Shrin Ville Parle Kelavani Mandal's

Institute of Technology, Dhule

Department of Information Technology

Design and Analysis of Algorithms Lab

Sr No.	Student Name	Roll No.	PRN	Class
1	Gaurav Shashikant Patil	36	2254491246037	SY-IT

Problem :

Write Strassen's algorithm to multiply two 2x2 matrices. Multiply the following matrices using strassen's Matrix multiplication.

$$A=\begin{bmatrix}3 & 2\\1 & 2\end{bmatrix} \qquad B=\begin{bmatrix}4 & 2\\2 & 3\end{bmatrix}$$

Solution:

Strassen’s Alorithm:

Strassen's Matrix Multiplication is the divide and conquer approach to solve the matrix multiplication problems. The usual matrix multiplication method multiplies each row with each column to achieve the product matrix.

Strassen's algorithm is a method for multiplying two matrices that is more efficient than the standard matrix multiplication algorithm, particularly for large matrices. It works by dividing the matrices into smaller submatrices and performing a series of mathematical operations to compute the product.

The complexity of Strassen's matrix multiplication algorithm is usually expressed using big O notation.

For conventional matrix multiplication of two ***n*n*** matrices, the complexity is $O(n^3)$

However, Strassen's algorithm reduces this complexity to $O(n^{\log_2 7})$, which is approximately $O(n^{2.81})$

Here's how Strassen's algorithm works for multiplying two 2x2 matrices, along with some theory and Example:

1. **Divide:**

Given two 2x2 matrices A and B, divide each matrix into four submatrices of size 1x1.

$$A = \begin{bmatrix}a & b\\c & d\end{bmatrix} \qquad B = \begin{bmatrix}e & f\\g & h\end{bmatrix}$$

A can be divided into: $A_{11} = [a]$, $A_{12} = [b]$, $A_{21} = [c]$, $A_{22} = [d]$

B can be divided into: $B_{11} = [e]$, $B_{12} = [f]$, $B_{21} = [g]$, $B_{22} = [h]$

Example:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$$

A can be divided into: $A_{11} = [3]$, $A_{12} = [2]$, $A_{21} = [1]$, $A_{22} = [2]$

B can be divided into: $B_{11} = [4]$, $B_{12} = [2]$, $B_{21} = [2]$, $B_{22} = [3]$

2. **Conquer:**

Recursively compute seven products (p1 to p7) using the submatrices.

p1 = a * (f - h)	p1 = 3 * (2 - 3) = -3
p2 = (a + b) * h	p2 = (3 + 2) * 3 = 15
p3 = (c + d) * e	p3 = (1 + 2) * 4 = 12
p4 = d * (g - e)	p4 = 2 * (2 - 4) = -4
p5 = (a + d) * (e + h)	p5 = (3 + 2) * (4 + 3) = 35
p6 = (b - d) * (g + h)	p6 = (2 - 2) * (2 + 3) = 0
p7 = (a - c) * (e + f)	p7 = (3 - 1) * (4 + 2) = 12

3. **Combine:** Combine the products to form the resulting matrix C.

C_{11} = p5 + p4 - p2 + p6 = 35 - 4 - 15 + 0 = 16

C_{12} = p1 + p2 = -3 + 15 = 12

C_{21} = p3 + p4 = 12 - 4 = 8

C_{22} = p5 + p1 - p3 - p7 = 35 - 3 - 12 - 12 = 8

Resultant Matrix :

$$C=\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad C=\begin{bmatrix} 16 & 12 \\ 8 & 8 \end{bmatrix}$$