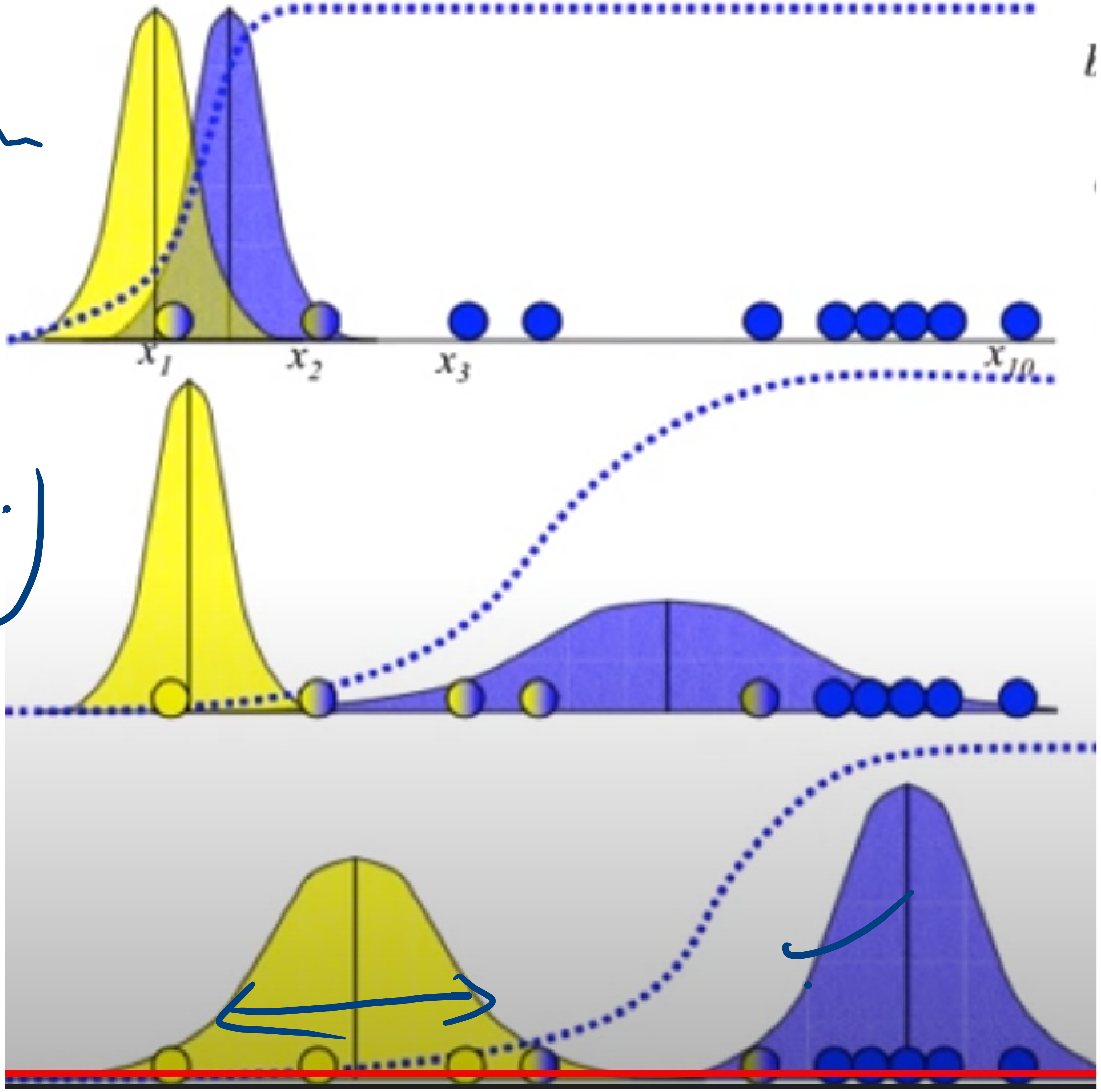
↳ 2 Gaussian

↳ $b_i = P(b/x_i)$

$y_i = P(y/x_i)$

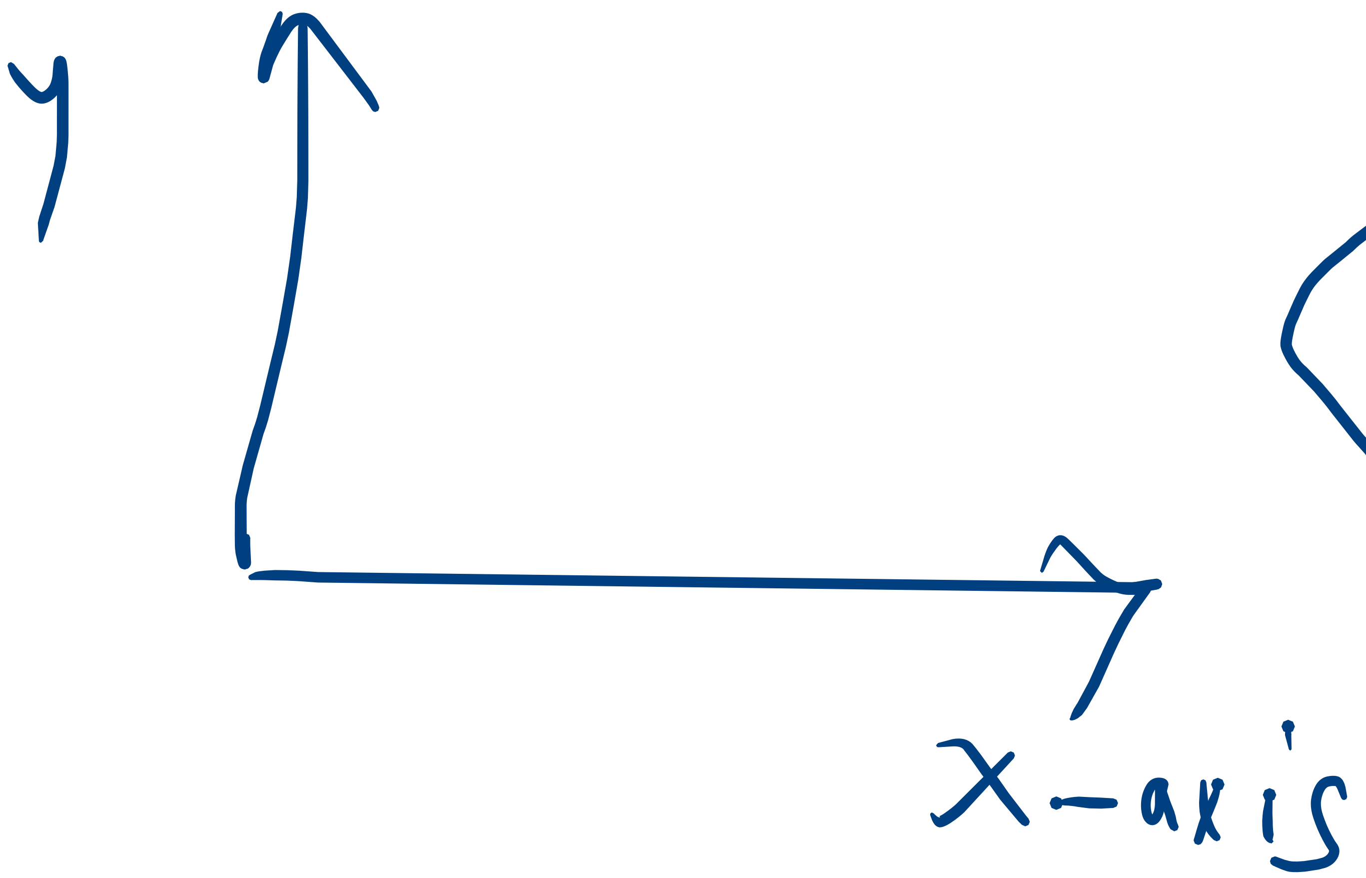
↳ σ_y, σ_b

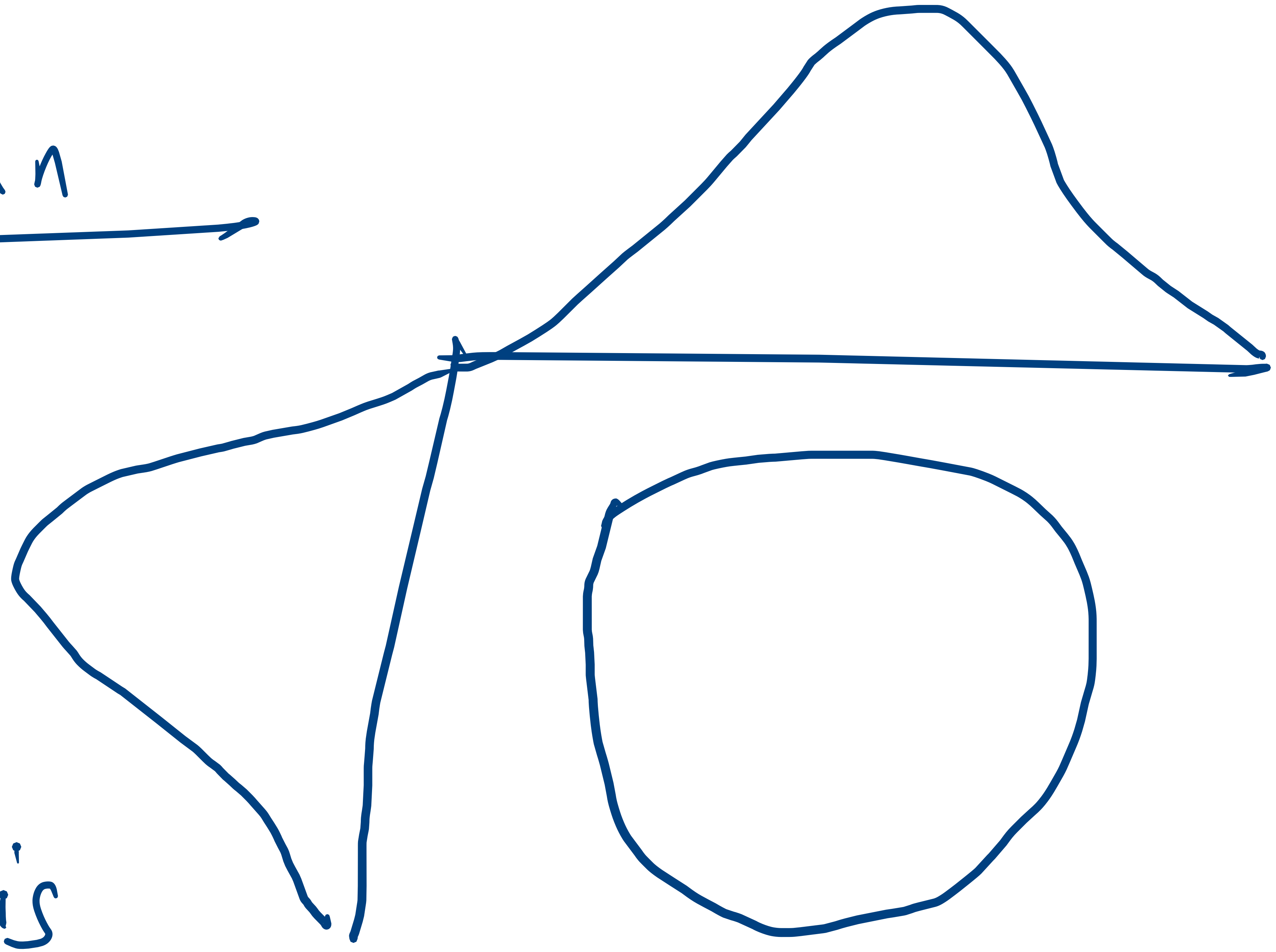
μ_y, μ_y



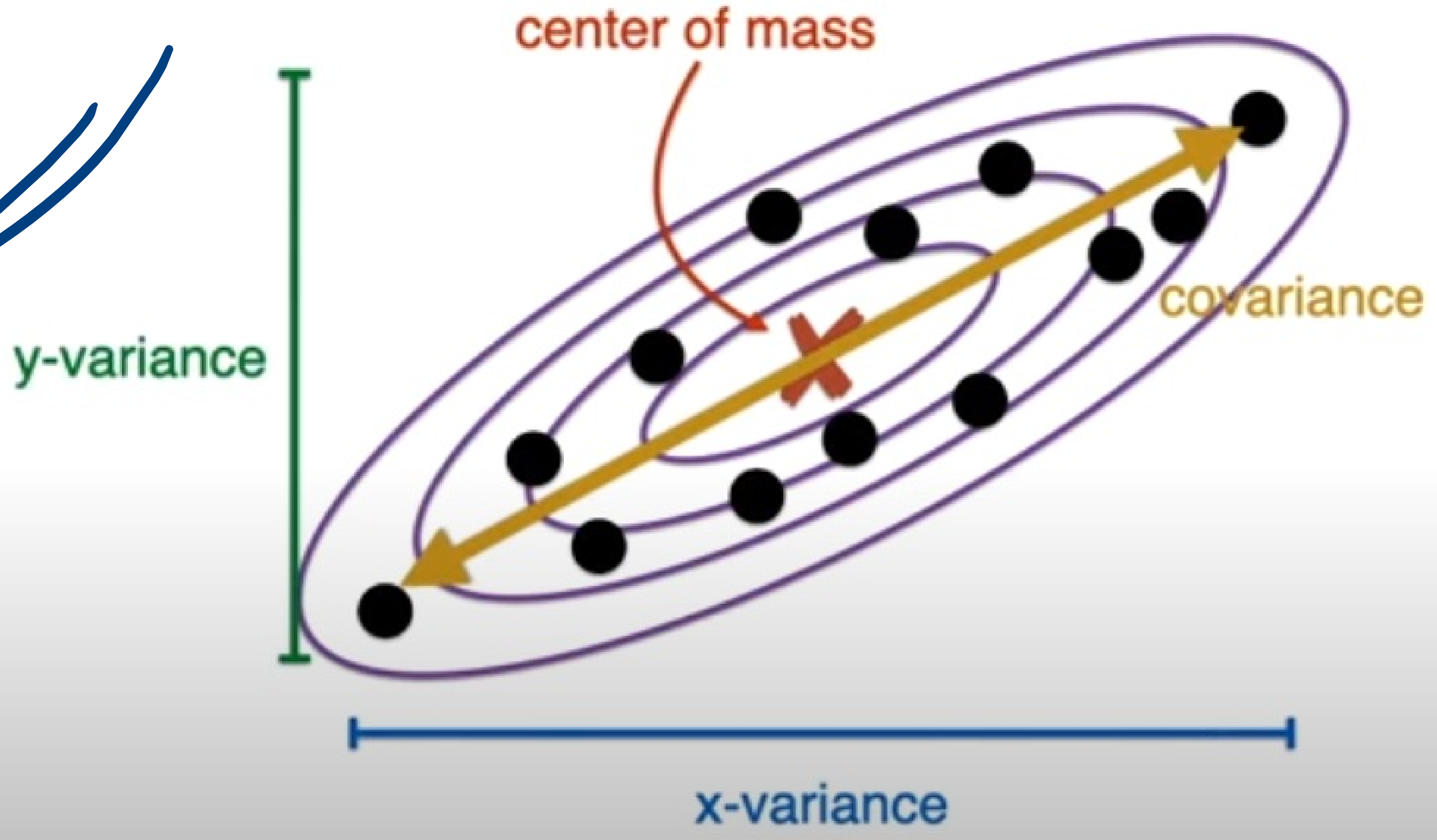
$\mu \quad \sigma^2$

Multivariate Gaussian





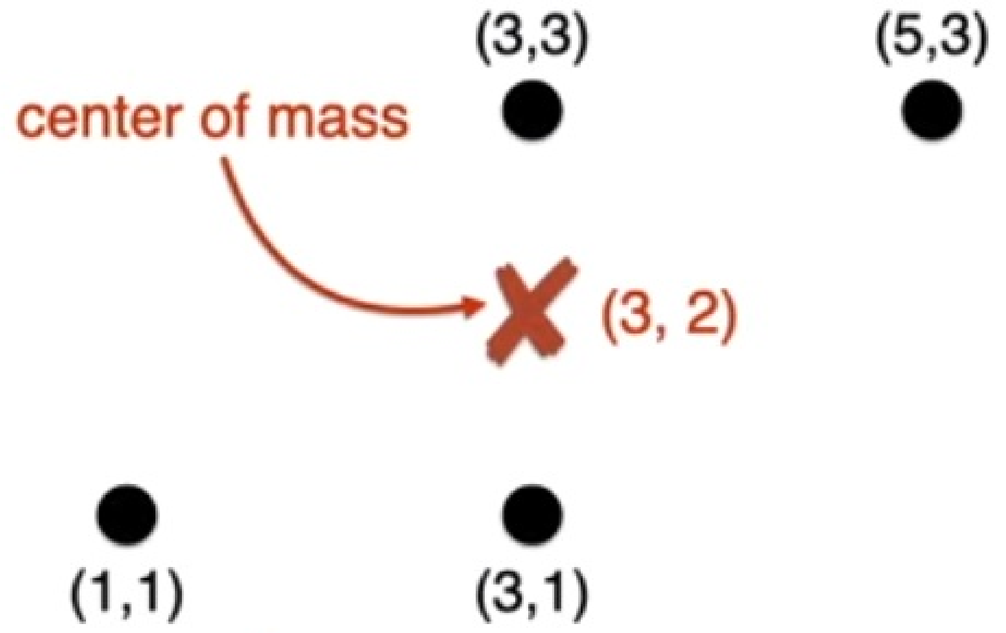
The covariance matrix



