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From sucking worms to Windkessel: The physics of an early eighteenth century firefighting device

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We describe the physics of the "Windkessel effect" and its role in smoothing the output of water produced by piston-driven pumps found in early fire engines and modern residential well houses. We also construct a simple, analytical model of its operation and apply this model to the Windkessel in Richard Newsham's 1725 fire engine. We find that Newsham's Windkessel reduces the variations in the pump output stream from a high of 80% to a low of 16%.
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I. INTRODUCTION

Probably neither the fanciful English phrase "sucking worm" nor the German loan word "Windkessel" are familiar to readers of the *American Journal of Physics*. Even so these terms refer to a successful, long-lived, currently used, physics-based, low-technology device that smooths out the pulses in a stream of pumped water. This regulating device originated as an in-line attachment to a manually operated, piston-driven pump that drew water from a reservoir. The need for these pumps in late 17th and early 18th century England was intensely felt after the Great Fire of London in 1666, which burned for four days and destroyed most of that closely built, wooden city of 300 000 to 400 000 inhabitants.

The first response to this need was the "sucking worm"—invented, patented, and marketed by the Dutch-born Englishman John Lofting (1659–1744). Apparently, "sucking" refers to a pump that pulls water from a reservoir while "worm" refers to the then newly available leather hoses that attach to the output of that pump. While it appears that no realizations of Lofting's sucking worm have survived to this day, contemporary advertisements tout its ability to pump a "continuous stream" of water "400 feet high."[1] The latter is surely an exaggeration; the former may be as well.

Contemporary engravings of the sucking worm in action (see Ref. 1) show a stream of water arching over the decoration on top of the newly rebuilt Royal Exchange, 160 ft (49 m) high. This would require a ground-level water speed of at least 22 m/s—a speed that, as we shall see, could be produced with a manually operated pump. However, without the addition of some device to regulate and smooth the stream of water, a piston-driven pump would produce water in spurts, rather than, as advertised, continuously. However, no such regulating device is mentioned in Lofting's sucking-worm patent of 1690[2] or illustrated in his advertisements.

Such a device may have been implemented in the fire-engine pumps designed and built by Jan van der Heyden in late 17th century Holland. van der Heyden's book *Fire Engines with Water Hoses and the Method of Fighting Fires now used in Amsterdam* (1690)[3] mentions that the "internal parts" of his new fire engines "are made in such a way that they continuously, without the slightest diminution, pump water." Unfortunately, van der Heyden never described these "internal parts"—possibly, in order to forestall patent infringement.[4]

A stream-regulating device was certainly implemented in the fire-engine pumps that the Englishman Richard Newsham patented in 1721 and 1724.[5] We know this because some of Newsham's fire engines survive to this day at the *Hall of Flame* fire-fighting museum in Phoenix, Arizona, at Colonial Williamsburg in Williamsburg, Virginia, and at several municipal museums in England. This device is an airtight chamber through which the pumped water enters before exiting into a manipulable hose or pipe. As water surges into the airtight chamber during the pump's high injection-rate phase, the air trapped in the upper part of the chamber is compressed and then, during its low injection-rate phase, the compressed air expands and expels water through a pipe or hose. We show a schematic of this device in Fig. 1.

However, according to the *Dictionary of National Biography*, Newsham "was not the inventor of that contrivance."[6] However, if not Richard Newsham, then who? Possibly, Jan van der Heyden. Or was it John Lofting? We simply do not know.

