Math Level 2.5 Handouts

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Contents

1 Counting Tricks and Geometric Probability

The following are a list of counting techniques that weren't suitable for this class, or weren't covered due to time constraints. They aren't super advanced, but they are extremely hard to explain without a good understanding of what we did in before.

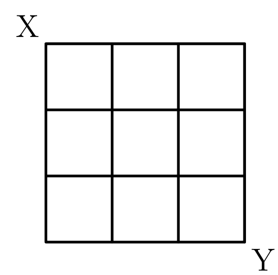
1.1 Basic Techniques

1.1.1 Committee Forming

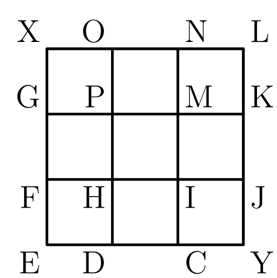
Forming committees is just a combination. If we wanted to form a committee of n members from m people, then the solution is $\binom{n}{m}$.

1.1.2 Path Finding

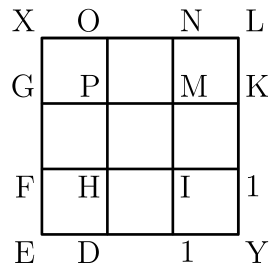
If we were to find how many paths could be made in a 5 by 5 grid if you could only go up and to the right, there would be $\binom{10}{5}$ ways because if we determine which steps will be to the right, then we can decide for the number of up ways. We choose from 10 because there will always be 10 steps.



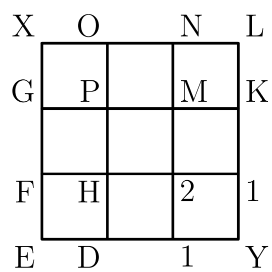
We are going to move either right or down. We start off with some easier means of this problem from A to B by labeling closer distances. Let's label these points with letters.



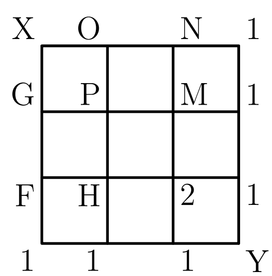
We first find that $\overline{YJ}$ and $\overline{YC}$ which is 1 for both of them "for $\overline{YJ}$ we have to move down, and for $\overline{YC}$ we move right 1".



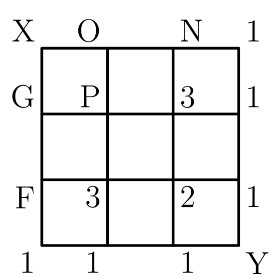
For YI she is going to have to either go right or down to start off with when we go right, we already know how many ways we have, which would be 1. For down, we again know how many ways we have, which is 1. Therefore for YI we have 2 total ways.



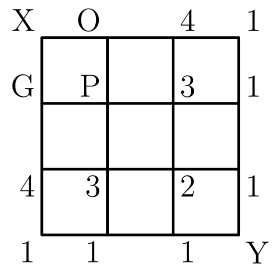
We also know for KY, LY, DY, EY we are going to have to go 2 or 3 down/right in both cases, which is a total of 1.



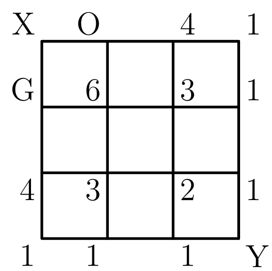
Using this same process, we find for MY and HY we are going to have to go down or right. From there, we already have the numbers filled in. For MY it is $1+2$, and for HY is it $2+1$ which in both cases is 3.



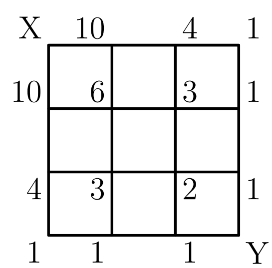
For NY and FY we have to move down or right, and find we have $3+1=4$ total ways once we move down/right.



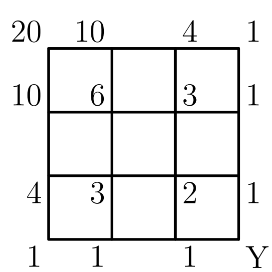
YP is going to have the two cases of going down/right again, which is going to give us $3+3=6$



For GY and OY we are going to have 6+4=10 total ways (since we start going down and right, and the cases



Now we evaluate XY. For XY, we have two cases. We can go down, or right. When we go right, we get O which has 10 possible ways from there. When we go down, we get G which has 10 possible ways. Therefore XY has $\boxed{20}$ possibles ways when only going right or down.



1.1.3 Permutations with Symmetries

Example 1.1. How many ways can 6 people be arranged around a table if two arrangements are the same if the person the the left/right of a person is the same.

Solution. 6! is obviously wrong. But if you were to look at the problem, there would be 6 rotations of the same thing so then $\frac{6!}{6} = 5! = 120$ is the answer.

As a general rule,

Theorem 1.1. (Permutations of a Line and Circle) For a line/row, the number of ways to arrange n people is $n!$. For a circle/round table, the number of ways to arrange n people where rotations are considered the same is $(n-1)!$.

1.2 Geometric Probability

In Geometric Probability, we do not count objects but rather lengths and areas.

Theorem 1.2. (Geometric Probability)

$$P(\text{good}) = \frac{\text{size of good region}}{\text{size of total region}}.$$

1.3 Problems

Problem 1.1. If two vertices of an octagon are chosen at random, what is the probability that they are adjacent?

Problem 1.2. How many ways are there to choose 3 people from 10 boys and 10 girls, given that not all boys and not all girls are chosen?

Problem 1.3. What is the probability you choose all boys or all girls when trying to choose 3 people from 10 boys and 10 girls?

Problem 1.4. If we go from $(0,0)$ to $(3,3)$ on a coordinate grid, how many paths are there of length 6?

Problem 1.5. From a standard deck, what is the probability that you draw two pairs of matching card ranks (example: 2244 or 55JJ)?

Problem 1.6. What is the probability that one randomly-selected card from a standard deck of cards is an ace or a two?

Problem 1.7. What is the probability that one randomly-selected card from a standard deck of cards is an ace or a club?

Problem 1.8. Two standard six-sided dice are rolled. What is the probability that they are both a perfect square?

Problem 1.9. The math club has 10 members, 5 boys and 5 girls. A 4 people committee is to be chosen at random. What is the probability that there is at least 1 boy and 1 girl?

Problem 1.10. There are two baseball teams, the Rhinos and the Lions, facing off against each other in 4 games. If the Rhinos have a 75% of winning each game, what is the probability that the Lions win every game?

Problem 1.11. There are 5 white marbles in a bag and 3 red marbles. What is the probability that you draw two marbles (without replacement) and they are both the same color?

Problem 1.12. Three cards are dealt from a standard deck of cards. What is the probability that the first card is a jack, the second card is a club, and the third card is a king?

Problem 1.13. There is a line segment AC with length 5, with Point B between A and C with the length of AB being 4. A point P is selected randomly in AC - what is the probability that P is closer to A than B?

Problem 1.14. Let CD be a line segment of length 6. A point P is chosen at random on CD. What is the probability that the distance from P to C is smaller than the square of the distance from P to D?

Problem 1.15. Point P is chosen at random atop a 5 foot by 5 foot table. A circular disk with radius 1 is placed on the table with its center directly on Point P. What is the probability that the entire disk is on top of the table?

Problem 1.16. My friend and I are hoping to meet for lunch. We each will arrive at the restaurant at a random time between 12 and 1, stay for 10 min, and then leave. What is the probability that we meet each other?