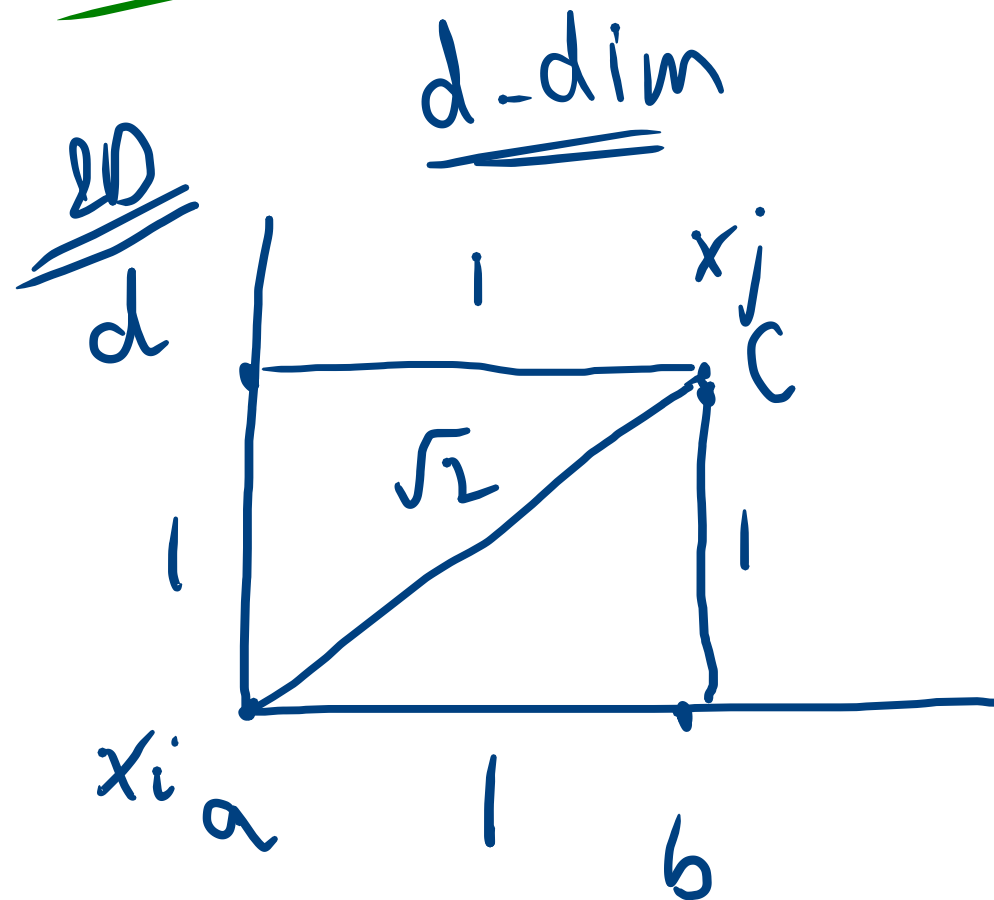


→ Class start at 9:05 pm

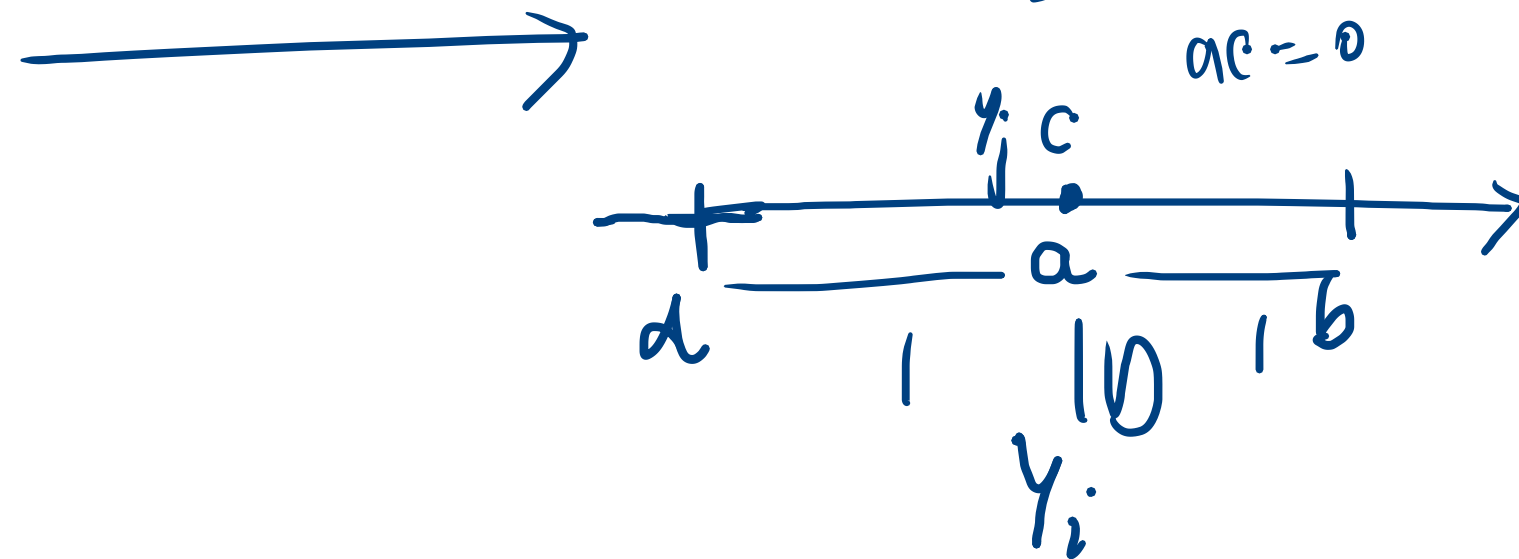
t-SNE (to preserve neigh. dist in  $d'$  dimension)

↳ t-dist noted Stochastic Neighbourhood  
Embedding

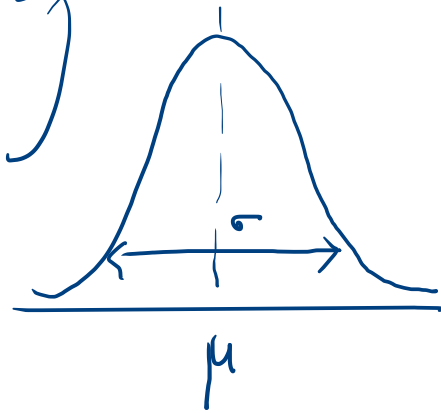
Crowding problem



$d' = d$  dim



$$p_i = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right)$$



$p_{ij}$  = dist b/w  $x_i$  &  $x_j$

$p_{ij} \uparrow$  dist( $x_i, x_j$ )  $\downarrow$

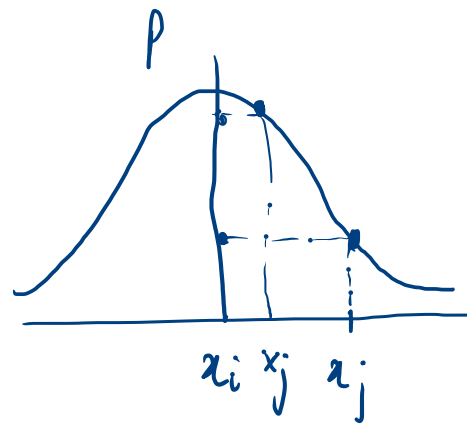
$p_{ij} \downarrow$  dist( $x_i, x_j$ )  $\uparrow$

$\begin{matrix} \cdot & x_i & \cdot \\ & \cdot & \\ \cdot & x_j & \cdot \end{matrix}$

