21/1/20

Analysis Of Continuous Random Variable.

A continuous random variable with an interval of real no. is called the range

Pro density fun.

For a continuous random X, a pro density funn is a funn such that p(

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $p(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx =$ area under f(x) a to b

for any a & b.

If f is a continuous random variable for any x_1 & x_2, then $p(x_1 \leq X \leq x_2) = p(x_1 < X \leq x_2) =$
$p(x_1 \leq X < x_2) = p(x_1 < x < x_2)$

Q6 Suppose that $f(x) = e^{-x}$ for $0 < x$. Determine the following pro.

a) $p(1<x) = \int_{1}^{\infty} e^{-x}\,dx$
$= (-e^{-x})_1^{\infty} = -e^{-\infty} + e^{-1}$
$= e^{-1}$
$= 0.3679$

b) $p(1 < x < 2.5)$
$= \int_{1}^{2.5} e^{-x}\,dx$
$= (-e^{-x})_1^{2.05}$
$= -e^{-2.5} + e^{-1}$
$= 0.285$

c) $p(x=3) = \int_{3}^{3} e^{-x}$
$= (-e^{-x})^3$
$= -e^{-3} + e^{-3}$
$= 0$

d) $p(x < 4) = \int_{4}^{\infty} e^{-x}\,dx$
$= \left[-e^{-x}\right]_4^{\infty}$
$= -e^{-\infty} + e^{-4}$
$= 0.0183$

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2)=

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③ $p(3 \le x) = p(x \ge 3)$

$= \int_3^\infty e^{-x}\,dx = \left[-e^{-x}\right]_3^\infty = -e^{-\infty} + e^{-3}$

$= 0.0497$

④ $p(x < X) = 0.1$

$\int_x^\infty e^{-x}\,dx = 0.1$

$\left[-e^{-x}\right]_x^\infty = 0.1$

$-e^{-\infty} + e^{-x} = 0.1$

$-x = \ln(0.1)$

$\boxed{x = 2.302}$

⑤ $p(X \le x)$

$\int_{-\infty}^{x} e^{-x}\,dx = 0.1$

$\left[-e^{-x}\right]_{-\infty}^{x} = 0.1$

$-e^{-x} + \cancel{e^{+\infty}} = 0.1$

$1 - e^{-x} = 0.1$

$-x \le -\ln(0.1)$

$e^{-x} = 0.9$

$-x = -0.105$

$\boxed{x = 0.105}$

2. $f(x) = x/8, \quad$ for $3 < x < 5$

① $p(x < 4)$

$= \int_3^4 \frac{x}{8}\,dx$

$= \left[\frac{x^3}{24}\right]_3^4$

$= \frac{4^3}{4} - \frac{3^3}{4}$

$= 4^2 - \frac{27}{4}$

$= 9.25$

② $p(x > 3.5)$

$f(x) = x/8$

$= \int_{3.5}^{5} \frac{x}{8}\,dx$

$= \left(\frac{x^2}{16}\right)_{3.5}^{5}$

$= 0.716$

③ $p(4 < x < 5) = \int_4^5 (x/8)\,dx$

$= \left(\frac{x^2}{16}\right)_4^5 = 0.5625$

④ $p(x < 4.5) = 0.7031$

⑤ $p(x < 3.5)$. ⑥ $p(x > 4.5)$

$$\int_3^{3.5} \frac{x}{8}\,dx + \int_{4.5}^{5} x/8\,dx$$

$$= \left(\frac{x^2}{16}\right)_3^{3.5} + \left[\frac{x^2}{16}\right]_{4.5}^{5} = 0.203 + 0.2968$$

$$= 0.4998$$

$$= \underline{0.5}$$

$\boxed{3}$ $f(x) = e^{-(x-4)}$, $\underline{4 \cancel{x} < x}$ means $x > 4$

$-\infty \quad 1 \quad 4 \quad +\infty$

① $P(1 < x)$

$$= \int_1^4 e^{-(x-4)}\,dx + \int_4^\infty e^{-(x-4)}\,dx$$

$$= -(x-4)\left[e^{-(x-4)}\right]_1^4 + -(x-4)\left[e^{-(x-4)}\right]_4^\infty$$

$$= \cancel{-(4-4)}\left[e^{\overset{0}{(4-4)}}\right] - (1-4)\left[e^{-(1-4)}\right]^1 +$$

$$-(\infty-4)\left[e^{-(\infty-4)}\right] + \cancel{(4-4)}\left[e^{-\overset{0}{(4-4)}}\right]$$

$$= -3\,e^{+3} - (\infty-4)\left[e^{-(\infty-4)}\right]$$

$$= \int e^{-(x-4)}\,dx = \int e^{-x} e^{4}\,dx = \underline{e^4 \int e^{-x}\,dx}$$

$$= e^4\left[e^{-x}\right]_1^4 + e^4\left[e^{-x}\right]_4^\infty$$

$$= e^4\left[e^{-4} - e^{-1}\right] + e^4\left[e^{-\infty} - e^{-4}\right]$$

$$= e^0 - e^3 + e^4 - e^0$$

$$= e^4 - e^3$$

$$= 54.59 -$$

2)

3)

4)

5)

2) $p(2 \le x < 5)$

$$= \int_4^5 e^{-(x-4)} dx = \left[-e^{-(x-4)}\right]_4^5$$

$$= -e^{-1} + e^{-0}$$

$$= 0.6321$$

3) $p(5 < x)$

$$= \int_5^\infty e^{-(x-4)} dx = \left[e^{-(x-4)}\right]_5^\infty$$

$$= \left[-e^{-\infty} + e^{-1}\right] = 0.3678$$

4) $p(8 < x < 12) = \int_8^{12} e^{-(x-4)} dx$

$$= \left[-e^{-(x-4)}\right]_8^{12} = -e^{-\infty} + e^{-4}$$

$$= 0.1798$$

5) $p(\cancel{X} < \cancel{x}) = 0.9$

$$\int_4^x e^{-(x-4)} dx = 0.9$$

$$\left[-e^{-(x-4)}\right]_4^x = 0.9$$

$$-e^{-(x-4)} + e^{-0} = 0.9$$

$$-e^{-(x-4)} + 1 = 0.9$$

$$-e^{-(x-4)} = -0.1$$

$$e^{-(x-4)} = 0.1$$

$$-(x-4) = -2.3$$

$$x - 4 = 2.3$$

$$x = 6.3$$

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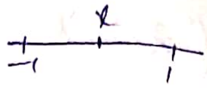
$f(x) = 1.5x^2$ for $-1 < x < 1$.



a) $p(0 < X)$

$= \int_0^1 1.5x^2 \, dx$

$= \left[\cancel{1.5} \frac{x^3}{\cancel{3}} \right]_0^1$ (0.5)

