Unit-IV

Pushdown Automata (PDA) :

1) Finite automata only accept finite language.
2) But some time language is more powerful and it contain some comparision that time Finite automata require extra memory.
3) Finite automata + extra stack memory is called as pushdown Automata (PDA)
4) This machine Handel any powerful Lang.

Ex) $a^n 1^n$ $n \geq 0$ This is a powerful Lang. If comparision occure then Lang is powerful.

5) If zero occure in I/P string then It is push into the stack. If one occure In I/P string that time pop first zero & then push one.
6) Context free lang recognised by PDA

Formal Definition of PDA :-

$P = (Q, \Sigma, \Gamma, \partial, q_0, Z_0, F)$

where.

Q – The set of states like FA.

Σ – The set of Finite I/P symbol

Γ – Stack Alphabet (push into stack)

∂ – Transition function

$\partial : Q \times (\Sigma \cup \underset{\text{epsilon}}{\varepsilon}) \times \Gamma \rightarrow Q \rightarrow \Gamma^*$

$q_0 \rightarrow$ Initial state.

$Z_0 \rightarrow$ Stack start symbol

$F \rightarrow$ Final state

There are two type of acceptance 1) Final state
2) empty state

☆ Differance betⁿ FA & PDA.

FA	PDA
1) The FA has no memory Hence it can not remember already read I/p.	1) The PDA has capacity of remembering already read I/p by means of stack.
2) It accept Regular Language.	2) It accepts ~~regular~~ & non Regular Lang. & CFG Lang.
3) It makes use of States & transition to model any Lang.	3) It makes use of Push & POP operation to model any Lang.
4) It is simple to represent.	4) It is more powerful than FSM.

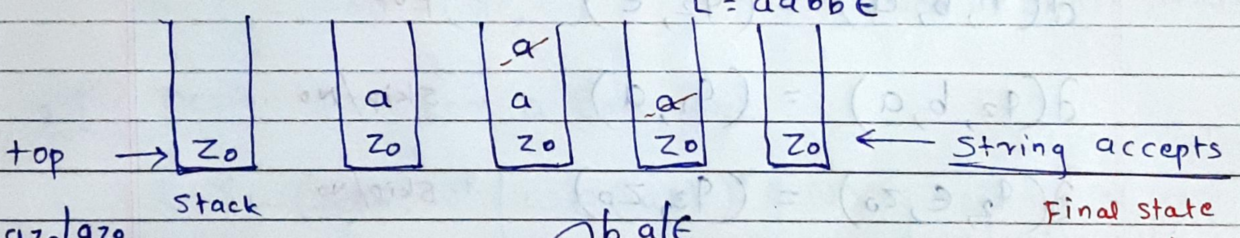
Imp.

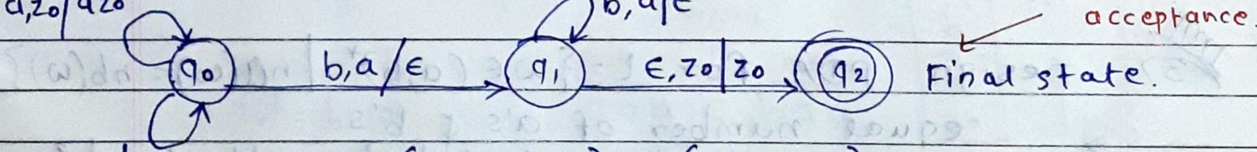
Ex 1) Design a PDA for accepting a lang $\{L = a^n b^n \mid n \geq 1\}$

[May-16, Dec-16, May-18-19 — Marks - 6]

→ This Lang contain equal number of a's & b's

$L = aabb\epsilon$





1) $\delta(q_0, a, Z_0) = (q_0, aZ_0)$ push

$\delta(q_0, a, a) = (q_0, aa)$ push

$\delta(q_0, b, a) = (q_1, \epsilon)$ pop

$\delta(q_1, b, a) = (q_1, \epsilon)$ pop

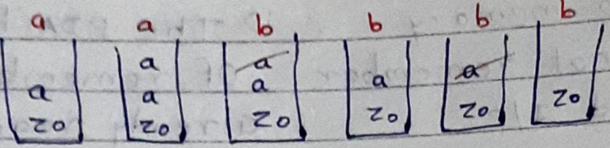
$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$ skip/No-operation

