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Course - B.Tech (C.S.E)

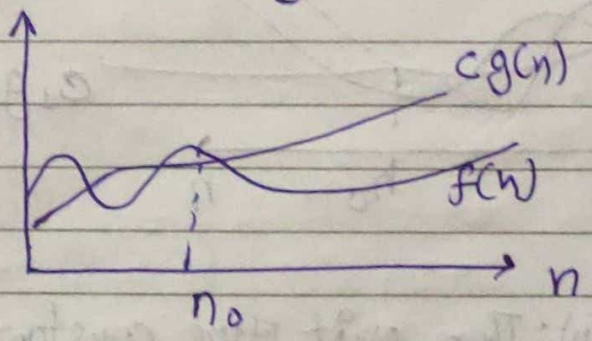
## Assignment - 1

Ans 1- Asymptotic notation are used to represent the complexities of algorithms for asymptotic analysis.

These notation are used for very large input.

1- **Big-oh (O) -**

It gives upper bound for a function $f(n)$ to within a constant factor.

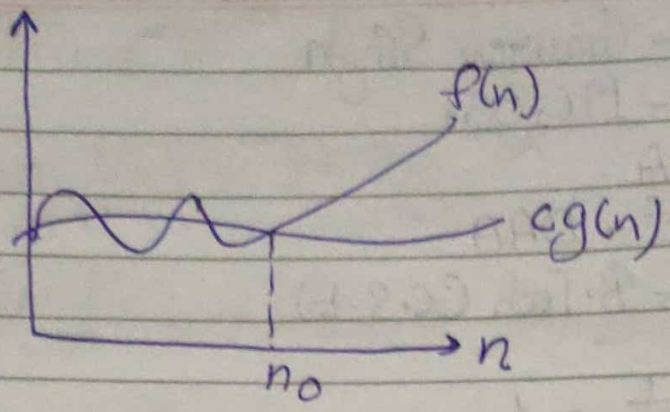



$$f(n) \leq c \cdot g(n) \quad \forall \; n \geq n_0, \; c > 0$$

eg - $O(n^2 + 3n) = O(n^2)$

2- **Big omega Notation ($\Omega$)**

Big-Omega ($\Omega$) notation gives a lower bound for a $f(n)$ to within a constant factor.

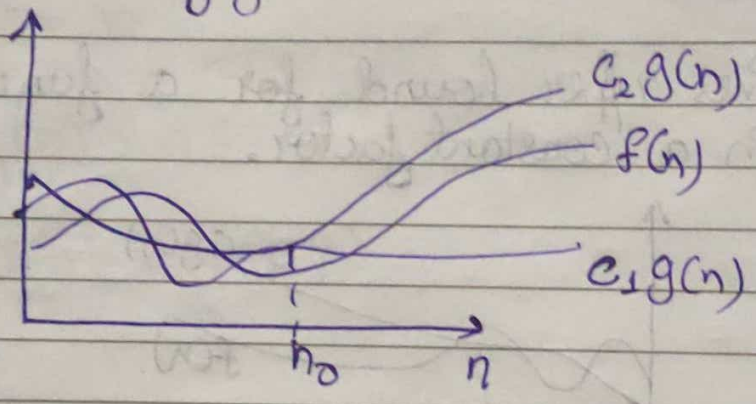



$\Omega(g(n)) = \{f(n)$ : There exist +ve constant c & no such that $0 \leq cg(n) \leq f(n) \quad \forall\ n \geq no\}$

eg - $\Omega(n \log n)$

3- Big theta Notation ($\Theta$)

it gives bound of function within a constant factor.




$\Theta(g(n)) = \{f(n)$ : There exist +ve constant $c_1, c_2$ and no such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n), \forall\ n \geq no\}$

eg - $\Theta(n^2)$

Ans 2- for (i=1 to n)
{
(i = i * 2);
}

1, 2, 4, 8, ..... n

$T(n) = O(\log_2 n)$

Ans 3- $T(n) = \begin{cases} 3T(n-1) & n>0 \\ 1 & n=0 \end{cases}$

$T(n) = 3T(n-1)$ — ①
$T(n-1) = 3T(n-2)$
$T(n) = 9T(n-2)$ — ②
$T(n) = 3^3 T(n-3)$ — ③
$T(k) = 3^k T(n-k)$ — ④
for $T(n-k) = T(0)$
$n-k = 0$
$n = k$

