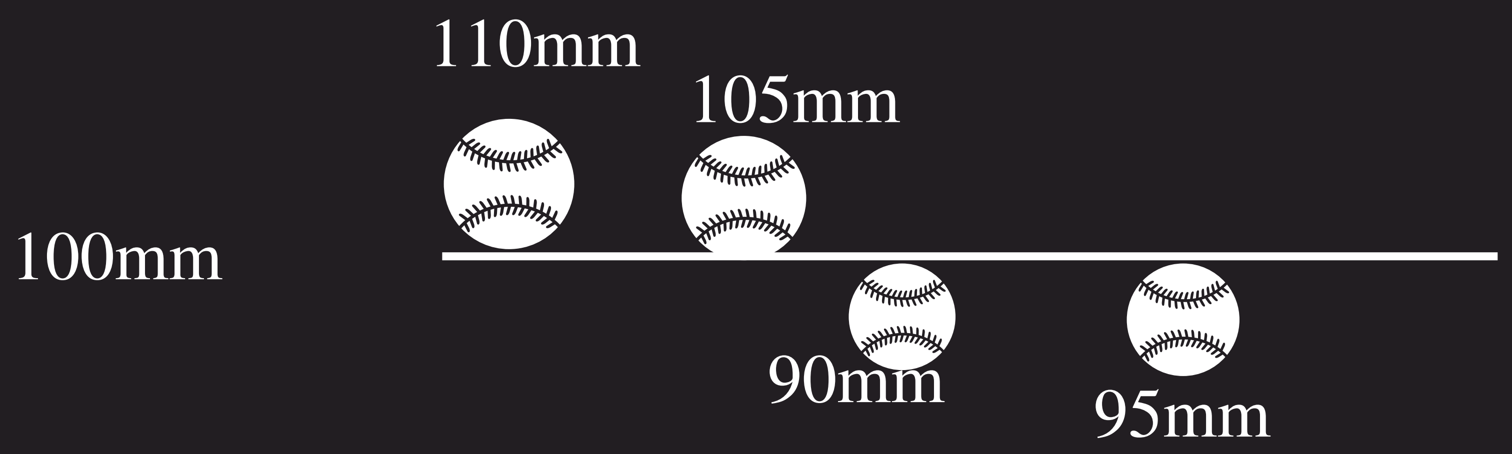
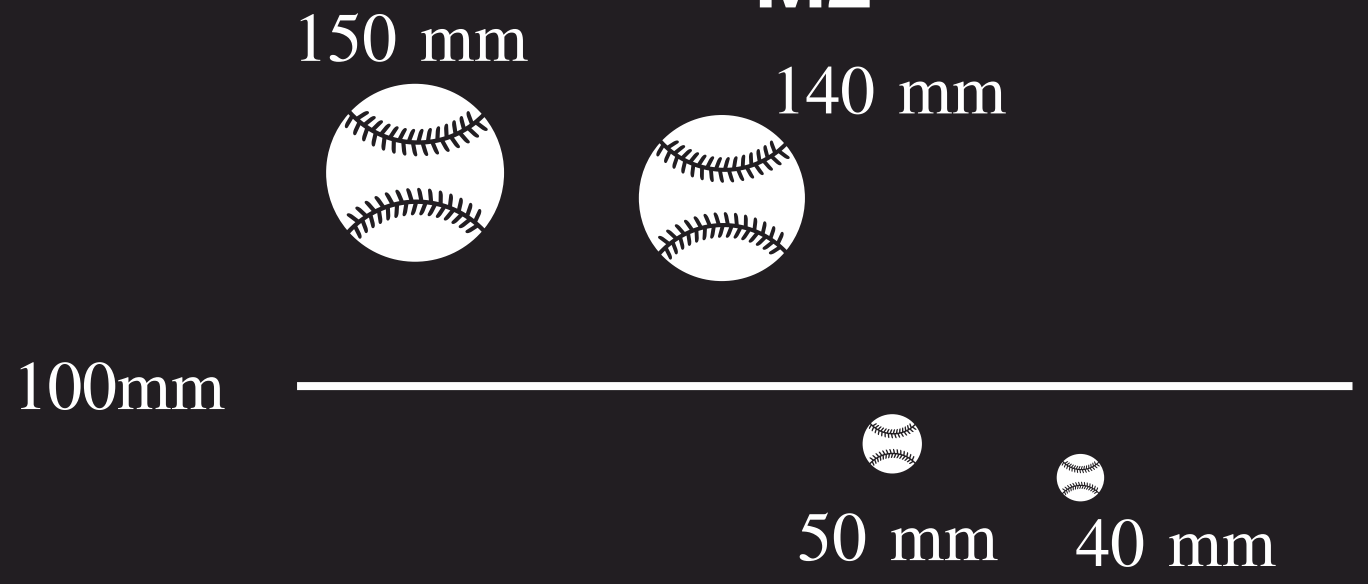
Variance

M1



M2



How to define Error?

