# Fundamental Analysis and Algebra

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# Contents

## Lecture 1: Introduction to Isometries

Thu 12 sep 2023 01:59

# 1 Geometry

**Definition 1.** Felix Klien's geometry is a set $S$, called a space on which a group $G$ operates. The study of properties invariant under the operations of $G$ is called geometry.

## 1.1 Geometries v. Groups

| Space | Tranf. grps | Geometry | Invariants | Linear groups |
|---|---|---|---|---|
| $\mathbb{R}^n$ | isometries | euclidean g | distance | $O_n\mathbb{R}$ |
| $\mathbb{R}^n$ | similarities | Similaritiy g. | angle | $\mathbb{R}_{>0}O_n\mathbb{R}$ |
| $\mathbb{R}^n$ | colllineations | affine g. | parallelism | $GL_n(\mathbb{R})$ |
| $\mathbb{R}^nP$ | projective transf | Projective g. | collinearity | $PGL_{n+1}(\mathbb{R})$ |

and many more... but these are the only ones we will talk about.

**Definition 2.** A transformation on $\mathbb{R}^n$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$. We will denote by $\mathscr{B}(\mathbb{R}^n)$ the set of all transformations on $\mathbb{R}^n$.

**Definition 3.** A transformation on the euclidean plane is called a plane transformation.

- A linear isomorphism is a transformation but a linear map may not.
- The identity map $1 : \mathbb{R}^n \to \mathbb{R}^n$ is a transformation.
- Fix $\boldsymbol{a}$.. The map $\boldsymbol{T_b} : \mathbb{R}^n \to \mathbb{R}^n$, is called a translation. and a translation is a transformation. A more geometric way to view a transformation is as follows: The map takes a point $P$ to the point $P'$ such that the vector $\overrightarrow{PP'} = \boldsymbol{b}$. We denote a translation that takes $P \to P'$ by $\tau_{PP'} \coloneqq T_b$

**Notation.** The map

$$T_{A\boldsymbol{b}} : \mathbb{R}^n \to \mathbb{R}^n, \boldsymbol{v} \mapsto A\boldsymbol{x} + \boldsymbol{b}.$$

## Lecture 2

Thu 12 sep 2023 19:44