$T(n) = 3^n T(0)$
$T(n) = 3^n$
$T(n) = \Theta(3^n)$ //

Ans 4- $T(n) = \begin{cases} 2T(n-1) - 1 & , n>0 \\ 1 & , n=0 \end{cases}$

$T(n) = 2T(n-1) - 1$ — ①
$T(n-1) = 2T(n-2) - 1$
$T(n) = 4T(n-2) - 1 - 2$ — ②
$T(n) = 8T(n-3) - (1+2+4)$ — ③

$$T(n) = 2^k T(n-k) - (\underbrace{1+2+4+\cdots 2^{k-1}}_{k \text{ terms}})$$

$$T(n-k) = T(0)$$

$$n = k$$

$$T(n) = 2^n T(0) - (\underbrace{1+2+4+\cdots}_{k \text{ terms}})$$

Its a G.P

$a = 1$

$r = 2$

$$T(n) = 2^n - \left(\frac{1(2^{n} - 1)}{2-1}\right)$$

$$T(n) = 2^n - 2^n + 1$$

$$T(n) = 1$$

$$T(n) = \Theta(1)$$ //

Ans 5-

```
int i=1, s=1;
while (s<=n) {
    i++;
    s=s+i;
    printf("#");
}
```

1, 3, 6, 10, 15, ---- n

<---- k terms ---->

Its $k^{th}$ term is $\frac{k(k+1)}{2} = n$

$K = \sqrt{n}$

$T(n) = O(\sqrt{n})$ //

Ans 6-

```
void function(int n) {
    int i, count = 0;
    for(int i = 1; i*i < n; i++)
        count ++
}
```

$T(n) = O(\sqrt{n})$

Ans 7- $T(n) = O(n * \log_2 n * \log_2 n)$

$T(n) = O(n * (\log_2 n)^2)$

$T(n) = O(n (\log n)^2)$

Ans 8-

```
function(int n) {                    T(n)
    if(n==1) return;
    for(i=1 to n) {                  n^2
        for(j=1 to n) {
            printf("*");
        }
    }
    function(n-3);                   T(n-3)
}
```

$T(n) = T(n-3) + n^2$ — ①

$T(n-1) = T(n-4) + (n-1)^2$

$T(n) = T(n-4) + n^2 + (n-1)^2$

$T(n) = T(n-5) + n^2 + (n-1)^2 + (n-2)^2$

$$T(n) = T(n-k) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$$
$$(K-2) \text{ terms}$$

for $T(n-k) = 1$

$k = n-1$

$$T(n) = T(1) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$$
$$(n-3) \text{ terms}$$

$$T(n) = T(1) + (4^2 + 5^2 + \cdots n^2)$$

$$T(n) = T(1) + \left(\frac{(n-3)(n-2)(2n-5)}{6}\right)$$

$$T(n) = 1 + \left(\frac{2n^3 + \cdots}{6}\right)$$

$$T(n) = n^3$$

$$T(n) = O(n^3)$$

Ans 9- void function(int n) {

```
    for(i = 1 to n) {
        for(j = 1; j <= n; j = j + i)
            printf("*");
    }
```

~~outer loop - n times~~ i=1 n times

~~Inner loop - 1,~~ i=2 1, 3, 5, ... n n

i=3 1, 4, 7 ... n n/2

⋮

i=n 0 n/3

$T(n) = \left( n + \frac{n}{2} + \frac{n}{3} - - \right)$ $n$ times

~~$T(n) = O(\log n)$~~ $T(n) = O(n \log n)$

Ans10- for the functions $n^k$ and $a^n$, what is the relation.

$K \geq 1$ & $a > 1$

relation is $n^k$ is $O(c^n)$

Ans 11-

```
void fun (int n)
{
    int j = 1, i = 0;
    while (i<n)
    {
        i = i + j;
        j++;
    }
}
```

0, 3, 6, 10, 15, ..... n

K terms.

So for this series is

$K^{th}$ term is $\frac{K(K+1)}{2}$

$$n = \frac{k^2+k}{2}$$

$$K \simeq \sqrt{n}$$

$$T = \Theta(\sqrt{n})$$ //

Ans 12- Recurrence relation of fabonacci series is

$$T(n) = \{T(n-1) + T(n-2) + 1\}$$

$$T(n) = 2T(n-2) + 1$$

$$T(n) = 4T(n-4) + 3$$

$T(n) = 8T(n-6) + 7$

$T(n) = 16T(n-8) + 15$

$T(n) = 2^k T(n-2k) + (2^k - 1)$

for $T(n-2k) = T(0)$

$n = 2k$

$k = \frac{n}{2}$

$T(n) = 2^{n/2} T(0) + (2^{n/2} - 1)$

$T(n) = 2^n - 1$

$T(n) = O(2^n)$

hence space complexity of fabinacci series is $O(n)$ as it depends on height of recursive tree & it is equal to n in fabinacci series.

