UNIT - 1 Analysis discrete random variable

Discrete random variable
Probability distribution and probability mass function
Cumulative distribution function
Mean and variables of discrete random variables
Discrete uniform distribution
Binomial distribution
Geometric distribution
Negative binomial distribution
Poisson distribution

Random variable

It is a function that assigns a real numbers to each outcome in the sample space in the random expt.

Ex: Tossing a coin twice
Tossing is an experiment. Getting head or tail i.e
{HH, HT, TH, TT} = Sample space.
Random variable can take the value 0, 1, 2
function: Getting number of heads.
There are two types of random variables
1. Discrete random variable
2. Continuous random variable

1. Discrete random variable

If a random variable takes a finite set of values, then it is called a discrete random variable i.e a random variable assume only a finite number or countabaly infinite number of possible discrete values.

Ex: Number of heads in tossing 4 coins
Number of CS students in SIT

Continuous random Variables

A random variable x is said to be continuous if it takes all the possible values between certain limits or an interval i.e a random variable assumes an infinite number of uncountable values is called Continuous random variables.

Ex: Electric circuit line, temperature etc

Analysis of discrete random variable

Many physical system can be modelled by the same similar random experiments and random variables. The distribution of random variables involved in each of these common system can be analyzed and the result of that analysis can be used in different applications and examples, the analysis of several random experiments & discrete random variables that frequently arise in applications.

Probability distribution & Probability mass function

The Probability distribution of a random variable X is a description of the probability associated with the possible value of X. For a discrete random variable, the distribution is often specified by just a list of the possible values along with probability of each. The values of X and P(X) are usually given in the form of a table called a Probability distribution table.

X	x_1	x_2	x_3	......	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	......	$P(x_n)$

Probability mass function

For a discrete random variable 'x' with possible values $x_1, x_2, \dots x_n$, A probability mass function is a function such that

i) $f(x_i) \geq 0$

2) $\sum_{i=1}^{n} f(x) = 1$

3) $f(x_i) = P(X = x_i)$

PROBLEMS

① Sample space of random experiment is $\{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows

Outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the probability mass function of x. Use the probability mass function to determine the following probabilities

a) $P(X = 1.5)$ b) $P(0.5 < x < 2.7)$ 3) $P(x > 3)$

4) $P(0 \leq x \leq 2)$ 5) $P(X = 0 \text{ or } X = 2)$

x	0	0	1.5	1.5	2	3
$P(x)$	1/6	1/6	1/6	1/6	1/6	1/6

a) $P(X = 1.5) = 1/6 + 1/6 = 1/3$

b) $P(0.5 < x < 2.7) = 1/6 + 1/6 + 1/6 = 1/2$

c) $P(x > 3) = 0$

d) $P(0 \leq x \leq 2) = 1/6 + 1/6 + 1/6 + 1/6 = 2/3$

e) $P(X = 0 \text{ or } X = 2) = 1/6 + 1/6 + 1/6 = 1/2$

...ify that the following functions are probability mass ...nctions & determine the required probabilities.

x	-2	-1	0	1	2
$f(x)$	$1/8$	$2/8$	$2/8$	$2/8$	$1/8$

1) $P(X=2)$ 2) $P(x>-2)$ 3) $P(-1\le x\le 1)$
4) $P(X\le -1$ or $x=2)$

$f(x)\ge 0$
and $\Sigma f(x) = 1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$
$\therefore$ It is Probability mass function

1) $P(x=2) = 1/8$
2) $P(X>-2) = 2/8 + 2/8 + 2/8 + 1/8 = 7/8$
3) $P(-1\le x\le 1) = 2/8 + 2/8 + 2/8 = 6/8$
4) $P(x\le -1$ or $x=2) = 1/8 + 2/8 + 1/8 = 1/2$
5) $P(x\le 2) = 1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$

write $P(X=$ something$)$ + something

③ Verify that the following functions are probability mass function & determine the required probabilities for $f(x) = (8/7)(1/2)^x$ where $x = 1, 2, 3$

a) $P(X\le 1)$ b) $P(X>1)$ c) $P(2<x<6)$
d) $P(X\le 1$ or $x>1)$

$f(1) = (8/7)(1/2) = 8/14 = 4/7$
$f(2) = (8/7)(1/2)^2 = \frac{8}{7}\times\frac{1}{4} = \frac{2}{7}$
$f(3) = (8/7)(1/2)^3 = \frac{8}{7}\times\frac{1}{8} = \frac{1}{7}$

$f(x)\ge 0$ and $\Sigma f(x) = \frac{4}{7}+\frac{2}{7}+\frac{1}{7} = 1$
$\therefore$ It is a Probability mass function

x	1	2	3
$f(x)$	$4/7$	$2/7$	$1/7$

a) $P(x\le 1) = P(x=1) = 4/7$
b) $P(x>1) = P(x=2) + P(x=3) = 3/7$
c) $P(2<x<6) = P(x=3) = 1/7$
d) $P(x\le 1$ or $x>1) = P(x=1) + P(x=2) + P(x=3)$
$= 1$

④ Verify that the following functions is probability mass function and determine the required probabilities for $f(x) = \frac{2x+1}{25}$, $x = 0, 1, 2, 3, 4$

1) $P(X=4)$ 2) $P(X\le 1)$ 3) $P(2\le x\le 4)$
4) $P(x>-10)$

$f(0) = \frac{2(0)+1}{25} = 1/25$
$f(1) = \frac{2(1)+1}{25} = 3/25$
$f(2) = \frac{2(2)+1}{25} = 5/25$
$f(3) = \frac{2(3)+1}{25} = 7/25$
$f(4) = \frac{2(4)+1}{25} = 9/25$

$f(x)\ge 0$ and $\Sigma f(x) = 1/25 + 3/25 + 5/25 + 7/25 + 9/25 = 1$
$\therefore$ It is a probability mass function

x	0	1	2	3	4
$f(x)$	$1/25$	$3/25$	$5/25$	$7/25$	$9/25$

1) $P(x=4) = 9/25$
2) $P(x\le 1) = P(x=0) + P(x=1)$
$= 1/25 + 3/25 = 4/25$
3) $P(2\le x\le 4) = P(x=2) + P(x=3) + P(x=4)$
$= 5/25 + 7/25 + 9/25$
$= 21/25$
4) $P(x>-10) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$
$= 1$

ify that the following functions are probability mass ...ctions & determine the required probabilities.

x	-2	-1	0	1	2
$f(x)$	$1/8$	$2/8$	$2/8$	$2/8$	$1/8$

1) $P(X=2)$ 2) $P(x>-2)$ 3) $P(-1\le x\le 1)$

4) $P(X\le -1$ or $x=2)$

$f(x)\ge 0$

and $\Sigma f(x) = 1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$

$\therefore$ It is Probability mass function

1) $P(x=2) = 1/8$

2) $P(x>-2) = 2/8 + 2/8 + 2/8 + 1/8 = 7/8$

3) $P(-1\le x\le 1) = 2/8 + 2/8 + 2/8 = 6/8$

4) $P(x\le -1$ or $x=2) = 1/8 + 2/8 + 1/8 = 1/2$

5) $P(x\le 2) = 1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$

write $P(X=$ something$)$ + something

③ Verify that the following functions are probability mass function & determine the required probabilities for $f(x) = (8/7)(1/2)^x$ where $x = 1, 2, 3$

a) $P(X\le 1)$ b) $P(X>1)$ c) $P(2<x<6)$

d) $P(x\le 1$ or $x>1)$

$f(1) = (8/7)(1/2) = 8/14 = 4/7$

$f(2) = (8/7)(1/2)^2 = \frac{8}{7}\times\frac{1}{4} = \frac{2}{7}$

$f(3) = (8/7)(1/2)^3 = \frac{8}{7}\times\frac{1}{8} = \frac{1}{7}$

$f(x)\ge 0$ and $\Sigma f(x) = \frac{4}{7}+\frac{2}{7}+\frac{1}{7} = 1$

$\therefore$ It is a Probability mass function

x	1	2	3
$f(x)$	$4/7$	$2/7$	$1/7$

a) $P(x\le 1) = P(x=1) = 4/7$

b) $P(x>1) = P(x=2) + P(x=3) = 3/7$

c) $P(2<x<6) = P(x=3) = 1/7$

d) $P(x\le 1$ or $x>1) = P(x=1) + P(x=2) + P(x=3)$

$= 1$

④ Verify that the following functions is probability mass function and determine the required probabilities for $f(x) = \frac{2x+1}{25}$, $x = 0, 1, 2, 3, 4$

1) $P(X=4)$ 2) $P(X\le 1)$ 3) $P(2\le X\le 4)$

4) $P(X>-10)$

$f(0) = \frac{2(0)+1}{25} = 1/25$

$f(1) = \frac{2(1)+1}{25} = 3/25$

$f(2) = \frac{2(2)+1}{25} = 5/25$

$f(3) = \frac{2(3)+1}{25} = 7/25$

$f(4) = \frac{2(4)+1}{25} = 9/25$

$f(x)\ge 0$ and $\Sigma f(x) = 1/25 + 3/25 + 5/25 + 7/25 + 9/25 = 1$

$\therefore$ It is a probability mass function

x	0	1	2	3	4
$f(x)$	$1/25$	$3/25$	$5/25$	$7/25$	$9/25$

1) $P(X=4) = 9/25$

2) $P(X\le 1) = P(X=0) + P(X=1)$

$= 1/25 + 3/25 = 4/25$

3) $P(2\le X\le 4) = P(X=2) + P(X=3) + P(X=4)$

$= 5/25 + 7/25 + 9/25$

$= 21/25$

4) $P(X>-10) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 1$

Verify the following function for Probability mass function & determine following probabilities for

