(No. of Printed Pages: 2)

Indian Statistical Institute

M.Tech. in Computer Science (2023-24)

Mid Semester Examination 2023

Paper: **Linear Algebra** Date: **20/09/2023**

Time : **2 Hrs.** Full Marks : **30**

Note: Symbols used have their usual meaning. *The figures in the margin indicate full marks.*

Answer any FIVE from the following questions. $5 \times 6 = 30$

1. (a) Consider the following system of equations:

$$\begin{aligned} x + sy &= a \\ x + ty &= b. \end{aligned}$$

Find the conditions for (i) consistency of the equations and (ii) uniqueness of the solution.

(b) Let A and B be $n \times n$ matrices and define $2n \times 2n$ matrices S and M by

$$S = \begin{bmatrix} I & A \\ O & I \end{bmatrix}, \quad M = \begin{bmatrix} AB & O \\ B & O \end{bmatrix}$$

Determine the block form of S^{-1} and use it to compute the block form of the product $S^{-1}MS$.

[3 + 3]

2. (a) Let V be a finite dimensional vector space over $\mathbb{C}$. Show that $|\langle v|w\rangle|^2 \leq \langle v|v\rangle\langle w|w\rangle$, for any two vectors $|v\rangle, |w\rangle \in V$. Derive the conditions for equality.

(b) Let T be the linear transformation which transforms $|0\rangle$ and $|1\rangle$ to $\alpha|0\rangle + \beta|1\rangle$ and $\gamma|0\rangle + \eta|1\rangle$ respectively. Derive the condition for which (i) 'null $T = \{0\}$', (ii) T orthogonal. Matrix

[3 + 3]

3. (a) Prove or disprove: if U_1, U_2, W are subspaces of V such that $V = U_1 \oplus W$ and $V = U_2 \oplus W$, then $U_1 = U_2$.

(b) Let $P_n[x]$ be the set of all polynomials of degree less or equal to n with real coefficient. Define a mapping $T : P_4[x] \to P_4[x]$ by

$$Tp(x) = (ax + b)\frac{d}{dx}p(x), \quad \text{for } p(x) \in P_4[x], \text{ where, } a, b \text{ are fixed complex number.}$$

Check T is linear or not. If, yes, then find the corresponding matrix representing T. Is T invertible? Justify your answer.

[2 + 4]

4. Let U, V, W are finite dimensional vector spaces over a field F and $T : U \to V$, $R : V \to W$ are linear transformations. Show that

P.T.O.

(a) rank of RT = rank of R, if T is surjective,

(b) rank of RT = rank of T, if R is injective.

(c) Consider $W = U$ and $RT = TR$. Does R and T both necessarily invertible? Justify your answer.

[2 + 2 + 2]

5. (a) $T : \mathbb{R}^2 \to \mathbb{R}$ be any transformation with $T(av) = aT(v)$ for all $a \in \mathbb{R}$ and all $v \in \mathbb{R}^2$. Does T necessarily linear? Justify your answer.

   (b) Derive the corresponding matrix P of the linear mapping that that projects every vector v in $\mathbb{R}^3$ onto the line in the direction of $a = (1, 2, 3)$. What are the column space and nullspace of P.

[2 + 4]

6. (a) Is it possible for a matrix to have the vector $(3, 1, 2)$ in its row space and $(2, 1, 1)^T$ in its null space? Explain.

   (b) Use the Gram-Schmidt process to find an orthonormal basis for the space spanned by $x_1 = (4, 2, 2, 2)^T, x_2 = (2, 0, 0, 2)^T, x_3 = (3, 2, 0, 2)^T, x_4 = (1, 1, -1, 1)^T$.

[2 + 4]

INDIAN STATISTICAL INSTITUTE

M.Tech (CS) I Year: 2023-24 (Semester II)

Statistical Methods: Mid-Semester Exam

Date: 27/02/2024 Time: 2:30 PM - 4:30 PM Full Marks: 30

Note: Show necessary steps to solve all of the following problems. There is no credit for a solution if the appropriate work is not shown even if the answer is correct. Use of calculator is allowed for this exam.

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1. The following data represents weights of 12 babies in kilogram.

