



General Information & Tips

You may notice that in a MathLeague test, there is always a “E) Other” choice. You choose this option when none of the other options are right. Guessing and checking is sometimes a good idea. For example, if the problem is:

Problem 1. If $a \times b$ is an even number, and a and b are positive integers, then what could (a, b) be?
A) (1,1) B) (5,7) C) (2,9) D) (-4,2) E) Other

Solution. By plugging in the values, we find that only (2, 9) and $(-4, 2)$ give us an even number. However, -4 is not a positive integer, so the answer is $(2, 9)$. \square

In this problem, we **plugged in values**, and **eliminated answer choices**. This is a good strategy for some problems.

Problem Types

There are a few problem types (note that the list below is not *all* of them, but does cover quite a few topics):

- Arithmetic (adding/subtracting/multiplying/dividing, fractions/percentages/decimals, etc.)
- Word problems
- Harder operations (square roots, exponents, logarithms, etc.)
- Algebraic manipulation
- Geometry formulas (perimeter/area/volume/surface area)
- Angles
- Simple counting
- Probability
- Bases
- Prime Factorization

Instructions This test will last 30 minutes. **Novice:** Do problems 1 to 30 in Sprint. **Advanced:** Do problems 21-30 in Sprint, and 1-8 in Target.

Remark 1. This is an actual MathLeague Sprint/Target test. Do not spread this material around. It is for ASE use only.

§ 1 Sprint

1. Simplify: $\frac{4000}{20}$.
A) 20 B) 100 C) 200 D) 400 E) Other
2. Find the number of diagonals in a square.
A) 0 B) 1 C) 2 D) 4 E) Other
3. Evaluate: $\frac{4 \times 6}{4 + 6}$.
A) 0.24 B) 1.2 C) 1.8 D) 2.4 E) Other
4. Simplify: $\frac{4}{7} + \frac{1}{5}$.
A) $\frac{5}{12}$ B) $\frac{18}{35}$ C) $\frac{22}{35}$ D) $\frac{27}{35}$ E) Other
5. The perimeter of a regular pentagon is 60 units. What is its side length?
A) 6 B) 10 C) 12 D) 30 E) Other
6. What is the third smallest positive multiple of 6?
A) 12 B) 18 C) 24 D) 30 E) Other
7. Evaluate: $18 + 42 \div 3$.
A) 4 B) 14 C) 20 D) 24 E) Other
8. Simplify: $92092 \div 1001$.
A) 46 B) 52 C) 92 D) 920 E) Other
9. What is 17×202 ?
A) 187 B) 374 C) 1717 D) 3434 E) Other
10. The angles of a triangle are 18° , 71° , and x° . Find x .
A) 61 B) 71 C) 81 D) 91 E) Other
11. What number is 28% more than 75?
A) 21 B) 96 C) 106 D) 141 E) Other
12. Mark can pack 3 boxes in 11 minutes. How many minutes does it take him to pack 18 boxes?
A) 33 B) 66 C) 70 D) 198 E) Other

13. There are 32 books on Andrew's bookshelf. Of these, 14 are about math, 12 are about history, and 3 are about math history (counted as both math and history books). The remainder of his books are dictionaries. How many dictionaries does Andrew have?
A) 3 B) 6 C) 10 D) 11 E) Other
14. Four red balls, three blue balls, and two green balls are in a bin. Six balls are removed at random from the bin. What is the probability that no red balls are removed?
A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) Other
15. If $2A + 7 = 25$, find $4A + 14$.
A) 50 B) 58 C) 64 D) 70 E) Other
16. Thrice the square of a positive integer equals 75. What is the value of two times the integer?
A) 2 B) 5 C) 10 D) 20 E) Other
17. Simplify: $\sqrt{2} \times \sqrt{4} \times \sqrt{8} \times \sqrt{16}$.
A) $8\sqrt{2}$ B) 16 C) $16\sqrt{2}$ D) 32 E) Other
18. Simplify: $2013 \times 2013 - 2012 \times 2013$.
A) 2012 B) 2013 C) 4024 D) 4025 E) Other
19. Convert 143_5 to base 10.
A) 25 B) 48 C) 92 D) 143 E) Other
20. A right cone has a circular base with radius 4 and a height of $\frac{6}{\pi}$. Find its volume.
A) 32 B) 96 C) 32π D) 96π E) Other
21. How many integers are greater than -2013 and less than 2013?
A) 2012 B) 2013 C) 4024 D) 4026 E) Other
22. How many positive integers are factors of 12?
A) 2 B) 4 C) 6 D) 8 E) Other
23. What is the largest integer n such that 2^n divides 640?
A) 6 B) 7 C) 8 D) 9 E) Other
24. Olivia is standing in a line. If 42% of the people in line are ahead of her and 56% of them are behind her, how many people are in the line?
A) 10 B) 25 C) 50 D) 100 E) Other
25. What is the next number in the sequence: 1, -2, 4, -8, 16, -32, ...?
A) -64 B) -48 C) -32 D) 32 E) Other
26. Kevin runs at $\frac{3}{4}$ of Evan's speed. How many feet can Evan run in the time Kevin takes to run 1 mile?
A) 3960 B) 4030 C) 7040 D) 8750 E) Other

