Image Processing and Computer Vision

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Histogram Processing

The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function of the form

$$\mathbf{H(rk)=nk}$$

where rk is the kth gray level and nk is the number of pixels in the image having the level rk..

A normalized histogram is given by the equation

$$p(rk)=nk/n \text{ for } k=0,1,2,\ldots,L-1$$

P(rk) gives the estimate of the probability of occurrence of gray level rk.

The sum of all components of a normalized histogram is equal to 1.

The histogram plots are simple plots of H(rk)=nk versus rk.

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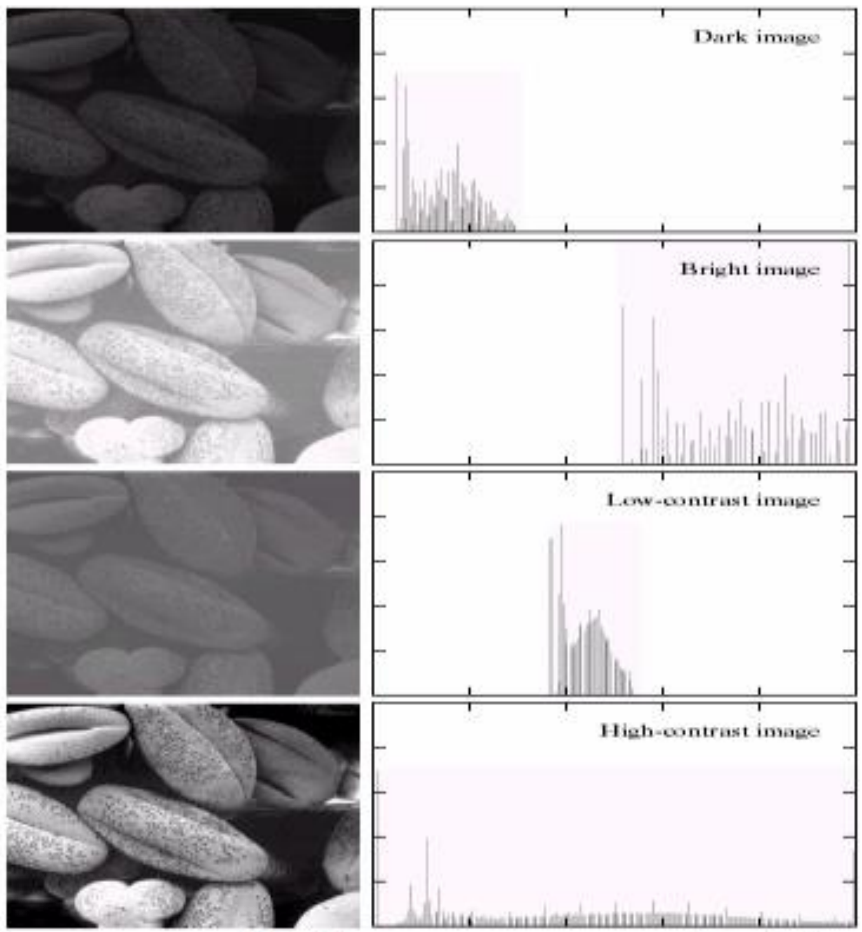
Histogram Processing



In the dark image the components of the histogram are concentrated on the low (dark) side of the gray scale.

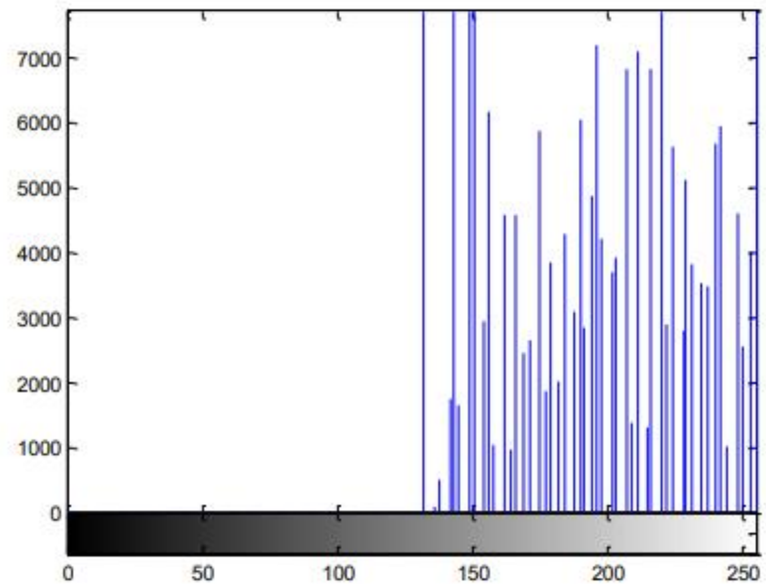
In case of bright image the histogram components are baised towards the high side of the gray scale.

