28th November, 2022.

DSML: CC Maths.

Probability 4 - Combinatorics.

Recap:
* Experiment.
* Outcome.
* Sample space
* Event.
* Conditional Probability.
* Multiplication Rule.
* Bayes' theorem.
* Law of total probability.
* Independence of events.

Class starts @ 9:05 p.m.

Today: Combinatorics through problem solving.

Recap: List of formulae:

(a) Conditional probability: $P[A|B] = \dfrac{P(A \cap B)}{P(B)}$

(b) Multiplication rule: $P(A \cap B) = P(A|B) \cdot P(B)$

(c) Bayes' Theorem: $P[B|A] = \dfrac{P[A|B] \cdot P(B)}{P(A)}$

(d) Law of total probability: $P(B) = P(B \cap A) + P(B \cap C)$

$= P[B|A] \cdot P[A] +$

$P[B|C] \cdot P(C)$.

(e) Independence: $P[A|B] = P(A$

(f) Mutually exclusive events: A and B are M.E. if $A \cap B = \emptyset$.

Q] How many ways are there of solving 2 T/F questions?

$2 \cdot 2 = 4.$

$\square \quad \square$

$\uparrow \quad\quad \uparrow$

T, F $\leftrightarrow$ T, F

$\downarrow$

T $\rightarrow$ T $\rightarrow$ T T

T $\rightarrow$ F $\rightarrow$ T F

F $\rightarrow$ T $\rightarrow$ F T

F $\rightarrow$ F $\rightarrow$ F F.

India and Pakistan play a 3-match series. How many results are possible? Note that we consider (Ind, Ind, Pak) different from (Ind, Pak, Ind) etc.

56 users have participated

A	6	18%
B	9	27%
C	8	55% ✓
D	4	0%

I P I P I P

$3 \times 3 \times 3 = 27.$

I → I → I
I → I → P
I → P → I
I → P → P
P → I → I
P → I → P
P → P → I
P → P → P

$3 \times 3 \times 3 = 27.$

In a bowl-out, for a specific ball you have to choose a bowler and a wicket keeper. Suppose you have 5 bowlers and 3 wicket keepers. How many ways can you select for a ball?

55 users have participated

A	8	5%
B	125	16%
C	243	2%
D ✓	15	76%
E	2	0%

$$5 \times 3 = 15$$

(B1, W1), (B2, W1), (B3, W1), (B4, W1), (B5, W1)
(B1, W2), (B2, W2), (B3, W2), (B4, W2), (B5, W2)
(B1, W3), (B2, W3), (B3, W3), (B4, W3), (B5, W3)

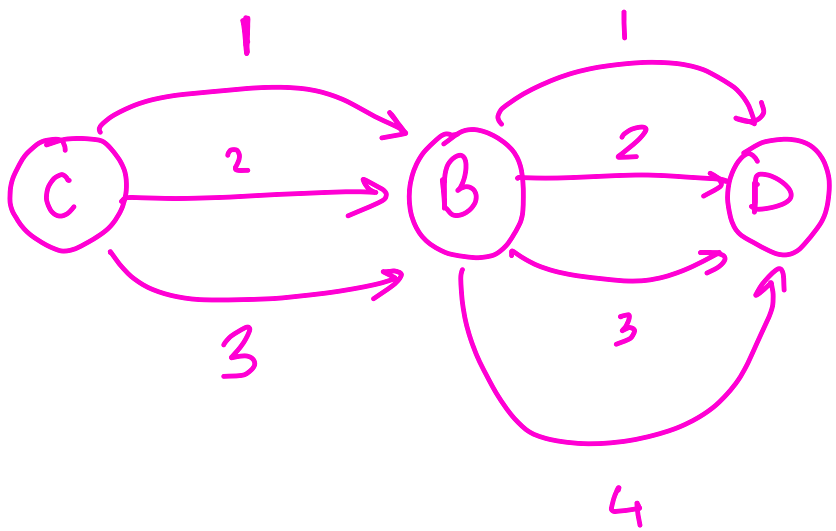
Quiz time!