Ans 13. → $n(\log n)$

```
void fun(){for(int j=0 ; j<n ; j++ {
    for( int i=0 ; i<n; i=i*2)
      {
        print ("*");
      }
  }
}
void main()
  {
    fun();
  }
```

→ $n^3$

```
#include <stdio.h>
void main()
  {
    int n;
    cin >> n;
    for(int i=0; i<n; i++) {
        for(int j=0; j<n; j++) {
            for(int k=0; k<n; k++) {
                x++;
            }
        }
    }
  }
```

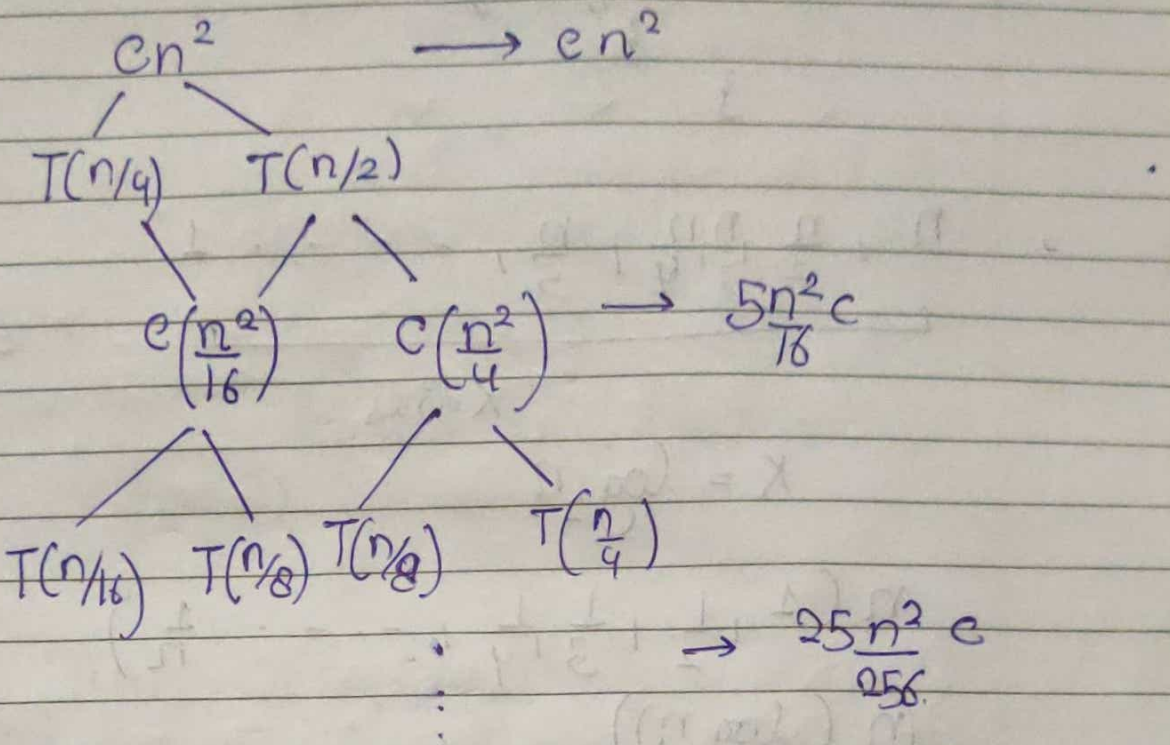
→ $\log(\log n)$

```
#include <bits/stdc++.h>
void fun(int n)
  {
    if(n == 2)
      return 1;
    else
      fun(sqrt(n));
  }
void main()
  {
    fun(100);
  }
```

Ans14- $T(n) = T(n/4) + T(n/2) + cn^2$

$T(1) = c$

$T(0) = 0$




$T(n)$ = Cost of each level

$$T(n) = cn^2 + \frac{5cn^2}{16} + \frac{25cn^2}{256} + \cdots\cdots$$

it is a G.P

with $a = n^2$

$r = \frac{5}{16}$

So sum of S/p.

