Number System and Computer Arithmetic

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Integer Representation

- **Integer**
  - **Signed**
    - Sign and Magnitude
    - 1's Complement
    - 2's Complement
  - **Unsigned**

- ✔ Computers uses *a fixed number of bits* to represent an integer.
- ✔ The commonly-used bit-lengths for integers are 8-bit, 16-bit, 32-bit or 64-bit.
- ✔ Besides bit-lengths, there are two representation schemes for integers:

1. ***Unsigned Integers*: can represent zero and positive integers.**
2. ***Signed Integers*: can represent zero, positive and negative integers.**
   Three representation schemes had been proposed for signed integers:
   1. **Sign-Magnitude representation**
   2. **1's Complement representation**
   3. **2's Complement representation**

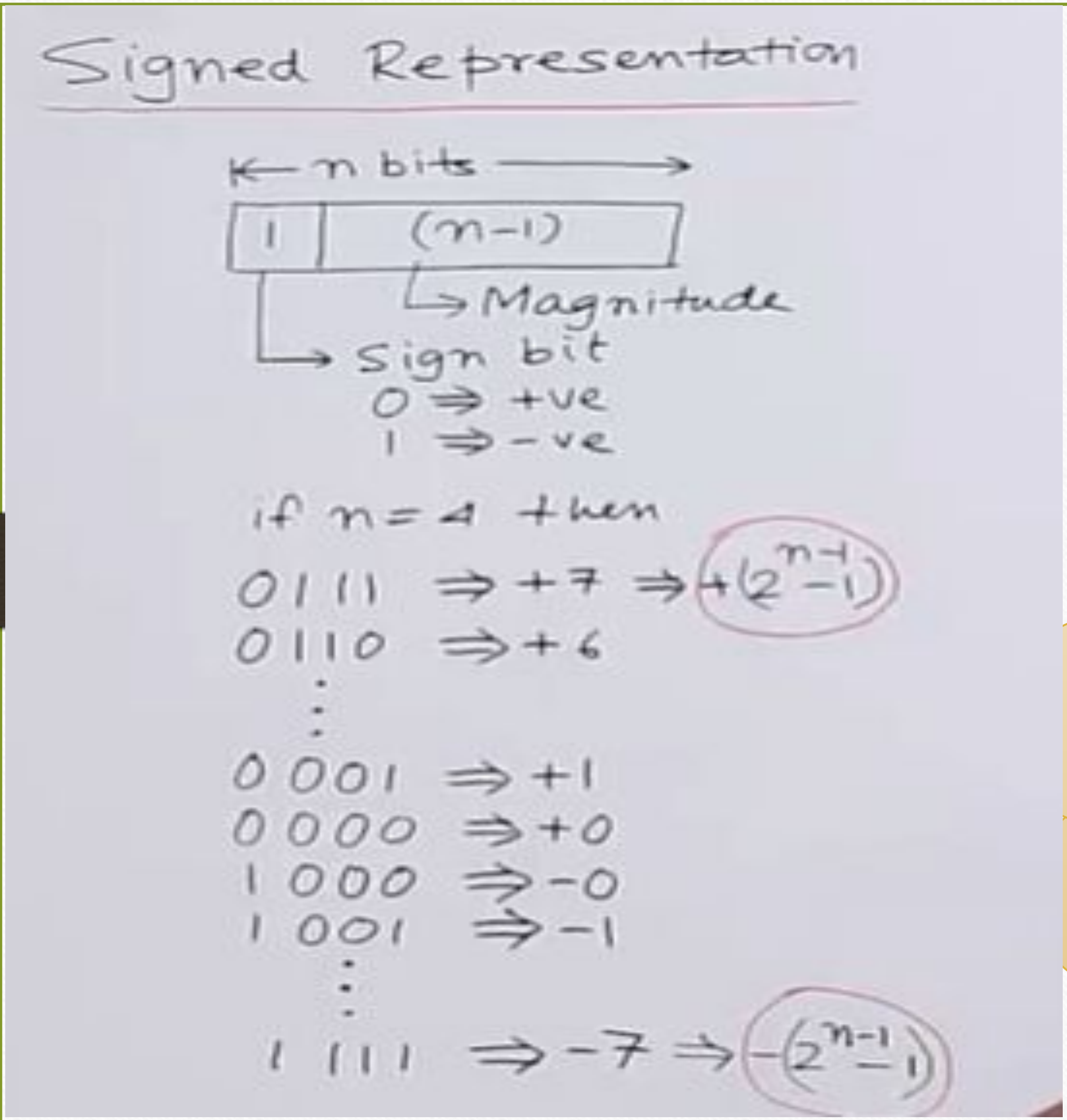
Unsigned Integers

- Unsigned integers can represent zero and positive integers, but not negative integers. An n-bit unsigned integer can represent integers from 0 to (2^n)-1