$$10\ \text{mm} + 5\ \text{mm} + (-5\ \text{mm}) + (-10\ \text{mm}) = 0\ \text{mm} \ ✗$$

$$(10\ \text{mm})^2 + (5\ \text{mm})^2 + (-10\ \text{mm})^2 + (-5\ \text{mm})^2 = 250\ \text{mm}^2 \ ✓$$

$$\text{Variance} = \frac{250}{4}\text{mm}^2$$

$$\text{Std dev} = \sqrt{\frac{250}{4}}\text{mm}$$

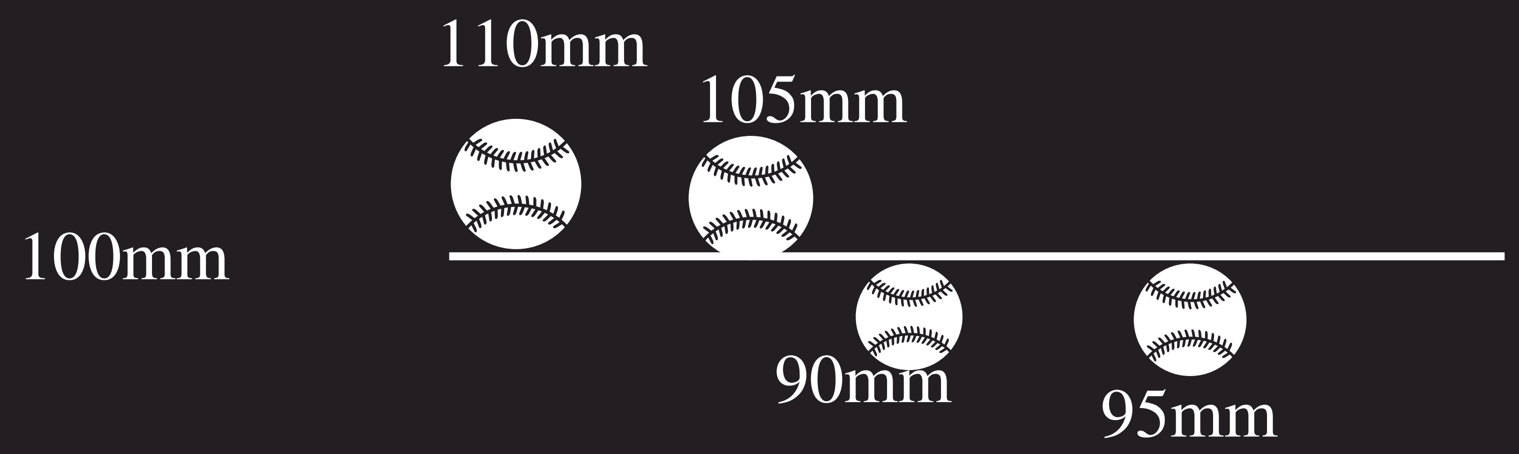
$$(50\ \text{mm})^2 + (40\ \text{mm})^2 + (-50\ \text{mm})^2 + (-40\ \text{mm})^2 = 8200$$

$$\text{Variance} = \frac{8200}{4}\text{mm}^2$$

$$\text{Std dev} = \sqrt{\frac{8200}{4}}\text{mm}$$

Variance

M1



x_1	110
x_2	105
x_3	95
x_4	90
$\bar{x}$	100

$$10\text{ mm} + 5\text{ mm} + (-5\text{ mm}) + (-10\text{ mm}) = 0\text{ mm}$$

$$(10\text{ mm})^2 + (5\text{ mm})^2 + (-10\text{ mm})^2 + (-5\text{ mm})^2 = 250\text{ mm}^2$$

$$\text{Variance} = \frac{250}{4}\text{mm}^2$$

$$\text{Std dev} = \sqrt{\frac{250}{4}}\text{mm}$$

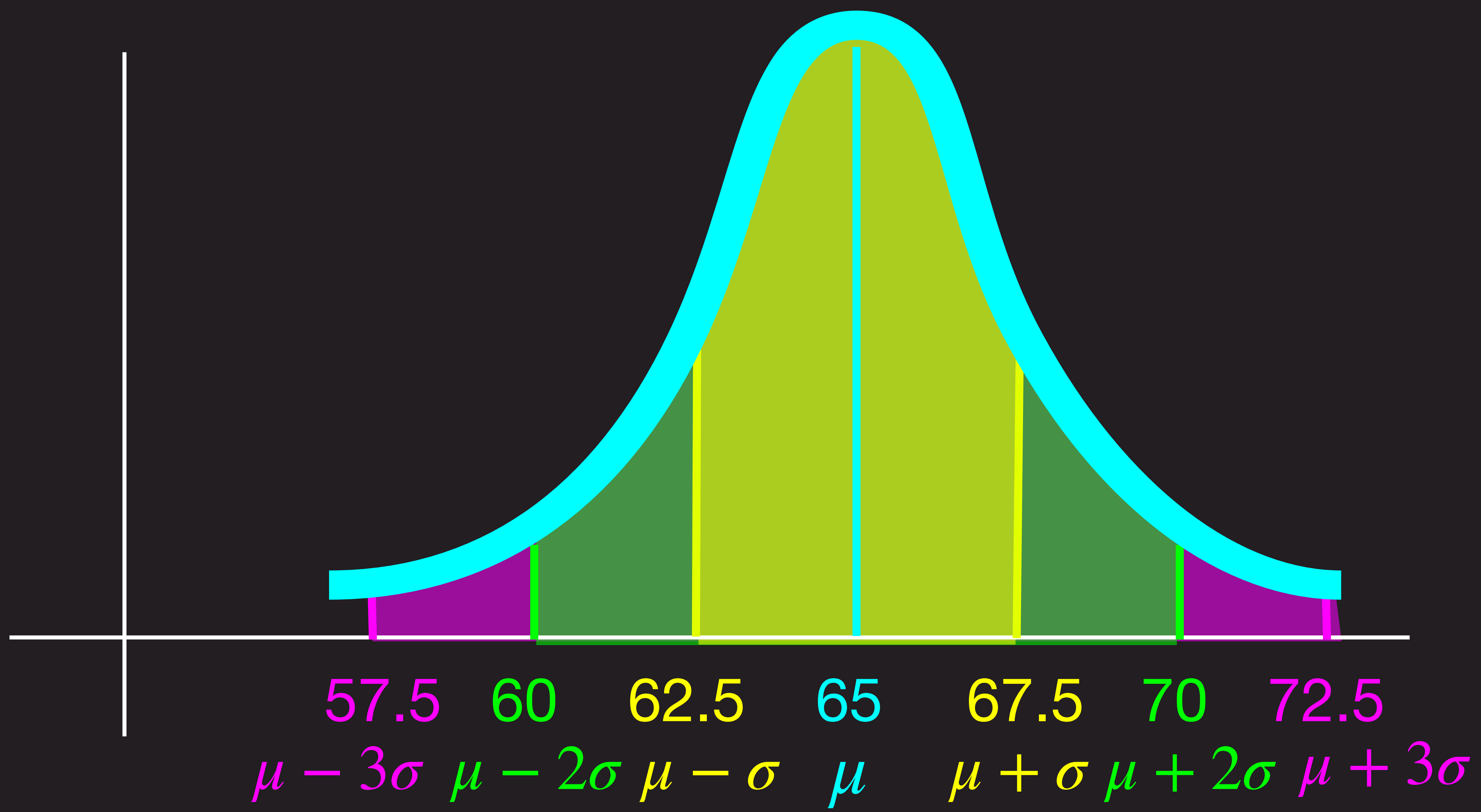
$$\text{Variance} = \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + (x_4-\bar{x})^2}{4}$$

