joined by an edge, the graph cannot be bipartite.

12.3 SUBGRAPHS

It may so happen that sometimes we may be interested only in that specific part of a large computer network which involves the computer centres. In the graph model, we can obtain that part by removing the vertices corresponding to the computer centre other than of those of our interest and also removing all the edges incident with the removed vertices. Thus, we obtain a smaller graph. Such a graph is known as a subgraph of the original graph.

Definition 12.16 Subgraphs

- A graph $H = (V_1, E_1)$ is called a subgraph of $G = (V, E)$ if $V_1 \subseteq V$ and $E_1 \subseteq E$.
- A graph $H = (V_1, E_1)$ is called a proper subgraph of $G = (V, E)$ if $V_1 \subset V$ and $E_1 \subset E$.
- H is called a spanning subgraph of G if $V_1 = V$.

Note 12.4 If H is a subgraph of G, then

- All the vertices of H are in G
- All the edges of H are in G
- Each edge of H has the same end points in H as in G.

Note 12.5 A spanning subgraph of G need not contain all the edges in G.

Example 12.18 Give some examples of subgraphs.

Solution Consider the graph G given in Fig. 12.31. Figure 12.32 shows the subgraphs of G.

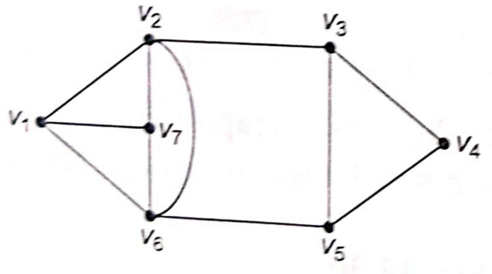

Figure 12.31 Graph G of Example 12.18

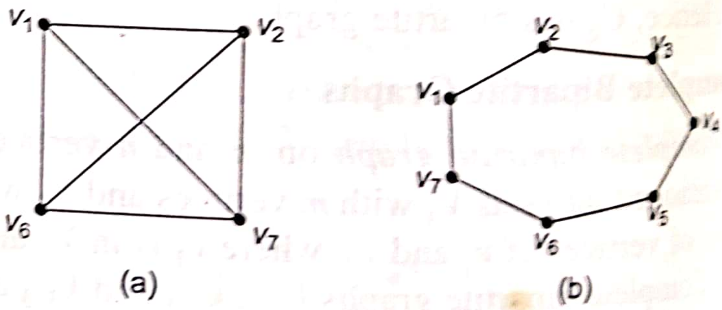

Figure 12.32 Subgraphs of G

Example 12.19 Consider the graph $G = (V, E)$ given in Fig. 12.34. Determine whether $H(V_1, E_1)$ is a subgraph of G or not, where

(i) $V_1 = \{v_1, v_2, v_4\}$; $E_1 = \{(v_1, v_2), (v_1, v_4)\}$

(ii) $V_1 = \{v_1, v_2, v_3, v_4\}$; $E_1 = \{(v_2, v_3), (v_2, v_4)\}$

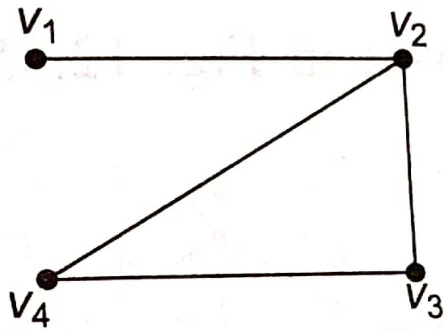

Figure 12.33 Graph G of Example 12.19

Solution (i) $H = (V_1, E_1)$ is not a subgraph of G since (v_1, v_4) is not an edge in G.

(ii) $H = (V_1, E_1)$ is a subgraph of G.

Note 12.6 Any subgraph of a graph G can be obtained by removing certain vertices and edges from G.