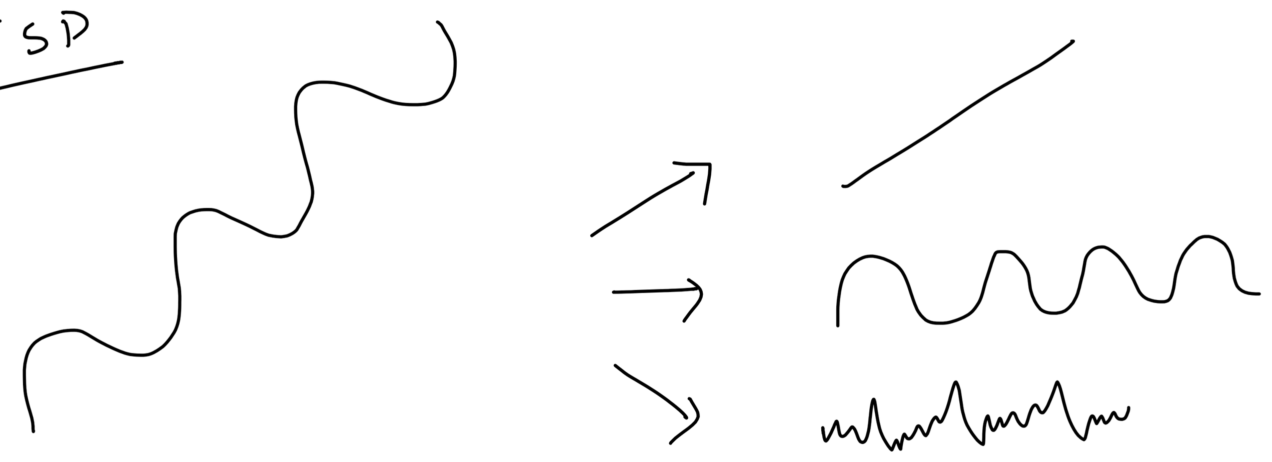
[ARIMA models] forecasting

TSP



Simple Methods
- naive
- mean
- Seasonal naive
- drift
- MA

Stationary T.S ✗

mean, variance,

Smoothing family

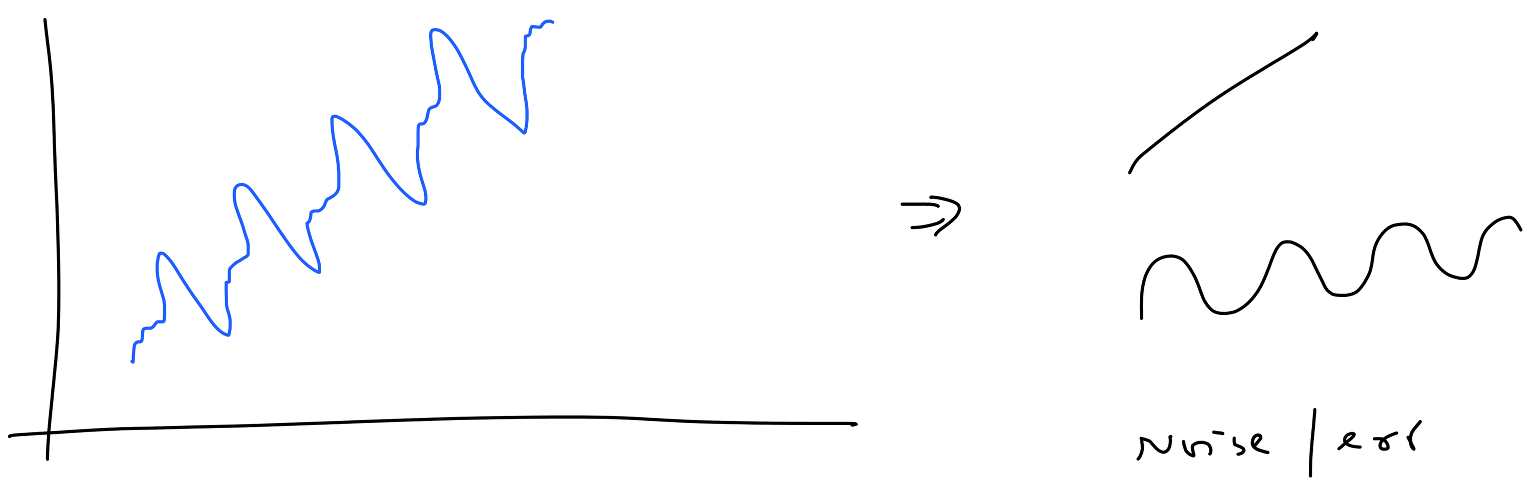
SES	level
DES	trend
TES	Seasonality

N.S TS → S. TS

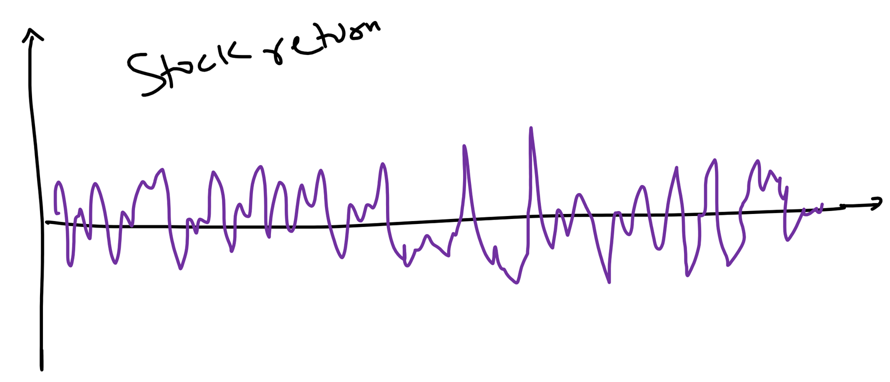
→ decomposition

→ differencing diff(1) $y_2 - y_1 \Rightarrow \frac{y_2 - y_1}{t_2 - t_1} = \frac{\Delta y}{\Delta t}$

12 − 11



Noise | error



$y(t) = b(t) + s(t) + e(t)$

$e(t) \to z(t)$, $e'(t)$ truly unpredictable

ARIMA family
- → AR
- → MA
- → ARMA
- → ARIMA
- → SARIMA
- → SARIMAX → New Problem statement

AR → Auto Regressive AR(r)

$$\boxed{\hat{y}_t = \alpha_1 \cdot y_{t-1} + \alpha_2 \cdot y_{t-2} + \alpha_0}$$ AR(2)

$$\hat{y}_t = \alpha_1 \cdot y_{t-1} + \alpha_2 \cdot y_{t-2} + \alpha_3 \cdot y_{t-3} + \alpha_0$$ AR(3)

$$\boxed{\hat{y}_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \cdots + \alpha_p \cdot y_{t-p} + \alpha_0}$$

Apply Linear Reg
learn α's.

Sales
27
14
15
20
25
30
37
26
19

lr.fit (X, y)

$X \to ? \to (m,n)$, $y \to ? \to (m,1)$

AR(3)

← input — output

α_3 y_{t-3}	α_2 y_{t-2}	α_1 y_{t-1}	y_t
27	14	15	20
14	15	20	25
15	20	25	30
20	25	30	37
⋮	⋮	⋮	⋮
30	37	26	19

