### .11.2 Shortest Path in Weighted Graphs

we have already discussed, the length of a path in a weighted graph is the sum of the weights of the ges of this path, and the shortest path between the two vertices is the minimum length of the path. In this section, we will describe the *shortest path algorithm*, also known as the *greedy algorithm*, ich was developed by Dijkstra. We assume that the graph under consideration is a simple and nnected weighted graph, in which the weights are positive real numbers.

Dijsktra's algorithm iteratively constructs the set $S$ that consists of all the vertices of $G$ for which length of a shortest path has been determined.

Let $G = (V, E)$ be a connected weighted graph. Let $a$ and $z$ be any two vertices where $a$ is the rting point and $z$ is the terminal point. Let $L(v)$ denote the label at vertex $v$. At any given point, me vertices will have temporary labels, while the rest will have permanent labels. It begins by belling the starting vertex $a$ with zero and other vertices with $\infty$. Next, label all neighbours $v$ of $a$ th $L(v)$, which is the weight of the edge from $a$ to $v$. Let $u$ be that vertex, among $v$, for which $L(u)$ is inimum. Now, find those neighbours of $w$ of $u$, and for those $w$ not already permanently labelled sign the label $L(w) + L(u) + w(e)$, where $w(e)$ is the weight of the edge from $u$ to $w$, while for those already labelled $L(w)$ change the label to $L(u) + w(e)$ if this is similar.

Each iteration of the algorithm changes the status of one label from temporary to permanent. Thus e terminate the algorithm when we recieves a permanent label.

**ijkstra's Algorithm**

**Input :** A connected weighted graph

**Output :** $L(z)$, the length of shortest distance from $a$ to $z$.

**Step (1) :** Let $L(a) = 0$ and $L(v) = \infty$ for all vertices $v \neq a$. Set $T = V$, where $T$ = set of vertice having temporary labels. $V$ = Vertex set of $G$.

**Step (2) :** Let $u$ be a vertex in $T$ for which $L(u)$ is minimum and hence the permanent label of

**Step (3) :** If $u = z$, then stop.

**Step (4) :** For every edge $e = (u, v)$, incident with $u$, if $v \in T$ then change $L(v)$ to $\min\{L(v), L(u) + w(e)\}$.

**Step (5) :** Change $T$ to $T - \{u\}$ and go to step (2).

**Example 12.74** Apply Dijkstra's algorithm to the graph given in Fig. 12.138 and find the shorte path from $a$ to $z$.

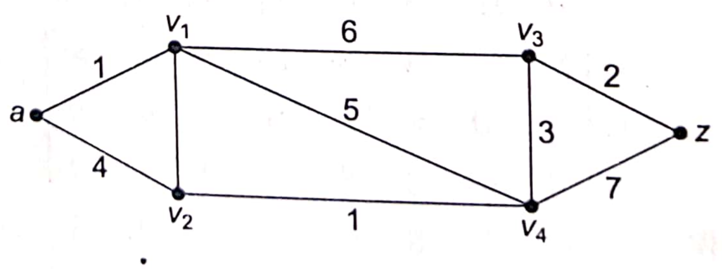


**Figure 12.138** Graph for Example 12.74

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| Vertex $V$ | $a_1$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $T$ | $\{a,$ | $v_1,$ | $v_2,$ | $v_3,$ | $v_4,$ | $z\}$ |

**Iteration (1)** $u = a$ has $L(u) = 0$. $T$ becomes $T - \{a\}$

There are two edges incident with $a$: $av_1$ and $av_2$, where $v_1, v_2 \in T$

$$L(v_1) = \min\{\text{old } L(v_1), L(a) + w(av_1)\}$$
$$= \min\{\infty, 0 + 1\} = 1$$
$$L(v_2) = \min\{\text{old } L(v_2), L(a) + w(av_2)\}$$
$$= \min\{\infty, 0 + 4\} = 4$$

Hence, the minimum label in $L(v_1) = 1$. The new labelling may now be written as

| Vertex $V$ | $a_1$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | 1 | 4 | $\infty$ | $\infty$ | $\infty$ |
| $T$ | $\{$ | $v_1,$ | $v_2,$ | $v_3,$ | $v_4,$ | $z\}$ |

