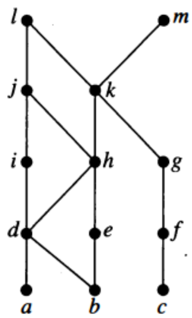
Discrete Mathematics (ES1030)
TUTORIAL 3 & 4

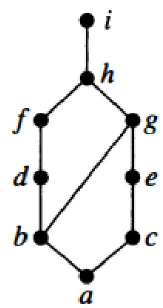
Q. 1	**Attempt the following**
1)	List the ordered pairs in the relation R from A ={0, 1 , 2, 3 , 4} to B = {0, 1 , 2, 3 } , where (a , b) E R if and only if a) a = b. b) a > b. c) gcd(a , b) = 1 . d) a + b = 4. e) a l b f) lcm(a , b) = 2. Also write the relation matrix of each relation.
2)	For each of these relations on the set {1 , 2, 3 , 4}, decide whether it is reflexive, whether it is symmetric, whether asymmetric, it is antisymmetric, and whether it is transitive. a) {(2, 2), (2, 3), (2, 4), (3 , 2), (3 , 3), (3 , 4)} b) {(1, 1), (1 , 2), (2, 1), (2, 2), (3 , 3), (4, 4)} c) {(2, 4), (4, 2)} d) {(1 , 2), (2, 3), (3 , 4)} e) {(1 , 1), (2, 2), (3 , 3), (4, 4)} f) {(1 , 3), (1 , 4), (2, 3), (2, 4), (3 , 1), (3 , 4)} Draw the diagraph. Also write the relation matrix
3)	Determine whether the relation R on the set of all people is reflexive, symmetric, asymmetric, antisymmetric, and/or transitive, only if where (a , b) E R if and only if a) a is taller than b. b) a and b were born on the same day. c) a has the same first name as b. d) a and b have a common grandparent.
4)	Determine whether the relation R on the set of all Web pages is reflexive, symmetric, anti symmetric, asymmetric, and/or transitive, where (a , b) E R if and only if a) everyone who has visited Web page a has also visited Web page b. b) there are no common links found on both Web page a and Web page b. c) there is at least one common link on Web page a and Web page b. d) there is a Web page that includes links to both Web page a and Web page b.
5)	Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if a) $x \neq y$ b) $xy \geq 1$ c) $x = y+1$ *or* $x = y-1$ d) $x \equiv y (\bmod 7)$ e) x is a multiple of y f) x and y are both negative or both nonnegative. h) $x \geq y^2$
6)	Let $R = \{(1,2),(2,3),(3,4)\}$ and $S = \{(1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)\}$ be relations from $\{1,2,3\}$ to $\{1,2,3,4\}$. Find a) $R \cup S$ b) $R \cap S$ c) $R - S$ d) $S - R$ e) $R \oplus S$ f) $R \circ S$ g) $S \circ R$ h) $\overline{R}$ i) $\overline{S}$
Q. 2	List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order). **a)** $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
Q. 3	Draw the Hasse diagram for divisibility on the set a) {1, 2, 3, 4, 5, 6} b) {3, 5, 7, 11, 13, 16, 17}. c) {2, 3, 5, 10, 11, 15, 25}. d) {1, 3, 9, 27, 81, 243}. Which of the poset is a lattice?
Q.4	Answer the following questions for the poset ({3, 5, 9, IS, 24, 45}, I). a) Find the maximal elements. b) Find the minimal elements. c) Is there a greatest element? d) Is there a least element? e) Find all upper bounds of {3, 5}. f) Find the least upper bound of {3, 5}, if it exists. g) Find all lower bounds of {15, 45}.

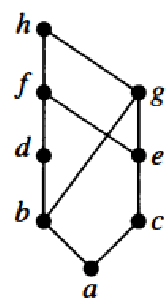
h) Find the greatest lower bound of {15, 45}, if it exists.
g) State Chains and Antichains

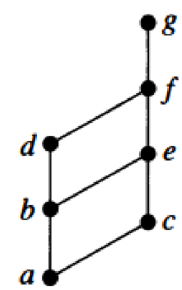
Q.5 Answer the following questions for the poset
({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, $\subseteq$).
a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {{2}, {4}}.
f) Find the least upper bound of {{2}, {4}}, if it exists.
g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exist
g) State Chains and Antichains

Q.6 Answer the following for the partial order represented by this Hasse diagram.
a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {a , b, c}.
f) Find the least upper bound of {a , b, c}, if it exists.
g) Find all lower bounds of {j, g, h}.
h) Find the greatest lower bound of {f, g, h}, if it exists.
g) State Chains and Antichains









Which of these Poset is a lattice?

