ASE 2020-21 Special Notes

Lecture Notes by Dylan Yu*

Last Updated September 25, 2020

Contents

§ 1 Decimals, Fractions, & Percentages

§ 1.1 Reading

1. Pages 23-29 and 163-166 of *Competitive Math for Middle School*, J. Batterson
2. Decimals, Fractions, and Percentages, Math is Fun

§ 1.2 Conversions

We can either convert between different units (like inches to feet), or convert between different forms (like fraction to decimal).

*The ASE playlist can be found here.

Problem 1 (ML MS Sprint 11411/4). The perimeter of a square is $\frac{1}{10}$ meters. How many centimeters is its side length? Express your answer as a decimal to the nearest tenth.

Problem 2 (ML MS Team 11411/5). A square with side length 3 feet has an area of how many square inches?

Problem 3 (ML MS Target 11411/1). Express $0.444\ldots$ as a common fraction.

§ 1.3 Decimals

Problem 4 (ML MS Sprint 11411/1). What is the product of 1.75 and 1.6?

Problem 5 (ML MS Countdown 11411/20). Express as a decimal: $\frac{65}{12.5}$.

§ 1.4 Fractions

Problem 6 (Mathcounts Ratio Warmup). What is $\frac{9}{30}$ as a common fraction?

Problem 7 (ML MS Sprint 11411/3). Express as a mixed number: $4\frac{1}{6} - 1\frac{5}{6}$.

Problem 8 (ML MS Sprint 11411/6). Simplify: $\frac{\frac{1}{x-1}+\frac{1}{x+1}}{\frac{1}{1-x}-\frac{1}{x+1}}$.

Problem 9 (ML MS Countdown 11411/7). Simplify: $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$. Express your answer as a common fraction.

Problem 10 (ML MS Countdown 11411/13). If $x = 2$ and $y = \frac{1}{2}$, what is the value of $\frac{xy}{2} + 3y$?

Problem 11 (ML MS Countdown 11411/19). Simplify: $\frac{2020}{101}$.

§ 1.5 Percentages

Problem 12 (ML MS Sprint 11411/2). The original price of a car is \$16,000. After two weeks, the price is reduced by 20%. Another two weeks later, the sale price is reduced by 20%. What is the final price of the car?

Problem 13 (ML MS Countdown 11411/58). If 1% of 10% of a number is 66, then what is the number?

§ 1.6 Ratios & Proportions

Problem 14 (ML MS Sprint 11411/13). Two numbers with a ratio of 2 to 9 have a sum of 143. What is their positive difference?

Problem 15 (ML MS Countdown 11413/10). In a map, $1\frac{5}{8}$ inches represents 91 miles. How many inches would 336 miles be represented by on the map?

Problem 16 (Mathcounts Ratio Warmup). In a pasture there are 12 white horses and the rest are black. If there are 52 horses in the pasture, what fraction are black? Express your answer as a common fraction.

Example 1. 10 moles dig 10 holes in 10 hours, how many holes do 20 moles dig in 20 hours?

Solution. There are twice as many moles, so that doubles the number of holes, and twice as many hours also doubles the number of holes. Thus, the answer is $10 \times 2 \times 2 = \boxed{40}$. □

Theorem 1 (Moles Digging Holes Formula). If there are m moles digging h holes in t time, then $h = rtm$, where r is the rate they dig at, and r is constant.

Theorem 2. Let person 1 work at a speed of one object per t_1 time, person 2 works at a speed of one object per t_2 time, and so on, all the way to person n who works at a speed of one object per t_n time. Then if they work together, they will finish in

$$\frac{1}{\frac{1}{t_1} + \frac{1}{t_2} + \ldots + \frac{1}{t_n}}$$

time.

Example 2. Dylan can paint a house in 3 hours. Cody can paint a house in x hours. If they work together, it takes then 2 hours to paint the house. What is x?

Solution. Using the theorem, we get

$$\frac{1}{\frac{1}{3} + \frac{1}{x}} = 2,$$

$$\frac{1}{x} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6},$$

$$x = \boxed{6}.$$

□

§ 1.7 Novice Problems

Problem 17 (ML MS Target 11411/3). A rectangular field is 40 meters long and 50 meters wide. Fence posts are placed every 5 meters along the perimeter of the field. How many fence posts are used?