The histogram of a low contrast image will be narrow and will be centered towards the middle of the gray scale.

The components of the histogram in the high contrast image cover a broad range of the gray scale.

The net effect of this will be an image that shows a great deal of gray levels details and has high dynamic range.

Histogram Processing



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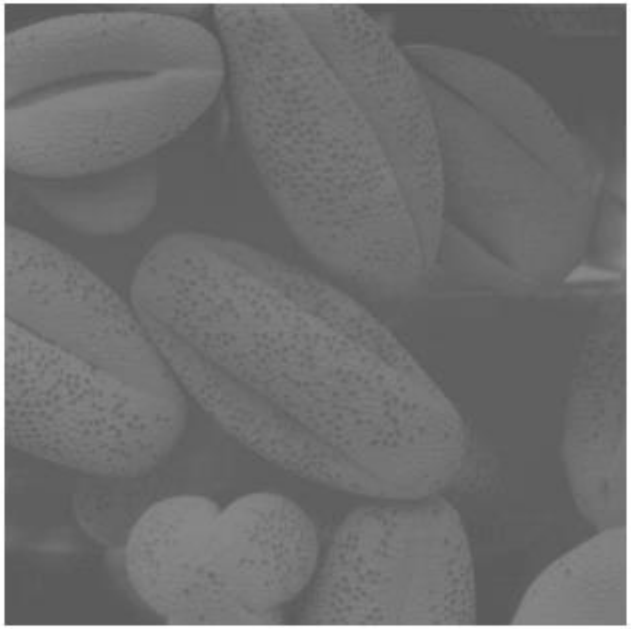
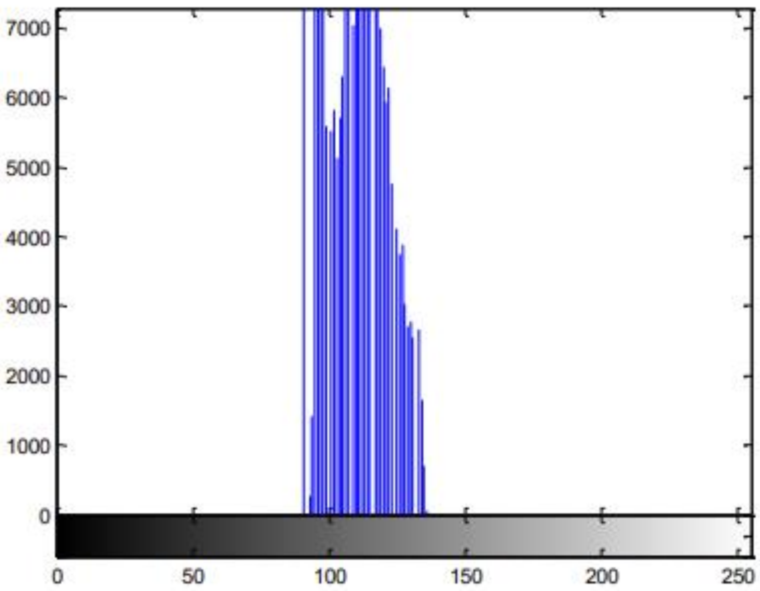
Histogram Processing