Quiz Ended!

There are 3 ways to move from Chennai to Bangalore.
There are 4 ways to move from Bangalore to Delhi.
What are the total ways of moving from Chennai to Delhi?

57 users have participated

A	7	4%
B ✓	12	95%
C	81	0%
D	64	2%



$\boxed{\cdot} \times \boxed{} = 12.$

3 4

1 1
2 2
3 3
4

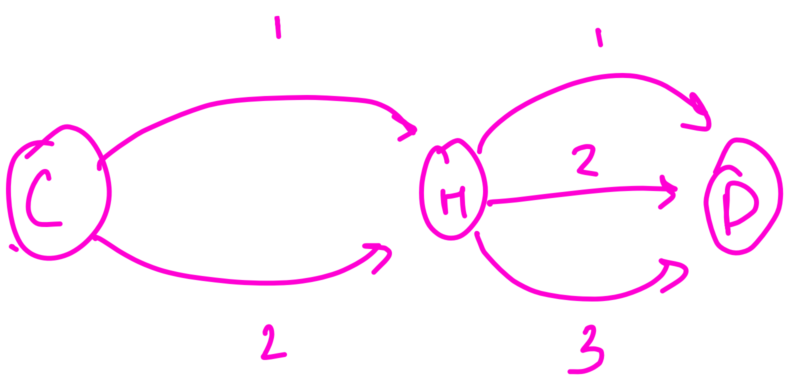
Quiz time!

Quiz Ended!

There are 2 ways to move from Chennai to Hyderabad.
There are 3 ways to move from Hyderabad to Delhi.
What are the total ways of moving from Chennai to Delhi?

60 users have participated

A	6	97%
B	5	3%
C	8	0%
D	9	0%

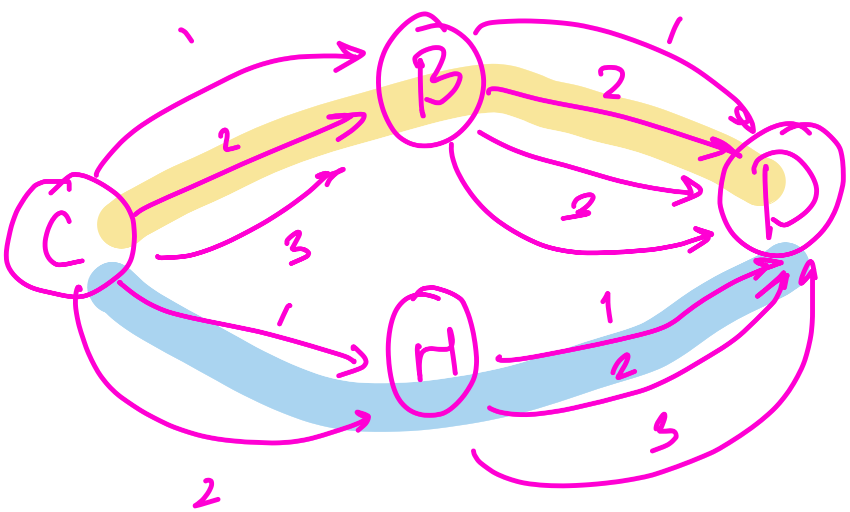


$2 \times 3 = 6$

There are 3 ways to move from Chennai to Bangalore, and 4 ways to move from Bangalore to Delhi.
There are 2 ways to move from Chennai to Hyderabad, and 3 ways to move from Hyderabad to Delhi.
In how many ways can we move from Chennai to Delhi?

