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Linking modern coexistence theory and contemporary niche theory

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Abstract

Modern coexistence theory and contemporary niche theory represent parallel frameworks for understanding the niche's role in species coexistence. Despite increasing prominence and shared goals, their compatibility and complementarity have received little attention. This paucity of overlap not only presents an obstacle to newcomers to the field, but it also precludes further conceptual advances at their interface. Here, we present a synthetic treatment of the two frameworks. We review their main concepts and explore their theoretical and empirical relationship, focusing on how the resource supply ratio, impact niche, and requirement niche of contemporary niche theory translate into the stabilizing and equalizing processes of modern coexistence theory. We show for a general consumer-resource model that varying resource supply ratios reflects an equalizing process; varying impact niche overlap reflects a stabilizing process; and varying requirement niche overlap may be both stabilizing and equalizing, but has no qualitative effect on coexistence. These generalisations provide mechanistic insight into modern coexistence theory, while also clarifying the role of contemporary niche theory's impacts and requirements in mediating coexistence. From an empirical perspective, we recommend a hierarchical approach, in which quantification of the strength of stabilizing mechanisms is used to guide more focused investigation into the underlying niche factors determining species coexistence. Future research that considers alternative assumptions, including different forms of species interaction, spatio-temporal heterogeneity, and priority effects, would facilitate a more complete synthesis of the two frameworks.

Keywords: mechanistic models, coexistence, competition, consumer-resource, fit-

25 ness differences, niche overlap, impact niche, requirement niche, Lotka-Volterra

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Introduction

The niche concept has an unstable history in community ecology. As recently as the turn of this century, Hubbell's unified neutral theory (Hubbell, 2001) came close to rendering the concept and its overburdened assortment of models, mechanisms, and processes largely obsolete. Yet over the past 10-15 years the niche has enjoyed a return to vogue. This is thanks in no small part to the independent development of two distinct frameworks for identifying the niche's role in species coexistence. The first of these, referred to here and elsewhere as "contemporary niche theory", has its origins in the mechanistic consumer-resource models pioneered by MacArthur (1970), popularised by Tilman (1982), and extended by Chase and Leibold (2003), among others. Following a similar nomenclature, the second, "modern coexistence theory" (Chesson, 2000b, Adler et al., 2007, HilleRis-Lambers et al., 2012), is more closely allied to the phenomenological Lotka-Volterra models familiar to all students of ecology. The emergence of these two frameworks has revitalised the field, and yet despite shared goals, their compatibility and complementarity has received little attention. Of the 2300+ citations (since 2003) of Chesson's 2000b paper formalizing modern coexistence theory, only ∼180 also cite contemporary niche theory's primary text (Chase and Leibold, 2003), while of the 1250+ citing the latter only ∼200 also cite the former. The paucity of overlap not only presents an obstacle to newcomers to the field seeking an entry point to the study of species coexistence, but it also precludes further conceptual advances taking advantage of the strengths of each. Here we offer a synthesis of contemporary niche theory and modern coexistence theory that strives to do both.

MacArthur's first forays into mechanistic models of competition were motivated

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by a desire to make the ecological processes behind Lotka-Volterra competition dynamics more transparent (MacArthur and Levins, 1964, MacArthur, 1970, 1972). The cost of translating Lotka-Volterra competition coefficients into consumer-resource dynamics was the need to make a series of assumptions about the processes and factors involved (Chesson, 1990). This trade-off between mechanistic precision (e.g. which resources are regulating coexistence?) and phenomenological accuracy (e.g. can they coexist?) has been inherited by the two frameworks discussed here. Inference using contemporary niche theory is limited by the investigators' knowledge of the natural history of system. On the other hand, the mathematically abstracted emergent properties of modern coexistence theory can all but obscure the underlying ecology. As yet, modern coexistence theory lacks an intuitive conceptual framework for translating the traits and life-history of interacting species into the niche overlap and fitness difference terms (see below) used to quantify coexistence. The few empirical studies to tackle this problem have largely documented weak or complex relationships between interacting species' niche and fitness differences and their evolutionary relatedness or functional similarity (Narwani et al., 2013, Godoy et al., 2014, Kraft et al., 2015, Germain et al., 2016).

A complementary pathway to the demystification of modern coexistence theory is to explore its relationship with the mechanistic framework of contemporary niche theory, which can provide a more explicit connection between species traits and the outcomes of competitive interactions. In an early paper foreshadowing the development of modern coexistence theory, Chesson (1990) showed how his niche overlap and fitness difference terms could be quantified directly from the pa-

rameters of MacArthur's consumer-resource model. More recently, Kleinhesselink and Adler (2015) have shown how Chesson's niche overlap could be derived from Tilman's (1982) consumer resource model. Nevertheless, to our knowledge, there has been no previous work explicitly examining how the criteria for coexistence under contemporary niche theory translate into both the niche overlap and fitness difference terms of modern coexistence theory.

A synthetic treatment of the two frameworks necessarily begins with an introduction to their fundamentals. In the interest of space, we have kept this section concise, and refer readers to several earlier articles and books for a more nuanced unpacking of their respective origins and development (e.g., contemporary niche theory: Tilman, 1982, Grover, 1997, Chase and Leibold, 2003; modern coexistence theory: Chesson, 2000b, Adler et al., 2007, HilleRisLambers et al., 2012, Chesson, 2013). This section is followed by an analytical exploration of the relationship between the two frameworks, complemented by analyses using empirical data from the literature. In the penultimate section, we discuss the relative merits of each framework in an empirical context. Finally, we conclude by highlighting some future directions and opportunities.

Modern coexistence theory and the niche

Under modern coexistence theory, two different classes of mechanisms mediate coexistence: equalizing mechanisms that reduce the average fitness difference between species; and stabilizing mechanisms that reduce niche overlap. Although equalizing and stabilizing mechanisms can theoretically be defined for any underlying model, here we show how they can be understood using a phenomenological

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97   Lotka-Volterra model. Under Lotka-Volterra, the complexity of organismal inter-

98   actions is reduced to summary parameters (competition coefficients) that capture

99   intra- and inter-specific effects on per capita growth rate. When Lotka-Volterra

100   competition is parameterised for two species in terms of absolute competition co-

101   efficients (as opposed to carrying capacities and relative competition coefficients),

102   the per capita growth rate of each species can be represented as:

$$\frac{1}{N_i}\frac{dN_i}{dt} = r_i(1 - \alpha_{ii}N_i - \alpha_{ij}N_j), \tag{1}$$

103   where r_i is the maximum per capita growth of species i, and α_{ij} represents the

104   impact of species j on the per capita growth rate of species i. Thus α_{ii} and α_{ij} rep-

105   resent the absolute intra- and inter-specific competition coefficients, respectively.

106   Two species can coexist when $\alpha_{11} > \alpha_{21}$ and $\alpha_{22} > \alpha_{12}$. This pair of inequalities,

107   sometimes referred to as the mutual invasibility criterion, means that for stable

108   coexistence each species must reduce its own growth more than it does that of its

109   competitor.

110   Chesson's key insight was that the mutual invasibility criterion can be defined

111   in terms that quantify the degree of niche overlap, ρ, between species and their

112   differences in average fitness, f_2/f_1 (Chesson, 2000b). Specifically, the ratio of

113   inter-specific to intra-specific competition coefficients is equal to the product of a

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114 niche overlap term and a fitness ratio term,

$$\frac{\alpha_{12}}{\alpha_{22}} = \frac{f_2}{f_1}\rho. \tag{2}$$

115 In Chesson's framework, niche overlap, ρ, is a measure of the relative strength of
116 intra- to inter-specific density dependence, where differentiation in resource use
117 (or shared predators in the case of apparent competition) leads to low values of
118 ρ. When $\rho = 0$ for a pair of species, they share none of the same resources, or
119 use them completely independently in space and time, and therefore impose no
120 density dependent feedbacks on each other (Chesson, 2000b, 2008). In contrast,
121 when species' resource use overlaps completely and each resource is of the same
122 relative value to each species, $\rho = 1$. This definition removes the overall level of
123 adaptedness to the environment from niche comparisons. Instead, this is captured
124 by the fitness ratio, f_2/f_1, which predicts the winner in competition under the
125 hypothetical scenario of complete niche overlap. Note the fitness ratio is also
126 commonly written as k_2/k_1 but we use lower case f to avoid confusion with Lotka-
127 Volterra carrying capacity, K.

128 From the mutual invasibility criterion, we know that the right hand side of the
129 equation above must be less than 1 for species 1 to be able to invade a population
130 of species 2 at equilibrium. This is the same as saying $f_2/f_1 < 1/\rho$. By the same
131 logic, for species 2 to be able to invade a population of species 1, $f_1/f_2 < 1/\rho$.
132 Therefore, satisfaction of the mutual invasibility criterion, i.e. stable coexistence,

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₁₃₃ requires:

$$\rho < \frac{f_2}{f_1} < \frac{1}{\rho}. \tag{3}$$

₁₃₄ As ρ goes to 1, the potential for two species to coexist is contingent on increasingly
₁₃₅ smaller fitness differences between them. Thus we can define an equalizing mech-
₁₃₆ anism as any process that reduces the average fitness difference between species;
₁₃₇ and a stabilizing mechanism as any process that reduces niche overlap and there-
₁₃₈ fore increases negative frequency dependence (Fig 1a). Stabilizing mechanisms are
₁₃₉ what cause a species at high density to buffer its own growth more than that of
₁₄₀ a competitor, while also allowing it to recover from low density due to low inter-
₁₄₁ specific competition. An example of a stabilizing mechanism would be coexisting
₁₄₂ plant species acquiring resources from different depths, assuming that the supply
₁₄₃ of resources is independent (Levine and HilleRisLambers, 2009). By contrast, an
₁₄₄ example of an equalizing mechanism would be a reduction in fitness differences
₁₄₅ due to an otherwise competitively dominant species experiencing higher density-
₁₄₆ independent rates of predation or infection by pathogens (Mordecai, 2011). Most
₁₄₇ of the relevant research on the relative strength of stabilizing and equalizing mech-
₁₄₈ anisms has focused on grassland plants as an empirical system (e.g. Levine and
₁₄₉ HilleRisLambers, 2009, Angert et al., 2009, Adler et al., 2010, Godoy et al., 2014,
₁₅₀ Germain et al., 2016), but work in other systems has also begun to emerge, in-
₁₅₁ cluding arthropods (Siepielski et al., 2011), green algae (Narwani et al., 2013) and
₁₅₂ bacteria (Zhao et al., 2015).

