Image Processing and Computer Vision

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Multiresolution Analysis

- The wavelet transform is the foundation of techniques for analysis, compression and transmission of images.

- Mallat (1987) showed that wavelets unify a number of techniques, including subband coding (signal processing), quadrature mirror filtering (speech processing) and pyramidal coding (image processing). The name multiresolution analysis has been used for these techniques.

Image Pyramid

Let A be an image of size $N \times N$ where $N = 2^J$.

Let A_{J-1} be formed by smoothing A and then downsampling.

Let $\tilde{A}$ be an approximation of A reconstructed by upsampling and interpolating.

Let $E_J = A - \tilde{A}$. If we record A_{J-1} and E_J we can perfectly reconstruct A.

The process can be repeated, leading to the construction of a pyramid.

The number of pixels in a pyramid with P levels is

$$N^2 \left(1 + \frac{1}{4} + \frac{1}{4^2} + \cdots + \frac{1}{4^P}\right) \leq \frac{4}{3} N^2$$

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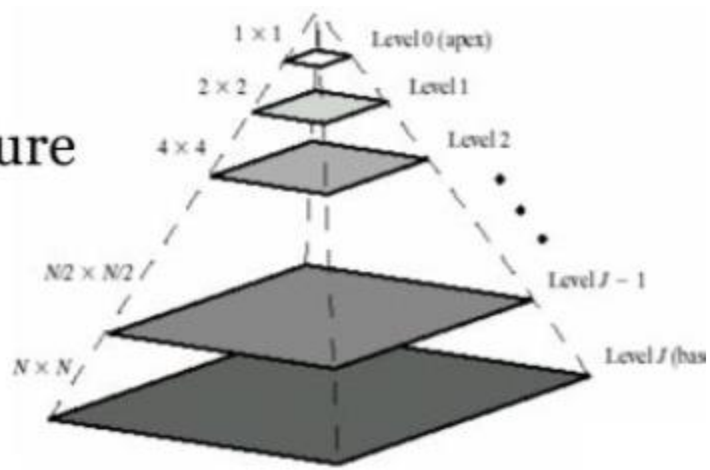
Image Pyramid

