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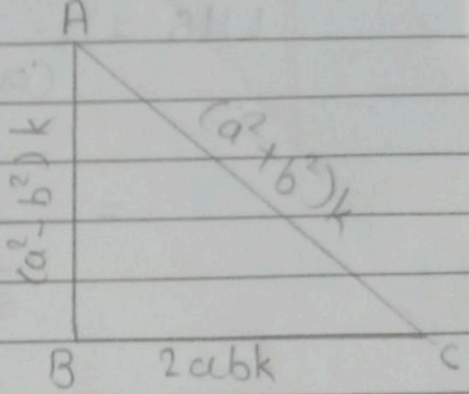
Trigonometry extra problems

1) If $\sin\theta = \dfrac{a^2-b^2}{a^2+b^2}$, then find the values of the other trigonometric ratios.

→ Given: $\sin\theta = \dfrac{a^2-b^2}{a^2+b^2}$



But $\sin\theta = \dfrac{AB}{AC}$

$\therefore AB = (a^2-b^2)k$ &

$AC = (a^2+b^2)k$

Now, $AC^2 = AB^2 + BC^2$ (Pyth. thm)

$$(a^2+b^2)^2k^2 = (a^2-b^2)^2k^2 + BC^2$$

$$(a^4+2a^2b^2+b^4)k^2 = (a^4-2a^2b^2+b^4)k^2 + BC^2$$

$$(\cancel{a^4}+2a^2b^2+\cancel{b^4}\cancel{-a^4}+2a^2b^2\cancel{-b^4})k^2 = BC^2$$

$$4a^2b^2k^2 = BC^2$$

$$BC = 2abk$$

$$\cos\theta = \frac{BC}{AC} = \frac{2abk}{(a^2+b^2)k} = \frac{2ab}{a^2+b^2}$$

$$\tan\theta = \frac{AB}{BC} = \frac{(a^2-b^2)k}{2abk} = \frac{a^2-b^2}{2ab}$$

$$\sec\theta = \frac{AC}{BC} = \frac{(a^2+b^2)k}{2abk} = \frac{a^2+b^2}{2ab}$$

$$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{(a^2+b^2)k}{(a^2-b^2)k} = \frac{a^2+b^2}{a^2-b^2}$$

$$\cot\theta = \frac{BC}{AB} = \frac{2abk}{(a^2-b^2)k} = \frac{2ab}{a^2-b^2}$$

1) Prove that

$$\frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}$$

$$LHS = \frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}$$

$$= \frac{1}{\frac{1}{\sin A}-\frac{\cos A}{\sin A}}-\frac{1}{\sin A}$$

$$= \frac{\sin A}{1-\cos A}-\frac{1}{\sin A}$$

$$= \frac{\sin^2 A-1(1-\cos A)}{(1-\cos A)\sin A}$$

$$= \frac{(1-\cos^2 A)-(1-\cos A)}{(1-\cos A)\sin A}$$

$$= \frac{(1+\cos A)(1-\cos A)-(1-\cos A)}{(1-\cos A)\sin A}$$

$$= \frac{\cancel{(1-\cos A)}\,[\cancel{1}+\cos A-\cancel{1}]}{\cancel{(1-\cos A)}\sin A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A \quad \text{———} \quad (1)$$

$$RHS = \frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}$$

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(3)

$$= \frac{1}{\sin A} - \frac{1}{\frac{1}{\sin A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\sin A} - \frac{\sin A}{1+\cos A}$$

$$= \frac{(1+\cos A) - \sin^2 A}{\sin A\,(1+\cos A)}$$

$$= \frac{(1+\cos A) - (1-\cos^2 A)}{\sin A\,(1+\cos A)}$$

$$= \frac{(1+\cos A) - (1+\cos A)(1-\cos A)}{\sin A\,(1+\cos A)}$$

$$= \frac{(1+\cos A)(1-1+\cos A)}{\sin A\,(1+\cos A)}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A \quad \text{———} (2)$$

from (1) & (2)
LHS = RHS

$$\therefore \frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

(4)

2) $\frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = 2\sec A$

$$L.H.S. = \frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A}$$

$$= \frac{(1+\sin A)^2 + \cos^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1 + 2\times\sin A + \sin^2 A + \cos^2 A}{\cos A\,(1+\sin A)}$$

$$= \frac{1 + 2\times\sin A + 1}{\cos A(1+\sin A)}$$

$$= \frac{2 + 2\times\sin A}{\cos A\,(1+\sin A)}$$

$$= \frac{2\,\cancel{(1+\sin A)}}{\cos A\,\cancel{(1+\sin A)}}$$

$$= \frac{2}{\cos A}$$

$$= 2 \times \frac{1}{\sec A}$$

$$= \frac{2}{\sec A}$$

$$\therefore \frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = 2\sec A$$

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3) $\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = 2\cot^2\theta \cdot \sec^2\theta$

→ $L.H.S. = \dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta}$

$= \dfrac{1-\cancel{\cos\theta}+1+\cancel{\cos\theta}}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{2}{1-\cos^2\theta}$

$= \dfrac{2}{\sin^2\theta}$

multiply & divide by $\cos^2\theta$

$= \dfrac{2\cos^2\theta}{\sin^2\theta \cdot \cos^2\theta}$

$= 2\times \dfrac{\cos^2\theta}{\sin^2\theta} \cdot \dfrac{1}{\cos^2\theta}$

$= 2\cot^2\theta \cdot \sec^2\theta$

$= R.H.S.$

$\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = 2\cot^2\theta \cdot \sec^2\theta.$

4) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

$\longrightarrow$

$$
\begin{aligned}
LHS &= \tan^4\theta + \tan^2\theta \\
&= (\tan^2\theta)^2 + \tan^2\theta \\
&= (\sec^2\theta - 1)^2 + (\sec^2\theta - 1) \\
&= (\sec^2\theta)^2 - 2\times\sec^2\theta + 1^2 + \sec^2\theta - 1 \\
&= \sec^4\theta - 2\sec^2\theta + \sec^2\theta \\
&= \sec^4\theta - \sec^2\theta
\end{aligned}
$$

$\therefore \tan^4\theta + \tan^2\theta = \sec^2\theta - \sec^2\theta.$

5) $\sqrt{\sec^2\theta + \csc^2\theta} = \tan\theta + \cot\theta$

$$
\begin{aligned}
LHS &= \sqrt{\sec^2\theta + \csc^2\theta} \\
&= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} \\
&= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cdot \cos^2\theta}} \\
&= \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}}
\end{aligned}
$$

$= \cancel{\dfrac{\sqrt{}}{\sin^2\theta \cdot \cos^2\theta}}$ $\qquad \dfrac{1}{\sin\theta \cdot \cos\theta}$