$$\mu = Average$$

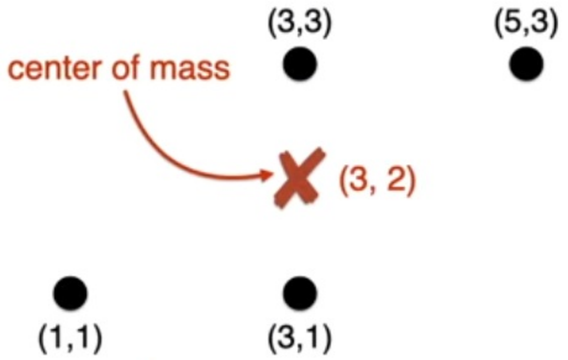
$$\Sigma = \begin{pmatrix} Var(x) & Cov(x,y) \\ Cov(x,y) & Var(y) \end{pmatrix}$$

Average



$$\left(\frac{1+3+3+5}{4}, \frac{1+1+3+3}{4}\right) = (3,2)$$

Average



$$\left(\frac{1+3+3+5}{4}, \frac{1+1+3+3}{4}\right) = (3,2)$$

Calculate $Var(x)$ and $Var(y)$

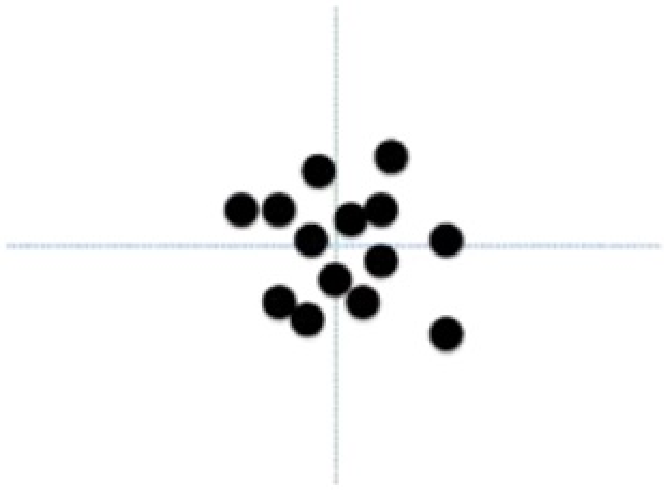