Doubt
Rem

$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^x$, $x = 0, 1, 2, 3, 4$ VVIP

1) $P(X=2)$ 2) $P(X \le 2)$ 3) $P(X > 2)$ 4) $P(X \ge 1)$

$f(0) = \frac{3}{4}\left(\frac{1}{4}\right)^0 = \frac{3}{4}$ $f(3) = \frac{3}{4} \times \frac{1}{64} = \frac{3}{256}$

$f(1) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ $f(4) = \frac{3}{4} \times \frac{1}{256} = \frac{3}{1024}$

$f(2) = \frac{3}{4} \times \frac{1}{16} = \frac{3}{64}$

$f(0) = \frac{768}{1024}$ $f(1) = \frac{192}{1024}$ $f(2) = \frac{48}{1024}$ $f(3) = \frac{12}{1024}$

$f(4) = \frac{3}{1024}$

$f(x) \ge 0$ and $\Sigma f(x) = \frac{1023}{1024} = 0.99989 = 1$

$\therefore$ This is ~~not~~ a valid probability mass function

x	0	1	2	3	4
$f(x)$	$\frac{768}{1024}$	$\frac{192}{1024}$	$\frac{48}{1024}$	$\frac{12}{1024}$	$\frac{3}{1024}$

$\Sigma f(x) = \frac{3}{4}\left[\frac{1}{4^x}\right] = \frac{3}{4}\sum_{x \ge 0}\frac{1}{4^x}$

$= \frac{3}{4}\left(\frac{1}{1-1/4}\right)$

$= \frac{3}{4} \times \frac{4}{3} = 1$

1) $P(X=2) = \frac{3}{64}$

2) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{3}{4} + \frac{3}{16} + \frac{3}{64}$

$= \frac{48}{64} + \frac{12}{64} + \frac{3}{64}$

$= \frac{53}{64}$

3) $P(X > 2) = P(X=3) + P(X=4) = \frac{3}{256} + \frac{3}{1024}$

$= \frac{15}{1024}$

4) $P(X \ge 1) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \frac{192}{1024} + \frac{48}{1024} + \frac{12}{1024} + \frac{3}{1024} = \frac{255}{1024}$

⑥ An article in knee surgery cides a success rate more than 90% for meniscal tears with rim width of less than 3mm but only 67% success rate for tears with 3 to 6mm. If you are unlucky enough to suffer a meniscal tear for less than 3mm on your left knee and one of the width 3 to 6mm on your right knee, what is the probability mass function of the number of successful surgeries? Assume surgeries are independent.

Left knee surgery successful probability is 90%.

$P_1 = 0.9$

Left knee surgery failure probability is $P_2 = 1 - P_1 = 0.1$

Right knee successfull $P_3 = 0.67$

Right knee failure $P_4 = 0.33$

Let the random variable X denotes the number of successful surgeries $X = 0, 1, 2$

$P(X=0)$ = surgery on both knees failure

$= 0.1 \times 0.33 = 0.033$

$P(X=1)$ = surgery on either of the knee fail

$= P_1 P_4 + P_2 P_3$

$= 0.9 \times 0.33 + 0.1 \times 0.67$

$= 0.364$

$P(X=2) = P_1 \times P_3$

$= 0.9 \times 0.67$

$= 0.603$

x	0	1	2
$f(x)$	0.033	0.364	0.603

⑦ An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are in spectate and that the classification are independent. Let the random variable 'x' denotes the number of parts that are correctly classified. Determine the probability mass function of 'x'

The Probability of successfully classifying any given part is 0.98. There are 3 parts in total and the classifications are independent.

The random variable 'x' is the number of correctly classified part out of total 3.

The range of random variable $x = 0, 1, 2$ and 3

Probability of unsuccessfull ~~P(x=0)~~
$= 1 - P(\text{success})$
$= 1 - 0.98$
$= \underline{0.02}$

$P(x=0) =$ Probability of unsuccess
$= (0.02)(0.02)(0.02)$
$= 8 \times 10^{-6}$.

follow this

$P(x=1) = (0.02)(0.02)(0.98)(3) \rightarrow \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c$
$= 3.92 \times 10^{-4} \times 3 = 0.00117 = \underline{0.0012}$

$P(x=2) = (0.98)(0.98)(0.02) \times 3$
$= 1.92 \times 10^{-2} \times 3 = \underline{0.0576}$

$P(x=3) = (0.98)(0.98)(0.98)$
$= \underline{0.941}$

$P(x=2) = [(0.98)(0.98)(0.02)]3 \quad (\bar{a}bc + a\bar{b}c + ab\bar{c})$
$= \underline{0.0576}$

X	0	1	2	3
f(X)	8×10^{-6}	0.0012	0.0576	0.941

In a semiconductor manufacturing process 3 waffers from a lot are tested. Each vapour is classified as pass or fail. Assume that the probability that a vapour passes the test is 0.8 and that the vapours are independent. Determine the probability mass function of the number of vapours from a lot that pass the test.

Probability of vapour passing test = 0.8

There are 3 vapours in total.

Let 'x' be the random variable that represents the number of vapours that passes the test.

The range of random variable $x = 0, 1, 2$ and 3.

Probability of success = 0.8

Probability of failure $= 1 - P(\text{success})$
$= 1 - 0.8$
$= 0.2$

$P(x=0) = (0.2)(0.2)(0.2)$
$= 0.008$

$P(x=1) = (0.2)(0.2)(0.8) + (0.2)(0.8)(0.2) + (0.8)(0.2)(0.2)$
$= 3(0.2)(0.2)(0.8)$
$= 0.096$

$P(x=2) = (0.2)(0.8)(0.8)(3)$
$= 0.384$

$P(x=3) = 0.8 \times 0.8 \times 0.8$
$= 0.512$

X	0	1	2	3
f(X)	0.008	0.096	0.384	0.512

The space shuttle control system called PASS uses 4 independent computers working in parallel. At each critical step, the computers "VOTE" to determine the appropriate step. The probability that a computer will ask for a roll to the left, when a roll to the right is appropriate is 0.0001. Let 'X' denotes the number of computers that VOTE for a left roll when a right roll is appropriate. What is the probability mass function of x?

Let X be the random variable that represent number of computers that VOTE (fail)

Range of random variable $X = 0, 1, 2, 3$ or 4.

The failures are independent and the probability of failure = 0.0001

Probability of success $= 1 - P(\text{failure})$
$= 1 - 0.0001$
$= \underline{0.9999}$

~~P(X=0) = (0.9999)(0.9999)(0.9999)(0.9999)~~

The probability that computer will ask for roll to left to right $P = 0.0001 \qquad q = 0.9999$

$P(X=0) = (0.9999)(0.9999)(0.9999)(0.9999)$
$= 0.99960006$

$P(X=1) = {}^4C_1[(0.0001)(0.9999)^3]$
$= 3.9988 \times 10^{-4}$

$P(X=2) = {}^4C_2[(0.0001)^2(0.9999)^2]$
$= 5.9988 \times 10^{-8}$

$P(X=3) = {}^4C_3[(0.0001)^3(0.9999)]$
$= 3.9996 \times 10^{-12}$

$P(X=4) = (0.0001)^4$
$= 1 \times 10^{-16}$

x	0	1	2	3	4
f(x)	0.9986	3.9988×10^{-4}	5.9988×10^{-8}	3.996×10^{-12}	1×10^{-16}

A disk drive manufacturer estimates that in a 5 years - a storage device with 1 tera byte of capacity will sell with probability 0.5, A storage device with 500 GB capacity will sell with a probability 0.3, and a storage device with 100 GB capacity will sell with probability 0.2. The revenue associated with the sales in that year are estimated to be \$50 million, \$25 million & \$10 million respectively. Let 'X' be the revenue of storage device during that year. Determine probability mass function of X.

Let X be the random variable that denote the revenue of storage devices in dollars during a particular year of capacities 1TB, 500GB, 100GB respectively.

$P(X = 10 \text{ million}) = 0.2$
$P(X = 25 \text{ million}) = 0.3$
$P(X = 50 \text{ million}) = 0.5$

Probability mass function

X	10	25	50
f(X)	0.2	0.3	0.5

Marketing estimates that a new instrument for the analysis of soil samples will be very successful, moderately successful or unsuccessful, with probabilities 0.3, 0.6 & 0.1 respectively. The yearly revenue associated with a very successful, moderately successful or unsuccessful product is \$10 million, \$5 million and \$1 million respectively. Let the random variable 'x' denote the yearly revenue of the product. Determine the probability mass function of X.