$= 0.5 \left[x^3 \right]_0^1$

$= \underline{0.5}$

b) $p(0.5 < X)$

$= \int_{0.5}^1 1.5x^2 \, dx$

$= \left[1.5 \frac{x^3}{3} \right]_{0.5}^1$

$= 0.5 \left[x^3 \right]_{0.5}^1$

$= 0.5 \left[1 - 0.125 \right]$

$= 0.4375$

c) $p(-0.5 \leq X \leq 0.5)$

$= \int_{-0.5}^{0.5} 1.5x^2 \, dx$

$= \left[1.5 \frac{x^3}{3} \right]_{-0.5}^{0.5}$

$= 0.5 \left[x^3 \right]_{-0.5}^{0.5}$

$= 0.5 \left[0.125 + 0.125 \right]$

$= 0.125$

d) $p(X < -2)$

$= \int$

$= 0$

e) $p(X < 0$ or $X > -0.50)$

$= \int_{-\cancel{1}}^{\cancel{0}} 1.5 \frac{x^3}{3}] + \int_{-0.5}^{1} 0.5]$

$= 0.5 \left[x^3 \right]_{-1}^{0} + 0.5 \left[x^3 \right]_{0.5}^{1}$

$= -0.5 + (0.5 - 0.125)$

$= -0.5 + 0.375$

$=$

$0.5 \left[x^3 \right]_0^1 + 0.5 \left[x^3 \right]_{-0.5}^{1}$

$0.5(1) + 0 + 0.5(1) - (-0.125)$

$= 0.5 + 0.5 + 0.125$

$= 1.125 \approx 1$

4) $p(X<x) = 0.05$

$$0.05 = \int_0^x 1.5x^2\,dx$$

$$= 0.5\left[x^3\right]_0^x$$

$$= 0.5\left[x^3 - 0\right]$$

$$= 0.5x^3$$

$$x^3 = \frac{0.05}{0.5}$$

$$x = 0.464$$

5] $f(x) = \frac{e^{\frac{-x}{1000}}}{1000}$ for $x>0$

a) a component lasts more than 3000 hrs before failure.

$$p(x>3000) = \int_{3000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000}$$

$$= \frac{1}{1000}\int_{3000}^{\infty} e^{-x/1000}$$

$$= \frac{-1}{1000}\left[e^{\frac{-x}{1000}}\right]_{3000}^{\infty}$$

$$= 0 \quad \frac{-1}{1000}\left[e^{-\frac{3000}{1000}}\right]$$

$$= \frac{-1000}{1000}(e^{-3})$$

$$= 0.049$$

b) $\int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000}$

$$= \frac{1000}{10000}\left[e^{\frac{-x}{1000}}\right]_{1000}^{2000}$$

$$= \frac{1}{10000}\left[e^{-2} + e^{-1}\right]$$

$$= [0.1353 + 0.367]$$

$$= 0.502$$

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c) $P(x<1000)$ $\int_0^{1000} \frac{e^{-x/1000}}{1000} dx$

$= \frac{1}{1000}\left[e^{-x/1000}\right]_0^{1000}$

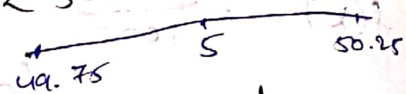
$= \frac{1}{1000}\left[e^{-1}\right] = 0.367$

d) $P(x>X) = 0.10$

$\int_0^{X} \frac{e^{-x/1000}}{1000} dx = 0.10$

$X = 105.36$ hours.

6] $f(x) = 2$ for $49.75 < x < 50.25$



a) $P(x>50)$

$\int_{50}^{50.25} 2\, dx$

$= 2\left[x\right]_{50}^{50.25}$

$= 0.5$

b) $0.9 = P(X>x)$

$0.9 = \int_x^{50.25} 2\, dx$

$= 2\left[50.25 - x\right]$

$0.9 = 100.5 - 2x$

$2x = 100.5 - 0.9$

$\boxed{x = 49.8}$

7] $f(x) = 1.25$ $74.6 < x < 75.4$

a) $P(X < 74.8)$

$\int_{74.6}^{74.8} 1.25\, dx$

$= 1.25\left[x\right]_{74.6}^{74.8}$

$= 1.25 \times 0.2$

$= 0.\cancel{0}25$

b)

Cumulative Distribution Function of a Continuous random variable.

$F(x) = P(X \leq x) = P(X < x) = \int_{-\infty}^{\infty} f(x)\, dx,$

for $-\infty < x < \infty$

Note $\frac{d}{dx}[F(x)] \doteq f(x)$

1) $F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 5 \\ 1 & 5 \leq x \end{cases}$

ⓐ $P(X < 2.8) = F(2.8) = 0.2(2.8)$
$= 0.56$

ⓑ $P(X > 1.5) = F(\cancel{2}.5)$
$= 1 - P(X < 1.5)$
$= 1 - 0.3$
$= 0.7$

ⓒ $P(X < -2) = 0$

ⓓ $P(X > 6) = 1$

2] $F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$

ⓐ $P(X < 1.8)$
$= 0.25x + 0.5$
$= 0.25(\cancel{0.18}\ 1.8) + 0.5 = 0.45$

ⓓ

ⓒ $P(X < -2) = 0$

ⓑ $P(\cancel{X > 1.5})$
$= 0.25(-1.5) + 0.5$
$= 0.125$

ⓑ $P(X > -1.5) = 1 - P(X \leq -1.5) = 1 - F(-1.5)$
$= 1 - (0.25(-1.5) + 0.5)$
$= 0.875$

④ $p(-1<X<1) = p(X<1) - p(X<-1)$

$= F(1) - F(-1)$

$= (0.25(1)+0.5) - (0.25(-1)+0.5)$

$= 0.5$

3] $f(x) = 0.5x \quad 0<x<2$

$F(X)=0$ for $0<X$ ⓐ $x>0$

& $F(x)$, $0<x<2$

i.e $F(x) = \int_0^2 f(x)\,dx = \int_0^2 0.5x\,dx$

$= 0.5\frac{x^2}{2}$

$F(x) = 0.25x^2$

③ $F(x)=1, \; 2\leq x$

CDF, $F(x) = \begin{cases} 0 & x<0 \\ 0.25x^2 & 0\leq x<2 \\ 1 & 2\leq x \end{cases}$

5] $f(x) = e^{-x}$ for $0<x$

$F(x)=0$; $x<0$

$F(x) = \int_0^x e^{-x}\,dx = -e^{-x}\Big|_0^x$

$= -e^{-x} + e^{-0}$

$= 1-e^{-x} \quad 0<x$

$F(x) = \begin{cases} 0 & x \\ 1-e^{-x} & x>0 \end{cases}$

6] $f(x) = x/8$ for $3<x<5$

$F(x)=0 \quad x<3$

$F(x) = \int_3^5 \frac{x}{8}$

$= \frac{x^2}{16}\Big|_3^5$

$=$

$F(x)=1 \quad x>5$

$F(x) = \begin{cases} 0, & x<3 \\ \frac{x^2}{16}, & 3<x<5 \\ 1, & 5\leq x \end{cases}$

$f(x) = e^{-(x-4)}$ for $4 < x$

$F(x) = 0$, $x < 4$

$F(x) = \int_4^x e^{-(x-4)}\, dx$

$= 1 - e^{-(x-4)}$

$F(x) = \begin{cases} 0 & x < 4 \\ 1 - e^{-(x-4)} & 4 \le x \end{cases}$

if 2 limits given 3 intervals

if 1 limit given 2 intervals.