$$\sigma_x^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{1}{4}\left((1-3)^2 + (3-3)^2 + (5-3)^2 + (3-3)^2\right)$$

$$= \frac{1}{4}\left(2^2 + 0 + 2^2 + 0\right) = 2$$

$$\sigma_y^2 = \frac{1}{n}\sum (y_i - \bar{y})^2$$

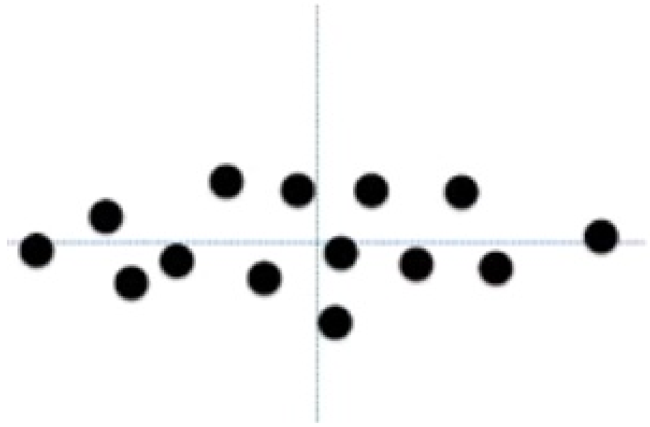
$$= \frac{1}{4}\left[(-1)^2 + 1^2 + 1^2 + (-1)^2\right] = 1$$