Let 'x' be the random variable of yearly revenue of product. Yearly revenue associated with products ranges as follows

Very successful = \$10 million
moderately successful = \$5 million
Unsuccessful = \$1 million

$\therefore P(X=10) = 0.3$
$P(X=5) = 0.6$
$P(X=1) = 0.1$

x	1	5	10
f(x)	0.1	0.6	0.3

Cumulative Distribution Function Read

The cumulative distribution function of a discrete random variable 'x' denoted by $F(x)$ and is defined as $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$

For a discrete random variable X, F(x) satisfies the following conditions.

i) $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$
ii) $0 \leq F(x) \leq 1$
iii) If $x \leq y$, then $F(x) \leq F(y)$

Mean and Variance for discrete random variable

Two numbers are often used to summarize a probability distribution for a random variable 'x'. The mean is a measure of the centre or middle of the probability distribution and variance is the measure of the dispersion or variability in the distribution. The two different distribution can have the same mean and variance.

The mean or expected value of the discrete random variable 'x', denoted as μ or $E(x)$ and is defined as

$$\boxed{\mu = E(x) = \sum x f(x)}$$

The variance of x denoted as σ^2 or $V(x)$ and is defined as

$$\boxed{\begin{aligned}\sigma^2 &= V(x) = E(X-\mu)^2 \\ \sigma^2 &= V(x) = \sum E(x-\mu)^2 f(x) \\ \sigma^2 &= V(x) = \sum x^2 f(x) - \mu^2\end{aligned}}$$

The standard deviation of x $(\sigma) = \sqrt{\sigma^2} = \sqrt{V(x)}$
$= \sqrt{\text{variance}}$

Problem

Determine the cumulative distribution function of random variable 'x' for sample space of a random experiment is $S = \{a, b, c, d, e, f\}$ and each outcome is equally likely. A random variable is defined as follows

Outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

x	0	0	1.5	1.5	2	3
f(x)	1/6	1/6	1/6	1/6	1/6	1/6

x	0	1.5	2	3
f(x)	1/3	1/3	1/6	1/6

The cumulative distribution function at $x = k$ is the sum of the probability mass function for x values smaller than $x = k$. Therefore

$F_X(0) = 1/3$

$F_X(1.5) = 1/3 + 1/3 = 2/3$

$F_X(2) = 2/3 + 1/6 = 5/6$

$F_X(3) = 5/6 + 1/6 = 1$

For values smaller than the first x value the cumulative distributive function is zero. In between two x values the cumulative distribution function is equal to the cumulative of the smaller x value. Then we obtain the following function

$$\therefore F_X(X) = \begin{cases} 0 & \text{if } x < 0 \\ 1/3 & 0 \le x < 1.5 \\ 2/3 & 1.5 \le x < 2 \\ 5/6 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

Determine the cumulative distribution function of random variable 'x' for sample space of random experiment is

x	-2	-1	0	1	2
f(x)	1/8	2/8	2/8	2/8	1/8

a) $P(X \le 1.25)$
b) $P(X \le 2.2)$
c) $P(-1.1 \le x \le 1)$
d) $P(X > 0)$

x	-2	-1	0	1	2
f(x)	1/8	2/8	2/8	2/8	1/8

The cumulative distribution function for

$F_X(-2) = 1/8$

$F_X(-1) = 3/8$

$F_X(0) = 3/8 + 2/8 = 5/8$

$F_X(1) = 5/8 + 2/8 = 7/8$

$F_X(2) = 7/8 + 1/8 = 1$

$$F_X(x) = \begin{cases} 0 & x < -2 \\ 1/8 & -2 \le x \le -1 \\ 3/8 & -1 \le x < 0 \\ 5/8 & 0 \le x < 1 \\ 7/8 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

a) $P(X \le 1.25) = 7/8$

b) $P(x \le 2.2) = 1$

c) $P(-1.1 \le x \le 1) = 7/8 - 1/8 = 6/8$
$= 3/4$

d) $P(x > 0) = \cancel{P(x \le \ldots) - P(x \le \ldots)}$
$= 3/8$

Determine cumulative distribution function for random variable 'x' for $f(x) = \frac{2x+1}{25}$, where $x = 0, 1, 2, 3, 4$

& also find i) $P(X<1.5)$ ii) $P(X \leq 3)$ iii) $P(x>2)$

iv) $P(1<x\leq 2)$

Also find mean & variance

$f(0) = 1/25$ $f(1) = 3/25$ $f(2) = 5/25$ $f(3) = 7/25$

$f(4) = 9/25$

Probability mass function

x	0	1	2	3	4
$f(x)$	1/25	3/25	5/25	7/25	9/25

$F_x(0) = 1/25$

$F_x(1) = 3/25 + 1/25 = 4/25$

$F_x(2) = 4/25 + 5/25 = 9/25$

$F_x(3) = 9/25 + 7/25 = 16/25$

$F_x(4) = 16/25 + 9/25 = 1$

$$F_x(x) = \begin{cases} 0 & x<0 \\ 1/25 & 0\leq x<1 \\ 4/25 & 1\leq x<2 \\ 9/25 & 2\leq x<3 \\ 16/25 & 3\leq x<4 \\ 1 & 4\leq x \end{cases}$$

i) $P(X<1.5) = P(0) + P(1)$
$= 4/25$

ii) $P(X\leq 3) = P(0) + P(1) + P(2) + P(3)$
$= 16/25$

iii) $P(x>2) = P(X=3) + P(X=4)$
$= 16/25$

iv) $P(1<x\leq 2) = P(2)$
$= 5/25$

mean (μ) = $E(x) = \sum x f(x)$

$= 0(1/25) + 1(3/25) + 2(5/25) + 3(7/25) + 4(9/25)$

$= 2.8$

Variance (σ^2) = $\sum (X-\mu)^2 f(x)$

$= (0-2.8)^2(1/25) + (1-2.8)^2(3/25) + (2-2.8)^2(5/25)$
$+ (3-2.8)^2(7/25) + (4-2.8)^2(9/25)$

$= 1.38$

Standard deviation $= \sqrt{\sigma^2} = \sqrt{1.36}$

$= 1.16$

Ex 3 (4) In a semiconductor manufacturing process, 3 wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent. Determine the probability mass function of the number of wafers from a lot that pass the test. Hence, find the cumulative distribution function of random variable. Find mean & variance

Probability of wafer passing test = 0.8

There are 3 wafers in total

Let 'x' be random variable that represent number of wafers that pass the test

The random variable $X = 0, 1, 2, 3$

$P(\text{success}) = 0.8$

$P(\text{failure}) = 0.2$

$P(X=0) = (0.2)(0.2)(0.2) = 0.008$

$P(X=1) = 3(0.2)(0.2)(0.8) = 0.096$

$P(X=2) = 3(0.2)(0.8)(0.8) = 0.384$

$P(X=3) = 0.8 \times 0.8 \times 0.8 = 0.512$

x	0	1	2	3
$f(x)$	0.008	0.096	0.384	0.512

$F_x(0) = 0.008$

$F_x(1) = 0.008 + 0.096 = 0.104$

$F_x(2) = 0.104 + 0.384 = 0.488$

$F_x(3) = 0.488 + 0.512 = 1$

$$F_x(x) = \begin{cases} 0 & x<0 \\ 0.008 & 0\leq x<1 \\ 0.104 & 1\leq x<2 \\ 0.488 & 2\leq x<3 \\ 1 & 3\leq x \end{cases}$$

mean (μ) = $E(x) = \sum x f(x)$

$= 0(0.008) + 1(0.096) + 2(0.384) + 3(0.512)$

$= 2.4$

Variance (σ^2) = $\sum (X-\mu)^2 f(x)$

$= (0-2.4)^2(0.008) + (1-2.4)^2(0.096) + (2-2.4)^2(0.384)$
$+ (3-2.4)^2(0.512)$

$= 0.48$

$SD = \sqrt{0.48} = 0.69$

Ex. 3 (5) The space shuttle flight control system called PASS uses 4 independent computers working in parallel. At each critical step, the computers 'vote' to determine appropriate step. The probability that a computer will ask for a roll

Sol: write all steps referring to previously solved question

X	0	1	2	3	4
f(x)	0.9996	3.9988×10^{-4}	5.9988×10^{-8}	3.9996×10^{-12}	1×10^{-16}

$F_x(0) = 0.9996$

$F_x(1) = 0.9996 + 3.9988 \times 10^{-4} = 0.99999988$

$F_x(2) = 0.99999988 + 5.9988 \times 10^{-8} = 0.99999994$

$F_x(3) = 0.99999994 + 3.9996 \times 10^{-12} = 0.99999994$

$F_x(4) = 0.99999994 + 1 \times 10^{-16} = 0.99999994$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ 0.9996 & 0 \le x < 1 \\ 0.99999988 & 1 \le x < 2 \\ 0.99999994 & 2 \le x < 3 \\ 0.99999994 & 3 \le x < 4 \\ 0.9999999421 & 4 \le x \end{cases}$$

mean$(\mu) = E(x) = \Sigma x f(x)$
$= 0(0.9996) + 1(3.9988 \times 10^{-4}) + 2(5.9988 \times 10^{-8})$
$+ 3(3.9996 \times 10^{-12}) + 4(1 \times 10^{-16})$
$= \underline{3.99999988 \times 10^{-4}}$

Variance $(\sigma^2) = \Sigma (x-\mu)^2 f(x)$
$= (0 - 3.9999 \times 10^{-4})^2 (0.9996) + (1 - 3.9999 \times 10^{-4})^2 (3.9988 \times 10^{-4})$
$+ (2 - 3.9999 \times 10^{-4})(5.9988 \times 10^{-8}) + (3 - 3.9999 \times 10^{-4})(3.9966 \times 10^{-12})$
$+ (4 - 3.9999 \times 10^{-4})(1 \times 10^{-16})$
$= \underline{3.9999 \times 10^{-4}}$

Ex 3 (6) desk e.o etc

Sol: $P(X=50) = 0.5$
$P(X=25) = 0.3$
$P(X=10) = 0.2$

X	10	25	50
f(x)	0.2	0.3	0.5

$F_x(10) = 0.2$

$F_x(25) = 0.5$

$F_x(50) = 1$

$$F_x(x) = \begin{cases} 0 & x < 10 \\ 0.2 & 10 \le x < 25 \\ 0.5 & 25 \le x < 50 \\ 1 & 50 \le x \end{cases}$$

mean$(\mu) = E(x) = \Sigma x f(x)$
$= 10(0.2) + 25(0.3) + 50(0.5)$
$= 34.5$

variance $(\sigma^2) = \Sigma (x-\mu)^2 f(x)$
$= (10 - 34.5)^2 (0.2) + (25 - 34.5)^2 (0.3)$
$+ (50 - 34.5)^2 (0.5)$
$= 267.25$

SD $= 16.347$

Discrete uniform distribution.

