

1. Test for convergence the series

a. $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$

b. $1 + \frac{1}{2^2} + \frac{2^2}{3^2} + \frac{3^2}{4^2} + \dots$

c. $\sum_{n=1}^{\infty} \sqrt{\frac{3^n - 1}{2^n + 1}}$

d. $\sum_{n=0}^{\infty} \frac{(\ln n)^2}{n^{\frac{3}{2}}}$

e. $\sum_{n=1}^{\infty} \frac{n! 3^n}{n^n}$

f. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2 + 1}} x^n$

2. Find the coefficient of x^{12} in the power series of each of these functions.

a. $\frac{1}{(1+2x)^4}$

b. $\frac{1}{(1+x)^8}$

c. $\frac{x^3}{(1+2x)^2}$

3. Solve by using generating function

a. $a_n = 3a_{n-1} + 4^{n-1}, a_0 = 1.$

b. $a_n = 3a_{n-1} + 2a_{n-2} + 2^n, a_0 = 4, a_1 = 12$

4. If n is a positive integer, then

a. $\left(-\frac{1}{2}\right)_n = \frac{1}{(-4)^n} \binom{2n}{n}.$

b. Show that the coefficient of x^n in the expansion of $(1 - 4x)^{-\frac{1}{2}}$ is $\binom{2n}{n}$ for all non-negative integer n .

5. Use division algorithm to establish the following

a. The square of any integer is of the form $3k$ or $3k+1$

b. The cube of any integer has one of the forms $9k, 9k+1$, or $9k+8$.

c. The fourth power of any integer is either of the form $5k$ or $5k+1$.

6. For an arbitrary integer a , verify the following:

a. $2|a(a+1)$, and $3|a(a+1)(a+2)$

b. $3|a(2a^2 + 7)$

c. If a is odd, then $32|(a^2 + 3)(a^2 + 7)$.

7. Use the Euclidean algorithm to obtain integers x and y satisfying the following

a. $\gcd(143, 227) = 143x + 227y$

b. $\gcd(306, 657) = 306x + 657y$

8. Find the integer solutions of the following

a. $56x + 5y = 48$

b. $221x + 35y = 11$

9. A theater charges Rs. 180 for adult admission and Rs. 75 for children. On the particular evening the total receipts were Rs. 9000. Assuming that more adults than children were present, how many people attended?

10. A customer brought a dozen pieces of fruit, apple and oranges, for Rs 132. If an apple costs 3 cents more than an orange and more apples than oranges were purchased, how many pieces of each kind were brought.