The term "Windkessel" (German for "wind chamber") was not used, at least in English, to describe the airtight chamber inserted into a pump line until 1899 when the German physiologist Otto Frank popularized the word.[7] Apparently, Frank's intention was to find a name he could use metaphorically to describe the heart-aorta blood delivery system. For the heart resembles a single-piston, two-stroke pump that pushes blood into the aorta during systole (injection) and relaxes during diastole (relaxation). Because the aorta is elastic it expands during systole and contracts during diastole, and, in this way, regulates the blood supply. Otto Frank modeled the physiological Windkessel effect with a linear equation, mathematically equivalent to a periodically forced, RC electrical circuit.[8]

While the physiological Windkessel effect is important, there is no literal, air-filled space in the heart-aorta system. Historical documents in English do refer to the "wind chamber" of a fire-engine pump or to its "pressure chamber" or its "pressure tank." Water-carrying fire-engines continued to use Windkessels until the middle of the 20th century when their piston pumps were replaced by centrifugal pumps. However, systems that pump ground water to residences still use piston pumps with pressure chambers that in this context are sometimes called a "hydraulic accumulator" or a "bladder

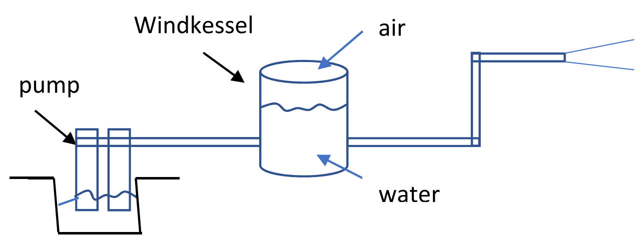

Fig. 1. Schematic of a Windkessel regulating the flow of water produced by a fire engine pump. The pump in Newsham's 1725 fire engine consists of two pistons.

chamber." They differ from fire-engine Windkessels in that their input is irregular. (We show a photo of a modern, residential Windkessel in Fig. 2.) In this way, Frank's usage failed to stick to its original reference. Even so, we find the word "Windkessel" useful as a term covering its various synonyms: wind chamber, pressure chamber, pressure tank, and bladder chamber.[9]

We have found no analytical model of the air-chamber Windkessel effect in the pedagogical literature and intend, in this paper, to provide one. While our primary purpose is to illustrate the Windkessel effect rather than to model its various applications, we do focus on one application because its structure is particularly simple: the Windkessel in Richard Newsham's 1725 fire engine. Incidentally, one of us (D.S.L.) was able to inspect this engine at the *Hall of Flame* museum in Phoenix, Arizona. See the photo in Fig. 3 for another realization of this engine residing in Colonial Williamsburg.

The Newsham fire engine was usually pulled to the scene of a fire by a team of men. Its narrow coffin-shaped, open-air, water container—which allowed it to go through doorways—was kept filled by bucket brigade as a pair of manually operated pistons pulled the water up from its reservoir and pushed it through an airtight pressure vessel (the

Fig. 2. Modern pressure tank or Windkessel in the well house of a system supplying water to a residence. The tape measure indicates a height of about 5 ft (3.3 m). Courtesy of Roger Frye, Santa Fe, New Mexico.

Fig. 3. Newsham fire engine circa 1750 residing at Colonial Williamsburg. Photo used by permission of Colonial Williamsburg.

Windkessel) before discharging the water through a short, approximately one meter-long, metal tube that could be aimed in the direction of a fire. Each of the two cylindrical pumps on the 1725 Newsham fire-engine are outfitted with leather flaps that allow water to be drawn up from the reservoir into one cylinder while water is expelled into the Windkessel from the other cylinder. Thus, the injection of water into the Newsham Windkessel never ceases except (possibly) instantaneously as the pistons simultaneously reverse their directions. Even so, the output of a two-piston pump varies with time.

In order to model the Windkessel effect on Newsham's 1725 fire-engine pump line, a question must be answered, and some assumptions made. Is the air trapped within the Windkessel compressed and expanded isothermally or adiabatically? We adopt, isothermal air partly for convenience and partly for pedagogical reasons. Isothermal compressions and expansions allow us to keep the mathematics relatively simple and to avoid solving a Chini equation[10] with hypergeometric functions that, we believe, would obscure rather than clarify the physics. For similar reasons, we assume there is no resistance to the flow within the relatively short pipe of the Newsham fire-engine—an assumption we will revisit after we numerically characterize the 1725 Newsham fire-engine in Sec. V.