$$\text{Std Dev} = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + (x_4-\bar{x})^2}{4}}$$

$$\text{Std Dev} = \sqrt{\frac{\sum_i (x_i-\bar{x})^2}{n}} = \sigma$$

$$\text{Variance} = \frac{\sum_i (x_i-\bar{x})^2}{n} = \sigma^2$$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$\mu = 65$

$\sigma = 2.5$

Fraction of people whose height is between 62.5 and 67.5 is 68%

$$P[62.5 < X < 67.5] = 0.68$$

$$P[\mu - \sigma < X < \mu + \sigma] = 0.68$$

Fraction of people whose height is between 60 and 70 is 95%

$$P[60 < X < 70] = 0.95$$

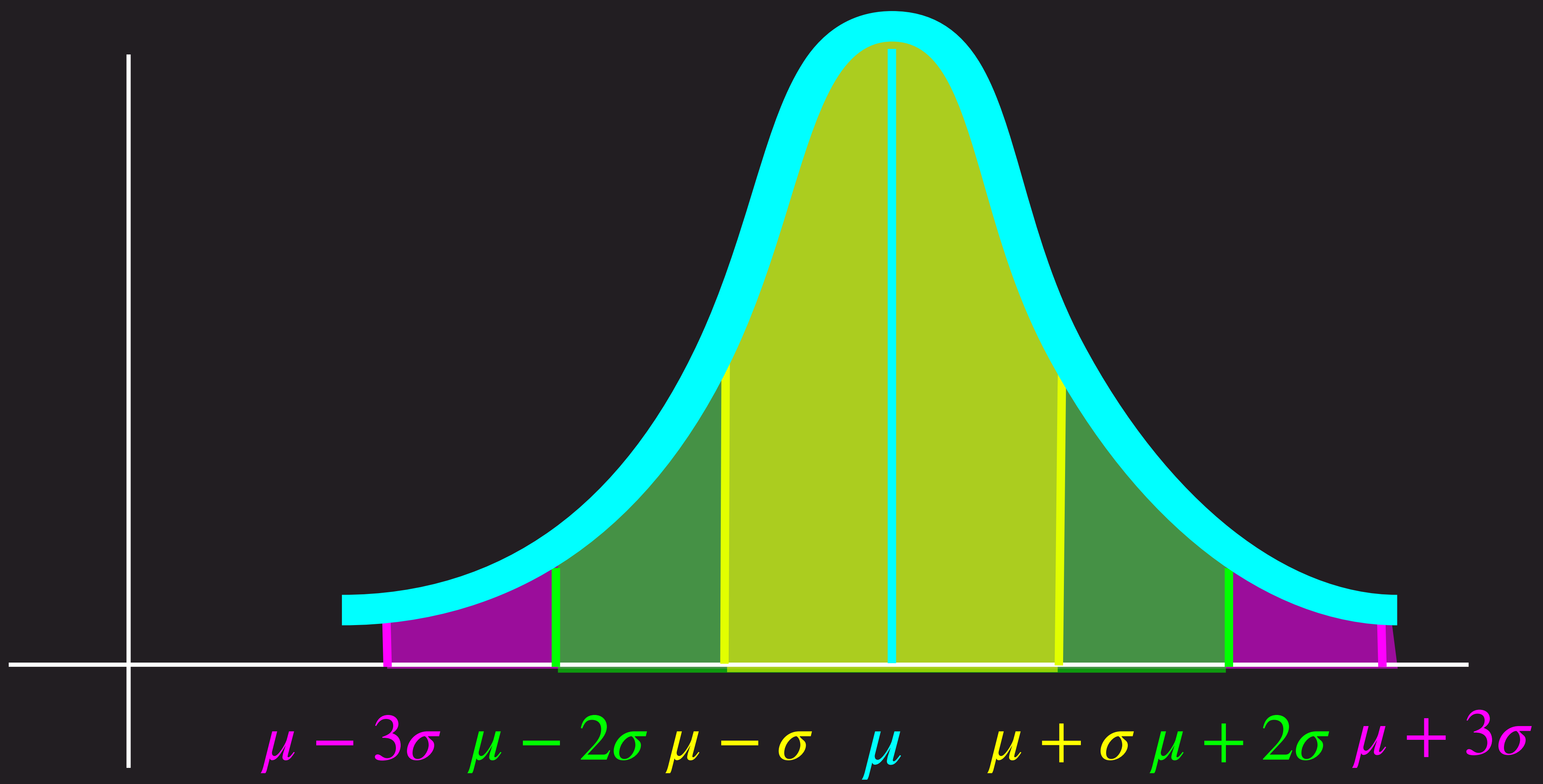
$$P[\mu - 2\sigma < X < \mu + 2\sigma] = 0.95$$