X Y

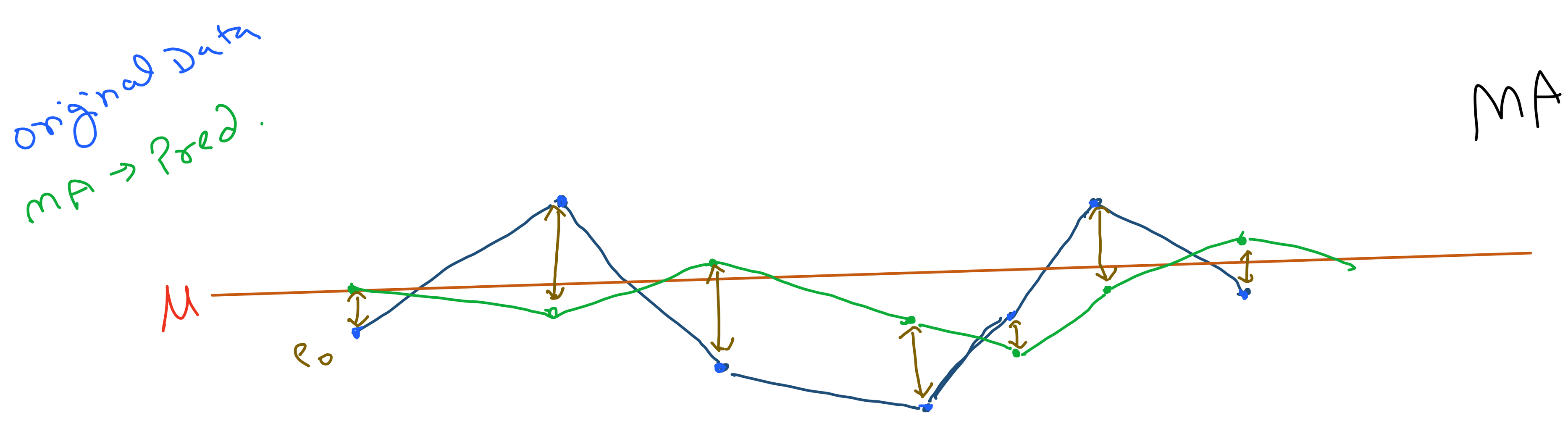
pd.Series.shift (3)

Better name!!

MA(q) → Moving Avg → forecasting

≠ rolling avg

MA(1)



"MA key Idea" :→ I should learn from my Previous errors

$$\hat{y}_t = \mu + \beta_1 e_{t-1}$$ MA(1)

$$\hat{y}_t = \mu + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \beta_3 e_{t-3}$$ MA(3)

e_0

↳ $\beta_1 \to \hat{y}_1 - y_1$

↓

e_1

↳ $\beta_2 \to \hat{y}_2 - y_2$

$\to \beta_2 \cdot e_2$

$\downarrow$

e_2

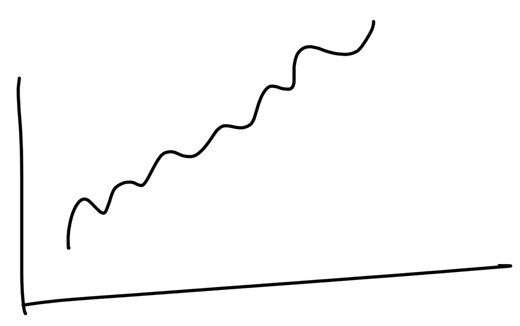
β_3

$e_3 \ \dots$

ARMA(p, q)

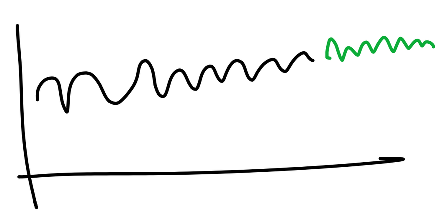
$$\boxed{\begin{aligned}\hat{y}_t = \ & c + \alpha_1 y_{t-1} + \alpha_2 \cdot y_{t-2} + \dots + \alpha_p \cdot y_{t-p} \\ & + \beta_1 \cdot e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q \cdot e_{t-q}\end{aligned}}$$

$y(t)$



diff

$\boxed{y'(t)}$



Stationary

$\downarrow$

AR(15)

$\downarrow$

$\boxed{\hat{y}'(t)} \xrightarrow[\text{integration}]{\text{go}} \hat{y}(t)$

	Cont.		discrete
$y \to y'$	$\frac{dy}{dt}$	$\Rightarrow$	$y_t - y_{t-1}$ y.diff(1)
$y' \to y$	$\int y' \cdot dt$	$\Rightarrow$	y'.cumsum()

