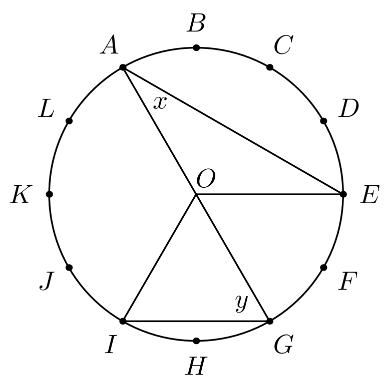
1 Problems

Please try problems on pages 190 and 192 of *Competitive Math for Middle School* before attempting the ones below.

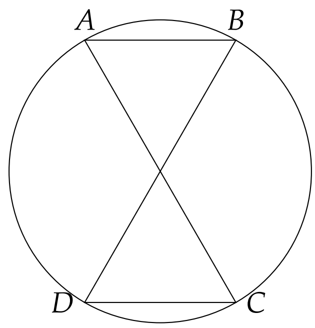
Problem 1 (AHSME 1950/33). The number of circular pipes with an inside diameter of 1 inch which will carry the same amount of water as a pipe with an inside diameter of 6 inches is n. Find n.

Problem 2 (AHSME 1973/25). A circular grass plot 12 feet in diameter is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. What is the number of square feet in the remaining grass area?

Problem 3 (AMC 8 2014/15). The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?

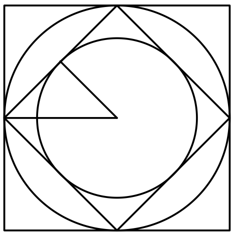


Problem 4. Chords $\overline{AC}$ and $\overline{BD}$ of a circle meet at P as shown. Given $AP = PB$, show that $\overline{AB} \parallel \overline{CD}$.

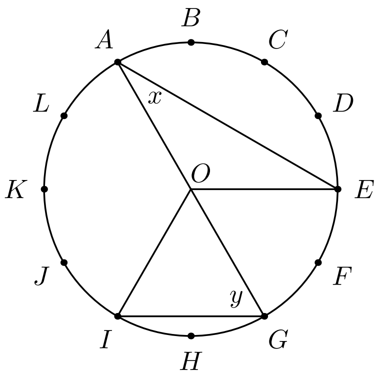


Problem 5 (AMC 12A 2002/7). A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

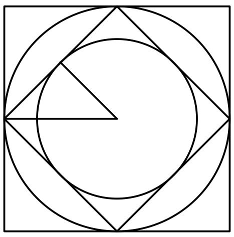
Problem 6 (AMC 10B 2005/7). A circle is inscribed in a square, then a square is inscribed in this circle, and finally, a circle is inscribed in this square. What is the ratio of the area of the smaller circle to the area of the larger square?



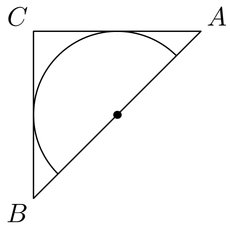
Problem 7 (AMC 8 2005/23). Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse $\overline{AB}$ and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?



Problem 8. Three circular arcs of radius 5 units bound the region shown. Arcs AB and AD are quarter-circles, and arc BCD is a semicircle. What is the area, in square units, of the region?

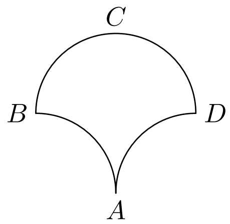


Problem 9. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?

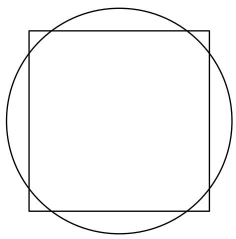


Problem 10 (AMC 8 2006/7). Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest radius.

Problem 11 (AMC 8 2008/25). Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Which of the following is closest to the percent of the design that is black?



