* ~~Ist~~ Following is the correct sol^n of Question 4) in the pdf which I have shared to you on 20 May 2023 *

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Q4) Find all sol^ns of the recurence rel^n

$a_n = 4a_{n-1} - 3a_{n-2} + 2^n + n + 3$ — (*)

with $a_0 = 1$ & $a_1 = 4$

sol^n Now, Associated Homo. recurence rel^n is

$a_n - 4a_{n-1} + 3a_{n-2} = 0$

$\Rightarrow$ Char eq^n is $r^2 - 4r + 3 = 0$

$\Rightarrow (r-3)(r-1) = 0$

$\Rightarrow r = 3, 1$

So $\boxed{a_n^{(h)} = \alpha_1(3^n) + \alpha_2(1^n)}$

Now, $f(n) = (2^n) + (n+3)$

Now corrosponding to $(1).2^n$ Perticular sol^n is $\cancel{(b_0) 2^n}$ $b_0(n^0)2^n = b_0(2^n)$

($\because$ 2 is Not root of char eq^n i.e. root of char eq^n of multiplicity 0)

& corrosponding to $(n+3)(1^n)$ Perticular sol^n is $(b_1 + b_2 n)(n^1)(1^n)$ because 1 is root of char eq^n of multiplicity 1

Hence $a_n^{(p)} = b_0(2^n) + (b_1 + b_2 \cdot n)(n)$ is — (**)

perticular sol^n of given recurence rel^n (*)

$b_0 2^n + (b_1 + b_2 n)(n) = 4[b_0 2^{n-1} + [b_1 + b_2(n-1)](n-1)]$
$- 3[b_0 2^{n-2} + (b_1 + b_2(n-2))(n-2)]$
$+ 2^n + n + 3$

$\Rightarrow b_0 2^n + b_2 n^2 + b_1 n = (2b_0 - \frac{3}{4}b_0 + 1)2^n$
$+ [4b_2 - 3b_2]n^2 + (-8b_2 + 4b_1 + 12b_2 - 3b_1 + 1)n + [-4b_1 + 4b_2 + 6b_1 - 12b_2 + 3]$

Comparing coefficients

$\Rightarrow 2b_0 - \frac{3}{4}b_0 + 1 = b_0$ — (1)

$b_2 = 4b_2 - 3b_2$ — (ii)

$b_1 = -8b_2 + 4b_1 + 12b_2 - 3b_1 + 1$ — (iii)

$0 = -4b_1 + 4b_2 + 6b_1 - 12b_2 + 3$ — (iv)

~~so,~~ eqn (i) $\Rightarrow \frac{b_0}{4} + 1 = 0 \Rightarrow \boxed{b_0 = -4}$

eqn (iii) $\Rightarrow b_1 = 4b_2 + b_1 + 1 \Rightarrow \boxed{b_2 = \frac{-1}{4}}$

eqn (iv) $\Rightarrow 2b_1 - 8b_2 = -3 \Rightarrow 2b_1 = -3 + 8\left(\frac{-1}{4}\right)$

$\Rightarrow \boxed{b_1 = \frac{-5}{2}}$

from eqn (**)

~~eqn~~ hence, $a_n^{(p)} = (-4)(2^n) + \left(\frac{-5}{2} + \left(\frac{-1}{4}\right)n\right)(n)$

$\Rightarrow$

$a_n = a_n^{(h)} + a_n^{(p)} = \alpha_1(3^n) + \alpha_2 - (4)(2^n)$
$- \left(\frac{5}{2} + \frac{n}{4}\right)(n)$

Now, put $n=0$ & $n=1$ in above eqn,

$\Rightarrow 1 = a_0 = \alpha_1 + \alpha_2 - 4$

$4 = a_1 = 3\alpha_1 + \alpha_2 - 8 - \frac{11}{4}$ $\left(\because a_0 = 1 \ \& \ a_1 = 4\right)$

$\Rightarrow \alpha_1 + \alpha_2 = 5$ — (v)

$3\alpha_1 + \alpha_2 = \frac{59}{4}$ — (vi)

eqn (v) - eqn (vi), $\Rightarrow -2\alpha_1 = \frac{-39}{4} \Rightarrow \boxed{\alpha_1 = \frac{39}{8}}$

eqn (v) $\Rightarrow \alpha_2 = 5 - \frac{39}{8} = \frac{1}{8} \Rightarrow \boxed{\alpha_2 = \frac{1}{8}}$

Hence. $a_n = \left(\frac{39}{8}\right)(3^n) + \frac{1}{8} - 2^{n+2} - \left(\frac{5}{2} + \frac{n}{4}\right)(n)$

is the soln of given recurrence reln $\forall n \geq 0$