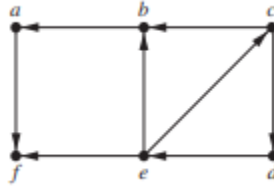
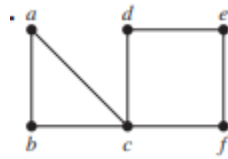


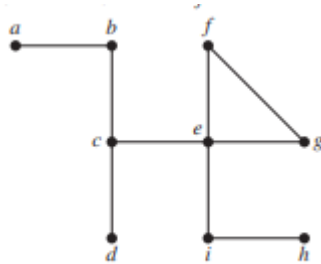
1. Show that a simple graph G with n vertices is connected if it has $\frac{(n-1)(n-2)}{2}$ edges.
2. Find the strongly connected component of the graph



3. Find the number of paths of length n between two vertices of K_4 where n is
 - a. 2
 - b. 3
 - c. 4
 - d. 5
4. Find the cut vertices of each of the graph
 - a.



b.



5. Show that each of the following graphs has no cut vertices
 - a. C_n , $n \geq 3$
 - b. W_n , $n \geq 3$
 - c. $K_{m,n}$, $m \geq 2$, and $n \geq 2$
 - d. Q_n , $n \geq 2$

6.

In an old puzzle attributed to Alcuin of York (735–804), a farmer needs to carry a wolf, a goat, and a cabbage across a river. The farmer only has a small boat, which can carry the farmer and only one object (an animal or a vegetable). He can cross the river repeatedly. However, if the farmer is on the other shore, the wolf will eat the goat, and, similarly, the goat will eat the cabbage. We can describe each state by listing what is on each shore. For example, we can use the pair (FG, WC) for the state where the farmer and goat are on the first shore and the wolf and cabbage are on the other shore. [The symbol \emptyset is used when nothing is on a shore, so that $(FWGC, \emptyset)$ is the initial state.]

- a) Find all allowable states of the puzzle, where neither the wolf and the goat nor the goat and the cabbage are left on the same shore without the farmer.
- b) Construct a graph such that each vertex of this graph represents an allowable state and the vertices representing two allowable states are connected by an edge if it is possible to move from one state to the other using one trip of the boat.
- c) Explain why finding a path from the vertex representing $(FWGC, \emptyset)$ to the vertex representing $(\emptyset, FWGC)$ solves the puzzle.
- d) Find two different solutions of the puzzle, each using seven crossings.
- e) Suppose that the farmer must pay a toll of one dollar whenever he crosses the river with an animal. Which solution of the puzzle should the farmer use to pay the least total toll?