Step 2: Identify the alphabets of DFA: The input alphabets of DFA are the input alphabets of NFA. So, $\Sigma = \{a, b\}$.

Step 3: Identify the transitions (i.e., δ_D) of DFA: For each state $\{q_i, q_j, \ldots, q_k\}$ in Q_D and for each input symbol a in Σ, the transition can be obtained as shown below:

$$\delta_D(\{q_i, q_j, \ldots, q_k\}, a) = \delta_N(q_i, a) \cup \delta_N(q_j, a) \cup \ldots \delta_N(q_k, a)$$

$$= \{q_l, q_m, \ldots, q_n\} \text{ say}$$

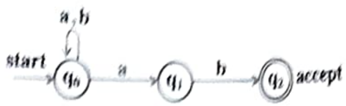
- Add the state $\{q_l, q_m, \ldots, q_n\}$ to Q_D, if it is not already in Q_D.
- Add the transitions from $\{q_i, q_j, \ldots, q_k\}$ to $\{q_l, q_m, \ldots, q_n\}$ on the input symbol a.

Note: The step 3 has to be repeated for each state that is added to Q_D.

Step 4: Identify the final states of DFA: If $\{q_i, q_j, \ldots q_k\}$ is a state in Q_D and if one of $q_i, q_j, \ldots q_k$ is the final state of NFA, then $\{q_i, q_j, \ldots q_k\}$ will be the final state of DFA.

Thus, DFA can be obtained using lazy evaluation method.

Example: Now, let us "Obtain the DFA for the following NFA using lazy evaluation method".



Solution: The transition table for the above DFA can be written as shown below:

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
$*q_2$	ϕ	ϕ

Step 1: Identify the start state of DFA: Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

Step 2: Identify the alphabets of DFA: The input alphabets of NFA are the input alphabets of DFA. So, $\Sigma = \{a, b\}$.

Step 3: Identify the transitions (i.e., δ_D) of DFA: Start from the start state q_0 and find the transitions as shown below:

For state $\{q_0\}$:

Input symbol = a

$$\delta_D(\{q_0\}, a) = \{q_0, q_1\}$$

Input symbol = b

$$\delta_D(\{q_0\}, b) = \{q_0\}$$

For state $\{q_0, q_1\}$:

Input symbol = a

$$\begin{aligned}\delta_D(\{q_0, q_1\}, a) &= \delta_N(\{q_0, q_1\}, a)\\ &= \delta_N(q_0, a) \cup \delta_N(q_1, a)\\ &= \{q_0, q_1\} \cup \phi\\ &= \{q_0, q_1\}\end{aligned}$$

Input symbol = b

$$\begin{aligned}\delta_D(\{q_0, q_1\}, b) &= \delta_N(\{q_0, q_1\}, b)\\ &= \delta_N(q_0, b) \cup \delta_N(q_1, b)\\ &= \{q_0\} \cup \{q_2\}\\ &= \{q_0, q_2\}\end{aligned}$$

For state $\{q_0, q_2\}$:

Input symbol = a

$$\begin{aligned}\delta_D(\{q_0, q_2\}, a) &= \delta_N(\{q_0, q_2\}, a)\\ &= \delta_N(q_0, a) \cup \delta_N(q_2, a)\\ &= \{q_0, q_1\} \cup \phi\\ &= \{q_0, q_1\}\end{aligned}$$

Input symbol = b

$$\begin{aligned}\delta_D(\{q_0, q_2\}, b) &= \delta_N(\{q_0, q_2\}, b)\\ &= \delta_N(q_0, b) \cup \delta_N(q_2, b)\\ &= \{q_0\} \cup \phi\\ &= \{q_0\}\end{aligned}$$

Since, no new state is generated this step is terminated.

Step 4: Identify the final states of DFA: Since q_2 is the final state of NFA in the above set, wherever q_2 is present as an element, the corresponding set is the final state of DFA. So, the final state is $\{q_0, q_2\}$.

Now, all the above transitions can be represented using transition table as shown below:

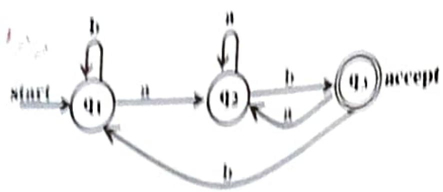
δ	a	b
$\rightarrow \{q_0\}$	$\{q_0,q_1\}$	$\{q_0\}$
$\{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$*\{q_0,q_2\}$	$\{q_0,q_1\}$	$\{q_0\}$

By renaming the states of DFA as A, B, C $\Longrightarrow$

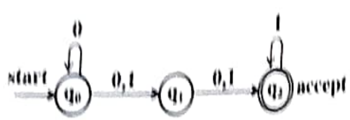
δ	a	b
$\rightarrow$ A	B	A
B	B	C
*C	B	A

So, the final DFA is given by $M = (Q, \Sigma, \delta, q_0, F)$ where

- $Q = \{A, B, C\}$
- $\Sigma = \{a, b\}$
- $q_0 = A$
- $F = \{C\}$
- δ is shown below using the transition table:



Example: Now, let us "convert the following NFA to its equivalent DFA".



