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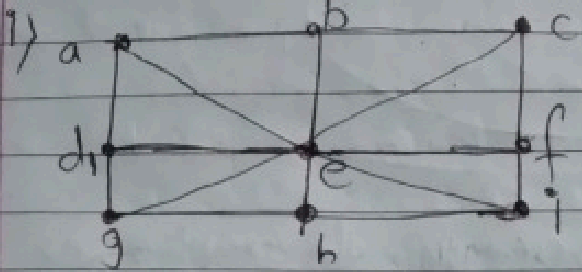
* some solved Problems on Eulerian, Hamilton graph, Planar *

Q) I) Determine whether the following graph is Eulerian / Hamilton / Planar

State the Eulerian Path / Hamilton circuit. If not Hamiltonian circuit, is there a Hamiltonian Path.

1)



G

Now, we know, The connected multigraph ~~is~~ contain Eulerian circuit / Eulerian graph iff degree of each vertex is even

~~Now~~ Hence G ~~is Not~~ does Not contain Eulerian circuit $(\because d(b) = 3)$

Also The connected multigraph contains Eulerian path but Not Eulerian circuit iff There exist exactly two vertices of odd degree.

Now, As in G contain 4 vertices of odd degree (i.e. of degree 3) so G does Not contain Eulerian path also.

G has Hamilton path ~~namely~~.

For example $\{a, d_1\}, \{d_1, g\}, \{g, h\}, \{h, i\}, \{i, f\}, \{f, c\}, \{c, b\}, \{b, e\}$. is one of the Hamilton path in G (ie $a d_1 g h i f c b e$)

G Also has Hamilton Circuit also, ~~ie~~ one of the Hamilton circuit is $\{d_1, g\} \{g, h\}. \{h, e\}. \{e, i\} \{i, f\}. \{f, c\}, \{c, b\}. \{b, a\}. \{a, d_1\}$

ie. $d_1 g h e i f c b a d_1$

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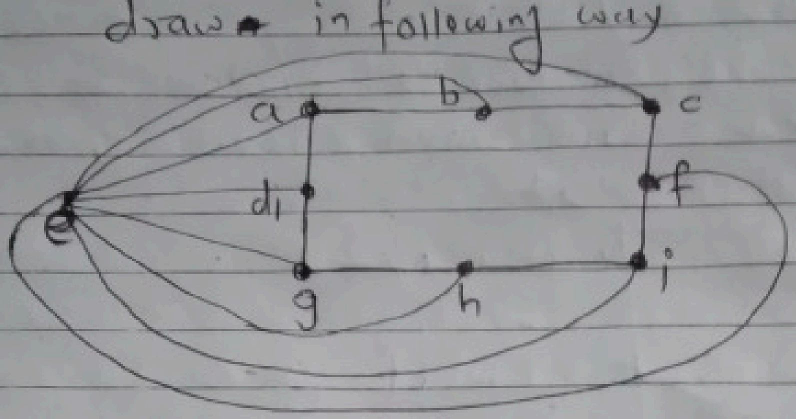
G is planor Graph because G can be drawn in following way

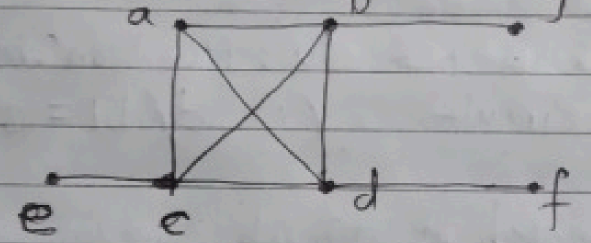


G

In this representation of Graph G no two edge are ~~cr~~ crosses each other (except at end Point)

so G is planar.

ii)



G_2

Now, As ~~each~~ degree of ~~each vertex~~ All vertices ~~is~~ are Not even (ie ∃ vertex of degree odd)

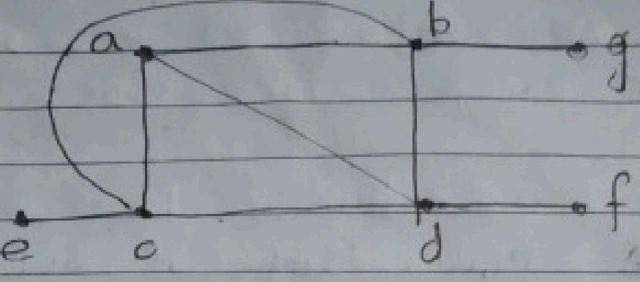
⇒ G_2 ~~is~~ ~~Not~~ does Not contain Eulerian circuit