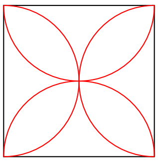
Problem 12 (AMC 8 2012/24). A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?



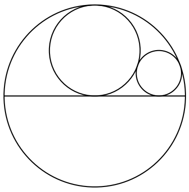
Problem 13. In a circle, chords AB and CD intersect at E. If $AE : BE = 1 : 4$ and $CE : DE = 4 : 9$, find $\frac{AB}{CD}$.

Problem 14. Chords AB and CD of a given circle are perpendicular to each other and intersect at a right angle. Given that $BE = 16$, $DE = 4$, and $AD = 5$, find CE.

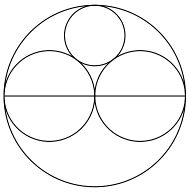
Problem 15. Find the area contained inside the red border below, if the square has a side length of 4:



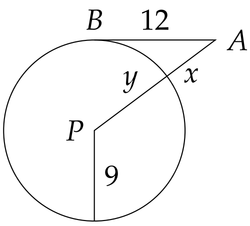
Problem 16. Find the radius of the smallest circle if the largest circle has a radius of 4.



Problem 17. Find the radius of the smallest circle if the largest circle has a radius of 4.



Problem 18 (Mathcounts State Sprint 2000/22). Given that AB is tangent to circle P at B, what is the ratio $\frac{x}{x+y}$? Express your answer as a common fraction.



Problem 19. Two parallel chords on the same side of the center of a circle are inches and 20 inches long and 2 inches apart. Find the radius of the circle. Express your answer in simplest radical form.

Problem 20 (Mathcounts State Sprint 1992). Triangle ABC is an equilateral triangle and O is the center of its inscribed circle. If the area of the circle is 4π cm^2, what is the area, in square centimeters, of triangle ABC? Express your answer in simplest radical form.

Problem 21 (Mathcounts National Sprint 1996). $\triangle ABC$ is an isosceles right triangle with hypotenuse AC. A circle is drawn through B that is tangent to AC at P. PB is a diameter of the circle. Given that AC is 8 units long, how many square units are in the area of the region inside the circle but outside the triangle? Express your answer in terms of π.

Problem 22. In a circle with center O, $AB = AC$. Chord AD cuts BC at E. If $AC = 12$ and $AE = 8$, find AD.

Problem 23 (Mathcounts National Team 1996). One side of a triangle is divided into segments of lengths 6 cm and 8 cm by the point of tangency of the inscribed circle. If the radius of the circle is 4 cm, what is the length, in centimeters, of the shortest side of the triangle?

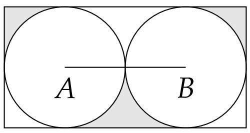
Problem 24 (Mathcounts National Target 1996). Square $SQUA$ with midpoints M and N of sides SQ and SA, respectively, has an area of 64 square units. WHat is the number of square units, rounded to the nearest integer, in the area of the largest circle which can be drawn in pentagon $MNAUQ$?

Problem 25 (Mathcounts National Team 2005). Square $ABCD$ has sides of length 1 cm. Triangle CFE is an isosceles right triangle tangent to arc BD at G. Arc BD is a quarter circle with its center at A. What is the total area of the two shaded regions? Express your answer as a decimal to the nearest thousandth.

Problem 26 (Mathcounts National Target 1994). A circular table is pushed into the corner of a square room so that a point P on the edge of the table is 8″ from one wall and 9″ from the other wall. Find the radius of the table in inches.

Problem 27 (Mathcounts National Team 1998). A round table is pushed into a corner. Point A is on the outer edge of the table and is 2 inches from one of the walls. Given that the radius of the table is 37 inches, how many inches is point A from the other wall?

Problem 28 (Mathcounts State Sprint 1990). Find the area of the shaded portion of the rectangle, where $AB = 9$. Express the answer in terms of π.