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Coexistence and contemporary niche theory

In their 2003 book, Chase and Leibold (2003) introduced contemporary niche theory as a synthesis of historically incompatible niche definitions. This they achieved through a conceptual extension of the graphical approach to the analysis of consumer-resource models popularised by Tilman (1982). Unlike phenomenological Lotka-Volterra models, consumer-resource models provide an explicit basis for coexistence. Tilman focused largely on the dynamics of consumer-resource systems, but Chase and Leibold (2003) showed how the graphical approach could be extended beyond consumer-resource dynamics to incorporate a broad spectrum of biotic and abiotic interactions. Thus, in addition to explaining the coexistence of consumers competing for limiting resources (e.g. algae competing for basic nutrients [Tilman, 1977, 1982] or invertebrates competing for algae [Rothhaupt, 1988]), contemporary niche theory can be invoked to explain community dynamics arising from a range of alternative niche factors (e.g. the response of marine bacteria to different stressors [Materna *et al.*, 2012], or the sensitivity of plants to the relative abundance of different pollinators, [Pauw 2013]).

Chase and Leibold (2003) redefined the consumption vectors of Tilman as the impact niche, encompassing not only the impact of consumers on resources, but also their potential impact on other factors such as the density of predators, pathogens or toxic chemicals (see also Grover, 1997). The impact niche can then be thought of as representing a species' Eltonian niche (Elton, 1927), i.e., the impact a species has on its environment. Similarly, Chase and Leibold (2003) suggested that the zero net growth isoclines (ZNGIs), which delineate the range of conditions in which a species maintains a positive growth rate, need not be defined only with respect

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¹⁷⁷ to consumable resources, but could be represented by, for example, the density of

¹⁷⁸ predators or the frequency of disturbance. Thus the ZNGIs can be considered akin

¹⁷⁹ to the Grinellian or Hutchinsonian view of the niche (Grinnell, 1917, Hutchinson,

¹⁸⁰ 1957), i.e., the biotic and abiotic requirements of a species, which Chase and Lei-

¹⁸¹ bold (2003) refer to as the requirement niche, following their earlier work (Leibold,

¹⁸² 1995).

¹⁸³ Under the mechanistic framework of contemporary niche theory, the conditions for

¹⁸⁴ local coexistence depend on three criteria (Fig 1b). Notwithstanding the broad

¹⁸⁵ scope of the framework, for heuristic purposes, we focus on the simplest and most

¹⁸⁶ well studied scenario: two species competing for two limiting resources in a spa-

¹⁸⁷ tially and temporally homogenous environment. First, their ZNGIs must intersect,

¹⁸⁸ the ecological implications of which is that each species is a better competitor for

¹⁸⁹ a different resource. Second, each species must have a relatively greater impact on

¹⁹⁰ the resource it finds most limiting. Third, the supply ratio of the two resources

¹⁹¹ must not disproportionately favour one species over the other. More precisely,

¹⁹² the supply ratio must be intermediate to the inverse of the impact vectors (the

¹⁹³ consumption vectors in a consumer-resource model).

¹⁹⁴ The second of these criteria is perhaps the most familiar in that it is the ba-

¹⁹⁵ sis for negative frequency dependent growth and therefore critical to coexistence.

¹⁹⁶ However, it is also the most easily misunderstood. A source of confusion is the

¹⁹⁷ conventional reference to the 'most limiting' resource, which although transparent

¹⁹⁸ in the case of essential resources (e.g., the inorganic nutrients obtained separately

¹⁹⁹ by plants [N, P, K etc.]), is less clear with respect to substitutable resources (e.g.

²⁰⁰ the more complex food forms consumed by animals). If we define the minimum

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₂₀₁ resource value a species needs to maintain a positive growth rate as its R^* (*sensu*
₂₀₂ Tilman, 1982), then for essential resources, the most limiting resource is the one
₂₀₃ for which a species has the higher R^*. However, if two resources are perfectly
₂₀₄ substitutable and a consumer can compensate for the complete absence of one
₂₀₅ resource by the consumption of another, then neither resource is physiologically
₂₀₆ limiting in the way we think of for plants consuming essential resources. So what
₂₀₇ is meant by most limiting? As best articulated by León and Tumpson (1975), we
₂₀₈ suggest that an intuitive way to think about this criterion of coexistence is that
₂₀₉ each competitor must have a greater impact on the resource for which a reduc-
₂₁₀ tion would have a greater impact on its growth. For substitutable resources it
₂₁₁ corresponds to the resource for which a species has the *lower* R^*. Note this runs
₂₁₂ counter to Fig 2.8 of Chase and Leibold (2003, see also discussion on p. 34), but
₂₁₃ is consistent with Leibold (1998).

Disentangling definitions of niche overlap

₂₁₅ Clearly the niche is central to both frameworks, but there are important subtle
₂₁₆ differences. Under contemporary niche theory, the impact niche and requirement
₂₁₇ niche can be understood as species-specific properties that are comparable across
₂₁₈ species. With respect to the impact niche, the cosine of the angle between the
₂₁₉ two impact vectors in Tilman's consumer-resource models is equivalent to Pianka's
₂₂₀ (1973) measure of niche overlap (Petraitis, 1989). With respect to the requirement
₂₂₁ niche, an explicit measure of niche overlap has yet to be defined, but a heuristic
₂₂₂ two dimensional definition that we adopt here is the cosine of the angle between
₂₂₃ the ZNGIs for substitutable resources, and the cosine of the angle between two

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224 lines from the origin to the corner of each ZNGI for essential resources.

225 Under modern coexistence theory, the niche can only be understood in light of
226 species interactions. It cannot be quantified for an individual species in isolation
227 from other species. Instead, the niche is a property of pairwise or one-to-many
228 comparisons between species, and therefore is only defined in terms of niche over-
229 lap or niche differences $(1 - \rho)$. Further, it is concerned solely with those aspects
230 of species' ecology that generate negative frequency dependence. The exclusion of
231 overall fitness from this niche definition may seem contrary to intuitive species-
232 specific interpretations of the niche (e.g., in species distribution modelling); for
233 instance, two species within a guild that are adapted to high temperature environ-
234 ments will not necessarily exhibit niche overlap in Chesson's framework. Never-
235 theless, if their fitness equivalence derives from a common adaptation to a limiting
236 resource, they will also exhibit high niche overlap. For example, oligotrophically
237 adapted plants may be expected to exhibit high niche overlap for the very same
238 reason they have high fitness in nutrient poor environments. Chesson's niche over-
239 lap is not equivalent to either the impact or requirement niche of contemporary
240 niche theory. However, they are still related, as we now show.

241 Translating impacts and requirements into stabilizing and
242 equalizing processes

243 Our starting point for a synthesis of the two frameworks is the translation of mech-
244 anistic consumer-resource models into a Lotka-Volterra form, following Tilman
245 (1982). This translation allows for the derivation of absolute competition coef-

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²⁴⁶ ficients in terms of consumer-resource parameters. We then quantify Chesson's

²⁴⁷ niche overlap and fitness ratio following Chesson and Kuang (2008) and Chesson

²⁴⁸ (2013). The analytical derivation is summarized for substitutable resources in Box

²⁴⁹ 1 and for essential resources in Appendix S1. As discussed below, coexistence dy-

²⁵⁰ namics under competition for substitutable resources and competition for essential

²⁵¹ resources are not always equivalent, hence they merit individual consideration. As

²⁵² pointed out by Kleinhesselink and Adler (2015), Tilman's model for substitutable

²⁵³ resources may result in negative resource equilibrium values for some regions of

²⁵⁴ parameter space. To address this issue, we performed all our analysis in parameter

²⁵⁵ regions that only resulted in positive resource equilibrium values (see Appendix

²⁵⁶ S2 for positivity criteria). Although for generality Chesson's equalizing and sta-

²⁵⁷ bilizing mechanisms are typically defined for resident and invader growth rates,

²⁵⁸ given that Tilman's consumer-resource model can be expected to reach a stable

²⁵⁹ equilibrium point, we derive niche overlap and fitness ratios at the equilibrium

²⁶⁰ (see also Kleinhesselink and Adler, 2015). This is also consistent with Chesson's

²⁶¹ (1990) derivations.

²⁶² In the following, we explore how changes to the three critical components of con-

²⁶³ temporary niche theory (resource supply ratio, impact vectors, and ZNGIs) trans-

²⁶⁴ late into Chesson's niche overlap and fitness ratio terms for both substitutable and

²⁶⁵ essential resources. In each case, we seek generalisations on the analytical rela-

²⁶⁶ tionship between the two frameworks. We also highlight those points where differ-

²⁶⁷ ent assumptions cause deviations from the generalisations we derive. Given that

²⁶⁸ modern coexistence theory rarely incorporates alternative stable states (but see

²⁶⁹ Chesson, 2008), as will sometimes arise when inter-specific competition is greater

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270 than intra-specific competition for two (or more) competitors (i.e. $\alpha_{12} > \alpha_{11}$ &

271 $\alpha_{21} > \alpha_{22}$), we constrain our analysis to the regions of parameter space that ex-

272 clude the possibility of alternative stable states (see Appendix S2 for parameter

273 values).

Resource ratios

275 In Figure 2, the ZNGIs and impact vectors are shown for two hypothetical species

276 competing for two substitutable (Fig 2a) or essential resources (Fig 2c). In the

277 case of substitutable resources, three different resource supply ratios result in three

278 qualitatively different equilibrium scenarios. At resource supply point 1, both

279 species have positive population growth in the absence of the other species, but

280 in competition red excludes blue. This is because supply point 1 is dominated by

281 resource 2, for which red is a better competitor (smaller R^*). When translated into

282 Chesson's framework, we see that supply point 1 represents a fitness ratio (solid

283 grey line in Fig 2b that is less than the niche overlap between the two species)

284 that, according to the inequality of modern coexistence theory (i.e. $\rho < f_b/f_r <$

285 $1/\rho$), results in competitive exclusion. Supply point 2 lies between the inverse

286 of the impact vectors in Fig 2a, and therefore gives rise to stable coexistence

287 (assuming each species has a greater impact on the resource from which it most

288 benefits). In the corresponding figure (Fig 2b), supply point 2 also lies in the

289 coexistence region bounded by ρ and $1/\rho$. The two species' fitnesses are not

290 equivalent owing to the resource supply ratio at point 2 favouring blue, but they

291 still coexist due to the strength of stabilization (Fig 2b). Supply point 3 is outside

292 the coexistence region in Fig 2a, with blue excluding red owing to its preference

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293 for resource 1 which is in abundant supply. Similarly in Fig 2b, supply point 3

294 corresponds to a fitness ratio that is now greater than $1/\rho$ and hence does not

295 meet the conditions for coexistence. An empirical illustration of the relationship

296 between the two frameworks for substitutable resources is provided in Box 2 using

297 data from Rothhaupt (1988).