As G_2 ~~contain~~ does Not contain Exactly two vertices of odd degree so G_2 does Not contain Eulerian Path also

G does Not contain Hamilton Path & Hamilton circuit becoz, If we comp at vertex e Then we can not go back otherwise c will be repeared so Eulerian circuit does Not exist in G_2

Also If we start from e then then either we can not come back from g ~~g2~~ ~~or~~ f because ~~we can not~~ otherwise vertex b or d ~~re~~peared.

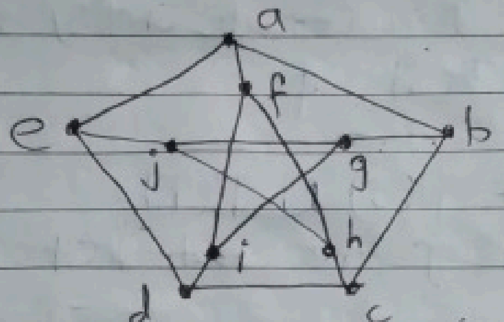
As We can draw G_2 in following way.



G_2

so No two edges are crossing in this representation so G_2 is Not planar.

iii)



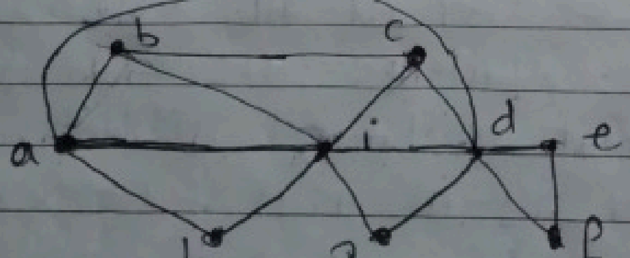
G_3

Not Eulerian (There exist vertices of odd degree
(ie Not contain Eulerian circuit)
Not contain Eulerian Path (There does Not exist exactly two vertices of odd degree)

For this graph G_3 Hamilton Path exist. one of the Hamilton Path is ~~{g,b}~~
{b,g}, {g,j} {j,e}, {e,a}, {a,f} {f,h}, {h,c}, {c,d} {d,i} ie b g j e a f h c d i

But G_3 does Not have Hamilton circuit
& also Not planar

iv)



G_5

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Now, the Graph G_4 does Not have Eulerian circuit.
~~Now, b/c are two~~ becoz in G_4 There are two vertices of odd degree.

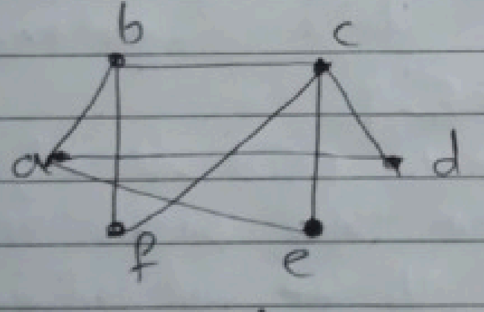
(b & c) So As G_4 contains exactly two vertices of odd degree so G_4 has Eulerian Path but Not has Eulerian circuit.

one of The Eulerian path is {b,a}, {a,i}, {i,c}, {c,b}, {b,i}, {i,g}, {g,d}, {d,a}, {a,h}, {h,i}, {i,d}, {d,e}, {e,f}, {f,d} (ie b a i c b i g d a h i d e f d)
(because all edges are coverges in this Path & also This path is simple ie with no repeated edges)

Hamilton Path is e f d g i h a b c
Hamilton circuit does Not exist.