2.8, 2.9, 2.0, 1.4, 1.7, 1.3, 2.3, 4.9, 4.0, 2.4, 1.8, 3.0

   (a) Find the upper fence, lower fence and outliers (if any) in the dataset. [3]

   (b) Construct a box plot and comment on the skewness of the data. [2+1]

   (c) Suppose the above observations are values of a variable x. What is the Spearman's rank correlation coefficient between x and y variables where $y = \log(2 + e^{2-2x})$? Justify your answer. [1+1]

2. Let $X_1, \ldots, X_n$ be i.i.d. samples with common density function

$$f(x \mid \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1}\, \mathbb{I}_{\{0 \le x \le \beta\}}, \text{ where } \alpha, \beta > 0 \text{ and } \mathbb{I}_{\{A\}} \text{ is the indicator function of an event } A.$$

   (a) Find a two-dimensional sufficient statistic for (α, β). [2]

   (b) Find the maximum likelihood estimator $(\widehat{\alpha}, \widehat{\beta})$ of (α, β). [4]

   (c) Show that $n(\beta - \widehat{\beta})$ converges in distribution, as $n \to \infty$, to an exponential distribution. [3]

   (d) If $\beta = 1$ in the above density function, write down the likelihood function based on the given samples and compute the Fisher Information $I_{\mathbf{X}}(\alpha)$ contained in $\mathbf{X} = (X_1, \ldots, X_n)$. [3]

3. Suppose $X_1, \ldots, X_n$ are i.i.d. observations having common density function

$$f(x \mid \theta) = \frac{4\theta^4}{x^5}\, \mathbb{I}_{\{x \ge \theta\}}, \text{ where } \theta > 0.$$

   (a) Find the method of moment estimator of θ, say $\widehat{\theta}_{MME}$. [2]

   (b) If $X_{(1)} = \min\{X_1, \ldots, X_n\}$, then show that $\delta(\mathbf{X}) = \frac{4n-1}{4n} X_{(1)}$ is both unbiased and consistent estimator of θ. [2+2]

   (c) What is the efficiency of $\widehat{\theta}_{MME}$ relative to $\delta(\mathbf{X})$? [4]

INDIAN STATISTICAL INSTITUTE

CLASS TEST
M.TECH(CS) I YEAR

DATA STRUCTURES

Date: 14.11.2022 Maximum marks: 40 Duration: 1 hour.

1. Give an algorithm to find the 3rd smallest element in a binary min-heap in constant time. [3]

2. State what you mean by AVL property of a node. Show that an AVL tree with n nodes should have $O(\lg n)$ height. [2 + 5 = 7]

3. Consider a binary search tree whose nodes are labeled by A, B, C, D, E, F, G, H. The in-order and pre-order travaersal of the tree yeilds the following sequence of nodes:

   In-order: H, D, I, B, E, J, A, F, C, G, K

   Pre-order: A, B, D, H, I, E, J, C, F, G, K

   Answer the following questions. You need not provide explanations.

   (a) Draw the tree.
   (b) Assume that the tree is a valid binary search tree. What is the successor of node A.
   (c) Is the tree an AVL tree?
   (d) Re-draw the tree after deleting node C.
   (e) Is this new tree an AVL tree?

   [5 + 1 + 1 + 2 + 1 = 10]

4. Consider a binary search tree, in which in addition to the standard fields (data, left and right) each node has an integer field called size, which stores the number of elements in the subtree rooted at this node. In a range query we are given two key values x_1, x_2, where $x_1 \leq x_2$, and wish to return a count of the number of nodes in the tree whose key value x satisfies $x_1 < x \leq x_2$. Give pseudocode for this operation, and briefly explain how your algorithm works. Your algorithm should run in $O(h)$ time, where h is the height of the tree.

   [10]

5. Consider a binary search tree implementation where each node have a key, a parent pointer and left and right child pointers. Given a node X in the tree, write a routine which does single right rotation at X. Adjust all pointers (including the parent pointers) appropriately.

   [10]

INDIAN STATISTICAL INSTITUTE

Mid Semester

M. Tech. CS 2023-24

DESIGN AND ANALYSIS OF ALGORITHMS

Date : 19.09.2023 Maximum Marks : 30

Duration : 2 Hours 30 minutes

(The paper carries four questions of ten marks each. You may answer all four questions. Your final marks will be the sum of highest three marks obtained.)