$$T(n) = cn^2\left(\frac{1}{1-\frac{5}{16}}\right) = \frac{16cn^2}{11} = \frac{16cn^2}{11}$$

$T(n) = \Theta(n^2)$ //

Ans 15 -

```
for (int i to n)
{
    for(int j=1 ; j<n ; j+=i)
    {
        //O(1)
    }
}
```

$n , \frac{n}{2} , \frac{n}{3} , \frac{n}{4} , \frac{n}{5} , - - - - 1$

k lines

$K = \log_2 n$

$n\left(1 , \frac{1}{2} , \frac{1}{3} , \frac{1}{4} , - - - \frac{1}{n}\right)$

$(n (\log n))$

$T(n) = O(n \log n)$ //

Ans 16 -

```
for (int i=2 ; i<=n ; i=pow(i,k)
{
    //O(1)
}
```

$2 , 2^k , 2^{(k)^2} , 2^{k^3} , - - - n$

If G.P $a = 2$

$r = 2^k$

$k^{th}$ term $= ar^{k-1}$

$$n = 2(2^k)^{k-1}$$

$$\log n = (k-1)\log 2^k$$

let $k^{(k-1)} = x$

$$k \log_k k = \log x$$

$$k = \log x \quad \text{---} (1)$$

$$n = 2^x$$

$$\log_2 n = x \log_2 2$$

$$x = \log_2 n$$

$$\log x = \log(\log n)$$

from (1)

$$k = \log(\log(n))$$

$$T(n) = O(\log(\log(n)))$$ //

Ans 17-

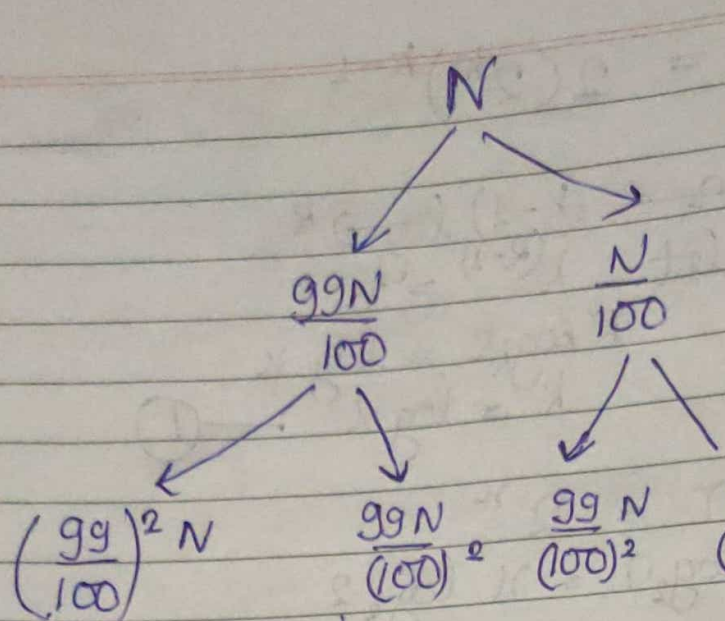
here pivot is divided in 99% & 1%

so

$$T(n) = T\left(\frac{99}{100} N\right) + T\left(\frac{N}{100}\right) + N$$

Now as here we can use 2 extreems of a tree.

where starting point is N




N

N

$N\left(\frac{(99)(99)}{100\times100} + \frac{99(1)}{100\times100}\right) + \frac{100}{100\times100}N = \frac{99}{100}N + \frac{N}{100}$

$= N$

$= N$

So cost of each level is N only.

Total cost = height * Cost of each level.

So for 1st stream - $N, \frac{99N}{100}, \left(\frac{99}{100}\right)^2 N, \ldots$

$\left(\frac{99}{100}\right)^{h-1} N = 1$

$\left(\frac{99}{100}\right)^{h-1} = \frac{1}{N}$

$N = \left(\frac{100}{99}\right)^{h-1}$

$\log N = h \log (1)$

$h = \log N$ or

$h = \frac{\log N}{\log(100/99)} + 1$ //

height of 2nd stream

$$N, \frac{N}{100}, \frac{N}{(100)^2}, \frac{N}{(100)^3}, \ldots 1$$

$$N\left(\frac{1}{100}\right)^{h-1} = 1$$

$$N = (100)^{h-1}$$

$$(h-1)\log 100 = \log N$$

$h = \frac{\log N}{\log 100} + 1$ & $h = \log N$ (approx)

$T(n) = O(N \log N)$

So time complexity is $O(N \log N)$

height of both extre is $\frac{\log N}{\log 100} + 1$ of $\left(\frac{1}{100}\right)$

and $\frac{\log N}{\log\left(\frac{100}{99}\right)} + 1$ of $\left(\frac{99}{100}\right)$

So we can conclude that if division is done more then height of tree will be more & and when division ratio is less then height is less.