Fraction of people whose height is between 57.5 and 72.5 is 99.7%

$$P[57.5 < X < 72.5] = 0.997$$

$$P[\mu - 3\sigma < X < \mu + 3\sigma] = 0.997$$

Gaussian Empirical Rule or 68/95/99 Rule

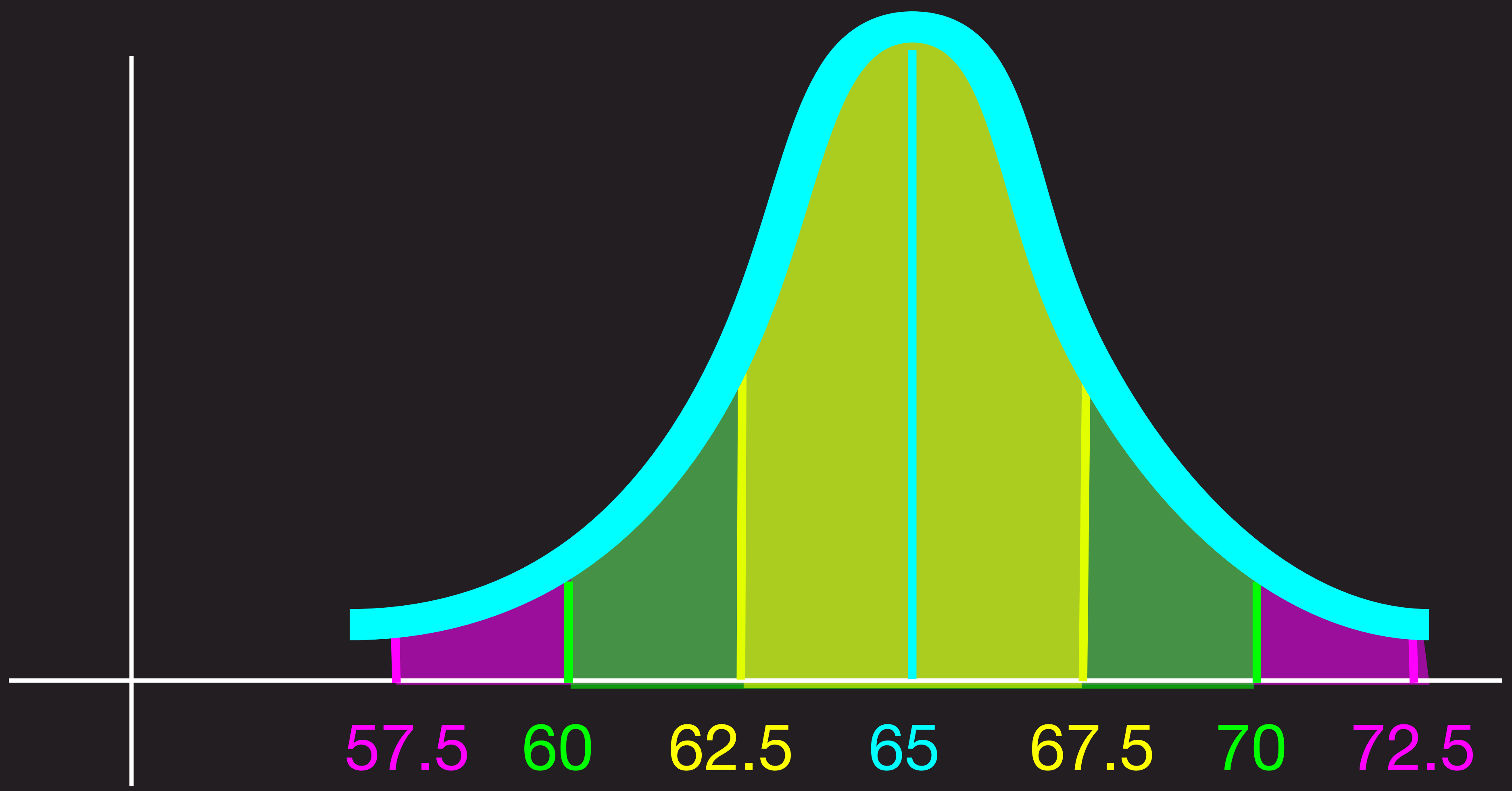


$$P[\mu - \sigma < X < \mu + \sigma] = 0.68$$

$$P[\mu - 2\sigma < X < \mu + 2\sigma] = 0.95$$

$$P[\mu - 3\sigma < X < \mu + 3\sigma] = 0.997$$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$$\mu = 65$$