1. Are the statements in $[(a) - (c)]$ True or False? Justify your answers.
   (a) $\frac{n^3}{\log n} = \Omega(n^3)$.
   (b) $n\sqrt{(n)} = O(n \log^3 n)$
   (c) $n\sqrt{(n)} = \Theta(n \log n)$
   (d) $\frac{O((n*2^n)}{2^n} = O(?)$, Justify your answer?
   (e) $GCD(a, b) = GCD(r_1 = (a \, mod \, b), b) = GCD(r_2 = (b \, mod \, r_1), r_1) = \ldots = GCD(r_3 = (r1 \, mod \, r_2), r_2) = GCD(r_n, 0) = r_n$. If $n = O(f(a))$ then find the value of $f(a)$.

[2 ×5]

2. Write a linear time algorithm to build a heap on n integers. Analyze running time complexity of your algorithm. [5 + 5]

3. Given a set of n integers which contain at most k distinct values. Write an $O(n \log k)$ time sorting algorithm. Analyze running time complexity of your algorithm. [4 + 6]

4. State and prove a non-trivial lower bound for constructing an AVL-tree with n integers. [2 + 8]

INDIAN STATISTICAL INSTITUTE

Mid Semester Examination

M.Tech CS, 2023-2024 (Semester – I)

Discrete Mathematics

Date: 22 September 2023 | Maximum Marks: 60 | Duration: 2 hours

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General comment. Answer as much as you can, but the maximum you can score from both Group-A and Group-B is 30.

Notations and definitions. $\mathbb{N}$ denotes the set of *natural numbers*. Given any set X, 2^X denotes the collection of all subsets of X.

Formally, given two partially ordered sets (posets) $(S, \leq)$ and $(T, \preceq)$ a function $f : S \to T$ is an *embedding* if for all x and y in S

$$x \leq y \text{ if and only if } f(x) \preceq f(y)$$

A $n \times n$ matrix $M = (M_{ij})_{1 \leq i,j \leq n}$ is a *permutation matrix* if $M_{ij} \in \{0, 1\}$, and every entry row and column of M contains exactly one 1.

Group-A

(AQ1) Show the equivalence between equivalence relations on a set Ω and different partitions of Ω. [10]

(AQ2) (a) Show that every connected graph contains a spanning tree.

(b) Let G be a connected graph on n vertices. If the number of edges of G is $n - 1$ then G is a tree.

[5+5 = 10]

(AQ3) Let $(X, \leq)$ be a poset where X is a finite set. If the length of any chain in X is less than $m + 1$ then X can be written as a union of m antichains. [10]

(AQ4) Let $G = (A \cup B, E)$ be a bipartite graph. Show that the size of the minimum vertex cover of G is equal to the size of the largest matching in G. (Can use Dilworth's Theorem for posets.) [10]

Group-B

(BQ1) Let G be a graph with n vertices, m edges, and T triangles. Show that

$$T \geq \frac{m}{3n}\left(4m - n^2\right).$$

[10]

(BQ2) Let A = $(A_{ij})_{1\leq i,j\leq n}$ be an $n \times n$ matrix with $A_{ij} \in \{0,1\}$, for all $1 \leq i, j \leq n$. If the sum of entries of any row or column of A is k then A can be written as a sum of k many permutation matrices.

[10]

(BQ3) Let $A_1, \ldots, A_n$ be distinct subsets of $\mathbb{N}$. Show that there exists a subset X of $\mathbb{N}$ of size at most $n-1$ such that for all $i \neq j$, we have $A_i \cap X \neq A_j \cap X$. [10]

(BQ4) Show that for every partially ordered set $(X, \preceq)$ there exists an embedding into the partially ordered set $(2^X, \subseteq)$.

[10]

INDIAN STATISTICAL INSTITUTE

MIDTERM EXAMINATION
M.TECH(CS) I YEAR

DATA AND FILE STRUCTURES

Date: 18.09.2023 Maximum marks: 60 Duration: 2.0 hours.

The paper contains 70 marks. Answer as much as you can, the maximum you can score is 60.