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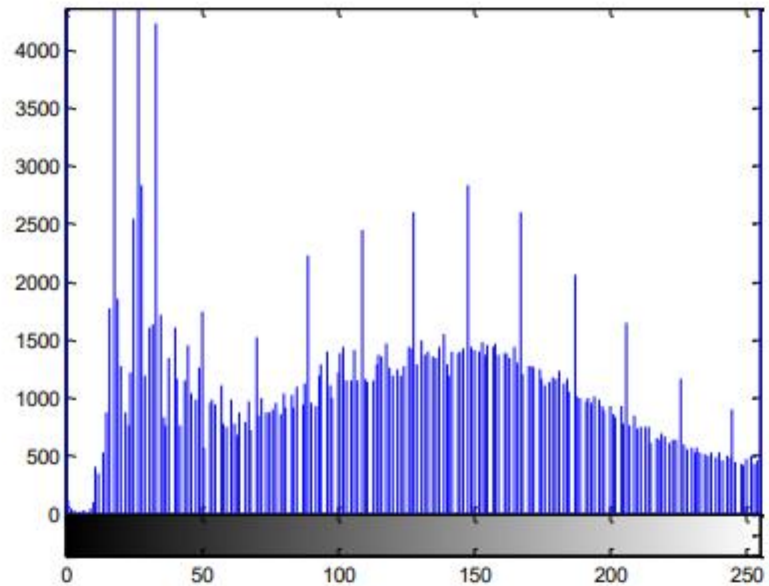
Histogram Processing

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Histogram Equalization

Let us denote r [0 L-1] as intensities of the image to be processed

r=0 corresponding to black and r=L-1 representing white.

Let the intensity transformation is defined by s=T(r) , where $0 \le r \le L-1$

- T(r) is monotonically increasing function in the interval $0 \le r \le L-1$
- $0 \le T(r) \le L-1$ and $0 \le r \le L-1$

Suppose we use the inverse operation as $r=T^{-1}(s)$, then the condition should be strictly monotonically increasing.

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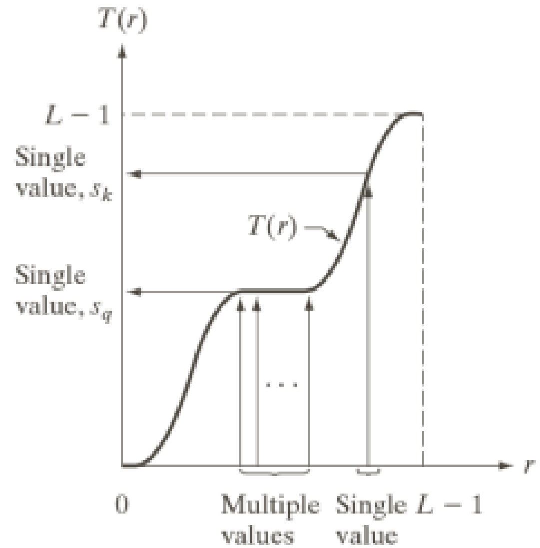
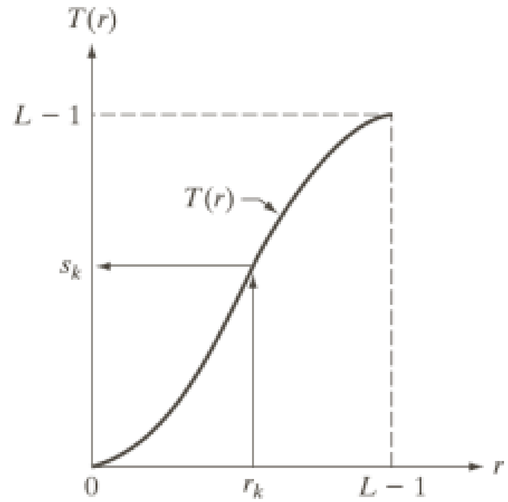
Histogram Equalization





Satisfies the condition T(r) is monotonically increasing function in the interval 0 ≤ r ≤ L-1 and 0 ≤ T(r) ≤ L-1 and 0 ≤ r ≤ L-1

Strictly monotonically increasing

Mapping is one to one in both the directions.

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Histogram Equalization

- Let us consider intensity levels in the image as random variables in the interval 0 to L-1.
- Let us defined the Probability Density Function (PDF) as $p_r(r)$ and $p_s(s)$ for r and s respectively.

- If $p_r(r)$ and T(r) is known, where T(r) is continuous and differentiable over the PDF range , then

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

- The transformation function is of the form

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Cumulative Distribution Function (CDF) of random variable r

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Histogram Equalization

- The transformation function of this form satisfies both the conditions we have seen.