8] $f(x) = \frac{e^{\frac{-x}{1000}}}{1000}$ for $x > 0$. Use the cumulative distribution fun to determine the pro that a component lasts more than 3000 hrs before failure.

$F(x) = 0$, $x < 0$

$F(x) = \int_0^x \frac{e^{-x/1000}}{1000}\, dx = 1 - e^{-\frac{x}{1000}}$

$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/1000} & x \ge 0 \end{cases}$

$F(x > 3000) = 1 - F(x \le 3000)$

$= 1 - \int_0^{3000} \frac{e^{-x/1000}}{1000}\, dx$

$= e^{-3}$

$= 0.0498$

9] $f(x) = 2$, $2.3 < x < 2.8$

$x < 2.3$

$F(x) =$

$F(x) = \int_{2.3}^{2.8} 2\, dx$

$= 2x$

$F(x) =$ $x > 2.8$

$F(x) = \begin{cases} 0 & , x < 2.3 \\ 2x & , 2.3 < x < 2.8 \\ 1 & , 2.8 \le x \end{cases}$

10) $P(x > 2.7) = \int_{2.7}^{2.8} 2 \, dx$

$= 2x \Big|_{2.7}^{2.8}$

$= 2(2.8) - 2(2.7)$

$= 0.2$

10) $F(x) = 1 - e^{-2x}, \quad x > 0$

$f(x) = \frac{d}{dx}[F(x)]$

$x > 0$

$-\infty \qquad 1 - e^{-2x}$

$$f(x) = \begin{cases} \frac{d}{dx}(0) & x \leq 0 \\ \frac{d}{dx}(1 - e^{-2x}) & x > 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 2e^{-2x} & x \geq 0 \end{cases}$$

11) $$F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 4 \\ 0.04x + 0.64 & 4 \leq x < 9 \\ 1 & 9 \leq x \end{cases}$$

$$f(x) = \begin{cases} \frac{d}{dx}(0) & x < 0 \\ \frac{d}{dx}(0.2x) & 0 \leq x < 4 \\ \frac{d}{dx}(0.04x + 0.64 & 4 \leq x < 9 \\ & 9 \leq x \end{cases}$$

$$f(x) = \begin{cases} 0 & x < -2 \\ 0.2 & 0 \leq x < 4 \\ 0.04 & 4 \leq x < 9 \\ 0 & 9 \leq x \end{cases}$$

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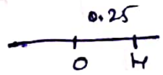
Mean & Variance.

Suppose x is a continuous random variable with pro density $f(x)$, the mean (or) expected value of x is

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x) \, dx$$

And the variance $(\sigma^2) = V(x) = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$

① Suppose $f(x) = 0.25$ for $0 < x < 4$



$$\mu = \int_0^4 x f(x) \, dx$$

$$= \int_0^4 0.25 x \, dx$$

$$= \left[0.25 \frac{x^2}{2} \right]_0^4$$

$$\cancel{\left[0.25 \frac{x^3}{3} \right]_0^4 - 2}$$

$$\mu = 2$$

$$\sigma^2 = \int_0^4 x^2 f(x) \, dx - \mu^2$$

$$= \int_0^4 0.25 x^2 \, dx - 2^2$$

$$= 0.25 \frac{x^3}{3} \Big|_0^4 - 2^2$$

$$\sigma^2 = 1.33$$

② $f(x) = 1.5 x^2 \quad -1 < x < 1$

$$\mu = \int_{-1}^{1} x (1.5 x^2) \, dx$$

$$= \int_{-1}^{1} 1.5 x^3 \, dx$$

$$= 1.5 \frac{x^4}{4} \Big|_{-1}^{1}$$

$$= 0$$

$$\sigma^2 = \int_{-1}^{1} x^2 f(x) \, dx - \mu^2$$

$$= \int_{-1}^{1} x^2 (1.5 x^2) \, dx - 0$$

$$= \int_{-1}^{1} 1.5 x^4 \, dx$$

$$= (1.5) \left(\frac{x^5}{5} \right)_{-1}^{1} =$$

$$= 0.6$$

③ $f(x) = x/8$ for $3 < x < 5$

$\mu = \int_3^5 x f(x) = \int_3^5 x \frac{x}{8} dx = \int_3^5 \frac{x^2}{8} dx = \left[\frac{x^3}{24}\right]_3^5$

$\frac{1}{24}\left[5^3 - 3^3\right] = 98/24 = 4.08$

$\sigma^2 = \int_3^8 x^2 f(x) - \mu^2$

$= \int_3^5 x^2 \left(\frac{x}{8}\right) dx - (4.08)^2$

$= \frac{1}{8}\left[\frac{x^4}{4}\right]_3^5 - (4.08)^2$

$= \frac{1}{32}\left[5^4 - 3^4\right] - (4.08)^2$

$= 0.3546$

④ $f(x) = 2x^{-3}, \ 1 < x$

$x > 1$

$\mu = \int_1^\infty x f(x) \, dx$

$= \int_1^\infty x \, 2x^{-3} dx = \int_1^\infty 2x^{-2} dx$

$= \left[2 \frac{x^{-1}}{-1}\right]_1^\infty$

$= 0 + 2\cancel{1}^{-1}$

$= 2$

⑤ $f(x) = 0.1$ $1200 < x < 1210$

ⓐ $\mu = \int_{1200}^{1210} x \, 0.1 \, dx$

$= \left[0.1 \frac{x^2}{2}\right]_{1200}^{1210}$

$= 0.05\left[x^2\right]_{1200}^{1210}$

$= 1205$

$\sigma^2 = \int_{1200}^{1210} x^2 f(x) dx - \mu^2$

$= 0.05\left[x^3\right.$

$= \left.\frac{x^3}{3}(0.1)\right]_{1200}^{1210} - 1205^2$

ⓑ $1195 < x < 1205$

$\mu = \int_{1195}^{1205} x \, 0.1 \, dx$

$= 0.05\left[x^2\right]_{1195}^{1205}$

$= 1200$

$\sigma^2 = 8.33$

$SD = \sqrt{\sigma^2}$

$= 2.886$

Cleary nearly centring the process at centre of specification results in the greater propotion of specification

