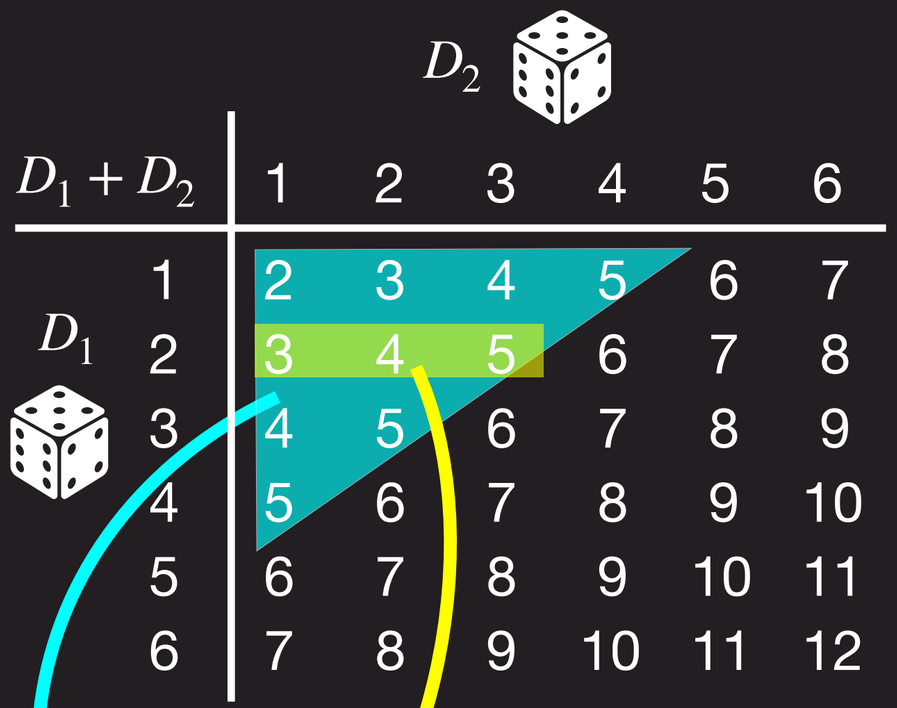

$$P\left[D_1 = 2\right] = \frac{6}{36}$$

$$P\left[\ D_1 + D_2 \le 5\ \right] = \frac{10}{36}$$

$$P\left[\ (D_1 = 2) \bigcap (D_1 + D_2 \le 5)\ \right] = \frac{3}{36}$$

$$P\left[\ D_1 = 2 \ \middle|\ D_1 + D_2 \le 5\ \right] = \frac{3}{10}$$

$$P\left[\ D_1 = 2 \ \middle|\ D_1 + D_2 \le 5\ \right] = \frac{P\left[\ (D_1 = 2) \bigcap (D_1 + D_2 \le 5)\ \right]}{P\left[\ D_1 + D_2 \le 5\ \right]} = \frac{3/36}{10/36} = \frac{3}{10}$$

$$\frac{3}{10}$$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \qquad P[A \cap B] = P[A \mid B]\ P[B]$$

Conditional Probability Multiplication Rule

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Century	Win: False	Win: True	
False	160	154	314
True	16	30	46
	176	184	360

$$P[W] = \frac{184}{360} \qquad P[C] = \frac{46}{360} \qquad P[W \cap C] = \frac{30}{360}$$

$$P[W \mid C] = \frac{30}{46}$$

$$P[C \mid W] = \frac{30}{184}$$

$$P[W \mid C] = \frac{P[W \cap C]}{P[C]} = \frac{30/360}{46/360} = \frac{30}{46}$$

$$P[C \mid W] = \frac{P[W \cap C]}{P[W]} = \frac{30/360}{184/360} = \frac{30}{184}$$

$$P[W \mid C] = \frac{P[W \cap C]}{P[C]}$$

$$P[C \mid W] = \frac{P[W \cap C]}{P[W]}$$

$$P[W \cap C] = P[W \mid C]\ P[C] = \frac{30}{46}\frac{46}{360} = \frac{30}{360}$$

$$P[W \cap C] = P[C \mid W]\ P[W] = \frac{30}{184}\frac{184}{360} = \frac{30}{360}$$

$$P[W \mid C]\ P[C] = P[C \mid W]\ P[W]$$

$$P[W \mid C] = \frac{P[C \mid W]\ P[W]}{P[C]}$$

$$P[B \mid A] = \frac{P[A \mid B]\ P[B]}{P[A]}$$

Bayes Theorem

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Among 30 faculty members in a department, 5 are females and 25 are males. 3 females and 12 males have a PhD

F M M M M M
F M M M M M
F M M M M M
F M M M M M
F M M M M M

$$P[F] = \frac{5}{30} \quad P[M] = \frac{25}{30} \quad P\left[F \bigcap \text{phd}\right] = \frac{3}{30} \quad P\left[M \bigcap \text{phd}\right] = \frac{12}{30} \quad P[\text{phd}] = \frac{15}{30}$$

Among those who have done PhD, what fraction are female?

$$P\left[F \mid \text{phd}\right] = \frac{3}{15} = \frac{3}{3+12}$$

$$P\left[F \mid \text{phd}\right] = \frac{P\left[\text{phd} \mid F\right] P[F]}{P[\text{phd}]} = \frac{\blacksquare}{\blacksquare + \blacksquare}$$

$$\blacksquare \longrightarrow P\left[\text{phd} \mid F\right] P[F] \longrightarrow P\left[F \bigcap \text{phd}\right]$$

$$\blacksquare \longrightarrow P\left[\text{phd} \mid M\right] P[M] \longrightarrow P\left[M \bigcap \text{phd}\right]$$

$$P[\text{phd}] = P\left[\text{phd} \mid F\right] P[F] + P\left[\text{phd} \mid M\right] P[M]$$

$$\frac{3}{5}\,\frac{5}{30} + \frac{12}{25}\,\frac{25}{30} = \frac{3}{30} + \frac{12}{30} = \frac{15}{30}$$

$$P[\text{phd}] = P\left[F \bigcap \text{phd}\right] + P\left[M \bigcap \text{phd}\right]$$

$$P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

Law of Total probability

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Law of Total probability $P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$

$P[B] = P[B \cap A] + P[B \cap A^c]$