A) X_var Y_var → Smal

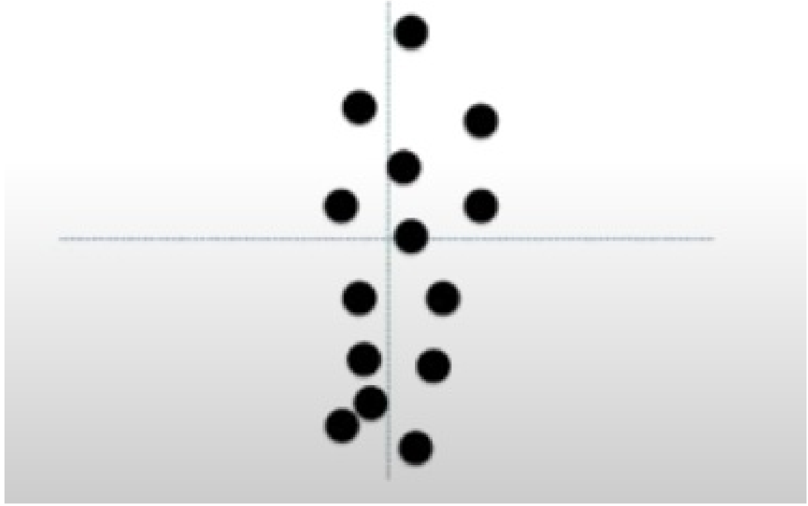


B) X_var → large
Y_var → small

X-variance
↳ Large or small

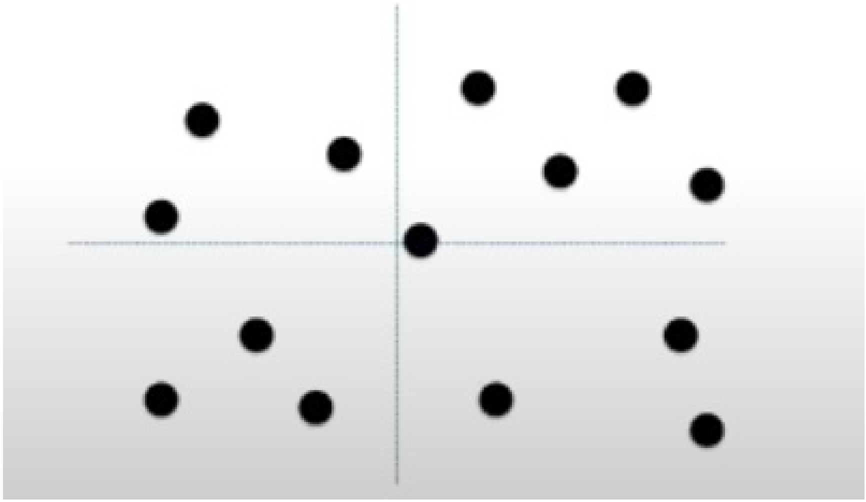


C) Y_var → large
X_var → smal



D) X_var
Y_var → large

Y-variance
↳ Large or small

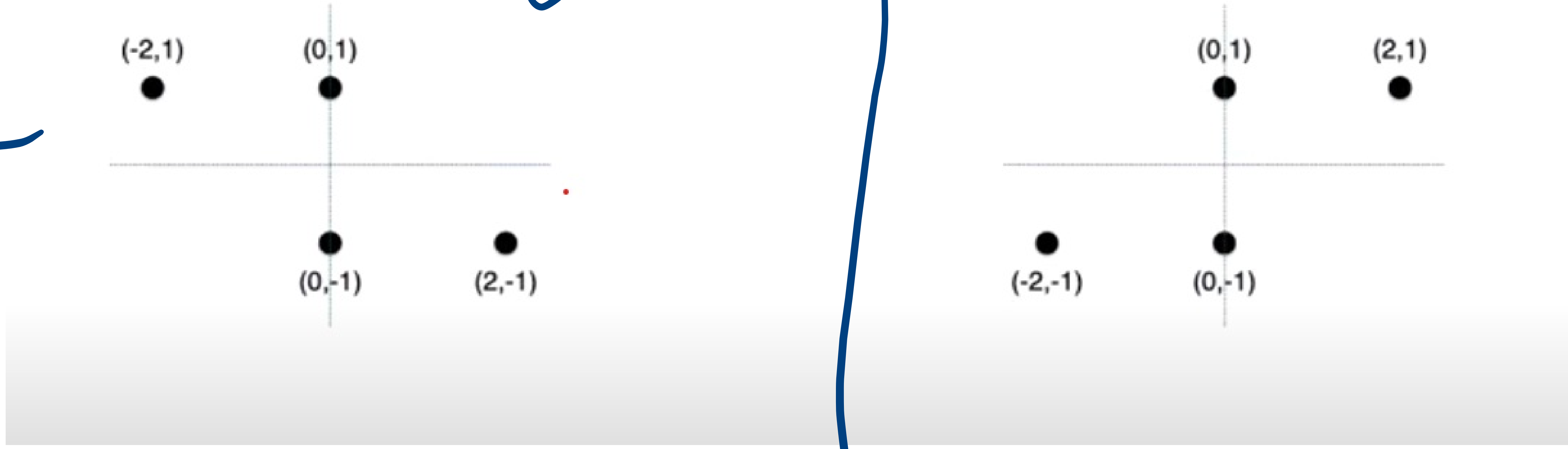


$$\mathrm{Var}_x = \frac{1}{4}\left((-2)^2 + 0^2 + 0^2 + 2^2\right) = 2$$

$$\mathrm{Var}_y = \frac{1}{4}\left(1^2 + 1^2 + (-1)^2 + (-1)^2\right) = 1$$

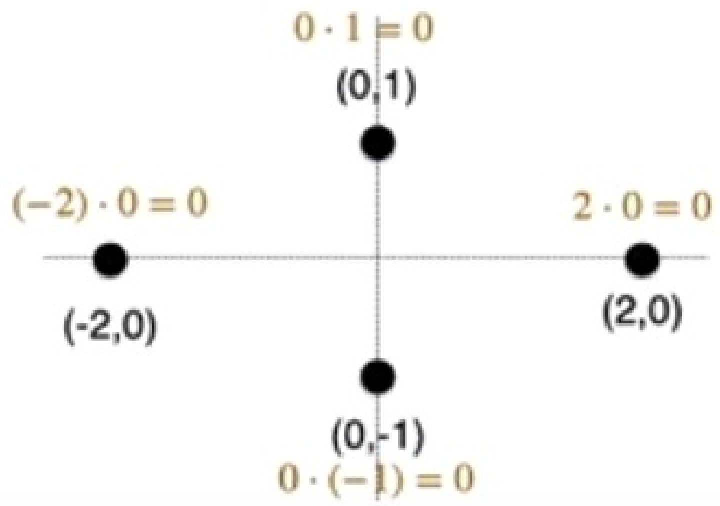
$$\mathrm{Var}_x = 2$$

$$\mathrm{Var}_y = 1$$



→ How can we distinguish b/w these?

$$\frac{1}{4}\left(\sum (x_i - \mu_x)(y_i - \mu_y)\right)$$

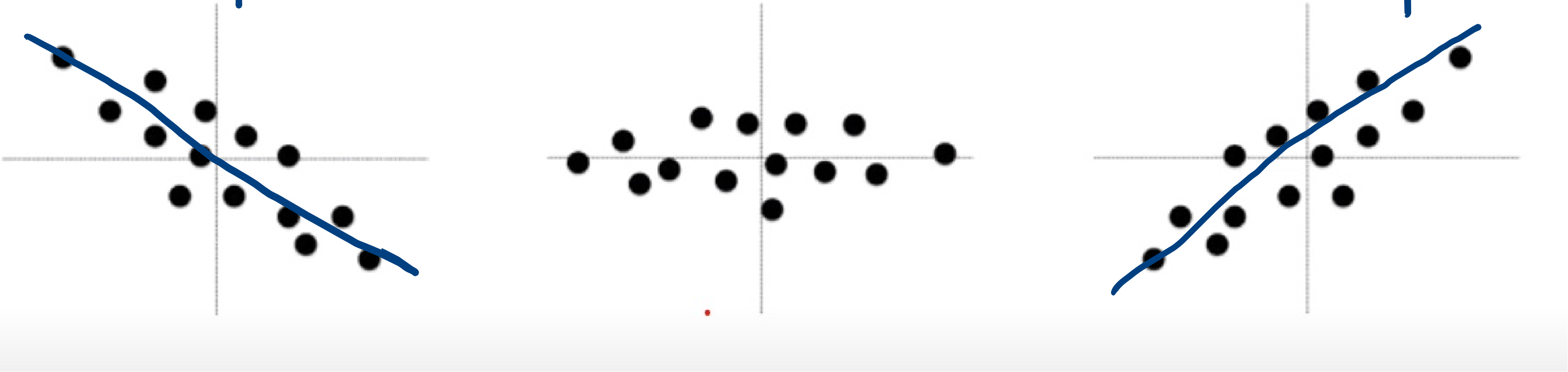


$$cov(x, y) = \frac{1}{4}(0 + 0 + 0 + 0)$$

$$cov(x, y) = 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a = b \qquad \begin{matrix} a > b \\ b \not> a \end{matrix} \rightarrow \text{Elliptical}$$

