Fundamental Analysis and Algebra

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Lecture 1: Introduction to groups

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1 Geometry

Definition 1. Felix Klien's geometry is a set S, called a space on which a group G operates. The study of properties invariant under the operations of G is called geometry.

1.1 Geometries v. Groups

Space	Tranf. grps	Geometry	Invariants	Linear groups
$\mathbb{R}^n$	isometries	euclidean g	distance	$O_n\mathbb{R}$
$\mathbb{R}^n$	similarities	Similaritiy g.	angle	$\mathbb{R}_{>0}O_n\mathbb{R}$
$\mathbb{R}^n$	colllineations	affine g.	parallelism	$GL_n(\mathbb{R})$
$\mathbb{R}^nP$	projective transf	Projective g.	collinearity	$PGL_{n+1}(\mathbb{R})$

and many more... but these are the only ones we will talk about.

Definition 2. A transformation on $\mathbb{R}^n$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$. We will denote by $\mathscr{B}(\mathbb{R}^n)$ the set of all transformations on $\mathbb{R}^n$.

Definition 3. A transformation on the euclidean plane is called a plane transformation.

- A linear isomorphism is a transformation but a linear map may not.
- The identity map $1 : \mathbb{R}^n \to \mathbb{R}^n$ is a transformation.
- Fix $\boldsymbol{a}$.. The map $\boldsymbol{T_b} : \mathbb{R}^n \to \mathbb{R}^n$, is called a translation. and a translation is a transformation. A more geometric way to view a transformation is as follows: The map takes a point P to the point P' such that the vector $\overrightarrow{PP'} = \boldsymbol{b}$. We denote a translation that takes $PtoP'$ by $\tau_{PP'} \coloneqq T_b$

Notation. The map

$$T_{A\boldsymbol{b}} : \mathbb{R}^n \to \mathbb{R}^n, \boldsymbol{v} \mapsto A\boldsymbol{x} + \boldsymbol{b}.$$

Lecture 2

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