$$\sigma = 2.5$$

$$P[62.5 < X < 67.5] = 0.68$$

$$P[60 < X < 70] = 0.95$$

$$P[57.5 < X < 72.5] = 0.997$$

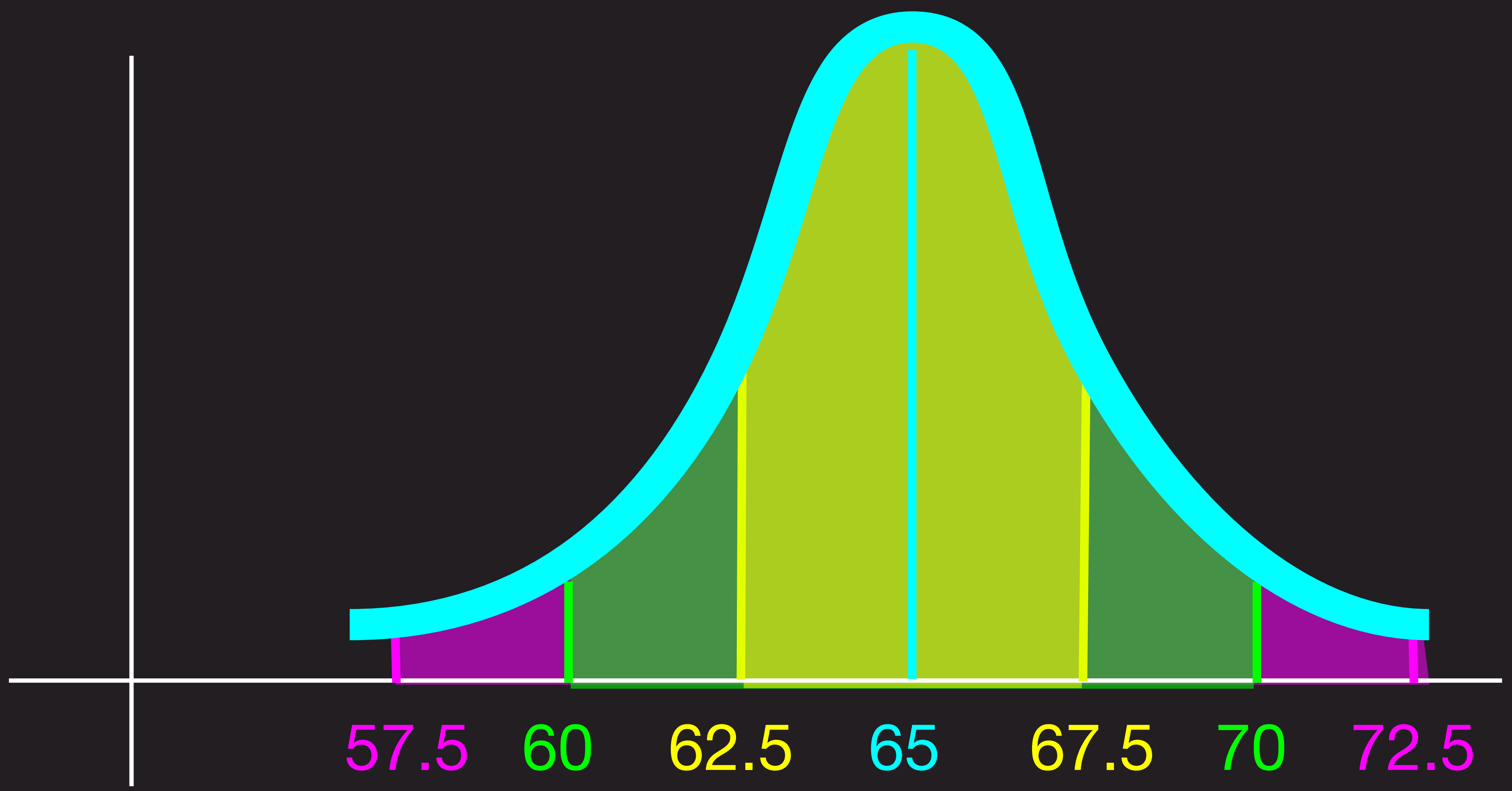
What is the fraction of people whose height is between 60 and 72.5?