- - - - train - - → ← - - - - - - test - - - - - →

$y = \boxed{100}$ 120 110 115 96

$y' \Rightarrow$ 20 −10 +5 −20

AR|MA|ARMA $\Rightarrow$ 19 −7 +3 −17

$\downarrow$

$\hat{y}'$

(Prediction)

$\downarrow$

$\hat{y} \Rightarrow$ 100 + 19 → 119 − 7 → 112 + 3 → 115 − 17

$\Rightarrow$ 119 112 115 98

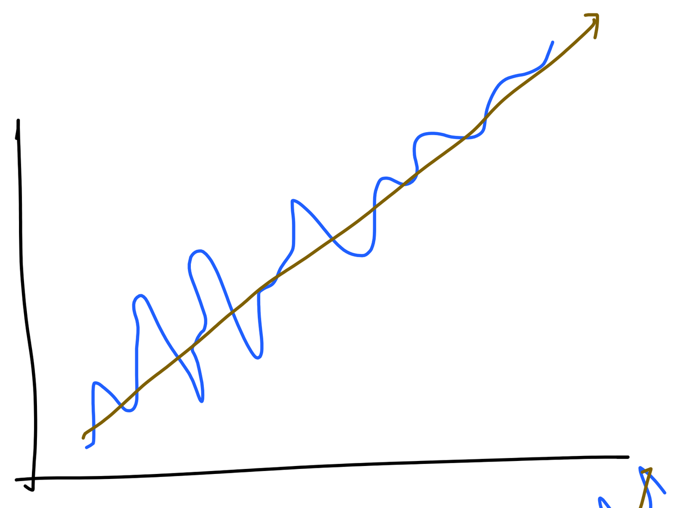
ARIMA (p, d, q)

I → integration

d → d = {0, 1, 2, 3}

diff
↳ AR, MA, ARMA → integration .

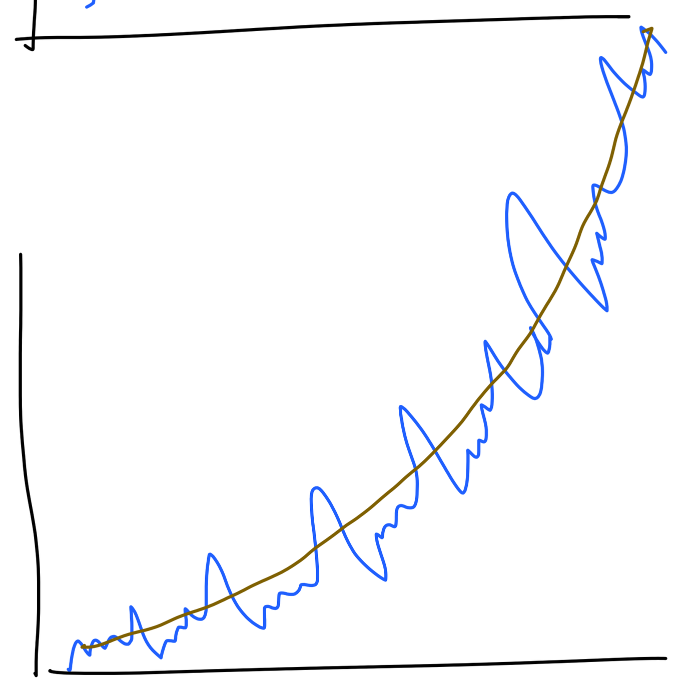
ARIMA



$b(t) = mt + c$

$b'(t) = m$

.diff(1)



