5th December, 2022

DSML : CC Maths

Probability 6 - Distributions - 1

Recap: (a) Probability theory.
(b) Bayes' theorem.
(c) Combinatorics.
(d) Descriptive stats.

Today: (a) numpy 'linear' confusion.
(b) Random variables.
(c) Empirical vs. Theoretical Probability.
(d) Distribution
(e) Expectation.
(f) Binomial distribution.

Class starts @ 9:05 p.m.

Percentile computation: The default method in numpy.

"linear interpolation"

$l = [1, 2, 3, 4, 5]$ length : 5

indices: 0 1 2 3 4

$p = 50.$ P/100.

$0.5 * (5-1) = 0.5 \times 4 = 2.$

$0.25 * (5-1) = 0.25 \times 4 = 1$

$0.75 * (5-1) = 0.75 \times 4 = 3$

$l = [1, 2.5, 3, 4, 5, 6.]$ length : 6

indices: 0 1 2 3 4 5

1.25, 3.5, 4.75

$p = 50, 25$

$0.5 * 5 = 2.5$

$0.25 * 5 = 1.25$

$0.75 * 5 = 3.75$

$2.5 + [(3-2.5) * 0.25]$

$2.5 + [0.5 * 0.25]$

$2.5 + 0.125 = 2.625.$

Agenda:

Casino case study.

① HTH → 2

② TTH → 1

outcome

a var. which cal. # heads

Random variable.

Key terms:

Random variable.

Mapping between outcome & some real number.

Empirical experiments vs. theory.

Empirical: Estimating from data.

Theory: without expts, rule based

* Distribution.

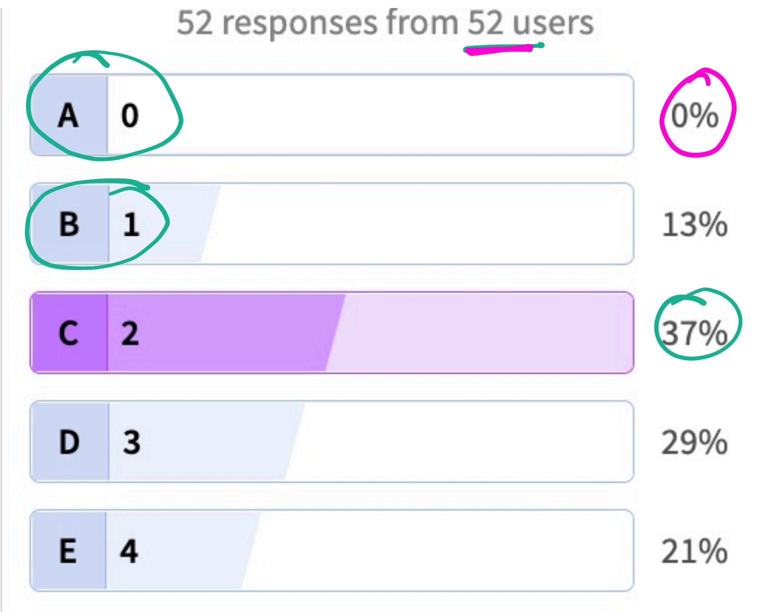
Rules.

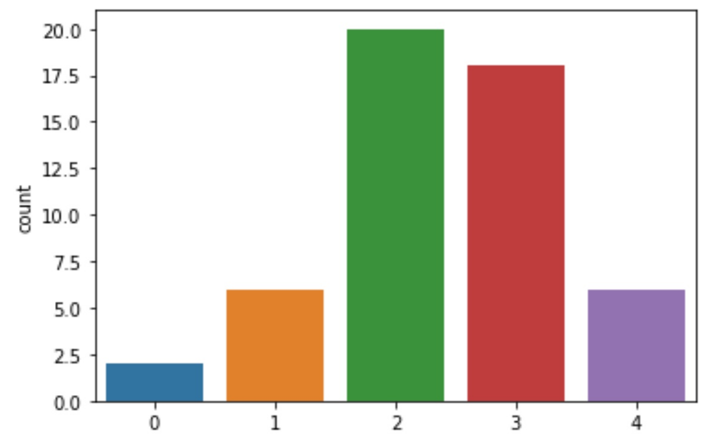
Expectation: weighted mean

↳ probability.

