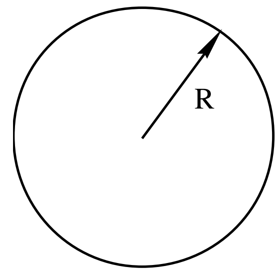
Appendix B: Geometry Formulas

Any reference book may be used to obtain the various formulas which contestants must know by memory to become proficient in solving geometric problems on the contest. As a convenience to coaches and contestants, we have assembled the formulas which we believe to be sufficient to solve geometric problems on the contest. Unless otherwise noted, angle measurement is given in degrees in the formulas, but on the contest both degree and radian measure are used.

CIRCLE

R = radius, D = diameter, C = circumference, K = area



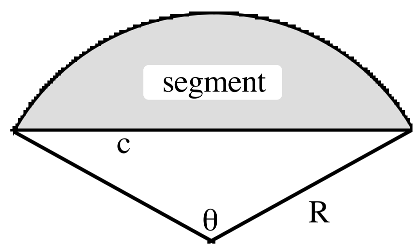
$D = 2R$

$C = 2\pi R = \pi D$

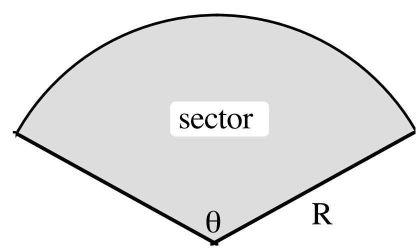
$K = \pi R^2 = \frac{\pi}{4} D^2 = \frac{C^2}{4\pi}$

SEGMENT AND SECTOR

R = radius, K = area, c = chord



$c = 2R \sin \frac{\theta}{2}$

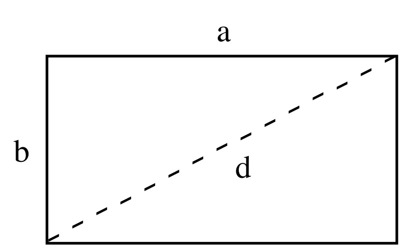


$K(\text{Segment}) = \frac{1}{2} R^2 (\theta - \sin\theta)$ [θ must be in radian measure]

$K(\text{Sector}) = \frac{1}{2} R^2\theta$ [θ must be in radian measure]

SQUARE AND RECTANGLE

a, b = side dimensions, p = perimeter, d = diagonal
K = area

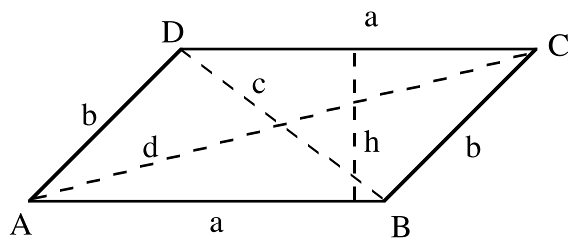


All internal angles = 90°

RECTANGLE	SQUARE
$a \neq b$	$a = b$
$d = \sqrt{a^2 + b^2}$	$d = a\sqrt{2}$
$p = 2(a + b)$	$p = 4a$
$K = ab$	$K = a^2$

RHOMBUS AND PARALLELOGRAM

a, b = side dimensions, p = perimeter
c, d = diagonals, h = altitude on side a
K = area, A, B, C, D = interior angles



PARALLELOGRAM

$A = C, B = D, a \neq b$

$A + B = 180°$

$c = \sqrt{a^2 + b^2 - 2ab\cos A} = \sqrt{a^2 + b^2 + 2ab\cos B}$

$d = \sqrt{a^2 + b^2 + 2ab\cos A} = \sqrt{a^2 + b^2 - 2ab\cos B}$

$p = 2(a + b)$

$h = b\sin A = b\sin B$

$K = ah = ab\sin A = ab\sin B$

RHOMBUS

$A = C, B = D, a = b$

$A + B = 180°$

$c = a\sqrt{2(1 - \cos A)} = a\sqrt{2(1 + \cos B)}$

$d = a\sqrt{2(1 + \cos A)} = a\sqrt{2(1 - \cos B)}$

$p = 4a$

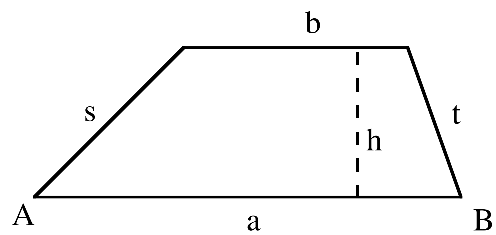
$h = b\sin A = b\sin B$

$K = ah = ab\sin A = ab\sin B$

TRAPEZOID

a, b, s, t = side dimensions, a and b are parallel
h = altitude on side a, K = area