$\rho = -ve$ $\qquad\qquad\qquad\qquad$ $\rho = +ve$

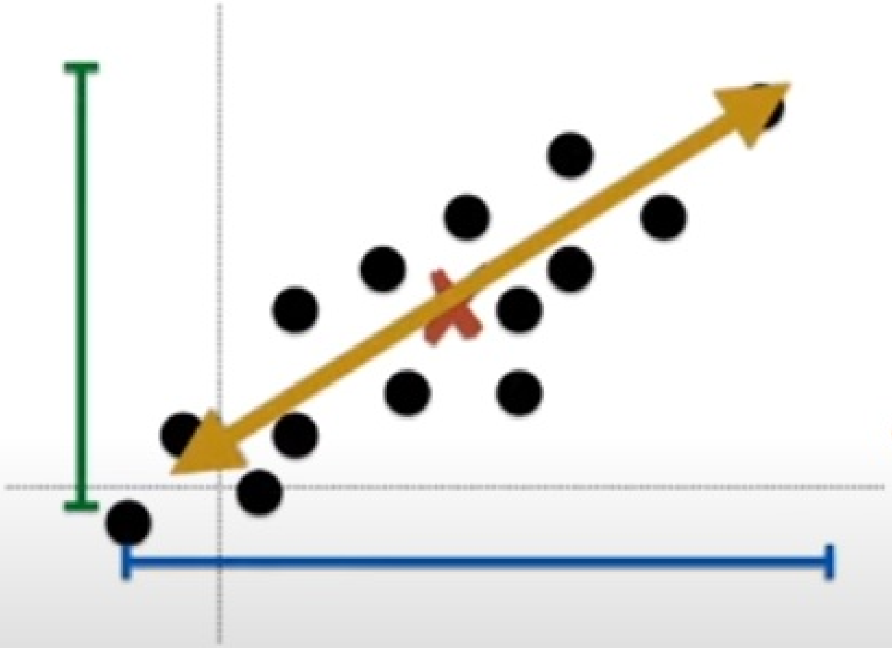


$$\Sigma = \begin{bmatrix} Var(x) & Cov(x,y) \\ Cov(x,y) & Var(y) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\boxed{\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}}$$

Formulas



Points

(x_1, y_1)
(x_2, y_2)
$\vdots$
(x_n, y_n)

Mean

$$(\mu_x, \mu_y) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i, \frac{1}{n}\sum_{i=1}^{n} y_i\right)$$

x-Variance

$$var(x) = \frac{1}{n}\sum_{i=1}^{n} (x_i - \mu_x)^2$$

y-Variance

$$var(y) = \frac{1}{n}\sum_{i=1}^{n} (y_i - \mu_y)^2$$

Covariance

$$cov(x, y) = \frac{1}{n}\sum_{i=1}^{n} (x_i - \mu_x)(y_i - \mu_y)$$

Covariance matrix

$$\Sigma = \begin{pmatrix} var(x) & cov(x, y) \\ cov(x, y) & var(y) \end{pmatrix}$$

Multivariate Gaussian

$$P(x_i/b) = \frac{1}{\sqrt{2\pi\,|\Sigma|}} \exp\left\{ -\frac{1}{2} (x_i - \mu)^T \Sigma^{-1} (x_i - \mu) \right\}$$

$b_i =$ ~~

Scalar (float)

$$\begin{bmatrix} x_i^1 \\ x_i^2 \end{bmatrix} \quad \begin{bmatrix} \mu^1 \\ \mu^2 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\mu_b = \frac{\sum b_i x_i}{\sum b_i}$$

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 \end{bmatrix}$$

n

Break until 10:25pm

→ K means can only form spherical

What can be shape of cluster in EM?