Binomial distribution.

Casino Case Study: A bag has 3 red and 2 blue balls. You pick a ball, see the color and put it back. You do this 4 times. I bet that if you get 4 red balls, I will give you Rs. 150. But for any other outcome, you pay me Rs. 10.





2
6
20
18
6

"X" → denote the number of red balls drawn.

X		Y	Y . P[·]
0	$P[X=0] = 2/52$	−10	$-10 \times 2/52$
1	$P[X=1] = 6/52$	−10	$+ -10 \times 6/52$
2	$P[X=2] = 20/52$	−10	$+ -10 \times 20/52$
3	$P[X=3] = 18/52$	−10	$+ -10 \times 18/52$
4	$P[X=4] = 6/52$	+150	$+ 150 \times 6/52$

E[Y]

Casino Case Study: A bag has 3 red and 2 blue balls. You pick a ball, see the color and put it back. You do this 4 times. I bet that if you get 4 red balls, I will give you Rs. 150. But for any other outcome, you pay me Rs. 10.

$X \rightarrow$ Counts the number of Red balls drawn.

	Count	
0 –	1	$= {}^4C_0$
1 –	4	$= {}^4C_1$
2 –	6	$= {}^4C_2$.
3 –	4	$= {}^4C_3$
4 –	1	$= {}^4C_4$

	X
(1) B B B B –	0
(2) R B B B –	1
(3) B R B B –	1
(4) B B R B –	1
(5) B B B R –	1

$P[R] = \frac{3}{5}$.

$P[B] = \frac{2}{5}$.

$P[BBBB] = \left(\frac{2}{5}\right)^4$

$P[RBBB] = \frac{3}{5} \times \left(\frac{2}{5}\right)^3$

$P[RRBB] = \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2$

$P[RRRB] = \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^1$

$P[RRRR] = \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^0$.

	Count		Prob
0 –	1	$= {}^4C_0$	$(2/5)^4 \cdot (3/5)^0$
1 –	4	$= {}^4C_1$	$(2/5)^3 \cdot (3/5)^1$
2 –	6	$= {}^4C_2$	$(2/5)^2 \cdot (3/5)^2$
3 –	4	$= {}^4C_3$	$(2/5)^1 \cdot (3/5)^3$
4 –	1	$= {}^4C_4$	$(2/5)^0 \cdot (3/5)^4$