1. Give short answers to the following questions.

   (a) Suppose we have a pointer to a node in a singly linked list that is guaranteed not to be the last node in the list. We do not have pointers to any other nodes (except by following links). Describe an $O(1)$ algorithm that logically removes the value stored in such a node from the linked list, maintaining the integrity of the linked list.

   (b) Describe a variant of Karatsuba algorithm to find squares of n-bit numbers using squares of three $n/2$-bit numbers. What is the running time of your algorithm?

   (c) Suppose we have a Skiplist with n elements where a promotion of a node to a list in higher level takes place with probability p. Find the expected height of such a skiplist.

   (d) Consider a data structure which is initially empty and can support the addition of one item at a time. Show that, at some point during the addition of n items, the data structure is wasting at least $\sqrt{n}$ amount of space.

[$3 + 6 + 6 + 5 = 20$]

2. Given two sorted singly linked lists with head pointers $h1$ and $h2$, describe a procedure to merge these two lists into another sorted linked list. Your procedure should take as input $h1$ and $h2$ and output $h3$ the head pointer of the merged linked list. Your procedure should take time $O(n_1 + n_2)$, where n_1, n_2 are the lengths of the two input lists.

[10]

3. Given an array A of size n. An index i ($0 \leq i \leq n-1$) of the array A is called GOOD if any one of the following are true

   - $i = n - 1$
   - $i < n - 1$ and $A[i] > A[j]$ for all $j > i$.

   Given an array A of size n describe an efficient algorithm to find all the GOOD indices of A. What is the running time of your algorithm. [*Hint: Consider using a stack*] [10]

4. Consider the Quack abstract data type which is a combination of a queue and a stack. A Quack essentially maintains a list with the following operations:

QPush(x): Add x at the front of the list.

QPop(): Remove and return the element in the front of the list.

QPull(): Remove the element in the back of the list.

Describe an implementation of a Quack using three stacks and $O(1)$ additional memory, so that the amortized time for any QPush, QPop, or QPull operation is $O(1)$. In particular, each element in the Quack must be stored in exactly one of the three stacks. You cannot access the component stacks except through the interface functions of the stacks, i.e., Push and Pop which runs in $O(1)$ time. Your solution should include the following.

(a) A high level description of your implementation in English.

(b) Pseudocodes for the QPush(x), QPop and QPull operations.

(c) A precise argument which proves that the amortized cost of each operation is $O(1)$.

[$3 + 6 + 6 = 15$]

5. The RankedSSet abstract data type is an extension of the Sorted-Set ADT that supports the following operations on a set

insert(x): Insert x in the set.

remove(x): Remove x from the set.

get(i): return the value x in the set that is larger than exactly i other elements in the set.

rank(x): return the number, i, of elements in the set that are less than x (note that x is not necessarily in the set).

Describe an implementation of RankedSSet where get(i) and rank(x) runs in expected $O(\lg(n))$ time. You can use any result or procedure which has been discussed in class without their detailed description.

[15]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination

M. Tech (CS), 2023-2024 (Semester – I)

Probability and Stochastic Processes

Date: 21.09.2023 | Maximum Marks: 75 | Duration: 3.0 Hours

Note:

Answer all questions in Group-A and answer as much as you can in Group-B but the maximum you can score in Group-B is 40.

$E[X]$ and var$[X]$ denote the expectation and variance of the random variable X, respectively.

Group A

(QA1) Let $A_1, A_2, \ldots$ be a decreasing sequence of events, so that $A_1 \supseteq A_2 \supseteq \ldots$. Now define a limiting event A as

$$A = \lim_{n\to\infty} A_n = \bigcap_{n=1}^{\infty} A_n$$

Then, show that

$$\Pr(A) = \Pr\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} \Pr(A_n)$$

[7]

(QA2) Two gamblers, G_1 and G_2 bet on the outcomes of successive flips of a coin. On each flip, if the coin comes up heads, G_1 collects Rs. 1 from G_2; and if the coin comes up tails, G_2 collects Rs. 1 from G_1. The gamblers continue to do this until one of them runs out of money. Assume that the successive flips of coins are independent and the probability of a coin flip resulting in a head is p, and resulting in a tail is $1-p$. Show that with probability 1, either G_1 or G_2 will wind up with all the money. [8]

(QA3) (i) State and prove Bayes' theorem.