$= \cancel{\dfrac{1}{\sin^2\theta \cdot \cos^2\theta}}$ $\qquad \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \dfrac{\cos^2\theta}{\sin\theta \cdot \cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$$= \tan\theta + \cot\theta.$$

$$\sqrt{\sec^2\theta + \cos^2\theta} = \tan\theta + \cot\theta$$

6) $\frac{\tan\theta}{\sec\theta+1} + \frac{\sec\theta+1}{\tan\theta} = 2\operatorname{cosec}\theta.$

$\rightarrow$ L.H.S. $= \frac{\tan\theta}{\sec\theta+1} + \frac{\sec\theta+1}{\tan\theta}$

$$= \frac{\tan^2\theta + (\sec\theta+1)^2}{(\sec\theta+1)\tan\theta}$$

$$= \frac{\tan^2\theta + \sec^2\theta + 2\sec\theta + 1}{(\sec\theta+1)\tan\theta}$$

$$= \frac{\sec^2\theta - \cancel{1} + \sec^2\theta + 2\sec\theta + \cancel{1}}{(\sec\theta+1)\tan\theta}$$

$$= \frac{2\sec^2\theta + 2\sec\theta}{(\sec\theta+1)\tan\theta}$$

$$= \frac{2\sec\theta\,\cancel{(\sec\theta+1)}}{\cancel{(\sec\theta+1)}\tan\theta}$$

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= \frac{2 \times \frac{1}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}$$

$$= \frac{2}{\sin\theta}$$

$$= 2 \times \operatorname{cosec}\theta$$

$\therefore \frac{\tan\theta}{\sec\theta+1} + \frac{\sec\theta+1}{\tan\theta} = 2\operatorname{cosec}\theta.$

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7) $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta.\cos\theta$

$\rightarrow$ L.H.S $= \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^2\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$(\because a^3-b^3 = (a^2+b^2+ab)(a-b))$

$= \frac{(\cos^2\theta+\sin^2\theta+\cos\theta.\sin\theta)\cancel{(\cos\theta-\sin\theta)}}{\cancel{(\cos\theta-\sin\theta)}}$

$= \underline{\cos^2\theta+\sin^2\theta} + \cos\theta.\sin\theta$

$= 1 + \cos\theta.\sin\theta$

$\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta.\cos\theta$

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2) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1+\sec\theta.\operatorname{cosec}\theta.$

→ $LHS = \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\sin\theta/\cos\theta}{1-\left(\frac{\cos\theta}{\sin\theta}\right)} + \frac{\cos\theta/\sin\theta}{1-\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin\theta/\cos\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\cos\theta/\sin\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta(\sin\theta-\cos\theta) - \cos^3\theta(\sin\theta-\cos\theta)}{\sin\theta.\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta\cdot(\sin\theta\cancel{-}\cos\theta)}{\sin\theta.\cos\theta\ (\sin\theta\cancel{-}\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta.\cos\theta+\cos^2\theta)}{\sin\theta.\cos\theta.}$$

$$= \frac{(\sin\theta-\cos\theta)(1+\sin\theta.\cos\theta)}{\sin\theta.\cos\theta.}$$

Taking LCM.

$$= \frac{\sin^3\theta}{\cos(\sin\theta-\cos\theta)} - \frac{\cos^3\theta}{(\sin\theta-\cos\theta)\sin\theta}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta.\cos\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{\cancel{(\sin\theta - \cos\theta)}\,(\sin^2\theta + \sin\theta.\cos\theta + \cos^2\theta)}{\sin\theta.\cos\theta\,\cancel{(\sin\theta - \cos\theta)}}$$

$$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1 + \sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1}{\sin\theta.\cos\theta} + \frac{\cancel{\sin\theta.\cos\theta}}{\cancel{\sin\theta.\cos\theta}}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta} + 1$$

$$= \operatorname{cosec}\theta \times \sec\theta + 1$$

$$= 1 + \sec\theta.\operatorname{cosec}\theta$$

$$\therefore \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\operatorname{cosec}\theta$$

9) $(1+\tan\theta)^2 + (1+\cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2$

$\rightarrow$ LHS $= (1+\tan\theta)^2 + (1+\cot\theta)^2$

$= 1 + 2\tan\theta + \tan^2\theta + 1 + 2\cot\theta + \cot^2\theta$

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$= \underline{1+\tan^2\theta} + 2\tan\theta + 2\cot\theta + \underline{1+\cot^2\theta}$

$= \sec^2\theta + 2(\tan\theta + \cot\theta) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{1}{\sin\theta.\cos\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2\left(\frac{1}{\sin\theta} \times \frac{1}{\cos\theta}\right) + \operatorname{cosec}^2\theta$

$= \sec^2\theta + 2(\sec\theta.\operatorname{cosec}\theta) + \operatorname{cosec}^2\theta$

$= (\sec\theta + \operatorname{cosec}\theta)^2$

$=$ RHS.

$\therefore (1+\tan\theta)^2 + (1+\cot\theta)^2 = (\sec\theta + \operatorname{cosec}\theta)^2$

18) $\left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$

$\rightarrow$

LHS $= \left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right)$

$= (1+\cot^2 A)(1+\tan^2 A)$

$= \operatorname{cosec}^2 A \cdot \sec^2 A$

$$= \frac{1}{\sin^2 A} \quad \frac{1}{\cos^2 A}.$$

$$= \frac{1}{\sin^2 A(1-\sin^2 A)}$$

$$= \frac{1}{\sin^2 A - \sin^4 A}$$

$$\therefore \left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$$

11) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta.$

$\longrightarrow$ $\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\cancel{\tan\theta}\ \frac{1}{\cot\theta}}{1-\cot\theta} + \frac{\cot\theta}{1-\frac{1}{\cot\theta}}$$

$$= \frac{\cancel{\tan}\ \frac{1}{\cot\theta}}{1-\cot\theta} + \frac{\cot^2\theta}{1-\cot\theta}$$

$$= \frac{\frac{1}{\cot\theta} + \cot^2\theta}{1-\cot\theta}$$

$$= \frac{\frac{1+\cot^3\theta}{\cot\theta}}{(1-\cot\theta)}$$

$$= \frac{1+\cot^3\theta}{\cot\theta(1-\cot\theta)}$$

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$$= \frac{\cancel{(1-\cot\theta)}(1+\cot^2\theta+\cot\theta)}{\cot\theta\cancel{(1-\cot\theta)}}$$

$$= \frac{1}{\cot\theta} + \frac{\cot^2\theta}{\cot\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \tan\theta + \cot\theta + 1$$

$$\therefore \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta.$$

(12) $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta.\cos^2\theta)$

$$LHS = \sin^8\theta - \cos^8\theta$$
$$= (\sin^4\theta)^2 - (\cos^4\theta)^2$$
$$= (\sin^4\theta + \cos^4\theta)(\sin^4\theta - \cos^4\theta)$$
$$= [(\sin^2\theta)^2 + (\cos^2\theta)^2][(\sin^2\theta)^2 - (\cos^2\theta)^2]$$