62 users have participated

A	12	3%
B	6	2%
C	72	11%
D	18	84%

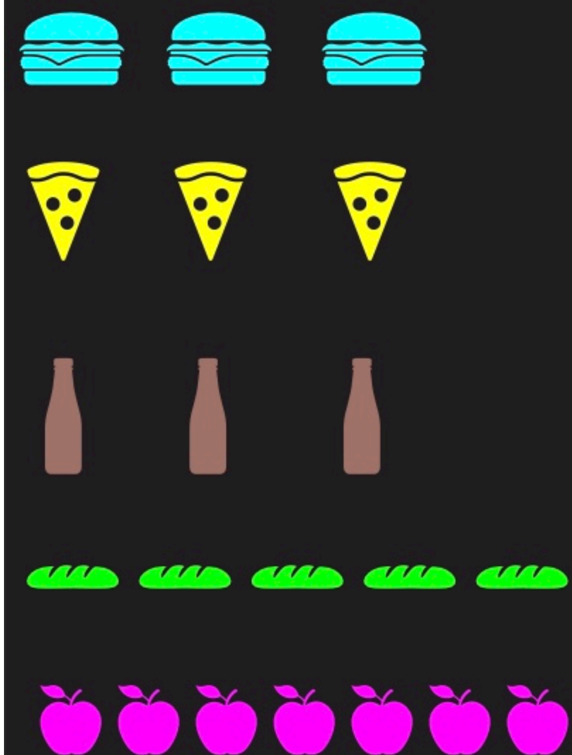


$3 \times 4 = 12$

$+$ $= 18.$

$2 \times 3 = 6.$

A fast food outlet has the following types of items in their menu



3 types of Burgers

3 types of Pizza

3 types of Drinks

5 types of Sandwiches

7 types of Fruits

You can choose one of the following combos:

- 1 Burger and 1 Sandwich → $3 \times 5 = 15$
- 1 Fruit and 1 Drink → $7 \times 3 = 21$
- 1 Pizza → 3

How many such combos can you make?

$15 + 21 + 3 = 39.$

Permutations

↳ Arrangement of objects.

↳ Order matters.

$$\underline{(i, j)} \neq \underline{(j, i)}$$

$$ab \neq ba.$$

Quiz time!

Quiz Ended!

a a a ×

Count the number of ways to arrange 3 characters a, b, & c

60 users have participated

A	8	17%
B	9	12%
C ✓	6	70%
D	3	2%

$$3 \times 2 \times 1 = 6$$

a → b → c (1)

a → c → b (2)

b → a → c (3)

b → c → a (4)

c → a → b (5)

c → b → a (6)

$3!$ → "3 factorial"

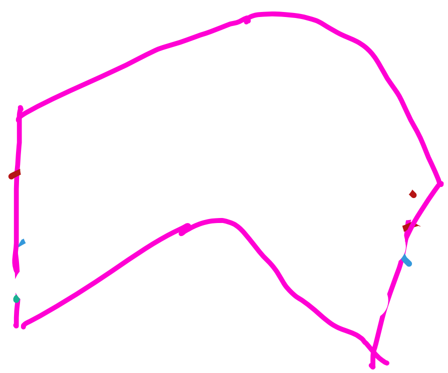
$$n! = n \cdot (n-1) \cdot (n-2) \cdots\cdots 1$$

Q] How many ways are there of arranging 4 distinct items?

$$\underline{4}\ \underbrace{\underline{3}\ \underline{2}}_{6}\ \underline{1} = \underline{\underline{24}}$$

$$2! = 2 \times 1 = 2.$$

Q]



$$0! = 1$$

Given 5 different characters, in how many ways can we arrange them in ~~2~~ 5. places?

57 users have participated

A	10	16%
B ✓	20	79% ✓
C	32	4%
D	120	2%

$$\rightarrow \frac{5!}{3!}$$

'A'

'B'

'C'

'D'

'E'

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\underline{5} \times \underline{4} \times \underbrace{\underline{3} \times \underline{2} \times \underline{1}}$$

A way to count the number of ways to arrange n items in r positions.

A → B, C, D, E

B

C

D

E

"AB CDE"

"AB DCE"

"AB CED"

"AB DEC"

"AB ECD"

"AB EDC"

There are 4 players P1, P2, P3, and P4 who can play in the top-order positions of 1, 2, and 3. How many arrangements of top-order can we make from 3 of these 4 players?

