The diameter of the graph is diam $(G) = 2$ Fig. 12.59.

Cut Vertex, Cut Set and Bridge

The concepts of cut vertex, cut set and bridge may be defined as given below.

Definition 12.42 Cut Vertex

A *cut vertex* of a connected graph G is a vertex whose removal increases the number of components.

Definition 12.43 Bridge

A *cut edge or bridge* of a connected graph G is an edge whose removal increases the number of components.

Definition 12.44 Cut Set

The set of all minimum number of edges of G whose removal disconnects a graph G is called a cut set of G. Thus, a cut set S of a graph G satisfies the following properties:

- S is a subset of the edge set E of G
- Removal of edges from a connected graph G disconnects G
- No proper subset of G satisfies the condition

Note 12.14 If v is a cut vertex of a connected graph G, then $G - v$ is disconnected. A cut vertex is also called a cut point.

Example 12.28 Find all the cut vertices of the graph given in Fig. 12.60.

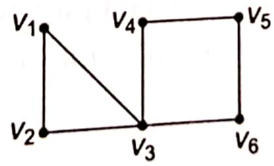

Figure 12.60 Graph for Example 12.28

Solution There is only one cut vertex in the graph, which is vertex V_3.

Example 12.29 Find all the bridges of the graph given in Fig. 12.61.

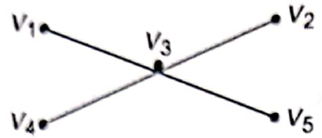

Figure 12.61 Graph for Example 12.29

Solution

$\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_4, v_3\}$, $\{v_5, v_3\}$ are all the bridges present in the given graph.

Example 12.30 Find the vertex sets of components, cut-vertices and cut-edges of the graph given in Fig. 12.62.

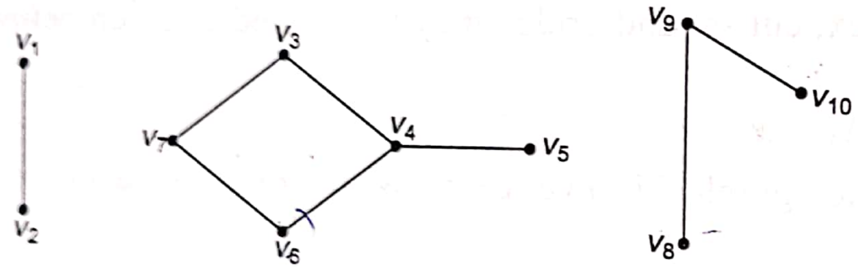

Figure 12.62 Graph for Example 12.30

Solution The graph shown in Fig. 12.62 has the following three components:

- The vertex set of the components are $\{v_1, v_2\}$, $\{v_3, v_4, v_5, v_6, v_7\}$ and $\{v_8, v_9, v_{10}\}$
- The cut vertices are v_6 and v_9.
- The cut-edges are $v_1\ v_2$, $v_7\ v_6$, $v_8\ v_9$ and $v_9\ v_{10}$.

Theorem 12.12 Let v be a vertex of a connected graph G. Then, the following statements are equivalent:

Example 12.44 Show that the graphs G_1 and G_2 (Fig. 12.91) are isomorphic.

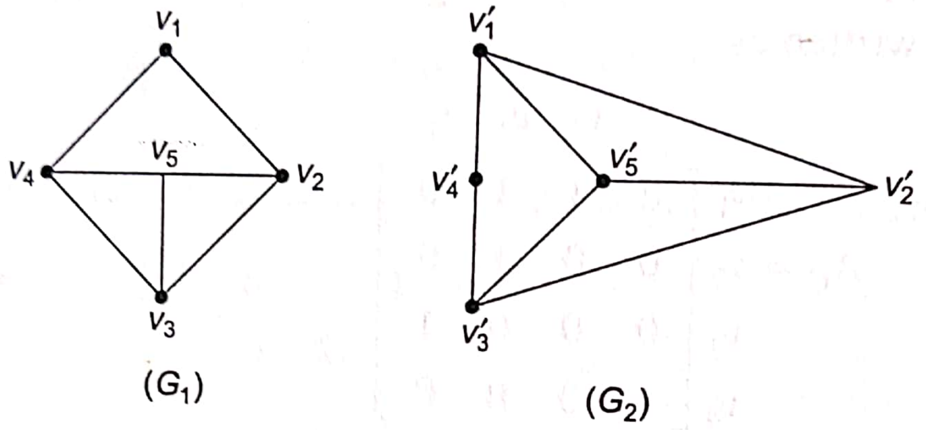

Figure 12.91 Graphs for Example 12.44

Solution Consider the map $f: G_1 \to G_2$ defined by

$$f(v_1) = v_4', f(v_2) = v_1', f(v_3) = v_2', f(v_4) = v_3' \text{ and } f(v_5) = v_5'.$$

The adjacency matrix of G_1 for the ordering of the vertices v_1, v_2, v_3, v_4 and v_5 is written as

$$A_{G_1} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The adjacency matrix of G_2 for the ordering of the vertices v_4', v_1', v_2', v_3' and v_5' is

$$A_{G_2} = \begin{array}{c c} & \begin{array}{ccccc} v_4' & v_1' & v_2' & v_3' & v_5' \end{array} \\ \begin{array}{c} v_4' \\ v_1' \\ v_2' \\ v_3' \\ v_5' \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$\Rightarrow A_{G_1} = A_{G_2}$

Therefore, G_1 and G_2 are isomorphic

Example 12.45 Find the adjacency matrix of the graph ...

Hence, Fig. 12.104 is a planar graph, though it appears to be a non-planar graph.

Example 12.53 Consider the graph in Fig. 12.106. Is it planar or non-planar?

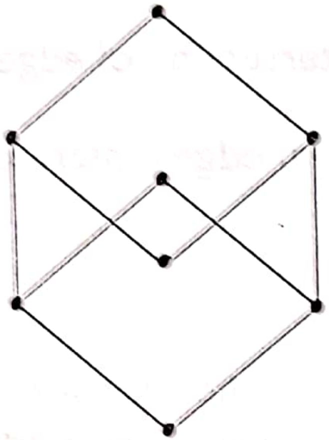

Figure 12.106 Graph for Example 12.53

Solution This graph can be redrawn as shown in Fig 12.107.

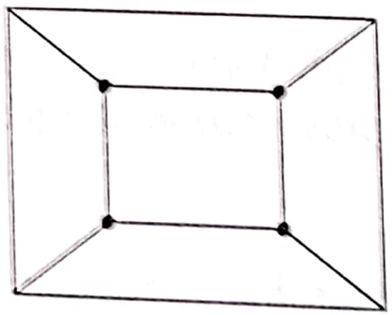

Figure 12.107 Redrawn graph

In this graph, no two edges intersect except at the vertices, which may be the common end verti
of the edges. Hence, the graph is planar.

Consider the planar repre-entation of a planar graph as shown in Fig. 12.108.

Let G denote the planar graph