Bansilal Ramnath Agarwal Charitable Trust's

VISHWAKARMA INSTITUTE OF TECHNOLOGY, PUNE –37

(An Autonomous Institute Affiliated to SPPU)

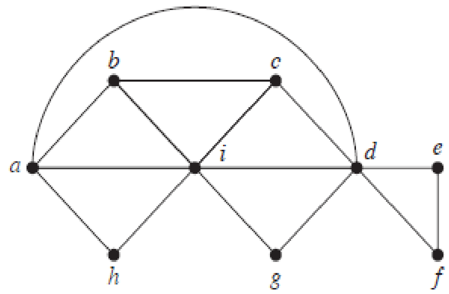
Discrete Mathematics (ES1030) HA 2, 3 and 4

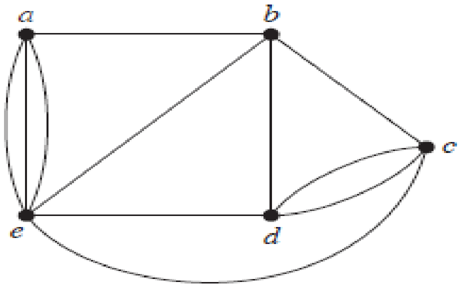
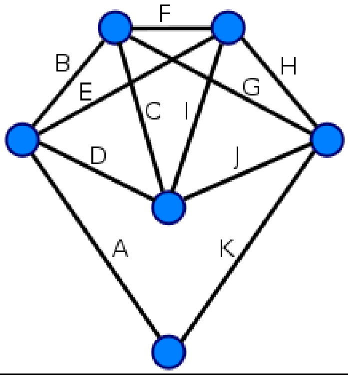
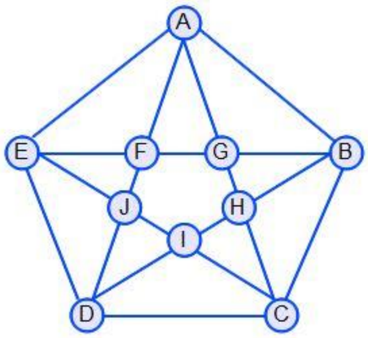
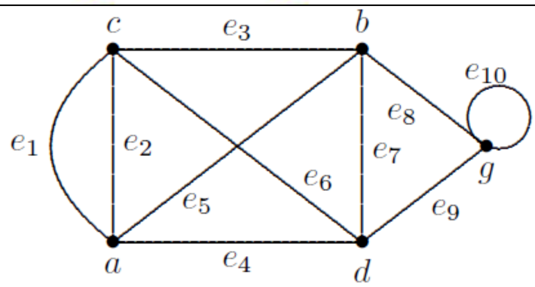
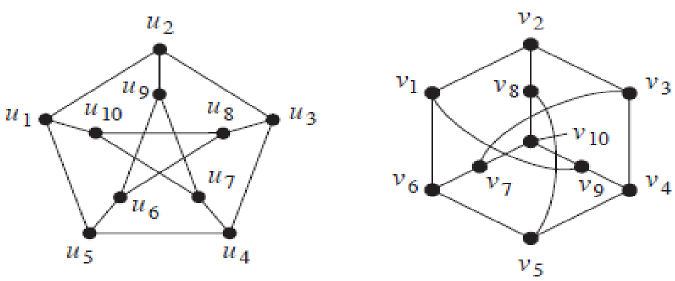
Attempt the sub questions shown against your roll numbers