RRBR

RRRB

$$P[X=0] = 1 * (2/5)^4 \cdot (3/5)^0$$

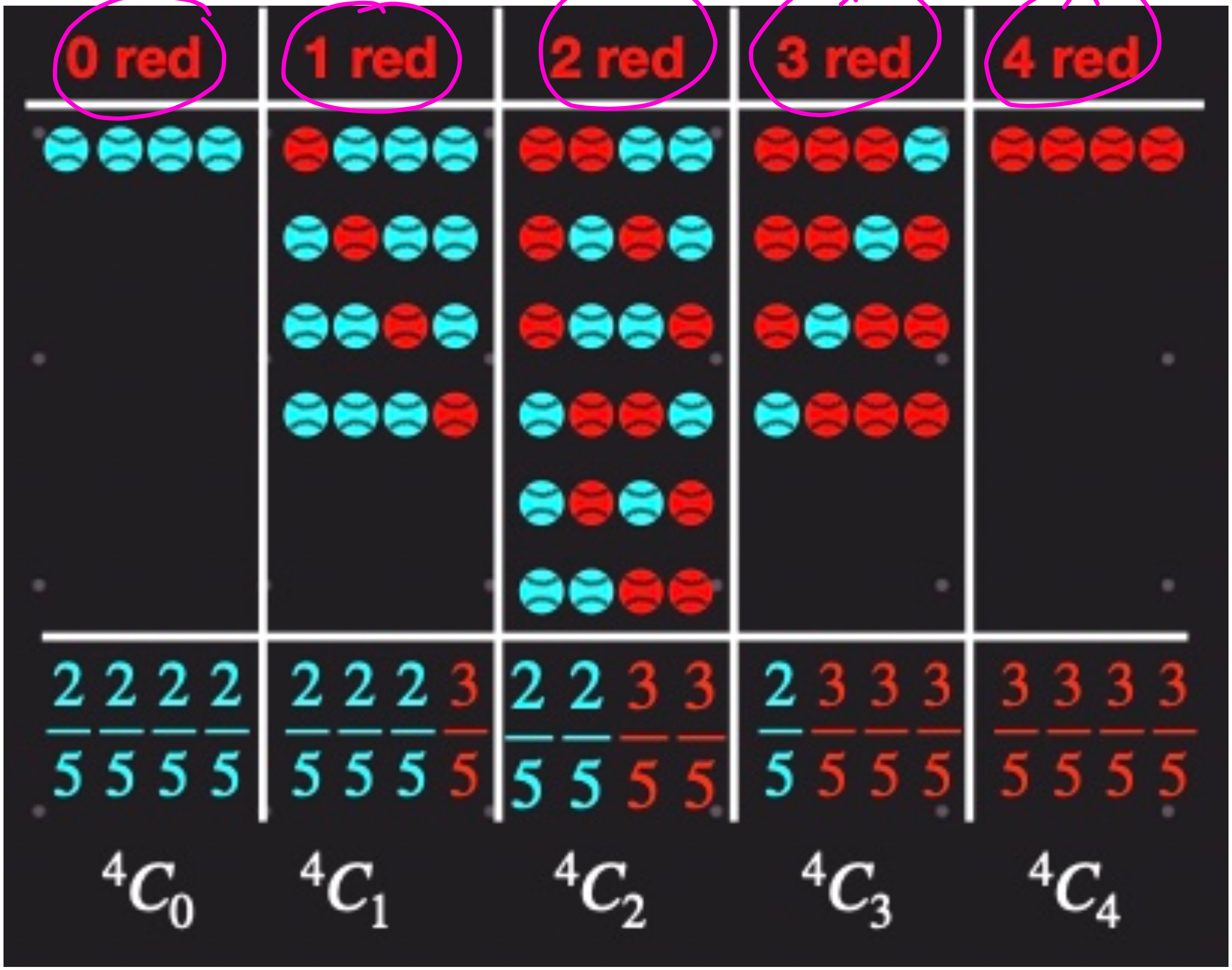
$$P[X=1] = 4 * (2/5)^3 \cdot (3/5)^1$$

$$P[X=2] = 6 * (2/5)^2 \cdot (3/5)^2$$

$$P[X=3] = 4 * (2/5)^1 \cdot (3/5)^3$$

$$P[X=4] = 1 * (2/5)^0 \cdot (3/5)^4$$

-10 $\quad -10$ $\quad -10$ $\quad -10$ $\quad +150.$



$E[Y].$

$P[Y=-10] \times -10$

$+ P[Y=150] \times 150.$

$Y \to$ R.V. that gives the winnings for a given outcome.

$$P[X = k] = {}^{4}C_{k} \cdot \left(2/5\right)^{4-k} \cdot \left(3/5\right)^{k}.$$

Binomial distribution.

$$\boxed{P[X = k] = {}^{n}C_{k}\,(p)^{k} \cdot (1-p)^{n-k}.}$$

casino example

$n \rightarrow$ total number of tries. 4

$k \rightarrow$ number of successes $\rightarrow$ 0, 1, 2, 3, 4

$p \rightarrow$ probability of success. $\rightarrow$ 3/5.

Examples of Binomial R.Vs.

i] Toss a coin 2 times. → 4.

① HH ② HT ③ TH ④ TT.

$p \cdot p$ $\quad p(1-p)$ $\quad (1-p)p$ $\quad (1-p)(1-p)$.

Coin is not fair! $P(H) = p$.

$$P[X=k] = {}^{2}C_{k}\, p^{k} (1-p)^{2-k}.$$

$k = 0, 1, 2$

X → counts the # of heads.

Outcome	Value of X	$P[X=k]$
HH	$k=2$ ✓	p^2
HT	$k=1$	$p(1-p)$
TH	$k=1$	$p(1-p)$
TT	$k=0$	$(1-p)^2$

$$P[X = k] = {}^{n}C_{k}\, p^{k} (1-p)^{n-k}.$$

↓

Binomial

n →

k →

p →

Avi : IND vs. NZ.