A random variable 'x' has a discrete uniform distribution if each of the 'n' values in its range say $x_1, x_2, x_3, \ldots x_n$ has equal probability then $f(x_i) = 1/n$

Mean and variance of discrete uniform distribution

Suppose X is a discrete uniform random variable on the consecutive integers $a, a+1, a+2, \ldots b$ for $a \le b$ then the mean of X is $\boxed{\mu = E(x) = \frac{b+a}{2}}$ and variance of $\boxed{X \; (\sigma^2) = \frac{(b-a+1)^2 - 1}{12}}$

① Let the random variable X have a discrete uniform distribution on integers $0 \le x \le 100$, Determine mean and variance of X.

$X = 0, \dots, 100$

Mean $(\mu) = \frac{a+b}{2} = \frac{0+100}{2} = 50$

Variance $(\sigma^2) = \frac{(b-a+1)^2 - 1}{12} = \frac{(100+1)^2 - 1}{12} = 850$

② $1 \le x \le 3$

$X = 01, 02, 03$

Mean $(\mu) = \frac{01+03}{2} = 2$

Variance $= \frac{(b-a+1)^2-1}{12} = \frac{(3-1+1)^2-1}{12} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$

③ Thickness measurements of a coating process are made to the nearest hundredth of a millimeter. The thickness measurements are uniformly distributed with values 0.15, 0.16, 0.17, 0.18, 0.19. Determine the mean and variance of coating thickness for this process.

$X = 0.15, 0.16, 0.17, 0.18, 0.19$

Mean $(\mu) = \frac{b+a}{2} = \frac{0.19+0.15}{2} = 0.17$

Variance $(\sigma^2) = \frac{(b-a+1)^2-1}{12} = \frac{(0.19-0.15+1)^2-1}{12} = 6.8 \times 10^{-3}$

④ Product codes of 2, 3 or 4 letters are equally likely. What is the mean and standard deviation of the number of letters in 100 codes?

$a = 2, \; b = 4$

Mean $= \frac{2+4}{2} = 3$

Variance $(\sigma^2) = \frac{(b-a+1)^2-1}{12} = \frac{(4-2+1)^2-1}{12} = \frac{2}{3}$

standard deviation $= \sqrt{2/3} = 0.81649$

⑤ The lengths of plate glass parts are measured to the nearest tenth of a millimeter. The lengths are uniform distributed, with values at every tenth of a millimeter starting at 590.0 and continuing through 590.9. Determine the mean & variance of length.

$a = 590.0 \quad b = 590.9$

Mean $(\mu) = \frac{a+b}{2} = \frac{590+590.9}{2} = 590.45$

Variance $(\sigma^2) = \frac{(b-a+1)^2-1}{12} = \frac{(590.9-590+1)^2-1}{12} = 0.2175$

⑥ Assume that the wavelength of photosynthetically active radiation (PAR) are uniformly distributed at integer nanometers in the red spectrum from 675 nm to 700 nm

a) what are mean & variance of wavelength distribution for this radiation?

$a = 675 \quad b = 700$

mean $(\mu) = \frac{675+700}{2} = 687.5$

Variance $(\sigma^2) = \frac{(700-675+1)^2-1}{12} = 56.25$

b)

$a = 75, \; b = 100$

mean $(\mu) = 87.5$

Variance $= 56.25$

Variance is same in both cases because range is same and mean is the midpoint so we are getting different means for both cases.

⑦ The probability of an operator entering alphanumeric data incorrectly into a field in a database is equally likely. The random variable (X) is the number of fields on data entry form with an error. The data entry form has 28 fields. Is X a DURV? Why or why not.

X is not a discrete uniform random variable

~~P(X)=0 ≠ P(X)=1~~ $P(X=0) \neq P(X=1)$.

⑧ Suppose that X has a discrete uniform distribution on the integers 0 through 9. Determine mean, variance and standard deviation of random variable $Y=5X$ and compare to the corresponding results for X.

$X = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$a=0, \quad b=9$

$\text{mean} = \frac{a+b}{2} = \frac{9}{2} = \underline{4.5}$

$\text{Variance } (\sigma^2) = \frac{(b-a+1)^2-1}{12} = \frac{99}{12} = \underline{8.25}$

$SD = \sqrt{\sigma^2} = \underline{2.87228}$

$Y=5X$

$Y = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45$

$a=0, \quad b=45$

$\text{mean} = \frac{a+b}{2} = \frac{45}{2} = \underline{22.5}$

$\text{variance } (\sigma^2) = \frac{(45-0+1)^2-1}{12} = \underline{176.25}$

$SD = \underline{13.2759}$

$\text{mean} = E(5X) = 5\times\mu = 5\times 4.5 = 22.5$

$\text{Variance } E(5X) = 5^2\times\sigma^2 = 25\times 8.25 = 206.25$

$SD = \underline{14.3614}$

---

⑨ Show that for a discrete uniform random variable X, if each of the values in the range of X is multiplied by the constant C, the effect is to multiply the mean of X by C and the variance of X by C^2. That is show that $E(CX) = CE(X)$ and $V(CX) = C^2\times V(X)$.

① $E(CX) = CE(X)$

we know

$E(X) = \Sigma X f(x)$

Replace X by CX

$E(CX) = \Sigma CX f(x)$

as C is constant,

$E(CX) = C\Sigma X f(x)$

$E(CX) = C\,E(X)$

② $V(CX) = C^2 V(X)$

$\text{variance } (\sigma^2) = E(X^2) - \mu^2$

$E(C^2X^2) = \Sigma C^2X^2 f(x)$

$= C^2x_1^2 f(x) + C^2x_2^2 f(x) + \dots$

$= C^2(x_1^2 f(x) + x_2^2 f(x) + \dots)$

$= C^2 E(X^2)$

$\therefore \text{variance} = C^2E(X^2) - (E(CX))^2$

$= C^2E(X^2) - (C\times E(X))^2$

$= C^2(E(X^2) - E(X)^2)$

$= C^2\times \text{Variance}$

$\therefore V(CX) = C^2V(X)$

Binomial distribution

A random experiment consists of 'n' bernoulli trials such that

① The trials are independent

② Each trial results in only 2 possible outcomes as "success" and "failure"

③ The probability of success in each trial denoted as P, remains constant.

The random variable X that equals the number of trials that results in a success as a binomial random variable with parameters $0<P<1$ and $n = 0, 1, 2, 3$ etc. The probability mass function of 'x' is $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$x = 0, 1, \dots, n$

$$\boxed{f(x) = {}^nC_x\, p^x q^{n-x}}$$

Mean & Variance of a Binomial distribution

Let 'x' be the Binomial function then

$\mu = E(X) = np$

& $\text{variance } (\sigma^2) = npq$

② Let X be a binomial random variable with $p = 0.2$ and $n = 20$. Determine the following probabilities

a) $P(X \le 3)$ b) $P(X > 10)$ c) $P(X = 6)$ d) $P(6 \le X \le 11)$

Given

$p = 0.2$, $n = 20$

$1 - p = q = 0.8$

$f(x) = {}^nC_x\, p^x q^{n-x}$

1) $P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{20}C_0 (0.2)^0 (0.8)^{20} + {}^{20}C_1 (0.2)^1 (0.8)^{19}$

$+ {}^{20}C_2 (0.2)^2 (0.8)^{18} + {}^{20}C_3 (0.2)^3 (0.8)^{17}$

$= 0.41145$

2) $P(X > 10) = P(X=11) + \cdots + P(X=20)$

$= 1 - P(X \le 10)$

$= 1 - [P(X=0) + \cdots + P(X=10)]$

$= 1 - 0.99943655E6$

$= 5.63 \times 10^{-4}$

3) $P(X=6) = {}^{20}C_6 (0.2)^6 (0.8)^{14}$

$= 0.10909$

4) $P(6 \le X \le 11) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10) + P(X=11)$

$= 0.1957$

③ Let X be binomial random variable with $p = 0.1$ $n = 10$. Calculate following probabilities from binomial probability mass function & also from binomial table & compare result

a) $P(X \le 2)$ b) $P(X > 8)$ c) $P(X = 4)$ d) $P(5 \le X \le 7)$

Given $p = 0.1$, $q = 0.9$, $n = 10$

$f(x) = {}^nC_x\, p^x q^{n-x}$

a) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{10}C_0 (0.1)^0 (0.9)^{10} + {}^{10}C_1 (0.1)(0.9)^9 + {}^{10}C_2 (0.1)^2 (0.9)^8$