(ii) Show that for any n events $A_1, A_2, \ldots, A_n$, $\Pr\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \Pr(A_i)$

[(1+3)+4=8]

(QA4) Let $Z = X_1 + X_2 + \cdots + X_N$ where N is a random variable that takes non-negative integer values and $X_1, X_2, \ldots$ are identically distributed random variables. If $N = 0$, we let $Z = 0$. Assume that $N, X_1, X_2, \ldots$ are mutually independent. Deduce an expression for $E[Z]$. [4]

(QA5) (i) State and prove Chebyshev's inequality.

(ii) Let X denote a random variable indicating the number of heads in a sequence of n independent fair coin flips. Compute a lower bound on $\Pr(X \leq 2n/3)$.

[(1+3)+4=8]

Group B

(QB1) (i) Let $\mathcal{F}$ be a σ-field of subsets of Ω and suppose that $B \in \mathcal{F}$. Show that $\mathcal{G} = \{A \cap B : A \in \mathcal{F}\}$ is a σ-field of subsets of B.

(ii) A fair coin is tossed repeatedly. Show that, with probability 1, a head turns up sooner or later.

[5+5=10]

(QB2) A coin that has probability of heads equal to p is tossed successively and independently until a head comes twice in a row or a tail comes twice in a row. Find the expected value of the number of tosses. [10]

(QB3) Let $X_1, \ldots X_n$ be independent random variables and let $Z = X_1 + \cdots + X_n$. Suppose that each X_i, $i = 1, \ldots, n$ is a Bernoulli with parameter p_i, and that $p_1, \ldots, p_n$ are chosen so that the mean of Z is a given $\mu > 0$. Deduce what p_i's should be so that the variance of Z is maximized. [10]

(QB4) Suppose that X and Y are independent, identically distributed geometric random variables with parameter p. Show that

$$\Pr(X = i \mid X + Y = n) = \frac{1}{n-1}, \quad i = 1, \ldots, n-1.$$

[10]

(QB5) (i) Consider n independent tosses of a coin with probability of a head equal to p. Let X and Y be the number of heads and of tails, respectively. Compute the correlation coefficient of X and Y.

(ii) Show that $\operatorname{Cov}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} \operatorname{Cov}(X_i, Y_j)$.

[5+5=10]

INDIAN STATISTICAL INSTITUTE

CLASS TEST
M.TECH(CS) I YEAR

DATA STRUCTURES

Date: 18.11.2023 Maximum marks: 40 Duration: 1 hour.

1. Consider a binary search tree whose nodes are labeled by A, B, C, D, E, F, G, H. The in-order and pre-order travaersal of the tree yields the following sequence of nodes:

   In-order: H, D, I, B, E, J, A, F, C, G, K

   Pre-order: A, B, D, H, I, E, J, C, F, G, K

   Answer the following questions. You need not provide explanations.

   (a) Draw the tree.

   (b) Assume that the tree is a valid binary search tree. What is the successor of node A.

   (c) Is the tree an AVL tree?

   (d) Re-draw the tree after deleting node C.

   (e) Is this new tree an AVL tree?

   $[5 + 1 + 1 + 2 + 1 = 10]$

2. Draw the binary min heap that results from inserting 9, 3, 7, 4, 8, 2, 6, 5 in that order into an initially empty binary min heap. You do not need to show the array representation of the heap. You are only required to show the final tree, although drawing intermediate trees may result in partial credit. If you draw intermediate trees, please circle your final result for any credit.

   [10]

3. Consider a binary search tree, in which in addition to the standard fields (data, left and right) each node has an integer field called size, which stores the number of elements in the subtree rooted at this node. In a range query we are given two key values x_1, x_2, where $x_1 \leq x_2$, and wish to return a count of the number of nodes in the tree whose key value x satisfies $x_1 < x \leq x_2$. Give pseudocode for this operation, and briefly explain how your algorithm works. Your algorithm should run in $O(h)$ time, where h is the height of the tree. [10]

4. Consider a binary search tree implementation where each node have a key, a parent pointer and left and right child pointers. Given a node X in the tree, write a routine which does single right rotation at X. Adjust all pointers (including the parent pointers) appropriately.

