1 Problems

Problem 1 (AMC 10 2002/6). Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

Problem 2 (UNB Grade 9 2016/12). A collection of coins was shared. Mary received $\frac{1}{3}$ of the coins, Amir received $\frac{1}{5}$ of the coins, and Samita received $\frac{1}{6}$ of the coins. The remaining 36 coins were given to Troy. How many coins were in the entire collection?

Problem 3 (CEMC Gauss 2000/13). Karl had his salary reduced by 10%. He was later promoted and his salary increased by 10%. If his original salary was $\$20,000$, what is his present salary?

Problem 4 (AMC 10A 2002/3). According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{(2^{(2^2)})} = 2^{16} = 65536.$$

If the order in which the exponentiations are performed is changed, how many other values are possible?

Problem 5. Compute the value of $(\sqrt{12} + \sqrt{75} + \sqrt{147})^2$.

Problem 6 (ML ES Target 11322/2). Find the units digit of 147^{148}.

Problem 7 (UNB Grade 9 2010/21). Farmer Fred said to Farmer John: "If you sell me 45 hectares of land, I will have twice as much land as you." Then Farmer John said to Farmer Fred: "If you sell me 45 hectares of land, I will have just as much land as you." How many hectares of land does farmer Fred have?

Problem 8 (AMC 8 2017/23). Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

Problem 9 (CEMC Gauss 2016/16). The mean (average) of a set of six numbers is 10. If the number 25 is removed from the set, what is the mean of the remaining numbers?

Problem 10 (Folklore). Compute

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{2019 \cdot 2020}.$$

Problem 11 (AMC 8 2008/12). A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?

Problem 12. Let $a\#b = a^2 - 2ab + b^2$. Find $(2\#4)\#3$.

Problem 13. Let m and n be the roots of the polynomial $x^2 - 2020x + 202$. Find the polynomial with roots $-m$ and $-n$ and leading coefficient 2.

Problem 14. Let s and t be the roots of $x^2 - 20x + 202 = 0$. What is $s^2 + t^2$?

Problem 15 (AMC 8 2002/2). How many different combinations of \$5 bills and \$2 bills can be used to make a total of \$17? Order does not matter in this problem.

Problem 16 (UNB Grade 9 2008/16). How many ways can the number $1, 2, 3, 4$ and 5 be placed in a line so that neither 1 nor 5 occupy either the first or the last place in the sequence?

Problem 17 (CEMC Gauss 2006/19). Bethany, Chun, Dominic, and Emily go to the movies. They choose a row with four consecutive empty seats. If Dominic and Emily must sit beside each other, in how many different ways can the four friends sit?

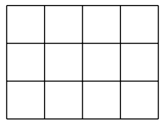
Problem 18. 10 books are labeled a number from 1-10. How many ways can Bob choose 3 books to bring to school?

Problem 19 (AMC 10A 2005/14). How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

Problem 20 (AMC 8 2016/17). An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence $9, 1, 1$, then how many passwords are possible?

Problem 21. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

Problem 22. How many rectangles are in the figure below?



Problem 23 (BmMT Team 2016/8). A seven digit number is called "bad" if exactly four of its digits are 0 and the rest are odd. How many seven digit numbers are bad?

Problem 24. How many positive integer solutions are there to $a+b+c+d=5$?

Problem 25. How many solutions are there to $a+b+c+d=100$, if a,b,c,d are all positive even integers? What if they are all odd?

Problem 26. How many solutions are there to $a+b+c+d \leq 10$, if a,b,c,d are all positive integers?

Problem 27. Simplify

$$\binom{100}{0}+2\binom{100}{1}+4\binom{100}{2}+\ldots+2^{100}\binom{100}{100}.$$

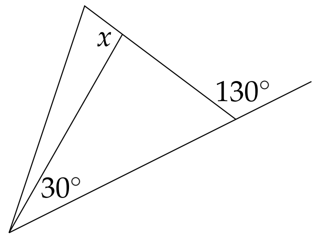
Problem 28. What is the 3rd element to the right in the 8th row of Pascal's triangle?

Problem 29. Four fair six-sided dice are rolled. What is the probability that the largest number rolled is at least 4? Express your answer as a common fraction in reduced form.

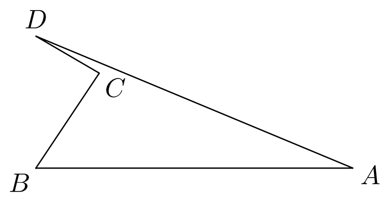
Problem 30 (AHSME 1973/23). There are two cards; one is red on both sides and the other is red on one side and blue on the other. The cards have the same probability $\left(\frac{1}{2}\right)$ of being chosen, and one is chosen and placed on the table. If the upper side of the card on the table is red, then what is the probability that the under-side is also red?