n= number of bits	Minimum	Maximum
8	0	(2^8)-1 (=255)
16	0	(2^16)-1 (=65,535)
32	0	(2^32)-1 (=4,294,967,295) (9+ digits)
64	0	(2^64)-1 (=18,446,744,073,709,551,615) (19+ digits)

Signed Integers
Sign Magnitude Representation

- The most-significant bit (msb) is the *sign bit*,
  - **0 representing positive integer**
  - **1 representing negative integer.**
- The remaining *n*-1 bits represents the magnitude (absolute value) of the integer.

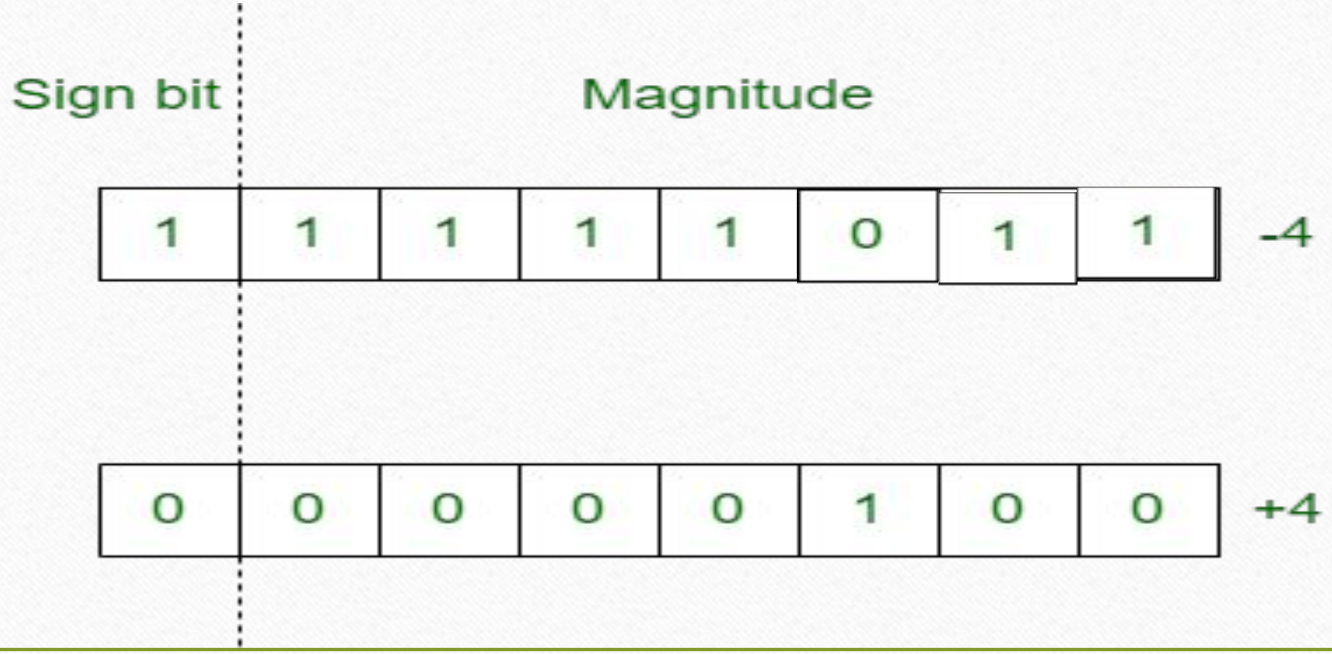


0 and -0 ?

- This is not possible in number system.
- so this is drawback of sign and magnitude representation method.
- Therefore we use 2's complement method to represent signed number

Signed Integers
1's Complement Representation

Sign Bit	1's complement of actual binary



1's Complement Representation

- ✔ Can represent numbers from -32,767 to 32,767.