Q.7 How many non-zero entries does the matrix representing the relation R on $A=\{1,2,\cdots,100\}$ consisting of the first 100 positive integers

i) $\{(a,b)\ /\ a>b\}$
ii) $\{(a,b)\ /\ a\neq b\}$
iii) $\{(a,b)\ /\ a=b+1\}$
iv) $\{(a,b)\ /\ a=1\}$
v) $\{(a,b)\ /\ ab=1\}$

Q.8 Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.
a) {(a, b) | a and b are the same age}
b) {(a, b) | a and b have the same parents}
c) {(a, b) | a and b share a common parent}
d) {(a, b) | a and b have met}
e) {(a, b) | a and b speak a common language}

Q.9 Which of these relations on the set of all functions from Z to Z are equivalence relations? Determine the properties of an equivalence relation that the others lack.
a) $\{(f,g) | f(1)=g(1)\}$
b) $\{(f,g) | f(0)=g(0)\ or\ \ f(1)=g(1)\}$

	c) $\{(f,g) \mid f(1)-g(1)=1 \forall x \in Z\}$ d) $\{(f,g) \mid \text{for some } C \in Z, \forall x \in Z, f(x)-g(x)=C\}$ e) $\{(f,g) \mid f(0)=g(1) \text{ and } f(1)=f(0)\}$
Q.10	Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs *(x, y)* such that $f(x)=f(y)$ a) Show that R is an equivalence relation on A. b) What are the equivalence classes of R?
Q.11	Show that the relation R consisting of all pairs *(x , y)* such that *x* and *y* are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more. What are the equivalence classes of these bit strings for the above equivalence relation? a) 010 b) 101 1 c) 11111 d) 01010101
Q.12	Which of these collections of subsets are partitions of the set of integers? a) the set of even integers and the set of odd integers b) the set of positive integers and the set of negative integers c) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3 d) the set of integers less than - 1 00, the set of integers with absolute value not exceeding 1 00, and the set of integers greater than 1 00 e) the set of integers not divisible by 3, the set of even integers, and the set of integers that leave a remainder of 3 when divided by 6
Q.13	Which of these are partitions of the set $Z \times Z$ of ordered pairs of integers? a) the set of pairs (x, y), where x *or* y is odd; the set of pairs (x, y), where x is even; and the set of pairs (x, y), where y is even b) the set of pairs (x, y), where both x *and* y are odd; the set of pairs (x, y), where exactly one of x *and* y is odd; and the set of pairs (x, y), where both x *and* y are even c) the set of pairs (x, y), where x is positive; the set of pairs (x, y), where y is positive; and the set of pairs (x, y), where both x *and* y are negative d) the set of pairs (x, y), where $3 \mid x$ and $3 \mid y$; the set of pairs (x, y), where $3 \mid x$ and 3 does not divide y ; the set of pairs (x, y), where 3 does not divide x and $3 \mid y$ and the set of pairs (x, y), where 3 does not divide both x *and* y . e) the set of pairs (x, y), where $x>0$ and $y>0$; the set of pairs (x, y), where $x>0$ and $y \le 0$; the set of pairs (x, y), where $x \le 0$ and $y>0$; and the set of pairs (x, y), where $x \le 0$ and $y \le 0$. f) the set of pairs (x, y), where $x \ne 0$ and $y \ne 0$; the set of pairs (x, y), where $x=0$ and $y \ne 0$; and the set of pairs (x , y), where $x \ne 0$ and $y=0$.
Q.14	State the equivalence relations produced by these partitions of {a , b, c, d, e, f, g}. a) {a , b}, {c, d}, {e, f, g} b) {a}, {b}, {c, d}, {e, f}' {g} c) {a, b, c, d}, {e, f, g} d) {a , c, e, g}, {b, d}, {f}
Q.15	Determine whether these posets are lattices. a) ({ 1, 3, 6, 9, 12}, |) b) ({1, 5, 2S, 125}, |) c) $(Z, \ge)$ d) $(\wp(S), \supseteq)$, where $\wp(S)$ is the power set of a set S

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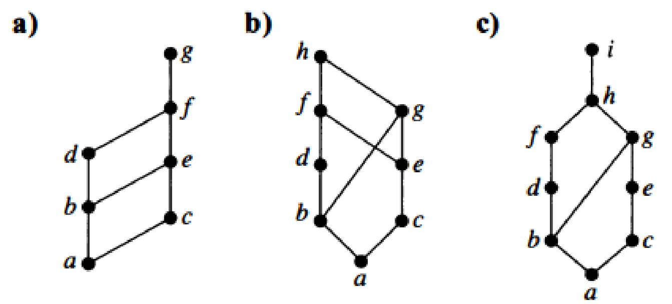
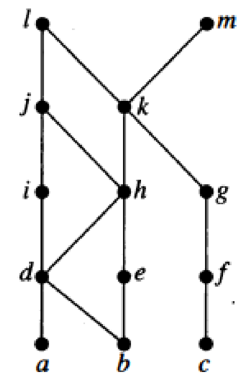
Q.16 Determine whether the posets with these Hasse diagrams are lattices.



Q.17 Answer these questions for the partial order represented by this Hasse diagram.

a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {a , b, c}.
f) Find the least upper bound of {a , b, c}, if it exists.
g) Find all lower bounds of {j, g, h}.
h) Find the greatest lower bound of {f, g, h}, if it exists.
i) State Chains and Antichains



Q.18 Find the lexicographic ordering of these strings of lower case English letters:
a) quack, quick, quicksilver, quicksand, quacking
b) open, opener, opera, operand, opened
c) zoo, zero, zoom, zoology, zoological

Q.19 Find the lexicographic ordering of the bit strings 0, 01, 11, 001, 010, 011 , 0001, and 0101 based on the ordering 0 < 1.

Q.20 Schedule the tasks represented in the following figures