$\begin{matrix} \cdot & & \cdot \\ & \cdot & \\ \cdot & x_j & \cdot \end{matrix} \quad x_i$

$$p_{ij} \propto \frac{1}{\text{dist}(x_i, x_j)}$$

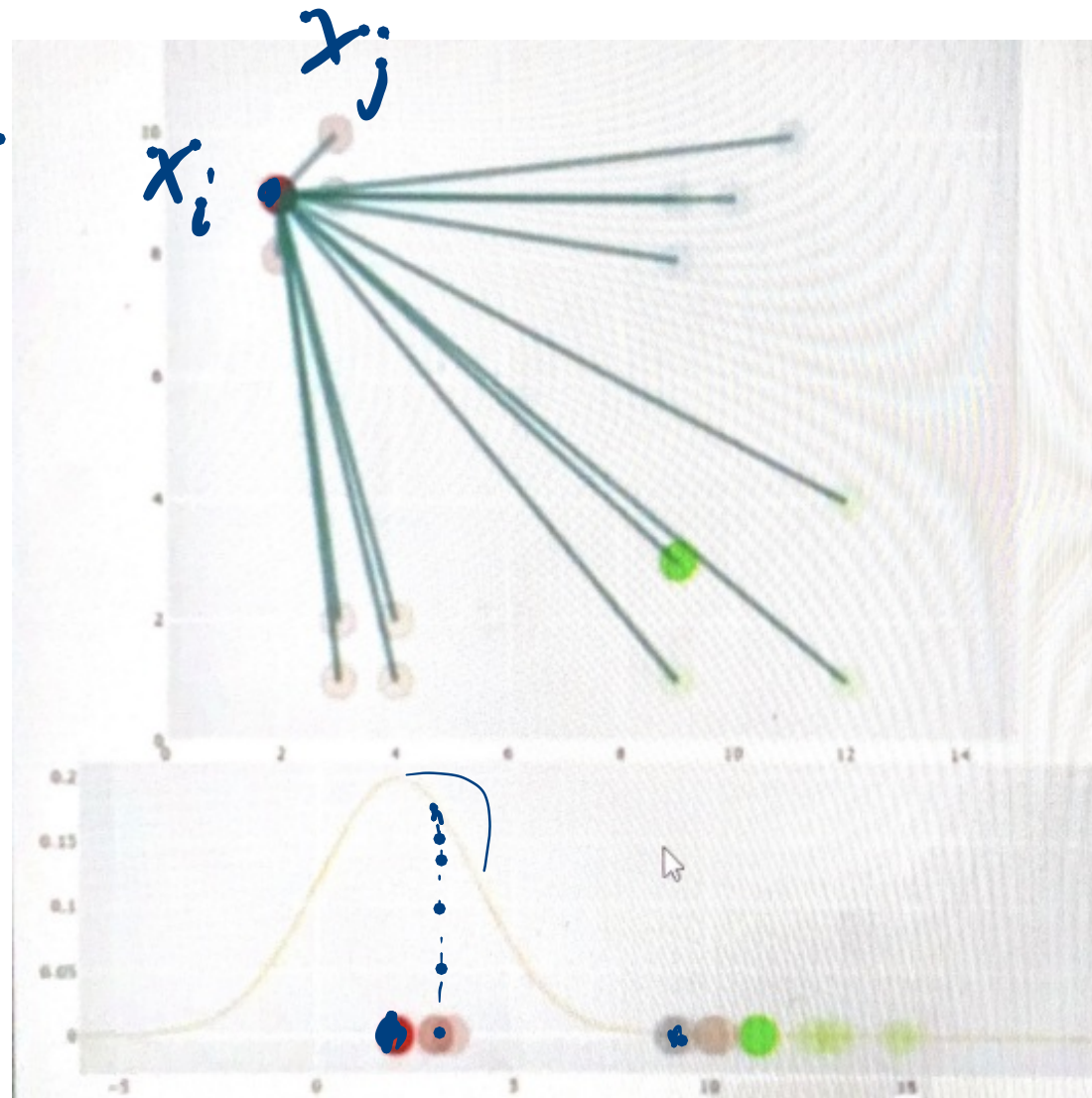
$p_{ij}$  =  $\exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma_i^2}\right)$



