Supplementary Numerical Results

June 15, 2020

1 Angular Synchronization Performance in the Presence of Zero Entries

This discussion concerns Theorem 3, which provides recovery guarantees for Algorithm 1. The statement of Theorem 3 assumes that $\widehat{\mathbf{x}}$ is non-vanishing for technical convenience. However, similar results can be derived in the case where $\widehat{\mathbf{x}}$ does vanish, as long as it does not have δ consecutive zeros. We verified this by running the angular synchronization algorithm in the case where $\widehat{\mathbf{x}}$ had a small number of zero entries and observed that the algorithm was still effective. A representative numerical simulation is show in Fig. 1 below, which plots the error in performing angular synchronization (Steps 3–7 in Algorithm 1) in the noiseless setting (for a signal size $d = 247$, and with $\rho = 10$), with a random selection of coefficients (not necessarily contiguous) in $\widehat{\mathbf{x}}$ set to zero. As can be seen, the proposed procedure is robust to the presence of a fairly large number of zeros (about 40% of the signal length), with angular synchronization failing only when a large number of zero entries are present.

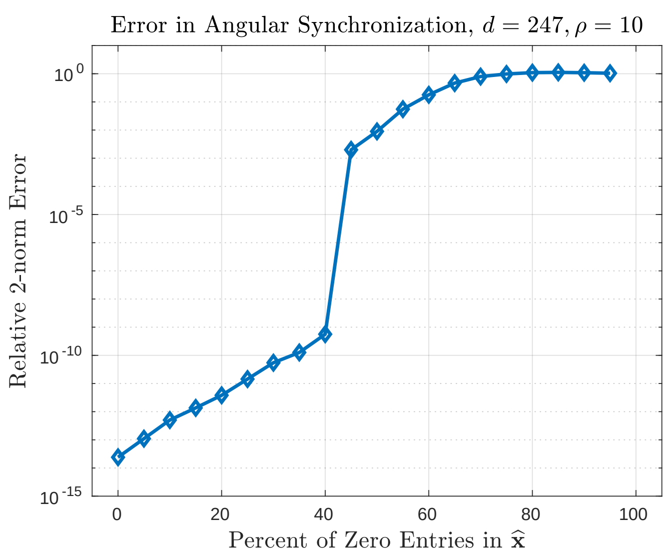


Figure 1: Robustness of Eigenvector-Based Angular Synchronization to the Presence of Zero Entries in $\widehat{\mathbf{x}}$

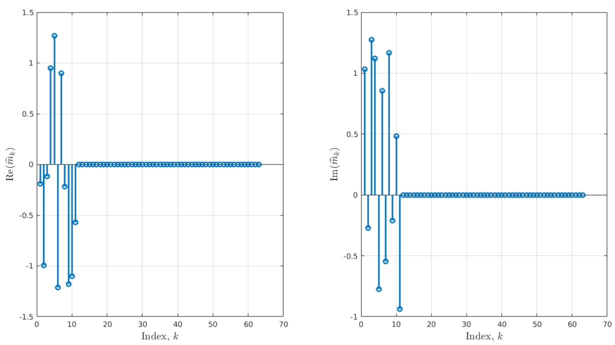
2 More General Masks

We note that the manuscript provides one possible deterministic mask construction for which it is provably true that constants μ_1 and μ_2 are strictly positive. Empirically, the conditions that μ_1 and μ_2 are strictly positive is fairly mild and is also satisfied if e.g. the nonzero entries of the $\mathbf{m}$ or $\widehat{\mathbf{m}}$ are generated i.i.d at random. In addition, some empirical results supporting this observation are provided below.

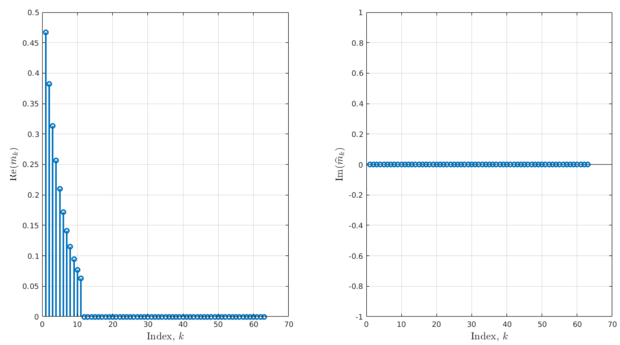
Tables 1 and 2 provide empirically computed μ_1 values (Mask constant for Algorithm 1) for several different parameter choices and four different types of masks (representative and illustrative plots of these masks are provided in Fig. 2):

1. Random masks (defined in (5.1) in the manuscript),
2. (Deterministic) Exponential masks (defined in (5.2) in the manuscript),
3. Random Symmetric masks – these are defined similar to (5.1), except exhibiting (conjugate) symmetry in the window of support,
4. (Deterministic) Gaussian masks – generated using Matlab's `gausswin` command and $\alpha = 2$ (reciprocal of the standard deviation).