↑ State ↑ I/P read from tape ↑ current stack top

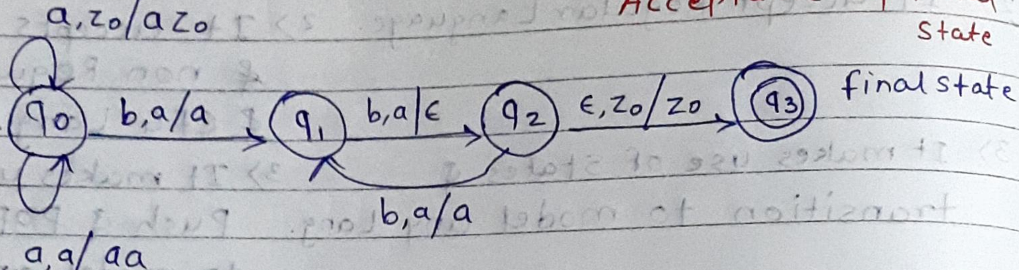
Imp

2) construct PDA for the Language $L = \{a^n b^{2n} \mid n \geq 1\}$

→ Ex string - aa bbbb



Acceptance by Final State



$\delta(q_0, a, z_0) = (q_0, az_0)$ push

$\delta(q_0, a, a) = (q_0, aa)$ push

$\delta(q_0, b, a) = (q_1, a)$ skip / NO

$\delta(q_1, b, a) = (q_2, \epsilon)$ pop

$\delta(q_2, b, a) = (q_1, a)$ skip/No

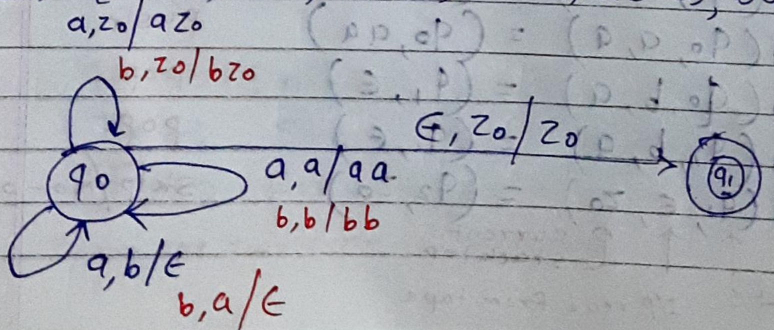
$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$ skip/No

Imp

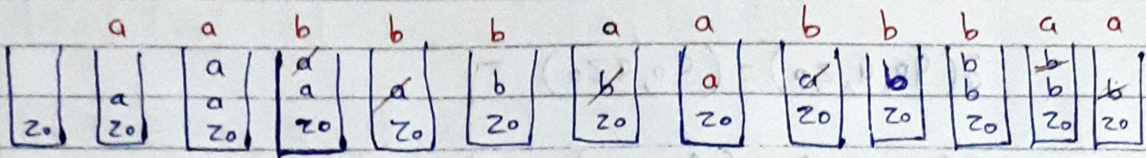
3) Design PDA for $L = \{w \in (a,b)^* \mid n_a(w) = n_b(w)\}$

→ equal number of a's & b's

Ex. {ab, ba, aabb, bbaa, abab, baba, abba, baab}



I/P - a, a, b, b, b, a, a, b, b, b, a, a



$\partial(q_0, a, z_0) = (q_0, az_0)$

$\partial(q_0, b, z_0) = (q_0, bz_0)$

$\partial(q_0, a, a) = (q_0, aa)$

$\partial(q_0, b, b) = (q_0, bb)$

push

$\partial(q_0, a, b) = (q_0, \epsilon)$

$\partial(q_0, b, a) = (q_0, \epsilon)$

pop

$\partial(q_0, \epsilon, z_0) = (q_1, z_0)$ → skip Acceptance by final state

or

$\partial(q_0, \epsilon, z_0) = (q_0, \epsilon)$ → skip Acceptace by empty state.