$a^2+b^2 = (a+b) - 2ab$

$$= \{\cancel{(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)}\}.$$
$$\{(\sin^4\theta + 2\sin^2\theta.\cos^2\theta + \cos^4\theta\}$$
$$= \{[(\sin^2\theta + \cos^2\theta)]^2 - 2\sin^2\theta.\cos^2\theta.\}$$
$$\{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)\}$$

$$= [(1)^2 - 2\sin^2\theta\cos^2\theta][\cdot 1(\sin^2\theta - \cos^2\theta)]$$

$$= [1 - 2\sin^2\theta.\cos^2\theta](\sin^2\theta - \cos^2\theta)$$

$$= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta.\cos^2\theta$$

$$\therefore \sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta.\cos^2\theta)$$

13) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$\rightarrow$ LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \cdot \sec A$

$= \underline{\sin^2 A + \cos^2 A} + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \times \frac{1}{\sin A} + 2\cos A \times \frac{1}{\cos A}$

$= 1 + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$

$= 5 + (1 + \cot^2 A) + (1 + \tan^2 A)$

$= 7 + \cot^2 A + \tan^2 A$

$\therefore (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

14) $\dfrac{1 - \sin\theta \cdot \cos\theta}{\cos\theta(\sec\theta - \operatorname{cosec}\theta)} \times \dfrac{(\sin^2\theta - \cos^2\theta)}{(\sin^3\theta + \cos^3\theta)} = \sin\theta$

LHS $= \dfrac{1 - \sin\theta \cdot \cos\theta}{\cos\theta(\sec\theta - \operatorname{cosec}\theta)} \times \dfrac{(\sin^2\theta - \cos^2\theta)}{(\sin^3\theta + \cos^3\theta)}$

$= \dfrac{1 - \sin\theta \cdot \cos\theta}{\cos\theta\left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)} \times \dfrac{\cancel{(\sin\theta + \cos\theta)}(\sin\theta - \cos\theta)}{\cancel{(\sin\theta + \cos\theta)}(\underline{\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta})}$

$= \dfrac{\cancel{1 - \sin\theta \cdot \cos\theta}}{\cancel{\cos\theta}\left(\frac{\cancel{\sin\theta - \cos\theta}}{\sin\theta \cdot \cancel{\cos\theta}}\right)} \times \dfrac{\cancel{(\sin\theta - \cos\theta)}}{\cancel{(1 - \sin\theta\cos\theta)}}$

$= \dfrac{1}{1/\sin\theta} = \sin\theta$

15) $\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2 \operatorname{cosec} A$.

$\rightarrow$ $LHS = \frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1}$

$= \frac{\tan A(\sec A + 1) + \tan A(\sec A - 1)}{(\sec^2 A - 1)}$

$= \frac{\cancel{\tan A}(\sec A \cancel{+1} + \sec A \cancel{-1})}{\tan^2 A}$

$= \frac{2 \sec A}{\tan A}$

$= \frac{2 \times \frac{1}{\cancel{\cos A}}}{\frac{\sin A}{\cancel{\cos A}}}$

$= 2 \times \frac{1}{\sin A}$

$= 2 \operatorname{cosec} \theta$.

$= RHS$

16) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

$\rightarrow$ $LHS = \frac{1 + \sec A}{\sec A}$

$= \frac{1 + \frac{1}{\cos A}}{1/\cos A} = \frac{\frac{\cos A + 1}{\cancel{\cos A}}}{1/\cancel{\cos A}}$

$= \cos A + 1$ —(1)

RHS $= \dfrac{\sin^2 A}{1-\cos A}$

$= \dfrac{1-\cos^2 A}{1-\cos A}$

$= \dfrac{\cancel{(1-\cos A)}(1+\cos A)}{\cancel{1-\cos A}}$

$= 1+\cos A$ —(2)

from (1) & (2)
LHS = RHS

17) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

→ LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$= \dfrac{\sin\theta(\sin^2\theta+\cos^2\theta-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-\sin^2\theta-\cos^2\theta)}$

$= \dfrac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

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18) $\frac{\sin\theta}{1+\cos\theta} = \frac{1-\sin\theta-\cos\theta}{\sin\theta-1-\cos\theta}$

$\rightarrow$

$$1-\cos^2\theta = \sin^2\theta.$$

$$(1-\cos\theta)(1+\cos\theta) = \sin\theta \cdot \sin\theta$$

$$\frac{1-\cos\theta}{\sin\theta} = = \frac{\sin\theta}{1+\cos\theta}$$

$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = k$$

by thm on equal ratios

$$k = \frac{1-\cos\theta-\sin\theta}{\sin\theta-1-\cos\theta}$$

$$\frac{\sin\theta}{1+\cos\theta} = \frac{1-\sin\theta-\cos\theta}{\sin\theta-1-\cos\theta}$$

19) $\frac{\cot\theta}{1-\cot\theta} + \frac{\tan\theta}{1-\tan\theta} = -1$

$\rightarrow$ LHS $= \frac{\cot\theta}{1-\cot\theta} + \frac{\tan\theta}{1-\tan\theta}$

$$= \frac{\cot\theta(1-\tan\theta)+\tan\theta(1-\cot\theta)}{(1-\cot\theta)(1-\tan\theta)}$$

$$= \frac{\cot\theta-\cot\theta.\tan\theta+\tan\theta-\tan\theta.\cot\theta}{1-\tan\theta-\cot\theta+\cot\theta.\tan\theta}$$

$$= \frac{\cot\theta - \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} + \tan\theta - \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}}{1-\tan\theta-\cot\theta+\cancel{\cot\theta}\times\frac{1}{\cancel{\cot\theta}}}$$

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$$= \frac{\cot\theta - 1 + \tan\theta - 1}{1 - \tan\theta - \cot\theta + 1}$$

$$= \frac{\cot\theta + \tan\theta - 2}{-\tan\theta - \cot\theta + 2}$$

$$= -\left(\frac{\cot\theta + \tan\theta - 2}{\tan\theta + \cot\theta - 2}\right)$$

$$= -1$$

$= RHS$

(20) $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

$\longrightarrow$

$LHS = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\left((\sin^2\theta)^3 + (\cos^2\theta)^3\right) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\left[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cdot\cos^2\theta + \cos^4\theta)\right]$
$\quad - 3(\sin^4\theta + \cos^4\theta) + 1$

$(a^3 + b^3) = (a+b)(a^2 + b^2 - ab)$

$= 2\left[1(\sin^4\theta - \sin^2\theta\cdot\cos^2\theta + \cos^4\theta)\right]$
$\quad - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\sin^4\theta - 2\sin^2\theta\cdot\cos^2\theta + 2\cos^4\theta - 3\sin^4\theta$
$\quad - 3\cos^4\theta + 1$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cdot\cos^2\theta + 1$

$= -(\sin^4\theta + 2\sin^2\theta \cdot \cos^2\theta + \cos^4\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1$

$= -(1)^2 + 1$

$= -1 + 1$

$= 0$

$=$ RHS.

