ASE 2020-21 Notes

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§1 Sunday, 08/02/20

§1.1 Number Sense: Squares Ending in 5

Here is the derivation for this trick. Let $a5$ represent any number ending in 5 (a could be any integer, not just restricted to a one-digit number).

$$\begin{aligned}(a5)^2 &= (10a+5)^2\\ &= 100a^2+100a+25\\ &= 100a(a+1)+25\end{aligned}$$

So you can tell from this that and number ending in 5 squared will have its last two digits equal to 25 and the remainder of the digits can be found from taking the leading digit(s) and multiplying it by one greater than itself. Here are a couple of examples:

	Tens/Ones:		**25**
$85^2 =$	Thousand/Hundreds:	$8 \times (8+1)$	**72**
	Answer:		**7225**

The next example shows how to compute 15^4 by applying the square ending in 5 trick twice, one time to get what 15^2 is then the other to get that result squared.

	Tens/Ones:	**25**		Tens/Ones:	**25**
$15^2 =$	Thousands/Hundreds:	$1 \times (1+1) = \mathbf{2}$	$225^2 =$	Rest of Answer:	$22 \times (23) = 11 \times 46 = \mathbf{506}$
	Answer:	**225**		Answer:	**50625**

In the above trick you *also* use the double/half trick *and* the 11's trick. This just shows that for some problems using multiple tricks might be necessary.

1. 25% of 25 = ____________________
2. $.35 \times 3.5 =$ ____________________
3. $12^2 + 2 \times 12 \times 13 + 13^2 =$ ____________________
4. $(115)^2 =$ ____________________
5. $\mathbf{f(x) = 9x^2 - 12x + 4,\ f(19)} =$ ____________________
6. **45% of 45 − 45** = ____________________
7. (*) $12^4 =$ ____________________
8. $505 \times 505 =$ ____________________
9. A square has an area of 12.25 sq. cm. It's perimeter is: ____________________

§1.2 Calculator: Exponents of Large Numbers

The following pages explain how to take exponents of large numbers. **This is very important.**

Calculator Underflow Example

Find $980{,}311^{-62{,}053}$ on a calculator. If you type in 980311^(-62053) on a TI-83 or TI-84 the answer given is 0. This is because your calculator cannot handle numbers very close to zero. Using logarithms and algebra we can solve for this answer. On the state calculator test in 2011 question number 28 asks:

11I - 28 What is $980{,}311^{-62{,}053}$?

Let $x = 980{,}311^{-62{,}053}$

Take the common logarithm of both sides

$\log x = \log(980{,}311^{-62{,}053})$

Use the exponent property of logarithms.

$\log x = -62{,}053 \log (980{,}311)$

Type -62,053 log (980,311) into the calculator

$\log x = -371{,}782.1026\ldots$

Subtract (-371,783) from -371,782.1026… to get a number between 0 and 1.

$(-371{,}783) - (-371{,}782.1026\ldots) = 0.89741171\ldots$

$\log x = 0.89741171\ldots$

$10^{\log x} = 10^{0.89741171\ldots}$

$x = 7.896083095\ldots$

Round to 3 decimal places.

Write $7.90 \times 10^{-371{,}783}$ (A number very close to zero)

Calculator Overflow Example

Find $83{,}946^{950{,}637}$ on your calculator. If you type 83946^950637 into the TI-83 or TI-84 calculator, the answer given is ERR:OVERFLOW because the number is too large for these calculators to handle. Using the properties of logarithms and algebra, we can solve for the correct answer. On the TMSCA state calculator contest in 2011 question number 28 is:

11E-28 What is $83{,}946^{950{,}637}$?

$$\text{Let } x = 83{,}946^{950{,}637}$$

Take the common logarithm of both sides.

$$\log x = 83{,}946^{950{,}637}$$

Use the property of logarithms for exponents.

$$\log x = 950{,}637\log(83{,}946)$$

Type 950637log(83946) into the calculator.

$$\log x = 470916.5887\ldots$$

Subtract 470916 from 470916.5887… to get 0.58866699…

$$\log x = 0.58866699\ldots$$

$$10^{\log x} = 10^{0.58866699\ldots}$$

$$x = 3.878528526\ldots$$

Round to 3 decimal places.

Write $3.88 \times 10^{470{,}916}$ (A very large number)

Overflow/Underflow Practice Problems

1. 09G-37 Calculate 0.0942^{-48285}

2. 09I -37. Calculate 72057^{5912}

3. 10B-36 What is $57{,}893^{453{,}562}$?

4. 10G-36 What is $349{,}441^{-902{,}521}$?