Solution: The transition table for the above DFA can be written as shown below:

δ	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$
$*q_1$	$\{q_2\}$	$\{q_2\}$
q_2	ϕ	$\{q_2\}$

Step 1: Identify the start state of DFA: Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

Step 2: Identify the alphabets of DFA: The input alphabets of NFA are the input alphabets of DFA. So, $\Sigma = \{0, 1\}$.

Step 3: Identify the transitions (i.e., δ_D) of DFA: Start from the start state q_0 and find the transitions as shown below:

For state $\{q_0\}$:

Input symbol = *0*

$$\delta_D(\{q_0\}, 0) = \delta_N(\{q_0\}, 0))$$
$$= \{q_0, q_1\}$$

Input symbol = *1*

$$\delta_D(\{q_0\}, 1) = \delta_N(\{q_0\}, 1)$$
$$= \{q_1\}$$

For state $\{q_0, q_1\}$:

Input symbol = *0*

$$\delta_D(\{q_0, q_1\}, 0) = \delta_N(\{q_0, q_1\}, 0)$$
$$= \delta_N(q_0, 0) \cup \delta_N(q_1, 0)$$
$$= \{q_0, q_1\} \cup \{q_2\}$$
$$= \{q_0, q_1, q_2\}$$

Input symbol = *1*

$$\delta_D(\{q_0, q_1\}, 1) = \delta_N(\{q_0, q_1\}, 1)$$
$$= \delta_N(q_0, 1) \cup \delta_N(q_1, 1)$$
$$= \{q_1\} \quad \cup \{q_2\}$$
$$= \{q_1, q_2\}$$

The above two states are added to Q_D shown in previous step. The resulting states are shown below:

$$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\} \}$$

For state $\{q_1\}$:

Input symbol = *0*

$$\delta_D(\{q_1\}, 0) = \delta_N(\{q_1\}, 0)$$
$$= \{q_2\}$$

Input symbol = *1*

$\delta_D(\{q_1\}, 1) = \delta_N(\{q_1\}, 1)$

$= \{q_2\}$

The above two states are added to Q_D obtained in previous step so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\} \}$

For state $\{q_0, q_1, q_2\}$:

Input symbol = *0*

$\delta_D(\{q_0, q_1, q_2\}, 0) = \delta_N(\{q_0, q_1, q_2\}, 0)$

$= \delta_N(q_0, 0) \cup \delta_N(q_1, 0) \cup \delta_N(q_2, 0)$

$= \{q_0, q_1\} \cup \{q_2\} \cup \{\phi\}$

$= \{q_0, q_1, q_2\}$

Input symbol = *1*

$\delta_D(\{q_0, q_1, q_2\}, 1) = \delta_N(\{q_0, q_1, q_2\}, 1)$

$= \delta_N(q_0, 1) \cup \delta_N(q_1, 1) \cup \delta_N(q_2, 1)$

$= \{q_1\} \cup \{q_2\} \cup \{q_2\}$

$= \{q_1, q_2\}$

The above two states are added to Q_D obtained in previous step so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\} \}$

For state $\{q_1, q_2\}$:

Input symbol = *0*

$\delta_D(\{q_1, q_2\}, 0) = \delta_N(\{q_1, q_2\}, 0)$

$= \delta_N(q_1, 0) \cup \delta_N(q_2, 0)$

$= \{q_2\} \cup \phi$

$= \{q_2\}$

Input symbol = *1*

$\delta_D(\{q_1, q_2\}, 1) = \delta_N(\{q_1, q_2\}, 1)$

$= \delta_N(q_1, 1) \cup \delta_N(q_2, 1)$

$= \{q_2\} \cup \{q_2\}$

$= \{q_2\}$

The above two states are added to Q_D obtained in previous step so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\} \}$

For state $\{q_2\}$:

Input symbol = *0*

$\delta_D(\{q_2\}, 0) = \delta_N(\{q_2\}, 0)$

$= \phi$

Input symbol = *1*

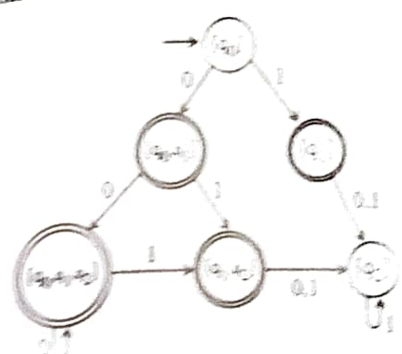
$\delta_D(\{q_2\}, 1) = \delta_N(\{q_2\}, 1)$