21) If $\sin\theta + \sin^2\theta = 1$ P.T. $\cos^2\theta + \cos^4\theta = 1$

→ $\sin\theta + \sin^2\theta = 1$ —(i)

$\sin\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta$

$\sin\theta = \cos^2\theta$

Substitute in (1)

$\cos^2\theta + \cos^4\theta = 1$

(20)

1) Verify $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$ for the values.

i) $A = 60^\circ$ & $B = 0^\circ$

ii) $A = 60^\circ$ & $B = 30^\circ$

→ i) $A = 60^\circ$ & $B = 0^\circ$

$$LHS = \tan(A-B)$$
$$= \tan(60-0)$$
$$= \tan 60$$
$$= \sqrt{3}$$

$$RHS = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\tan 60 - \tan 0}{1+\tan 60 \cdot \tan 0}$$

$$= \frac{\sqrt{3} - 0}{1+\sqrt{3}\times 0}$$

$$= \frac{\sqrt{3}}{1+\sqrt{3}}$$

$$= \cancel{\frac{\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}}$$

$$= \cancel{\frac{\sqrt{3}-3}{1-3}}$$

$$= \frac{\sqrt{3}}{1+0}$$

$$= \sqrt{3}$$

L.H.S = R.H.S

$$\therefore \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

ii) for $A = 60^\circ$ & $B = 30^\circ$

$LHS = \tan(A-B)$

$= \tan(60-30)$

$= \tan 30^\circ$

$= \dfrac{1}{\sqrt{3}}$

$RHS = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$= \dfrac{\tan 60 - \tan 30}{1 + \tan 60 \cdot \tan 30}$

$= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \cancel{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{3}}}}$

$= \dfrac{\frac{3-1}{\sqrt{3}}}{2}$

$= \dfrac{2}{\sqrt{3}} \times \dfrac{1}{2}$

$= 1/\sqrt{3}$

$\therefore LHS = RHS$.

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

(22)

2) If $\tan\theta + \sin\theta = m$ & $\tan\theta - \sin\theta = n$
Show that $m^2 - n^2 = 4\sqrt{mn}$

$\longrightarrow$

$$LHS = m^2 - n^2$$
$$= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$$
$$= \tan^2\theta + \sin^2\theta + 2\tan\theta\cdot\sin\theta.$$
$$- (\tan^2\theta + \sin^2\theta - 2\tan\theta\cdot\sin\theta)$$

$$= \cancel{\tan^2\theta} + \cancel{\sin^2\theta} + 2\tan\theta\cdot\sin\theta$$
$$- \cancel{\tan^2\theta} - \cancel{\sin^2\theta} + 2\tan\theta\cdot\sin\theta$$

$$= 4\tan\theta\cdot\sin\theta \quad \text{--- (1)}$$

$$RHS = 4\sqrt{mn}$$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta\cdot\cos^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta\cdot\sin^2\theta}{\cos^2\theta}}$$

$$= 4\,\frac{\sin\theta\cdot\sin\theta}{\cos\theta}$$

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$= 4 \tan\theta . \sin\theta$. — (2)

from (1) & (2)

LHS = RHS

$m^2 - n^2 = 4\sqrt{mn}$

3) If $\sec\theta + \tan\theta = p$ show that $\frac{p^2-1}{p^2+1} = \sin\theta$

→ $\sec\theta + \tan\theta = p$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$

$\frac{1+\sin\theta}{\cos\theta} = p$

$\frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{p^2}{1}$

$\frac{(1+\sin\theta)^2+\cos^2\theta}{(1+\sin\theta)^2-\cos^2\theta} = \frac{p^2+1}{p^2-1}$ (componendo-dividendo)

$\frac{p^2-1}{p^2+1} = \frac{(1+\sin\theta)^2-\cos^2\theta}{(1+\sin\theta)^2+\cos^2\theta}$ (Invertendo)

$= \frac{1+2\sin\theta+\sin^2\theta-\cos^2\theta}{1+2\sin\theta+\sin^2\theta+\cos^2\theta}$

$= \frac{\sin^2\theta+2\sin\theta+\sin^2\theta}{1+2\sin\theta+1}$

$= \frac{2\sin\theta+2\sin^2\theta}{2+2\sin\theta}$

$$= \frac{2\sin\theta(1+\sin\theta)}{2(1+\sin\theta)}$$

$= \sin\theta$

$= RHS.$

$$\frac{p^2-1}{p^2+1} = \sin\theta$$

4) If $a\cos\theta + b\sin\theta = m$ & $a\sin\theta - b\cos\theta = n$
P.T. $a^2+b^2 = m^2+n^2$

$\longrightarrow$

$a\cos\theta + b\sin\theta = m$
$(a\cos\theta + b\sin\theta)^2 = m^2$

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2$ —(1)

$a\sin\theta - b\cos\theta = n$
$(a\sin\theta - b\cos\theta)^2 = n^2$

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\cdot\sin\theta = n^2$ —(2)

Adding (1) & (2).

$a^2\cos^2\theta + b^2\sin^2\theta + \cancel{2ab\cos\theta\cdot\sin\theta}$
$+ a^2\sin^2\theta + b^2\cos^2\theta - \cancel{2ab\cos\theta\cdot\sin\theta} = m^2+n^2$

$\underline{a^2\cos^2\theta} + b^2\sin^2\theta + \underline{a^2\sin^2\theta} + b^2\cos^2\theta = m^2+n^2$

$(a^2+b^2)(\sin^2\theta + \cos^2\theta) = m^2+n^2$

$a^2+b^2 = m^2+n^2$

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5) If $\sqrt{3}\tan\theta = 3\sin\theta$ find value of $\sin^2\theta - \cos^2\theta$, where $\theta \neq 0$

→

$$\sqrt{3}\tan\theta = 3\sin\theta$$

$$\theta \neq 0$$

$$\sqrt{3}\frac{\cancel{\sin\theta}}{\cos\theta} = 3\cancel{\sin\theta}$$

$$\frac{\sqrt{3}}{\cos\theta} = 3$$

$$\sqrt{3} = 3\cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{3} = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\sqrt{3}}$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$\cos^2\theta = 1/3 \quad -(1)$$