Image Pyramid

- A pyramid image structure



- System block diagram

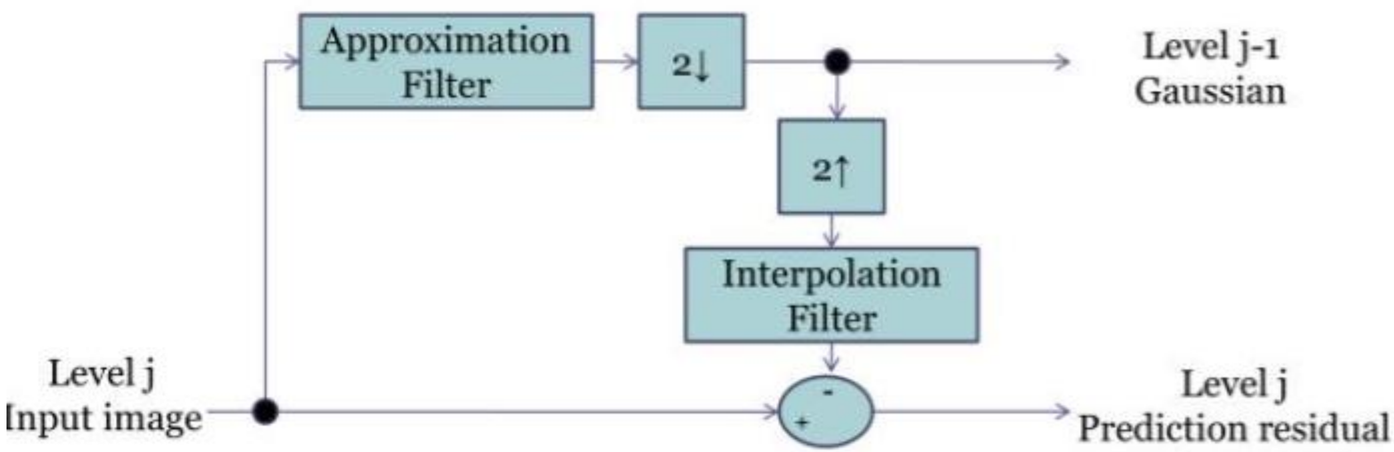


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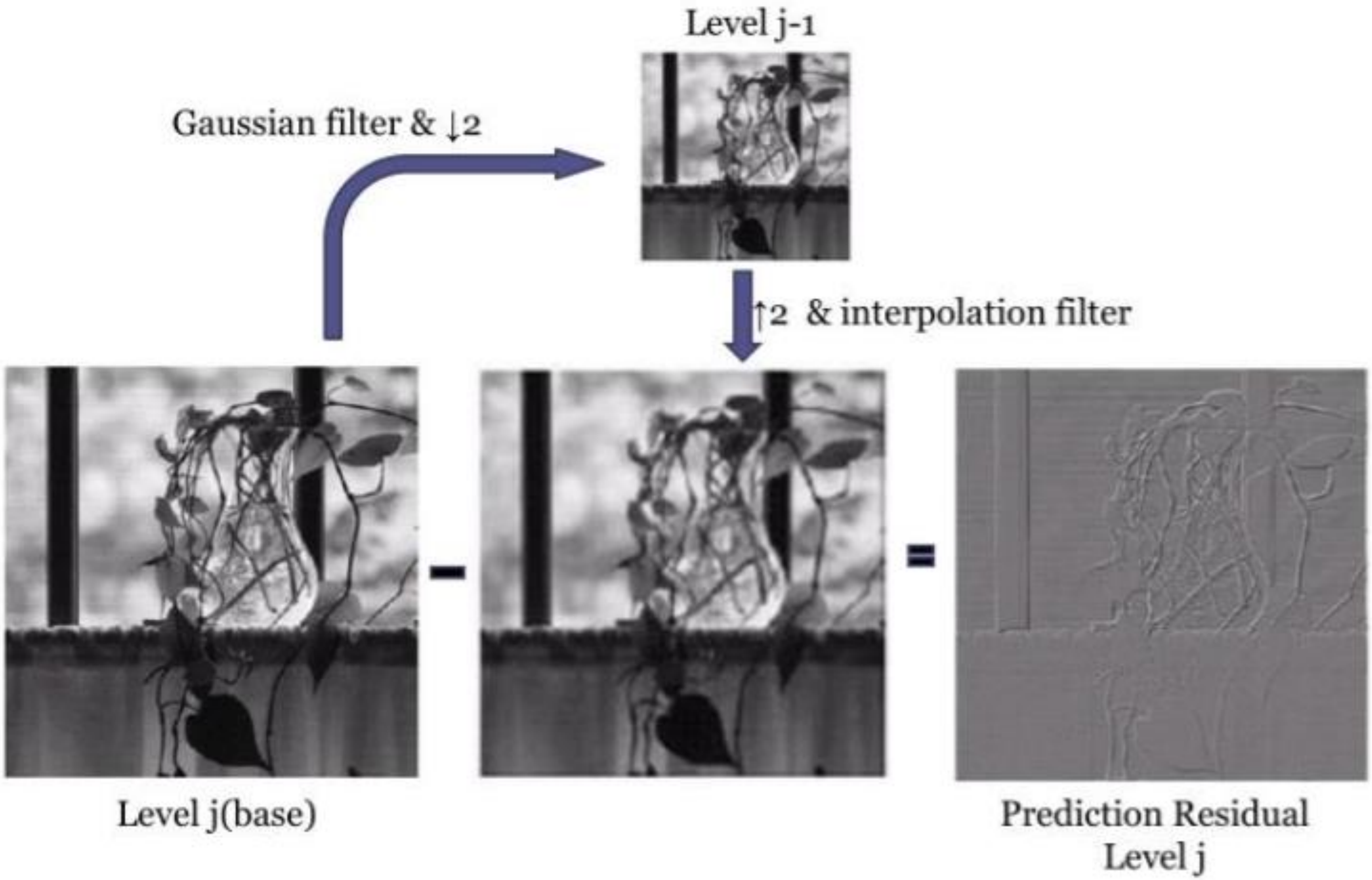
Image Pyramid