55 users have participated

A	12	11%
B	16	0%
C	9	2%
D	4	15%
E ✓	24	73%
F	48	0%

$${}^{4}P_{3} = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \underline{24.}$$

$\underline{4}\ \underline{3}\ \underline{2} = 24.$

In how many ways can we select 3 batsmen from a pool of 4 cricketers.

50 users have participated

A	24	70%
B	4	28%
C	3	2%
D	32	0%

$$\frac{24}{6} = \underline{\underline{4}}$$

P1 P2 P3.
P1 P3 P2
P2 P1 P3
P2 P3 P1
P3 P1 P2
P3 P2 P1

$\rightarrow$ 1

4 items out of which 9 have to make combinations of 2 items.

I_1, I_2, I_3, I_4

How many ways to repeat a combination of 2 items?
2!

(I_1, I_3) and (I_3, I_1) → counted as the same combination.

^nP_r → Count the number of ways of arranging n items in r slots.

$$\frac{4!}{(4-2)!} = \frac{4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} = \frac{\cancel{12}}{\cancel{2}} = \underline{\underline{6}}$$

$$\underline{{}^{n}C_{r}} = \frac{{}^{n}P_{r}}{(r!)} = \frac{n!}{(n-r)! \cdot r!}$$

total number of
way I can repeat/permute
r items.

$${}^{n}C_{r} = {}^{5}C_{2} = \frac{5!}{3! \times 2!} = \frac{5 \times \cancel{4}}{\cancel{2}} = 5 \times 2$$

$$= \underline{\underline{10}}$$

Suppose we have to select 2 players out of 5 players in our team. In how many ways can we do this?

56 users have participated

A	10	82% ✓
B	20	14%
C	9	2%
D	32	2%

End Quiz Now

~ 1000

J F M . . . D

Three students are randomly chosen from across cohorts at Scaler. Each student is equally likely to belong to any of the 12 cohorts starting in each month: January, Feb, ... , December $\rightarrow 12$

What is the probability that no two students belong to the same cohort?

Probability : $\left\{ \frac{\text{No. of ways of making combinations}}{\text{Total no. of ways.}} \right.$

$\underline{12} \times \underline{12} \times \underline{12} = 12^3.$

$\underline{12} \times \underline{11} \times \underline{10}$

J
F
M
⋮
D

$\frac{\cancel{12} \times 11 \times \cancel{10}^{5}}{\cancel{12} \times 12 \times \cancel{12}_{6}} = \frac{55}{72} \approx \boxed{0.76}$

s.t. no two student is in the same cohort.

Nadal Vs Federer on Clay

Nadal wins 70% of the points he plays against Federer on clay

Suppose a tennis game between Nadal and Federer is on Deuce.

What is the probability of Nadal winning the game?

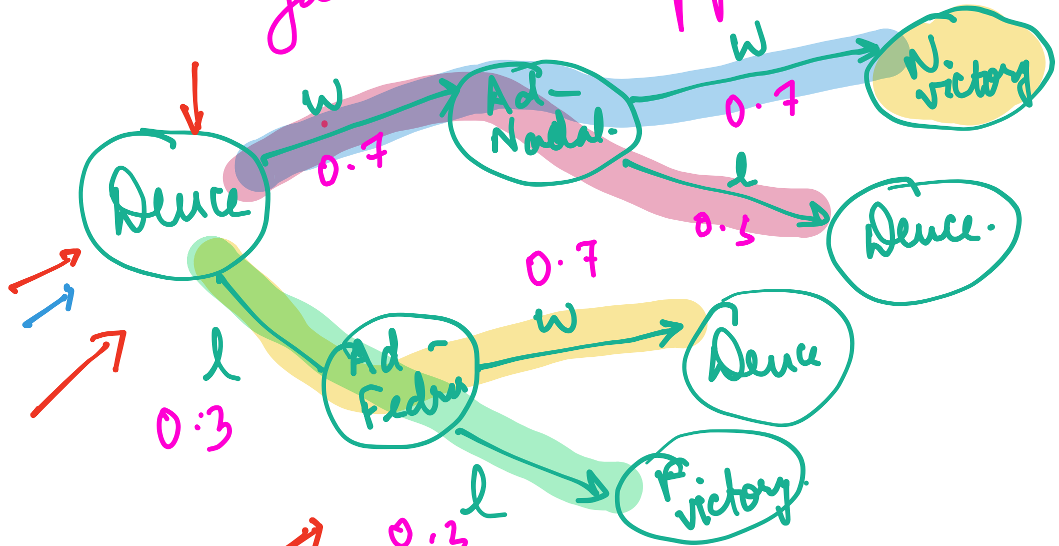
N → event that Nadal wins.