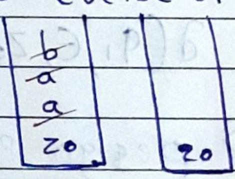
Imp 4) Design PDA for a Language

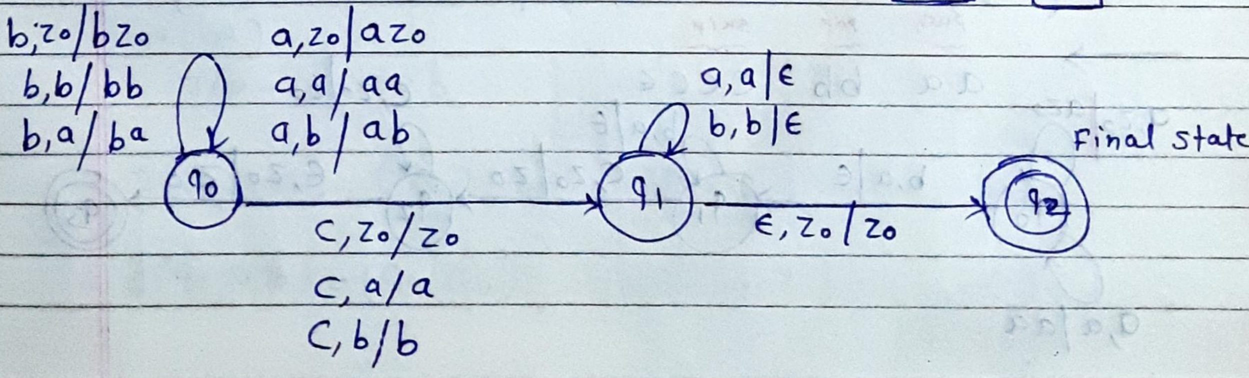
$L = \{ w \cancel{(a+b)} c\, w^R \mid w \in (a+b)^* \text{ \& } w^R \text{ is reverse of } w \}$

→ $w = aab$

$w^R = baa$

push — aab, No/skip — c, pop — baa





Skip

$\underbrace{W\,(\text{push})}$ ↓ $\underbrace{W^R\,(\text{pop})}$

I/P – a b b a a c a a b b a

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, b, a) = (q_0, ba)$

W, push

$\delta(q_0, c, z_0) = (q_1, (z_0))$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

skip / No operation for 'c' I/P

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

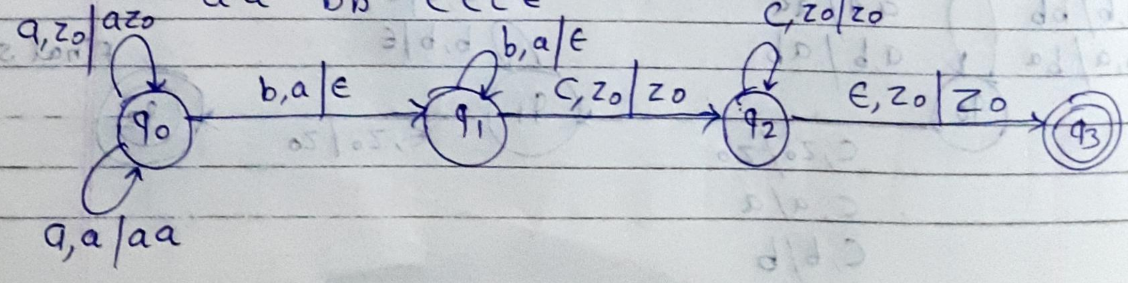
pop operation for W^R I/P

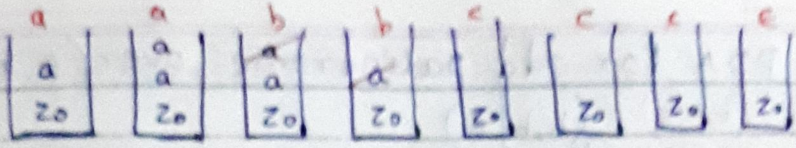
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$ skip / No operation (Final state)