This analysis is partly a case study in constructing and testing a model. To do so we collect data from early documents, from current videos of historical equipment in operation, and from a visual inspection of a surviving Windkessel. The analysis is also an exercise in the art of problem solving in which modeling choices are made and reasonable approximations are applied in order to highlight the essentials of the Windkessel effect.

We note that while Bernoulli's principle (which helps frame our analysis) was established in 1738, the Guy–Lussac law (upon which the ideal gas equation of state depends) was not formalized until the beginning of the 19th century. Therefore, this kind of analysis could not have been completed before the early 19th century.

This paper is organized as follows: After this Sec. I we establish, in Sec. II, a dynamical equation based on the conservation of water as it flows through the Windkessel. We make this dynamical equation dimensionless in Sec. III and plot its time-dependent, output profile in Sec. IV. Section V is devoted to numerical data extracted from patents, advertisements, photos, and a visual inspection of an actual 1725 Newsham fire engine. These data are used to estimate the Windkessel's regulating effect. Section VI is a summary and conclusion.

II. THE DYNAMICAL EQUATION

The dynamics of the Windkessel effect is that of conservation of water in the pressure chamber. Accordingly, water, an incompressible fluid, enters the Windkessel at a given volumetric flow rate $\dot{V}_i(t)$ where the dot indicates a time derivative. Then the volume of the water in the Windkessel increases (or decreases) at a rate $\dot{V}_w$ and leaves at a rate $\dot{V}_o$. In these terms, conservation of water volume is expressed by

$$\dot{V}_i(t) = \dot{V}_w(t) + \dot{V}_o(t), \tag{1}$$

where these rates are signed quantities, and, as noted, the function $\dot{V}_i(t)$ is given.

The Windkessel container is rigid and airtight. Hence, its volume is constant, and the sum of the volume of the water $V_w(t)$ and of the air $V_a(t)$ in the Windkessel is constant. Symbolically,

$$V_w(t) + V_a(t) = \text{const.} \tag{2}$$

Consequently,

$$\dot{V}_w(t) = -\dot{V}_a(t). \tag{3}$$

Since air is typically an ideal gas, $V_a = NkT/P$, where T and P are the temperature and pressure of the air. We assume that the pressure P is uniform throughout the Windkessel and equal to the pressure of the water in the Windkessel, as these fluids must be in mechanical equilibrium. Therefore,

$$\dot{V}_w = -\frac{d}{dt}\left(\frac{NkT}{P}\right), \tag{4}$$

and the conservation equation (1) becomes

$$\dot{V}_i(t) = -\frac{d}{dt}\left(\frac{NkT}{P}\right) + \dot{V}_o. \tag{5}$$

This is our general dynamical equation. That of our specific model follows.

When the air temperature T and the air particle number N are constant during the pump-induced compressions and expansions, dynamical equation (5) becomes

$$\dot{V}_i(t) = -NkT\frac{d}{dt}\left(\frac{1}{P}\right) + \dot{V}_o. \tag{6}$$

Furthermore, the speed u with which the water exits the Windkessel is determined by Bernoulli's principle, that is, by

$$P = \rho\frac{u^2}{2} + P_a, \tag{7}$$

where P is the pressure in the Windkessel, P_a is the atmospheric pressure or, equivalently, the pressure within the short output tube, and ρ is the (constant) density of water. Since $uA = \dot{V}_o$ where A is the cross-sectional area of the Windkessel output pipe or hose, Eq. (7) becomes

$$P = \frac{\rho\dot{V}_o^2}{2A^2} + P_a. \tag{8}$$

Consequently, dynamical equation (6) becomes

$$\dot{V}_i(t) = -\frac{NkT}{P_a}\frac{d}{dt}\left(\frac{1}{\rho\dot{V}_o^2/2A^2P_a + 1}\right) + \dot{V}_o, \tag{9}$$

which could be solved numerically for a given injection rate $\dot{V}_i(t)$.