$P[N \text{ scoring a point}] = 0.7$.

W → Event that Nadal wins a point.

Deuce

A player has to score 2 consecutive points against the opponent.



$$
\begin{aligned}
P[N] &= P[N \mid ww] \cdot P[ww] \\
&+ P[N \mid wl] \cdot P[wl] \\
&+ P[N \mid lw] \cdot P[lw] \\
&+ P[N \mid ll] \cdot P[ll].
\end{aligned}
$$

$P[N] \rightarrow$ probability that Nadal wins.

$\{ww, wl, lw, ll\}$

$w \rightarrow$ Nadal scores a point.

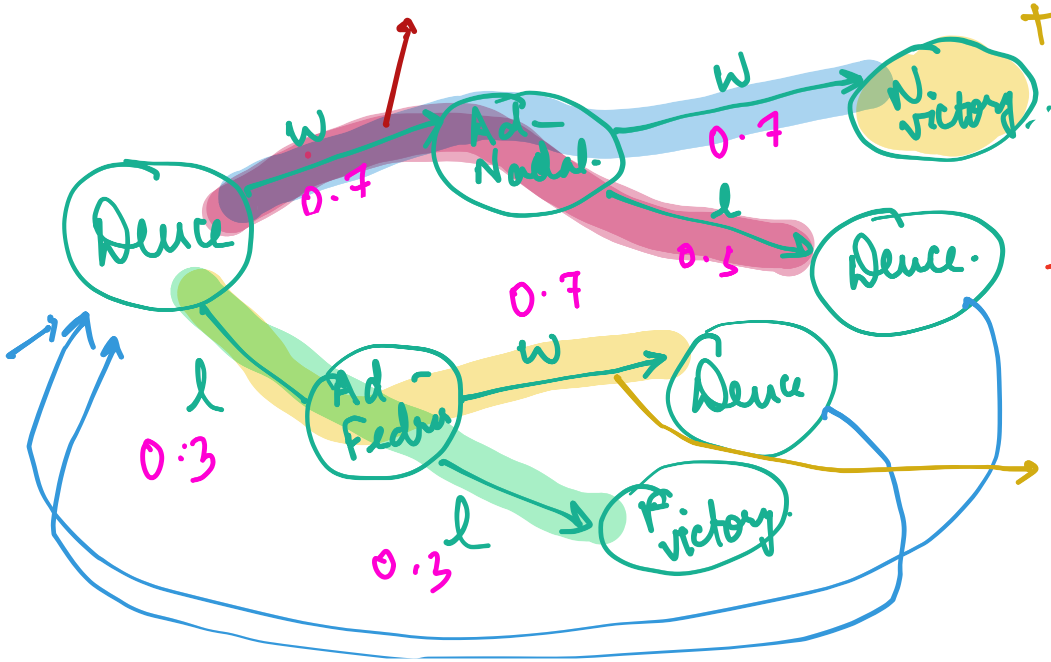
$l \rightarrow$ Federer scores a point.