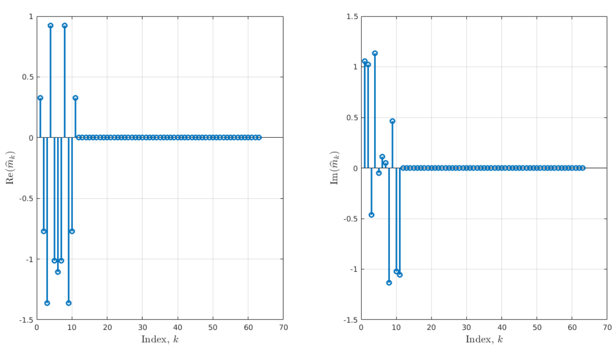
As can be observed from the table of μ_1 values, the proposed method allows for a wide variety of mask prescriptions. Similar results hold for Algorithm 2 and μ_2 values.



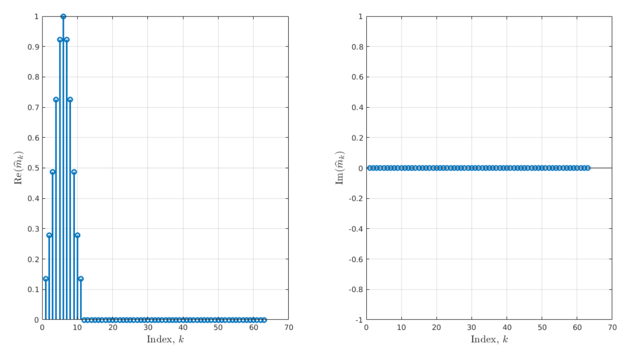
(a) Random Mask



(b) (Deterministic) Exponential Mask



(c) Random Symmetric (within window of support) Mask



(d) (Deterministic) Gaussian Mask

Figure 2: Representative plots (of $\widehat{\mathbf{m}}$, with $\rho = 11$) for different types of masks. Corresponding mask constants (μ_1) are provided in Table 1.

(d, ρ)	Random	Exponential	Random Symmetric	Gaussian
$(21, 11)$	1.9720×10^{-1}	1.2685×10^{-2}	1.3955×10^{-1}	1.2011×10^{-3}
$(63, 11)$	1.2140×10^{-1}	1.1983×10^{-2}	5.5436×10^{-2}	3.9680×10^{-4}
$(1029, 11)$	6.3642×10^{-2}	1.1892×10^{-2}	3.4183×10^{-3}	3.4520×10^{-5}

Table 1: Empirically evaluated μ_1 (mask constant for Algorithm 1) values for different types of masks (with ρ fixed and increasing d).

(d, ρ)	Random	Exponential	Random Symmetric	Gaussian
$(63, 2)$	4.0464×10^{-1}	2.2827×10^{-1}	7.7457×10^{-2}	9.1324×10^{-4}
$(65, 3)$	2.1832×10^{-1}	1.3767×10^{-1}	2.5399×10^{-2}	6.5404×10^{-3}
$(63, 4)$	1.9319×10^{-1}	9.0640×10^{-2}	5.8460×10^{-2}	5.4034×10^{-3}
$(63, 5)$	1.7196×10^{-1}	6.2255×10^{-2}	5.0854×10^{-2}	4.0929×10^{-3}
$(65, 7)$	1.4867×10^{-1}	3.1409×10^{-2}	4.9980×10^{-2}	2.6888×10^{-3}
$(57, 10)$	1.4545×10^{-1}	1.4018×10^{-2}	6.1526×10^{-2}	1.1799×10^{-4}
$(69, 12)$	1.2449×10^{-1}	1.0398×10^{-2}	4.8719×10^{-2}	2.4892×10^{-4}

Table 2: Empirically evaluated μ_1 (mask constant for Algorithm 1) values for different types of masks (with d (approximately) fixed (subject to divisibility constraints) and increasing ρ.)