$h = s\sin A$
$h = t\sin B$
$K = \frac{1}{2}(a + b)h$

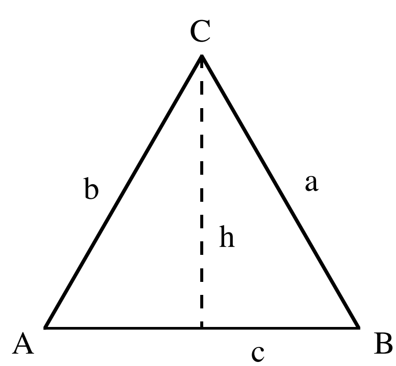


EQUILATERAL TRIANGLE

a = b = c = leg dimensions
$A = B = C = 60°$
h = altitude, K = area

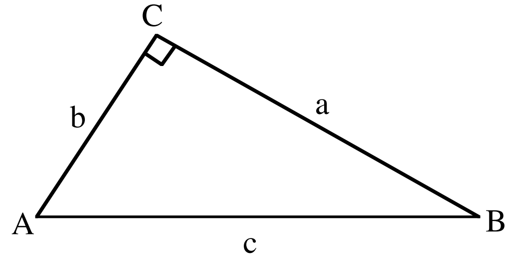
$h = \frac{\sqrt{3}}{2}a$

$K = \frac{\sqrt{3}}{4}a^2$



RIGHT TRIANGLE

a,b = leg dimensions, c = hypotenuse
A,B = acute angles, C = 90°, K = area



$A + B + C = 180°$

$c^2 = a^2 + b^2$ (Pythagorean Theorem)

$K = \frac{1}{2}\ ab$

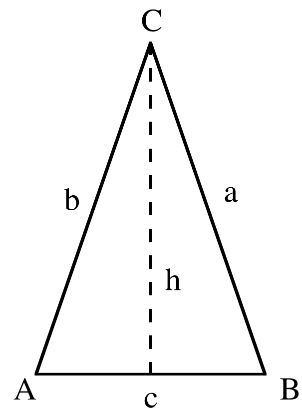
$\sin(A) \equiv \sin A = a/c$

$\cos(A) \equiv \cos A = b/c$

$\tan(A) \equiv \tan A = a/b$ $[\sin(A)]^2 + [\cos(A)]^2 \equiv \sin^2 A + \cos^2 A = 1$

ISOSCELES TRIANGLE

a, b, c = leg dimensions
A, B, C = angles
h = altitude on side c
K = area



$A = B$ $a = b$ $2A + C = 180°$

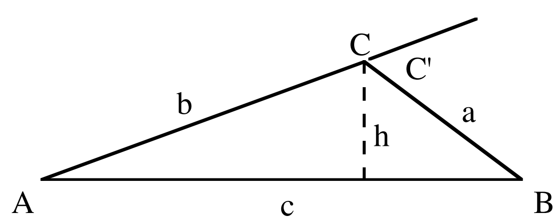
$h = a \sin A = \frac{c}{2}\ \tan A$

$c^2 = 4(a^2 + h^2) = 4\ (1 + \sin^2 A)\ a^2$

$K = \frac{1}{2}\ ch = \frac{1}{4}\ c^2 \tan A$

SCALENE TRIANGLE

a, b, c = leg dimensions, h = altitude on side c
A, B, C, C' = angles, K = area



$A + B + C = 180°$

$h = a \sin B = b \sin A$

$K = \frac{1}{2}\ bc \sin A$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin C'}{c}$ (Law of Sines)