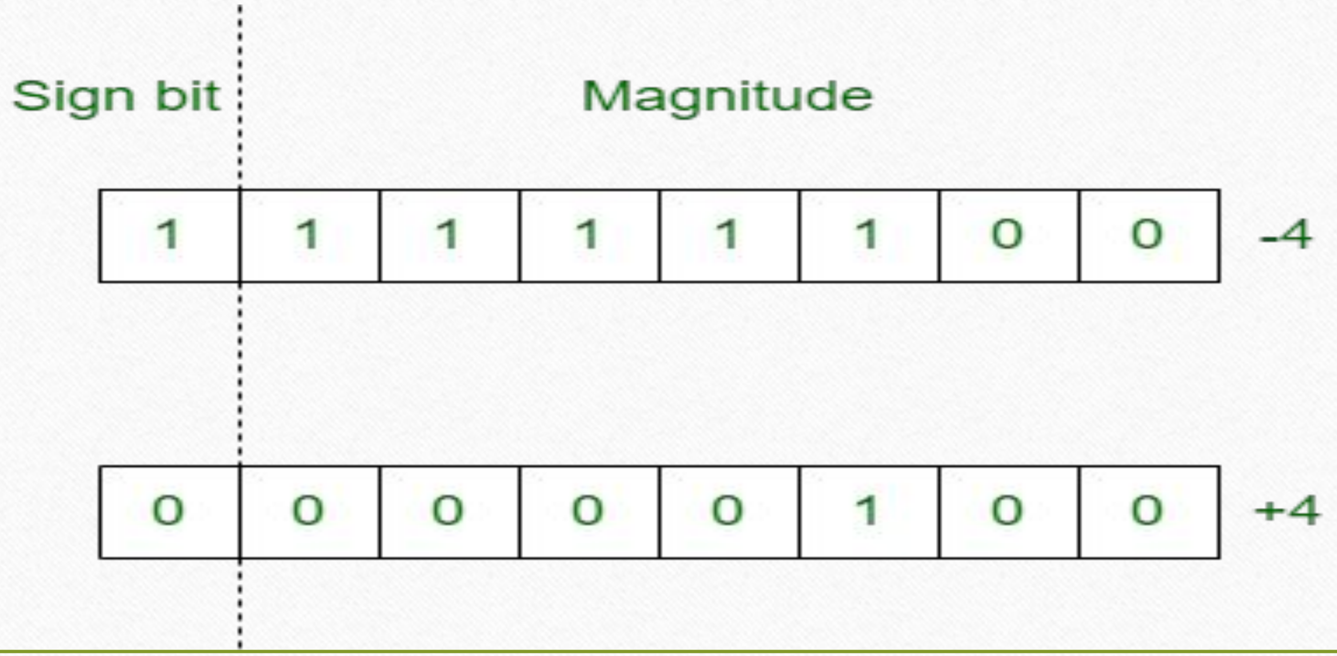
  -215+1 .. 215-1
- ✔ Arithmetic is easier than sign-magnitude.
- ✔ But, still have two representations for zero:

  0 = 00000000 00000000

  -0 = 11111111 11111111

Signed Integers
2’s Complement Representation

Sign Bit	2’s complement of actual binary



Signed Integers
2's Complement Representation

Signed Representation (Using 2's complement)

if $n = 4$ then

$0111 \Rightarrow +7 \Rightarrow +(2^{n-1} - 1)$

$0110 \Rightarrow +6$

$\vdots$

$0001 \Rightarrow +1$

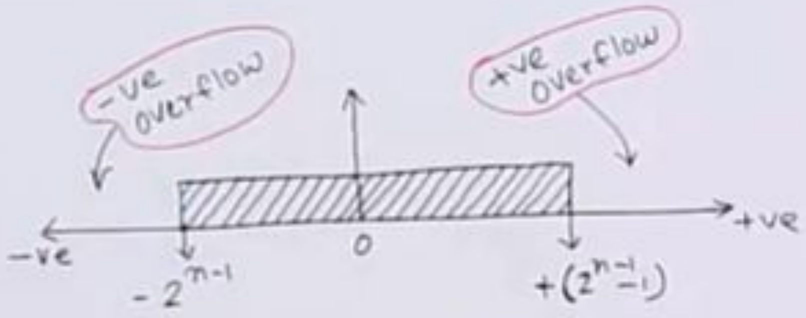
$0000 \Rightarrow 0$

$1111 \Rightarrow -1$

$1110 \Rightarrow -2$

$\vdots$

$1000 \Rightarrow -8 \Rightarrow -(2^{n-1})$



Binary Addition

Input Bits		Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

1+1+1=11

1+1+1+1=100