**Iteration (2)** $u = v_1$, the permanent label of $v_1$ is 1. $T$ becomes $T - \{v_1\}$

There are three edges incident with $v_1$: $v_1v_2$, $v_1v_3$ and $v_1v_4$, where $v_2, v_3, v_4 \in T$

$$L(v_2) = \min\{\text{old } L(v_2), L(v_1) + w(v_1v_2)\}$$
$$= \min\{4, 1 + 2\} = 3$$
$$L(v_3) = \min\{\text{old } L(v_3), L(v_1) + w(v_1 v_3)\}$$
$$= \min\{\infty, 1 + 6\} = 7$$
$$L(v_4) = \min\{\text{old } L(v_4), L(v_1) + w(v_iv_4)\}$$
$$= \min\{\infty, 1 + 5\} = 6$$

The minimum label is $L(v_2) = 3$. The new labelling is now written as

| Vertex $V$ | $a$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | 1 | 3 | 7 | 6 | $\infty$ |
| $T$ | $\{$ | | $v_2,$ | $v_3,$ | $v_4,$ | $z\}$ |

**Iteration (3)** $u = v_2$, the permanent label of $v_2$ is 3. $T$ becomes $T - \{v_2\}$.

There is one edge incident with $v_2$: $v_2 v_4$, where $v_4 \in T$.

$$L(v_4) = \min\{\text{old } L(v_4), L(v_2) + w(v_2 v_4)\}$$
$$= \min\{6, 3 + 1\} = 4$$

Thus, the minimum label is $L(v_4) = 4$. The new labelling is written as

| Vertex $V$ | $a$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | 1 | 3 | 7 | 4 | $\infty$ |
| $T$ | $\{$ | | | $d$ | $e$ | $f\}$ |

**Iteration (4)** $u = v_4$, the permanent label of $v_4$ is 4. $T$ becomes $T - \{v_4\}$.
There are two edges incident with $v_4$: $v_4\ v_3$ and $v_4 z$

$$L(v_3) = \min\ \{\text{old } L(v_3), L(v_4) + w(v_4\ v_3)\}$$
$$= \min\ \{7, 4 + 3\} = 7$$
$$L(z) = \min\ \{\text{old } L(z), L(z) + w(z v_4)\}$$
$$= \min\ \{\infty, 4 + 7\} = 11$$

The minimum label is $L(v_3) = 7$
The new labelling is written as

| Vertex $V$ | $a$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | 1 | 3 | 7 | 4 | 11 |
| $T$ | { | | | $d$ | | $f$} |

**Iteration (5)** $u = v_3$, the permanent label of $v_4$ is 7. $T$ becomes $T - \{v_3\}$.
There is one edge incident with $v_4$: $z v_3$, where $v_3$, $z \in T$

$$L(z) = \min\ \{\text{old } L(z), L(v_3) + w(z v_3)\}$$
$$= \min\ \{11, 7 + 2\} = 9$$

The minimum label is $L(z) = 9$. The new labelling is now written as

| Vertex $x$ | $a$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $z$ |
|---|---|---|---|---|---|---|
| $L(v)$ | 0 | 1 | 3 | 7 | 4 | 11 |
| $T$ | { | | | | | $z$} |

Since $u = z$ is the only choice, iteration stops. Thus, the shortest distance between $a$ and $z$ is 9 and the shortest path is $(a, v_1, v_2, v_4, v_3, z)$.

**Example 12.75** Determine the shortest path between the vertices $a$ to $z$ of the graph given in Fig. 12.139.

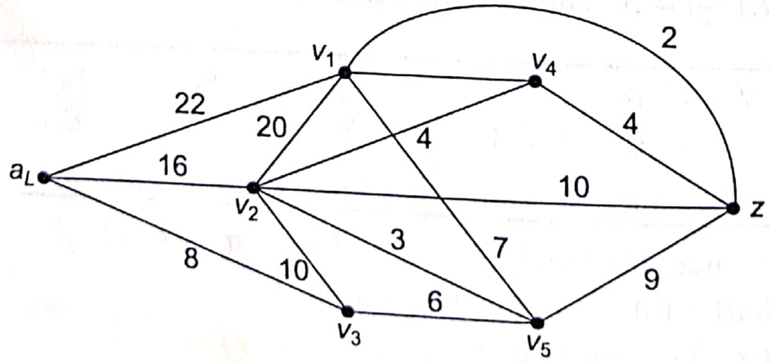


**Figure 12.139** Graph for Example 12.75

***Solution*** The initial labelling is given by