   [10]

INDIAN STATISTICAL INSTITUTE

M.Tech (CS) I Year: 2023-24 (Semester II)

Statistical Methods: Class Test 1

Date: 21/02/24 Time: 60 minutes Full Marks: 20

Note: Show necessary steps to solve all of the following problems. There is no credit for a solution if the appropriate work is not shown even if the answer is correct.

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1. Let $X_1, \ldots, X_n$ be independent samples from the common distribution with density

$$f(x \mid \theta) = \frac{3\theta^3}{x^4} \mathbb{I}_{\{x \geq \theta\}}, \text{ where } \theta > 0 \text{ and } \mathbb{I}_{\{A\}} \text{ is the indicator function of an event } A.$$

Consider estimators $\delta_1 = \frac{3n-1}{3n} X_{(1)}$ and $\delta_2 = \frac{2}{3}\overline{X}$ of θ, where $\overline{X}$ is the sample mean and $X_{(1)} = \min\{X_1, \ldots, X_n\}$.

(a) Are δ_1 and δ_2 unbiased estimators of θ? [3]

(b) Verify whether δ_1 and δ_2 are consistent estimators of θ. [4]

(c) Find the efficiency of δ_1 relative to δ_2 and comment whether δ_1 or δ_2 is preferred. [4]

2. Suppose $X_1, \ldots, X_n$ are i.i.d. samples from $Gamma(\alpha, \beta)$ distribution with density function

$$f(x \mid \alpha, \beta) = \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha} \mathbb{I}_{\{x>0\}} \quad \text{where } \alpha > 0,\ \beta > 0.$$

(a) Find a two-dimensional sufficient statistic for (α, β). [2]

(b) Find the method of moment estimator of (α, β). [4]

(c) If $\alpha = 2$ in the above density function, find the maximum likelihood estimator of β. [3]

INDIAN STATISTICAL INSTITUTE

M.Tech (CS) I Year: 2023-24 (Semester II)

Statistical Methods: Class Test 2

Date: 06/04/2024 | Time: 60 minutes | Full Marks: 20

Note: Show necessary steps to solve all of the following problems. There is no credit for a solution if the appropriate work is not shown even if the answer is correct.

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1. Suppose X has the probability mass function $f(x|\theta) = (1-\theta)\,\theta^x$ for $x = 0, 1, 2, \ldots$, where $0 < \theta < 1$. For $\alpha = 0.15$, derive a most powerful size-α test for testing $H_0 : \theta = 0.9$ vs $H_1 : \theta = 0.8$ based on the single observation X. [6]

2. Suppose $X_1, \ldots, X_n$ are i.i.d. observations from Rayleigh distribution with the density function
$$f(x|\theta) = \frac{2}{\theta}\, x\, e^{-x^2/\theta}\, \mathbb{I}_{\{x>0\}} \text{ where } \theta > 0.$$
   (a) For a prefixed $\alpha \in (0, 1)$, find the UMP size-α test for testing $H_0 : \theta \le \theta_0$ vs $H_1 : \theta > \theta_0$ where θ_0 is known. You may use the fact that $\frac{2X_1^2}{\theta} \sim \chi_2^2$. [5]
   (b) Derive the p-value of this test in terms of the CDF of a chi-squared distribution. [2]

3. A psychological study was conducted to compare the reaction times of men and women to a stimulus. Independent random samples of 10 men and 12 women were employed in the experiment. The sample means for men and women were 3.4 and 3.8 (in seconds) respectively. The sample variances for men and women were 0.18 and 0.14 respectively. The reaction times of men and women are assumed to be independent and normally distributed with same population variance. Do the data present sufficient evidence to suggest a difference between the mean reaction times for men and women? State the null and alternative hypotheses to be tested and draw your conclusion at 5% level of significance. [7]

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

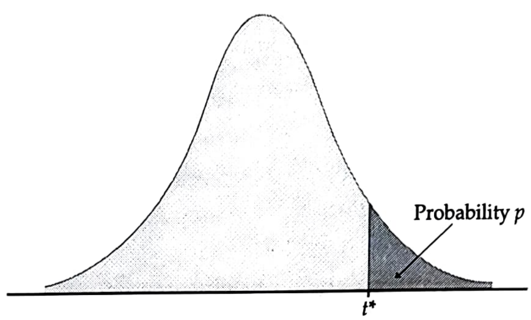

TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											