Bansilal Ramnath Agarwal Charitable Trust's

VISHWAKARMA INSTITUTE OF TECHNOLOGY, PUNE –37

(An Autonomous Institute Affiliated to SPPU)

Discrete Mathematics (ES1030)

TUTORIAL 6 (Number Theory)

Q. 1	**Attempt the following**
A)	Which of the following are true? i) $446 \equiv 278 \pmod 7$ ii) $269 \equiv 413 \pmod{12}$ iii) $445 \equiv 536 \pmod{18}$ iv) $793 \equiv 682 \pmod 9$ v) $473 \equiv 369 \pmod{26}$ vi) $383 \equiv 126 \pmod{15}$ vii) $224 \equiv 762 \pmod 8$ viii) $582 \equiv 263 \pmod{11}$ ix) $156 \equiv 369 \pmod 7$ x) $-238 \equiv 483 \pmod{13}$
B)	Evaluate these quantities. i) -17 mod 2 ii) - 101 mod 13 17 iii) 13 mod 3 iv) 155 mod 19 v) 144 mod 7 vi) 1 99 mod 19 vii) -97 mod 11 viii) -22 1 mod 23
C)	Show that if $n \mid m$, where n and m are positive integers greater than 1, and if $a \equiv b (\bmod m)$, a, b are integers then $a \equiv b (\bmod n)$.
D)	Solve each linear congruence equation: i) $3x \equiv 2 \pmod 8$ ii) $6x \equiv 5 \pmod 9$ iii) $4x \equiv 6 \pmod{10}$ iv) $455x \equiv 204 \pmod{469}$
E)	Find the smallest positive solution of each system of congruence equations: i) $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 4 \pmod{11}$ ii) $x \equiv 3 \pmod 5$, $x \equiv 4 \pmod 7$, $x \equiv 6 \pmod 9$ iii) $x \equiv 5 \pmod{45}$, $x \equiv 6 \pmod{49}$, $x \equiv 7 \pmod{52}$ iv) $x \equiv 2 \pmod 3$, $x \equiv 1 \pmod 4$, $x \equiv 3 \pmod 5$ v) $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 1$, $x \equiv 3 \pmod 5$, $x \equiv 4 \pmod{11}$ vi) $x \equiv 5 \pmod 7$, $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{11}$ vii) $x \equiv 7 \pmod 8$, $x \equiv 4 \pmod 9$, $x \equiv 16 \pmod{13}$
F)	Prove that if n is an odd positive integer, then n2 = 1 (mod 8).
G)	Find the prime factorization of each of these integers. i) 88 ii) 126 iii) 729 iv) 1001 v) 1111 vi) 909,090
H)	What are the greatest common divisors and least common multiples of these pairs of integers? i) $a = 2^2 \cdot 3^3 \cdot 5^5 ; b = 2^5 \cdot 3^3 \cdot 5^2$ ii) $a = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13; b = 2^{11} \cdot 3^9 \cdot 11 \cdot 17^4$ iii) $a = 17 ; b = 17^{17}$ iv) $a = 2^2 \cdot 7; b = 5^3 \cdot 13$ v) $a = 0; b = 5$ vi) $a = 2 \cdot 3 \cdot 5 \cdot 7; b = 2 \cdot 3 \cdot 5$ vii) $a = 3^7 \cdot 5^3 \cdot 7^3; b = 2^{11} \cdot 3^5 \cdot 5^9$ viii) $a = 41 \cdot 43 \cdot 53 \; ; \; b = 41 \cdot 43$
I)	i) Find an inverse of 4 modulo 9. ii) Find an inverse of 19 modulo 141. iii) Find an inverse of 13 modulo 2436. iv) Find an inverse of 7 modulo 26. v) Find an inverse of 4 modulo 9. vi) Find an inverse of 144 modulo 233.
Q.2	**Attempt the following**
A)	Find the value of n if ${}^{n}C_{12} = {}^{n}C_{8}$.
B)	Show that ${}^{2n}C_2 = 2\,{}^{n}C_2 + n^2$
C)	Prove that ${}^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$
D)	Find the coefficient of $a^5 b^4$ in the expansion of $(a+b)^9$
E)	Use binomial theorem to show that $\sum_{r=0}^{n} (-1)^r \, {}^{n}C_r = 0$
F)	Use binomial theorem to show that $\sum_{r=0}^{n} 2^r \, {}^{n}C_r = 3^n$
G)	What is the coefficient of $x^{12} y^{13}$ in the expansion of $(2x - 3y)^{25}$?
Q.3	**Attempt the following**
A)	Find these terms of the sequence $\{a_n\}$, where $a_n = 2(-3)^n + 5^n$

Bansilal Ramnath Agarwal Charitable Trust's

VISHWAKARMA INSTITUTE OF TECHNOLOGY, PUNE –37

(An Autonomous Institute Affiliated to SPPU)

Discrete Mathematics (ES1030)

TUTORIAL 6 (Number Theory)

	a) a_0 b) a_1 c) a_4 d) a_5
B)	What is the term a_8 of the sequence $\{a_n\}$ if a_n equals a) 2^{n-1} b) 7 c) $1+(-1)^n$ d) $-(-2)^n$
C)	Find : (a) ϕ (10); (b) ϕ (12); (c) ϕ (15); (d) ϕ (37); (e) ϕ (56); (f) ϕ(24·76·133).
D)	Find : (a) $\lceil 7.5 \rceil$ (b) $\lceil 2.8 \rceil$ (c) $\lceil -3.3 \rceil$ (d) $\lceil -0.5 \rceil$ (e) $\lceil 0.5 \rceil$
E)	Find : (a) $\lfloor 7.5 \rfloor$ (b) $\lfloor 2.8 \rfloor$ (c) $\lfloor -3.3 \rfloor$ (d) $\lfloor -0.5 \rfloor$ (e) $\lfloor 0.5 \rfloor$
F)	Let a and b be positive integers, and suppose Q is defined recursively as follows: $Q(a,b)=\begin{cases} 0 & \text{if } a<b \\ Q(a-b,b)+1 & \text{if } b \le a \end{cases}$. Find (a) $Q(2,5)$ (b) $Q(12,5)$.
G)	Let a and b be positive integers, and suppose Q is defined recursively as follows: $Q(a,b)=\begin{cases} 5 & \text{if } a<b \\ Q(a-b,b+2)+a & \text{if } b \le a \end{cases}$. Find (a) $Q(2,7)$ (b) $Q(5,3)$ (c) $Q(15,2)$.