🍂1 Problems

Problem 1. If $2^{3x+5} = 8^{-23} \cdot 4^x$, find x.

Problem 2. If $3^{9x} = \frac{9^{24}}{81^{14}}$.

Problem 3. A bacteria colony doubles its population every 10 seconds. If it has a population of 200 right now, how many seconds ago did it have a population of 100?

Problem 4 (ML ES Target 11322/2). Find the units digit of 147^{148}.

Problem 5. Find the last two digits of 6^{2020}.

Problem 6. Compute

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}.$$

Problem 7 (ML ES Sprint 11927/20). If $16^{3x-2} = 36$, what is the value of 8^x?

Problem 8 (ML ES Sprint 12029/21). If $16^{x+21} = 64^{\frac{x}{2}-1}$, then what is the value of $2^{-\frac{x}{2}}$?

Problem 9 (ML ES Sprint 12027/22). If $9^{3x-2} = 243$, then what is the value of 2^{4x+3}?

Problem 10 (ML ES Sprint 11721/23). Simplify $\frac{9^6}{6^9}$.

Problem 11 (ML ES Sprint 11923/23). If $4^{x+3} = 192$, what is the value of 2^{4x+1}?

Problem 12 (ML ES Sprint 12121/24). If $12^y = 32$, then what is the value of $144^{\frac{y}{4}}$?

Problem 13 (ML ES Sprint 12020.2/25). If $9^{6x} = 27^{1-x}$, what is the value of $\frac{1}{x}$?

Problem 14 (ML ES Sprint 12121/27). What is the least whole number value of x for which 2^x is greater than 5^{27}?

Problem 15 (ML ES Sprint 11923/27). What is the remainder when 21^{22} is divided by 23?

Problem 16 (ML ES Team 12027/6). What is the units digit of $2^{2020} + 3^{2020} + 5^{2020} + 7^{2020}$?

Problem 17 (ML ES Team 11927/7). What is the units digit of $2^{2019} + 0^{2019} + 1^{2019} + 9^{2019}$?

Problem 18 (ML ES Team 12029/7). What is the sum of the hundreds, tens, and units digit of 18^{2020}?

Problem 19 (ML ES Team 12020.2/8). What is the units digit of $2^{2021} + 0^{2021} + 1^{2021} + 9^{2021}$?

> *Remark 1.1.* Does the above problem look familiar?