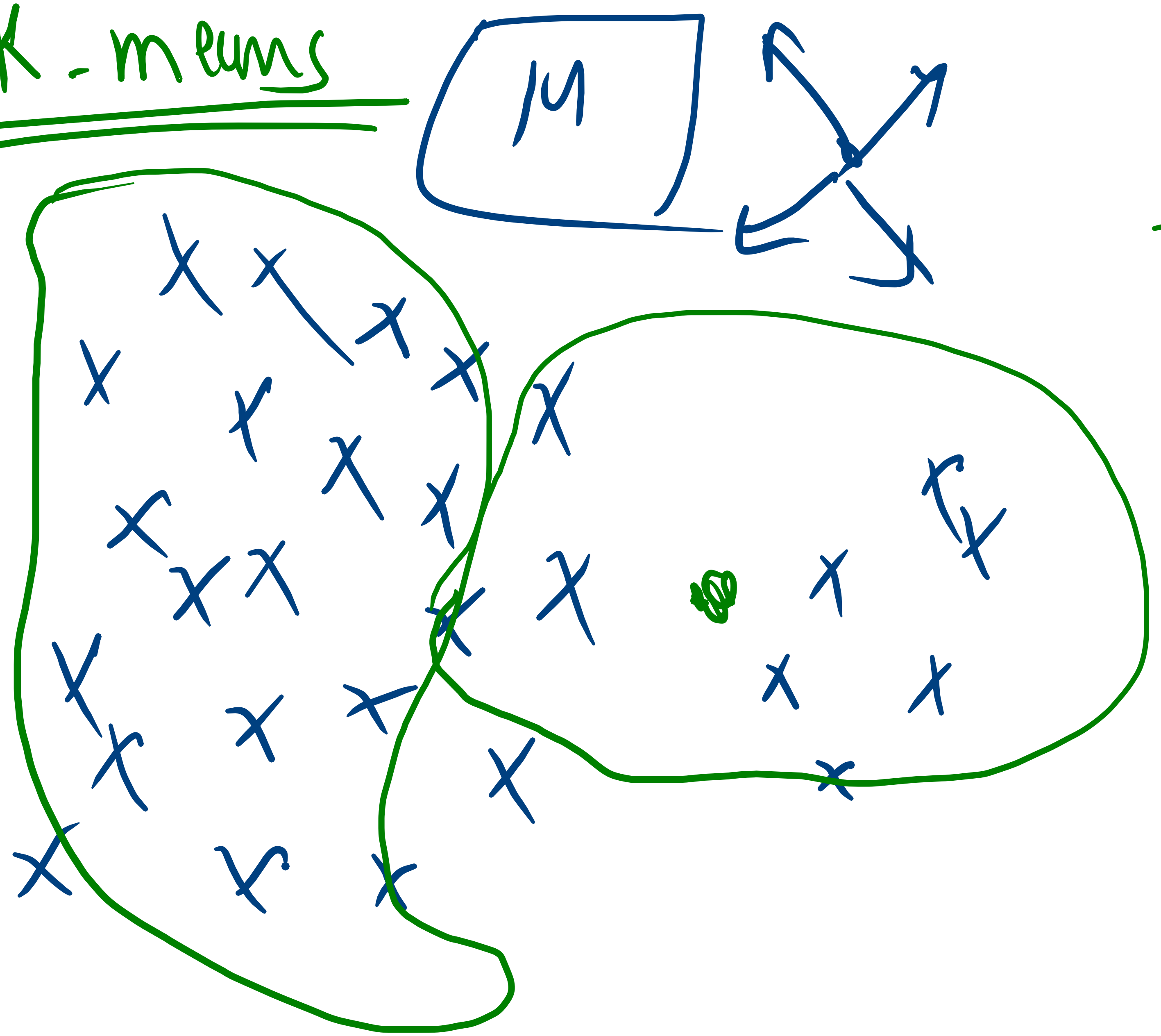
↳ Can form non-spherical

→ Similarity w.r.t K-means

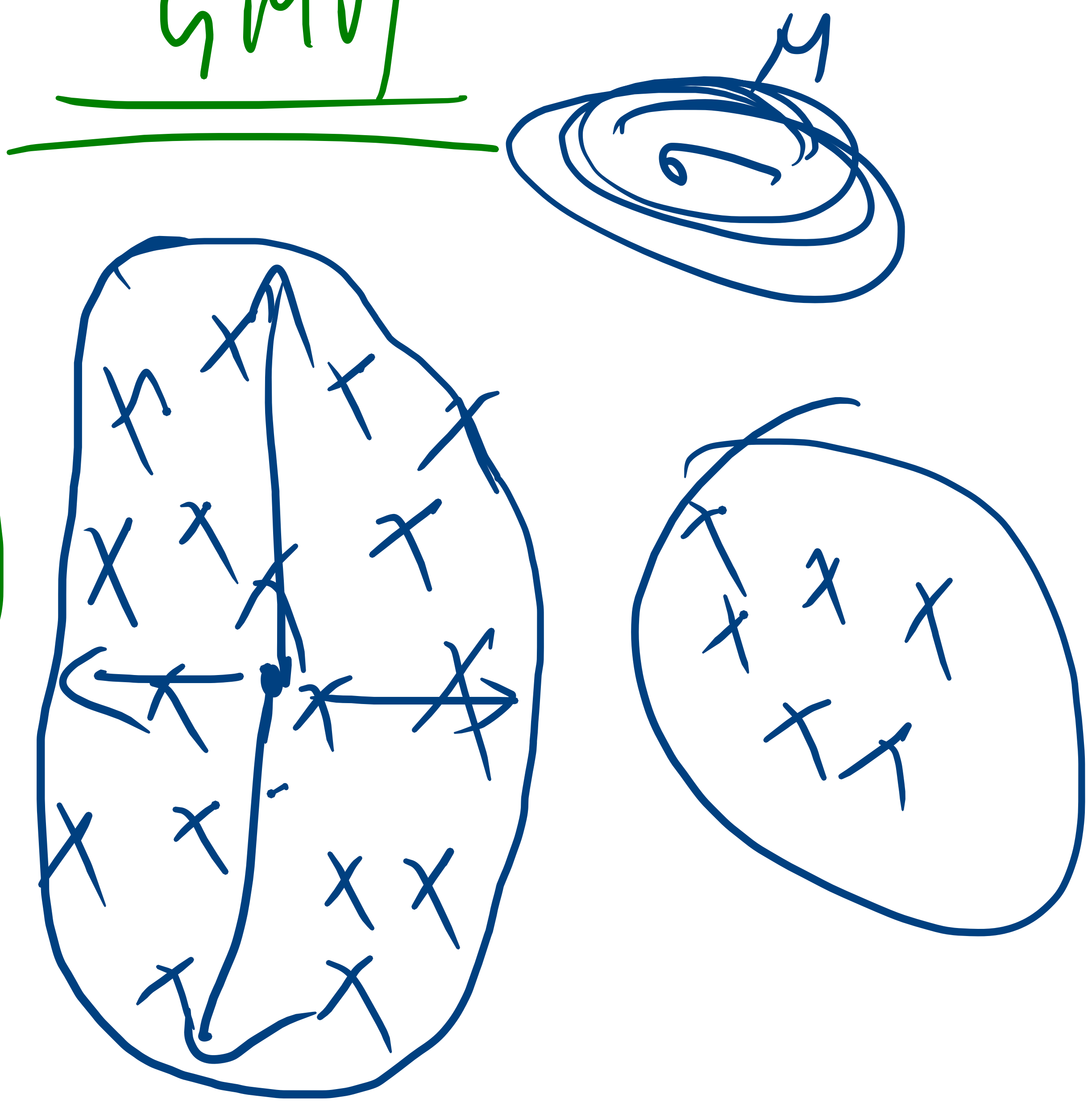
- No of cluster K
- Sensitive to initialization
- Local minimum

K-means	EM
→ Hard clustering ↳ A point belong to only 1 cluster. Assign a point to a cluster	Soft clustering ↳ A point can belong to multiple cluster Find the probab. of point to each cluster
→ Spherical	→ Non-spherical
→ Mean or Centroid	→ Mean, Variance, Priors
→ Normal mean	→ Weighted mean

