PART

I

Lesson Notes

1 Sunday, 04.26.20

1.1 MathLeague 12024

The following is an outline of the ideas discussed in class for problems 19-30.

19. Do casework on if no digits are the same or two digits are the same.

20. First calculate the number of minutes it takes 12 machines to produce 2500 rivets, then calculate the number of minutes it takes 12 machines to produce 50 rivets, then calculate the number of minutes it takes 12 machines to produce 48000 rivets.

21. Compare the numbers individually.

22. Use the formula
$$\frac{\text{angle of the sector}}{360^\circ} = \frac{\text{area of the sector}}{\text{area of the circle}}.$$

23. Convert the numbers to base 10 and equate them to get $7A + B = 9B + A$. Then solve for A, B. Notice that they are less than 7.

24. First calculate how much time has passed after losing 1 minute (60 hours). Then find how much time has passed after losing 1 hour (3600 hours). Convert this to days (150 days) and calculate the date.

25. Factor out 7! to get $8 \cdot 7! + 7! = 9 \cdot 7!$. Then factorize.

26. Do casework on which two digits are identical.

27. First prove that the other vertex also lies on the square. Then realize the side length of the square is one-third the length of the hypotenuse, and calculate the ratio of areas.

28. Note that for all real a,
$$\frac{2}{\sqrt{a}+\sqrt{a+1}} = \frac{2(\sqrt{a+1}-\sqrt{a})}{(\sqrt{a}+\sqrt{a+1})(\sqrt{a+1}-\sqrt{a})},$$
so by difference of squares, the denominator is $(\sqrt{a+1})^2 - (\sqrt{a})^2 = 1$, so it simplifies to $2(\sqrt{a+1}-\sqrt{a})$. Then we have a **telescoping series**[1], which becomes makes it easy to find the answer from.

29. Draw a figure. Since we are looking for the ratio, we can let the side length be whatever we want.[2] Thus, $AP = 4, BP = 2$. Furthermore, since $AQ = BQ \cdot 2$, this is obviously a $30^\circ - 60^\circ - 90^\circ$ triangle, so $BQ = 2\sqrt{3}$. From here we can derive $PQ = 4$, and then we just need to find the ratio, which is easy.

30. Since $x^2 - 6x + 5$ divides this polynomial, we know that
$$x^4 + ax^2 + b = p(x)(x^2 - 6x + 5)$$

[1] A *telescoping series* just a sum that has a bunch of terms cancel out.

[2] I used 6, since it was a nice number.

for some polynomial $p(x)$. Since $x^2 - 6x + 5 = (x-1)(x-5)$, if we plug in 1 we get

$$1 + a + b = 0,$$

and if we plug in 5 we get

$$625 + 25a + b = 0.$$

We can solve these two equations and get the answer.