What are derivatives (Abteilungen)?

Let’s imagine we have a mountain like in figure XXa and you want to known the slope of that mountain. One option is to take the height of the summit and divided it by the length of its base (see figue XXb). This gives you the overal denivelé that you will have achieved once you have reached the summit. However, if you are climbing up that mountain and you want to mentally prepare for the ascention and modulate your efforts, you would want a more precise measure. Indeed, if you compare the three green points in figure XXc, you see that their slopes are different from the overall slope. You can divide your mountain into two subsets, like in figure XXd, which gives you a slope already closer from the slope you would experience but not quite that precise. You can further continue subseting into smaller and smaller interval getting a more and more accurate slope for the position you are interested in. If the mountain can be described by a function, then the function that describe the slope in all points of our mountain function is the derivative. It is the slope of the tangeant on that point.

Let us now look at the axes of figure XX and say that the mountain can be described by a function f(x)=y. The slope is given by the height over the base so we need to find the length of the base and of the height. The length of the base is dx: dx=(x+dx)-x. And the length of the height is given by the difference between yx of position x and yx+dx of position x+dx. Remember that y in any position is given by the the function y=f(x), so

dy=f(x+dx)-f(x)

We are interested in the slope so height divided by base: . EqXX is the slope between the two points in figure XX. But what we would like, is to make the difference between the two values of x as small as possible to increase the precision of our slope in our point of interest. In other words, we want dx to tend towards zero. Thus we are interested in the limit of eqXX when dx tends towards zero:

Equation XX is the formal definition of a derivative.

Let’s look at an example: f(x)=x2,

By distributing the square function:

By simplification you get:

But when dx tends towards zero you get:

=2x

We do not demonstrate this here but it can extend to:

xn’=nxn-1

product rule (Produktregel)

(fg)’ = f’ g + fg’

Chain rule (Kettenregel)

Integrals

An integral is the area under the curve.

Integrals are the reverse operation of derivatives. Let’s look back at our mountain in chapter XX, figure XX. Assume we want to take down that mountain and use the rocks to build house. How much building material will be have? Well, this is the area under the mountain curve. A shape from which it is easy to find the area is rectangle; the area of a rectalangle is side x side.To approximate the area under the mountain curve, we can split it into small rectangular subsets like in figure XX. However, you can see that not all the area under the curve is covered and that some part of the area of the rectangles are dépace the line of the mountain curve. To increase precision, we can make smaller bins like in figure XX.