Rather than solve (9) numerically, we capture the essence of its solution by exploiting the fact that typically $\rho\dot{V}_o^2/2A^2 \gg P_a$. This is equivalent to dropping the second term on the right-hand side of Bernoulli's principle (7) or (8), and, thus, of assuming that the pressure P inside the Windkessel and the kinetic energy density of the water $\rho\dot{V}_0^2/2A^2$ exiting the Windkessel are both much larger than atmospheric pressure P_a. This approximation is like one sometimes used when determining how far or how high a jet of water exiting a hole in a tank, with a pressure head, will go—a problem familiar to introductory physics students studying elementary fluid dynamics. We will find, in Sec. V, that the approximation $\rho\dot{V}_o^2/2A^2 \gg P_a$ is at least approximately met for parameters that describe the 1725 Newsham fire engine.

Adopting this approximation preserves the Windkessel effect and allows us to replace (9) with

$$\dot{V}_i(t) = -\frac{2A^2NkT}{\rho}\frac{d}{dt}\left(\frac{1}{\dot{V}_0}\right)^2 + \dot{V}_o. \tag{10}$$

Equation (10) is a relatively simple, first order, non-linear, inhomogeneous, ordinary, differential equation that can, when $\dot{V}_i$ is constant, be solved in terms of simple functions.

III. INPUT MODEL AND DIMENSIONLESS VARIABLES

A two-cylinder pump would produce an input profile $\dot{V}_i(t)$ whose time dependence is close to $\sin^2\omega t$ where ω is the frequency of each of the two identical pistons. Thus, the input flow would oscillate between maximum (i.e., one) and minimum (i.e., zero) values. A simple way to model this flow is to assume a step-up and step-down volumetric inflow profile $\dot{V}_i(t)$. Accordingly, during the first half period $T_{1/2}$ of the injection cycle

$$\dot{V}_i(t) = \dot{V}_{i,high} \quad \text{when} \quad 0 \le t < T_{1/2} \tag{11}$$

and, during the following half-period $T_{1/2}$,

$$\dot{V}_i(t) = \dot{V}_{i,low} \quad \text{when} \quad T_{1/2} \le t < 2T_{1/2}. \tag{12}$$

Additionally, we denote the ratio $\dot{V}_{i,\text{low}}/\dot{V}_{i,\text{high}}[<1]$ as r. The value $r = 0.2$ approximates the $\sin^2\omega t$ profile.[11]

We make the quantities in Eq. (10) dimensionless by using the high pump volumetric inflow rate $\dot V_{i,\text{high}}$ to normalize the time-dependent inflow $\dot V_i(t)$ and outflow $\dot V_o(t)$ rates and use the pump period $2T_{1/2}$ to normalize the independent time variable t. As a preliminary step, we reframe (10) so that it becomes

$$\frac{\dot V_i(t)}{\dot V_{i,\text{high}}} = -\frac{2A^2NkT}{\rho \dot V^3_{i,\text{high}}}\frac{d}{dt}\left(\frac{\dot V_{i,\text{high}}}{\dot V_o}\right)^2 + \frac{\dot V_o}{\dot V_{i,\text{high}}}. \tag{13}$$

Consequently, the normalizations

$$y(t) = \frac{\dot V_o}{\dot V_{i,\text{high}}}, \tag{14a}$$

$$y_i(t) = \frac{\dot V_i(t)}{\dot V_{i,\text{high}}}, \tag{14b}$$

and

$$\tau = \frac{2\pi t}{2T_{1/2}}, \tag{14c}$$

transform (13) into

$$y_i(\tau) = -W\frac{d}{d\tau}y^{-2} + y, \tag{15}$$

where

$$W = \frac{2\pi A^2NkT}{\rho T_{1/2}\dot V^3_{i,\text{high}}}. \tag{16}$$

The input function $y_i(\tau) = 1$ during the high pump phase when $0 \le \tau \le \pi$, and the input function $y_i(\tau) = r[<1]$ during the low pump phase when $\pi \le \tau \le 2\pi$.