$c^2 = a^2 + b^2 - 2ab \cos C = a^2 + b^2 + 2ab \cos C'$ (Law of Cosines)

Circumscribed and Inscribed Circles

$$s \equiv \frac{1}{2}(a+b+c)$$

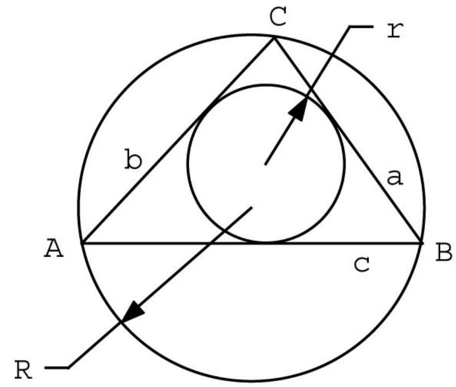
$$K = area = \sqrt{s(s-a)(s-b)(s-c)}$$ (Heron's Formula)

General (Scalene Triangles)

$$r = (s-c)\tan\left(\frac{C}{2}\right) = \frac{(a+b-c)}{2}\tan\left(\frac{C}{2}\right)$$

$$R = \frac{abc}{4K}$$

$$r = 4R\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$



Right Triangles (c = hypotenuse)

$$r = s - c = \frac{a+b-c}{2}$$

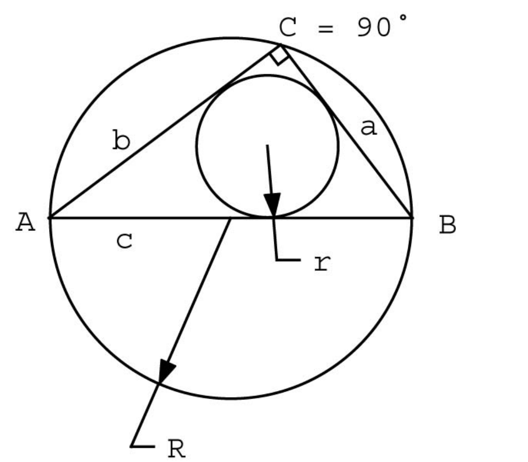
$$R = \frac{c}{2}$$

$$r = R\left(\frac{a+b-c}{c}\right)$$

Given r,c: $a,b = \frac{2r+c}{2} \pm \frac{\sqrt{8c^2 - 4(2r+c)^2}}{4}$

Given r,a: $b = \frac{2r(a-r)}{a-2r}$ and

$$c = (a-2r) + \frac{2r(a-r)}{a-2r} = a + b - 2r$$

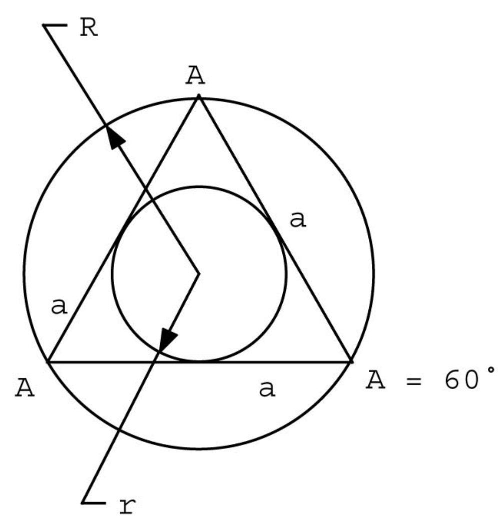


Equilateral Triangles

$$r = \frac{a\sqrt{3}}{6}$$

$$R = \frac{a\sqrt{3}}{3}$$

$$r = \frac{R}{2}$$

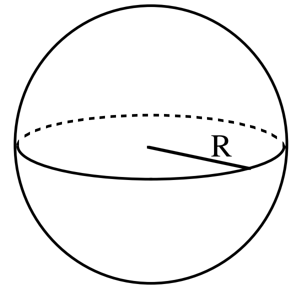


SPHERE

R = radius, D = diameter = 2R
S = surface area, V = volume

$S = 4\pi R^2 = \pi D^2$

$V = \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$

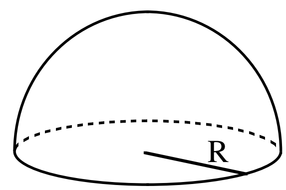


HEMISPHERE

R = radius, D = diameter
S = spherical surface area, T = total surface area
V = volume

