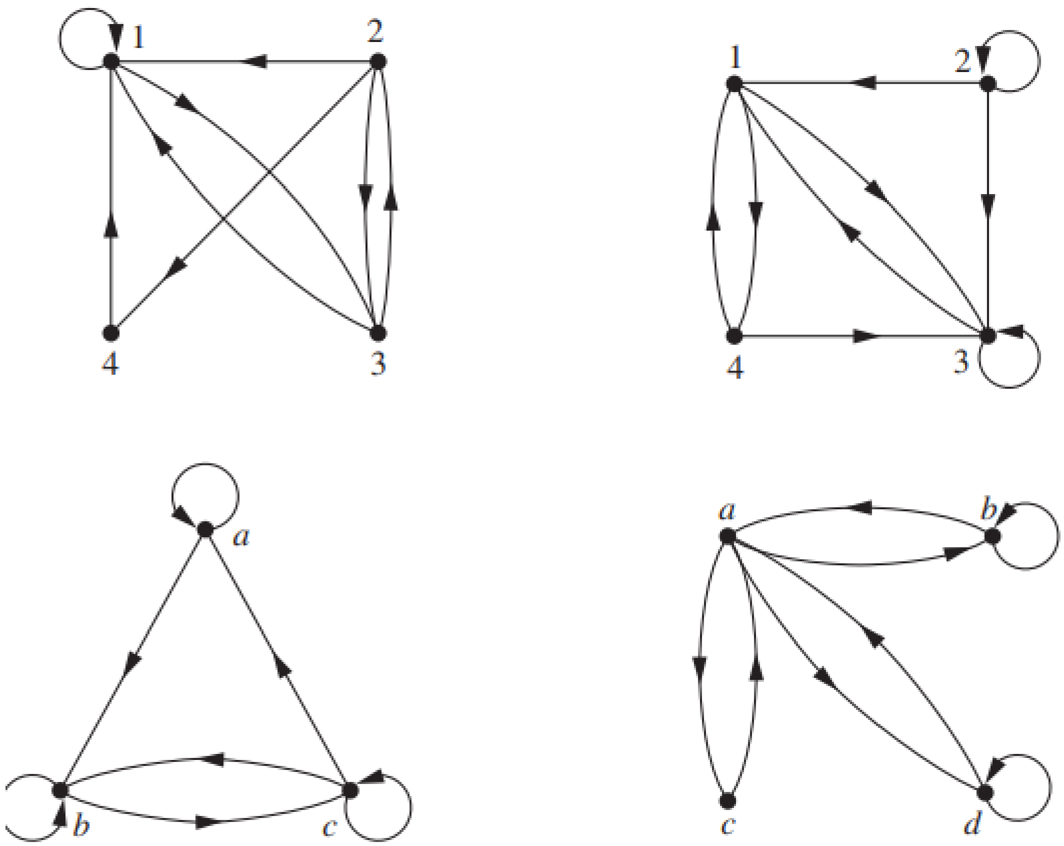
1. Let R be the relation on the set $A = \{0,1,2,3\}$ and the relation on A is $R = \{(0,1),(1,1),(1,2),(2,0),(2,2),(3,0)\}$. Find the
   a. Reflexive closure of R
   b. Symmetric closure of R
   c. Transitive closure of R
2. Determine whether the directed graphs shown in the figure are reflexive symmetric, antisymmetric, and/or transitive



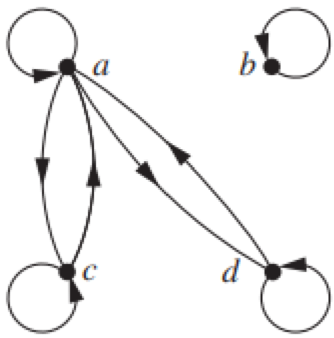
3. On $\mathbb{R}^2$, define a binary relation as follows

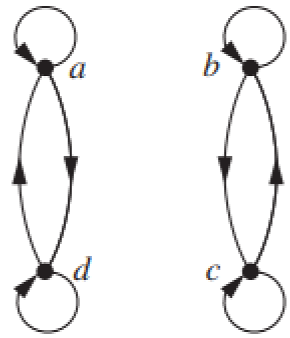
$$R = \{((a,b),(c,d)) \mid a^2 + b^2 = c^2 + d^2\}$$

