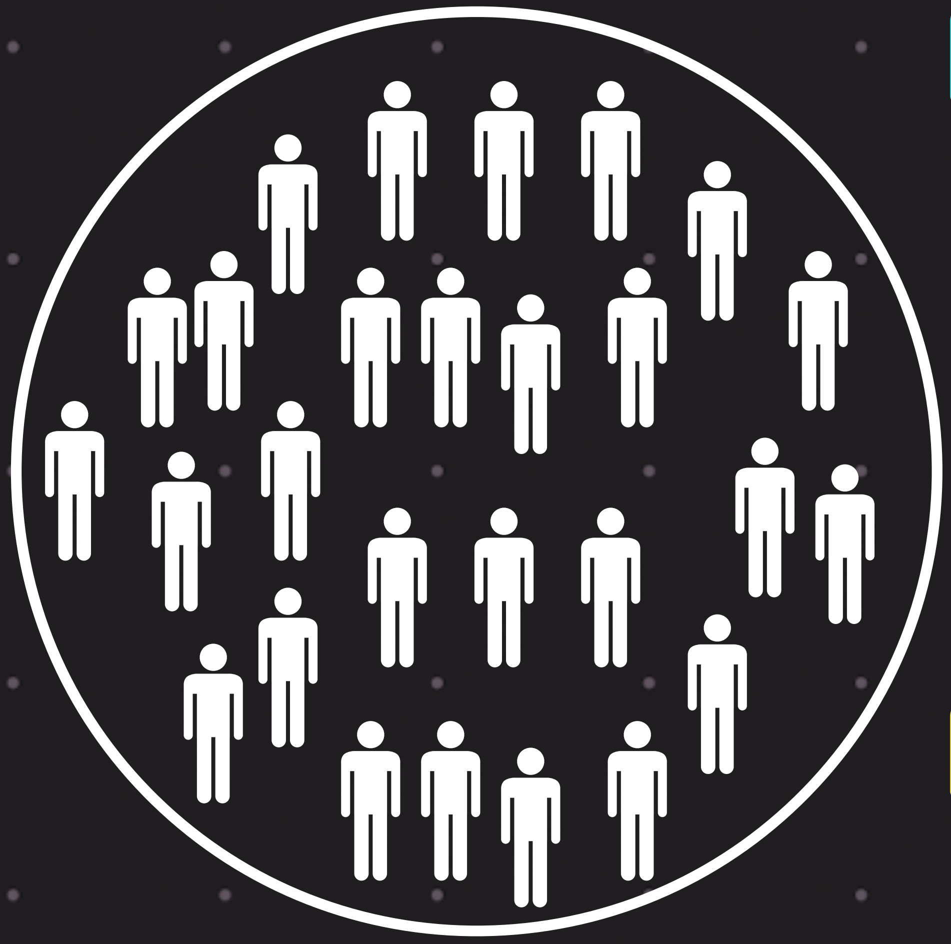
SDE-2 Salary



- Average of these three numbers is 35 L
- What is the unknown number?

34 L

If we know the sample mean of 3 numbers, then knowing 2 numbers is enough to know everything

- Average of these four numbers is 37 L
- What is the unknown number?

39 L

If we know the sample mean of 4 numbers, then knowing 3 numbers is enough to know everything

If we know the sample mean of n numbers, then knowing $n - 1$ numbers is enough to know everything

Degree of freedom is said to be $n - 1$

DF = $n - 1$

Height and Weight

	Height (inches)	Weight (kg)
	73	85
	68	73
	74	96
	71	82
	62	70
Average	71	81.2

- We know the average height and weight of 5 people

  We want to fill the table

- How many minimum numbers in the table should we know?

  We need minimum 8 numbers DF = 8

  The number 8 comes as (5-1) + (5-1)

- In general, DF = n1 + n2 - 2

Sachin - Centuries and winning

		Win		
		False	True	
Century	False	160	154	314
	True	16	30	46
		176	184	360

Sachin has scored 46 centuries in 360 matches.

Of these 360 matches, India has won 184.

We want to construct the contingency table with centuries and win

- We know these 5 numbers from data

  We want to fill the contingency table

- If we know this one number, can we fill the table with the other three?

  Yes

  One number is all we need!