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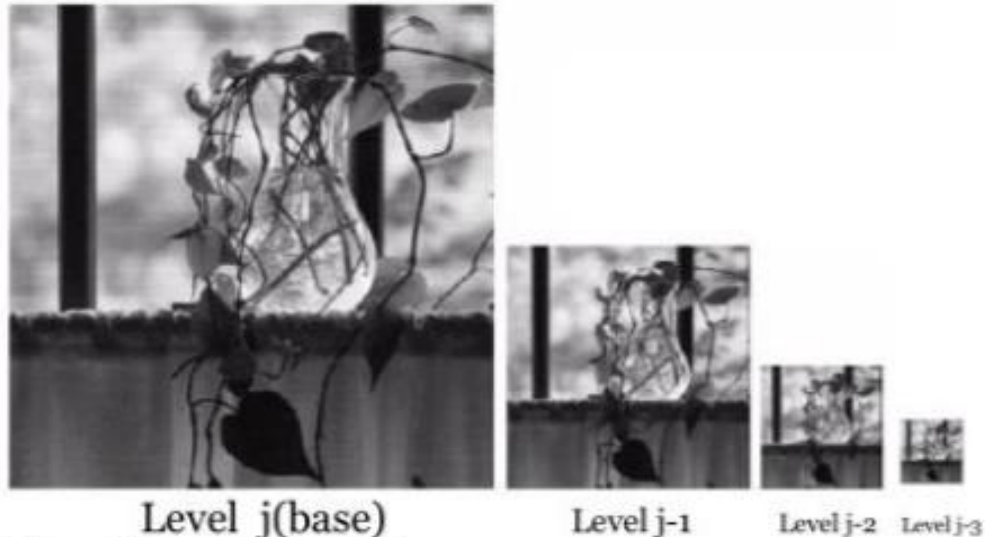
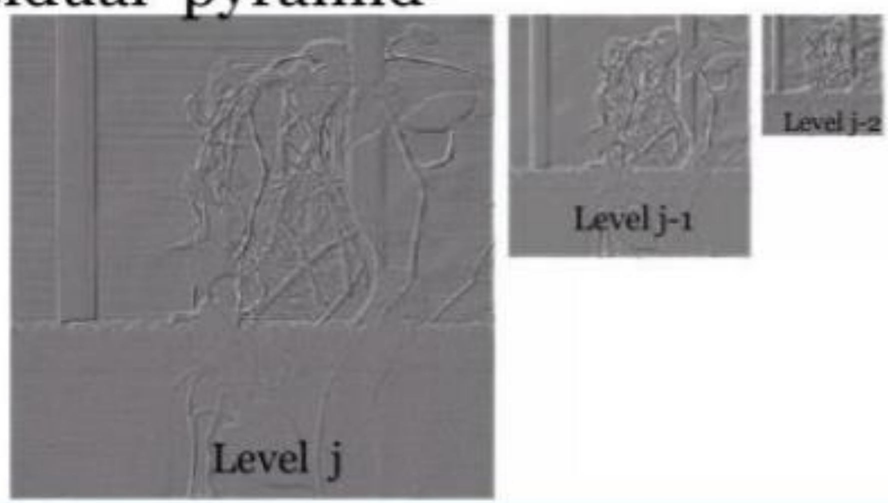
- A Gaussian pyramid



- A prediction residual pyramid



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Approximation Filter

- Using Gaussian Filter
  - Mask (3x3)

3/33	4/33	3/33
4/33	**5/33**	4/33
3/33	4/33	3/33

$\otimes$

0,0	0,1	0,2	0,3	0,4	0,5
1,0	1,1	1,2	1,3	1,4	1,5
2,0	2,1	2,2	2,3	2,4	2,5
3,0	3,1	3,2	3,3	3,4	3,5
4,0	4,1	4,2	4,3	4,4	4,5
5,0	5,1	5,2	5,3	5,4	5,5

- Ex:

1,1	1,0	1,1	1,2	1,3	1,4	1,5	1,4
0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,4
1,1	1,0	1,1	1,2	1,3	1,4	1,5	1,4
2,1	2,0	2,1	2,2	2,3	2,4	2,5	2,4
3,1	3,0	3,1	3,2	3,3	3,4	3,5	3,4
4,1	4,0	4,1	4,2	4,3	4,4	4,5	4,4
5,1	5,0	5,1	5,2	5,3	5,4	5,5	5,4
4,1	4,0	4,1	4,2	4,3	4,4	4,5	4,4