27. The number 1827394 is squared. What is the units digit of the result?
A) 2 B) 3 C) 4 D) 5 E) Other
28. The side lengths of a triangle are 9, 40, and 41. Find the area of the triangle.
A) 20 B) 41 C) 45 D) 90 E) Other
29. In a certain game, players flip a fair coin. If the coin lands heads, the player wins 2 points. If the coin lands tails, the player loses 1 point. If Bob plays the game 1000 times (starting with 0 points), what is the expected number of points he ends with?
A) 250 B) 500 C) 1000 D) 1500 E) Other
30. Simplify: $100^2 - 99^2 + 98^2 - 97^2 + \dots + 2^2 - 1^1$.
A) 0 B) 100 C) 4950 D) 5050 E) Other

§ 2 Target

- Find the average of the numbers in the set $\{8, 19, 21, 36\}$.
- How many fluid ounces are in 1 gallon?
- What is the sum of the cubes of the first 5 positive integers?
- March 1, 2013 falls on a Friday. What day of the week does June 1, 2013 fall on?
- The ratio between two positive integers is $4 : 3$, and the difference between the two integers is 7. Find the sum of the two integers.
- A *palindrome* is a number that reads the same forwards and backwards. For example, 14541 and 11 are palindromes, while 18 and 100 are not. How many positive integers less than 10^6 are palindromes?
- In how many different orders can Adam, Beth, Carl, Dan, and Ed stand in a row, if Adam must stand ahead of Beth?
- Two of the side lengths of a right triangle are 7 and 9. What is the sum of the possible lengths of the third side? Express your answer as a decimal to the nearest hundredth.

§ 3 Answer Key

§ 3.1 Sprint

- | | | | | | |
|------|-------|-------|-------|-------|-------|
| 1. C | 6. B | 11. B | 16. C | 21. E | 26. C |
| 2. C | 7. E | 12. B | 17. D | 22. C | 27. E |
| 3. D | 8. C | 13. E | 18. B | 23. B | 28. E |
| 4. D | 9. D | 14. E | 19. B | 24. C | 29. B |
| 5. C | 10. D | 15. A | 20. A | 25. E | 30. D |

§ 3.2 Target

- | | | | |
|--------|-------------|---------|----------|
| 1. 21 | 3. 225 | 5. 49 | 7. 60 |
| 2. 128 | 4. Saturday | 6. 1998 | 8. 17.06 |

§ 4 Solutions

§ 4.1 Sprint

- We can just divide it out to get $\boxed{20}$.
- Let's draw a diagram:



From the figure we can easily tell there are $\boxed{2}$ diagonals.

- The numerator is 24 and the denominator is 10, so the answer is $\frac{24}{10} = \boxed{2.4}$.
- Let's take the common denominator of both fractions. the first one becomes $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$ and the second one becomes $\frac{1}{5} = \frac{1 \times 7}{5 \times 7} = \frac{7}{35}$, so the answer is $\frac{20}{35} + \frac{7}{35} = \boxed{\frac{27}{35}}$.
- A regular pentagon has 5 equals sides, so $\frac{60}{5} = \boxed{12}$.
- The smallest is 6, the second smallest is 12, so the third smallest is $\boxed{18}$.
- Using PEMDAS, we must do $42 \div 3$ first. Thus, $18 + 42 \div 3 = 18 + 14 = \boxed{32}$.
- We know that any two-digit number \overline{AB} times 1001 gives us $\overline{AB0AB}$, so this two-digit number must be $\boxed{92}$.