$\therefore 1200 < x < 1205$

$$= \int_{1195}^{1200} 0 \, dx + \int_{1200}^{1205} 0.1 \, dx$$

$$= 0.1 [x]_{1200}^{1205}$$

$$= 0.5$$

⑥ $f(x) = 600 x^{-2}$ $\quad 100 \mu m < x < 120 \mu m$

$$\mu (\text{mean}) = \int_{100}^{120} x (600 x^{-2}) dx \Rightarrow \int_{100}^{120} x \frac{600}{x^2} dx$$

$$= \int_{100}^{120} 600 x^{-1} dx$$

$$= 600 x^0$$

$$= \left[600 \ln (x) \right]_{100}^{120}$$

$$= 109.39$$

$$\text{Variance} = \int_{100}^{110} x^2 \frac{600}{x^2} dx - 109.39^2$$

$$= 33.82$$

ⓑ cost → \$ 0.50 per part have to find average cost

$$\text{avg cost} = \text{Mean} \times \text{cost per part}$$

$$= 109.39 \times 0.50$$

$$= \$ 54.7$$

$$\int \frac{1}{x} = \ln x$$

7] $f(x) = \frac{70}{69x^2}$ $\quad 1 < x < 70$

$$\mu = \int_1^{70} x \frac{70}{69x^2} dx$$

$$= \frac{70}{69} [\cancel{\log} \ln x]_1^{70}$$

$$= 4.310$$

$$\sigma^2 = \int_1^{70} x^2 \frac{70}{69x^2}\, dx - \mu^2$$

$$= \int_1^{70} \frac{70}{69}\, dx - (4.31)^2$$

$$= \frac{70}{69}[x]_1^{70} - (4.31)^2$$

5/2/2020

Continuous Uniform distribution

A continuous random variable X with pro. density fun $f(x) = \frac{1}{(b-a)}$, $a \le x \le b$ is continuous uniform random variable

The mean & variance of of a CUD is

$$\mu = E(x) = \frac{a+b}{2} \text{ \&}$$

$$\sigma^2 = V(x) = \frac{(b-a)^2}{12}$$

The cumulative distribution of continuous uniform distribution of random variable X is

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \end{cases}$$

I) Interval $[1.5, 5.5]$

Determine mean, σ^2, SD

$p(x < 2.5)$

$a = 1.5 \quad b = 5.5$

$$\text{mean} = \frac{1.5+5.5}{2} = 7/2 = 3.5$$

$$V = \frac{(5.5-1.5)^2}{12} = \frac{\cancel{16}^{4}}{\cancel{12}_{3}} = 1.33$$

$$SD = \sqrt{1.33}$$

$$= 1.154$$

(b) $P(X < 2.5)$

$$= \int_{1.5}^{2.5} \frac{1}{b-a} \, dx = \int_{1.5}^{2.5} \frac{1}{5.5-1.5} \, dx$$

$$= \frac{1}{4} (x)_{1.5}^{2.5}$$

$$= \frac{1}{4} (2.5 - 1.5) = \frac{1}{4}$$

$$F(x) = \begin{cases} 0 & x \leq 1.5 \\ \dfrac{x-1.5}{5.5-1.5} & 1.5 \leq x \leq 5.5 \\ 1 & 5.5 \leq x \end{cases}$$

$$= \begin{cases} 0 & x \leq 1.5 \\ \dfrac{x-1.5}{4} & 1.5 \leq x \leq 5.5 \\ 1 & 5.5 \leq x \end{cases}$$

(2) (a) $a = -1$, $b = 1$

$\mu = \frac{-1+1}{2} = 0$

$V = \frac{(1+1)^2}{12} = \frac{4}{12} = \frac{1}{3}$

$SD = \sqrt{1/3} = 0.577$

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{1+1} & -1 \leq x \leq 1 \\ 1 & 1 \leq x \end{cases}$$

$$= \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{2} & -1 \leq x \leq 1 \\ 1 & 1 \leq x \end{cases}$$

(b) $P(-x < X < x) = 0.90$

$\int_{-x}^{x} \frac{1}{b-a} \, dx = 0.9$

$\int_{-x}^{x} \frac{1}{1+1} \, dx = 0.9$

$= \frac{1}{2} (x)_{-x}^{x} = 0.9$

$= \frac{1}{2} (2x) = 0.9$

$\boxed{x = 0.9}$

(3) (a) $a = 49.75 \quad b = 90.25$

$$\mu = \frac{49.75 + 50.25}{2} = 50$$

$$V = \frac{(50.25 - 49.75)^2}{12} = 0.0208$$

$$SD = \sqrt{0.0208} = 0.144$$

(b)
$$F(x) = \begin{cases} 0 & , x < 49.75 \\ \dfrac{x - 49.75}{0.5} & , 49.75 \le x \le 50.25 \\ 1 & , 50.25 \le x \end{cases}$$

(c) $P(X < 50.1)$

$$\int_{49.75}^{50.1} \frac{1}{0.5}\, dx$$

$$= \frac{1}{0.5} [x]_{49.75}^{50.1}$$

$$= \frac{1}{0.5} (50.1 - 49.75)$$

$$= 0.7$$

(4) $a = 0.95 \quad b = 1.05$

a)
$$F(x) = \begin{cases} 0 & , x < 0.95 \\ \dfrac{x - 0.95}{1.05 - 0.95} & , 0.95 \le x \le 1.05 \\ 1 & , 1.05 \le x \end{cases}$$

b) $P(X > 1.02)$

$$\int_{1.02}^{1.05} \frac{1}{1.05 - 0.95}\, dx$$

$$\int_{1.02}^{1.05} \frac{1}{0.1}\, dx$$

$$= \frac{1}{0.1} [x]_{1.02}^{1.05}$$

$$= \frac{1}{0.1} [1.05 - 1.02]$$

$$= 0.3$$

c) $P(X > x) = 0.9$

$1 - P(X \le x) = 0.9$

$1 - 0.9 = P(X \le x)$

$P(X \le x) = 0.1$

$$\int_{0.95}^{x} \frac{1}{0.1}\, dx = 0.1$$

$$\frac{1}{0.1} [x]_{0.95}^{x} = 0.1$$

$$\frac{1}{0.1} [x - 0.95] = 0.1$$

$$\boxed{x = 0.96}$$

(d) $\mu = 0.95 + 1.05/2 = 1$

$V = (1.08 - 0.95)^2/12 = 8.33 \times 10^{-4}$

(5) $a = 1.5$, $b = 2.2$

(a)
$$F(x) = \begin{cases} 0 & x < 1.5 \\ \dfrac{x - 1.5}{0.7} & 1.5 \le x \le 2.2 \\ 1 & 2.2 < x \end{cases}$$