298 In the case of essential resources, we highlight five different resource supply ratios

299 that correspond to five distinct combinations of Chesson's fitness and niche overlap

300 terms. Supply points 2-4 result in niche overlap and fitness ratio scenarios that

301 correspond with supply points 1-3 for substitutable resources: red excludes blue

302 when resource supply is at point 2 ($f_b/f_r < \rho$); blue and red coexist when resource

303 supply is at point 3 ($\rho < f_b/f_r < 1/\rho$); and blue excludes red when resource supply

304 is at point 4 ($f_b/f_r > 1/\rho$). Supply points 1 and 5 have the same competitive

305 outcomes as 2 and 4 respectively, however at these comparatively extreme supply

306 points, both species are limited by the same resources and therefore Chesson's

307 niche overlap term jumps to one. In Box 2, we show that in Tilman's classic

308 experiments on resource competition between algal species, owing to the closeness

309 of the species' ZNGIs, the region in which one species competitively excludes the

310 other even when limited by different resources is very narrow. As such, all but

311 four of Tilman's 13 experimental supply points were in regions where both species

312 are limited by the same resource and thus have Chesson's niche overlap = 1.

313 Given the preceding discussion, can we make any generalisations on the effect of

314 shifting resource supply ratios on fitness and niche differences? The most conspic-

315 uous result is that it affects fitness differences between competing species (irre-

316 spective of whether the resources are substitutable or essential). In terms of the

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³¹⁷ Lotka-Volterra parameters, this effect is reflected in changing intra-specific compe-

³¹⁸ tition coefficients. In contrast, changing the resource supply ratio independently

³¹⁹ of other parameters appears to have no effect on Chesson's niche overlap, except

³²⁰ at the extremes of the resource ratio gradient when species competing for essential

³²¹ resources are no longer limited by different resources and niche overlap jumps to

³²² one. The implication is that for a large region of parameter space, changes in

³²³ the ratio of resource supply can only act as an equalizing mechanism, but when

³²⁴ resources are essential, changes in resource ratio can also be destabilizing. In other

³²⁵ words, the fitness difference between a pair of differentially adapted species will

³²⁶ shift across a heterogenous landscape but their degree of Chesson's niche overlap

³²⁷ will remain constant except when resource ratios become highly skewed (assuming

³²⁸ all else remains equal).

³²⁹ How do these predictions fit with empirical studies on resource ratio manipula-

³³⁰ tions? HilleRisLambers et al. (2012) suggested that studies demonstrating changes

³³¹ in relative abundance following the manipulation of limiting resources are consis-

³³² tent with changes in fitness differences, whilst those demonstrating a loss of diver-

³³³ sity indicate a change in niche overlap. Although we would argue that a loss of

³³⁴ diversity can arise solely from a change in fitness differences, these observations

³³⁵ are broadly consistent with our results. For example, within the shaded coexis-

³³⁶ tence regions in Fig 2b&d, shifts in the fitness ratio are associated with changing

³³⁷ relative abundances of the two competitors, up until the point when the fitness

³³⁸ ratio exceeds niche overlap. Nevertheless, in plants studies, changes to niche over-

³³⁹ lap may also contribute to diversity loss following the manipulation of essential

³⁴⁰ resources (Harpole and Tilman, 2007, Clark et al., 2007, Hautier et al., 2009). For

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³⁴¹ example, Harpole and Tilman (2007) attributed a loss of grassland plant diversity

³⁴² to reduced niche dimensionality following the addition of surplus nitrogen. This

³⁴³ reduction in niche dimension is equivalent to the jump in niche overlap that occurs

³⁴⁴ when species go from being limited by different resources to being limited by the

³⁴⁵ same resource (illustrated in Fig 2d).

³⁴⁶ **Impact niche overlap**

³⁴⁷ Following Pianka (1973), Petraitis (1989), and Chase and Leibold (2003), we define

³⁴⁸ impact niche overlap as the cosine of the angle between the impact vectors, θ

³⁴⁹ (where complete niche overlap $= cosine(0) = 1$). For substitutable resources,

³⁵⁰ Chessons's niche overlap, ρ, has a positive monotonic relationship with impact

³⁵¹ niche overlap (Fig. 5). As species' impacts converge, the stabilizing term in

³⁵² Chesson's framework decreases (i.e. the range of allowed fitness ratios enclosed by

³⁵³ the inequality decreases). In contrast, the fitness ratio, f_b/f_r remains constant.

³⁵⁴ When impact niche overlap corresponds with the angle given by θ_1 in Fig. 5a, the

³⁵⁵ fitness ratio, $\rho < f_b/f_r < 1/\rho$, is compatible with stable coexistence. When impact

³⁵⁶ niche overlap narrows symmetrically to θ_2, blue and red still coexist, but blue is

³⁵⁷ on the verge of excluding red. This transpires when impact niche overlap narrows

³⁵⁸ to θ_3; the supply point now falls outside the coexistence region (Fig. 5a), and

³⁵⁹ $f_b/f_r > 1/\rho$. The only difference for essential resources is that once the impact

³⁶⁰ niche overlap exceeds a threshold, there is a one-time jump in the fitness ratio,

³⁶¹ and Chesson's niche overlap jumps to 1. This occurs when one of the lines of

³⁶² limitation, which parallels the inverse of the associated impact vector, crosses the

³⁶³ resource supply point (θ_3 in Fig. 5c), causing species to become limited by the

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³⁶⁴ same resource.

³⁶⁵ The first generalisation we can make for the impact niche is that an increase in
³⁶⁶ impact niche overlap results in an increase in Chesson's niche overlap. Second,
³⁶⁷ with the exception of the step-function in the fitness ratio that arises for essential
³⁶⁸ resources, changing impact niche overlap has no effect on the fitness difference. As
³⁶⁹ such, it does not reflect an equalizing mechanism. Even in the event of a jump
³⁷⁰ in the fitness ratio following a switch in resource limitation under competition
³⁷¹ for essential resources, the jump happens in conjunction with an instantaneous
³⁷² jump in niche overlap, and thus has no qualitative effect on coexistence. We
³⁷³ note, however, that fitness ratio changes can accompany changes in impact niche
³⁷⁴ overlap when the impact vectors shift asymmetrically (i.e., if one species' impacts
³⁷⁵ are held constant whilst a competitor's change) or the ZNGIs and impact vectors
³⁷⁶ are asymmetric to begin with (Appendix S3, Fig. S1). In these circumstances it is
³⁷⁷ not the degree of impact niche overlap that affects the fitness ratio, but a change
³⁷⁸ in the relationship between the impact vectors and the ZNGIs that favours one
³⁷⁹ species over the other (Appendix S4, Fig. S1).

³⁸⁰ We have shown above that changing impact niche overlap reflects a stabilizing
³⁸¹ mechanism, but the question remains what does this mean biologically? The
³⁸² most obvious impact organisms have on their environment is via their depletion of
³⁸³ resources, which is mediated by their rate of consumption. To change impact niche
³⁸⁴ overlap in Fig. 5, we modified the consumption rate parameters, c_{ij}, that enter
³⁸⁵ into the resource equation, which in Tilman's (1982) model are independent of the
³⁸⁶ resource value parameters, w_{ij}, that enter the consumer equations (see Appendix
³⁸⁷ S1). Nevertheless, it should be expected that consumption rate contributes to some

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degree to resource value, and therefore the ZNGIs (see the *Requirement niche overlap* section below), as it does explicitly in some consumer-resource models (e.g., in the analytical form given by Chase and Leibold, 2003). This suggests that the degree of orthogonality between requirements and impacts depends on the magnitude of the contribution of consumption rate to the ZNGIs, relative to other factors (e.g., assimilation efficiency).

Owing to the broad life history strategies exhibited in the natural world, it is difficult to make any broad conclusions on the likelihood of changing consumption rates having either a large or negligible impact on ZNGIs. Nevertheless, we speculate that the degree of orthogonality between requirements and impacts is greater for species competing for essential resources than those competing for substitutable resources. For optimal foraging, organisms should expend greater effort foraging for the resource that is most beneficial to their growth (Tilman, 1982, Vincent et al., 1996, Chase and Leibold, 2003). For organisms competing for essential resources (e.g., some plants), this resource is the one for which they have the larger R^*. As a result, traits that define the ZNGIs (those that determine resource value, e.g., consumption rate and assimilation efficiency) may cancel each other out. In contrast, for organisms competing for substitutable resources (e.g., some animals), the most beneficial resource is the one for which they have the smaller R^*. In this case, their impacts and requirements will be positively correlated.

The stabilizing effect of diverging impacts is also linked to the concept of character displacement (Brown and Wilson, 1956, Pfennig and Pfennig, 2012). For example, adaptive divergence in beak size in Darwin's finches translates into divergence in consumption rates; small beaked finches spend more time feeding on small seeds

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₄₁₂ while large beaked finches spend more time feeding on large seeds (Grant and

₄₁₃ Grant, 2006). This divergence is stabilizing (not to be confused with stabilizing

₄₁₄ selection in the evolutionary context). However, divergence in consumption rates

₄₁₅ is itself associated with a shift in the value of small vs. large seeds to finches

₄₁₆ with different sized beaks, and thus may be expected to also affect the ZNGIs.

₄₁₇ Whether this translates into a shift in fitness differences depends on the supply

₄₁₈ rate and relative value of the resources being partitioned (i.e. small and large

₄₁₉ seeds) and the degree of asymmetry during divergence. Nevertheless, in order for

₄₂₀ character displacement to evolve, the benefits must outweigh the costs (Pfennig

₄₂₁ and Pfennig, 2012). As such, the shift in niche overlap should be expected to

₄₂₂ exceed any corresponding shift in the fitness ratio.