  DF = 1

Regional support for politicians

	A	B	C	D	Total
X	90	60	104	95	349
Y	30	50	51	20	151
Z	30	40	45	35	150
Total	150	150	200	150	650

- We know the total numbers from data

  We want to fill the contingency table

- How many minimum numbers in the table should we know?

  If we know these 6 numbers, can we fill the table?

  Yes

  DF = 6

- In general, DF = (#rows - 1) (#columns - 1)

4 politicians

A B C D

3 cities

X Y Z

Degrees of Freedom

- If we know the sample mean of n numbers, then knowing $n - 1$ numbers is enough to know everything

  DF = $n - 1$

- If we know the sample means of two sets of numbers n_1 and n_2 numbers, then knowing $n_1 + n_2 - 2$ numbers is enough to know everything

  DF = $n_1 + n_2 - 2$

- In a contingency table, if we know the row sums and column sums, then

  DF = (#rows - 1) (#columns - 1)

Chi-Square Test

(A favourite word used by product managers)

- Suppose we have a lot of features in a machine learning model x_1, x_2, x_3, x_4
- We may have very big equation in these features

$$y = ax_1^2 + bx_2 + \cdots +$$

- Often you will be asked to do chi-squared test to remove variables that are not significant
- "This feature (say x_3) is not relevant, we have done chi-squared test. Let us remove this feature"

Going forward, the model will only use x_1, x_2, x_4

Chi-Square Test   Coin toss 50 times

Let us set up the null and alternate hypothesis

H_0 : Fair coin H_a : Biased coin

We shall use a new test statistic called

χ^2 Test statistic ("chi-squared")

$$\chi^2 = \frac{(28-25)^2}{25} + \frac{(22-25)^2}{25} = 0.72$$

If the coin is fair, should this number be large or small?

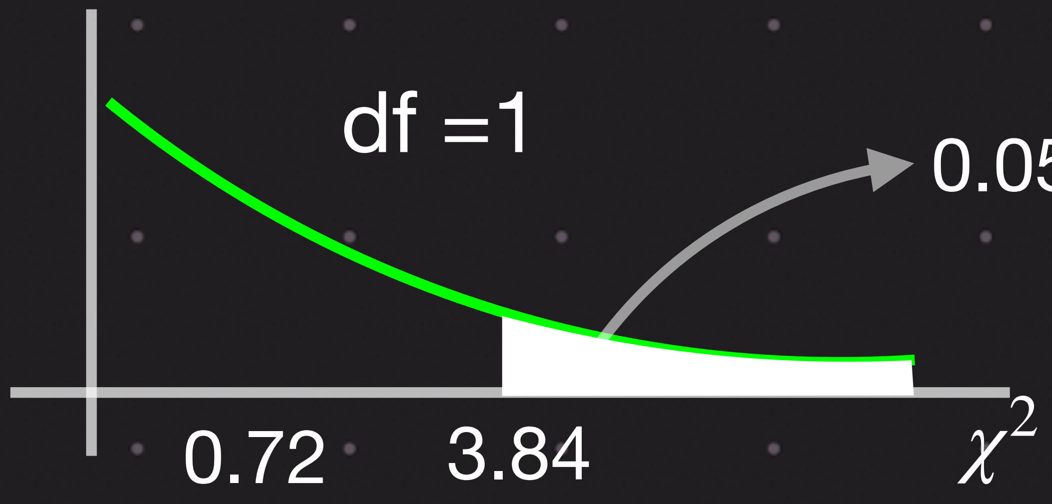
Small

	Heads	Tails
Expected	25	25
Actual	28	22

Knowing one number, we know the full table   DF = 1

DF = (#rows - 1) (#cols - 1) $= (2-1)(2-1) = 1$

Let us see the distribution of the χ^2 test statistic with df = 1



Critical region for 95% confidence

```
from scipy.stats import chi2
cr = chi2.ppf(q=0.95, df=1)
```

cr = 3.84

```
from scipy.stats import chisquare
chi_stat, p_value = chisquare(
    [28, 22], [25, 25]
)
```

```
chi_stat = 0.72
p_value = 0.396
```

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

$$\chi^2 = \sum_i \frac{(o_i - e_i)^2}{e_i}$$

Fail to reject H_0 since observed χ^2 0.72 is less than 3.84

p-value > 0.05

Chi-Square Test   Coin toss 50 times

Let us set up the null and alternate hypothesis

H_0 : Fair coin H_a : Biased coin

We shall use a new test statistic called

χ^2 Test statistic ("chi-squared")

$$\chi^2 = \frac{(45-25)^2}{25} + \frac{(5-25)^2}{25} = 32$$

If the coin is fair, should this number be large or small?