=  $\exp\left(-\frac{d_{ij}^2}{2\sigma_i^2}\right)$   $d_{ij} = \|x_i - x_j\|$

$$p_{ij} = \frac{1}{\exp\left(\frac{d_{ij}^2}{2\sigma_i^2}\right)}$$

$$\underline{k=4}$$



$$P_{ij} = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma_i^2}\right)$$

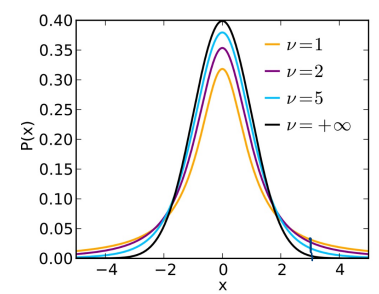
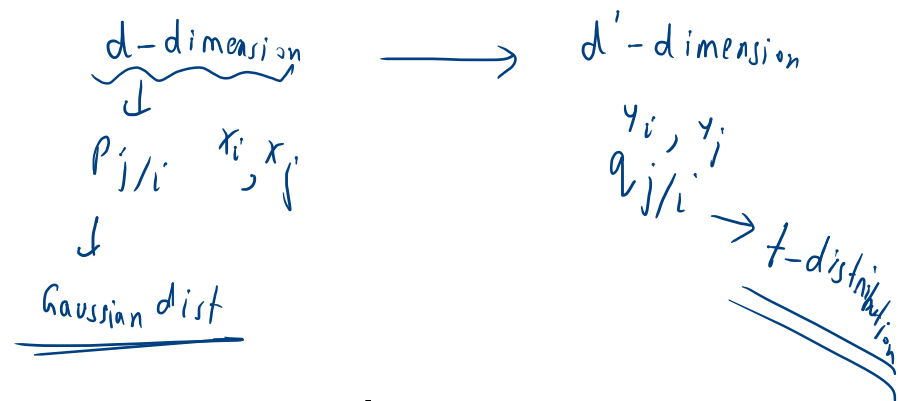
$$d_{ij} = \|x_i - x_j\|^2$$

is a scalar quantity

$$P_{ij} = \frac{\exp(-\frac{\|x_i - x_j\|^2}{2\sigma^2})}{\sum_{k \neq l} \exp(-\frac{\|x_k - x_l\|^2}{2\sigma^2})}$$

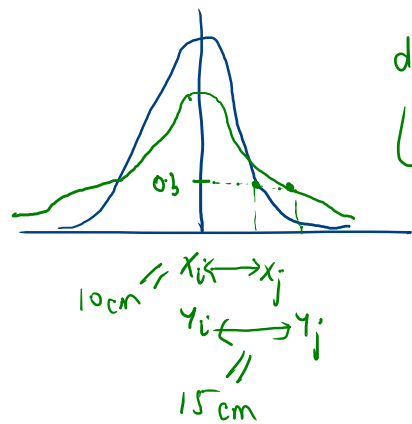
Pairwise similarity

$$\sum \sum P_{ij} = 1$$



$\rightarrow$  t-dist has smaller peak and larger tail

$\rightarrow$  t-dist with dof=1



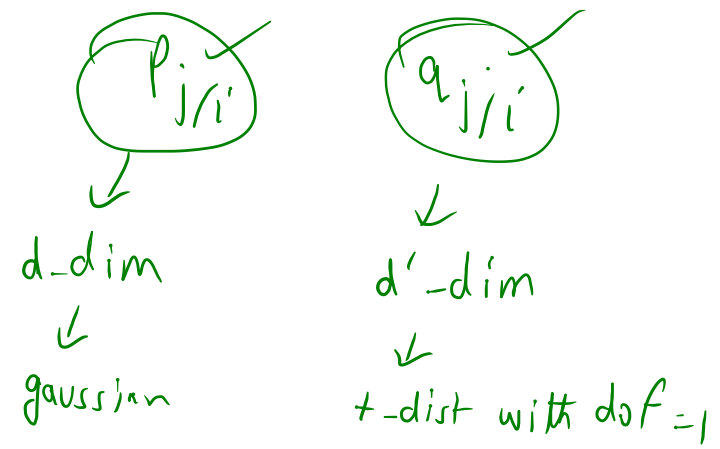
dij b/w  $x_i, x_j$

$\hookrightarrow p_{j/i} = 0.3 \rightarrow$  d-dim

$q_{j/i} = 0.3 \rightarrow$  d'-dim

$\hookrightarrow$  dij b/w  $y_i, y_j$  is larger

$\rightarrow$  Help somewhat overcome crowding problem.