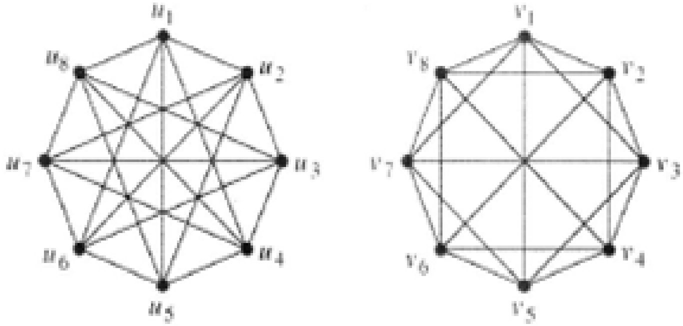
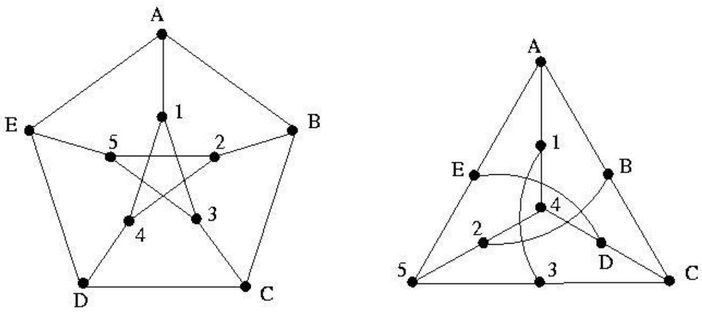
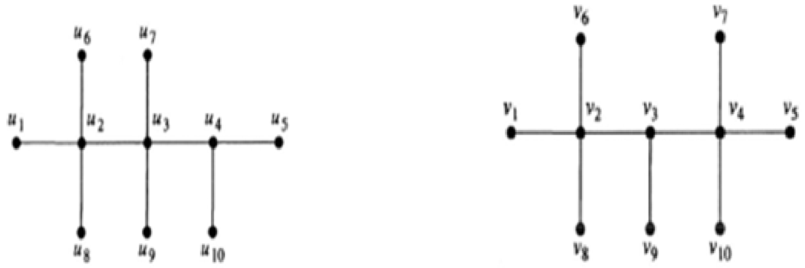
Question	Sub Question	Statement
Q.1		**Attempt the following**
	1)	How many relations are there on a set with n elements that are symmetric? Justify
	2)	How many relations are there on a set with n elements that are irreflexive? Justify
	3)	How many relations are there on a set with n elements that are asymmetric? Justify
	4)	How many relations are there on a set with n elements that are reflexive? Justify
	5)	How many relations are there on a set with n elements that are reflexive and symmetric? Justify
	6)	Let R and S be the following relations on $\text{A}=\{a,b,c,d\}$ and $\text{R}=\{(a,a),(a,c),(c,b),(c,d),(d,b)\}$ and $\text{S}=\{(b,a),(c,c),(c,d),(d,a)\}$ Find the following composition relations: i) $\text{R}\circ\text{S}$ ii) $\text{R}\circ\text{R}$ iii) $\text{S}\circ\text{R}$ iv) $\text{S}\circ\text{R}$
	7)	Let R be the relation on N defined by $\text{R}=\{(x,y)\mid x+3y=12\}$ i) Write R as a set of ordered pairs. ii) Find R^{-1} iii) Find the domain and range of R iv) Find $\text{R}\circ\text{R}$
	8)	Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack. i) {(a, b) | a and b are the same age} ii) {(a, b) | a and b have the same parents} iii) {(a, b) | a and b share a common parent} iv) {(a, b) | a and b have met} v) {(a, b) | a and b speak a common language}
	9)	Which of these relations on the set of all functions from Z to Z are equivalence relations? Determine the properties of an equivalence relation that the others lack. i) $\{(f,g)\mid f(1)=g(1)\}$ ii) $\{(f,g)\mid f(0)=g(0)\ or\ f(1)=g(1)\}$ iii) $\{(f,g)\mid f(1)-g(1)=1\forall x\in Z\}$ iv) $\{(f,g)\mid for\ some\ C\in Z,\forall x\in Z, f(x)-g(x)=C\}$ v) $\{(f,g)\mid f(0)=g(1)\ and\ f(1)=f(0)\}$
	10)	Show that the relation R consisting of all pairs *(x , y)* such that *x* and *y* are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more. What are the equivalence classes of these bit strings for the above equivalence relation? i) 010 ii) 101 1 iii) 11111 iv) 01010101
	11)	Which of these collections of subsets are partitions of the set of integers? i) the set of even integers and the set of odd integers ii) the set of positive integers and the set of negative integers iii) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3 iv) the set of integers less than - 1 00, the set of integers with absolute value not exceeding 1 00, and the set of integers greater than 1 00 v) the set of integers not divisible by 3, the set of even integers, and the set of integers that leave a remainder of 3 when divided by 6