Definition (16) for W parameterizes the Windkessel effect. For, if $W = 0$, the middle term of Eq. (15) vanishes, the Windkessel effect vanishes, and the output rate $y(\tau)$ equals the input rate $y_i(\tau)$. Furthermore, as we shall see, the larger the Windkessel parameter W, the less variation in the output flow; and the smaller W, the more variation in the output flow.

IV. OUTPUT PROFILES

The normalized dynamical equation (15) can be solved analytically, with the help of partial fractions, in a region in which the input function $y_i(\tau)$ is constant. Accordingly,

$$\frac{\tau}{W} = \text{const.} - \frac{1}{y^2y_i} - \frac{2}{yy_i^2} + \frac{2}{y_i^3}ln\left[\frac{y}{y-y_i}\right], \tag{17}$$

where y_i is either 1 (during the first half of the input cycle) or $r[<1]$ (during the second half of the input cycle).

Another approach is to start with an initial value $y(0)$ and numerically integrate (15) over multiple input cycles with the piecewise-constant, input function (11) and (12). We choose a Windkessel parameter $W = 1.27$ in order to match that of the Windkessel in Newsham's 1725 fire engine—as will be estimated in Sec. V. This produces the profile in Fig. 4. Here, the initial conditions $y(0) = 0.2$ and 0.4 are set

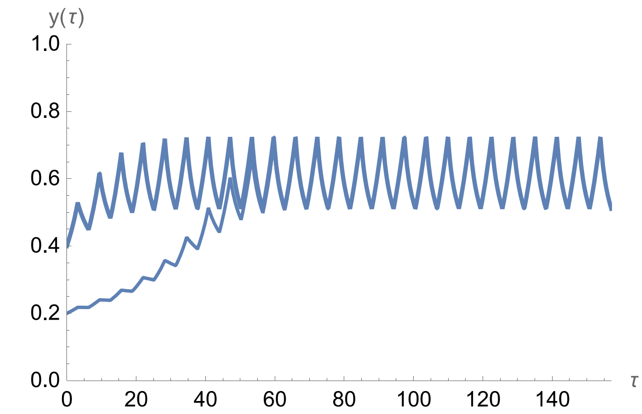

Fig. 4. Normalized Windkessel output $y(\tau)$ vs normalized time τ when the Windkessel has two different initial conditions, $y(0) = 0.2$ and $y(0) = 0.4$, and the parameter $W = 1.27$ is chosen to match that of Newsham's 1725 fire engine. In both cases, $r = 0.2$.

low but non-zero[12] and $r = 0.2$. Apparently, profiles with different initial conditions and the same Windkessel parameter $W = 1.27$ achieve the same steady-state oscillations. However, the smaller the initial condition, the longer the "build-up" time.

We set the Windkessel parameter W significantly lower in Fig. 5 than in Fig. 4, that is, so that $W = 0.271$. Because, in this case, the two initial condition profiles, $y(0) = 0.2$ and 0.4, are quite close to one another, we include only the $y(0) = 0.2$ profile. Apparently, the smaller the Windkessel parameter W, the shorter the build-up time and the larger the relative variations in the steady-state oscillations.

In this way, Figs. 4 and 5 illustrate the Windkessel effect. Defining a relative variation in the steady state oscillations as [(maximum output−minimum output)/maximum output], we find that this variation is 16% for the case displayed in Fig. 4—probably not large enough to be noticed in the water coming out of the pipe of the 1725 Newsham fire engine. Without a Windkessel effect, that is, with $W = 0$, the output variation is 80% $\left[= (1.0\text{–}0.2)/1.0\right]$.