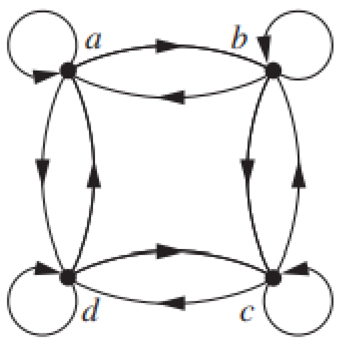
   Prove that R is an equivalence relation. Find equivalence classes of R.

4. Determine whether the relations represented by these matrices are equivalence relations.

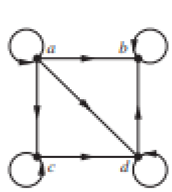
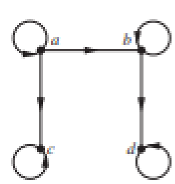
   a. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b. $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. Determine whether the relations represented by directed graphs are equivalence relations.
   a.


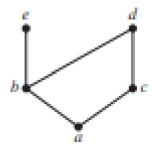
   b.


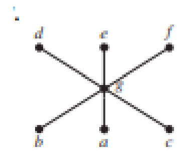
   c.


Partial Order Relation

6. Which of these relations on $\{0,1,2,3\}$ are partial order relations? Determine the properties of a partial ordering that the others lack.
   a. {(0,0), (1,1),(1,2),(1,3),(2,2),(2,3),(3,3)}
   b. {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)}
7. Determine whether the relations represented by these zero one matrices are partial orders.
   a. $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$
8. Determine whether the relation with the directed graph shown in figure is a partial order.
   a.

   b.

9. Draw the Hasse diagram for the divisibility on the set
   a. {1,2,3,4,6,7,8}
   b. {1,2,3,6,12,24,36,48}

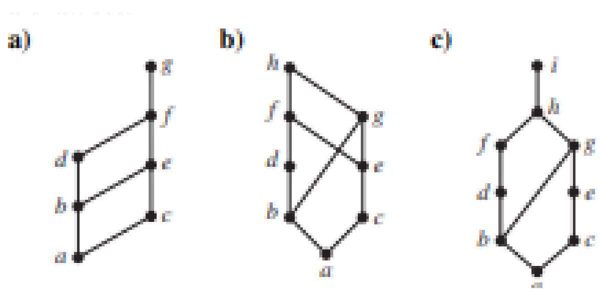
10. List all order pairs in the partial ordering with the accompanying Hasse diagram

a. 

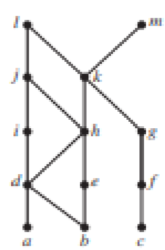
b. 

11. Answer the following questions for the poset $\{\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\},\subseteq\}$
    a. Find the maximal elements
    b. Find the minimal elements
    c. Is there a greatest element?
    d. Is there a least element?
    e. Find all the upper bounds of $\{\{2\},\{4\}\}$.
    f. Find the least upper bounds of $\{\{2\},\{4\}\}$.
    g. Find all lower bound of $\{\{1,3,4\},\{2,3,4\}\}$.
    h. Find greatest lower bound of $\{\{1,3,4\},\{2,3,4\}\}$.

12. Determine whether the posets with the Hasse diagram are lattices.



13. Answer the questions for the partial order represented by this Hasse Diagram.



    a. Find the maximal elements
    b. Find the minimal elements
    c. Is there a greatest element?
    d. Is there a least element?
    e. Find all the upper bounds of $\{a, b, c\}$.
    f. Find the least upper bound of $\{a, b, c\}$. If it exists.
    g. Find all lower bound of $\{f, g, h\}$.
    h. Find greatest lower bound of $\{f, g, h\}$. If it exists.

14. Determine whether these posets are lattices.
    a. $\{1,3,6,9,12,\ |\}$
    b. $\{Z,\ \geq\}$

c. $\{P(S),\ \supseteq\}$, where P(S) is a power set of Set S.