↓

✓ $n = 5$

✓ $k =$

✓ p → prob. of india winning.

X : Count of IND victories

↓

Binomial distribution.

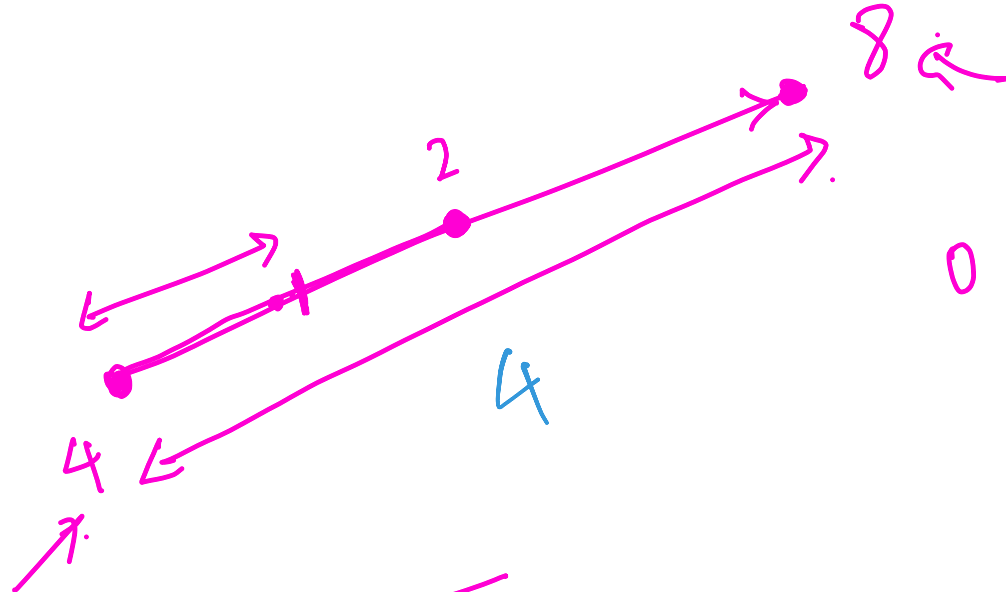
" What is the probability that in a

$n = 5$ series match, India wins

$k = 3$ times, given that India has a

$p = 0.6$ chance of winning?

$$P[X = k] = {}^{5}C_{k} \cdot (0.6)^{k} \cdot (0.4)^{5-k}$$



$0.3 \times 4 = 1.2.$

$(0.3 \times 4) + 4$

$1.2 + 4$

$= 5.2.$

$p \in [0, 1].$

$p = (0.5 \times 4) + 4$

[1 37 43 . 64 128 256]

0 1 2 ⊙ 3 4 5

6 .

$$\frac{5}{2} = 2.5$$

Q] In a bag, I have 3 R, 2 B.
How many ways are there
for me to select 3 R and 1 B
in 4 repetitions?

B_1, B_2 R_1, R_2, R_3

$\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$
① ② ③ ④

1, 2, 3 RRRB
1, 2, 4 RRBR
2, 3, 4 BRRR
1, 3, 4. RBRR.

$^4C_3 \rightarrow \frac{4!}{3! \times 1!} = 4.$

5 items, how many ways to make permutations of 4.

$$^5P_4.$$

→ 3 R, 2 B,, how many ways to make permutations of 4

$B_1 \; B_2 \; B_3 \; R_1$ $\qquad$ _ _ _ _ .

$\equiv B_1 \; R_1 \; B_2 \; B_3$

$\equiv B_1 \; B_2 \; R_1 \; B_3$

Q] We are playing the casino game.
where we do n tries of picking.
4 red balls → victory.

100

S – 5

U – 6.

R – 2

$$\frac{2\times5\times6}{13\times2} = \frac{5\times\cancel{2}}{2\cancel{6}} = \frac{5}{13}.$$

total ways I can select 2 subjects.

13×12

$\underset{5}{\_\_}\ \underset{6}{\_\_}$ or $\underset{6}{\_\_}\ \underset{5}{\_\_}$.

$Y \rightarrow$ number of eggs which are safe.

$+1$ when egg safe.

$+0$ when egg breaks.