Small

	Heads	Tails
Expected	25	25
Actual	45	5

Knowing one number, we know the full table   DF = 1

DF = (#rows - 1) (#cols - 1) $= (2-1)(2-1) = 1$

Let us see the distribution of the χ^2 test statistic with df = 1

Critical region for 95% confidence

```
from scipy.stats import chi2
cr = chi2.ppf(q=0.95, df=1)
```

cr = 3.84

```
from scipy.stats import chisquare
chi_stat, p_value = chisquare(
    [45, 5], [25, 25]
)
```

```
chi_stat = 32
p_value = 1.54e-08
```

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

$$\chi^2 = \sum_i \frac{(o_i - e_i)^2}{e_i}$$

Reject H_0 since observed χ^2 32 is greater than 3.84

p-value < 0.05

Chi-Square Test

Dice, 36 times

	1	2	3	4	5	6
Expected	6	6	6	6	6	6
Actual	2	4	8	9	3	10

H_0 : Fair dice H_a : Biased dice

Test statistic

$$\chi^2 = \frac{(2-6)^2}{6} + \frac{(4-6)^2}{6} \cdots + \frac{(10-6)^2}{6} = 9.66$$

Degrees of freedom

DF = (#rows - 1) (#cols - 1)

DF = (2 - 1)(6 - 1) = 5

Critical region for 90% confidence

$\alpha = 0.1$

```
from scipy.stats import chi2
cr = chi2.ppf(q=0.90, df=5)
cr = 9.24
```

```
from scipy.stats import chisquare
chi_stat, p_value = chisquare(
    [2, 4, 8, 9, 3, 10],
    [6, 6, 6, 6, 6, 6]
)
```

```
chi_stat = 9.66
p_value = 0.0852
```

Reject H_0 since observed χ^2 9.66 is greater than 9.24

p-value < 0.1

Online Vs Offline shopping   Does gender effect this?

Observed

	Male	Female		
Offline	527	72	599	66%
Online	206	102	308	34%
	733	174	907	

Expected

	Male	Female	
Offline	484	115	599
Online	249	59	308
	733	174	907

All these are observed values

To compute χ^2 test statistic, what do we need?   The expected values

What percent people prefer offline?   66%

Among 733 males, how many are expected to prefer offline?   733 * 0.66 = 484

Among 174 females, how many are expected to prefer offline?   174 * 0.66 = 115

What percent people prefer online?   34%

Among 733 males, how many are expected to prefer online?   733 * 0.34 = 249

Among 174 females, how many are expected to prefer online?   174 * 0.34 = 59

Online Vs Offline shopping — Does gender effect this?

Observed

	Male	Female		
Offline	527	72	599	66%
Online	206	102	308	34%
	733	174	907	

Expected

	Male	Female	
Offline	484	115	599
Online	249	59	308
	733	174	907

DF = (2-1) * (2-1) = 1

$$\chi^2 = \frac{(527-484)^2}{484} + \frac{(72-115)^2}{115} + \frac{(206-249)^2}{249} + \frac{(102-59)^2}{59} = 59$$

Critical region for 90% confidence

```python
from scipy.stats import chi2
chi2.ppf(q=0.9, df=1)
```

cr = 2.7

Reject H_0 since χ^2 is greater than 2.7

$$\alpha = 0.1$$

```python
from scipy.stats import chi2_contingency
observed = [
    [527, 72],
    [206, 102]
]
chi_stat, p_value, df, exp_freq = chi2_contingency(observed)
chi_stat = 57.04
p_value = 4e-14
```

p-value < 0.1

Assumptions of Chi2 test

Variables are categorical

Observations are independent

Each cell is mutually exclusive

Expected value in each cell is greater than 5 (at least in 80% of cells)

ANOVA - Analysis of Variance

So far, we compared two sets of samples, or two groups

Let us develop an intuitive way of comparing across multiple groups

Imagine we have data of heights and weights of three different groups

Our goal is to say whether these three groups have statistically the same height/weight

ANOVA - Analysis of Variance

Setup 1

American Basketball players	Very low variance within this group
Indonesian college students	Very low variance within this group
Indian cricket team	Maybe not too low

Setup 2

Suppose we take all these three groups and sort their names alphabetically

Names from A to G

Names from H to N

Names from O to Z

Which setup will have higher F-ratio?