9. Notice that $202 = 2 \times 101$, so $17 \times 202 = 17 \times 2 \times 101 = 34 \times 101 = \boxed{3434}$.
10. The angles of a triangle add up to 180° , so $18^\circ + 71^\circ + x^\circ = 180^\circ \implies x^\circ = 180^\circ - 18^\circ - 71^\circ = \boxed{91^\circ}$.
11. We know that 28% of 75 is $28\% \cdot 75 = \frac{28}{100} \cdot 75 = \frac{2100}{100} = 21$, so 21 more than 75 is $21 + 75 = \boxed{96}$.
12. If he can pack 3 boxes in 11 minutes, he can pack $3 \times 6 = 18$ boxes in $11 \times 6 = \boxed{66}$ minutes.
13. If we add the number of math and history books, we get $14 + 12 = 26$. However, 3 of them (the math history books) are counted twice, since they were counted once for math and once for history. This means we have to subtract the extra 3 we counted, giving us $26 - 3 = \boxed{23}$.
14. Let's try to pick as little red balls as possible. If we pick the 3 blue balls and 2 green balls, we have 1 ball left over to pick. We must pick one of the red balls then, which means we can never pick 0 red balls. This means the answer is $\boxed{0}$.
15. If we multiply the equation by 2, we get $2(2A + 7 = 25) \implies 4A + 14 = \boxed{50}$.
16. Let's say the integer is n . If we take the square of that, we get n^2 , and if we take thrice (which means three times) of that, we get $3n^2$, and this equals 75. This means $n^2 = 25$, and since n is positive, $n = \boxed{5}$.
17. Notice that $\sqrt{4} = 2$ and $\sqrt{16} = 4$. This means we just need to find $\sqrt{2} \times \sqrt{8}$. The rule is that $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, so $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$. This means the answer is $2 \times 4 \times 4 = \boxed{32}$.
18. If we factor out the 2013, we get $2013(2013 - 2012) = 2013 \times 1 = \boxed{2013}$.
19. We know that $\overline{a_1 a_2 \dots a_n b} = a_1 \times b^{n-1} + a_2 \times b^{n-2} + a_3 \times b^{n-3} + \dots + a_n \times b^0$, so $143_5 = 1 \times 5^2 + 4 \times 5^1 + 3 \times 5^0 = 25 + 20 + 3 = \boxed{48}$.
20. The volume of a cone with radius r and height h is $\frac{\pi r^2 h}{3}$, so the answer is $\frac{\pi \cdot 4^2 \cdot \frac{6}{\pi}}{3} = \boxed{32}$.
21. The smallest integer greater than -2013 is -2012, and the largest integer less than 2013 is 2012, so the range is -2012 to 2012. If we add 2012 to each number, we get the range 0 to 4024, which is $\boxed{4025}$ numbers.
22. If we prime factorize 12, we get $12 = 2^2 \cdot 3^1$. This means that the number of prime factors is $(2 + 1) \cdot (1 + 1) = \boxed{6}$.
23. If we prime factorize 640, we get $640 = 2^7 \cdot 5$, so the largest power of 2 that divides 640 is 2^7 , which means $n = \boxed{7}$.
24. $42\% + 56\% = 98\%$ of people are the people other than her, so $100\% - 98\% = 2\%$ of the people are her. This means that she is 2% of everyone in line, which means there must be $\boxed{50}$ people in line.
25. The sequence is a geometric sequence with a common ratio of -2 , so the next term is $-32 \times (-2) = \boxed{64}$.