5) Design PDA for CFL $L = \{a^n b^n c^m \mid n, m \geq 1\}$

push pop skip

aa bb cccε





$\partial(q_0, a, z_0) = (q_0, az_0)$

$\partial(q_0, a, a) = (q_0, aa)$ — push

$\partial(q_0, b, a) = (q_1, \epsilon)$

$\partial(q_1, b, a) = (q_1, \epsilon)$ — pop

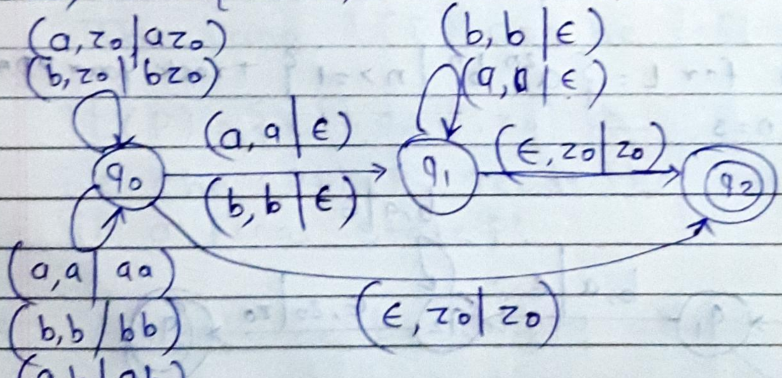
$\partial(q_1, c, z_0) = (q_2, z_0)$

$\partial(q_2, c, z_0) = (q_2, z_0)$ — skip

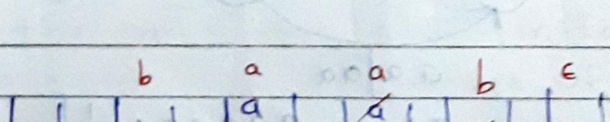
$\partial(q_2, \epsilon, z_0) = (q_3, z_0)$ ← skip acceptance by final state

6) Draw PDA for even palindrome over $(a,b)^*$

→ $L = \{\epsilon, a, b, ab, ba, bb, aa, \ldots\}$

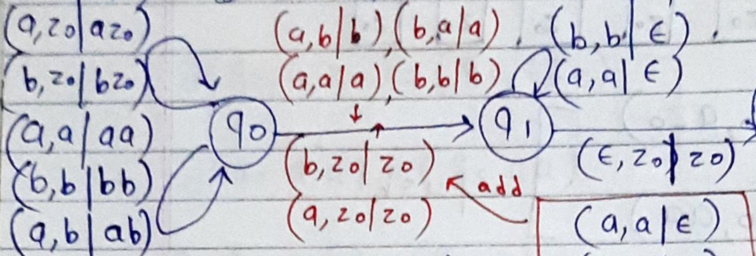


I/P - baab



$\partial(q_0, b, z_0) = (q_0, bz_0)$	$\partial(q_0, a, a) = (q_1, \epsilon)$
$\partial(q_0, a, b) = (q_0, ab)$	$\partial(q_1, b, b) = (q_1, \epsilon)$
	$\partial(q_1, \epsilon, z_0) = (q_2, z_0)$

7) Draw PDA for odd palindrome over (a, b)



I/p = aba

obtain final ans for ex 8

$\partial(q_0, a, z_0) = \partial(q_0, az_0)$ push

$\partial(q_0, b, a) = \partial(q_1, a)$ ~~pop~~ skip

$\partial(q_1, a, a) = \partial(q_1, \epsilon)$ pop

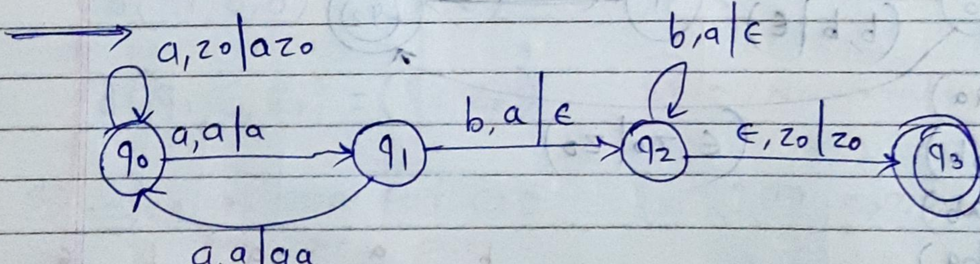
$\partial(q_1, \epsilon, z_0) = \partial(q_2, \epsilon)$ skip

8) The set of palindromes over alphabet (a, b)