V. THE 1725 NEWSHAM WINDKESSEL

The most appropriate figure of merit of a fire-engine Windkessel is how well it approaches steady flow. To evaluate this figure of merit and to illustrate Windkessel design in

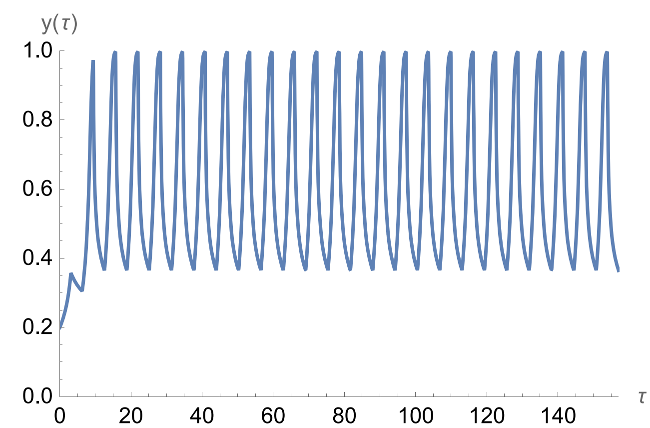

Fig. 5. Normalized Windkessel output $y(\tau)$ vs normalized time τ when the Windkessel parameter $W = 0.271$ is significantly smaller than that in Fig. 4 where $W = 1.27$. The initial condition $y(0) = 0.2$ and the input ratio $r = 0.2$.

a particular case, we appeal to the construction of the 1725 Newsham fire-engine as estimated from patents, advertisements, photographs, a visit to the *Hall of Flame* in Phoenix, Arizona, and a video,[13] produced by the *Hall of Flame*. Our numerical findings, some of which are motivated estimates, are listed in Table I.

These numbers with the relation $NkT = P_a V_{\text{air}}$, where V_{air} is the volume of the air trapped in the Windkessel when it is at atmospheric pressure P_a, are enough to determine the Windkessel parameter W, that is,

$$W = \frac{2\pi A^2 P_a V_{\text{air}}}{\rho T_{1/2} \dot{V}_{i,\text{high}}^3}. \tag{18}$$

The parameters $P_a\left[= 1.01 \times 10^5\, P_a\right]$ and $\rho[= 9.97 \times 10^2\,\text{kg/m}^3]$ in Table I are standard sea level values, while the half period $T_{1/2}$ is 0.250 s. (Note that a complete cycle of the pump handle is 1 s. However, because the pump contains two pistons, a complete cycle of one piston is 0.5 s and its half cycle is 0.25 s.) The radius of the outlet hole of 1.0 cm and the volume of air within the Windkessel of 4 l are approximate values taken from photos and a visual inspection of the 1725 Newsham Windkessel. The average volumetric inflow (and outflow) rate $3\dot{V}_{i,\text{high}}/5$ is the number on which the Windkessel parameter W depends most sensitively. According to the *Hall of Flame* website, $3\dot{V}_{i,\text{high}}/5 = 60\,\text{Gal/min}$,[14] while the Colonial Williamsburg website describes a (somewhat larger) 1750 Newsham fire engine as capable of producing a stream of water at a rate of 125 Gal/min.[14]

A larger average Windkessel volumetric input and output rate $3\dot{V}_{i,\text{high}}/5$ decreases the Windkessel parameter W and, consequently, increases the relative variation of the steady-state oscillations, while a larger cross-sectional area A of the outlet pipe and a larger volume of the air V_{air} in the Windkessel decreases the relative variation (Fig. 6). Figure 5 illustrates the dependence of the relative variation in the steady-state output vs Windkessel parameter W. The $W = 1.27$, relative variation $= 16\%$ point describes the 1725 Newsham fire engine as characterized by the values collected in Table I.

The dimensional factor $\rho \dot{V}_o^2/2A^2$ that we assumed to be large compared to P_a, in order to turn the dynamical equation (9) into (10), is, given the numbers in Table I, such that

$$\frac{\rho \dot{V}_o^2}{2A^2 P_a} \approx 2.26 \gg 1, \tag{19}$$

Table I. Parameters and values describing the pump and Windkessel of the 1725 Newsham fire engine.