	12)	Which of these are partitions of the set $Z \times Z$ of ordered pairs of integers? i) the set of pairs (x, y), where x *or* y is odd; the set of pairs (x, y), where x is even; and the set of pairs (x, y), where y is even ii) the set of pairs (x, y), where both x *and* y are odd; the set of pairs (x, y), where exactly one of x *and* y is odd; and the set of pairs (x, y), where both x *and* y are even iii) the set of pairs (x, y), where x is positive; the set of pairs (x, y), where y is positive; and the set of pairs (x, y), where both x *and* y are negative iv) the set of pairs (x, y), where $3 \mid x$ and $3 \mid y$; the set of pairs (x, y), where $3 \mid x$ and 3 does not divide y ; the set of pairs (x, y), where 3 does not divide x and $3 \mid y$ and the set of pairs (x, y), where 3 does not divide both x *and* y. v) the set of pairs (x, y), where $x > 0$ and $y > 0$; the set of pairs (x, y), where $x > 0$ and $y \le 0$; the set of pairs (x, y), where $x \le 0$ and $y > 0$; and the set of pairs (x, y), where $x \le 0$ and $y \le 0$.
	13)	Consider the set of words $W = \{sheet, last, sky, wash, dash, wind, sit, bit, quit\}$. Find W/R where R is the equivalence relation on W defined by (a) 'has the same number of letters ' (b) 'begins with the same letter'.
	14)	How many non-zero entries does the matrix representing the relation R on $A = \{1, 2, \cdots, 100\}$ consisting of the first 100 positive integers i) $\{(a,b) \mid a > b\}$ ii) $\{(a,b) \mid a \ne b\}$ iii) $\{(a,b) \mid a = b+1\}$ iv) $\{(a,b) \mid a = 1\}$ v) $\{(a,b) \mid ab = 1\}$
	15)	Show that the relation R consisting of all pairs *(x , y)* such that *x* and *y* are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more. What are the equivalence classes of these bit strings for the above equivalence relation? a) 010 b) 101 1 c) 11111 d) 01010101
Q. 2		**Attempt the following**
	1)	A computer password consist of a letter of the alphabet followed by 3 or 4 digits. Find the total number of passwords that can be created and the number of passwords in which no digit repeats.
	2)	A Label identifier for a computer program consists of one letter followed by three digits. If repetitions are allowed, how many distinct label identifiers are possible?
	3)	If telephone area codes are three digit numbers whose middle digit must be 0 or 1. Codes whose last two digits are $1's$ are being used for some other purposes. With these conditions, how many area codes are available?
	4)	On an English test, a student must write two essays. For the first essay, the student must select from topics A, B, and C. For the second essay, the student must select from topics 1, 2, 3, and 4. How many different ways can the student select the two essay topics? Show all possibilities using tree diagram.
	5)	i) Determine the number of distinguishable arrangements for MISSISSIPPI. ii) How many seven letter words can be formed using the letters of the word BENZENE?

	6)	Out of 12 employees, a group of four trainees is to be sent for 'Software testing and QA' training of one month. a) In how many ways can the four employees be selected? b) In how many ways a group can be selected if two of the 12 employees refuse to go together?
	7)	How many permutations of the letters ABCDEFGH contain a) the string ED? b) the string CDE? c) the strings BA and FGH? d) the strings AB, DE, and GH? e) the strings CAB and BED? f) the strings BCA and ABF?
	8)	If repetitions are not permitted, how many four digit numbers can be formed from digits $1,2,3,5,7$ and 8? How many of these numbers are less than 5000? How many of these numbers contain both the digits 3 and 5?
	9)	13 people on a softball team show up for a game. How many ways are there to choose 10 players to take the field? How many ways are there to assign the 10 positions by selecting players from 13 people who show up? Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a women?
	10)	A bag contains six white marbles, and five white marbles. Find the number of ways four marbles can be drawn from the bag a) if they can be of any color b) two must be white and two red c) they must be of the same color.
	11)	How many strings of eight English letters are there a) if letters can be repeated? b) if no letter can be repeated? c) that start with X, if letters can be repeated? d) that start with X, if no letter can be repeated? e) that start and end with X, if letters can be repeated? f) that start with the letters BO (in that order), if letters can be repeated? g) that start and end with the letters BO (in that order), if letters can be repeated? h) that start or end with the letters BO (in that order), if letters can be repeated?
	12)	In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if a) the bride must be next to the groom? b) the bride is not next to the groom? c) the bride is positioned somewhere to the left of the groom?
	13)	How many bit strings of length 10 contain a) exactly four Is? b) at most four Is? c) at least four Is? d) an equal number of Os and 1 s?
	14)	Draw a tree diagram to show the number of ways so that four friends Amit, Bob, Chetan and Danny sit, so that Bob and Danny always sit together. Hence find the possible ways of this sitting arrangement.
	15)	How many different signals, each consisting of eight flags in a vertical line, can be formed from a set of four indistinguishable red flags, three indistinguishable white flags and a blue flag?
Q. 3		**Solve the following difference equations by the method of substitution or iteration**
	1)	$a_n = a_{n-1} + 3, n \geq 2$ and $a_n = 2 \ for \ n = 1$.
	2)	$a_n = a_{n-1} + (n-1), \ n \geq 2$.
	3)	$a_n = 2a_{n-1} + 1, \ n \geq 2$ and $a_1 = 1$
	4)	$a_n = 2a_{n-1}, \ n \geq 1$ and $a_0 = 1$