Between 60 and 65? $\frac{95}{2} = 47.5$

Between 65 and 72.5? $\frac{99.7}{2} = 49.85$

Totally, 47.5 + 49.85 = 97.35

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$\mu = 65$

$\sigma = 2.5$

$P[62.5 < X < 67.5] = 0.68$

$P[60 < X < 70] = 0.95$

$P[57.5 < X < 72.5] = 0.997$

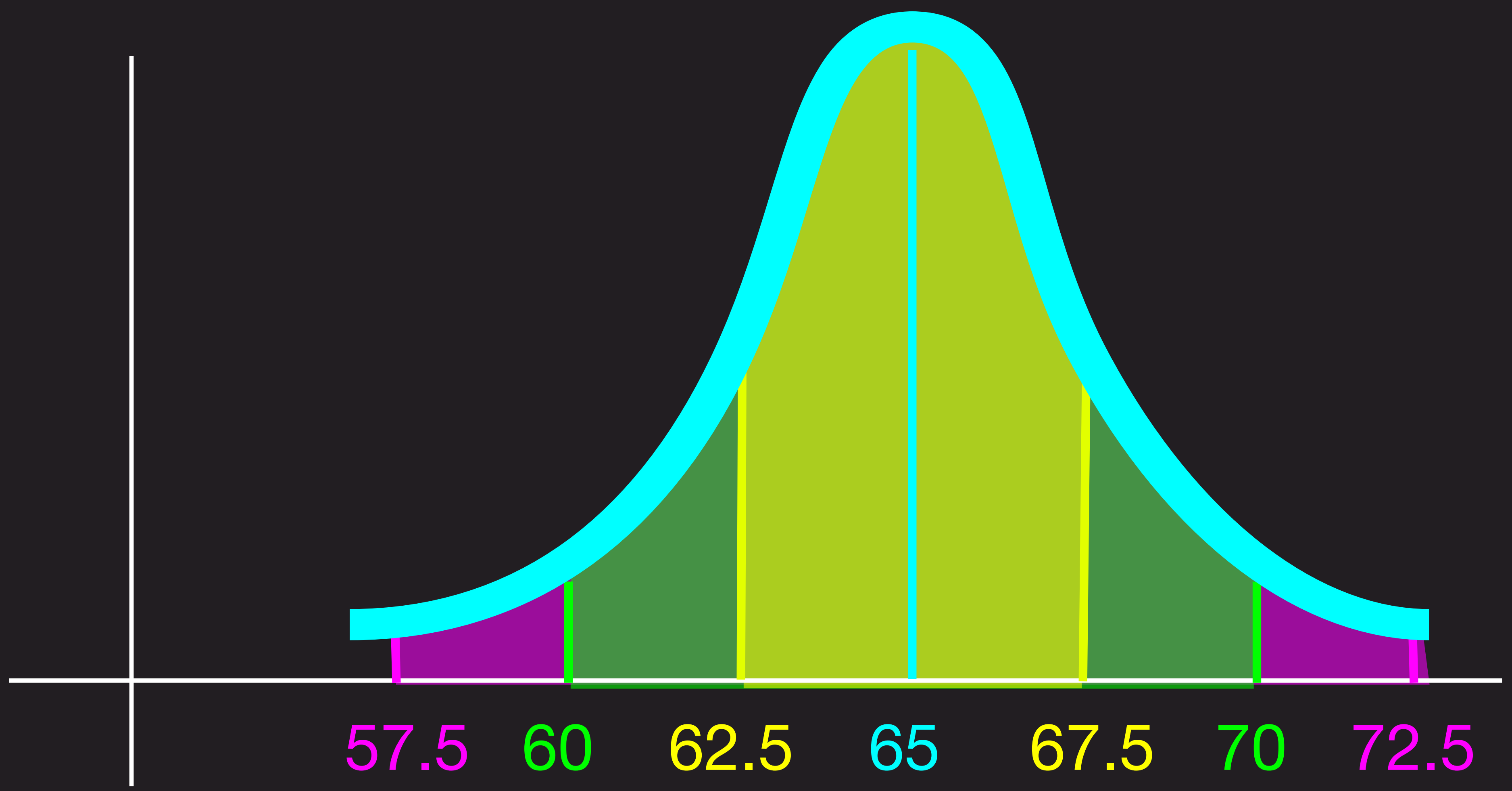
What fraction of people are shorter than 67.5?

What fraction of people are shorter 65? 50%

What fraction of people are in between 65 and 67.5? 68/2 = 34%

Totally 50 + 34 = 84% $P[X < 67.5] = P[X < 65] + P[65 < X < 67.5] = 0.5 + 0.34 = 0.84$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$$\mu = 65$$

$$\sigma = 2.5$$

$$P[62.5 < X < 67.5] = 0.68$$

$$P[60 < X < 70] = 0.95$$

$$P[57.5 < X < 72.5] = 0.997$$

What fraction of people are shorter than 69.1?

How many σ (std devs) away from 65 is this number?