Now let us compute $p_s(s)$, we know $s=T(r)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$= (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right]$$

$$= (L-1)p_r(r)$$

Substituting this for $p_s(s)$, we get

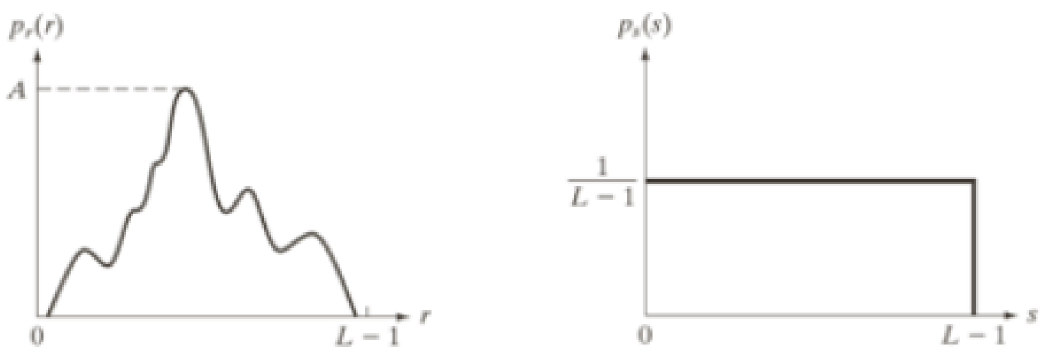
$$p_s(s) = p_r(r)\frac{dr}{ds}$$

$$= p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right|$$

$$= \frac{1}{L-1} \quad 0 \le s \le L-1$$

Histogram Equalization

- Which is a uniform probability density function, this means , performing intensity transformation yields a random variable s characterized by uniform PDF.
- It can be noted that T(r) depends on $p_r(r)$ but ps(s) is always uniform and independently of the form of $p_r(r)$.



Histogram Equalization(Example)

Suppose intensity values in an image have the PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & for\ 0 \le r \le (L-1) \\ 0 & otherwise \end{cases}$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{(L-1)}\int_0^r w dw = \frac{r^2}{(L-1)}$$

Suppose L=9 and pixel at location say (x,y) has the value $r=3$, then

$s=T(r)= r^2/9=1$

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Histogram Equalization(Example)

The PDF of the intensities in the new image is

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\frac{(L-1)}{2r}\right| = \frac{1}{L-1}$$

Assume r is positive and L>1

Result is uniform PDF

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Histogram Equalization

- For the discrete values of the histogram , we deal with summation instead of integration

$$p(r_k) = \frac{n_k}{MN} \quad k = 0,1,2,......L-1$$

The discrete form of transformation is given by

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \quad k = 0,1,2,......L-1$$

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Histogram Equalization

- The input pixel r_k is mapped to output pixel s_k
- The transformation (mapping) T(r_k) is called as histogram equalization or histogram linearization.

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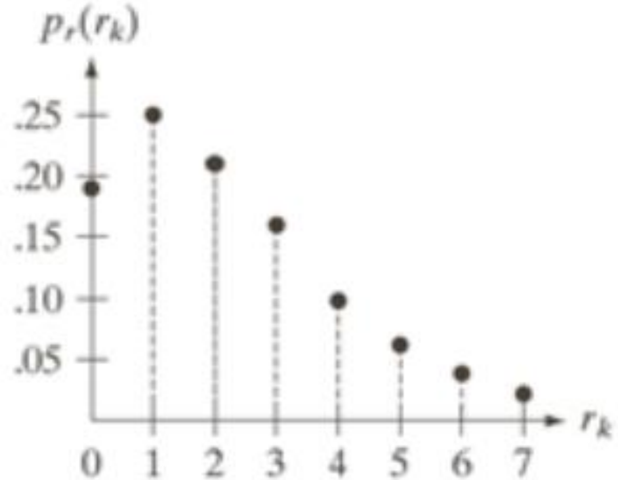
Histogram Equalization(Example)

Let us consider a 3 bit image (L=8) of 64 x 64 (MN=4096), has the intensity distribution shown below.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



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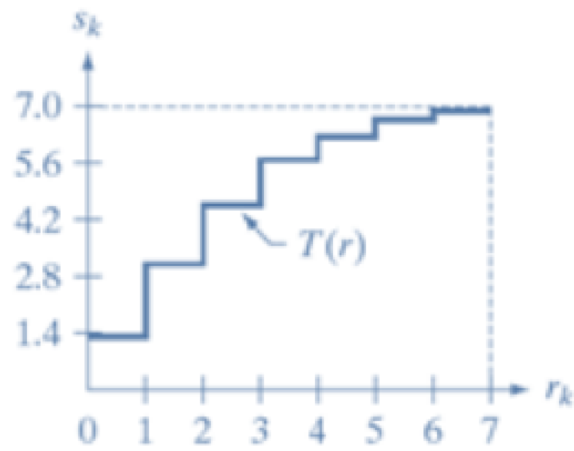
Histogram Equalization(Example)