Problem 18 (ML MS Countdown 11411/28). The angles of a triangle are in the ratio $2:3:4$. What is the degree measure of the largest angle in the triangle?

Problem 19 (ML MS Countdown 11411/29). How many positive integers less than 100 are divisible by 3?

Problem 20 (ML MS Countdown 11411/31). Evaluate: $\left(2\frac{4}{9}\right)\left(4\frac{4}{9}\right)$. Express your answer as a mixed number.

Problem 21 (ML MS Countdown 11411/45). Express in simplest form: $0.444\ldots+0.555\ldots$.

Problem 22 (ML MS Countdown 11411/61). Express in scientific notation: $\frac{1\times10^4}{4\times10^7}$.

Problem 23 (ML MS Countdown 11411/67). The four numbers a, b, c, and d satisfy the relations $\frac{a}{b}=4$, $\frac{b}{c}=\frac{2}{3}$, and $\frac{c}{d}=12$. What is $\frac{a}{d}$?

Problem 24 (ML MS Countdown 11411/71). The perimeter of a regular pentagon is 35 cm. What is its side length in cm?

Problem 25 (ML MS Countdown 11411/74). A baseball team lost 37.5% of the 80 games they played. How many games did they win?

Problem 26 (ML MS Countdown 11411/78). Express as a decimal: $4\times100+2\times1+\frac{4}{10}$.

Problem 27 (ML MS Sprint 11412/19). Simplify: $\frac{1}{2+\frac{2}{4+\frac{1}{3}}}$. Express your answer as a common fraction.

Problem 28 (ML MS Target 11412/2). If $\frac{1}{3}+\frac{1}{7}=\frac{1}{x}$, then express x as a decimal to the nearest tenth.

Problem 29 (Mathcounts Ratio Warmup). There are three times the number of orange fish as blue fish in a tank at the pet store, and there are no other fish. What percentage of the fish are orange?

Problem 30 (Mathcounts Ratio Stretch). Fairy Godmother has granted wishes to Aurora, Belle and Cindi in the ratio $6 : 8 : 11$. What fraction of the ratios were granted to Belle? What percent of the wishes granted by Fairy Godmother were not granted to Aurora? What is the absolute difference between the percents of wishes Fairy Godmother has granted to Aurora and to Cindi?

Problem 31 (Mathcounts Chapter 2019/8). After a brisk workout, Felicia counts 32 heartbeats in 15 seconds. Based on this count, what is Felicia's expected number of heartbeats in one minute?

Problem 32 (ML MS Team 11412/3). Let $r(n)$ denote the reciprocal of the number n. For example, $r(3) = \frac{1}{3}$. Express $r(1) + r(2) + r(3)r + r(r(4)) + r(r(5)) + r(6)$ in simplest form.

Problem 33 (ML MS Countdown 11412/40). Simplify: $\frac{\frac{3}{5}}{\frac{12}{4}}$.

Problem 34 (ML MS Countdown 11412/59). Compute: $\left(\frac{36}{81}\right)^{-\frac{1}{2}}$. Express as a fraction.

Problem 35 (ML MS Countdown 11413/19). Express $0.\overline{246}$ as a common fraction.

Problem 36 (ML MS Sprint 11415/2). If $4 = \frac{k}{8} - 4$, find k.

Problem 37 (ML MS Team 11415/2). A car is driven 400000 miles during 2014, and each subsequent year it drives 20% fewer miles than the year before. Which year will the car first drive fewer than 20000 miles? Express your answer as an integer.

Problem 38 (MOEMS 3/4). Express the following sum as a simple fraction in lowest terms.

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6}$$

Problem 39 (MOEMS 4/4). If $\frac{1}{3} = \frac{1}{A} + \frac{1}{B}$ where A and B are different whole numbers, find the value of A and the value of B.

Problem 40 (MOEMS 5/3). Express the extended fraction as a simple fraction in lowest terms.

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}$$

Problem 41 (MOEMS 79/5). What simple fraction is equal to the complex fraction shown?

$$\frac{1}{4 + \frac{1}{2 + \frac{1}{3}}}$$

§ 1.8 Advanced Problems

Problem 42 (ML MS Countdown 11411/57). Simplify: $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{12}\right)$. Express your answer as a common fraction.

Problem 43 (ML MS Target 11412/7). A basketball team won 40% of the first 50 games it played in an 82-game season. What is the minimum number of the remaining games the team must win to finish with a winning percentage of at least 55%.