$b(t) = m_1 t^2 + m_2 t + c$

$b'(t) = 2m_1 t + m_2$

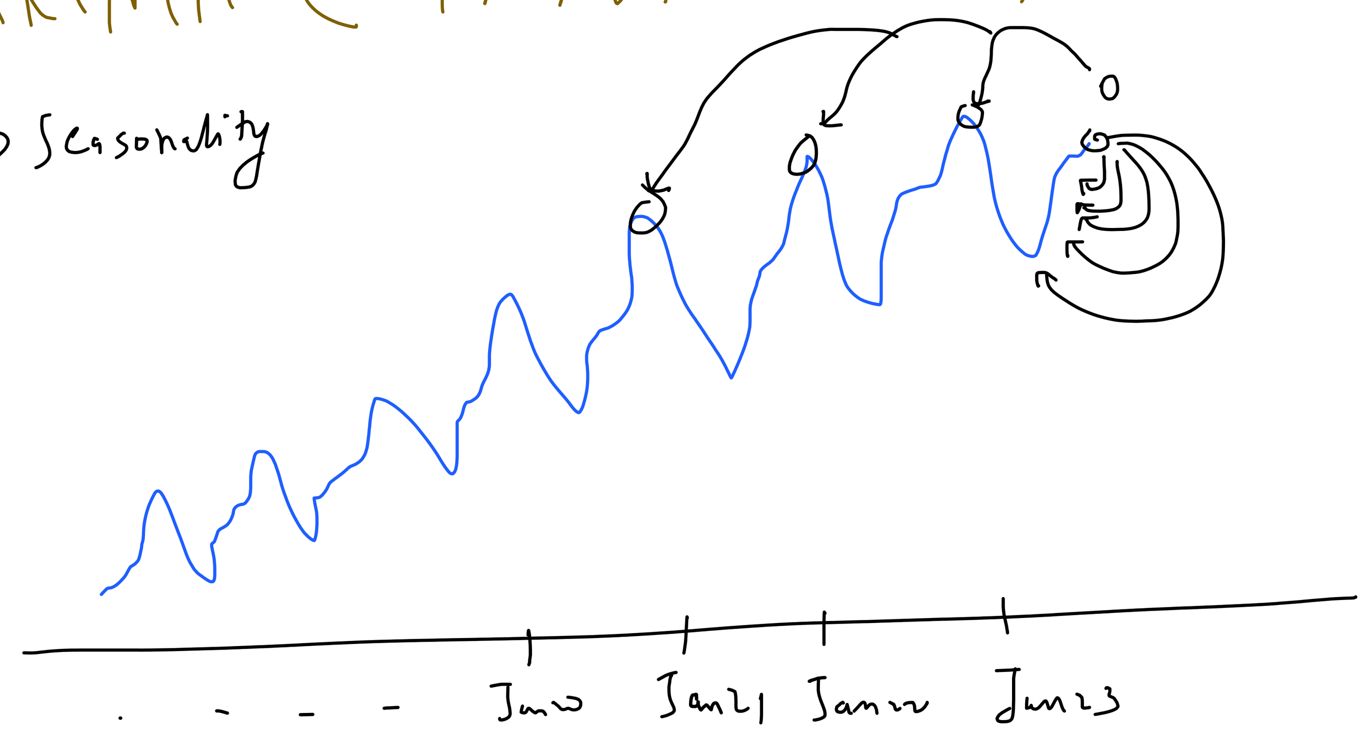
$\boxed{b''(t) = 2m_1}$

.diff( ) .dif( )

$y \longrightarrow y' \longrightarrow y''$

N.S N.S [Stationary]

SARIMA (p, d, q, P, D, Q, S)
↳ Seasonality



$y_{t-1}\ y_{t-2} \ldots t-p$

$e_{t-1}\ e_{t-2} \ldots t-q$

AR(p)

$$\hat{y}_t = \alpha_1 \cdot y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_p \cdot y_{t-p}$$

SAR(P)

$$\hat{y}_t = \gamma_1 y_{t-s} + \gamma_2 y_{t-2s} + \dots + \gamma_P y_{t-Ps} + \gamma_0$$

MA(q)

SMA(Q)

$$\hat{y}_t = \delta_1 e_{t-s} + \delta_2 e_{t-2s} + \dots + \delta_Q e_{t-Qs} + \delta_0$$

Finally SARIMA

$$\begin{aligned}\hat{y}_t = C &+ \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} \\ &+ \beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q e_{t-q} \\ &+ \gamma_1 y_{t-s} + \gamma_2 y_{t-2s} + \dots + \gamma_P y_{t-Ps} \\ &+ \delta_1 e_{t-s} + \delta_2 e_{t-2s} + \dots + \delta_Q e_{t-Qs}\end{aligned}$$