From the equation of histogram equalization , we have

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1)$$



Similarly compute $s_2, s_3, s_4, s_5, s_6, s_7$

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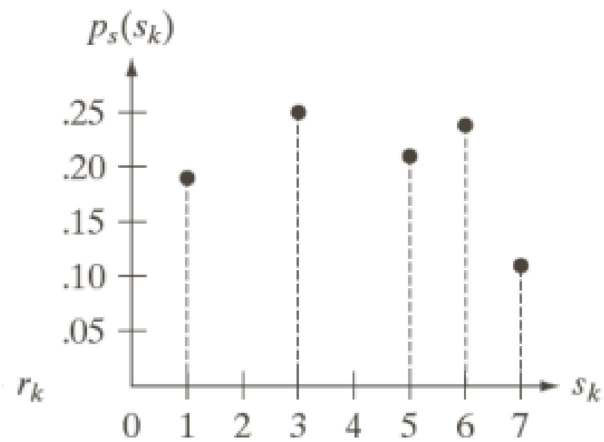
Histogram Equalization(Example)

s_0	1.33	1
s_1	3.08	3
s_2	4.55	5
s_3	5.67	6
s_4	6.23	6
s_5	6.65	7
s_6	6.86	7
s_7	7.00	7



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Histogram matching (Specification)

- Histogram equalization is an automatic enhancement.
- Some times shape of the histogram can be specified based on the requirement.
- The method used to generate a processed image that has a specified histogram is called histogram matching or histogram specification

$$p(r_k) = \frac{n_k}{MN} \quad k = 0,1,2,\ldots\ldots L-1$$

The discrete form of transformation is given by

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \quad k = 0,1,2,\ldots\ldots L-1$$

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Histogram matching (Specification)

Let $p_z(z)$ is the specified PDF, which is going to be the PDF of the output image. So we have

$$G(z_q) = (L-1)\sum_{j=0}^{q} p_z(z_i) = s_k$$

Desired value $z_q = G^{-1}(s_k)$

This will give value of z for each value of s, by performing mapping of s to z

Let us understand it by an example

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Histogram matching (Specification)

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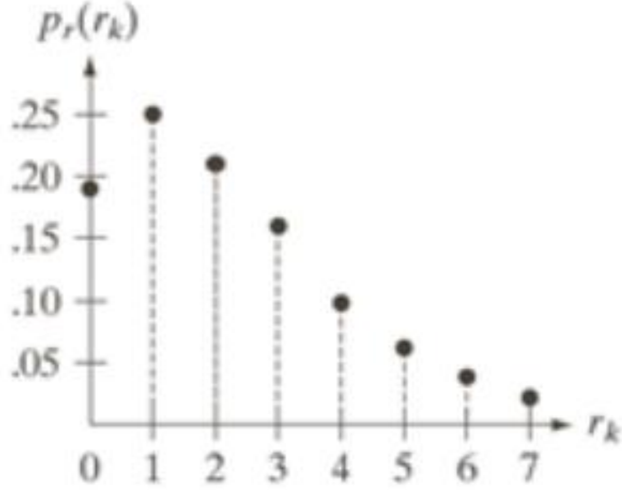

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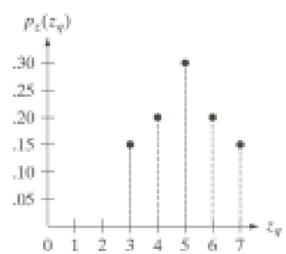
Histogram matching (Specification)