$8X8$

$$
\begin{aligned}
I(0,0)' = {} & 3/33 \cdot I(1,1) + 4/33 \cdot I(1,0) + 3/33 \cdot I(1,1) \\
& + 4/33 \cdot I(0,1) + 5/33 \cdot I(0,0) + 4/33 \cdot I(0,1) \\
& + 3/33 \cdot I(1,1) + 4/33 \cdot I(1,0) + 3/33 \cdot I(1,1)
\end{aligned}
$$

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Downsampling & Upsampling

- Downsampling

0,0	0,1	0,2	0,3	0,4	0,5
1,0	1,1	1,2	1,3	1,4	1,5
2,0	2,1	2,2	2,3	2,4	2,5
3,0	3,1	3,2	3,3	3,4	3,5
4,0	4,1	4,2	4,3	4,4	4,5
5,0	5,1	5,2	5,3	5,4	5,5

6X6

$\xrightarrow{\downarrow 2}$

0,0	0,2	0,4
2,0	2,2	2,4
4,0	4,2	4,4

3X3

- Upsampling

0,0	0,2	0,4
2,0	2,2	2,4
4,0	4,2	4,4

3X3

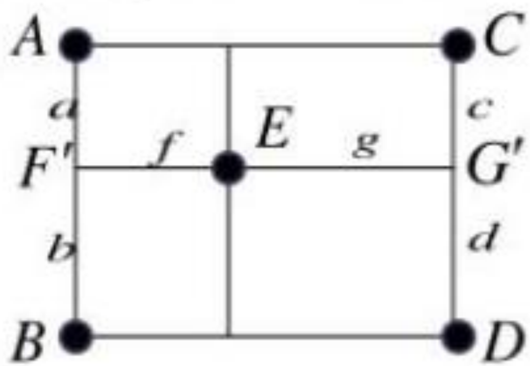
$\xrightarrow{\uparrow 2}$

0,0	0	0,2	0	0,4	0
0	0	0	0	0	0
2,0	0	2,2	0	2,4	0
0	0	0	0	0	0
4,0	0	4,2	0	4,4	0
0	0	0	0	0	0

6X6

Interpolation Filter

- Using bilinear interpolation



$$F' = \frac{b}{a+b} \times A + \frac{a}{a+b} \times B$$

$$G' = \frac{d}{c+d} \times C + \frac{c}{c+d} \times D$$

$$\Rightarrow E = \frac{g}{f+g} \times F' + \frac{f}{f+g} \times G'$$

0,0	0	0,2	0	0,4	0
0	0	1,2	0	1,4	0
2,0	0	2,2	0	2,4	0
0	0	3,2	0	3,4	0
4,0	0	4,2	0	4,4	0
0	0	0	0	0	0

6X6

Interpolation →

0,0	0,1	0,2	0,3	0,4	0,5
1,0	1,1	1,2	1,3	1,4	1,5
2,0	2,1	2,2	2,3	2,4	2,5
3,0	3,1	3,2	3,3	3,4	3,5
4,0	4,1	4,2	4,3	4,4	4,5
5,0	5,1	5,2	5,3	5,4	5,5

6X6

Interpolation Order: 1 2

Ex:

$\boxed{0,1} = (\boxed{0,0} + \boxed{0,2})/2$

$\boxed{1,1} = (\boxed{0,1} + \boxed{1,0} + \boxed{2,1} + \boxed{1,2})/4$

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Multiresolution Expansions

- Multiresolution expansions in wavelets refer to a a mathematical framework for decomposing signals or data into different scales or resolutions.
- Multiresolution analysis provides a systematic way of decomposing signals into different levels of detail or resolutions. The idea is to represent a signal using a set of basis functions, known as wavelet functions, that capture different frequency components and time-localized features.
- The multiresolution expansion is typically obtained by applying a series of filtering and down sampling operations to the signal. At each level or resolution, the signal is decomposed into two components: an approximation component that represents the coarse-scale information, and a detail component that represents the high-frequency or fine-scale details.

Multiresolution Expansions

- This process is repeated iteratively, resulting in a hierarchical representation of the signal with multiple levels of approximation and detail coefficients. The approximation coefficients gives the low-frequency components of the signal(scaling function), while the detail coefficients gives the high-frequency components(wavelet function).
- Multiresolution processing refers to the manipulation and analysis of signals or data in the multiresolution domain.
- Wavelet-based methods have found applications in various fields, including signal processing, image and video compression, data analysis, and pattern recognition.