$= 0.9298$

b) $P(X > 8) = P(X=9) + P(X=10)$

$= 9.1 \times 10^{-9}$

c) $P(X=4) = {}^{10}C_4 (0.1)^4 (0.9)^6$

$= 0.011160$

d) $P(5 \le X \le 7) = P(X=5) + P(X=6) + P(X=7)$

$= 1.6345 \times 10^{-3}$

④ $n = 10$, $p = 0.5$, $q = 0.5$

$f(x) = {}^nC_x\, p^x q^{n-x}$

a) $P(X=5) = {}^{10}C_5 (0.5)^5 (0.5)^5 = 0.24609$

b) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2)(0.5)^{10}$

$= 0.0546875$

c) $P(X \ge 9) = P(X=9) + P(X=10)$

$= ({}^{10}C_9 + {}^{10}C_{10})(0.5)^{10}$

$= 0.010742$

d) $P(3 \le X \le 5) = P(X=3) + P(X=4) + P(X=5)$

$= ({}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5)(0.5)^{10}$

$= 0.568359$

⑤ $n = 10$, $p = 0.01$, $q = 0.99$

$f(x) = {}^nC_x\, p^x q^{n-x}$

a) $P(X=5) = {}^{10}C_5 (0.01)^5 (0.99)^5$

$= 2.39649 \times 10^{-8}$

b) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.999$

c) $P(X \ge 9) = P(X=9) + P(X=10)$

$= 9.91 \times 10^{-18}$

d) $P(3 \le X \le 5) = P(X=3) + P(X=4)$

$= 1.1438 \times 10^{-4}$

⑥ $n = 10$, $p = 0.5$, $q = 0.5$

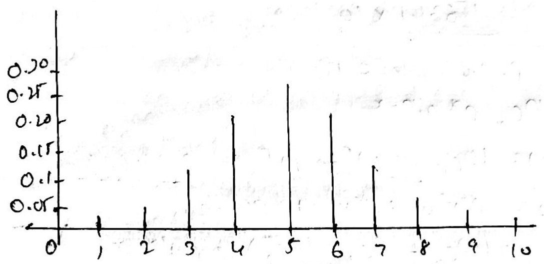
$f(x) = {}^nC_x\, p^x q^{n-x}$

$f(0) = {}^{10}C_0 (0.5)^{10} = 9.76 \times 10^{-4}$	$f(4) = {}^{10}C_4 (0.5)^{10} = 0.2050$
$f(1) = {}^{10}C_1 (0.5)^{10} = 9.76 \times 10^{-3}$	$f(5) = {}^{10}C_5 (0.5)^{10} = 0.246$
$f(2) = {}^{10}C_2 (0.5)^{10} = 0.04394$	$f(6) = {}^{10}C_6 (0.5)^{10} = 0.2053$
$f(3) = {}^{10}C_3 (0.5)^{10} = 0.11718$	$f(7) = {}^{10}C_7 (0.5)^{10} = 0.1176$

$f(8) = {}^{10}C_8 (0.5)^{10} = 0.0441$

$f(9) = {}^{10}C_9 (0.5)^{10} = 0.0098$

$f(10) = {}^{10}C_{10} (0.5)^{10} = 0.00098$



$f(X=5) = 0.2468$ is the most likely

$f(X=0)$ & $f(X=10)$ are least likely.

Mean (μ) and variance (σ^2) $= np = 10(0.5) = 5$

---

⑦ $n=10$, $p=0.01$

$f(x) = {}^nC_x p^x q^{n-x} = {}^{10}C_x (0.01)^x (0.99)^{10-x}$

$p = 0.01$ $q = 0.99$

$f(0) =$

$f(1) =$

$f(2) =$

$f(3) =$

$f(4) =$

$f(5) =$

$f(6) =$

$f(7) =$

$f(8) =$

$f(9) =$

$f(10) =$

Most likely value is 0

least likely value is 10

⑧ Determine the cumulative distributive function of a binomi. random variable with $n=3$, $p = 1/2$

$f(x) = {}^nC_x p^x q^{n-x}$

$p = 1/2$ $q = 1/2$

$f(0) = {}^3C_0 (0.5)^3 = 0.125$

$f(1) = {}^3C_1 (0.5)^3 = 0.375$

$f(2) = {}^3C_2 (0.5)^3 = 0.375$

$f(3) = {}^3C_3 (0.5)^3 = 0.125$

x	0	1	2	3
$f(x)$	0.125	0.375	0.375	0.125

$F_X(0) = 0.125$

$F_X(1) = 0.125 + 0.375 = 0.5$

$F_X(2) = 0.5 + 0.375 = 0.875$

$F_X(3) = 0.875 + 0.125 = 1$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.125 & 0 \le x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.875 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

⑨ $n=3$, $p = 1/4$

$p = 0.25$ $q = 0.75$

$f(x) = {}^nC_x p^x q^{n-x}$

$= {}^3C_x (0.25)^x (0.75)^{3-x}$

$f(0) = {}^3C_0 (0.25)^0 (0.75)^3 = 0.421875$

$f(1) = {}^3C_1 (0.25)(0.75)^2 = 0.421875$

$f(2) = {}^3C_2 (0.25)^2 (0.75)^1 = 0.140625$

$f(3) = {}^3C_3 (0.25)^3 = 0.015625$

x	0	1	2	3
$f(x)$	0.421875	0.421875	0.140625	0.015625

$F_X(0) = 0.421875$

$F_X(1) = 0.84375$

$F_X(2) = 0.984375$

$F_X(3) = 1$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.421875 & 0 \le x < 1 \\ 0.84375 & 1 \le x < 2 \\ 0.984375 & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

(10) An electronic product contains 40 IC. The probability that any IC is defective is 0.01. & IC are independent. The product operates only if there are no defective IC. What is the probability that the products operate?

$n=40, \quad p=0.01, \quad q=0.99$

The product operates if there is no defective IC in it

$f(x)= {}^nC_x\, p^x q^{n-x}$

$f(0)= {}^{40}C_0 (0.01)^0 (0.99)^{40}$
$= \underline{0.66897}$

(11) 'x' be the binomial random variable as $n=10$

$p=0.4 \quad q=0.6$

$f(x)= {}^nC_x\, p^x q^{n-x}$
$= {}^{10}C_x (0.4)^x (0.6)^{10-x}$

a) $P(X=3) = f(3) = {}^{10}C_3 (0.4)^3 (0.6)^7$
$= \underline{0.21499}$

~~b) $P(X\geq 1) = 1 - P(X=0)$~~
~~$= 1-({}^{10}C_0 (0.4)^0 (0.6)^{10})$~~
~~$= 0.993953$~~

~~c) $P(X=10) = {}^{10}C_{10} (0.4)^{10} (0.6)^0$ $\cancel{\mu = E(x) =}$~~
~~$= 0.0001048576$~~

b) $P(x\leq 9) = 1-P(x>9)$
$= 1-P(X=10)$
$= 1-{}^{10}C_{10} (0.4)^{10}$
$= \underline{0.999895}$

c) $\mu = E(X) = np$
$= 10\times 0.4$
$= 4$

(12) $n=25, \quad p=0.25, \quad q=0.75$

$f(x)= {}^nC_x\, p^x q^{n-x}$
$= {}^{25}C_x (0.25)^x (0.75)^{25-x}$

a) $P(X>20) = f(21)+f(22)+f(23)+f(24)+f(25)$
$= 9.6769\times 10^{-10}$

b) $P(X<5) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$
$= \underline{0.213740}$

(13)

(14) $n=15, \; p=0.01, \; q=0.99$

$f(x)= {}^nC_x\, p^x q^{n-x}$

$P(X=0) = 0.86$

$P(X\leq 1) = P(X=0)+P(X=1) = 0.99$

$P(X>8) = 1-P(X\leq 7) = 0$

(15) $n=20, \; p=0.13, \; q=0.87$

a) $P(X=3) = {}^{20}C_3 (0.13)^3 (0.87)^{17} = \underline{0.2347}$

b) $P(X\geq 3) = 1-P(X<3)$
$= 1-[P(X=0)+P(X=1)+P(X=2)]$
$= 1-[{}^{20}C_0 (0.87)^{20} + {}^{20}C_1 (0.13)(0.87)^{19} + {}^{20}C_2 (0.13)^2 (0.87)^{18}]$
$= 1-0.50795 = \underline{0.49205}$

c) Mean $= np = 20\times 0.13 = 2.6$

Variance $= npq = 2.262$

Standard deviation $= \sqrt{2.262} =$ ~~[illegible]~~ 1.50399

(16) Let X be the random variable of number of passwords hits (out of 1 billion), $n=10^9$

$f(x)= {}^nC_x\, p^x q^{n-x}, \quad x=0,1,2\ldots n$

The total number of available passwords easily calculated knowing there are 36 characters (0-9 & a-z) of which 6 are chosen with replacement.