26. Let's say the speed of Kevin is k and the speed of Evan is e . Then $k = \frac{3}{4}e$. If Kevin runs a mile in t time, then $kt = 1$. If we substitute in $k = \frac{3}{4}e$, we get $\frac{3}{4}et = 1 \implies et = \frac{4}{3}$. This means in the same time, Evan can run $\frac{4}{3}$ miles, but since 1 mile is 5280 feet, he can run $\frac{4}{3} \times 5280 = \boxed{7040}$ feet.
27. The units digit of a number squared is just the last digit of the square of the units digit of the number. This may sound confusing, but it just means that if the number ends in 4, then the units digit is the units digit of $4^2 = 16$, which is 6. This means that 1827394^2 ends in $\boxed{6}$.
28. Using Pythagorean Theorem, we know that $9^2 + 40^2 = 41^2$, so the angle between the sides of length 9 and 40 is a 90° angle. This means the area is $\frac{1}{2} \times 9 \times 40 = \boxed{180}$.
29. If Bob plays the game 1000 times, we expect he will get heads 500 times and tails 500 times. This means that he will win $500 \times 2 = 1000$ points but lose $500 \times 1 = 500$ points, giving us $1000 - 500 = \boxed{500}$.
30. First, let's replace 1^1 with 1^2 , since they are the same thing. If we compute $100^2 - 99^2$ using difference of squares, we get $(100 - 99)(100 + 99) = 100 + 99$. If we compute $98^2 - 97^2$, we get $(98 - 97)(98 + 97) = 98 + 97$, and doing this with every pair of squares, we will eventually get $100 + 99 + 98 + 97 + \dots + 2 + 1 = \frac{100 \cdot 101}{2} = \boxed{5050}$.

§ 4.2 Target

- The average of a set of numbers is the sum of the numbers divided by the number of numbers, so we get $\frac{8+19+21+36}{4} = \boxed{21}$.
- This is just a memorization question – there are 8 fluid ounces in one cup, and there is 2 cups in one pint, and 2 pints in one quart, and 4 quarts in one gallon, so $8 \cdot 2 \cdot 2 \cdot 4 = \boxed{128}$ fluid ounces are in 1 gallon.
- We can just compute this one out: $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 1 + 8 + 27 + 64 + 125 = \boxed{225}$.
- April 1 is 31 days from March 1, May 1 is 30 days from April 1, and June 1 is 31 days from May 1. This means March 1 is $31 + 30 + 31 = 92$ days from June 1, but $92 = 7 \cdot 13 + 1$, which means we will go through 13 weeks then 1 day, but adding a week doesn't change the day of the week, so we are technically only going to add 1 day, so the answer is **Saturday** (which is one day after Friday).
- Let's say the integers are $4k$ and $3k$ (notice this is because the ratio is $4 : 3 = 4k : 3k$). If we subtract them, we get $4k - 3k = k = 7$, so one of the integers is 28 and the other is 21. This means the sum is $28 + 21 = \boxed{49}$.
- Let's do some casework. If it is a one-digit number, 1 through 9 are all palindromes, so there are 9 of those. If it is a two-digit number, 11, 22, 33, \dots , 99 are all palindromes, so there are 9 of those. If it is a three-digit number, it has to be in the form \overline{aba} , but a is a positive integer, so there are 9 choices for that, and there are 10 choices for b , so there are $9 \cdot 10 = 90$ three-digit palindromes. A four-digit

palindrome is in the form \overline{abba} , and there are again $9 \cdot 10 = 90$ palindromes. For five-digit palindromes, they must be in the form \overline{abcba} , so there are $9 \cdot 10 \cdot 10 = 900$ palindromes. The six-digit palindromes are in the form \overline{abccba} , so there are $9 \cdot 10 \cdot 10 = 900$ palindromes again. These are all the possibilities under $10^6 = 1,000,000$, so the answer is $9 + 9 + 90 + 90 + 900 + 900 = \boxed{1998}$.

7. The number of ways where Adam is ahead of Beth is the same as the number of ways Beth is ahead of Adam. These are the only two possibilities, so if there are X possibilities of Adam being ahead of Beth, there are also X possibilities of Beth being ahead of Adam. Again, these are the only two cases (we can't have that they are in the same position!), so $X + X = 2X = \text{total \# of possibilities}$, which is $5! = 120$, so $2X = 120 \implies X = \boxed{60}$.
8. If 9 is the largest side, and x is the other side, then $x^2 + 7^2 = 9^2 \implies x = 4\sqrt{2}$. If x is the largest side, then $7^2 + 9^2 = x^2 \implies x = \sqrt{130}$. There is no way 7 is the largest side (since 9 is already larger than 7), so the two possibilities summed together give us $\sqrt{2} + \sqrt{130} \approx \boxed{17.06}$.