Substituting into the percent increase formula, the answer is obtained.

$$\text{Percent Increase} = 100\left[\frac{\text{Larger Number}}{\text{Smaller Number}} - 1\right] = 100\left[\frac{1.1798\ V_o}{V_o} - 1\right] = 18.0\%$$

<u>Function Example 14.</u> A wire under tension stretches 10%. What was the percent decrease in radius, assuming constant volume? (%)

Assume that the wire has a radius R and length L. For a cylindrical shape,

$$V = \pi R^2 L$$

If the wire is stretched 10%, the length increases to a new value $L_s = 1.1L$, and the radius decreases to R_s. R_s may be obtained from the equality of volume.

$$V = \pi R^2 L = \pi R_s^2 L_s = \pi R_s^2 (1.1\ L)$$

$$\pi R^2 L = \pi R_s^2 (1.1\ L)$$

$$R^2 = 1.1\ R_s^2$$

$$R_s = 0.9535\ R$$

The answer then comes from use of the percent decrease formula.

$$\text{Percent Decrease} = 100\left[1 - \frac{\text{Smaller Number}}{\text{Larger Number}}\right] = 100\left[1 - \frac{0.9535\ R}{R}\right] = 4.65\%$$

vi. Logarithmic Solutions

Calculators have problems with extremely small or extremely large numbers. Usually, the calculator substitutes "0" for small numbers and the largest number it knows for large numbers, often $9.99\text{x}10^{99}$. Some calculators print a message, like "Out of Range" or "Underflow". You can see what your calculator does by trying to obtain the answer to 577^{3094} or 1.45^{-7692}. The purpose of logarithmic solutions is to extend the capacity of your calculator beyond its intrinsic range. To do this, knowledge of two properties of logarithms is necessary. These properties are

$$\text{Log}_{10}(K^m) = m\ \text{Log}_{10}(K) \qquad \text{and} \qquad 10^{\log_{10}(A)} = A \quad .$$

We use these relationships to split a large number into two workable numbers, the digits part of the number and the power of ten. To evaluate a number of the form K^m using the logarithmic solution, we write an equation where the answer A is given.

$$A = K^m$$

Next, we take the base 10 logarithm of both sides. With use of the property of logarithms, this yields:

$$\text{Log}_{10}(A) = \text{Log}_{10}(K^m) = m\ \text{Log}_{10}(K) = M + C,$$

where the last term has been split into two parts: a number M, called the "mantissa", which must lie between zero and one, and an *integer* remainder C, called the "characteristic". When A is greater than 1, M is the fractional part and C is the integer part of $m\ \text{Log}_{10}(K)$. When A is less than one, C must be one less than the integer part of $m\ \text{Log}_{10}(K)$. The motivation for this will become apparent in the example problems below. We now raise 10 to a power equal to the left and right terms of the above equation. Another way to say this is to take the "anti-logarithm" of both sides of the equation.

$$10^{\log_{10}(A)} = 10^{(M + C)}$$

$$A = 10^M x 10^C$$

The only remaining task is to evaluate 10^M on the calculator, leaving 10^C as the power-of-tens part of a scientific notation answer. The best way to demonstrate the utility of the logarithmic solution is through several illustrative examples.

Function Example 15. What is 5050^{505}?

Following the steps above, an answer is obtained.

$$A = 5050^{505}$$

$$\mathrm{Log}_{10}(A) = \mathrm{Log}_{10}(5050^{505}) = 505\ \mathrm{Log}_{10}(5050) = 1870.16215 = 0.16215 + 1870$$

$$10^{\log_{10}(A)} = 10^{(0.16215 + 1870)}$$

$$A = 10^{(0.16215 + 1870)} = (10^{0.16215})10^{1870}$$

$$A = 1.45 x 10^{1870}$$

Function Example 16. What is 7080^{-655}?

The solution is identical to the previous example, excepting that care must be taken in choosing the correct value of the mantissa M, the fractional part of the logarithm of A.

$$A = 7080^{-655}$$

$$\mathrm{Log}_{10}(A) = \mathrm{Log}_{10}(7080^{-655}) = -655\ \mathrm{Log}_{10}(7080) = -2521.772 = 0.228 + (-2522)$$

Note that M must be between 0 and 1, so –0.772 is not the correct value for M. We must add one to –0.772 to obtain 0.228 and maintain equality by subtracting one from –2521 to obtain C = –2522.

$$10^{\log_{10}(A)} = 10^{[0.228 + (-2522)]}$$

$$A = 10^{[0.228 + (-2522)]} = (10^{0.228})10^{-2522}$$

$$A = 1.69 x 10^{-2522}$$

G. Problems Involving Transcendental Functions (Solver Problems)

There are a large number of engineering equations that involve one variable and which do not have an algebraic solution. Consider x+sinx=2 where x is in radians. There are advanced methods for solving this using series expansions of the sin function, but these methods lie outside the purview of the Calculator Applications Contest. Equations like this can be solved using the solver (or equivalent) feature on a calculator.

The solver approach is to guess a value for x and see how close we come to making the equation work. For the function x+sinx=2, we might guess x=1.5. Then x+sinx=2.50. If we guess x=2, we get x+sinx=2.91. Based on these two values, we surmise that x might be less than 1.5, so we guess x=1 and get x+sinx=1.84. So, it looks like x lies between 1 and 1.5. By guessing1.25, etc., to continue to refine the value of x, we could with patience arrive at x=1.11, for which x+sinx =2.000. This educated-guess approach to solving an equation is called *iteration*.

Engineering calculators have a built-in program (the solver) that works by doing the educated guessing for us very quickly. The method varies depending on the calculator, but the general steps to using a solver are (a) enter the equation into the calculator, (b) setting a first guess for the variable we want to solve, and (c) instructing the calculator to solve the equation for the desired variable. To learn specifics on the use of the solver for your calculator, you must refer to the operator's manual or ask your coach or another person who knows how to use the solver.