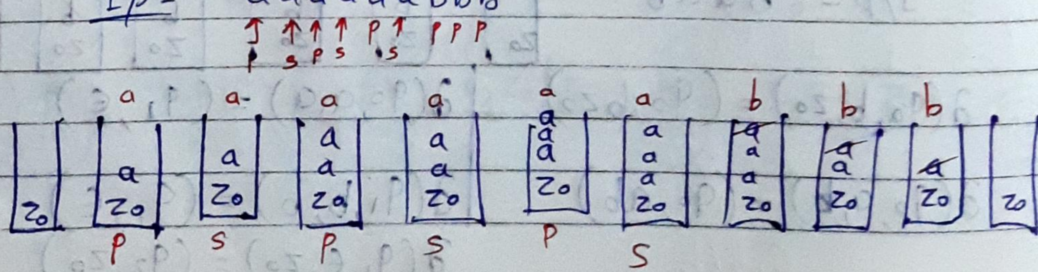
→ soln → Draw odd paindrom digram add only two formula.

9) Draw PDA for $L = \{a^{2n} b^n \mid n >= 1\}$ track your PDA for I/p with n=3.

→



IP - aaaaaabbb



$\partial(q_0, a, z_0) = (q_0, az_0)$ push

$\partial(q_0, a, a) = (q_1, a)$ skip

$\partial(q_1, a, a) = (q_0, aa)$ push

$\partial(q_1, b, a) = (q_1, \epsilon)$ pop

$\partial(q_2, b, a) = (q_2, \epsilon)$ pop

$\partial(q_2, \epsilon, z_0) = (q_3, z_0)$ skip

☆ Acceptance by Final state:-

The PDA accepts its I/P by consuming it & then enters in the final state. The formal defⁿ is

Let $P = (Q, \Sigma, \Gamma, \partial, q_0, z_0, F)$ be a PDA. Then the Lang $L(P)$ is called the Language accepted by final state.

The Lang $L(P)$ can be defined as

$$L(P) = \{ \omega \mid (q_0, \omega, z_0) \vdash^{*} (P, \varepsilon, \alpha) \}$$

i.e the PDA 'P' reads the entire i/p ω & enters in a final state 'p'.

☆ Acceptance by empty stack:-

on reading the i/p string from intial configuration for some PDA, the stack of PDA gets empty.

The Lang accepted by empty stack $N(P)$ is defined as

$$N(P) = \{ \omega \mid (q_0, \omega, z_0) \vdash^{*} (P, \varepsilon, \varepsilon) \}$$

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☆ Convert PDA to CFG

$M = \{P, q\}, \{0, 1\}, \{x, y\}, \partial, q, z$

1) $\partial(q, 1, z) = (q, xz)$
2) $\partial(q, 1, x) = (q, xx)$
3) $\partial(q, \epsilon, x) = (q, \epsilon)$
4) $\partial(q, 0, x) = (P, x)$
5) $\partial(P, 1, x) = (P, \epsilon)$
6) $\partial(P, 0, z) = (q, z)$

CFG

$S \to [q\,z\,q]$	$S \to A$
$S \to [q\,z\,P]$	$S \to B$

Initial state.

$\partial(q, 1, z) \Rightarrow (q, xz)$ push

$[q, z, q] = 1\,[q, x, q]\,[q, z, q]$	$A \to 1EA$
$[q, z, q] = 1\,[q, x, P]\,[P, z, q]$	$A \to 1FC$
$[q, z, P] = 1\,[q, x, q]\,[q, z, P]$	$B \to 1EB$
$[q, z, P] = 1\,[q, x, P]\,[P, z, P]$	$B \to 1FD$

only two state

$\partial(q, 1, x) \Rightarrow (q, xx)$ push

$[q, x, q] = 1\,[q, x, q]\,[q, x, q]$	$E \to 1EE$
$[q, x, q] = 1\,[q, x, P]\,[P, x, q]$	$E \to 1FG$
$[q, x, P] = 1\,[q, x, q]\,[q, x, P]$	$F \to 1EF$
$[q, x, P] = 1\,[q, x, P]\,[q, x, P]$	$F \to 1FH$

3) $\delta(q, \epsilon, x) \ni (q, \epsilon)$ pop

$[q, x, q] \to \epsilon$ — $E \to \epsilon$

4) $\delta(q, 0, x) \ni (P, x)$ skip

$[q, x, q] \to 0[P, x, q]$ — $E \to 0G$

$[q, x, P] \to 0[P, x, P]$ — $F \to 0H$

5) $\delta(P, 1, x) \ni (P, \epsilon)$ pop

$[P, x, P] \to [1]$ — $H \to 1$

6) $\delta(P, 0, z) \ni (q, z)$ skip

$[P, z, q] \to 0[q, z, q]$ — $C \to 0A$

$[P, z, P] \to 0[q, z, P]$ — $D \to 0B$

Rename production.