	5)	$a_n = a_{n-1} + n,\ n \geq 1$ and $a_0 = 1$
	6)	$a_n = a_{n-1} + 4n,\ n \geq 1$ and $a_0 = 0$
	7)	$a_n = 2a_{n-1} + 3 \cdot 2^n,\ n \geq 2$ and $a_1 = 5$
	8)	$a_n = a_{n-1} - 1,\ n \geq 2$ and $a_1 = 1$
	9)	$a_n = a_{n-1} - 3n,\ n \geq 1$ and $a_0 = 0$
	10)	$a_n = a_{n-1},\ n \geq 1$ and $a_0 = 7$
Q. 4		**Solve the following**
	1)	$a_n = 5a_{n-1} - 8a_{n-2} + 4a_{n-3} + 1, n \geq 3$ and $a_0 = 0, a_1 = 1, a_2 = 2$.
	2)	$a_n = 5a_{n-1} - 6a_{n-2} + 2^n, n \geq 2$ and $a_0 = 7, a_1 = 16$
	3)	$a_n = 7a_{n-1} - 10a_{n-2} + n, n \geq 2$ and $a_0 = 5, a_1 = 16$
	4)	$a_n = 2a_{n-1} + 8a_{n-2} + 5, n \geq 2$ and $a_0 = 4, a_1 = 10$
	5)	$a_n = 6a_{n-1} - 9a_{n-2} - 1, n \geq 2$ and $a_0 = 1, a_1 = 6$
	6)	$2a_n - 7a_{n-1} + 3a_{n-2} = 2^n, n \geq 2$ and $a_0 = 1, a_1 = 2$
	7)	$a_n + 2a_{n-1} - 3a_{n-2} = 4.2^n, n \geq 2$ and $a_0 = 1, a_1 = 2$
	8)	$a_n - a_{n-1} - 6a_{n-2} = 36n, n \geq 2$ and $a_0 = 0, a_1 = 1$
	9)	$a_n + 2a_{n-1} - 8a_{n-2} = 4 - 5n - 9(-1)^n, n \geq 2$ and $a_0 = -2, a_1 = -3$
	10)	$a_n - 4a_{n-2} = -3n + 4, n \geq 2, a_0 = a_1 = 0$
	11)	$a_n + 2a_{n-1} + a_{n-2} = 2n + 6, n \geq 2, a_0 = 0, a_1 = 1$
	12)	$a_n + a_{n-1} - 12a_{n-2} = n2^n, n \geq 2, a_0 = 0, a_1 = 1$
	13)	$a_n + 2a_{n-1} = n, n \geq 1, a_0 = 0$
	14)	$a_n = 4a_{n-1} - 4a_{n-2} + (n + 1)2^n$, where $a_0 = 0$ and $a_1 = 1$
	15)	$a_n = -5a_{n-1} - 6a_{n-2} + 42 .4^n$, $a_1 = 56$ and $a_2 = 278$.
Q. 5		**Attempt the following**
		For the following graphs State degree of each vertex, Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists. Find spanning tree for given graph. Also verify the handshaking lemma. Is there any cut vertex? If so, state it.
	1)	

	2)	
	3)	
	4)	
	5)	
Q. 6		Determine whether the given pair of graphs below is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.
	1)	

	2)	
	3)	
	4)	
	5)