The number of passwords $= (10+26)^6 = (36)^6$

The probability of hitting some user's password at random is then evidently $p = \frac{10000}{(36)^6} = 4.50\times 10^{-6}$

$q = 1-p = \underline{0.9999955}$

b) $P(X=0) = \left(1-\frac{10000}{(36)^6}\right)^{10^9} \times \left(\frac{10000}{36}\right)^0 \times {}^{10^9}C_0$
$= 0$

mean $= np = 4.59\times 10^3$

variance $= npq \approx 4.4\times 10^3$

(17) $n=125$, $p=0.1$, $q=0.9$

$f(x) = {}^nC_x\, p^x q^{n-x}$

Let X be the random variable of number of people who do not show up for flight (out of 125 people who brought tickets)

a) If every body who showup can take the flight then it must be

$P(X\geq 5) = 1-P(<5)$

$= 1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)]$

$= 0.9961$

b) $P(X>5) = 1-P(X\leq 5)$ 0.9986

$= 1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)]$

Geometric distribution (Binomial)

In a series of bernoulli trial, let the random variable 'x' denote the numbers of trials until the first success. Then X is a geometric random variable with parameters $0<p<1$ and $f(x) = (1-p)^{x-1}p$ where $x=1,2,3\ldots$

Mean and variance of the Geometric random variable

If X is a geometric random variable with parameter 'p'. Therefore mean $(\mu) = E(x) = (1/p)$ and variance $(\sigma^2) = V(x) = \left(\frac{1-p}{p^2}\right)$

Negative binomial distribution

In a series of bernoulli trial, let the random variable X denote the numbers of trials until 'r' successes occurs. Then X is a negative binomial random variable with parameters $0<p<1$ and $r=1,2,3\ldots$ etc

Therefore $f(x) = {}^{x-1}C_{r-1}(1-p)^{x-r}p^r$, where $x=r, r+1, r+2, \ldots$

Mean & variance of negative binomial distribution

If X is a negative binomial random variable with parameters p and r. mean $(\mu) = E(x) = r/p$

variance $(\sigma^2) = V(x) = \frac{r(1-p)}{p^2}$

Problems

① Suppose that the random variable X has a Geometric distribution with $p=0.5$. Determine the following probabilities a) $P(X=1)$ b) $P(X=4)$ c) $P(X=8)$ d) $P(X\leq 2)$ e) $P(X>2)$

Let 'X' is a Geometric random variable with $P=0.5$

$f(x) = (1-p)^{x-1}p$

The probability mass function is

$f(x) = (1-1/2)^{x-1}(1/2)$

$= (1/2)^{x-1}(1/2)$

$= (1/2)^x = 1/2^x$

① $P(X=1) = 1/2^1 = 1/2$

② $P(X=4) = 1/2^4 = 1/16$

③ $P(X=8) = 1/2^8 = 1/256$

④ $P(X\leq 2) = P(X=1)+P(X=2)$

$= 1/2+1/4$

$= 3/4$

⑤ $P(X>2) = 1-P(X\leq 2)$

$= 1-3/4 = 1/4$

---

② mean = 2.5

$P(X=1)$ $P(X=4)$ $P(X=5)$ $P(X\leq 3)$ $P(X>3)$

$\mu = 1/p = 2.5$

$p = 1/2.5 = 0.4$

$f(x) = (1-p)^{x-1}p$

$= (0.6)^{x-1}(0.4)$

$P(X=1) = (0.6)^0(0.4) = 0.4$

$P(X=4) = (0.6)^3(0.4) = 0.0864$

$P(X=5) = (0.6)^4(0.4) = 0.05184$

$P(X\leq 3) = P(X=1)+P(X=2)+P(X=3)$

$= 0.4+0.24+0.144 = 0.784$

$P(X>3) = 1-P(X\leq 3)$

$= 1-0.784 = 0.216$

③ Consider a sequence of independent Bernoulli trials with $p=0.2$

a) What is the expected number of trials to obtain first success?

b) After the 8th success occurs, what is the expected number of trials to obtain 9th success

$p=0.2$

a) $E(x) = 1/p = 1/0.2 = 5$

Expected number of trials to obtain first success $= 5$

b) Because the trials are independent, the count of numbers of trials until the next success can be started at any trial without changing the probability distribution of the random variable. So after 8th success occurs, the expected numbers of trials to obtain 9th success is $E(x) = 5$.

⑤ $P = .8$

Let 'x' is the successful optical alignment of product.

$\therefore$ Its $P=0.8$.

Probability mass function $f(x) = (1-p)^{x-1} p$

$f(x) = (1-p)^{x-1} \cdot p \quad x = 1, 2, 3, \ldots$

a) $P(x=4) = (0.8)(1-0.8)^3$
$= 0.0064$

b) $P(X<=4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$= (0.8) + (0.8)(0.2) + (0.8)(0.2)^2 + (0.8)(0.2)^3$
$= 0.9984$

c) $P(X \geq 4) = 1 - P(x<4)$
$= 1 - [P(X=1) + P(X=2) + P(X=3)]$
$= 0.008$

⑦ $P=0.02$ Let X is random variable that the call will connect

Probability mass function is
$f(x) = (1-p)^{x-1} p$

a) $P(X=10) = 0.016674$

b) $P(X>5) = 1 - P(X \leq 5)$
$= 0.9224$

c) mean $= 1/p = 50$

⑧ Let 'x' denote numbers of oponents.

A player has 80% probability of defeating each opponent. Therefore $P=0.2$

a) Probability mass function
$f(x) = (1-p)^{x-1} p$
$= (0.2)(0.8)^{x-1}$

b) $P(X \geq 2) = 1 - P(X<2)$
$= 0.80$

c) $E(x) = 1/p = 1/0.2 = 5$

d) $P(X \geq 4) = 1 - P(X<4)$
$= 0.512$

e) mean $= \dfrac{1}{P(X \geq 4)} = \dfrac{1}{0.512} = 1.953125$

⑨ $P=0.13 \quad f(x) = (1-p)^{x-1} p$

a) $P(X=1) = 0.13$

b) $P(X=3) = 0.0983$

c) $1/p = 1/0.13 = 7.6923$ → mean for outside factors

$1/p = 1/0.87 = 1.149425$

⑩ Total characters $= 10 + 26 = 36$

Total possible 6 digit passwords $= (36)^6 =$

a) Probability that hacker selects user password randomly

$p = \dfrac{9900}{(36)^6} = 4.54799 \times 10^{-6}$

mean $E(x) = E(x) = 1/p = \dfrac{(36)^6}{9900} = 219877.0036$

variance $= \dfrac{1-p}{p^2} = 4.83458 \times 10^{10}$

Standard deviation $= \sqrt{\text{variance}}$
$= 219876.8824$

b) Total number of 3 digit password = $(36)^3$

The hacker selects user password randomly

probability, $p = \frac{100}{(36)^3} = 2.143347 \times 10^{-4}$

mean $(\mu) = 1/p = \frac{(36)^3}{100} = 466.56$

Variance $= \frac{1-p}{p^2} = 217211.6736$

Standard deviation $= 466.0597318$

c) 6 digit passwords are more secure than 3 digit passwords because attempt of number of users passwords is higher in case of 6 digit passwords

---

(11) $p = 0.005$

a) $P(X=8) = (0.005)^8$

$= 3.90625 \times 10^{-19}$

b) $1/p = 1/0.005 = 200$

c) $\frac{1}{P(X=8)} = 2.56 \times 10^{18}$

---

(12) Let 'X' be the random variable that the number of parts that require rework.

The sample size $n = 20$, $p = 0.01$

Probability mass function

$P(x) = (1-p)^{x-1} p$ $\quad$ B.D $= {}^nC_x\, p^x q^{n-x}$

(1) $P(x>1) = 1 - P(x \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}^{20}C_0 (0.01)^0 (0.99)^{20} + {}^{20}C_1 (0.01)(0.99)^{19}]$

$= 0.016$

$P(X=10) = P(1-p)^{x-1}$

$= (0.016)(1-0.016)^9$

$= 0.0138$

b) $n=20$, $p=0.04$

$f(x) = (1-p)^{x-1} p$

$P(x>1) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}^{20}C_0 (0.04)^0 (0.96)^{20} + {}^{20}C_1 (0.04)(0.96)^{19}]$

$= 0.1897$

$P(X=10) = P.(1-P)^{x-1}$

$= (0.1897)(1-0.1897)^9$

$= 0.0286$

c) $\mu = 1/p = 1/0.1897 = 5.27148$

---

(4) $r=4$, $p = 0.2$

Probability mass function $= {}^{x-1}C_{r-1} (1-p)^{x-r} p^r$

a) $E(x) = r/p = 4/0.2 = 20$

b) $P(X=20) = {}^{19}C_3 (0.8)^{16} (0.2)^4$

$= 0.043639$

c) $P(X=19) = 0.0459$

d) $P(X=21) = 0.04109$

e) 19

---

(6) Let 'x' be the random variable that number of people need to be tested to detect gene

$p = 0.1$, $r = 2$

Probability mass function $= {}^{x-1}C_{r-1} (1-p)^{x-r} p^r$

(1) $P(x \geq 4) = 1 - P(X<4)$

$= 1 - [P(X=2) + P(X=3)]$

$= 1 - [{}^1C_1 (0.9)^0 (0.1)^2 + {}^2C_1 (0.9)^1 (0.1)^2]$

$= 0.972$

(2) $\mu = E(x) = r/p = 2/0.1 = 20$

Poisson distribution

Given an interval of real numbers, assume events occurs at random throughout the interval.