$S = 2\pi R^2 = \frac{\pi}{2} D^2 \qquad T = 3\pi R^2 = \frac{3\pi}{4} D^2$

$V = \frac{2}{3}\pi R^3 = \frac{1}{12}\pi D^3$



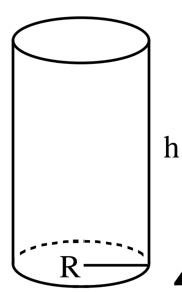
CYLINDERS

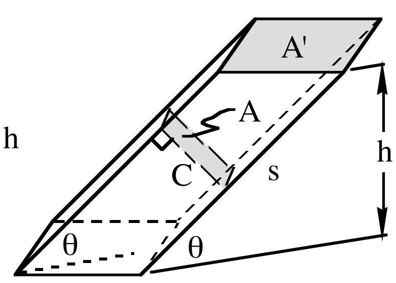
R = radius, h = altitude, s = slant height
C = right section perimeter, θ = slant angle
A = right section area, A' = slant face area
S = lateral surface area, T = total surface area
V = volume

For right circular cylinders,
$S = 2\pi Rh \qquad T = 2\pi R(R + h) \qquad V = \pi R^2 h$

For slant or right cylinders of any shape,

$S = sC = \frac{hC}{\sin\theta} \qquad A = A' \sin\theta \qquad V = hA' = sA$



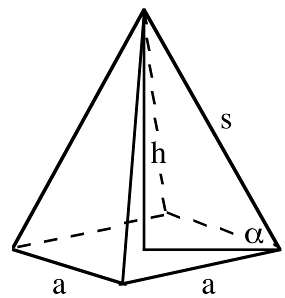


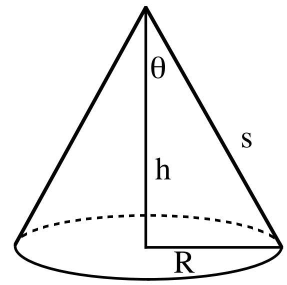
CONES AND PYRAMIDS

R = radius, a = side dimension
s = maximum slant height, h = altitude
θ = semicone angle
S = lateral surface area, T = total surface area
V = volume

For any right cone or pyramid,

$$V = \frac{h}{3} \text{ x (area of the base)}$$





SQUARE-BASE PYRAMID

$$h = \frac{a \tan\alpha}{\sqrt{2}} \qquad s = \frac{h}{\sin\alpha} = \frac{a}{\sqrt{2}\cos\alpha}$$

$$S = \frac{a^2}{\cos\alpha}\sqrt{1 + \sin^2\alpha} \qquad T = S + a^2$$

$$V = \frac{a^2 h}{3}$$

RIGHT CIRCULAR CONE

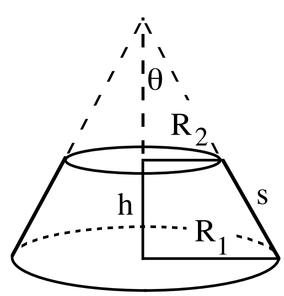
$$s = \sqrt{R^2 + h^2} = \frac{h}{\cos\theta} = \frac{R}{\sin\theta}$$

$$S = \pi R s \qquad T = \pi R(R + s)$$

$$V = \frac{1}{3}\pi R^2 h$$

FRUSTUM OF A RIGHT CIRCULAR CONE

R_1 = radius of lower base
R_2 = radius of upper base
s = slant height, h = altitude
θ = semicone angle
S = lateral surface area, T = total surface area
V = volume



$$s = \sqrt{(R_1 - R_2)^2 + h^2} = \frac{h}{\cos\theta} = \frac{R_1 - R_2}{\sin\theta}$$

$$S = \pi(R_1 + R_2)s$$

$$T = \pi[R_1^2 + R_2^2 + (R_1 + R_2)s]$$

$$V = \frac{1}{3}\pi h(R_1^2 + R_2^2 + R_1 R_2)$$