₄₂₃ We can conclude that when impact niche overlap alone changes, the outcome is

₄₂₄ (de)stabilizing. However, the underlying biological traits that make up a species'

₄₂₅ impacts (e.g., consumption rate) may also affect its requirements. As we show in

₄₂₆ the following section, changing requirements (i.e., ZNGIs) will typically result in

₄₂₇ changes in relative fitness, in which case we can also expect an equalizing compo-

₄₂₈ nent.

₄₂₉ **Requirement niche overlap**

₄₃₀ Due to the different shapes of the ZNGIs under competition for substitutable and

₄₃₁ essential resources, manipulating the degree of requirement niche overlap requires

₄₃₂ slightly different approaches. For substitutable resources we define the cosine of

₄₃₃ the angle, φ, between the ZNGIs as the requirement niche overlap, where complete

₄₃₄ requirement niche overlap $= cosine(0) = 1$. For essential resources we define it as

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435 the cosine of the angle, φ, between two lines from the origin to the corner of each

436 ZNGI (R^*s in monoculture).

437 For substitutable resources, converging ZNGIs has no effect on coexistence (Fig.

438 6a), because the position of the ZNGIs has no bearing on the position of the

439 supply point in relation to the inverse of the impact vectors. Converging ZNGIs

440 does, however, make the species' carrying capacities more equivalent (inverse of

441 the intra-specific competition coefficients), which in turn leads to a decline in

442 the fitness ratio (Fig. 6b). At the same time, convergence of the ZNGIs also

443 results in competing species requiring increasingly more of the resource that its

444 competitor is having a greater impact on. Consequently, Chesson's niche overlap

445 also increases. Nevertheless, this destabilizing effect never occurs to a sufficiently

446 large degree to surpass the equalizing effect of converging ZNGIs. At the extreme

447 when species' ZNGIs are identical, both the fitness ratio and Chesson's niche

448 overlap = 1, highlighting an important distinction between the two frameworks:

449 impact niche overlap can be < 1 when Chesson's niche overlap = 1. In other

450 words, a pair of species can experience zero negative frequency dependence even

451 when their impacts on the environment are different. This apparent paradox arises

452 because any decrease in the availability of resources results in the same reduction in

453 species per capita growth irrespective of species-specific impacts. When expressed

454 in terms of absolute competition coefficients, we see that $\alpha_{11} = \alpha_{21}$ and $\alpha_{22} = \alpha_{12}$,

455 which causes $\rho = f2/f1 = 1$ (see Box 1), i.e, complete neutrality. In contrast,

456 when impact niches are identical, $\alpha_{11} = \alpha_{12}$ and $\alpha_{21} = \alpha_{12}$, which means fitnesses

457 can be different ($f2/f1 \neq 1$) even when $\rho = 1$, and therefore one species always

458 excludes the other.

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Under competition for essential resources, converging ZNGIs do not affect either Chesson's fitness ratio or niche overlap terms (Fig. 6d). The stability of Chesson's niche overlap can be explained analytically by the absence of resource value terms (w_{ij}) in the mechanistic derivations of the competition coefficients for essential resources (Appendix S1, Eqns. 16.1-16.4). This in turn can be understood ecologically as a consequence of a focal species being physiologically limited by one resource at a time, and thus insensitive to a competitor's consumption of a non-limiting resource for which the focal species' requirements are changing. However, when the ZNGIs completely overlap, i.e. $\varphi = 1$, both Chesson's niche overlap and the fitness ratio would jump to 1, as in the case of substitutable resources. As such, in spite of the apparent stability of the fitness ratio and Chesson's niche overlap, the closer the ZNGIs are together the more sensitive the interacting pair will be to other deterministic or stochastic phenomena that tip them one way or the other. The constancy of Chesson's fitness ratio in Fig. 6d is an artefact of maintaining a constant equilibrium point. If the equilibrium point is allowed to change (i.e., by bringing the ZNGIs together along a straight line joining the corners), then the fitness ratio will change as the ZNGI axis relating to the most limiting resource moves with respect to the fixed resource supply point (Appendix S5, Fig. S1).

If we focus on substitutable resources, the observed relationship between requirement niche overlap and Chesson's fitness ratio and niche overlap offers some insight into ongoing debate on the role of limiting similarity in structuring communities. Both frameworks have been invoked as arguments that coexisting species can be more similar than expected by chance. Under modern coexistence theory, there is

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a tension between the stabilizing mechanisms, which are consistent with the notion of limiting similarity, and the equalizing mechanisms, which suggest a simultaneous limiting dissimilarity (Chesson, 2000b, Mayfield and Levine, 2010). Following a similar logic, in their 2003 book, Chase and Leibold argue that coexisting species should be more dissimilar in their impacts but more similar in their requirements (also see Leibold, 1998). Our analysis here, however, reveals that these two lines of reasoning are not wholly compatible. While the convergence of ZNGIs is indeed equalizing, it also has a destabilizing effect, which decreases opportunities for coexistence. To unpack this relationship fully we would want to explore divergence of impacts and convergence of ZNGIs simultaneously. Based on the current work, coexistence may not necessarily be more common between species with similar ZNGIs, but rather between species with similar intercepts on the resource axis that plays a larger role in determining competitive dominance.

We have assumed that requirement niche overlap can change independently of, or at least more rapidly than, impact niche overlap. For this assumption to be true, an organismal trait that affects species needs for a resource (e.g., assimilation efficiency) must change independently, or more rapidly, than traits that affect their impacts on that resource. This decoupling of impacts and requirements may arise due to evolutionary trade-offs and/or environmental change. For example, in a meta-analysis of thermal responses for more than 1,000 microbes, plants and animals, Dell et al. (2011) found that weakly regulated traits, such as basal metabolic rate, were more sensitive to temperature than those under active control, such as consumption (see also Lemoine and Burkepile, 2012). Experimental evidence from individual communities supports this result (e.g., in rocky intertidal invertebrates

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[Iles, 2014], and forest floor spiders and beetles [Rall et al., 2010, Vucic-Pestic et al., 2011]). These studies suggest that increased temperatures can result in requirement niche overlap changing faster than impact niche overlap.

The empiricist's dilemma: which framework?

Having explored the theoretical relationship between the two frameworks we now consider their empirical strengths and weaknesses. In Box 2, we see that in both Tilman's (1977, 1982) and Rothhaupt's (1988) consumer-resource experiments that the outcome of competition was not always consistent with the precise predictions of contemporary niche theory. This inconsistency may have been simply due to stochastic phenomena or alternatively it may have been due to hidden niche factors contributing to the strength of stabilizing or equalizing mechanisms over and above those captured by the two resource axes under investigation. This latter explanation points to an important distinction between the two frameworks as they are applied to empirical tests of coexistence.

With modern coexistence theory, it is not necessary to have *a priori* knowledge of the underlying mechanisms (other than consideration of the spatial and temporal scale at which stabilization could be in effect) (HilleRisLambers et al., 2012). Instead, measurements of invader growth rates of a focal individual grown at varying densities in monoculture and with heterospecifics is sufficient to obtain competition coefficients (Inouye, 2001), which in turn can be used to calculate niche overlap and average fitness differences (Chesson, 2013, Godoy et al., 2014). As such, an empirical strength of modern coexistence theory is that it is possible to test for stable coexistence even when multiple processes (e.g. multiple limiting factors)

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ultimately contribute to the stability of the system. This is to say that modern coexistence theory represents an integrated approach that is agnostic with respect to the particular resource or non-resource factors involved. Its utility in establishing the cumulative strength of niche stabilisation has been demonstrated by several recent empirical studies (e.g. Sears and Chesson, 2007, Levine and HilleRisLambers, 2009, Adler et al., 2010, Godoy et al., 2014).

In contrast, when the emphasis is on delineating the niche factors fostering coexistence, contemporary niche theory offers a preferable approach. The difficulty in adopting a more mechanistic approach is it requires considerable *a priori* knowledge of the niche factors likely to be regulating coexistence in the community under study and how the community members relate to these factors (HilleRisLambers et al., 2012). Robust tests require investigators to quantify impact vectors (e.g., consumption rates), ZNGIs (minimum resource requirements), and resource supply rates along a minimum of two axes. If the factors selected are not those regulating coexistence, the investigator can come to a misleading conclusion. At the same time, notwithstanding the experimental resources required to quantify species' minimum resource requirements, the number of required tests scales additively with community size. This is a potential advantage over modern coexistence theory, for which ideal tests necessitate pairwise comparisons, and therefore scale multiplicatively with community size (but see Carroll et al., 2011, for an approach for multispecies competition). Similarly, once the niche characteristics for a set of species have been characterised, predictions on their dynamics with respect to new species necessitate only characterisation of the new species. Under modern coexistence theory, the introduction of a new species would require testing its dynamics

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554   with respect to all the species under study.

555   To summarize, the choice of frameworks for empirically testing coexistence depends
556   on both the question at hand and the investigators' knowledge of the system. The
557   strength of empirical tests using modern coexistence theory is that the equalizing
558   and stabilizing terms integrate across all resource and non-resource factors con-
559   tributing to coexistence. Modern coexistence theory is a useful framework when
560   i) the primary aim is to test coexistence irrespective of the underlying processes,
561   or ii) little prior information is available on key resources and non-resource factors
562   at play. In contrast, contemporary niche theory offers a more direct approach to
563   disentangling the specific niche factors involved. The risk, however, is falsely ruling
564   out coexistence when the 'wrong' combination of resource/non-resource factors are
565   tested. In Vellend's (2016) terminology, empirical tests using modern coexistence
566   theory represent tests of high-level processes: what is the relative contribution
567   of negative frequency dependence to species coexistence? In contrast, empirical
568   tests using contemporary niche theory represent tests of low-level processes: what
569   are the specific modes of selection that contribute to coexistence, e.g. resource-
570   competition vs. predation vs. disturbance? This hierarchical distinction points
571   to the potential complementarity of the two approaches, which we discuss further
572   below.