If the interval can be partitioned into sub intervals of small enough length such that

1. The probability of more than one event in a subinterval is zero
2. The probability of one event in a sub interval is the same for all sub intervals and proportional to the length of the sub interval
3. The event in each subinterval is independent of other subintervals, the random experiment is called a poisson process.

The random variable 'x' that equals the number of event in the interval is a poisson random variable with $0<\mu$ and probability mass function of x is

$$\boxed{f(x) = \frac{e^{-\mu}\mu^x}{x!} \quad x = 0, 1, 2, \ldots}$$

If x is a poisson random variable with parameters μ then

$$\boxed{\begin{array}{l} \text{mean } (\mu) = E(x) = \mu \\ \text{Variance } (\sigma^2) = V(x) = \mu \end{array}}$$

1. Given $\mu = 4$

Probability mass function $= \frac{e^{-\mu}\mu^x}{x!}$

a) $P(X=0) = \frac{e^{-4}\mu^0}{0!} = e^{-4} = 0.01831$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$
$= 0.2381$

c) $P(X=4) = \frac{e^{-4}4^4}{4!} = 0.1953$

d) $P(X=8) = \frac{e^{-4}4^8}{8!} = 0.0297$

2. $\mu = 0.4$

Probability mass function $= \frac{e^{-\mu}\mu^x}{x!}$

a) $P(X=0) = \frac{e^{-0.4}0.4^0}{0!} = 0.67032$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$
$= e^{-0.4}\left(\frac{0.4^0}{0!} + \frac{0.4^1}{1!} + \frac{0.4^2}{2}\right)$
$= e^{-0.4}(1 + 0.4 + 0.08)$
$= 0.992$

c) $P(X=4) = \frac{e^{-0.4}0.4^4}{4!} = 7.15 \times 10^{-4}$

d) $P(X=8) = \frac{e^{-0.4}0.4^8}{8!} = 1.0895 \times 10^{-08}$

3. 'X' is a poisson random variable with customers entering a bank in an hour

$P(X=0) = 0.05$

Probability mass function $= \frac{e^{-\mu}\mu^x}{x!}$

$\frac{e^{-\mu}\mu^0}{0!} = 0.05$

$e^{-\mu} = 0.05$

$e^{\mu} = 20$

$\mu = \ln 20$

$\mu = 2.9957 \approx 3$

variance $=$ mean $= 2.9957 \approx 3$

4. Mean $= \mu = 10$ let $X =$ Number of calls per hour

a) $P(X=5) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-10}10^{05}}{5!} = 0.03783$

b) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= e^{-10}\left(\frac{10^0}{0!} + \frac{10^1}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!}\right)$
$= e^{-10}(1 + 10 + 50 + 166.667)$
$= 0.010336$

c) $\mu = 2 \times 10 = 20/2\text{hr}$

$P(X=15) = \frac{e^{-20}(20)^{15}}{15!} = 0.051648$

d) $\mu = 10/2 = 5$ @ 30min

$P(X=5) = \frac{e^{-5}(5)^5}{5!} = 0.175467$

⑤ Let X be the random variable that represent the number of stars in given volume of space

a) $\mu = 1$ per 16 cubic light years

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1) + \cancel{P(X=2)}]$

$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} + \right]$

Probability mass function $= \frac{e^{-\mu}\mu^x}{x!}$

$= 1 - [e^{-1} + e^{-1}]$

$= 0.26424$

b) $P(X>=1) = 0.95$

$0.95 = \frac{e^{-\mu}\mu^x}{x!}$ $(x=0)$

$0.95 = 1 - \frac{e^{-\mu}\mu^0}{0!} = 1 - e^{-\mu}$

$e^{-\mu} = 0.05$

$e^{+\mu} = 20$

$\mu = 3$

$V = 16 \times 3 = 48$ cubic light years.

⑥ X = Number of insect fragment in 225 gram chocolate bars

$\mu = 14.4$ in 225g   P.M.F $f(x) = \frac{e^{-\mu}\mu^x}{x!}$

a) $P(X=0) = \frac{e^{-14.4}(14.4)^0}{0!} = 5.57 \times 10^{-7}$

b) Mean in 45gram $= \frac{14.4 \times 45}{225} = 2.88$

$f(X=0) = \frac{e^{-2.88}(2.88)^0}{0!} = 0.05613$

c) Mean in 28.35 $= \frac{14.4}{225} \times 28.35 = 1.8144$

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-1.8144}(1.8144)^0}{0!}$

$= 0.837$

$n = 7$ $P = 0.837$

$Y \geq 1$ $= 1 - P(Y < 1)$

$= 1 - P(Y=0)$ use Binomial distribution

or

mean in 7 28.35g $= \frac{28.35}{225} \times 14.4 \times 7$

$= 12.7008$

$P(X \geq 1) = 1 - e^{-12.7008}$

$= 0.999996$

d) $14.4 \times 2 = 28.8$

$P(X > 28.8) = 1 - P(X \leq 28.8)$

$= 1 - \left[\sum_{x=0}^{28} \frac{e^{-14.4}(14.4)^x}{x!}\right]$

$= 0.00046$

Probability of contamination twice the mean is unusual

⑦

Unit II

Analysis of continuous random variable

A continuous random variable is a random variable with an interval of real numbers called its range.

Probability density function

For a continuous random variable 'x' a probability density function is a function such that

① $f(x) \geq 0$

② $\int_{-\infty}^{+\infty} f(x) \cdot dx = 1$

③ $P(a \leq x \leq b) = \int_a^b f(x) dx = f(x)$ a to b

= Area under $f(x)$ from a to b for any a & b

If X is a continuous random variable for any x_1 & x_2 then $P(x_1 \leq x \leq x_2) = P(x_1 < x \leq x_2) = P(x_1 \leq x < x_2) = P(x_1 < x < x_2)$

Section 1

① $f(x) = e^{-x} \quad 0 < x$

$\int_{-\infty}^{\infty} f(x)\, dx = 1$

a) $P(1 < x) = \int_1^{\infty} e^{-x} dx$

$= -(e^{-x})]_1^{\infty}$

$= -e^{-\infty} + e^{-1}$

$= e^{-1} = 0.3679$

b) $P(1 < x < 2.5) = \int_1^{2.5} f(x) dx = \int_1^{2.5} e^{-x} dx$

$= -e^{-x}]_1^{2.5} = -e^{-2.5} + e^{-1}$

$= 0.28579$

c) $P(X = 3) = \int_3^3 e^{-x} dx = 0$

d) $P(X < 4) = \int_0^4 e^{-x} dx = -e^{-x}]_0^4 = -e^{-4} + e^{-0}$

$= 0.981684$

e) $P(3 \leq x) = \int_3^{\infty} e^{-x} dx = -(e^{-x})]_3^{\infty} = -e^{-\infty} + e^{-3}$

$= e^{-3}$

$= 0.04978$

f) $P(x < X) = 0.1$

$x \geq X$

$X > x$

$\int_x^{\infty} e^{-x} dx = 0.1$

$-e^{-\infty} + e^{-x} = 0.1$

$e^{-x} = 0.1$

$-x = \ln(0.1)$

~~x = 4.605~~ $x = 2.30258$

g) $P(X \leq x) = 0.1$

$\int_0^x e^{-x} dx = 0.1$

$-e^{-x} + e^{-0} = 0.1$

$-e^{-x} + 1 = 0.1$

$e^{-x} = 0.9$

$x = 0.1053$

---

② $f(x) = \frac{x}{8} \quad 3 < x < 5.$

a) $P(X < 4) = \int_3^4 \frac{x}{8} dx = \frac{1}{8}\left[\frac{x^2}{2}\right]_3^4$

$= \frac{1}{8}\left[\frac{16}{2} - \frac{9}{2}\right] = \frac{7}{16}$

b) $P(X > 3.5) = \int_{3.5}^{5} x/8 \, dx = \frac{1}{8}\left(\frac{x^2}{2}\right)_{3.5}^{5}$

$= \frac{1}{8}\left(\frac{25}{2} - \frac{12.25}{2}\right) = 0.796875$

c) $P(4 < x < 5) = \int_4^5 x/8 \, dx = \frac{1}{8}\left(\frac{25}{2} - \frac{16}{2}\right) = \frac{9}{16}$

d) $P(X<4.5] = \int_3^{4.5} x/8 \, dx$

$= \frac{1}{8}\left(\frac{x^2}{2}\right)_3^{4.5}$

$= \frac{1}{8}\left(\frac{4.5^2}{2} - \frac{3^2}{2}\right) = \underline{0.703125}$

e) $P(X<3.5 \text{ or } X>4.5)$

$= P(X<3.5) + P(X>4.5)$

$= \int_3^{3.5} x/8 \, dx + \int_{4.5}^{5} x/8 \, dx$

$= \frac{1}{8}\left[\frac{3.5^2}{2} - \frac{3^2}{2}\right] + \frac{1}{8}\left[\frac{5^2}{2} - \frac{4.5^2}{2}\right]$

$= \frac{1}{8}\left[\frac{3.25}{2}\right] + \frac{1}{8}\left[\frac{4.75}{2}\right]$

$= \underline{0.5}$

---

3. $f(x) = e^{-(x-4)} \quad 4<X$

$\int_4^{\infty} f(x)\, dx$

a) $P(1<x) = \int_4^{\infty} e^{(4-x)} dx = \left.\frac{e^{4-x}}{-1}\right]_4^{\infty}$