$= \{q_2\}$

The above two states are added to Q_D obtained in previous step so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\} \}$

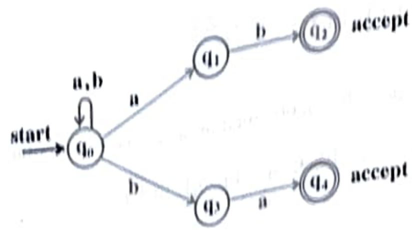
The final transition table along with transition diagram is shown below:



δ	0	1
→ $\{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
*$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	ϕ	$\{q_2\}$

Example: now, let us "Obtain an NFA to accept strings of a's and b's ending with ab or ba. From this obtain an equivalent DFA".

Solution: The NFA to accept strings of a's and b's ending *ab* or *ba* is shown below:



The transition table for the above transition diagram is shown below:

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	ϕ	$\{q_2\}$
$*q_2$	ϕ	ϕ
q_3	$\{q_4\}$	ϕ
$*q_4$	ϕ	ϕ

Step 1: Identify the start state of DFA: Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA. So, $Q_D = \{ \{q_0\} \}$.

Step 2: Identify the alphabets of DFA: The input alphabets of NFA are the input alphabets of DFA. So, $\Sigma = \{a, b\}$.

Step 3: Identify the transitions (i.e., δ_D) of DFA: Start from the start state $\{q_0\}$ and find the transitions as shown below:

For state $\{q_0\}$:

Input symbol $= a$

$\delta_D(\{q_0\}, a) = \{q_0, q_1\}$

Input symbol $= b$

$\delta_D(\{q_0\}, b) = \{q_0, q_3\}$

The above two states are added to Q_D obtained in step 1 so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_0, q_3\} \}$

For state $\{q_0, q_1\}$:

Input symbol $= a$

$\delta_D(\{q_0, q_1\}, a) = \delta_N(\{q_0, q_1\}, a)$

$= \delta_N(q_0, a) \cup \delta_N(q_1, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

Input symbol $= b$

$\delta_D(\{q_0, q_1\}, b) = \delta_N(\{q_0, q_1\}, b)$

$= \delta_N(q_0, b) \cup \delta_N(q_1, b)$

$= \{q_0, q_3\} \cup \{q_2\}$

$= \{q_0, q_2, q_3\}$

The above two states are added to Q_D so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_0, q_3\}, \{q_0, q_2, q_3\} \}$

For state $\{q_0, q_3\}$:

Input symbol $= a$

$\delta_D(\{q_0, q_3\}, a) = \delta_N(\{q_0, q_3\}, a)$

$= \delta_N(q_0, a) \cup \delta_N(q_3, a)$

$= \{q_0, q_1\} \cup \{q_2\}$

$= \{q_0, q_2, q_1\}$

Input symbol $= b$

$\delta_D(\{q_0, q_3\}, b) = \delta_N(\{q_0, q_3\}, b)$

$= \delta_N(q_0, b) \cup \delta_N(q_3, b)$

$= \{q_0, q_3\} \cup \phi$

$= \{q_0, q_3\}$

The above two states are added to Q_D so that

$Q_D = \{ \{q_0\}, \{q_0, q_1\}, \{q_0, q_3\}, \{q_0, q_2, q_3\}, \{q_0, q_1, q_4\} \}$

On similar lines, the reader is supposed to find the transitions for other states specified in Q_D. The reader is advised to verify the following answers:

$\delta_D(\{q_0, q_2, q_3\}, a) = \{q_0, q_1, q_4\}$

$\delta_D(\{q_0, q_2, q_3\}, b) = \{q_0, q_3\}$

$\delta_D(\{q_0, q_1, q_3\}, a) = \{q_0, q_1\}$

$\delta_D(\{q_0, q_1, q_4\}, a) = \{q_0, q_2, q_3\}$

The final DFA obtained along with trabsition diagram and transition table is shown below:

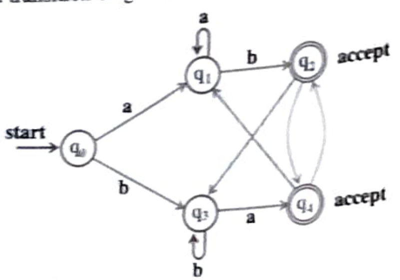
δ	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$\{q_0, q_2, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$

The states of the above DFA are:

$\{q_0\}, \{q_0, q_1\}, \{q_0, q_2, q_3\}, \{q_0, q_3\}, \{q_0, q_1, q_4\}$

By renaming q_0, q_1, q_2, q_3, q_4

The final transition diagram and transition table are shown below:



Transition diagram

δ	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
$*q_2$	q_4	q_3
q_3	q_4	q_3
$*q_4$	q_1	q_2

Transition table

Now, it is observed that for every NFA there exists some DFA that accepts the same language accepted by NFA. Now, let us "Formally prove that every NFA N can be converted into a DFA such that L(D) = L(M)".