Q) H) Determine whether the Graph is bipartite or Not

i)



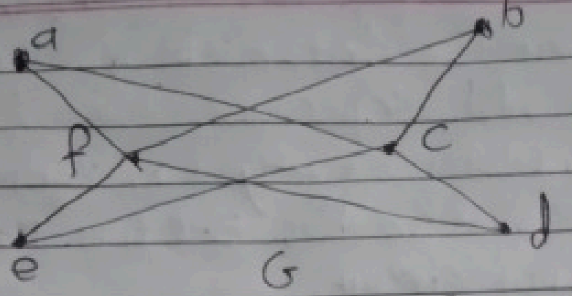
Now, $V = \{a, b, c, d, e, f\}$

Now, If we get $V_1, V_2 \subseteq V$ such that $V_1 \cap V_2 = \phi$ & $V_1 \cup V_2 = V$ & each edge in graph is betn vertex of V_1 & vertex of V_2

If we take $V_1 = \{a, f\}$ Then $V_2 = \{b, e, c, d\}$

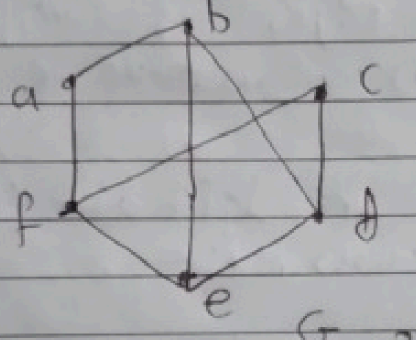
~~so~~ & There is edge betn c & d.

So Not bipartite (because we can not take ~~ext~~ one more element in V_1 otherwise there will be edge betn two vertices of V_1)

ii) 

Now, take $V_1 = \{a, b, d, e\}$ & $V_2 = \{f, c\}$

so Any edge in G is from one vertex of V_1 & one vertex of V_2
so G is bipartite

iii) 

Now take $V_1 = \{b, f\}$ (vertices in V_1 are pairwise Non adjucent)

$V_2 = \{a, e, c, d\}$

so Here in V_2 c & d are adjucent.
& we can not take another vecter in V_1
(otherwise V_1 will contain two adjucent vertices so)
so G is Not bipartite.

Q (x) ~~How many~~ Find the Number of paths of length n betn two different vertices in K_4 if n is a) 2, b) 3, c) 4

soln: Now, In General If A is Adjucency matrix of G Then The Number of paths of length n betn i & j vertices is $(i,j)^{th}$ entry of A^n

so Here for
a) $n=2$

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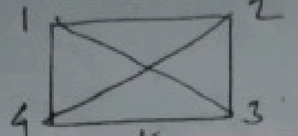


K_4

Now, ~~Adjucency m~~

a) Adjuecency Matrix (K_4) =

$$A = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix} \end{array}$$

~~$A^3 =$~~

a) ~~If~~ Total No. of Path of length 2 betn ith & jth vertices $= (ij)^{th}$ entry of A^2 $= 2$ if $i \neq j$

$= 3$ if $i = j$

For $n = 3$

b) Now, ~~$A^3 = [17 \; 17 \; 17 \; 17]$~~

$$A^3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 21 & 20 & 20 & 20 \\ 20 & 21 & 20 & 20 \\ 20 & 20 & 21 & 20 \\ 20 & 20 & 20 & 21 \end{bmatrix} \end{array}$$

Total No. of Paths of length 3 betn i^{th} & j^{th} vertices

$= (ij)^{th}$ entry of A^3

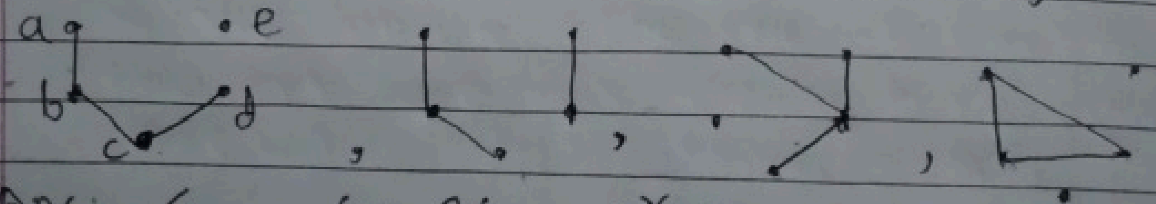
$= 21$ if $i = j$

20 if $i \neq j$

For $n = 4$

c) Similary Find A^4 & Find total No. of paths of length 4 betn i^{th} & j^{th} vertex of G from entry $(ij)^{th}$ of A^4.

Q. c) ix) How many Non isomorphic simple Graphs are there with five vertices & three edges.



Ans: 4 (∵ This are total such Non isomorphic graphs)