Now,

$$\sin^2\theta - \cos^2\theta = (1-\cos^2\theta) - \cos^2\theta$$

$$= 1 - 2\cos^2\theta$$

$$= 1 - 2(1/3)$$

$$= 1 - 2/3$$

$$= \frac{3-2}{3}$$

$$= 1/3$$

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6) If $\sqrt{1+x^2}\sin\theta = x$ prove that

$$\tan^2\theta + \cot^2\theta = x^2 + \frac{1}{x^2}$$

$\longrightarrow$

$$\sqrt{1+x^2}\sin\theta = x$$

$$\sin\theta = \frac{x}{\sqrt{1+x^2}} \quad \text{—(1)}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{x}{\sqrt{1+x^2}}\right)^2 + \cos^2\theta = 1 \quad \text{(from 1)}$$

$$\cos^2\theta = 1 - \left(\frac{x}{\sqrt{1+x^2}}\right)^2$$

$$= 1 - \frac{x^2}{1+x^2}$$

$$= \frac{1\cancel{+x^2}-\cancel{x^2}}{1-x^2}$$

$$= \frac{1}{1-x^2} \quad \text{—(2)}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \quad \text{(from (1) \& (2))}$$

$$= \frac{\frac{x^2}{1-x^2}}{1/1-x^2}$$

$$= \frac{x^2}{\cancel{1-x^2}} \times \frac{\cancel{1-x^2}}{1}$$

$$= x^2$$

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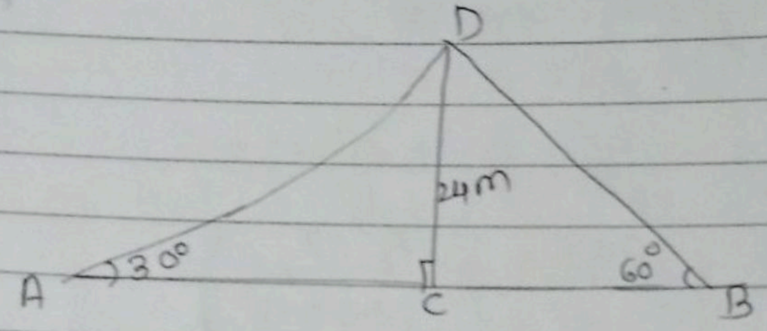
$$\cot^2\theta = \frac{1}{\tan^2\theta}$$

$$= \frac{1}{x^2}$$

$$\therefore \tan^2\theta + \cot^2\theta = x^2 + \frac{1}{x^2}$$

1) Two men standing on opposite sides of a flagstaff measure the angle of elevation of the top of flagstaff as $30°$ & $60°$. If the height of the flagstaff is 24 m. then find the distance between the two men.
$(\sqrt{3} = 1.73)$

→



let A & B be two men standing on opposite sides of a flagstaff.
let CD be a flagstaff = 24 m.

In ΔACD, $\angle ACD = 90°$

$$\tan 30° = \frac{DC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{24}{AC}$$

$$AC = 24\sqrt{3} \text{ m} \quad -(1)$$

In ΔBCD, $\angle BCD = 90°$

$$\tan 60° = \frac{DC}{BC}$$

$$\sqrt{3} = \frac{24}{BC}$$

$$BC = \frac{24}{\sqrt{3}}$$

$$= \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$= \frac{24\sqrt{3}}{3}$

$= 8\sqrt{3}$ m —(ii)

$AB = AC + BC$ ($\because$ A-C-B)

$= 24\sqrt{3} + 8\sqrt{3}$ (from (i) & (ii))

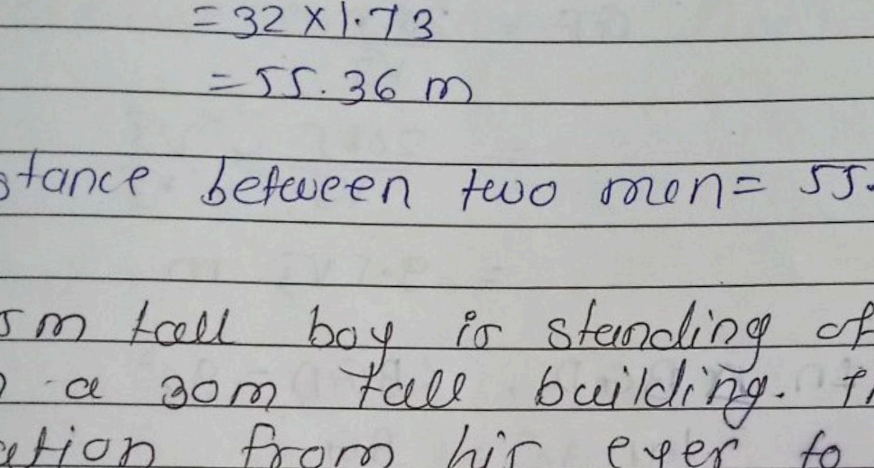
$= 32\sqrt{3}$

$= 32 \times 1.73$

$= 55.36$ m

$\therefore$ Distance between two men = 55.36 m

2) A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.



→ AB = building of height 30 m
CD = first position of boy of height 1.5 m
EF = second position of boy.
let the distance travelled ~~by~~ be x m.

$CD = EF = AG = 1.5$ m

$BG = AB - AG$ (A-G-B)

$= 30 - 1.5$

30

$= 28.5 \text{ m}$

In $\triangle BGF$, $\angle BGF = 90^\circ$

$\tan 60^\circ = \frac{BG}{GF}$

$\sqrt{3} = \frac{28.5}{GF}$

$GF = \frac{28.5}{\sqrt{3}}$

$= \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 9.5\sqrt{3} \text{ m}$ — (i)

In $\triangle BGD$, $\angle BGD = 90^\circ$

$\tan 30^\circ = \frac{BG}{GD}$

$\frac{1}{\sqrt{3}} = \frac{28.5}{GD}$

$GD = 28.5\sqrt{3} \text{ m}$ — (ii)

$AC = GD = 28.5\sqrt{3}$ — (from ii)

$AE = GF = 9.5\sqrt{3}$ — (From i)

$AC = AE + CE$

$28.5\sqrt{3} = x + 9.5\sqrt{3}$

$x = 28.5\sqrt{3} - 9.5\sqrt{3}$

$x = 19\sqrt{3} \text{ m}$

$\therefore$ Distance he walked towards the building is $19\sqrt{3}$ m

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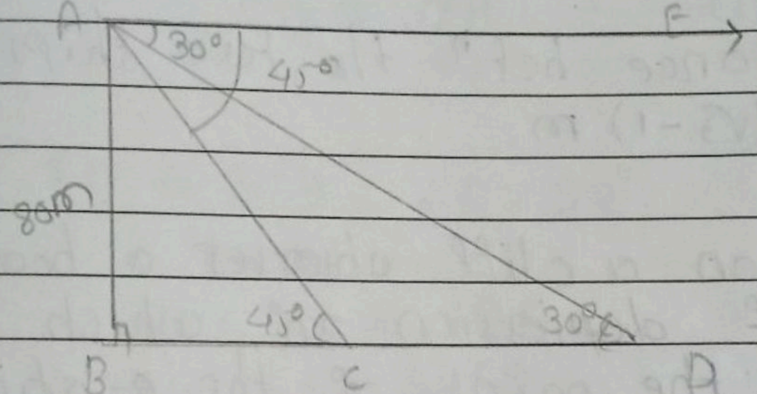
3) From the top of a lighthouse, 80 m high, two ships on the same side of light house are observed. The angle of depression of the ships as seen from the light house are found to be of 45° & 30°. Find the distance between the two ships.
(Assume that the two ships & the bottom of the lighthouse are in a line)