Answer 18 -

a) $\sqrt{n}$, $n!$, $\log n$, $\log\log n$, $\text{root}(n)$, $n\log n$
$2^n$, $2^{2n}$, $4^n$, $n^2$, $100$

Ans -

$O(100) < O(\log\log n) < O(\log n) < O(\sqrt{n}) < O(n) <$
$O(n\log n) < O(n^2) < O(2^n) < O(2^{2n}) < O(4^n)$.

b) $2(2^n)$, $4n$, $2n$, $1$, $\log(n)$, $\log(\log(n))$, $\sqrt{\log(n)}$,
$\log 2n$, $2\log n$, $\sqrt{n}$, $\log(n!)$, $n!$, $n^2$,
$n\log(n)$

$O(1) < O(\log(\log(n))) < O(\log(n)) < O(\log 2n) < O(2\log n)$
$< O(n) < O(n\log(n)) < O(\log(n!)) < O(2n)$
$< O(4n) < O(n^2) < O(n!) < O(2(2^n))$.

c) $8^{\wedge 2n}$, $\log_2\sqrt{n}$, $n\log_6(n)$, $n\log_2(n)$, $\log(n!)$, $n$
$\log_8(n)$, $96$, $8n^2$, $7n^3$, $5n$.

Ans $O(96) < O(\log_8(n) < O(\log_2 n) < O(\log(n!) <$
$O(n\log_8(n)) < O(n\log_2(n)) < O(5n) < O(8n^3)$
$< O(7n^3) < O(n!) < O(8^{\wedge 2n})$.

Ans 19-

```
void Linear Search ( int arr[], int n, int key)
{
  for(i=0 to i=n )
      if arr[i] == key
          cout << "found";
      else
          continue

}
```

Ans 20- Iterative Insertion Sort

```
void Insertion sort( arr , n ) {

  int i, temp, j
   for i - 1 to n
    {
      temp = arr[i]
      j = i - 1
      while j >= 0 && arr[j] > temp
        {
          arr[j+1] = arr[j]
            j--
        }
      arr[j+1] = temp.

    }.
}
```

## Recursive Insertion sort

```
insertion sort (arr, n)
{
    if n<=1
    return;

    insertionsort (arr, n-1),
    last = arr[n-1];
     j = n-2

    while (j>=0 and arr[j] > last)
    {
        arr[j+1] = arr[j]
          j--
    }
    arr[j+1] = last;

}
```

Insertion sort is called online sorting because it don't know the whole input, it might make decision that later turn out to be not optimal.

Other algorithm are off-line algorithms. that are discussed in lectures.

Ans 21 -

| | Time complexity | | | Space |
|---|---|---|---|---|
| | Best | Avg | worst | |
| Bubble sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Selection sort | $O(n^2)$ | $O(n^2$ | $O(n^2)$ | $O(1)$ |
| Insertion sort | $(n)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Merge sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(n)$ {due to recursion} |
| Quick sort | $O(n \log n)$ | $O(n \log n)$ | $O(n^2)$ | $O(n)$ |
| Heap sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(1)$ |

Ans 22 -

| | inplace | Stable | Online sorting |
|---|---|---|---|
| Bubble Sort | Yes | Yes | No |
| Selection Sort | Yes | No | No |
| Insertion Sort | Yes | Yes | Yes |
| Merge Sort | No | Yes | No |
| Quick Sort | Yes | No | No |
| Heap Sort | Yes. | No. | No. |

Ans 23-

```
BinarySearch (arr, int n, key)
{
   beg = 0
   end = n-1
   while (beg <= end)
    {
      mid = (beg + end)/2

      if [arr[mid] == key]
         found
      else if arr[mid] < key
            beg = mid + 1

      else
            end = mid - 1

    }

}
```

Time complexity of Linear search - O(n)
Space complexity of Linear search - O(1)

Time complexity of Binary search = O(log n)
Space complexity of Binary search = O(n)

Ans 24- $T(n) = T\left(\frac{n}{2}\right) + 1$ //