$$65 + z\,(2.5) = 69.1$$

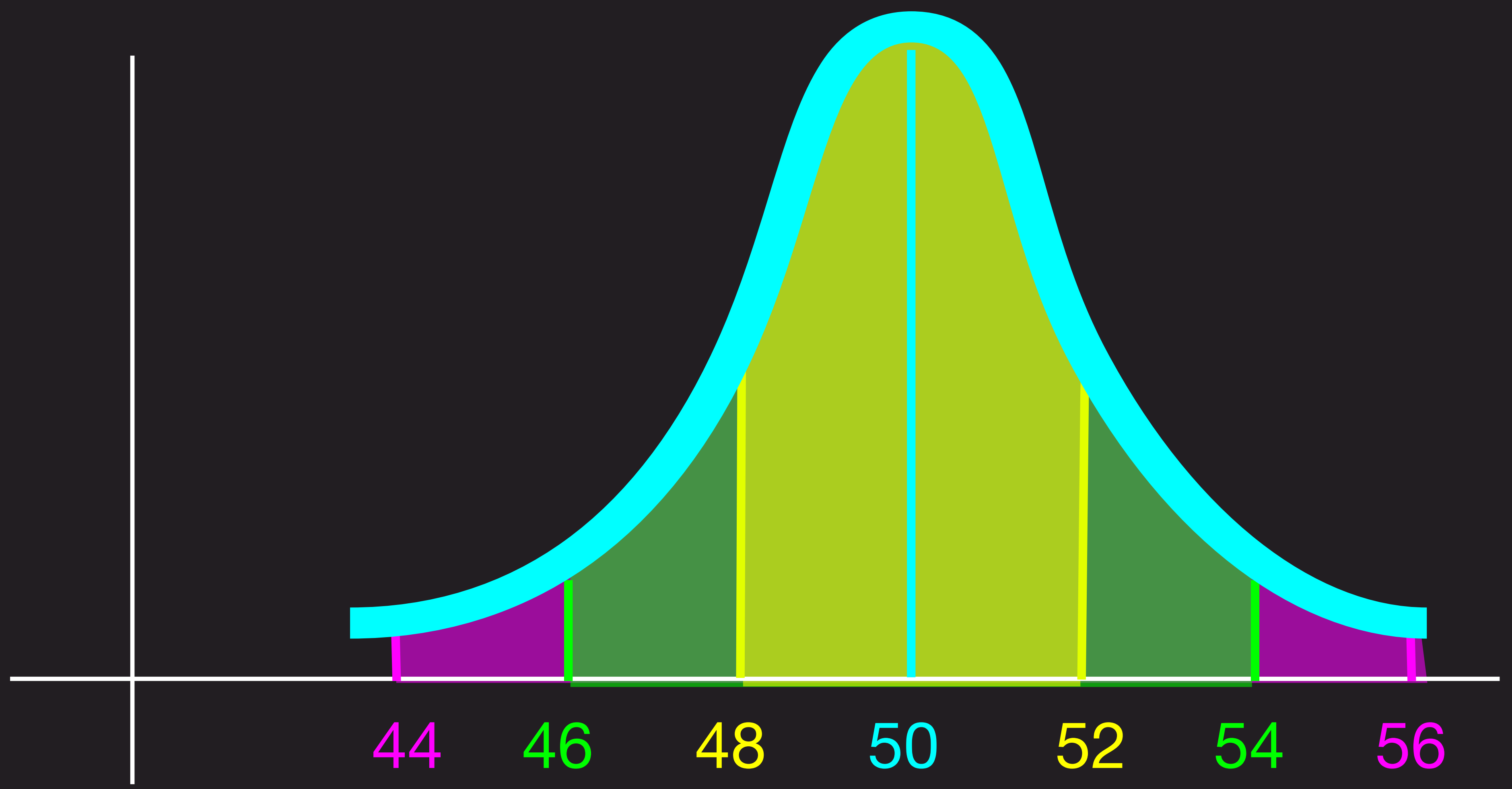
$$z = \frac{(69.1 - 65)}{2.5} = 1.64$$

Z-Score

$$z = \frac{(x - \mu)}{\sigma}$$

To find this probability, we use the Z-table 94.9%

Balls produced by manufacturer have mean 50 mm and std dev 2 mm

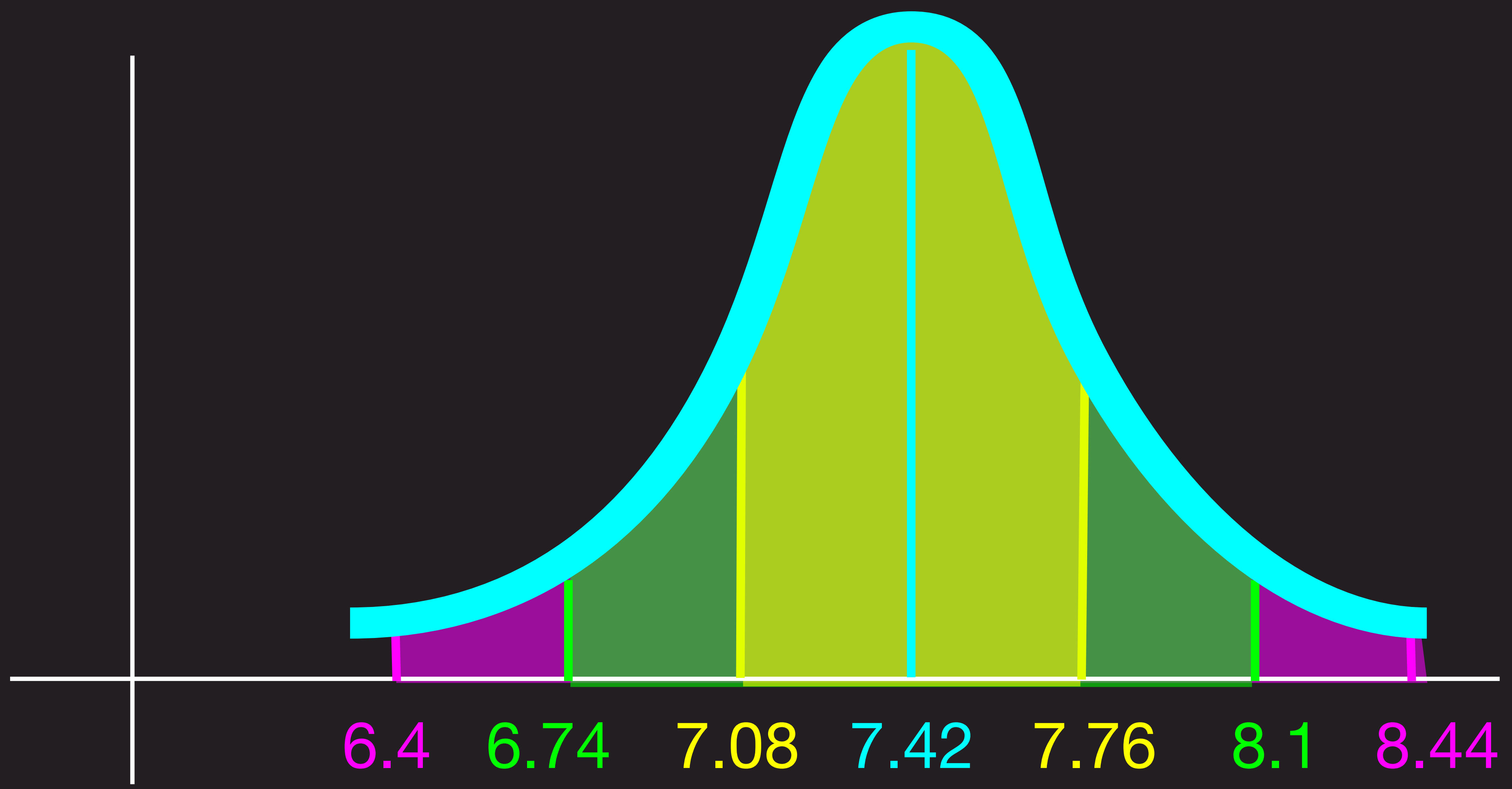


What fraction of balls are smaller than 53 mm?