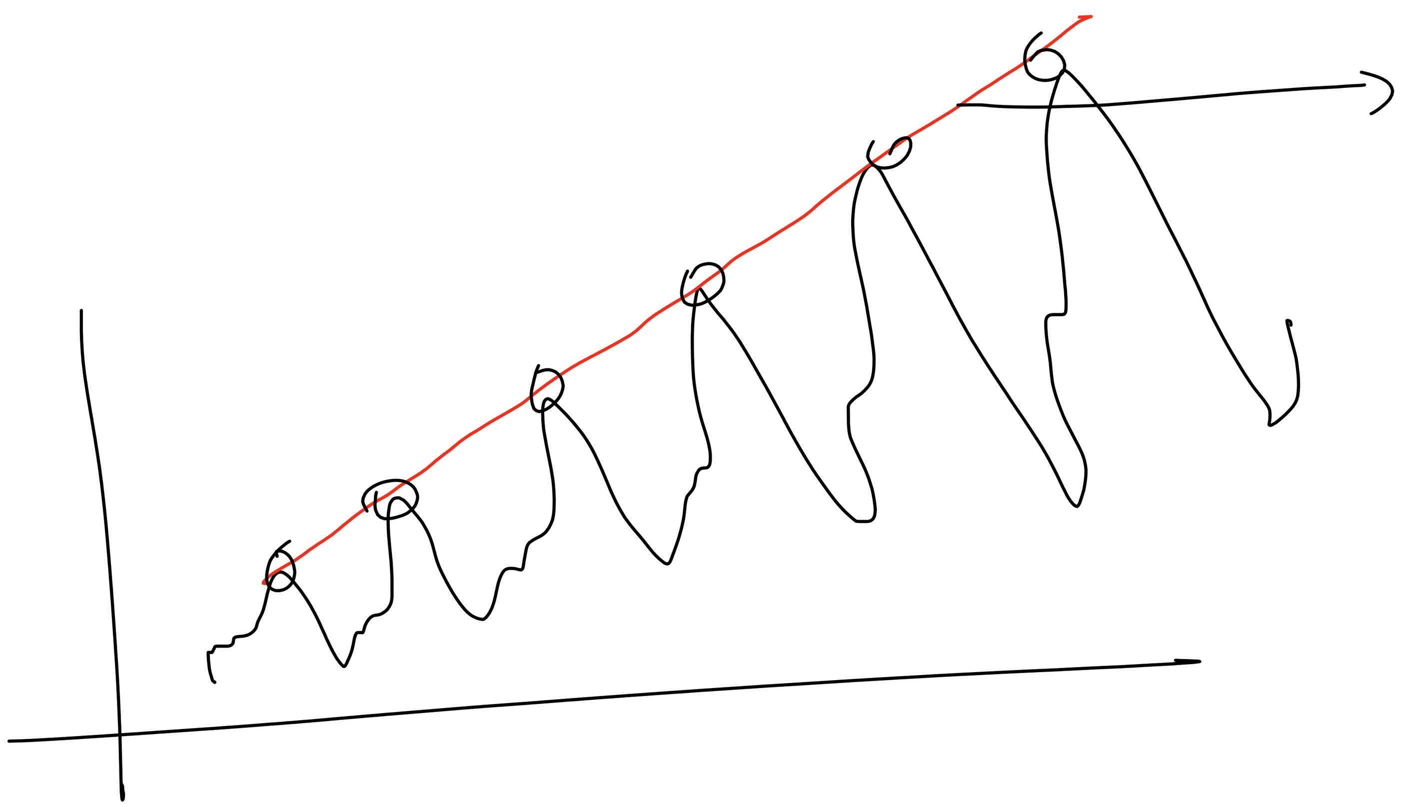
AR

MA

SAR

SMA

'D' → why to differentiate Seasonality



→ trend.