AB = lighthouse of height 80 m
D & C = position of two ships.
$\angle EAD$ & $\angle EAC$ are angle of depression

$\angle EAD = 30^\circ = \angle ADB$ --(i)
$\angle EAC = 45^\circ = \angle ACB$
(Alternate angles)

In $\triangle ABD$, $\angle B = 90^\circ$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{BD}$$

$$BD = 80\sqrt{3}\ m \quad \text{—(ii)}$$

In $\triangle ABC$, $\angle ABC = 90^\circ$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{80}{BC}$$

$$BC = 80 \text{ m} \qquad \text{—(ii)}$$

$$BD = BC + CD \qquad -(B-C-D)$$

$$80\sqrt{3} = 80 + CD$$

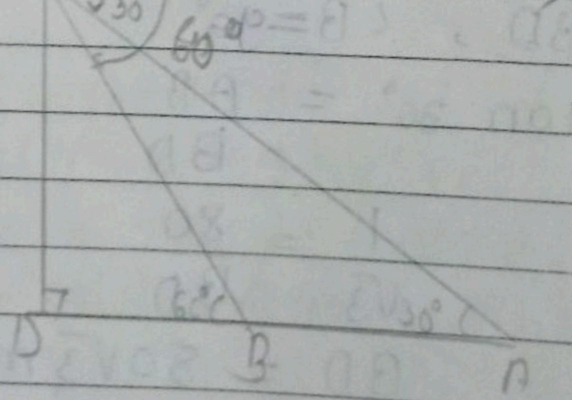
$$CD = 80\sqrt{3} - 80$$

$$CD = 80(\sqrt{3}-1) \text{ m}$$

∴ The distance betⁿ the two ships is $80(\sqrt{3}-1)$ m.

4) A man on a cliff observes a boat at an angle of depression 30°, which is sailing towards the point of the shore immediately beneath him. Three minutes later the angle of depression of the boat is found to be 60°. Assuming that the boat sails at a uniform speed, determine how much more time it will take to reach the shore.

→



CD = cliff.

A - initial position of the boat &

B - position after 3 minute.

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$\angle CAD = \angle HCA = 30^\circ$
$\angle CBD = \angle HCB = 60^\circ$ — (Alternate angles)

In $\triangle ACD$, $\angle D = 90^\circ$

$$\tan 30^\circ = \frac{CD}{DA}$$

$$\frac{1}{\sqrt{3}} = \frac{CD}{DA}$$

$$CD = \frac{DA}{\sqrt{3}} \quad \text{— (1)}$$

In $\triangle BCD$, $\angle D = 90^\circ$

$$\tan 60^\circ = \frac{CD}{DB}$$

$$\sqrt{3} = \frac{CD}{DB}$$

$$CD = \sqrt{3}\,DB \quad \text{— (2)}$$

from (1) & (2)

$$\frac{DA}{\sqrt{3}} = \sqrt{3}\,DB$$

$$DA = 3DB$$

$$DB + BA = 3DB \quad \text{— (D-B-A)}$$

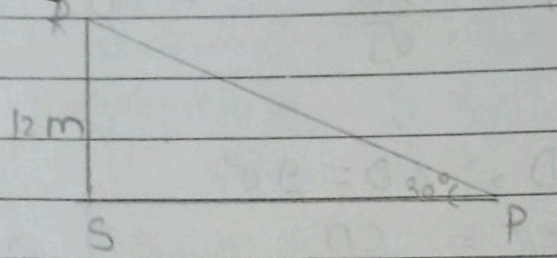
$$BA = 2DB$$

$$DB = \frac{1}{2} BA$$

Now, the boat has taken 3 minutes to sail from A to B it will take $\frac{1}{2} \times 3 = 1.5$ minuts to sail B to D.

Further time taken to reach the shore is 1.5 minutes i.e. 1 minute 30 seconds.

5) A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole & tied to a peg at the ground. the hight of the pole is 12 m & the angle made by the rope with the ground level is 30°. calculate the distance covered by the artist in climbing to the top of the pole.



RS = be the vertical pole
RP, the rope pegged at P
The distance covered by the circus artist is RP

In ΔRSP, $\angle S = 90°$

$$\sin 30° = \frac{RS}{RP}$$

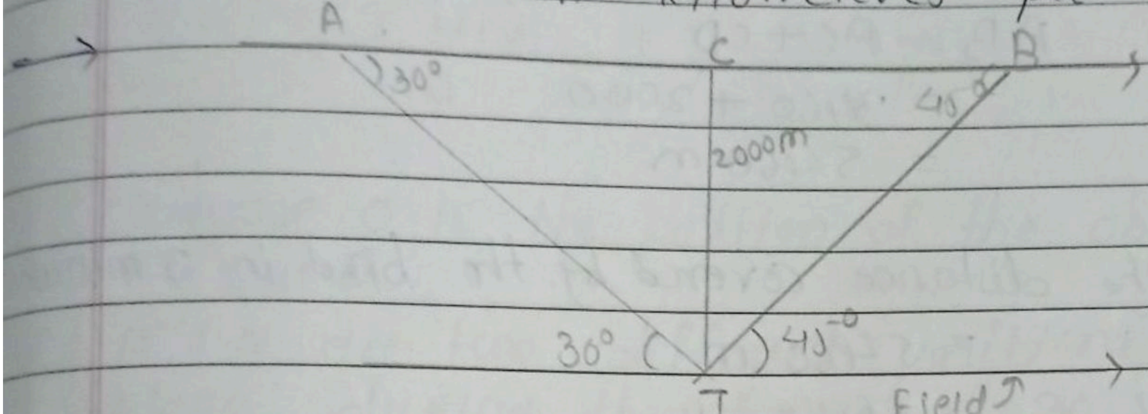
$$\frac{1}{2} = \frac{12}{RP}$$

$$RP = 24\,m$$

∴ The distance covered by the artist in climbing to the top of the pole is 24 m

6) A bird was flying in a line parallel to the ground from north to south at a height of 2000 m. Tom, standing in the

middle of the field, first observed the bird in the north at an angle of 30°. After 3 minutes, he again observed it in the south at an angle of 45°. Find the speed of the bird in kilometeres per hour ($\sqrt{3}=1.73$)



Let T be the position of Tom.
A is the initial position of the bird
B is the final position of the bird
CT = 2000 m.