(a) $\mu = \dfrac{1.5 + 2.2}{2} = 1.8$

$\sigma^2 = \dfrac{(2.2 - 1.5)^2}{12} = 0.0408$

(b) $P(X < 2)$

$$\int_{1.5}^{2} \frac{1}{0.7}$$

$= 0.714$

6) $a = 0.2050 \quad b = 0.2150$

(a)
$$F(x) = \begin{cases} 0 & x < 0.2050 \\ \dfrac{x - 0.2050}{0.0100} & 0.2050 \leq x \leq 0.2150 \\ 1 & 0.2150 < x \end{cases}$$

(b) $P(x > 0.2125)$

$$\int_{0.2125}^{0.2150} \frac{1}{0.01}\, dx = \frac{1}{0.01}\left[0.2150 - 0.2125\right]$$

$=$

(c) $F(X > x) = 0.1$

$1 - P(X \leq x) = 0.1$

$P(X \leq x) = 0.9$

$$\int_{-x}^{x} \frac{1}{0.1}\, dx = 0.9$$

$$\frac{2x}{0.1} = 0.9$$

$$\int_{0.2050}^{x} \frac{1}{0.01}\, dx = 0.9$$

$$\frac{1}{0.01}\left[x - 0.2050\right] = 0.9$$

$x =$

(d) $$\mu = \frac{0.2050 + 0.2150}{2}$$

$$\sigma^2 = \frac{(0.2150 - 0.2050)^2}{12}$$

7) $a = -2, \ b = 2$

$SD = \sqrt{4/3}$

$= 1.15$

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8] 8:30 AM – 10:30 AM (hours).
(0 – 90) minutes

ⓐ $f(x) = \frac{1}{b-a} = \frac{1}{90}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x-0}{90} & 0 \leq x \leq 90. \\ 1 & 90 \leq x \end{cases}$$

ⓑ $\mu = \frac{90}{2} = 45$ $\quad \sigma^2 = \frac{90^2}{12} = 675$

ⓒ $p(x < 10) = 3\int_0^{10} \frac{1}{90}\, dx$

$= \frac{3}{90}[x]_0^{10}$

$= \frac{1}{3}$

(or) $p(x<10) = \int_{8.5}^{9.0} \frac{1}{90}\, dx + \int_{9.20}^{9.30} \frac{1}{90}\, dx + \int_{9.50}^{10} \frac{1}{90}\, dx$

=

ⓓ $p(x>20) = 3\int_{20} \frac{1}{90}\, dx = 1 - p(x \leq 20)$

$= 1 - \int_0^{20} \frac{1}{90}\, dx$

=

Normal Distribution.

A random variable with prob density

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty \le x < \infty$$

at $-\infty < \mu < \infty$ & $\sigma > 0$

is a normal random variable with parameter μ where $-\infty < \mu < \infty$ & $\sigma > 0$

the mean & variance of X are equal to

$E(X) = \mu$ & $V(X) = \sigma^2$

Standard normal Variable.

A Normal random variable with $\mu = 0$ & $\sigma^2 = 1$ is called Standard normal variable & is denoted as Z.

The cumulative distribution fun of a standard normal random variable is denoted as

$\phi(z) = P(Z \le z)$

Standardizing a Normal Random Variable.

If X is a normal random variable with $E(X) = \mu$ & $V(X) = \sigma^2$ the random Variable $Z = \frac{X-\mu}{\sigma}$ is a normal random Variable with $E(Z) = 0$, $V(Z) = 1$

i.e Z is a standard Normal random Variable

Standardizing to calculate probability

Suppose X is a normal random variable with mean μ & variance σ^2

then $P(X \le x) = P\left(\frac{X-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right)$

$= P(Z \le z)$

where Z ... of ... curve the from above

i) ii)

b) $P(Z$

a) $P($

c) $P($

$P($

b) P

$-P$

$P($

$P($

$P($

where z is a standard normal variable & $Z = \frac{X-\mu}{\sigma}$ is the Z value obtained by Stardadizing X.

The normal curve is a bell shaped curve & is symmetrical the line $X=\mu$. The area under normal ~~pro~~ density fun from $-\infty < x < \infty$ is 1 & is symmetric about the line $Z=1$