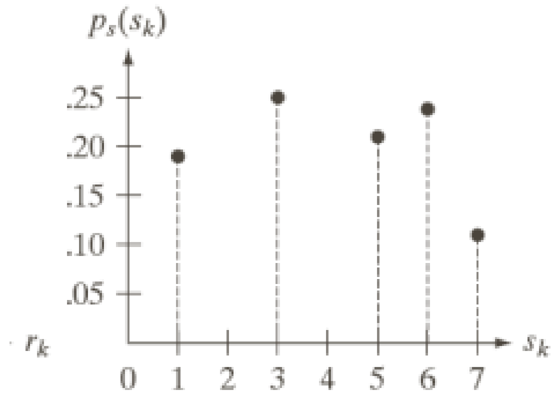
Specified histogram is given as follows

z_q	$P_z(z_q)$
$Z_0=0$	0.00
$Z_1=1$	0.00
$Z_2=2$	0.00
$Z_3=3$	0.15
$Z_4=4$	0.20
$Z_5=5$	0.30
$Z_6=6$	0.20
$Z_7=7$	0.15



STEP 1 : Scaled histogram-equalized values

s_0	1.33	1
s_1	3.08	3
s_2	4.55	5
s_3	5.67	6
s_4	6.23	6
s_5	6.65	7
s_6	6.86	7
s_7	7.00	7



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Histogram matching (Specification)

STEP 2 : Compute all the values of transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j)$$

$$G(z_1) = 7\sum_{j=0}^{0} p_z(z_j) = 7[p(z_0) + p(z_1)]$$

$$G(z_2) = 0.00 \quad G(z_3) = 1.05 \quad G(z_4) = 2.45 \quad G(z_5) = 4.55$$

$$G(z_6) = 5.95 \quad G(z_7) = 7.00$$

These fractional values are converted to integer values as shown

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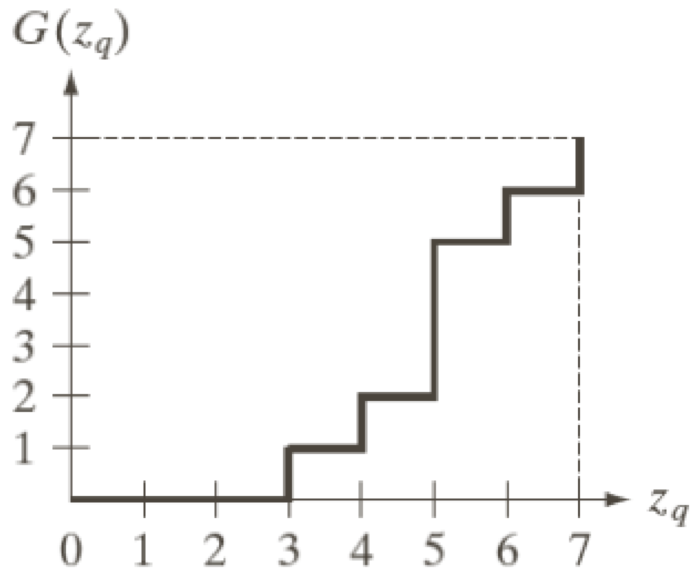
Histogram matching (Specification)

$G(z_0)$	0.00	0
$G(z_1)$	0.00	0
$G(z_2)$	0.00	0
$G(z_3)$	1.05	1
$G(z_4)$	2.45	2
$G(z_5)$	4.55	5
$G(z_6)$	5.95	6
$G(z_7)$	7.00	7



- The condition of strictly monotonic is violated

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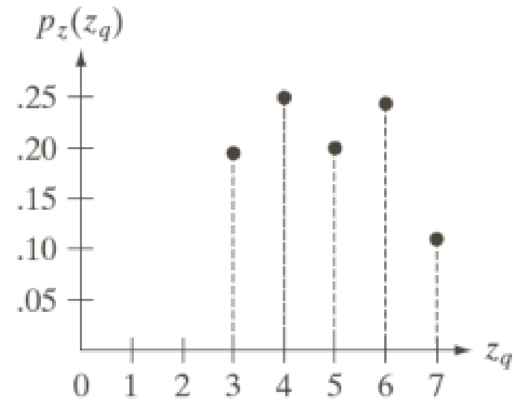
Histogram matching (Specification)

To handle this situation following procedure is used

Find the smallest value of z_q so that the value $G(z_q)$ is closest to s_k.

For example $s_0=1$, and $G(z_3)=1$, which is a perfect match for this case, here $s_0 \rightarrow z_3$,i. e every pixel whose value is 1 in the histogram equalized image is mapped to pixel valued 3 in the histogram specified image. Continuing this we get,

s_k	z_q
1	3
3	4
5	5
6	6
7	7



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