$\angle CAT = 30^\circ$
$\angle CBT = 45^\circ$ } (Alternate angles)

In ΔCAT, $\angle C = 90^\circ$

$$\tan 30^\circ = \frac{CT}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{2000}{AC}$$

$$AC = 2000\sqrt{3}$$
$$= 2000 \times 1.73$$
$$= 3460 \text{ m} \quad -(1)$$

In ΔCBT, $\angle C = 90^\circ$

$$\tan 45^\circ = \frac{CT}{BC}$$

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$$1 = \frac{2000}{BC}$$

$$BC = 2000 \text{ m} \qquad -(2)$$

from (1) & (2)

$AB = AC + CD$

$= 3460 + 2000$

$= 5460$ m

So, the distance covered by the bird in 3 minutes

$= 5460$ m

$= \frac{5460}{1000}$ km

∴ The distance covered by the bird in 1 hour

$= \frac{5460}{1000 \times 3} \times 60$

$= \frac{5460 \times 20}{1000}$ km/h

$= 109.2$ km/hr

∴ The speed of the bird is 109.2 km/hr

7) The angle of elevation of a jet plane from a point on the ground is $60°$. After a flight of 30 seconds, the angle of elevation changes to $30°$. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane.

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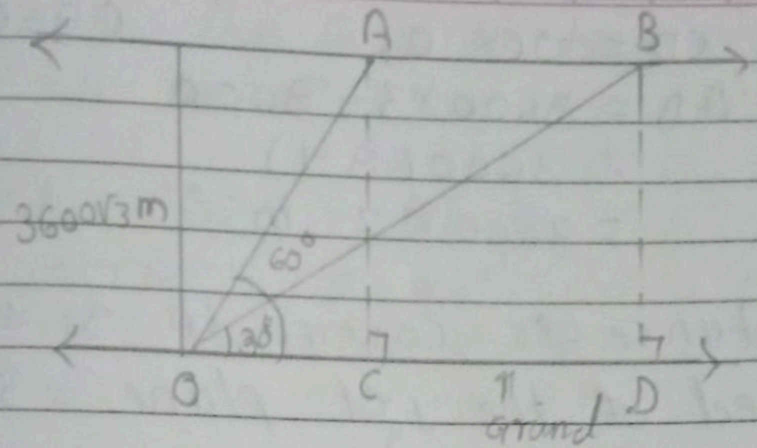


suppose O is the position of the observer on the ground.

A & B are two different positions of the jet plane during the interval of 30 seconds.

OCD is the horizontal line on the ground.

Draw AC & BD ⊥ to the ground.

$\angle AOC = 60°$ & $\angle BOD = 30°$

$AC = BD = 3600\sqrt{3}$ m

In ΔOAC, $\angle A = 90°$

$$\tan 60° = \frac{AC}{OC}$$

$$\sqrt{3} = \frac{3600\sqrt{3}}{OC}$$

$$OC = 3600 \text{ m} \quad -(1)$$

In ΔBOD, $\angle D = 90°$.

$$\tan 30° = \frac{BD}{OD}$$

$$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{OD}$$

$$OD = 3600 \times 3$$

$$OD = 3600 \times 3 \text{ m} \quad -(2)$$

$AB = CD = OC - OC$ $(O - C - D)$

$AB = 3600 \times 3 - 3600$

$= 3600(3-1)$

$= 3600 \times 2$ m

this distance is covered in 30 seconds

$\therefore$ The speed of the jet plane $= \frac{3600 \times 2}{30}$

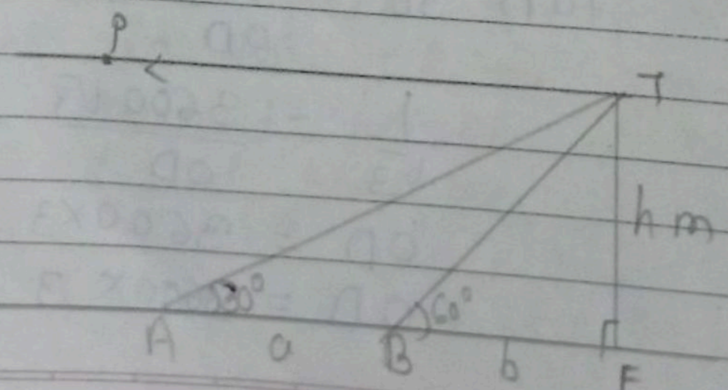
$= 240$ m/s

$= \frac{240}{1000} \times 60 \times 60 \cdot$ km/hr

$= 864$ km/hr

$\therefore$ The speed of the jet plane is 864 km/hr

8) A straigh highway leads to the foot of the tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.



let F be the foot of tower FT of height h meter.

let $AB = a$, $BF = b$.

In ΔTAF, $F = 90^\circ$

$$\tan 30^\circ = \frac{TA}{AF}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{a+b}$$

$$h = \frac{a+b}{\sqrt{3}} \quad -(1)$$

In ΔTBF, $F = 90^\circ$

$$\tan 60^\circ = \frac{TF}{BF}$$

$$\sqrt{3} = \frac{h}{b}$$

$$h = \sqrt{3}\, b \quad -(2)$$

$$\frac{a+b}{\sqrt{3}} = \sqrt{3}\, b \quad \text{(from (1) \& (2))}$$

$$a + b = 3b$$

$$a = 2b$$

Now, the car covers the distance $AB = a$ in 6 seconds.

$$a = 2b = 6$$

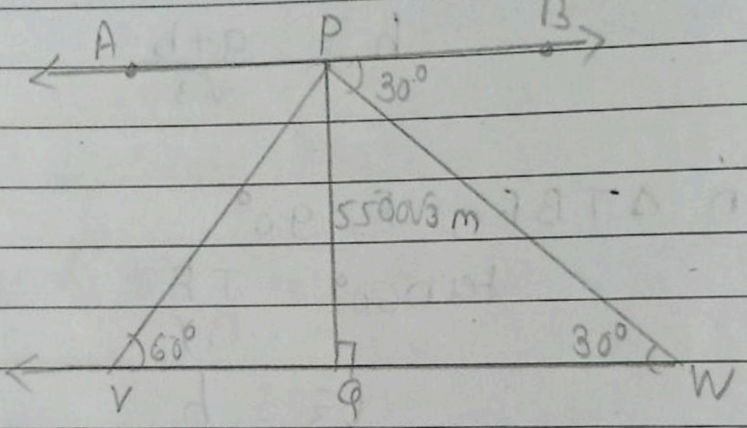
$$b = \frac{6}{2}$$

$$b = 3$$

∴ The car took 3 seconds to reach the foot of the tower from point B.