1] ⓐ $p(Z<z)=0.9$

$z = 1.2 + 0.08 = 1.28$

ⓑ $p(Z<z)=0.5$

$Z=0$

ⓒ $p(Z>z)=0.1 = 1.28$

$1-p(Z<z)=0.9$

$p(Z<z)=-1.29$

ⓓ $p(Z>z)=0.9$ $z = 1.29$

ⓔ $p(-1.24<Z<z)=0.8$

$p(Z<z) - p(Z<-1.24) = 0.8$

$p(Z<z) = 0.8 + p(Z<-1.24)$

$= 0.8 + 0.107488$

$p(Z<z) = 0.907488$

$z = 1.33$

ⓕ $p(-z<Z<z)=0.95$

$-p(Z<-z) + p(Z<z) = 0.95$

Note

$\phi(z) + \phi(-z) = 1$

$p(Z<z) - p(Z<-z) = 0.95$

$p(Z<z) - (1-p(Z<z)) = 0.95$

$p(Z<z) - 1 + p(Z<z) = 0.95$

$2p(Z<z) = 1.95$

$p(Z<z) = 0.975$

$z = 1.96$

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⑦ $p(-z < Z < z) = 0.99$

$p(Z < z) - p(Z < -z) = 0.99$

$\phi(z) - (1 - \phi(+z)) = 0.99$

$\phi(z) = 1 + 0.99$

$\phi(z) = 1.99$

$= 0.995$

$z = 2.58$

⑧ $p(-z < Z < z) = 0.68$

$\phi(z) = 1.68$

⑨ $p(-z < Z < z) = 0.9973$

2 $\mu = 10$, $SD = 2$

$p(X > x) = 0.5$

$p\left(\frac{x-\mu}{\sigma} \geq \frac{x-\mu}{\sigma}\right) = 0.5$

$1 - p\left(z \leq \frac{x-10}{2}\right) = 0.5$

$p\left(z \leq \frac{x-10}{2}\right) = 1 - 0.95 = 0.05$

$\frac{x-10}{2} = 0$

$\boxed{x = 10}$

② $p(X > x) = 0.95$

$1 - p(X \leq x) = 0.95$

$p\left(\frac{x-\mu}{\sigma} \geq \frac{x-\mu}{\sigma}\right) = 1 - 0.95 = 0.05$

$p\left(z \leq \frac{x-\mu}{\sigma}\right) = 0.05$

$p(z \leq z) = 0.05$

$\boxed{z = 1.64}$

$$\frac{x-10}{2} = -1.62$$

$$\boxed{x = 6.27}$$

③ $P(x < X < 10) = 0.2$

$$P\left(\frac{x-10}{2} \le \frac{X-10}{2} \le \frac{10-10}{2}\right) = 0.2$$

$$P\left(\frac{x-10}{2} \le Z \le 0\right) = 0.2$$

$$P(Z \le 0) - P\left(Z \le \frac{x-10}{2}\right) = 0.2$$

$$0.5 - P\left(Z \le \frac{x-10}{2}\right) = 0.2$$

$$P\left(Z \le \frac{x-10}{2}\right) = 0.3$$

$$\frac{x-10}{2} = -0.52$$

$$\boxed{x = 8.96}$$

④ $P(-x < X-10 < x) = 0.95$

$$P(-x+10 < X-10+10 < x+10) = 0.95$$

$$P(-x+10 < X < x+10) = 0.95$$

$$P\left(\frac{-x+10-10}{2} < \frac{X-\mu}{\sigma} < \frac{x+10-10}{2}\right) = 0.95$$

$$P\left(\frac{-x}{2} < Z < \frac{x}{2}\right) = 0.95$$

$$P(-z < Z < z) = 0.95$$

$$P(Z < z) - P(Z < -z) = 0.95$$

$$\phi(z) - \phi(-z) = 0.95$$

$$\phi(z) - 1 + \phi(-z) = 0.95$$

$$2\phi(z) = 1 + 0.95$$

$$\phi(z) = 0.975$$

$$z = 1.96$$

$$\frac{x}{2} = 1.96$$

$$\boxed{x = 3.92}$$

5 $p(-x < X-10 < x) = 0.99$

$P(-x+10 < X-10+10 < x+10) = 0.99$

$P\left(\frac{-x+10-10}{2} < \frac{X-10}{2} < \frac{x+10-10}{2}\right) = 0.99$

$P\left(-x/2 < Z < x/2\right) = 0.99$

$P(-z < Z < z) = 0.99$

$P(Z < z) - P(Z < -z) = 0.99$

$\phi(z) - \phi(-z) = 0.99$

$\phi(z) - (1 - \phi(+z)) = 0.99$

$2\phi(z) = 1 + 0.99$

$\phi(z) = 0.995$

$x/2 = 2.58$

$\boxed{x = 5.16}$

3 $\mu = 6000 \quad \sigma = 100$

$Z = \frac{X-\mu}{\sigma}$

a) $P(X < 6250)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{6250-\mu}{\sigma}\right) = P\left(Z < \frac{6250-6000}{100}\right)$

$= P(Z < 2.5) = 0.99379$

$\phi(2.5) = 0.99329$

b) $P\left(\frac{5800-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{5900-\mu}{\sigma}\right)$

$\hookrightarrow P(5800 < X < 5900)$

$P\left(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}\right)$

$= P(-2 < Z < -1) = P(Z < -1) - P(Z < -2)$

$= 0.15866 - 0.02275 = 0.13591$

$P(X > x) = 0.95$

$1 - P(X < x) = 0.95$

$P\left(\frac{X-6000}{100} < \frac{x-6000}{100}\right) = 1 - 0.95 = 0.05$

$P(Z < z) = 0.05$

$z = -1.64$

$\frac{x-6000}{100} = -1.64$

$x = 5836$

$\boxed{4}$ $\mu = 260$, $\sigma = 50$ min

$Z = \frac{X-\mu}{\sigma}$

① $P(X > 4h) = P(X > 4 \times 60)$

$= P(X > 240)$

$= 1 - P(X < 240)$

$= 1 - P\left(\frac{X-\mu}{\sigma} < \frac{240-\mu}{\sigma}\right)$

$= 1 - P\left(Z < \frac{240-250}{50}\right)$

$= 1 - P(Z < -0.4)$

$= 1 - 0.344576$

$= 0.655422$

②

5. $\mu = 129$, $\sigma = 14$

ⓐ $p(x > 2\sigma + \mu) = p(x > 2 \times 14 + 129)$

$= p(x > 28 + 129)$

$= p\left(\frac{x-\mu}{\sigma} > \frac{28+129-\mu}{\sigma}\right)$

$= p\left(z > \frac{28+129-129}{14}\right)$

$= p(z > 2)$

$= 1 - p(z < 2)$ from table

$= 1 - 0.977250$

$= 0.02275$

ⓑ $p(x < 100) = p\left(\frac{x-\mu}{\sigma} < \frac{100-\mu}{\sigma}\right)$

$= p\left(z < \frac{100-129}{14}\right)$

$= p(z < -2.071)$ from table

$= 0.019226$

ⓒ $p(X < x) = 0.95$

$p\left(\frac{X-129}{14} < \frac{x-129}{14}\right) = 0.95$ from T

$\frac{X-129}{14} = 1.65$

$\boxed{X = 152.1}$

ⓓ $p\left(z \geq \frac{199-129}{14}\right)$

$= p(z \geq 5)$

$= 0$

This is very unlikely for high volume hostiptal that the surgery was 199 minutes

(a) large.

So it is small volume of such surgery.

6] $\mu = 159.2$ & $\sigma = \sigma$

(a) $p(X < 200) = 0.841$

$p\left(\frac{X-\mu}{\sigma} < \frac{200-\mu}{\sigma}\right) = 0.41$

$p\left(Z < \frac{200-159.2}{\sigma}\right) = 0.84$ fT

$\frac{40.8}{\sigma} = 1$

$\boxed{\sigma = 40.8}$

(b) $p(X < x) = 0.25$

$p\left(Z < \frac{x-159.2}{40.8}\right) = 0.25$ fT

$\frac{x-159.2}{40.8} = -0.69$

$\boxed{x = 131.864}$

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$P(X < x) = 0.75$

$P\left(Z < \dfrac{x - 159.2}{40.8}\right) = 0.75$ (FT)

$\dfrac{x - 159.2}{40.8} = 0.68$

$x = 186.944$

ⓒ $P(X > x) = 0.9$

$1 - P(X < x) = 0.9$

$P\left(\dfrac{X-\mu}{\sigma} < \dfrac{x-\mu}{\sigma}\right) = 0.1$

$P\left(Z < \dfrac{x - 159.2}{40.8}\right) = 0.1$ FT

$\dfrac{x - 159.2}{40.8} = -1.28$

$\boxed{x = 106.976}$

ⓓ $P(x < X < x)$

$P(\mu + \sigma < X < \mu + 2\sigma)$

$P\left(\dfrac{\mu+\sigma-\mu}{\sigma} < \dfrac{X-\mu}{\sigma} < \dfrac{\mu+2\sigma-\mu}{\sigma}\right)$