Problem 31. x and y are two positive numbers chosen randomly and uniformly in the interval $[0,1]$. What is the probability that $x^2+y^2 \leq 1$ and $x \leq y$?

Problem 32 (UNB Grade 9 2018/3). Find the measure of the angle labeled x in the diagram.

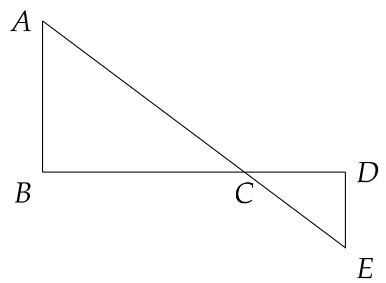


Problem 33 (AMC 8 2017/18). In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$.



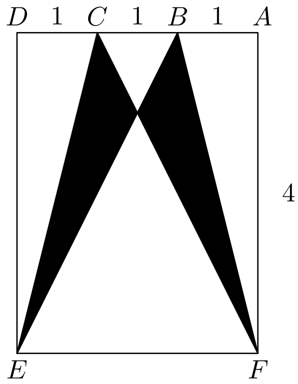
What is the area of quadrilateral $ABCD$?

Problem 34 (CEMC Gauss 2011/15). In the diagram, AE and BD are straight lines that intersect at C. If $BD = 16$, $AB = 9$, $CE = 5$, and $DE = 3$, then what is the length of AC?



Problem 35 (AMC 8 2018/22). Point E is the midpoint of side $\overline{CD}$ in square $ABCD$, and $\overline{BE}$ meets diagonal $\overline{AC}$ at F. The area of quadrilateral $AFED$ is 45. What is the area of $ABCD$?

Problem 36. Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA = 1$. The area of the "bat wings" (shaded area) is



Problem 37 (AMC 10B 2007/21). Right $\triangle ABC$ has $AB = 3, BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}$, W on $\overline{AB}$, and Z on $\overline{BC}$. What is the side length of the square?

Problem 38. A triangle has perimeter 14. What is the largest possible integer side length of the triangle?

Problem 39. What is the perimeter of a right triangle with legs 15 and 20?

Problem 40. There are two chords BC and DE in a circle that intersect at point A. If $AB = 1, AC = 8$, and $AE = 2$, what is the length of chord DE?

Problem 41. The area of a rectangle is 27. If the length is three times the length of the width, What is the length of the rectangle?

Problem 42 (CEMC Gauss 2002/24). $PQRS$ is a square with side length 8. X is the midpoint of side PQ, and Y and Z are the midpoints of XS and XR, respectively, as shown. What is the area of trapezoid $YZRS$?

Problem 43. A circle is inscribed inside a square of side length 8. What is the total area of the region inside the square but outside the circle?

Problem 44 (BmMT Ciphering 2012/26). The lines $y = 3x$ and $x = 4$ form a right triangle with the x-axis. Find the slope of a line through the origin that bisects the triangle into two portions of equal area.

Problem 45 (PPP Volume 5/11). Find the coordinates of all points in the Cartesian plane that are equidistant from the x-axis, y-axis, and the point $(2, 1)$.

Problem 46 (BmMT Ciphering 2012/20). What is the surface area of a cube inscribed in a sphere with surface area 8π?

Problem 47 (BmMT Ciphering 2012/24). An 8×11 sheet of paper is rolled up so that the 11-inches align. Find the volume of the resulting cylinder.

Problem 48 (CEMC Gauss 2008/7). The product of three *different* positive integers is 72. What is the smallest possible sum of these integers?

Problem 49. Does 3 divide 11111199?

Problem 50. Find the number of divisors of 210.

Problem 51. What is the sum of all odd factors of 162?

Problem 52. The four-digit number $AB37$ is divisible by 99. What is the product AB?

Problem 53. What is 993×1007?

Problem 54. Compute $726_8 + 145_8$. Express your answer in base 8.

Problem 55. Compute the greatest common divisor of 2021 and 2881.

Problem 56 (PiMC Final Round Team 2018/8). What is the least common multiple of $24, 28, 32$, and 36?

Problem 57. Find the last digit of $1^{2020} + 2^{2020} + 3^{2020} + 4^{2020} + 5^{2020} + 6^{2020} + 7^{2020} + 8^{2020} + 9^{2020}$.