9) A pilot in an aeroplane observes that vashi bridge is on one side of the plane & worli sea-link is just on the opposite side. The angle of depression of vashi bridge & worli sea-link are 60° & 30° resp. If the aeroplane is at a height of $5500\sqrt{3}$ m at that time, what is the distance between vashi bridge & worli sea-link?

→



let P be the position of the plane
Q is a point on ground

$QP = 5500\sqrt{3}$ m

V = vashi bridge position
W = position of worli sea-link

$\angle PVQ = \angle APV = 60°$
$\angle PWQ = \angle BPW = 30°$ — (Alternate angle)

In ΔPVQ, $\angle Q = 90°$

$$\tan 60° = \frac{PQ}{VQ}$$

$$\sqrt{3} = \frac{5500\sqrt{3}}{VQ}$$

$$VQ = \frac{5500\sqrt{3}}{\sqrt{3}}$$

$$VQ = 5500 \text{ m}$$

ΔPWQ, $\angle Q = 90^\circ$.

$$\tan 30^\circ = \frac{PQ}{WQ}$$

$$\frac{1}{\sqrt{3}} = \frac{5500\sqrt{3}}{WQ}$$

$$WQ = 5500 \times 3$$

$$WQ = 16500 \text{ m}$$

$$VW = VQ + QW \qquad (V-Q-W)$$

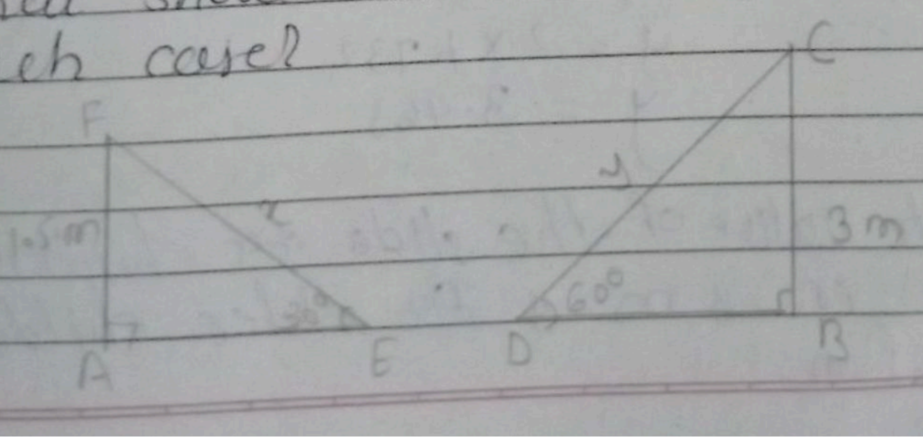
$$= 5500 + 16500$$

$$= 22000 \text{ m}$$

$$= 22 \text{ km}$$

$\therefore$ The distance between the vashi bridge & worli sea-link is 22 km.

10) A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m & is inclined at an angle of 30° to the ground. Whereas for elder children she wants to have a steep slide at a height of 3 m & inclined at an angle of 60° to the ground. What should be the length of the slide in each case?



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suppose x m is the length of Slides for children below 5 years & the length of Slides for elders children be y m.

Given that, $AF = 1.5$ m, $BC = 3$ m.

$\angle AEF = 30^\circ$, $\angle BDC = 60^\circ$

ΔEAF, $\angle A = 90^\circ$

$$\sin 30^\circ = \frac{AF}{EF}$$

$$\frac{1}{2} = \frac{1.5}{x}$$

$$x = 3 \text{ m}$$

ll'y ΔCDB, $\angle B = 90^\circ$

$$\sin 60^\circ = \frac{3}{y}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{y}$$

$$y = \frac{6}{\sqrt{3}}$$

$$y = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$y = \frac{\cancel{6}^{2}\sqrt{3}}{\cancel{3}_{1}}$$

$$y = 2\sqrt{3}$$

$$y = 2 \times 1.732$$

$$y = 3.468$$

Hence the value of the slide for children below 5 years is 3 m & for elder children is 3.468 m.

1) $9\sec^2 A - 9\tan^2 A = ?$

0 3 -9 9

2) $\frac{1}{1+\tan^2\theta} = ?$

$\sec^2\theta$ $\cot^2\theta$ $\cos^2\theta$ $\operatorname{cosec}^2\theta$

3) $\sin 45^\circ + \cos 45^\circ = ?$

$\sqrt{2}$ $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$ $\frac{2}{\sqrt{3}}$

4) If $\tan\theta = \frac{12}{5}$ then $5\sin\theta - 12\cos\theta = ?$

$\frac{119}{13}$ 0 1 $\frac{1}{13}$

5) ~~2se~~ If $\sin\theta = \cos\theta$, θ is an acute angle, then $\tan\theta = ?$

2 $\frac{1}{\sqrt{2}}$ $\sqrt{2}$ 1

6) $\cos^2\theta(1+\tan^2\theta) = ?$

2 $\sin^2\theta$ $2\cos^2\theta$ 1

7) $1+\tan^2\theta = ?$

(a) $\sin^2\theta$; (b) $\sec^2\theta$ (c) $\operatorname{cosec}^2\theta$; (d) $\cot^2\theta$

8) $\sin\theta \times \operatorname{cosec}\theta = ?$

$\sqrt{2}$ $\frac{1}{2}$ 0 1

9) $\sin 60^\circ = ?$

0 $\frac{1}{2}$ $\frac{1}{\sqrt{2}}$ $\cos 30^\circ$

10) If $5\sin\theta - 12\cos\theta = 0$, then $\tan\theta = ?$

$\frac{5}{12}$ $\frac{12}{5}$ $\frac{-5}{12}$ $\frac{-12}{5}$

11) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = ?$

$\tan 30^\circ$ $\tan 60^\circ$ $\cot 60^\circ$ $\cot 90^\circ$

12) $2\sin 30^\circ \cdot \cos 30^\circ = ?$

$\frac{\sqrt{3}}{2}$ $\frac{1}{2}$ 1 $\frac{2}{\sqrt{3}}$

13) $\cos^2\theta + \cos^2(90-\theta) = ?$

1 0 θ -1

14) which of the following is not a fundamental trigonometric identity?

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$1 + \sec^2\theta = \operatorname{cosec}^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

9) If $4\sin^2\theta - 1 = 0$ & $\theta < 90°$.
which of the following statement is true.

$\theta = 0°$ $\theta = 30°$ $\theta = 45°$ $\theta = 60°$

10) $\csc^2 45° + \sec 60°$

4 3 1 $\frac{3}{4}$

11) when we see at higher level, from the horizontal line, angle formed is ——.

angle of depression angle of elevation
0 straigh angle

12) $\sec^2\theta - 1 =$

$\tan^2\theta$ $\cot^2\theta$ $\sin^2\theta$ $\csc^2\theta$