Theorem: If there exists NFA $M_N = (Q_N, \Sigma, \delta_D, q_0, F_N)$ which accepts the language $L(M_N)$, there exists an equivalent DFA $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L(M_D) = L(M_N)$.

Proof: It is required to prove that

$$\delta^*_D(q_0, w) = \delta^*_N(q_0, w)$$

We know that if Q_N represents states of NFA then power set of Q_N which contains the set of subsets of set Q_N are the states of DFA denoted by Q_D. The DFA interprets each set as a single state in DFA.

Basis: Consider a string $w = \varepsilon$ where $|w| = 0$

$\delta^*_D(\{q_0\}, \varepsilon) = \{q_0\}$ by definition of extended transition function of DFA

$\delta^*_N(\{q_0\}, \varepsilon) = \{q_0\}$ by definition of extended transition function of NFA

Hence $\delta^*_D(q_0, w) = \delta^*_N(q_0, w)$ is proved when $w = \varepsilon$.

Induction hypotheses: Now, let us assume that

$\delta^*_D(q_0, w) = \delta^*_N(q_0, w)$ for some w where $|w| = n$

Now, it is required to prove that the statement:

$\delta^*_D(q_0, w) = \delta^*_N(q_0, w)$ is true for some w where $|w| = n + 1$

Inductive proof: Let $w = xa$ where a is the last symbol of w and x is the remaining string of w:

So, $|x| = n$ and $|xa| = |w| = n + 1$

By extended definition δ^* of NFA, we know that:

$$\begin{aligned}\delta^*_N(q_0, w) &= \delta^*_N(q_0, xa)\\ &= \delta_N(\underbrace{\delta^*_N(q_0, x)}, a)\end{aligned} \qquad (1)$$

Now, x is the string to be processed and after consuming the string x, let the states of the machine be $\{p_i, p_j, \ldots, p_k\}$

i.e., $\delta^*_N(q_0, x) = \{p_i, p_j, \ldots, p_k\}$

Substituting this in Eq. (1), we have

$$\begin{aligned}\delta^*_N(q_0, w) &= \delta_N(\{p_i, p_j, \ldots, p_k\}, a)\\ &= \delta_N(p_i, a) \cup \delta_N(p_j, a) \cup \ldots \delta_N(p_k, a)\end{aligned} \qquad (2)$$

By extended definition δ^* of DFA, we know that

$$\begin{aligned}\delta^*_N(q_0, w) &= \delta^*_N(q_0, xa)\\ &= \delta_N(\underbrace{\delta^*_N(q_0, x)}, a)\end{aligned} \qquad (3)$$

Now, x is the string to be processed and after consuming the string x, let the states of the mach be $\{p_i, p_j, \ldots, p_k\}$.

i.e., $\delta^*_D(q_0, x) = \{p_i, p_j, \ldots, p_k\}$

Substituting this in Eq. (3), we have

$$\delta^*_D(q_0, w) = \delta_D(\{p_i, p_j, \ldots, p_k\}, a)$$

$$= \delta_N(p_i, a) \cup \delta_N(p_j, a) \cup \ldots \delta_N(p_k, a) \quad (4)$$

By comparing Eqs. (2) and (4), we have

$$\delta^*_D(\{q_0\}, w) = \delta^*_N(\{q_0\}, w)$$

So, if $\delta^*_D(\{q_0\}, w)$ is in F_N and $\delta^*_N(\{q_0\}, w)$ is in F_N, then both enters into final state accepting th same language. Thus, $L(M_N) = L(M_D)$. Hence, the proof.

Theorem: Now, let us "Prove that a language L is accepted by some DFA if and only if L i accepted some NFA".

Proof: The above statement can be proved as shown below:

Note: Write the subset construction method of converting an NFA to DFA + proof of previou theorem.

Since in the subset construction, all the transitions defined for NFA are also defined for DFA, the language accepted by NFA is same as the language accepted by DFA. So, *w* is accepted by DFA M_D if and only if *w* is accepted by M_N, i.e., $L(M_N) = L(M_D)$. Hence the proof.

Exercises

1. What is an NFA? Explain with example.
2. What is the need for an NFA?
3. What is the difference between DFA and NFA?
4. Give a general procedure to convert as NFA to DFA.

5. Convert the following NFA into an equivalent DFA.

6. Draw an NFA to accept the string of a's and b's such that it can accept either the string consisting of one a followed by any number of a's or one b followed by any number of b's (i.e. aa* | bb*) and obtain the corresponding DFA.