$P(1 < Z < 2) = P(Z < 2) - P(Z < 1)$

$= 0.977250 - 0.841345$

$= 0.1354$

7] $\mu = 0.5 \quad \sigma = 0.05$

(a) $p(x > 0.62) = 1 - p(x < 0.62)$

~~$p(x>0$~~ $= 1 - p\left(z < \frac{0.62 - 0.5}{0.05}\right)$

$= 1 - p(z < 2.4)$

$= 1 - 0.991802$

$= 0.008192$

(b) $p(0.47 < X < 0.63)$

$= p\left(\frac{0.47 - 0.5}{0.05} < z < \frac{0.63 - 0.5}{0.05}\right)$

$= p(-0.6 < z < 2.6)$

$= p(z < 2.6) - p(z < -0.6)$

$= 0.995334 - 0.274253$

$= 0.721086$

(c) $p(X < x) = 0.9$

$p\left(z < \frac{x - 0.5}{0.05}\right) = 0.9$

$\frac{x - 0.5}{0.05} = 1.29$

$\boxed{x = 0.5645}$

8] $\mu = 12.4$, $\sigma = 0.1$

(a) $p(x < 12) = 0$

(b) $p(x < 12.1$ (or) $x > 12.6)$

$p(x < 12.1) + p(x > 12.6)$

$p\left(z < \frac{12.1 - 12.4}{0.1}\right) + 1 - p\left(z < \frac{12.6 - 12.4}{0.1}\right)$

$= p(z < -3) + 1 - p(z < 2)$

$= 0.001350 + 1 - 0.977250$

$= 0.0241$

ⓒ $P(\mu - x < X < \mu + x) = 0.199$

$P(12.4 - x < X < 12.4 + x) = 0.99$

$P\left(\frac{-x}{0.1} < z < \frac{x}{0.1}\right) = 0.99 + \frac{1-0.99}{2}$

$$\phi\left(\frac{x}{0.1}\right) = 0.99 + \frac{1-0.99}{2}$$

$$= 0.995$$

$$\frac{x}{0.1} = 2.58$$

$$x = 0.258$$

Normal approximation to Binomial distribution

If x is a binomial random variable with n & p

$$Z = \frac{x-\mu}{\sigma} = \frac{x-np}{\sqrt{npq}}$$

is approximately standard normal variable to approximate the binomial distribution to normal distribution is a continuity correction is applied as follows.

$$P(X=x) = P(X \le x + 0.5)$$

$$= P\left(Z \le \frac{x+0.5-\mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{x+0.5-np}{\sqrt{npq}}\right)$$

$$P(X \ge x) = P(X \ge x - 0.5) =$$

$$= P\left(Z \ge \frac{x-0.5-\mu}{\sigma}\right)$$

$$= P\left(Z \ge \frac{x-0.5-np}{\sqrt{npq}}\right)$$

This approximation is good for

$np > 5$ & $n(1-p) = nq > 5$

I] $n = 200$, $p = 0.4$

ⓐ $X < 70$ 39

$P(X \le 70) = P(X \le 70 + 0.5)$

$= P(X \le 70.5)$

$= P\left(Z \le \frac{70.5 - \mu}{\sigma}\right)$ $\mu = np$

$= 200 \times 0.4$

$= 80$

$= P\left(Z \le \frac{70.5 - 80}{6.92}\right)$ $\sigma = \sqrt{npq}$

$= P(Z \le -1.37)$ $= \sqrt{200 \times 0.4 \times 0.6}$

$= 6.92$

$= 0.08534$

ⓑ $x > 70$ $x < 90$

$x + 0.5 < x < x - 0.5$ → concept

$P(70 \le x \le 90)$

1 ae. 2

$= P(70.5 \le x \le 89.5)$

$= P\left(\left(\frac{70.5 - \mu}{\sigma}\right) \le Z \le \frac{89.5 - \mu}{\sigma}\right)$

$= P\left(\frac{70.5 - 80}{0.92} \le Z \le \frac{89.5 - 80}{0.92}\right)$

$= P(-1.37 \le Z \le 1.37)$

$= P(Z < 1.37) - P(Z < -1.37)$ $1 = P(Z < x) + P(Z < -x)$

$= 2\phi(1.37) - 1$

$= 0.8294$

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(c) $X = 80$

$P(X=80) = P(79.5 \le x \le 80.5)$

$= P(x < 80.5) - P(x < 79.5)$

$= P\left(Z \le \frac{80.5-80}{0.92}\right) - P\left(Z \le \frac{79.5-80}{0.92}\right)$

$= P(Z \le 0.07) - P(Z \le -0.07)$

$= 0.527903 - 0.472097$

$= 0.0558$

2] defective chips $p = 0.02$, $n = 1000$

$\mu = np$

$= 1000 \times 0.02$

$= 20$

$\sigma = \sqrt{npq} = \sqrt{1000 \times 0.02 \times 0.98}$

$= 4.43$

(1) $P(X > 25) = 1 - P(X \le 25)$

$= 1 - P(x \le 25.5)$

$= 1 - P\left(Z \le \frac{25.5-20}{4.45}\right)$

$= 1 - P(Z \le 1.24)$

$= 1 - 0.892512$

$= 0.107488.$

(2) $P(20 \le x \le 30) = P(20.5 \le x \le 29.5)$

$= P(x \le 29.5) - P(x \le 20.5)$

$= P\left(Z \le \frac{29.5-20}{4.43}\right) - P\left(Z \le \frac{20.5-20}{4.43}\right)$

$= 1.44$

3] $n = 1000$, $p = 0.193$

$\mu = np = 1000 \times 0.193$

$= 193$

$\sigma = \sqrt{npq} = 12.18$

① $p(x > 200) = 1 - p(x \leq 200.5)$

$= 1 - p\left(z \leq \frac{200.5 - \mu}{\sigma}\right)$

$= 1 - p\left(z \leq \frac{200.5 - 193}{12.18}\right)$

$= 1 - p(z \leq 0.61576)$

$= 1 - 0.729069$

4] $P = 0.001$ $\quad n = 362000$

$(\mu) = np = 0.001 \times 362000$

$= 362$

$p(\sigma) = \sqrt{npq} = \sqrt{362000 \times 0.001 \times (1 - 0.001)}$

$= 19.02$

(b) $p(X < 350) = p(X \leq 349)$

$= p\left(X \leq \frac{349 + 0.5 - \mu}{\sigma = 19.02}\right)$

$= p(z \leq -0.66)$

$= 0.257846$ (-0.65) value

(c) $p(X > x) = 0.05$

$1 - p(X < x) = 0.05$

$p(X < x) = 0.95$

$p\left(z \leq \frac{x + 0.5 - 362}{19.02}\right) = 0.95$

$\frac{x + 0.5 - 362}{19.02} = 1.65$

$\boxed{x = 392.89}$

(a) $(p > 400)$

Calculate for one month.