Parameters describing the 1725 Newsham fire engine		
Atmospheric pressure	P_a	1.01×10^5 Pa
Area of outlet	A	$3.14 \times 10^{-4}\,\text{m}^2$ (1 cm radius)
Density of water	ρ	$9.97 \times 10^2\,\text{kg/m}^3$
Volume of Windkessel air at atmospheric pressure	V_{air}	$4.00 \times 10^{-3}\,\text{m}^3$ (4 l)
Half period of cycle	$T_{1/2}$	0.250 s
Average volumetric output rate	$\frac{3\dot{V}_{i,\text{high}}}{5}$	$3.78 \times 10^{-3}\,\text{m}^3/\text{s}$ (60 Gal/min)

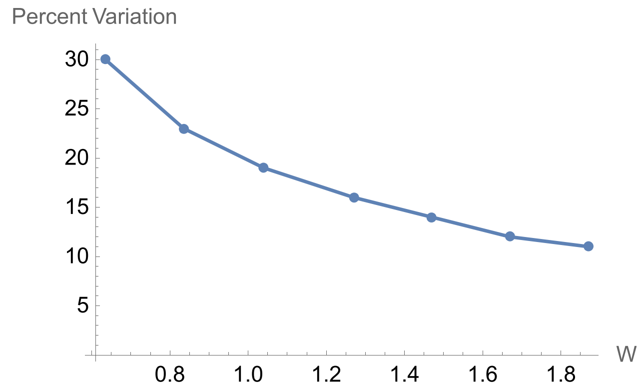

Fig. 6. Percent variation in the outflow rate $\dot{V}_o$ vs Windkessel parameter W for selected values. The percent variation corresponding to the Windkessel parameter $(W = 1.27)$, which describes the 1725 Newsham Windkessel, is 16%.

where we have used the rate $3\dot{V}_{i,\text{high}}/5$ for the average output flow rate $\dot{V}_o$. This result is clearly a marginal observance but one that preserves the Windkessel effect.

The pipe which guides the output on the 1725 Newsham fire engine is about one meter long, probably, as we shall see, too short to significantly resist flow. The average output speed $u\left[= \dot{V}_o/A\right]$ is determined by the average volumetric input flow rate $3\dot{V}_{i,\text{high}}/5$ and pipe cross-sectional area A which means that the outflow speed is 12 m/s. This leads to a pipe-flow Reynolds number $Re\,[= 2Ru/\nu]$ of 2.4×10^5. Here, u is the average outflow speed (12 m/s), R is the outflow pipe radius (0.01 m), and ν is the kinetic viscosity of water $\left(10^{-6}\,\text{m}^2/\text{s}\right)$. The value $Re = 2.4 \times 10^5$ is about a factor of 100 larger than the critical Reynolds number 2300 required for turbulence in a pipe.

However, pipe turbulence has an entrance length, that is, a length over which turbulence develops in flowing fluid. Two formulae exist for this entrance length L_c, namely, $L_c/2R = 1.6Re^{1/4}$ and $L_c/2R = 4.4Re^{1/6}$.[15] Both lead to an entrance length given by $L_c/2R = 35$ or, when $R = 0.01$, to $L_c = 0.7$ m. For this reason, the turbulence may not be fully developed by the time the water reaches the end of the pipe. Note, however, that if the nozzle of the outflow pipe is pinched, as is suggested by the photo in Fig. 3, the water speed exiting the end of the pipe could easily be double or triple that of 12 m/s.

VI. SUMMARY AND CONCLUSION

While the word "Windkessel" refers most famously to the elastic action of the aorta in regulating the flow of blood from the heart into the vascular system, its original application is to an airtight chamber inserted into the line of a piston-driven, water pump in order that it smooth the output of that pump. We construct and solve an analytical model of this original application of the Windkessel effect that highlights its physics, avoids possible complications, and applies to a 1725 Newsham fire-engine. Of course, we could not experiment with the 1725 Newsham Windkessel. However, the results of our analysis are consistent with contemporary descriptions and with modern descriptions of its operation.