$= \frac{e^{-\infty}}{-1} + e^0 = \underline{1}$

b) $P(2 \le x < 5) = \int_4^{5} e^{(4-x)} dx = \left.\frac{e^{4-x}}{-1}\right]_4^{5}$

$= \frac{e^{-1}}{-1} + e^0 = \underline{0.6321}$

c) $P(5<X) = \int_5^{\infty} e^{(4-x)} dx = \left.\frac{e^{4-x}}{-1}\right]_5^{\infty}$

$= 0 + e^{-1} = \underline{0.367879}$

d) $P(8<x<12) = \int_8^{12} e^{4-x} dx = \left.\frac{e^{4-x}}{-1}\right]_8^{12}$

$= -e^{-8} + e^{-4} = \underline{0.01798}$

e) $P(X<x) = 0.90$

$\int_4^{x} e^{4-x} dx = 0.9$

$\frac{e^{4-x}}{-1} + e^0 = 0.9$

$e^{4-x} = 0.1$

$4-x = -2.3025$

$\underline{x = 6.3025}$

---

4. $f(x) = 1.5x^2 \quad -1<x<1$

$\int_{-1}^{1} f(x)\, dx$

a) $P(0<x) = \int_0^1 1.5x^2 = 1.5\left(\frac{x^3}{3}\right)\Big|_0^1$

$= \underline{0.5}$

b) $P(0.5<x) = \int_{0.5}^{1} 1.5x^2 = 1.5\left[\frac{1}{3} - \frac{0.125}{3}\right]$

$= \underline{0.4375}$

c) $P(-0.5 \le x \le 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 1.5\left[\frac{0.125}{3} + \frac{0.125}{3}\right]$

$= \underline{0.125}$

d) $P(X<-2) = 0$

e) $P(X<0 \text{ or } X>-0.5) = P(X<0) + P(X>0.5)$ $(-P(-0.5 \le x \le 0))$ $\cancel{-P(0 \le x \le 0.5)}$

$= \int_{-1}^{0} f(x)dx + \int_{-0.5}^{1} f(x)dx \left(-\int_{-0.5}^{0} f(x)dx\right)$

$= 1.5\left[\frac{1}{3}\right] + 1.5\left[\frac{1}{3} + \frac{0.125}{3}\right]\left(-1.5\left[0 + \frac{1.25}{3}\right]\right)$

$= 0.5 + \cancel{0.4375} + 0.5625 - 0.0625$

$= \cancel{0.9375} \quad 1.0625 = \underline{1.0625} = 1$

$1.0625 \approx 1$

f) $P(X<x) = 0.05$

$\int_{-1}^{x} f(x)dx = 0.05$

$\int_{-1}^{x} 1.5x^2 = 0.05$

$1.5\left[\frac{x^3}{3} + \frac{1}{3}\right] = 0.05$

$x^3 + 1 = \frac{0.05 \times 3}{1.5} = -0.9 \quad x = \underline{-0.9654}$ $\cancel{0.4641}$

⑤ $f(x) = \dfrac{e^{\frac{-x}{1000}}}{1000}$ $x > 0$

a) $P(X > 3000) = \int_{3000}^{\infty} \dfrac{e^{-x/1000}}{1000} dx$

$= \dfrac{-e^{-x/1000}}{\frac{1}{1000} \times 1000} \Big]_{3000}^{\infty} = -e^{-\infty} + e^{-3} = 0.049$

b) $P(1000 < x < 2000) = \int_{1000}^{2000} \dfrac{e^{-x/1000}}{1000} dx$

$= e^{-2} + e^{-1} = 0.232$

c) $P(X < 1000) = \int_{0}^{1000} f(x) dx = e^{-1} + 1 = 0.632$

d) $P(X < x) = \int_{0}^{x} f(x)\, dx = 0.1$

$= -e^{-x/1000} + e^{-0} = 0.1$

$\underline{x = 105.4}$

0.5

$\frac{-x}{1000} =$

---

⑥ $f(x) = 2.0$ for $49.75 < x < 50.25$

a) $P(X > 50) = \int_{50}^{50.25} 2.0\, dx = 2x \Big]_{50}^{50.25} = 2 \times 0.25 = \underline{0.5}$

b) $P(X < x) = 0.9 = \int_{49.75}^{x} 2 \cdot dx = 2x \Big]_{49.75}^{x}$

$2x - 99.5 = 0.9$

$2x = 100.4$

$\underline{x = 50.2}$

---

⑦ $f(x) = 1.25$ $74.6 < x < 75.4$

a) $P(X < 74.8) = \int_{74.6}^{74.8} 1.25 dx = 1.25\ x \Big]_{74.6}^{74.8}$

$= \underline{0.25}$

b) $P(X < 74.8$ or $x > 75.2)$

$= 0.25 + \int_{75.2}^{75.6} 1.25\, dx = \underline{0.5}$

c)

Cumulative distribution function.

$F(x) = P(X \leq x) = P(X < x) = \int_{-\infty}^{x} f(x)\,dx$ for $-\infty < x < \infty$

NOTE

$\frac{d}{dx}[F(x)] = f(x)$

① $F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 5 \\ 1 & 5 \leq x \end{cases}$

a) $P(X < 2.8) = F(2.8) = 0.2 \times 2.8 = 0.56$

b) $P(X > 1.5) = 1 - P(X < 1.5)$

$= 1 - [F(1.5)]$

$= 1 - 0.2(1.5) = 0.7$

c) $P(X < -2) = 0$

d) $P(X > 6) = 1 - P(X < 6)$

$= 1 - 1$

$= 0$

② $F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$

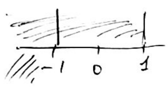
a) $P(X < 1.8) = 0.25(1.8) + 0.5 = 0.95$

b) $P(X > -1.5) = 1 - P(X \leq 1.5)$

$= 1 - F(1.5) = 1 - (0.25(1.5) + 0.5)$

$= \underline{0.125}$

c) $P(X < -2) = \cancel{F(0)} = 0$



d) $P(-1 < x < 1) = P(X < 1) - P(X < -1)$

$= F(1) - F(-1)$

$= (0.25(1) + 0.5) - (0.25(-1) + 0.5)$

$= \underline{0.5}$

③ $f(x) = 0.5x \qquad 0 < x < 2$

$\underline{(D)}$

$F(x) = 0$ for $0 > x$ or $x \not< 0$

& $F(x)$

$F(x) = \int_0^2 f(x)\,dx = \int_0^2 0.5x\,dx = 0.5\left(\frac{4}{2}\right)$

$= 0.25x^2 \qquad = 1$

$F(x) = 1, \quad 2 \leq x$

CDF

$F(x) = \begin{cases} 0 & x < 0 \\ 0.25x^2 & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$

④ $f(x) = \frac{e^{-x/10}}{10} \qquad 0 < x$

⑤ $f(x) = e^{-x} \qquad 0 < x$

$F(x) = 0$ for $x < 0$

$F(x) = \int_0^x e^{-x}\,dx = -e^{-x}\Big]_0^x = 1 - e^{-x} \qquad 0 < x$

$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x} & x > 0 \end{cases}$

⑥ $f(x) = x/8$ for $3 < x < 5$

$e^4 \cdot e^{-x}$

$e^4(-e^{-x})$

$e^4(-e^{-x})_4^x$

$F(x) = 0 \qquad x < 3$

$F(x) = \int_3^5 x/8\,dx = \frac{x^2}{16}\Big)_2^5$

$F(x) = 1 \qquad x \geq 5$

$F(x) = \begin{cases} 0 & x < 3 \\ x^2/16 & 3 < x < 5 \\ 1 & x \geq 5 \end{cases}$

⑦ $f(x) = e^{-(x-4)} = e^{4-x} \qquad 4 < x$

$F(x) = 0$ for $x < 4$

$F(x) = \int_4^x e^{(4-x)}\,dx = -\underline{e^{4-x}}\Big]_4^x = 1 - e^{4-x}$

$$F(x) = \begin{cases} 0 & x < 4 \\ 1 - e^{4-x} & x \geq 4 \end{cases}$$

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(8) $f(x) = \dfrac{e^{-x/1000}}{1000}$ for $x > 0$

$F(x) = 0$ for $x < 0$

$$F(x) = \int_0^x \frac{e^{-x/1000}}{1000}\,dx = \frac{1}{1000} \times \frac{e^{-x/1000}}{\frac{-1}{1000}} = -e^{-x/1}$$

$$= 1 - e^{-x/1000}$$

CDF $F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/1000} & x > 0 \end{cases}$

$$P(X > 3000) = 1 - P(X \leq 3000)$$

$$= 1 - \int_0^{3000} \frac{e^{-x/1000}}{1000}\,dx$$

$$= 1 - 1 + e^{-3}$$

$$= \underline{0.0498}$$

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(9) $f(x) = 2 \quad 2.3 < x < 2.8$

$F(x) = 0$ for $x < 2.3$

$$F(x) = \int_{2.3}^{2.8} 2\,dx = 2x \Big|_{2.3}^{2.8}$$

$F(x) = 1$ for $x > 2.8$

$$F(x) = \begin{cases} 0 & x < 2.3 \\ 2x & 2.3 < x < 2.8 \\ 1 & x > 2.8 \end{cases}$$