573   **Future directions**

574   The broad theoretical appeal of both modern coexistence theory and contempo-
575   rary niche theory belies the dearth of rigorous empirical treatments. The key
576   papers, including Tilman (1982), Chase and Leibold (2003) and Chesson (2000b),

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are cited heavily in the empirical literature but typically only in passing reference to concepts explaining diversity maintenance. Indeed, tests of modern coexistence theory have only recently begun to accumulate (e.g. Adler et al., 2007, Levine and HilleRisLambers, 2009, Angert et al., 2009, Adler et al., 2010, Kraft et al., 2015, Mordecai et al., 2016). Similarly, the most recent comprehensive review of Tilman's resource-ratio theory reported as few as eight robust tests of the effect of consumption rates and supply ratios on coexistence (Miller et al., 2005), and only a small number appear to have been conducted since then (e.g. Passarge et al., 2006). While further empirical tests of coexistence using either framework independently is of undoubted value, we suggest that future investigations would benefit from the adoption of a hierarchical approach. First, establish the strength of niche stabilization, and second, use this information as a yardstick for guiding low-level tests of coexistence under contemporary niche theory. Although this hierarchical approach necessitates a considerable number of inter- and mono-specific growth experiments, it should be tractable in some systems, such as microbial communities where population dynamics are rapid and the experimental units are small (e.g., Fox, 2002, Jiang and Morin, 2007, Violle et al., 2011, Vannette and Fukami, 2014, Tucker and Fukami, 2014). HilleRisLambers et al. (2012) recommended a similar approach, drawing on both experimental manipulations of niche factors and more phenomenological demographic approaches to understand equalizing and stabilizing mechanisms. We agree that adopting complementary approaches is useful, but because the two frameworks focus on two different levels of ecological processes (*sensu* Vellend, 2016), we propose that phenomenological approaches provide a logical starting point to guide more focused mechanistic work.

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⁶⁰¹ In the current work, we constrained ourselves to stable equilibria when examining

⁶⁰² the relationship between the two frameworks, ignoring priority effects and alterna-

⁶⁰³ tive stable states (*sensu* Fukami, 2015). These phenomena are easily understood

⁶⁰⁴ within the graphical framework of contemporary niche theory, but have received

⁶⁰⁵ less explicit attention in the context of modern coexistence theory. Chesson (2008)

⁶⁰⁶ has discussed the potential for negative stabilizing mechanisms, and more recently

⁶⁰⁷ Mordecai (2011) provided a conceptual model for the integration of priority effects

⁶⁰⁸ into Chesson's framework (see also Fukami et al., 2016). The synthetic approach

⁶⁰⁹ we have taken here has the potential to provide further insights. When the direc-

⁶¹⁰ tion of the impact vectors are switched such that each species has a greater impact

⁶¹¹ on the resource that its competitor most benefits from, it can then be shown that

⁶¹² the criteria for alternative stable states in Chesson's framework is the inverse of

⁶¹³ the stable coexistence inequality (Eqn. 3), i.e $\rho > f_2/f_1 > 1/\rho$. In two recent em-

⁶¹⁴ pirical studies quantifying niche overlap and fitness differences, negative values for

⁶¹⁵ ρ (indicating negative stabilization) were truncated to zero because the focus was

⁶¹⁶ on regions of coexistence (Godoy et al., 2014, Germain et al., 2016), but in future

⁶¹⁷ empirical work it would be informative to test explicitly for negative stablisation

⁶¹⁸ and its effects on the outcome of competition.

⁶¹⁹ It would also be valuable to explore the robustness of the above stated generalisa-

⁶²⁰ tions with respect to niche factors other than resources (e.g. predators, pathogens

⁶²¹ or stressors), under spatio-temporal heterogeneity, and under more complex formu-

⁶²² lations of consumer-resource interactions. There is a rich theoretical and empirical

⁶²³ literature incorporating added complexity into both frameworks (Grover, 1997 chs

⁶²⁴ 5 & 7, Chase and Leibold, 2003 chs 5 & 6, Chesson and Kuang, 2008, Chesson,

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625 2013), and several of the most prominent mechanisms under modern coexistence

626 theory rely on variability in resource or non-resource factors (e.g., the storage ef-

627 fect and relative non-linearity of competition) (Chesson, 1994, 2000a,b). To our

628 knowledge, however, little research has examined the consistency of the two frame-

629 works under more complex scenarios. For instance, we showed above that, all else

630 being equal, changing resource supply ratios only affects fitness differences (except

631 at the extreme of the essential resource ratio gradient [Fig 2d]). However, our

632 analysis assumes that species' impacts are constant under changing resource avail-

633 ability, and ignores the potential effect of a third spatially or temporally varying

634 factor (e.g. temperature) on impacts traits, such as consumption rates. If con-

635 sumption rates change across space or time, niche overlap can also be expected

636 to change. A valuable direction for future work would be to investigate the rela-

637 tionship between the two frameworks when the three components of contemporary

638 niche theory are changed simultaneously. Further, by focusing on the local scale,

639 we ignore the potential for spatial heterogeneity to act as a stabilizing mechanism

640 at the regional scale (Adler et al., 2006, Sears and Chesson, 2007, Angert et al.,

641 2009). An additional extension to the current work could include an analysis of

642 how changing resource supply ratios across space (or time) translate into fitness

643 and niche differences when quantified at the regional scale (or across time).

644 Finally, an outstanding question that applies equally to both frameworks is the

645 degree to which the impacts and requirements of contemporary niche theory or

646 the fitness difference and niche overlap of modern coexistence theory are indepen-

647 dent of each other. Both frameworks have their origins in simplified mathematical

648 models of competitive interactions from which the above components can be or-

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thogonally partitioned out, but there have been few empirical studies attempting to quantify the relative contribution of a focal trait to different coexistence components (but see Kraft et al., 2015). We have argued here that weakly correlated impacts and requirements are likely rare, and that a single trait can contribute to both niche overlap and fitness difference (Fig 6b). Nevertheless, many traits may contribute more to one component than the other. For example, Kraft et al. (2015) showed that even though several traits contributed to both fitness differences and niche overlap amongst competing annual plants, single functional traits were strongly correlated with fitness differences. A trait-based ecology will benefit from further efforts to quantify the relative contribution of focal traits to different coexistence components.

Conclusions

By exploring the relationship between the two frameworks, we have sought to make a stronger connection between the ecological attributes of species that cause them to interact and the processes that determine the outcomes of those interactions. We have shown that variation in resource supply rates affects species' fitness differences and therefore reflects an equalizing process; variation in impact niche overlap affects the magnitude of niche overlap and therefore reflects a stabilizing process; and variation in overlap in species' requirements affects fitness differences and niche overlap, and therefore reflects both a stabilizing and an equalizing process. The analysis we have presented here points to the utility of a unified approach, drawing on the strengths of both frameworks, for an improved understanding of species coexistence.

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846 **Supporting Information**

847 Appendices S1-S5

Figure legends

Figure 1: Graphical illustration of the criteria for coexistence under modern coexistence theory (**a**) and contemporary niche theory (**b**). In **a** the lower and upper bounding black lines denote the point where the fitness ratio is equal to niche overlap and the inverse of niche overlap, respectively. Thus, according to the inequality $\rho < f_2/f_1 < 1/\rho$, two species can coexist in the shaded region but exclude each other above or below these bounding lines. The asymmetry in **a** is due to the y-axis being a ratio, and therefore would appear symmetrical on a log scale i.e., contrary to their appearance on the ratio scale, the unshaded regions of parameter space corresponding to exclusion are equal in size for both species. In **b** coexistence of two species competing for two substitutable resources depends on three criteria: intersecting ZNGIs (solid red and blue lines connecting the x- and y-axes); each species having a greater impact on the resource from which it most benefits (impact vectors denoted by the red and blue arrows); and a resource supply ratio that is intermediate to the inverse of the impact vectors (dashed red and blue lines).

Figure 2: Translating changing resource supply ratios in contemporary niche theory (**a** & **c**) into the equalizing and stabilizing terms of modern coexistence theory (**b** & **d**), under pairwise competition for substitutable (**a** & **b**) and essential resources (**c** & **d**). In **a** & **c**, the solid red and blue lines are the ZNGIs for each species; the solid lines with arrow heads are the respective impact vectors; and the dashed lines are the inverse of the impact vectors, defining regions of stable coexistence. The additional dotted lines in **c** denote the regions in which species switch from being limited by different resources (above blue and below red) to

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being limited by the same resource (below blue or above red). In **b** & **d**, the x-axis represents the resource ratio moving along the grey lines in **a** & **c** from top left to bottom right. The y-axis gives the values of the fitness ratio, f_b/f_r (solid grey line), and the degree of niche overlap, ρ (solid black line) and $1/\rho$ (dashed black line). The grey shaded area indicates the coexistence region, where $\rho < f_b/f_r < 1/\rho$. For reference, equal fitness, where $f_b/f_r = 1$, is illustrated by the horizontal dashed grey line. Numbers 1-3 in **a** and 1-5 in **c** correspond to the respective numbers in **b** and **d**.

Figure 3: Illustrating the equivalence of predictions from contemporary niche theory and modern coexistence theory using data from Rothhaupt (1988). In the top panel (**a**) ZNGIs and consumption vectors for *B. rubens* and *B. calyciflorus* are shown in blue and red respectively. The relationship between different supply point ratios (circles) and the inverse of the consumption vectors are shown in the middle panel (**b**), where filled circles indicate supply points where both species are predicted to coexist and empty circles indicate regions where one species is predicted to competitively exclude the other. The corresponding niche overlap, ρ, and fitness ratio, f_{Br}/f_{Bc}, values are shown in the bottom panel (**c**). Solid and dashed black lines indicate ρ and $1/\rho$ respectively; dashed grey line indicates equal fitness; filled circles indicate regions of coexistence where $\rho < f_{Br}/f_{Bc} < 1/\rho$; empty circles indicate regions of competitive exclusion.