$P[N] =$ 0.49

$P[N|wl]$

$+ P[N] \cdot 0.21$

$+ P[N] \cdot 0.21$



$P[N] = 0.49 + P[N](0.21 + 0.21)$

$P[N|lw]$

$$P[N] = 0.49 + P[N] \cdot 0.21 + P[N] \cdot 0.21$$

$$P[N] = 0.49 + P[N] \cdot (0.21 + 0.21)$$

$$P[N] - 0.42\, P[N] = 0.49$$

$$\Rightarrow P[N] = \frac{0.49}{1 - 0.42} = 0.84.$$

'a', 'b', 'c' → 6.

_ a _
① ②

n=1 ['a']

n=2 ['ab', 'ba']

n=3 [ ]

_ a _ b _
① ② ③
[cab, acb, abc]

_ b _ a _
① ② ③
[cba, bca, bac]

$\boxed{0! = 1.}$

"exactly" vs "at least"

$\boxed{N == k}$

$\boxed{N \leq k.}$

Numerator

12 — 11 — 10

Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sept.
Oct.
Nov.
Dec.

Feb

Jan — Feb — ~~Jan~~.

Belong to the same

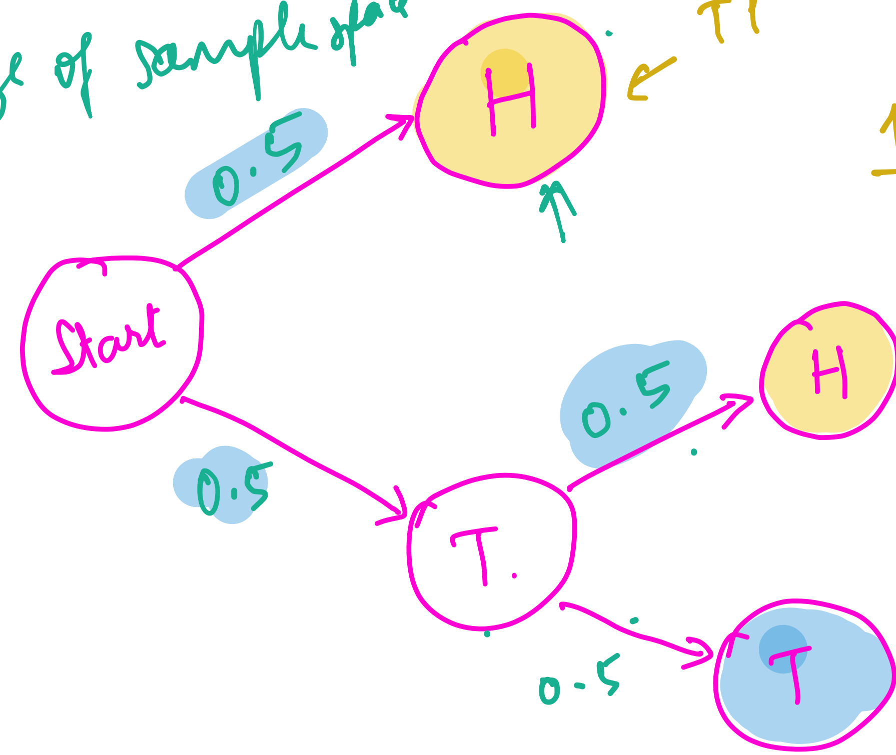
$$\frac{12 \times 11 \times 10}{\cancel{3!}} = 1320.$$

12 — 12 — 12

$$\frac{\cancel{12}^{3}}{\cancel{3!}}$$

JFM ≠ MFJ.

$$\frac{\text{Size of event}}{\text{Size of sample space}}$$



T1

W: Event that the gambler wins.

W: {H, TH}

T2

{H, TH, TT}

sample space

$$P[W] = P(W \cap T1) + P(W \cap T2).$$

$$= \underbrace{P[T1]}_{0.5} \cdot \underbrace{P[W \mid T1]}_{1} + \underbrace{P[T2]}_{0.5 \times 0.5} \cdot \underbrace{P[W \mid T2]}_{1}$$

$$= 0.5 \quad 1 \quad + \quad 0.5 \times 0.5 \quad 1$$

$$= 0.5 + 0.25 = \underline{\underline{0.75}}$$