Problem 44 (ML MS Sprint 11413/21). Leopold has nearly-perfect pitch. He can't hear a discrepancy within 1% of a "perfect" note, but a variation between 1% and 2% makes him twitch. He winces if he hears a note that is between 2% and 3% off of "true". A tone that is between 3% and 4% too flat or too sharp makes him squirm, and a variation of 4% to 5% makes him cringe. A note that is off by 5% or more makes him cry. At his sister's school concert, a soloist attempts to hit a "high A" (880 Hertz). The nervous soloist is singing flat: she produces a tone that is actually 850 Hertz. What level of discomfort does she cause to poor Leopold? Does he notice nothing, twitch, wince, squirm, cringe, or cry?

Problem 45 (ML MS Team 11413/8). In a shared minecraft game, Henry and eleanor construct a lovely 12th century Norman castle. Henry begins construction at 9:00 AM. At noon, Eleanor logs in and begins to help. They finish the last turret at 5:00 PM. Assuming Eleanor works twice as fast as Henry, what fraction of the castle did she build? Express your answer as a common fraction.

Problem 46 (ML MS Sprint 11415/20). Cliff is 13 years old, Steve is 16 years old, and Britney is 21 years old. They need to work together to mow the law. THe lawn is 15m by 20m, but they each need to mow an amount of the lawn proportional to their age. If only three quarters of the lawn needs to be mowed, how many more m^2 does Steve mow than Cliff? Express your answer as a decimal.

Problem 47 (MOEMS 6/1). X and Y are two different numbers selected from the first fifty counting numbers from 1 to 50 inclusive. What is the largest value that $\frac{X+Y}{X-Y}$ can have?

Problem 48 (MOEMS 23/5). A jar filled with water weighs 10 pounds. When one-half of the water is poured out, the jar and remaining water weigh $5\frac{3}{4}$ pounds. How much does the jar weigh?

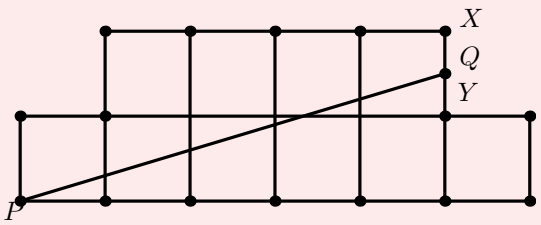
Problem 49 (AMC 8 2018/1). An amusement park has a collection of scale models, with ratio 1 : 20, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica to the nearest whole number?

Problem 50 (AMC 8 2017/6). If the degree measures of the angles of a triangle are in the ratio $3:3:4$, what is the degree measure of the largest angle of the triangle?

Problem 51 (AMC 8 2010/11). The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

Problem 52 (AMC 8 2010/16). A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

Problem 53 (AMC 8 2010/17). The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



Problem 54 (AMC 8 2004/3). Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have just enough food for the 12 of them?

Problem 55. If 3 people can build 5 walls in 2 days, how many days would it take for 10 people to build 150 walls?

Problem 56. Suppose that x, y, z satisfy the following system of equations:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{3}$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{5}$$

$$\frac{1}{y}+\frac{1}{z}=\frac{1}{7}$$

What is the value of the ratio $\frac{z}{y}$?

Problem 57. How many digits to the right of the decimal point does $\frac{1}{2^{2020}}$ have when written in decimal form?

Problem 58 (Mathcounts). On a True/False test, Amy answered the first three questions wrong but answered the rest of the questions correctly. On the same test, Scott answered exactly two questions wrong. He answered the last question wrong, the 26 questions before it correctly, and the question before that wrong. If Amy and Scott were both incorrect on the same question exactly once, what is the greatest possible percent of the total number of questions that Amy could have answered correctly?

Problem 59. For how many k between 1 and 30, inclusive, does the decimal representation of $\frac{k}{30}$ terminate?

Problem 60 (Mathcounts School Sprint 2014). What is the absolute difference between the additive inverse and the multiplicative inverse of $\frac{2}{3}$?

Problem 61 (AMC 12B 2011/1). What is

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}?$$

Problem 62 (AMC 12B 2011/3). LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

Problem 63 (∗). Evaluate $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+\ldots+\frac{1}{97\cdot 100}$.

Problem 64 (∗). Evaluate $1-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}-\frac{13}{42}+\frac{15}{56}$