## KL-divergence

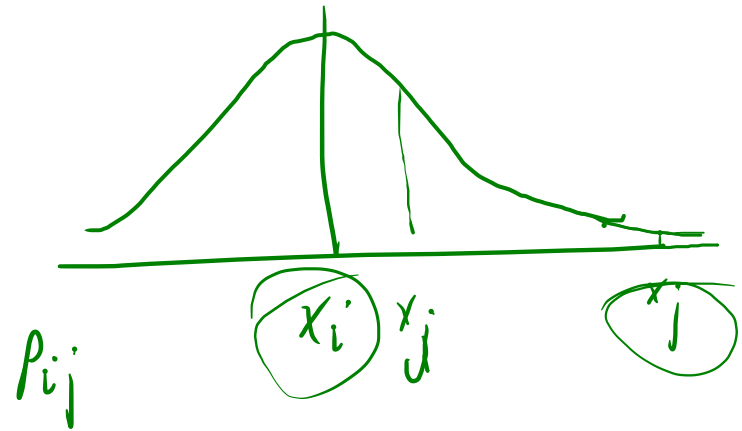
$$KL(P, Q) = \sum_i \sum_j \textcircled{p_{ij}} \log \left( \frac{p_{ij}}{q_{ij}} \right)$$

Compare 2 distribution  
and measure  
dissimilarity

$$\sum_{i=1}^m \sum_{j=1}^c p_{ij} \log \hat{p}_{ij}$$

①  $p_{ij} \approx q_{ij}$        $KL(P, Q) = 0$

②  $KL \approx 0$        $p_{ij} \approx 0$   
 $\downarrow$   
 $x_i$  &  $x_j$   
 are far apart



$$p_{ij} \approx 0$$

Perplexity → hyperparameter

↳ Effective no. of neigh who  
distance we want to preserve.

↳ 5-50

↳ Controlling local aspect  
and global aspect

## Disadvantage

→ Very slow algorithm

→ Does not have predict function only

$$D = \{x_i\}_{i=1}^n \rightarrow D' = \{y_i\}_{i=1}^n$$

$x_{\text{test}}$

TSNE again

$y_{\text{test}}$

PCA

$p_{ij}$

top-K eigen vecs

$q_{ij}$

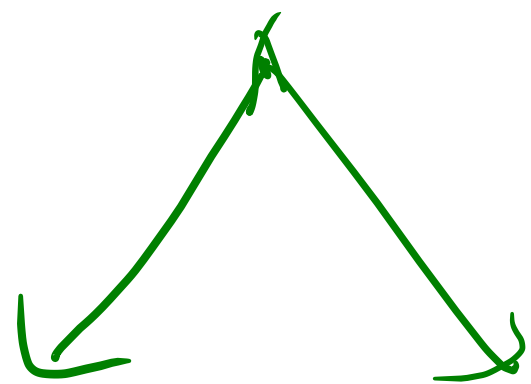
have top-K eigen values

$$x_{\text{test}} \begin{bmatrix} \uparrow \\ v_1 \\ \downarrow \end{bmatrix} \quad \text{---} \quad \begin{bmatrix} \uparrow \\ u_K \\ \downarrow \end{bmatrix}$$



→ Break until 10:25pm

UMAP  $\rightarrow$  Uniform Manifold  
Approximation &  
Projection



Algebraic  
topology  
(Manifold)

Algorithmic  
+  
Optimization

$\rightarrow$  Tries to preserve the structure  
as best as possible