Figure 4: Illustrating the equivalence of predictions from contemporary niche theory and modern coexistence theory using data from Tilman (1977, 1982). In

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the top panel (**a**) ZNGIs and consumption vectors for *Asterionella* and *Cyclotella*) are shown in blue and red respectively. The relationship between different supply point ratios and the inverse of the consumption vectors are shown in the middle panel (**b**). Corresponding with Fig 31.A of Tilman (1982), the supply point symbols indicate the outcome of competition experiments where stars denote the exclusion of *Cyclotella* by *Asterionella*, dots denote coexistence, and diamonds denote the exclusion of *Asterionella* by *Cyclotella*. The corresponding niche overlap, ρ, and fitness ratio, f_{Af}/f_{Cm}, values are shown in the bottom panel (**c**). Solid and dashed black lines indicate ρ and $1/\rho$ respectively; dashed grey line indicates equal fitness; shaded area indicates region of coexistence where $\rho < f_{Af}/f_{Cm} < 1/\rho$. To aid visualisation, both axes in (c) have been log-transformed.

Figure 5: Translating changing impact niche overlap in contemporary niche theory (**a** & **c**) into the equalizing and stabilizing terms of modern coexistence theory (**b** & **d**), under pairwise competition for substitutable (**a** & **b**) and essential resources (**c** & **d**). In **a** & **c**, the solid red and blue lines are the ZNGIs for each species; the solid lines with arrow heads are the respective impact vectors; and the dashed lines are the inverse of the impact vectors, defining regions of stable coexistence. The additional dotted lines in **c** denote the regions in which species switch from being limited by different resources (above blue and below red) to being limited by the same resource (below blue or above red). In **b** & **d**, the x-axis represents the impact niche overlap starting in the position given by the bold dashed lines and ending at complete overlap. The y-axis gives the values of the fitness ratio, f_b/f_r (solid grey line), and the degree of niche overlap, ρ (solid black line) and $1/\rho$

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920 (dashed black line). The grey shaded area indicates the coexistence region, where

921 $\rho < f_b/f_r < 1/\rho$. For reference, equal fitness, where $f_b/f_r = 1$, is illustrated by

922 the horizontal dashed grey line. The angles given by θ_{1-3} in **a** and **c** correspond to

923 the respective θ_{1-3} in **b** and **d**. Impact niche overlap is defined here as the cosine

924 of the angle between species impact vectors, $cosine\ \theta = \frac{c_{11}c_{21}+c_{12}c_{22}}{\sqrt{c_{11}^2+c_{12}^2}\times\sqrt{c_{21}^2+c_{22}^2}}$, where

925 (c_{i1}, c_{i2}) is the consumption vector for species i (see Appendix S1 and Box 1 for

926 parameter definition).

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928 **Figure 6**: Translating changing requirement niche overlap in contemporary niche

929 theory (**a** & **c**) into the equalizing and stabilizing terms of modern coexistence

930 theory (**b** & **d**), under pairwise competition for substitutable (**a** & **b**) and essential

931 resources (**c** & **d**). In **a** & **c**, the solid red and blue lines are the ZNGIs for each

932 species; the solid lines with arrow heads are the respective impact vectors; and

933 the dashed lines are the inverse of the impact vectors, defining regions of stable

934 coexistence. The additional dotted lines in **c** denote the regions in which species

935 switch from being limited by different resources (above blue and below red) to

936 being limited by the same resource (below blue or above red). In **b** & **d**, the

937 x-axis represents the requirement niche overlap starting in the position given by

938 the solid bold ZNGIs and ending at complete overlap. The y-axis gives the values

939 of the fitness ratio, f_b/f_r (solid grey line), and the degree of niche overlap, ρ

940 (solid black line) and $1/\rho$ (dashed black line). The grey shaded area indicates the

941 coexistence region, where $\rho < f_b/f_r < 1/\rho$. For reference, equal fitness, where

942 $f_b/f_r = 1$, is illustrated by the horizontal dashed grey line. The angles given

943 by φ_{1-3} in **a** and **c** correspond to the respective φ_{1-3} in **b** and **d**. Requirement

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⁹⁴⁴ niche overlap for substitutable resources is defined here as the cosine of the angle

⁹⁴⁵ between species ZNGIs, $cosine\ \phi = \frac{w_{12}w_{22}+w_{11}w_{21}}{\sqrt{w_{11}^2+w_{12}^2}\times\sqrt{w_{21}^2+w_{22}^2}}$, where $\frac{-w_{i1}}{w_{i2}}$ is the ZNGIs

⁹⁴⁶ slope for species i. Requirement niche overlap for essential resources is defined

⁹⁴⁷ as the cosine of the angle between two lines from the origin to the corner of each

⁹⁴⁸ ZNGI, $cosine\ \phi = \frac{R_{11}^*R_{21}^*+R_{12}^*R_{22}^*}{\sqrt{R_{11}^{*2}+R_{12}^{*2}}\times\sqrt{R_{21}^{*2}+R_{22}^{*2}}}$, where (R_{i1}^*, R_{i2}^*) is the coordinate of corner

⁹⁴⁹ of the ZNGI for species i (see main text, Appendix S1 and Box 1 for parameter
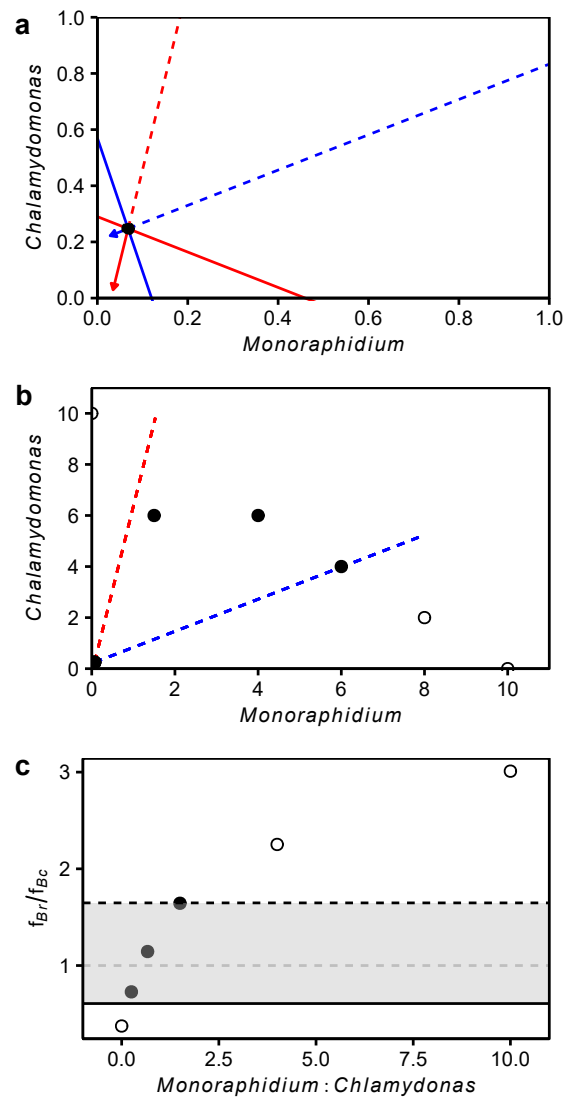
⁹⁵⁰ definition).

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Figure 1

Figure 2

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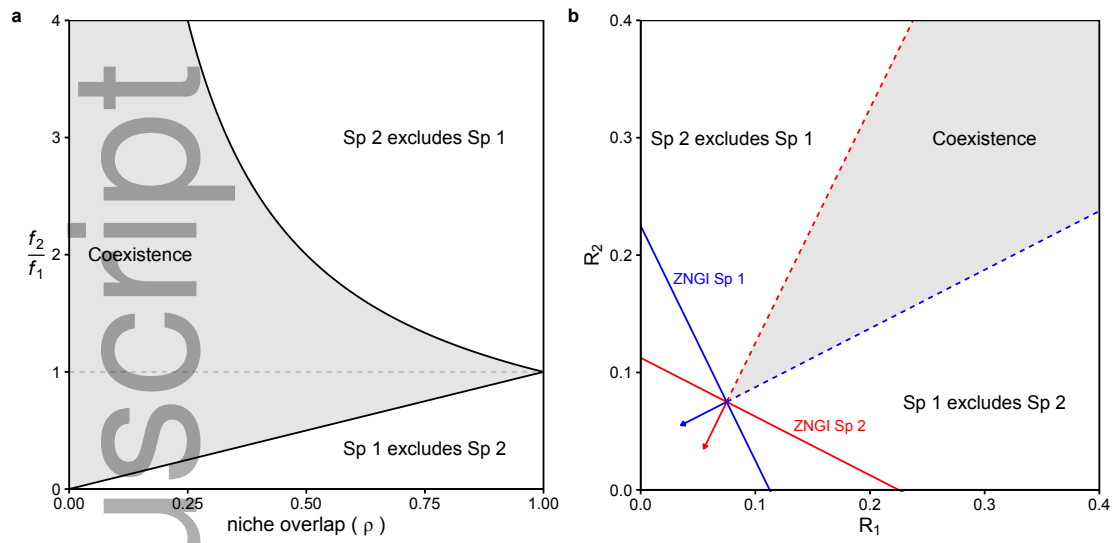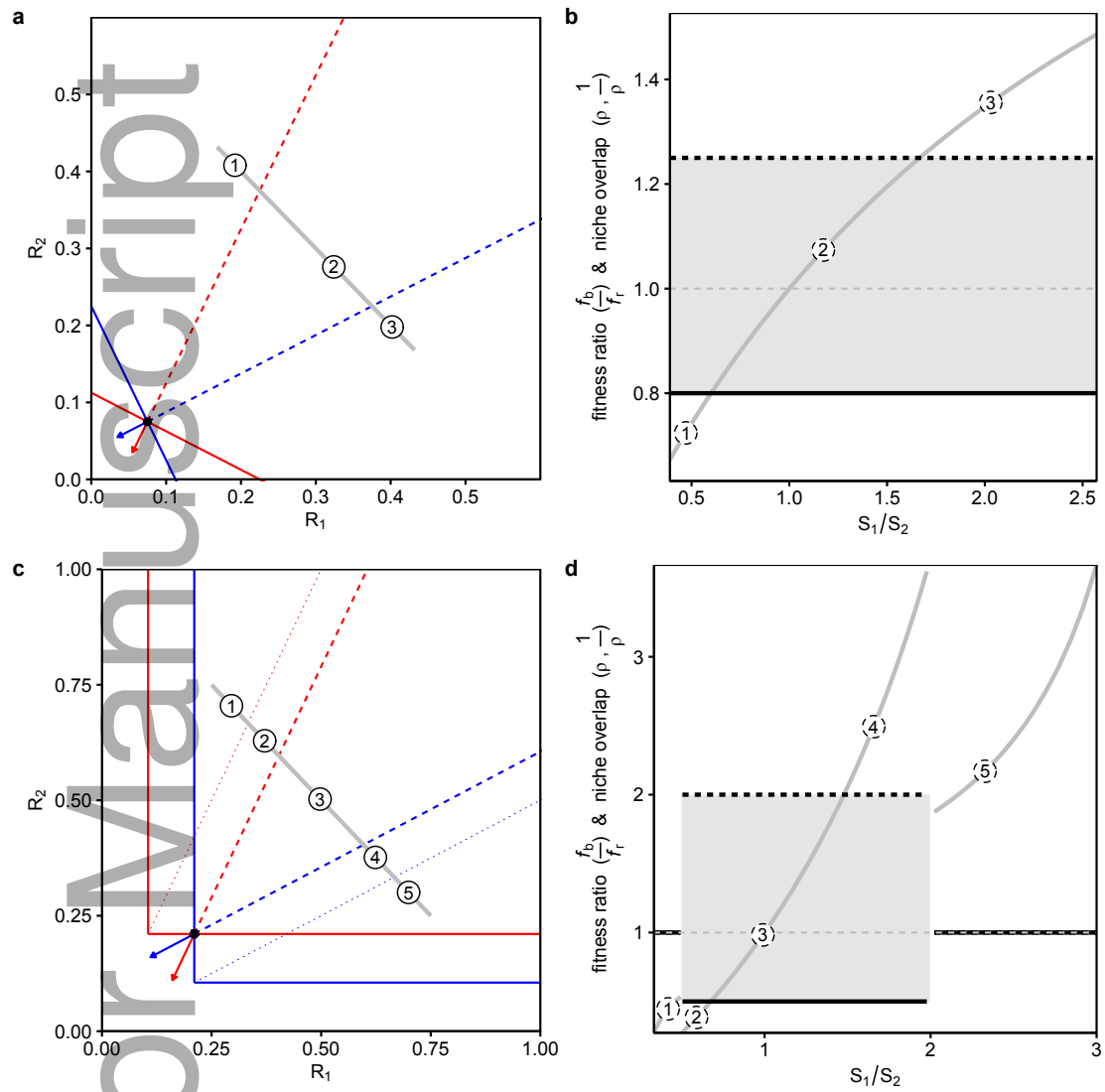
Figure 3 (to be placed near Box 2)

