Reg.No. | | | | | |

Bansilal Ramnath Agarwal Charitable Trust's
VISHWAKARMA INSTITUTE OF TECHNOLOGY, PUNE – 411037.
(An Autonomous Institute Affiliated to Savitribai Phule Pune University)

Examination: ESE

Year: SY **Branch:** IT

Subject: Probability and Calculus **Subject Code:** IT2273

Max. Marks: 100 **Total Pages of Question Paper: 2**

Day & Date: Friday, 24/11/23 **Time:** 10:30 am to 1:30 pm

Instructions to Candidate

1. All questions are compulsory.
2. Neat diagrams must be drawn wherever necessary.
3. Figures to the right indicate full marks.

<table>
<tr><th>Q.N.</th><th>CO No.</th><th>BT* No.</th><th></th><th>Max marks</th></tr>
<tr><td>Q. 1. A)</td><td>1</td><td>2</td><td>What are the types of probabilities? Explain any two with examples.</td><td>08</td></tr>
<tr><td>Q. 1. B)</td><td>1</td><td>3</td><td>In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. Apply multiplication theorem of probability for calculating probabilities that it was manufactured by machines A, B and C?</td><td>08</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td></tr>
<tr><td>Q. 1. B)</td><td>1</td><td>3</td><td>The contents of urns I, II and 11/ are as follows:
1 white, 2 black and 3 red balls
2 white, 1 black and 1 red balls
4 white, 5 black and 3 red balls
One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II or III?</td><td>08</td></tr>
<tr><td>Q. 2. A)</td><td>2</td><td>5</td><td>For discrete random variable X evaluate the following :
<table><tr><td>Value</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>Probability</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2+k$</td></tr></table>a) Find k
b) Mean of discrete random variable X
c) Standard Deviation of discrete random variable X</td><td>09</td></tr>
<tr><td>Q. 2. B)</td><td>2</td><td>4</td><td>The independent continuous random variables X and Y have the probability density function given by:
$f(x) = 8ax, \quad 0 \le x \le 1$
$= 0, \quad \text{otherwise}$
$f(y) = 8by, \quad 0 \le y \le 1$
$= 0, \quad \text{otherwise}$
Prove that mathematical expectation E (XY) =E(X)*E(Y).</td><td>08</td></tr>
<tr><td>Q. 3. A)</td><td>3</td><td>5</td><td>Let X be a discrete random variable with the following PMF (Probability Mass Function)</td><td>08</td></tr>
</table>

			$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$ a) Find R_X, the range of the random variable X b) Find $P(x \leq 0.5)$ c) Find $P(0.25 < x < 0.75)$ d) Find $P(x = 0.2 \mid x < 0.6)$	
Q. 3. B)	3	5	Given that 4% of items produced by a firm are defective. Find the probability with Poisson distribution that a box containing 300 items has: a) At least 1 defective items b) At least 2 defective items c) 3 or more defective items	09
			OR	
Q. 3. B)	3	5	If a basket containing balls which are defective with a Beta distribution of $\alpha=5$ and $\beta=2$. Compute the probability of defective balls in the basket from 20% to 30%.	09
Q. 4. A)	4	2	Explain the following with example: a) Uniform distribution b) Log-normal distribution c) Beta distribution d) Gamma distribution	08
Q. 4. B)	4	5	Joint probability of two continuous random variables X and Y is as follows: $f(x) = 4xy, \quad 0 \le x \le 1, 0 \le y \le 1$ $=0, \quad$ otherwise a) Prove that these variables has valid PDF(Probability Density Function) b) Find the marginal probabilities c) $p(x<0.5, y<0.5)$	09
Q. 5. A)	5	5	Calculate the partial derivative of the following function: $u=\log(x^3+y^3+z^3-3xyz)$	09
Q. 5. B)	5	5	Find the maxima and minima of the function: $f(x, y)=x^3+y^3+x^2+4y^2+6$	08
Q. 6. A)	6	5	Calculate the fourier series of for the periodic function: $f(x)=x^2, \quad -\pi < x < +\pi$	08
Q. 6. B)	6	5	Calculate the fourier series of for the periodic function by n interval of length 2 method: $f(x)=x^3, \quad -1 < x < 1$	08

CO Statements:

CO1: Understand basics of probability and Bayes rule.
CO2: Solve problems related to random variables and mathematical expectation.
CO3: Apply discrete and continuous probability distributions in analyzing the probability models arising in engineering field.
CO4: Understand and analyze various probability densities.
CO5: Apply partial differentiation for two or more variables.
CO6: Identify Fourier concepts and techniques to provide mathematical models of real world situations.

***Blooms Taxonomy (BT) Level No:**

- Remembering; 2. Understanding; 3. Applying; 4. Analyzing; 5. Evaluating; 6. Creating