$[q, z, q]$	A
$[q, z, P]$	B
$[P, z, q]$	C
$[P, z, P]$	D
$[q, x, q]$	E
$[q, x, P]$	F
$[P, x, q]$	G
$[P, x, P]$	H

Final Grammer

$S \to A \mid B$

$A \to 1EA \mid 1FC$

$B \to 1EB \mid 1FD$

$E \to 1EE \mid 1FG \mid \epsilon \mid 0G$

$F \to 1EF \mid 1FH \mid 0H$

$H \to 1$

$C \to 0A$

$D \to 0B$

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2) consider the following PDA moves

$\delta(q_0, a, Z_0) = (q_0, a, Z_0)$

$\delta(q_0, a, a) = (q_0, a, a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_0, \epsilon, Z_0) = (q_1, \epsilon)$ obtain CFG.

→

PDA	CFG
$S \to (q_0, Z_0, q_0)$	$S \to A$
$S \to (q_0, Z_0, q_1)$	$S \to B$
1) $\delta(q_0, a, Z_0) = (q_0, a, Z_0)$	
$[q_0, Z_0, q_0] = a[q_0, a, q_0][q_0, Z_0, q_0]$	$A \to \cancel{aAA}$ aEA
$[q_0, Z_0, q_0] = a[q_0, a, q_1][q_1, Z_0, q_0]$	$A \to aFC$
$[q_0, Z_0, q_1] = a[q_0, a, q_0][q_0, Z_0, q_1]$	$B \to aEB$
$[q_0, Z_0, q_1] = a[q_0, a, q_1][q_1, Z_0, q_1]$	$B \to aFD$
2) $\delta(q_0, a, a) = (q_0, aa)$	
$[q_0, a, q_0] = a[q_0, a, q_0][q_0, a, q_0]$	$E \to aEE$
$[q_0, a, q_0] = a[q_0, a, q_1][q_1, a, q_0]$	$E \to aEG$
$[q_0, a, q_1] = a[q_0, a, q_0][q_0, a, q_1]$	$F \to aEF$
$[q_0, a, q_1] = a[q_0, a, q_1][q_1, a, q_1]$	$F \to aFH$
3) $\delta(q_0, b, a) = (q_1, \epsilon)$	
$[q_0, a, q_1] \to b$	$F \to b$

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4) $\partial(q_1, b, a) = (q_1, \epsilon)$

$[q_1, a, q_1] = b$ | $H \to b$

5) $\partial(q_0, \epsilon, z_0) = (q_1, \epsilon)$

$[q_0, z_0, q_1] = \epsilon$ | $B \to \epsilon$

Rename Production		Final Grammer
$[q_0, z_0, q_0]$	A	$S \to A \mid B$
$[q_0, z_0, q_1]$	B	$A \to aEA \mid aFC$
$[q_1, z_0, q_0]$	C	$B \to aEB \mid aFD \mid \epsilon$
$[q_1, z_0, q_1]$	D	$E \to aEE \mid aFG$
$[q_0, a, q_0]$	E	$F \to aEF \mid aFH \mid b$
$[q_0, a, q_1]$	F	$H \to b$
$[q_1, a, q_0]$	G	
$[q_1, a, q_1]$	H	

★ CFG to PDA

Rule-1:-

For each non-terminal symbols.

$$\partial(q, \epsilon, A) = (q, \beta)$$

where the Production Rule is

$$A \to \alpha$$

Rule-2:-

For each terminal symbols.

$$\partial(q, a, a) = (q, \epsilon)$$

For every terminal symbols.