$$z = \frac{(53-50)}{2} = 1.5$$

From Z-table, we see that the answer is 93.32%

Skaters take a mean of 7.42 seconds and std dev of 0.34 seconds for 500 meters.
What should his speed be such that he is faster than 95% of his competitors?



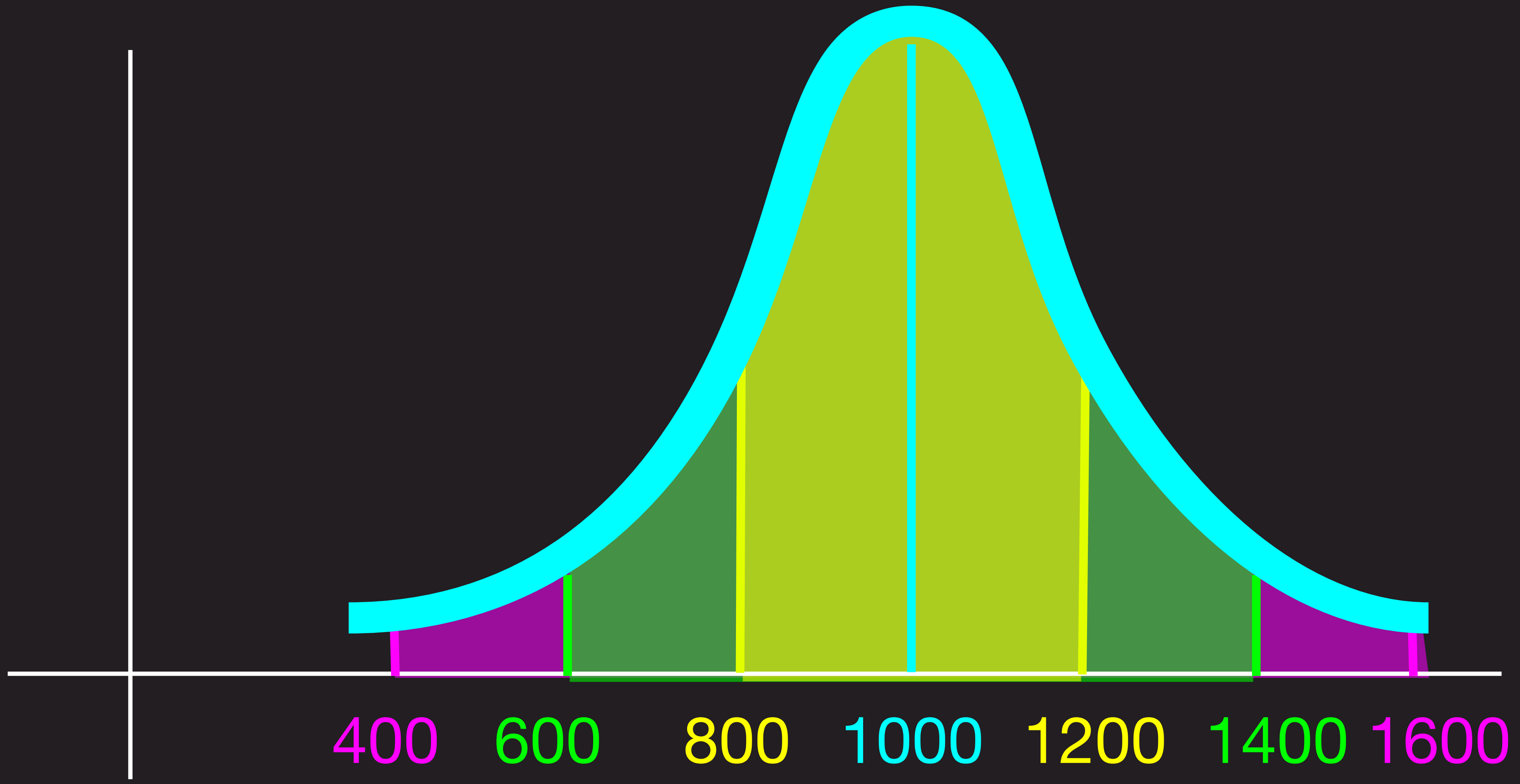
Unlike earlier examples, here the fraction is given, and we have to find Z-score

Let us use the Z-table   We need the Z-score of the area corresponding to 0.05

From Z-table, z-score is -1.65

$$z = \frac{(x - \mu)}{\sigma} \qquad x = \sigma\, z + \mu = (0.34)\,(-1.65) + 7.42 \;= 6.859$$

A retail outlet sells around 1000 toothpastes a week, with std dev = 200.
If the on-hand inventory is 1300, what is the need for replenishment within the week?



Let X denote the weekly sales. The questions asks for the probability that X > 1300

What is the Z-score of 1300?

$$z = \frac{1300 - 1000}{200} = 1.5$$

From Z-table, we see that $P[X \leq 1300] = 0.933$

$$P[X > 1300] = 1 - 0.933 = 0.067$$