Of course, much remains to be done if one's purpose is to accurately model a particular Windkessel system. For instance, the air within Newsham's Windkessel is probably neither isothermal nor adiabatic but, rather, bounded by these two modeling idealizations. Furthermore, the approximation that turns the dynamical equation (9) into (10) could be eliminated. Eliminating these modeling assumptions would make the analysis more dependent on numerical computation. However, the construction and interpretation of such a numerical computation could be guided by the simpler analysis presented here.

Finally, we find it interesting that the simple physics of the Windkessel has been useful for over 300 years and yet, somehow, has escaped the notice of physics professors, textbook writers, and AJP authors.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

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[2]Great Britain, Patent Office, *Titles of Patents Chronologically Arranged, 1617–1852. Page 52, Number 263 (1690)* (Internet Archive, Digitized by the University of Illinois, Champagne-Urbana, 1854).

[3]Jan Van der Heyden, *Fire Engines with Water Hoses and the Methods of Fighting Fires Now Used in Amsterdam* (Science History Publications, Canton, MA, 2009) (Translated from the Dutch into English by Lette Stibbe Multhauf).

[4]Peter M. Molloy, who wrote a Forward to Lette Stibbe Multhauf's 2009 translation of van der Heyden's book, writes that "An air chamber allows water to be pumped in a continuous stream. This provides more water and makes aiming a stream much easier. The air chamber was known to the ancient Greeks, but apparently no European engine builders incorporated this feature until the van der Heydens."

[5]Great Britain, Patent Office, *Titles of Patents Chronologically Arranged, 1617–1852, Page 87, Number 479 (1721) and Page 90, Number 494 (1725)* (Internet Archive, Digitized by the University of Illinois, Champagne-Urbana, 1854).

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[7]Kim H. Parker, "A brief history of arterial wave mechanics," Med. Biol. Eng. Comput. **47**(2), 111–118 (2009).

[8]Otto Frank, "The basic shape of the arterial pulse. First treatise: mathematical analysis 1899," J. Mol. Cell. Cardiol. **22**(3), 255–277 (1990).

[9]It is easy to confuse an "expansion tank" with a "pressure tank." Only the latter is a true Windkessel. The former refers to a tank connected to hot water systems that allows the volume of water it contains to expand when heated. "Pressure tanks" are attached to pump systems that draw water from the ground for residential purposes. Unfortunately, the devices seem not to have a uniform nomenclature. See the link <https://yourh2home.com/expansion-tank-vs-pressure-tank-which-one-do-you-need/>.

[10]Erich Kamke, *Differentialgleichungen, Lösungsmethoden Und Lösungen* (Akademische Verlagsgesellschaft, Geest & Prtig, Leibzig, 1951), pp. 26, 302, 303, 326, 327, and 545.

[11]In particular, the ratio of the height $\dot{V}_{i,\mathrm{low}}$ of the step consisting of the lower half period of the continuous and smooth profile $\dot{V}_i(t) = \sin^2\left(2\pi t/2T_{1/2}\right)$ to the height $\dot{V}_{i,\mathrm{high}}$ consisting of its higher half-period that conserves the net injection rate during one period $2T_{1/2}$ is given by $\dot{V}_{i,\mathrm{low}}/\dot{V}_{i,\mathrm{high}} = 0.22$.

[12]An initial condition $y(0) = 0$ is not allowed by our model because terms in the dynamical equation (14) and its predecessor (9) become undefined when there is no output. Another approach that would allow for $y(0) = 0$ is to numerically solve (9).

[13]See the website of the *Hall of Flame* in Phoenix, Arizona USA, <https://hallofflame.org>. See also the YouTube video at <https://youtu.be/KvBHxfyRnu8?si=ge_n1VKLEkeLgQrD>

[14]See the Colonial Williamsburg, <https://emuseum.history.org>

[15]Fabien Anselmet, Fabien Ternat, Muriel Amielh, Olivier Boiron, Patrick Boyer, Laurence Pietri *et al.*, "Axial development of the mean flow in the entrance region of turbulent pipe and duct flows," C. R. Mec. **337**(8), 573–584 (2009).