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1> construct PDA for given Grammer

$S \to 0S1 \mid 00 \mid 11$

$\longrightarrow$ PDA

N.T → $\partial(q, \epsilon, S) = (q, 0S1), (q, 00), (q, 11)$ — I

T → $\partial(q, 0, 0) = (q, \epsilon)$ — II

T → $\partial(q, 1, 1) = (q, \epsilon)$ — III

I/P - 0111

$\partial(q, 0111, S)$ — I

$\partial(q, \cancel{0}111, \cancel{0}S1)$ — II

$\partial(q, 111, S1)$ — I pop operation perform

$\partial(q, \cancel{1}11, \cancel{1}11)$ — III

$\partial(q, \cancel{1}1, \cancel{1}1)$ — III

$\partial(q, \cancel{1}, \cancel{1})$ — III

$\partial(q, \epsilon, \epsilon)$ — Accept.

2> construct PDA for given Grammer.

$S \to 0BB$

$B \to 0S \mid 1S \mid 0$

Test whether 010^4 is acceptable by PDA

$\longrightarrow$

N.T $\partial(q, \epsilon, S) = (q, 0BB)$ — I

N.T $\partial(q, \epsilon, B) = (q, 0S) (q, 1S) (q, 0)$ — II

T $\partial(q, 0, 0) = (q, \epsilon)$ — III

T $\partial(q, 1, 1) = (q, \epsilon)$ — IV

I/P string - 010000

$\partial(q, 010000, S)$ — (1)
$\partial(q, \cancel{0}10000, \cancel{0}BB)$ — (III)
$\partial(q, 10000, BB)$ — (2)
$\partial(q, \cancel{1}0000, \cancel{1}SB)$ — (4)
$\partial(q, 0000, SB)$ — (1)
$\partial(q, \cancel{0}000, \cancel{0}BBB)$ — (3)
$\partial(q, 000, BBB)$ — (2)
$\partial(q, \cancel{0}00, \cancel{0}BB)$ — (3)
$\partial(q, 00, BB)$ — (2)
$\partial(q, \cancel{0}0, \cancel{0}B)$ — (3) → $\partial(q, 0, B)$ — (2)
$\partial(q, 0, 0)$ — (3)
$\partial(q, \epsilon, \epsilon)$ — Accept

3) convert the grammer
$S \rightarrow 0S1 \mid A$
$A \rightarrow 1A0 \mid S \mid \epsilon$
to the PDA that accepts the same lang by empty stack.

→ $\partial(q, \epsilon, S) = (q, 0S1), (q, A)$

$\partial(q, \epsilon, A) = (q, 1A0), (q, S), (q, \epsilon)$

$\partial(q, 0, 0) = (q, \epsilon)$

$\partial(q, 1, 1) = (q, \epsilon)$

★ Application of PDA.
→ 1) Top down parser.
2) Bottom up parser.

PDA → ① only Read operation
② Head move only one direction

☆ Non-deterministic PDA (NPDA) :-

1> construct NPDA that accepts the Lang generated by $S = S+S \mid S*S \mid 4$

→ The PDA can be.

$$A = \{ q, \{*, 4, +\}, \{S, 4, +, *\}, \partial, q, S, \phi \}$$

$R_1 \rightarrow \partial(q, \epsilon, S) = (q, S+S), (q, S*S), (q, 4)$
$R_2 \rightarrow \partial(q, \epsilon, 4) = (q, \epsilon)$
$R_3 \rightarrow \partial(q, \epsilon, +) = (q, \epsilon)$
$R_4 \rightarrow \partial(q, \epsilon, *) = (q, \epsilon)$

It is NPDA because there are more than one moves for same I/P symbol & from same current state.

2> construct NPDA that accept Lang $L = \{ a^{2n} \mid n > 0 \}$.

$$\left.\begin{aligned} &\partial(q_0, a, z_0) = (q_1, a z_0) \\ &\partial(q_1, a, a) = (q_1, aa) \end{aligned}\right\} \text{push}$$

$$\left.\begin{aligned} &\partial(q_1, a, a) = (q_2, \epsilon) \\ &\partial(q_2, a, a) = (q_3, \epsilon) \\ &\partial(q_2, \epsilon, z_0) = (q_3, \epsilon) \end{aligned}\right\} \text{pop}$$

$\partial(q_3, \epsilon, z_0) = (q_3, \epsilon) \leftarrow$ Accept

☆ Equivalence of Acceptance by Final & empty state:-

1> The Lang accepted by empty state PDA is also acceptable by Final state PDA.

2> The Lang accepted by final state PDA is also acceptable by empty state PDA.