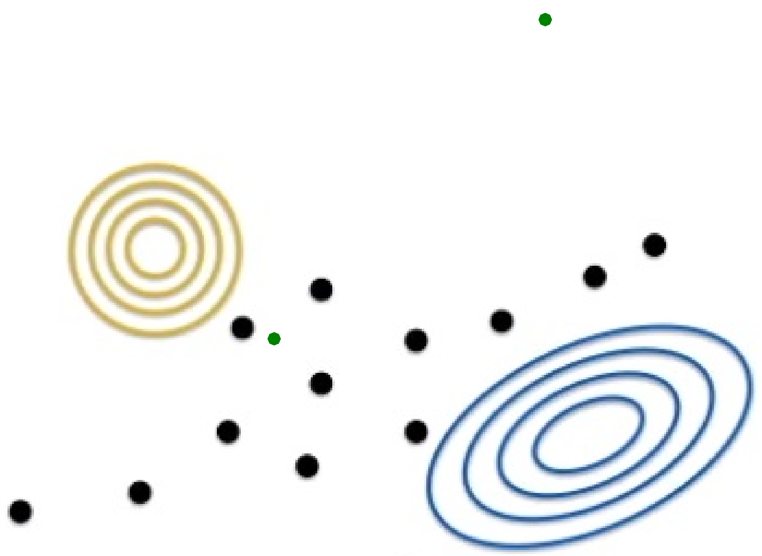
K-means



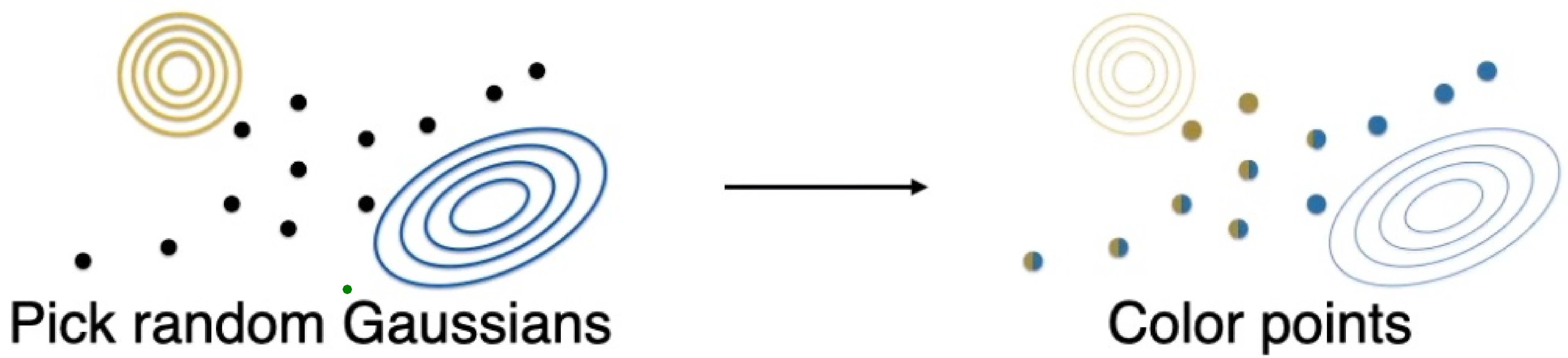
GMM

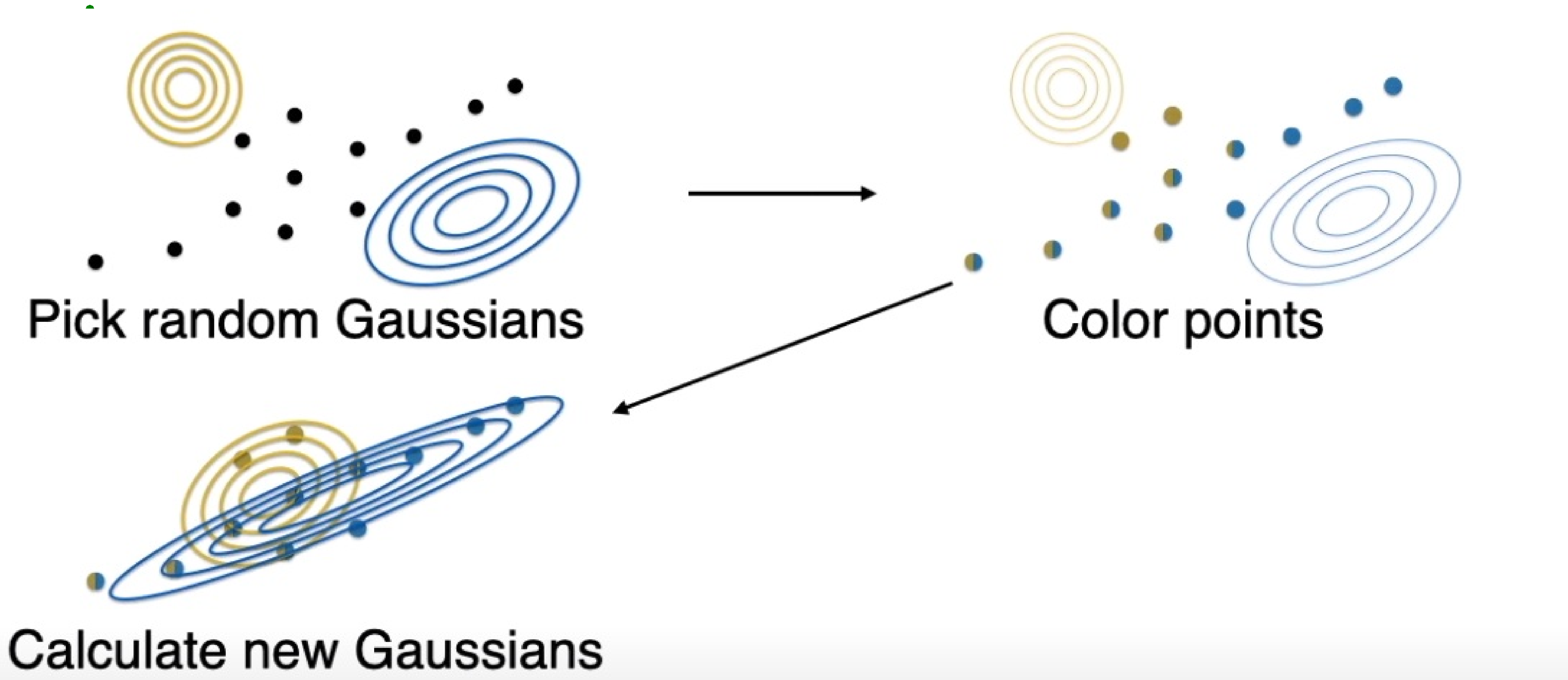


$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

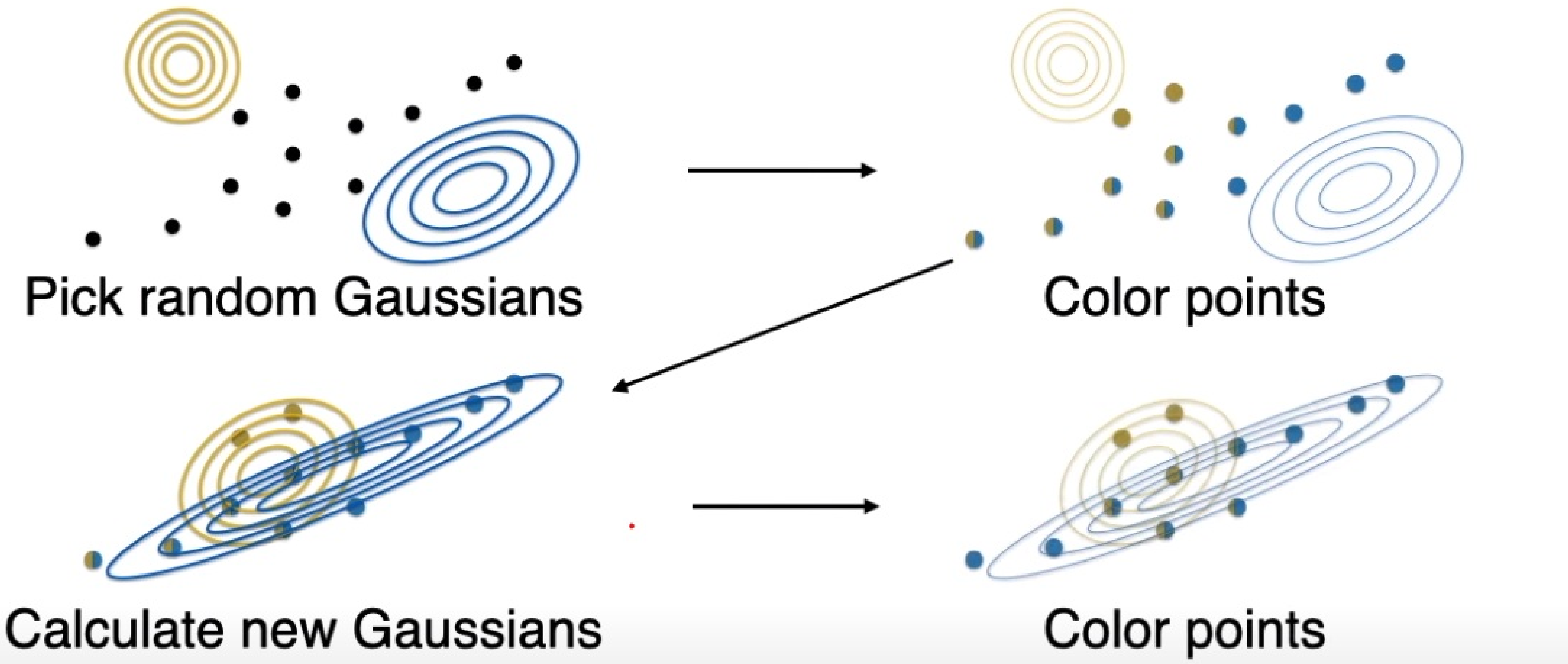


Pick random Gaussians

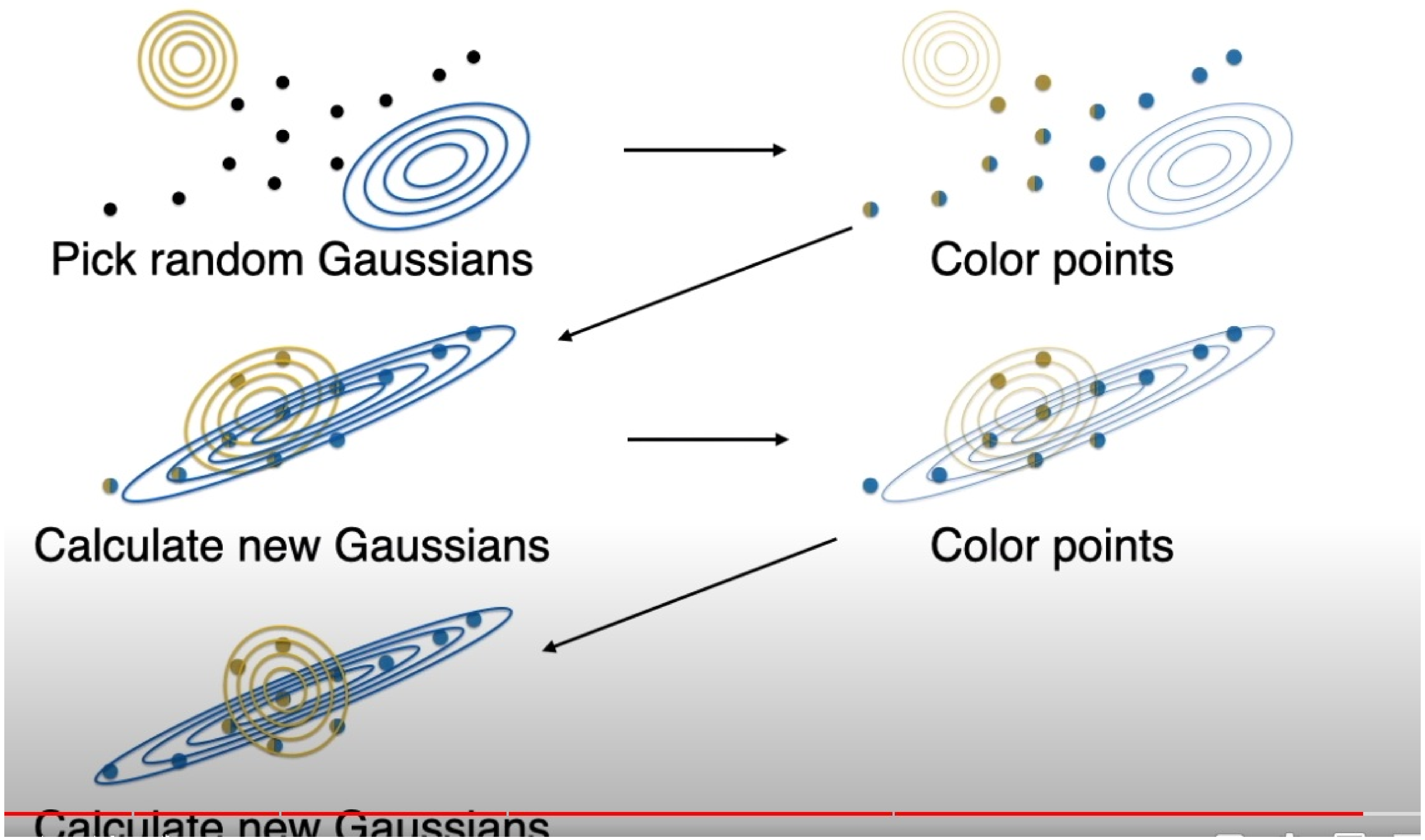

Pick random Gaussians
Color points

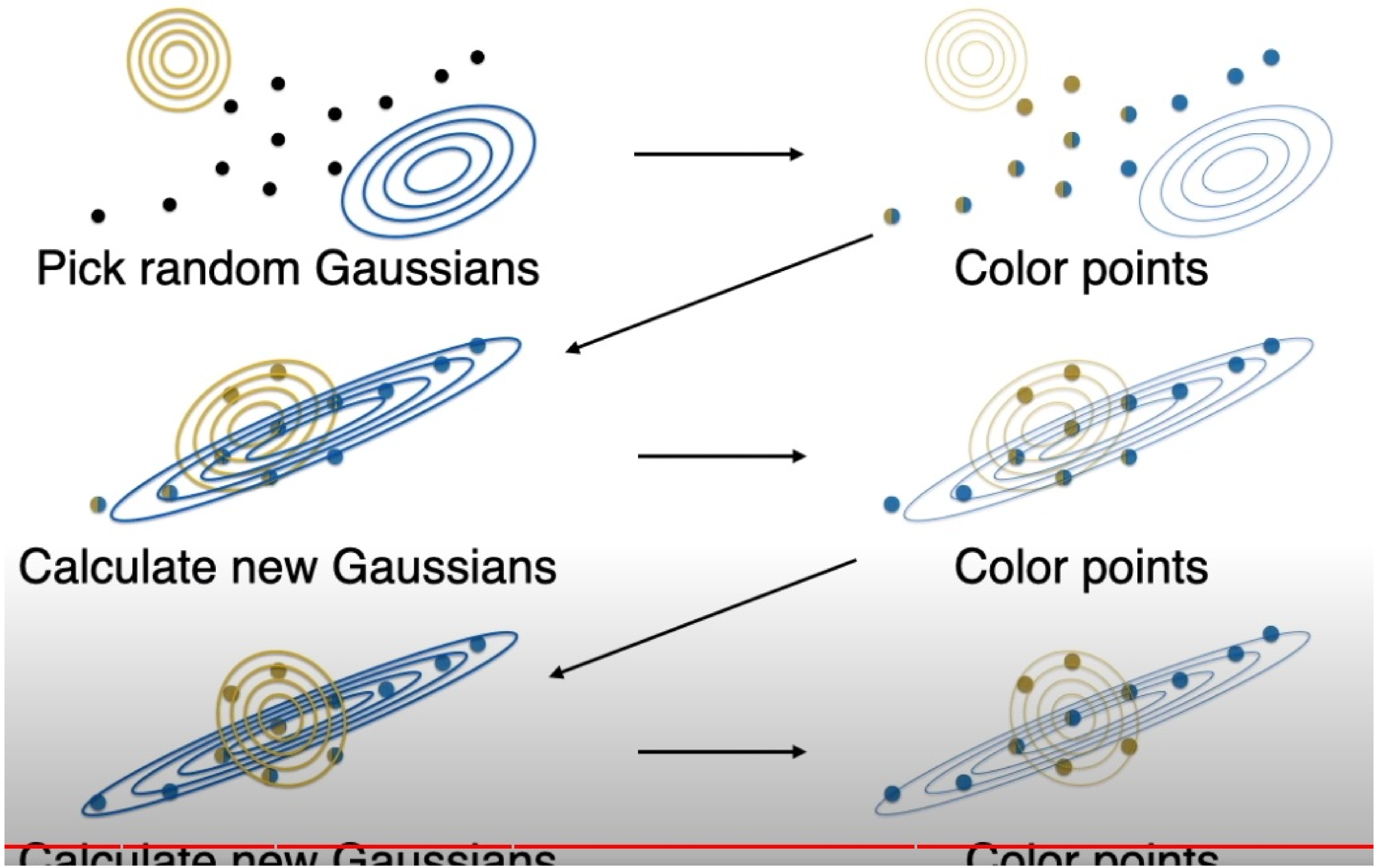


Pick random Gaussians
Color points
Calculate new Gaussians



Pick random Gaussians
Color points
Calculate new Gaussians
Color points


Pick random Gaussians
Color points
Calculate new Gaussians
Color points
Calculate new Gaussians


Pick random Gaussians
Color points
Calculate new Gaussians
Color points
Calculate new Gaussians
Color points