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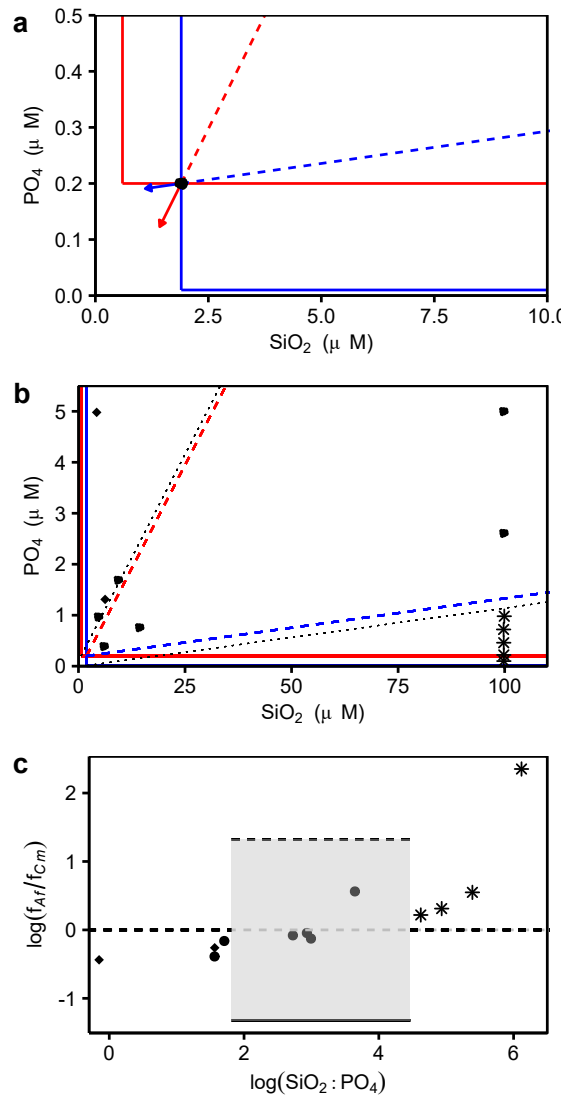**Figure 4** (to be placed near Box 2)
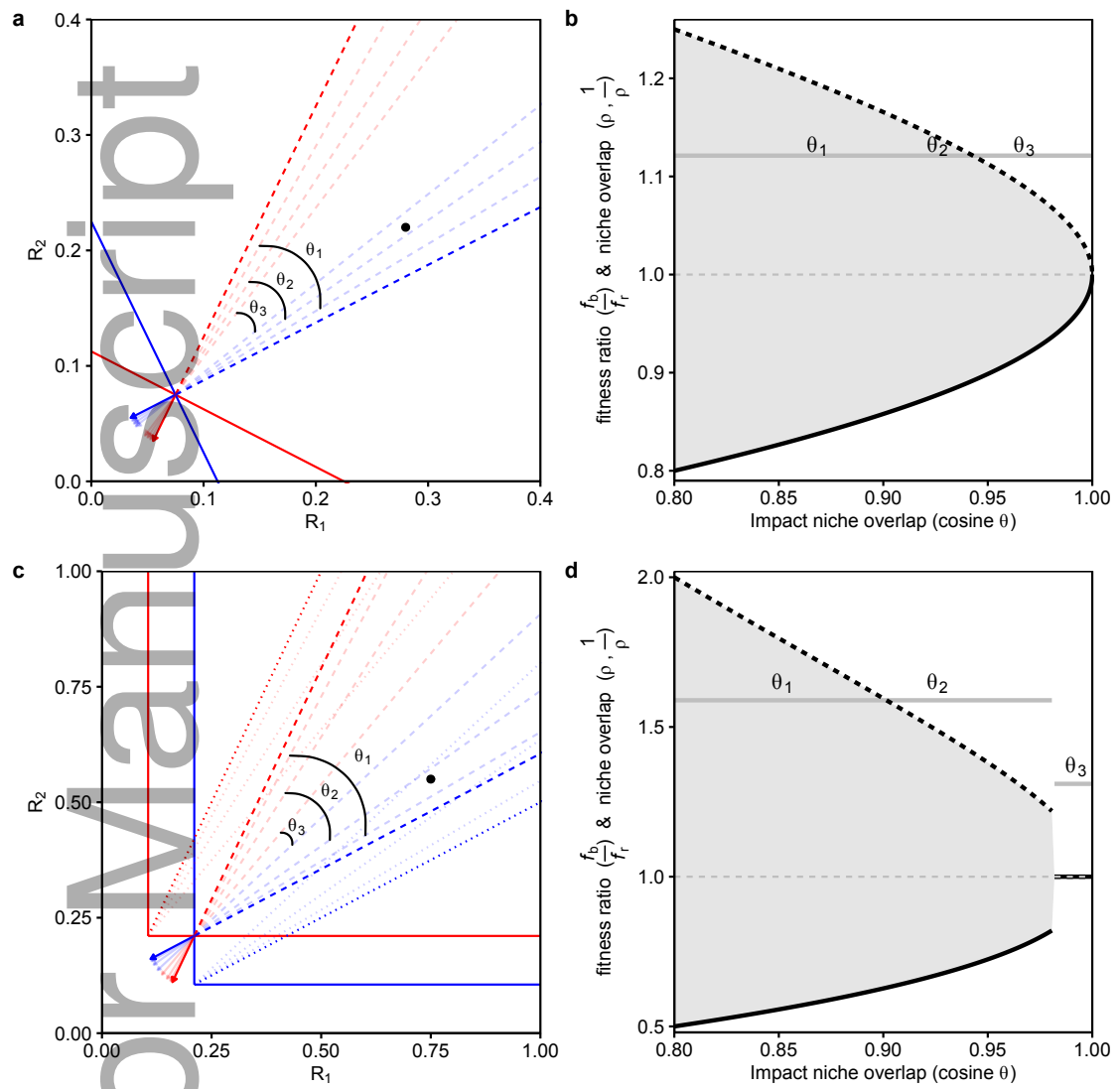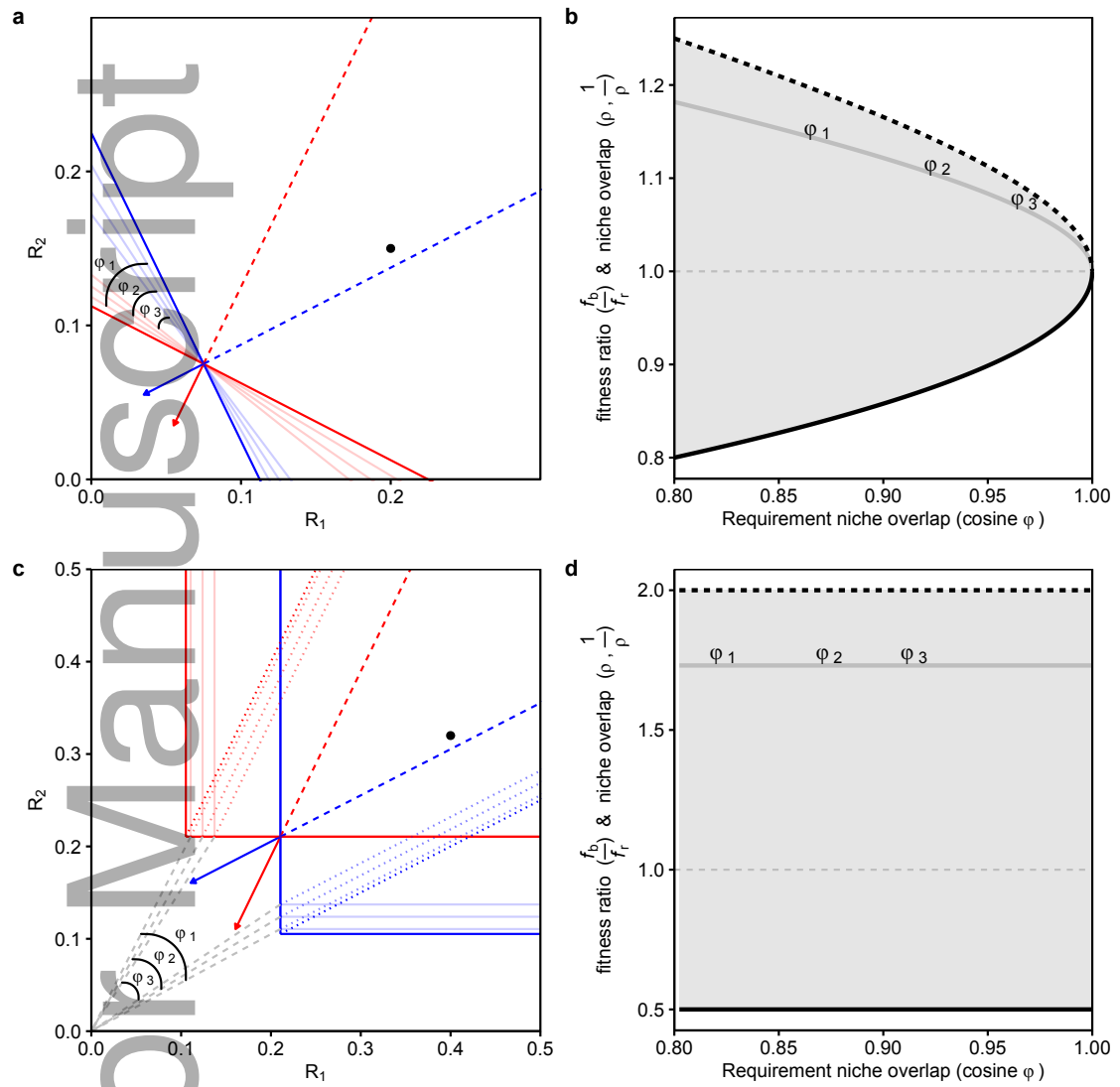