$P(p > 400) = 1 - (p \le 400)$

$= 1 - \left(p \le \left\{\frac{400 + 0.5 - 362}{19.02}\right\}\right)$

$= 1 - p(\le 2.024)$

$= 1 - \cancel{0.979325}\ 0.978308$ (take)

$= 0.020675$

4.274×10^{-4}

5. $p = 0.999 \quad n = 5000$

$\mu = np = 5000 \times 0.999$

$=$

$\sigma = \sqrt{npq} = \sqrt{5000 \times 0.999 \times (1 - 0.999)}$ Not fail

$=$

For fail

$q = 1 - p$

$= 1 - 0.999$

$= 0.001$

$\mu = np = 5 \quad \& \quad \sigma = 2.24$

$= 5000 \times 0.001$

$\sigma = \sqrt{npq}$

$= \sqrt{5000 \times 0.001 \times 0.999}$

$P(x \ge 10) = 1 - p(x < 10)$

$= 1 - p(x \le 9)$

$= 1 - p\left(x \le \frac{9 + 0.5 - 5}{2.24}\right)$

$= 1 - p\left(x \le \frac{4.5}{2.24}\right)$

$= 1 - p(z \le 2.014)$

$= 1 - 0.977784$

$= 0.022216$

(6) $p = \frac{50}{1000} = 0.05$

$\mu = np = 100 \times 0.5 = 5$

$\sigma = \sqrt{npq} = \sqrt{\quad}$

18/2/20

Normal approximation to poisson distribution

If X is a poisson random variable then $\mu = \lambda$ & $\sigma = \sqrt{\lambda}$ then

$Z = \frac{X - \mu}{\sigma} = \frac{X - \lambda}{\sqrt{\lambda}}$ is approximately a standard normal variable, to approximate poisson pro with normal distribution. a continuity correction is applied as follows.

$P(X \le x) = P(X \le x + 0.5) = P\left(Z \le \frac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$

$$\boxed{p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}}$$

I $\lambda = 6$

(a) $P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{e^{-6} 6^{0}}{0!} + \frac{e^{-6} 6^{1}}{1!} + \frac{e^{-6} 6^{2}}{2!} + \frac{e^{-6} 6^{3}}{3!}$

$=$

2] $\mu = 64$

a) $p(X > 72)$

$$p(X > 72) = 1 - p(X \le 72)$$
$$= 1 - p\left(Z \le \frac{72 + 0.5 - 44}{\sqrt{64}}\right)$$
$$= 1 - p(Z \le 1.06)$$
$$= 1 - 0.855428$$
$$= 0.144571$$

b) $p(X < 64) = p(X \le 63)$
$$= p\left(Z \le \frac{63 + 0.5 - 64}{\sqrt{64}}\right)$$
$$= p(Z \le -0.06)$$
$$= 0.476078$$

$60 < X \Rightarrow X > 60$

$X \ge 61$

$\Rightarrow 61 \le X$

c) $p(60 < X \le 68)$

$p(\underset{-0.5}{61} \le X \le \underset{+0.5}{68})$ for altering

$$p\left(\frac{61 - 0.5 - 64}{\sqrt{64}} \le Z \le \frac{68 + 0.5 - 64}{\sqrt{64}}\right)$$
$$p(-0.44 \le Z \le 0.56)$$
$$= p(Z \le 0.56) - p(Z \le -0.44)$$
$$= 0.712260 - 0.329967$$
$$= 0.382291$$

3] $\mu = 1000$ for $1cm^2$

→ For $10cm^2$ $\mu = 1000 \times 10 = 10,000$

$$p(X > 10,000) = 1 - p(X \le 10,000)$$
$$= 1 - p\left(Z \le \frac{10000 + 0.5 - \mu}{\sqrt{\mu}}\right)$$
$$= 1 - p(Z \le 0.0005)$$
$$= 1 - 0.5000$$
$$= 0.5$$

4]

ⓐ No. of events in no. of independent disjoint, the pages are (disjoint intervals & consecutively the errors count is independent)

ⓑ $p(X \geq 1)$

$= 1 - p(X < 1)$

$= 1 - [p(X = 0)]$

$= 1 - \left[\frac{e^{-0.4}(0.4)^0}{0!}\right] = 1 - 0.670 = 0.330$

The mean no. of pages with 1 or more errors in $0.33 \times 1000 = 330$ pages $= \mu$

ⓒ $p(x > 350) = 1 - p(x \leq 350)$

$= 1 - p\left(z \leq \frac{350 + 0.5 - 330}{\sqrt{330}}\right)$

$= 1 - p(z \leq 1.128)$

$= 1 - 0.868643$

$= 0.131357$

5] $\mu = 10,000$

a) $p(x > 20,000) = 1 - p(x \leq 20,000)$

$= 1 - p\left(z \leq \frac{20000 + 0.5 - 10000}{\sqrt{10000}}\right)$

$= 1 - \cancel{100.5}\ p(z \leq 100)$

$= \cancel{-99.5}\ 1 - 1$

$= 0$

b) $p(x < 9900)$

$= p\left(x \leq \frac{9899 + 0.5 -}{\quad}\right)$

3] $\lambda = 3$ minute

(a) mean $\mu = 1/\lambda = 1/3 = 20$ sec

(b) variance $\sigma^2 = 1/\lambda^2 = 1/9$

$SD = \sqrt{1/9} = 1/3 = 20$ sec

(c) $p(X<x) = 0.95$

$$\int_0^x 3e^{-3x}\,dx$$

$$= 3 \quad -e^{-3x}\Big|_0^x$$

$$= -e^{-3x} + 1 = 0.316$$

$e^{-3x} = 0.684$ $\boxed{e = 1.139}$

4] $\mu = 15 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{15} = 0.066$

(1) $p(x>30) = \int_{30}^{\infty} \lambda e^{-\lambda x}\,dx$

$$= \int_{30}^{\infty} \frac{1}{15} e^{-x/15}\,dx$$

$$= e^{-2} = 0.135$$

(2) $p(x<10) = \int_0^{10} \frac{1}{15}$

(3) $p(5<x<10)$

(4) $p(x<x) = 0.9$