Setup 1 will have higher F-ratio

If there is a difference, then F-ratio will be high.

If there is no difference, then F-ratio will be small.

H_0 : all groups have same mean

Under H_0, F-ratio will be very low

If F-ratio is high, we reject H_0

$$\text{F-ratio} = \frac{\text{Variance between groups}}{\text{Variance within groups}}$$

iPhone sales in 3 stores

	A	B	C	
	25	30	18	
	25	30	30	
	27	25	29	
	30	24	29	
	23	26	24	
	20	28	26	
	25	26.5	26	25.83
	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{Y}$

$$F = \frac{3.49}{14.9} = 0.23$$

$$F = \frac{MSB}{MSW}$$

H_0: All means are equal $\qquad H_a$: Means are different

Step 1 Compute individual group means $\bar{Y}_1 = 25 \quad \bar{Y}_2 = 26.5 \quad \bar{Y}_3 = 26.5$

Step 2 Compute mean of these 3 values $\bar{Y} = \dfrac{25 + 26.5 + 26}{3} = 25.83$

Step 3 Between groups

$$SSB = 6(25 - 25.83)^2 + 6(26.5 - 25.83)^2 + 6(26 - 25.83)^2 = 6.9$$

$$DF = 3 - 1 = 2$$

$$MSB = \frac{SSB}{DF} = \frac{6.9}{2} = 3.49$$

Step 4 Within groups

$$SSW = (25-25)^2 + (25-25)^2 + (27-25)^2 + \cdots + (20-25)^2$$
$$+$$
$$(30-26.5)^2 + (30-26.5)^2 + (25-26.5)^2 + \cdots + (28-26.5)^2$$
$$+$$
$$(18-26)^2 + (30-26)^2 + (29-26)^2 + \cdots + (26-26)^2$$
$$= 223$$

$$DF = 18 - 3 = 15$$

$$MSW = \frac{SSW}{DF} = \frac{223}{15} = 14.9$$

iPhone sales in 3 stores

A	B	C	
25	30	18	
25	30	30	
27	25	29	
30	24	29	
23	26	24	
20	28	26	
25	26.5	26	25.83
$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{Y}$

$$F = \frac{3.49}{14.9} = 0.23$$

$$F = \frac{MSB}{MSW}$$

H_0: All means are equal

H_a: Means are different

Critical region for 95% confidence

```python
from scipy.stats import f
cr = f.ppf(0.95, dfn=2, dfd=15)
cr = 3.68
```

Fail to reject H_0 since observed F statistic 0.23 is less than 3.68

$\alpha = 0.05$

```python
from scipy.stats import f_oneway
a = [25, 25, 27, 30, 23, 20]
b = [30, 30, 21, 24, 26, 28]
c = [18, 30, 29, 29, 24, 26]
f_stat, p_value = f_oneway(a,b,c)
f_stat = 0.234
p_value = 0.793
```

p_value > 0.1

Assumptions of ANOVA

Normality, independent, equal variances

Normality – that each sample is taken from a normally distributed population (Gaussian)

Independence - each sample is drawn independently of the other samples

Equal variance of data in different groups

When assumptions of ANOVA don't hold, we use the Kruskal Wallis test

```
from scipy.stats import f_oneway
a = [25, 25, 27, 30, 23, 20]
b = [30, 30, 21, 24, 26, 28]
c = [18, 30, 29, 29, 24, 26]
f_stat, p_value = f_oneway(a,b,c)
f_stat = 0.234
p_value = 0.793
```

```
from scipy.stats import kruskal
  a = [25, 25, 27, 30, 23, 20]
  b = [30, 30, 21, 24, 26, 28]
  c = [18, 30, 29, 29, 24, 26]
  kruskal_stat, p_value = kruskal(a, b, c)
  kruskal_stat = 0.679
  p_value = 0.711
```