Problem 29 (Mathcounts State Sprint 1990). Circles A (with center A) and D (with center D) of radius 10 inches intersect in such a way that their centers are 10 inches apart. BC is drawn such that B lies on circle A and C lies on circle D, and BC is tangent to both circles. What is the area of the region inside $ABCD$ and outside the two circles?

Problem 30 (Mathcounts National Sprint 2001/18). Sector OAB is a quarter of a circle (with center O) of radius 3 cm. A circle is drawn inside this sector, tangent to the arc AB and sides AO and BO. What is the number of centimeters in the radius of the inscribed circle? Express your answer in simplest radical form.

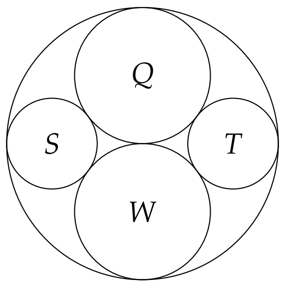
Problem 31 (Mathcounts National Team 1997/9). Two semicircles are inscribed in a rectangle so their diameters are opposite sides of the long edge of the rectangle. What is the probability that a point randomly selected inside the rectangle will also be inside both semicircles?

Problem 32 (Mathcounts National Team 1984/8). The radii of 3 mutually tangent circles are consecutive positive integers. The triangle formed by connecting the centers of the circles has perimeter 30 units. Find the circumference of the largest circle. Express the answer in terms of π.

Problem 33 (Mathcounts National Sprint 1987/38). Three circles with radii 2, 4, and 6 are tangent to each other as shown, and $\angle ABC = 90°$. Find the number of square units in the area of the triangle whose vertices are A, B, and C, the centers of the circles.

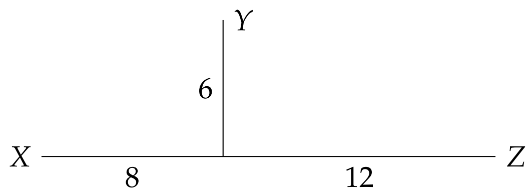
Problem 34 (Mathcounts National Sprint 2008/29). In the circle with center Q, radii AQ and BQ formed a right angle. Semicircles with diameter AQ and BP are tangent, where P lies on BQ. The radius of the circle with center Q is 14 inches. What is the radius of the smaller semicircle? Express your answer as a common fraction.

Problem 35 (Mathcounts National Sprint 1996/25). In the diagram, circle Q is congruent to circle W, and both are tangent to circle O and to each other. Circle S and circle T are congruent and are tangent to circle O, to circle Q and to circle W. Find the ratio of the area of the smallest circle to the largest circle.



Problem 36 (Mathcounts State Sprint 1995). An isosceles triangle is inscribed in a circle so that one of its sides is a diameter. The ratio of the area of the triangle to the area of the circle is $1 : a$. Express a in terms of π.

Problem 37 (Mathcounts National Team 1997). How many units are in the length of the radius of the circle which passes through points X, Y, and Z? Express your answer as a decimal rounded to the nearest tenth.



Problem 38 (Mathcounts State Target 1992/6). An isosceles triangle with equal sides of 5 inches and a base of 6 inches is inscribed in a circle. What is the radius, in inches, of the circle? Express your answer as a mixed number.

Problem 39 (Mathcounts National Sprint 1999). The diameter of a circle is 26 centimeters. A chord is drawn such that its midpoint is 12 cm from the center of the circle. What is the number of centimeters in the length of the chord?

Problem 40 (Mathcounts National Sprint 1997). Equilateral $\triangle ABC$ is inscribed in circle O. The radius of circle O is 12 inches. How many square inches are in the area of $\triangle ABC$? Express your answer in simplest radical form.

Problem 41 (Mathcounts State Team 1988). A semicircle has center O and longest chord AB. A rectangle whose length is twice its width has two vertices on AB and two vertices on the semicircle. Find the ratio of the area of the semicircle to the area of the rectangle. Express your answer in terms of π.