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Figure 5

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Figure 6

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Box 1: Deriving niche overlap and fitness differences in terms of consumer-resource parameters

In order to explore the analytical relationship between modern coexistence theory (Chesson, 2000b) and contemporary niche theory (Chase and Leibold, 2003) we first translated a consumer-resource model into Lotka-Volterra form following Tilman (1982, Ch. 7). This was done by solving the equilibrium of the consumer-resource model, and rearranging the equilibrium algebraically (for an alternative approach see Meszéna et al., 2006, Barabás et al., 2014). A summary of the mathematical derivation for substitutable resources is provided here, and in full for both substitutable and essential resources in Appendix S1.

Tilman's (1982) consumer-resource model consists of two resources (R_1 and R_2) that are perfectly nutritionally substitutable for two consumers (N_1 and N_2). By setting the right hand side of the consumer equations to zero, Tilman (1982) solved for the equilibrium and rearranged the consumer equilibrium density to a form comparable to the Lotka-Volterra model (i.e. $N_1^* = \frac{1}{a_{11}} - \frac{a_{12}}{a_{11}}N_2^*$ and $N_2^* = \frac{1}{a_{22}} - \frac{a_{21}}{a_{22}}N_1^*$). The equilibrium density for N_1 and N_2 can be subsequently written as:

$$N_1^* = \left[\frac{D\left(S_2 + \frac{w_{11}}{w_{12}}S_1 - B_1\right)}{c_{12} + c_{11}\frac{w_{11}}{w_{12}}}\right] - \left[\frac{c_{22} + c_{21}\frac{w_{11}}{w_{12}}}{c_{12} + c_{11}\frac{w_{11}}{w_{12}}}\right]N_2^*. \tag{4.1}$$

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$$N_2^* = \left[\frac{D\left(S_2 + \frac{w_{21}}{w_{22}}S_1 - B_2\right)}{c_{22} + c_{21}\frac{w_{21}}{w_{22}}} \right] - \left[\frac{c_{12} + c_{11}\frac{w_{21}}{w_{22}}}{c_{22} + c_{21}\frac{w_{21}}{w_{22}}} \right] N_1^*. \tag{4.2}$$

Here, r_i represents the maximum population growth rate for species i ($i = 1$ or 2) and D represents the constant mortality of the consumers and turnover rate of resources. Per capita resource consumption rate of consumer N_i on resource R_j ($j = 1$ or 2) is represented by c_{ij}, whereas w_{ij} represents a weighting factor that converts availability of R_j into its value for consumer N_i. Following a Monod growth model, k_i is the half-saturation constant for N_i resource consumption, and T_i is the minimum amount of total resource required for N_i to grow. Finally, S_1 and S_2 represents the resource supply concentration for R_1 and R_2, respectively.

Eqns. (4.1) and (4.2) consists of two parts. The first bracket represents a density-independent component with only N_1- and N_2-related parameters and are the algebraic equivalent of $\frac{1}{a_{11}}$ and $\frac{1}{a_{22}}$, respectively. The second bracket represents a heterospecific density-dependent component that decreases with its competitors' density, and is the algebraic equivalent of $\frac{a_{12}}{a_{11}}$ and $\frac{a_{21}}{a_{22}}$ in the Lotka-Volterra model, respectively.

Chesson defines niche overlap as $\rho = \sqrt{\frac{a_{12}a_{21}}{a_{11}a_{22}}}$ and average fitness difference of N_2 over N_1 as $\frac{f_2}{f_1} = \sqrt{\frac{a_{11}a_{12}}{a_{22}a_{21}}}$ (Chesson and Kuang, 2008, Chesson, 2013). Thus niche overlap for two consumers competing for substitutable resources can be expressed

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985 as:

$$\rho = \sqrt{\frac{a_{12}a_{21}}{a_{11}a_{22}}} = \sqrt{\frac{\left(c_{22} + c_{21}\frac{w_{11}}{w_{12}}\right)\left(c_{12} + c_{11}\frac{w_{21}}{w_{22}}\right)}{\left(c_{12} + c_{11}\frac{w_{11}}{w_{12}}\right)\left(c_{22} + c_{21}\frac{w_{21}}{w_{22}}\right)}}, \tag{5.1}$$

and the absolute fitness difference of N_2 over N_1 is:

$$\frac{f_2}{f_1} = \sqrt{\frac{a_{11}a_{12}}{a_{22}a_{21}}} = \frac{\left(S_2 + \frac{w_{21}}{w_{22}}S_1 - B_2\right)}{\left(S_2 + \frac{w_{11}}{w_{12}}S_1 - B_1\right)}\sqrt{\frac{\left(c_{12} + c_{11}\frac{w_{11}}{w_{12}}\right)\left(c_{22} + c_{21}\frac{w_{11}}{w_{12}}\right)}{\left(c_{22} + c_{21}\frac{w_{21}}{w_{22}}\right)\left(c_{12} + c_{11}\frac{w_{21}}{w_{22}}\right)}}. \tag{5.2}$$

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Box 2: Empirical tests of consumer-resource competition through the lens of modern coexistence theory

In order to illustrate further the transformation between contemporary niche theory and modern coexistence theory, we extracted data from two seminal experimental works on resource competition, Rothhaupt (1988) and Tilman (1977).

Rothhaupt (1988) investigated the effect of modifying the ratio of two substitutable resources on the competitive dynamics of two species of herbivorous zooplankton, the rotifers *Brachionus rubens* and *B. calyciflorus*. To quantity the parameters defining each species' ZNGI and consumption vectors, Rothhaupt (1988) first measured the per capita growth rate of *B. rubens* and *B. calyciflorus* across a range of concentrations of two algae species (*Monoraphidium minutum* and *Chlamydonas sphaeroides*). These were subsequently used to make predictions on the outcomes of competition between the two rotifers at different supply ratios of the two resources and at two dilution rates (the nutrient independent mortality rate in a chemostat).

We extracted the relevant parameters at the lower dilution rate (either from the text or from figure 2a) and used these to quantify Chesson's niche overlap and fitness ratio terms following the approach outlined in Box 1. Fig. 3a shows each species' ZNGI and associated consumption vector, and corresponds with Figure 2a of Rothhaupt (1988). Notably the intersecting ZNGIs and positively correlated consumption vectors satisfy two of the three criteria for stable coexistence. Fig. 3b, which is drawn on a different scale to Fig. 3a, shows the manipulated resource supply ratios, where black dots satisfy the third criteria for stable coexistence,

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intermediate supply rates. Fig. 3c shows the equivalent coexistence predictions when translated into Chesson's niche overlap and fitness ratio. In accordance with the logic of the main text, manipulating resource supply ratio only affects fitness differences, and the regions of stable coexistence correspond with those identified by Tilman's graphical method. In subsequent competition experiments, Rothhaupt (1988) found the results to be in agreement with theoretical predictions in all but one of the scenarios outlined above.

Tilman (1977, 1982) investigated the effect of modifying the ratios of two essential resources, phosphate and silicate, on the coexistence of two algal species, *Asterionella formosa* and *Cyclotella meneghiniana*. As for the Rothhaupt data, we used the parameters given in Tilman (1977) and extracted the supply point ratios from Fig 31.A of Tilman (1982) to quantify Chesson's niche overlap and fitness ratio terms. Fig. 4a shows a zoomed-in view of each species' ZNGIs and impact vectors. Fig. 4b, which corresponds with Fig 31.A of Tilman (1982), shows the position of the resource supply points and predicted outcomes of competition. When translated into Chesson's niche overlap and fitness ratio terms (Fig. 4c) we see that the four supply points compatible with coexistence in Fig. 4b all correspond with fitness ratios that are bounded by ρ & $1/\rho$. As highlighted in the main text, all of the supply points that fall outside the coexistence region are sufficiently extreme such that both species are limited by the same resource. This is reflected in the superimposition of $1/\rho$ and ρ in Fig. 4c. The results of subsequent competition experiments were in agreement with all but two of the predictions, where both species coexisted despite falling just outside the coexistence region identified in

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1033 Fig. 4b.

1034 Note that the equivalence of the coexistence predictions in both examples is a
1035 natural result of their deriving from the same underlying data. It would be valuable
1036 to conduct a more thorough comparative study using data collected independently
1037 for analysis under each framework, where inconsistent predictions could serve to
1038 highlight inappropriate assumptions.

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