5. What is 777^{888}?

6. What is 888^{-999}?

Overflow/Underflow Answers

1. $8.96x10^{49,537}$

2. $3.82x10^{28,718}$

3. $1.58x10^{2,160,146}$

4. $4.80x10^{-5,003,012}$

5. $4.94x10^{2566}$

6. $3.43x10^{-2946}$

1. $0.2442^{-285} =$ ____________________
2. $727^{592} =$ ____________________
3. $53^{4532} =$ ____________________
4. $34141^{-901} =$ ____________________
5. What is 774^{228}?
6. What is 442^{442}?
7. What is 558^{558}?
8. What is 223^{332}?
9. What is 23242^{32}?
10. What is 221^{332}?
11. What is 0.123^{34342}?
12. What is 0.463^{-332}?
13. What is 1245^{32}?
14. What is 12^{33332}?
15. What is 475^{-665}?
16. What is 581^{-329}?
17. What is 370^{-292}?
18. What is 726^{280}?
19. What is 380^{-438}?
20. What is 608^{66}?

§1.3 General Math: Trigonometry

We'll start out with a right triangle. It's a nice triangle – we know an angle of 90°. What about the other angles? Let's call one θ and the other one will be $90° - \theta$:

The big question arises: how does θ even relate to a, b, c? That's why we introduce trigonometric functions:

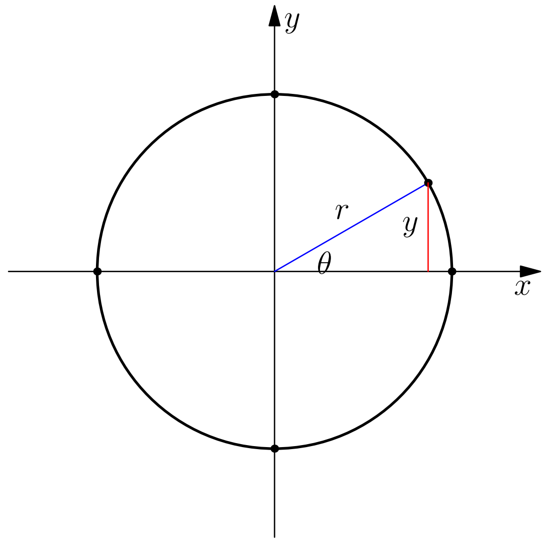
§1.3.1 Definitions of Trigonometric Functions

Let us first start with a quick definition of a few important parts of a right triangle:

Definition 1 (Hypotenuse). The **hypotenuse** of a right triangle is the side across from the right angle.

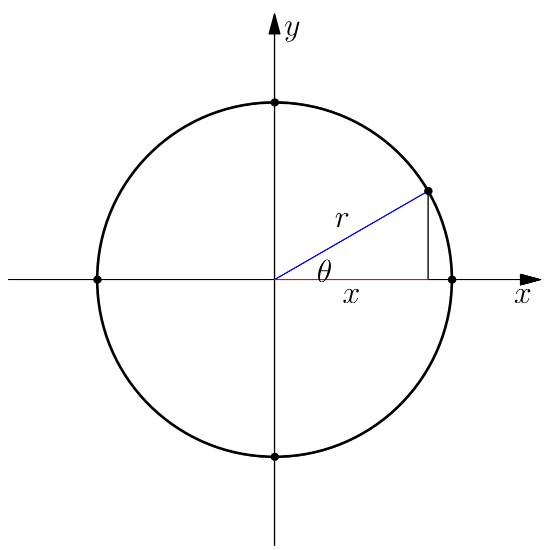
Definition 2 (Leg). A **leg** of a right triangle is a side adjacent to the right angle and not the hypotenuse.

Definition 3 (Sine). The **sine** of an angle θ is written as $\sin(\theta)$, and is equivalent to the ratio of the length of the side across from the angle to the length of the hypotenuse.



Note that when this altitude to the x-axis is below the x-axis the sine of the angle is negative. When θ is between 0° and 180° or 0 rad and π rad, then $\sin(\theta)$ is positive. In addition, when θ is between 0° and 90°, $\sin(\theta)$ can be viewed in the context of a right triangle as the ratio of the length side opposite the angle to the length of the hypotenuse (think about how the radius of the unit circle is the hypotenuse of the triangle in the first definition and how from there we can scale it up for larger hypotenuses without changing the value of the sine).

Definition 4 (Cosine). The **cosine** of an angle θ is written as $\cos(\theta)$, and is equivalent to the ratio of the length of the side adjacent to the angle (not the hypotenuse) to the length of the hypotenuse.



Similar to the sine, the cosine is negative when the point is to the left of the y axis (i.e. for $90° < \theta < 270°$). In addition, for angles angles between 0° and 90°, the cosine can be seen in the context of a right triangle as the ratio of the lengths of the side adjacent to the angle over the hypotenuse of the triangle (again, think about scaling up the unit circle).

Definition 5 (Tangent). The **tangent** of an angle θ is written as $\tan(\theta)$ and is equivalent to the ratio of the length of the line segment opposite the angle to the length of the line segment adjacent to the angle (that is not the radius of the circle, i.e. the hypotenuse).