# ASE 2020-21 Notes

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# §1 Sunday, 07/19/20

## §1.1 Number Sense: Multiplying by Any Fraction of 100, 1000, etc.

You can take what we learned from the 25's and 75's trick (converting them to fractions of 100) with a variety of potential fractions. $\frac{1}{8}$'s are chosen often because:

$$125 = \frac{1}{8} \cdot 1000 \qquad 37.5 = \frac{3}{8} \cdot 100 \qquad 6.25 = \frac{5}{8} \cdot 10$$

In addition, you see $\frac{1}{6}$'s, $\frac{1}{3}$'s, $\frac{1}{9}$'s, and sometimes even $\frac{1}{12}$'s for approximation problems (because they do not go evenly into $100, 1000$, etc..., they have to be approximated usually).

$$223 \approx \frac{2}{9} \cdot 1000 \qquad 8333.3 \approx \frac{5}{6} \cdot 10000 \approx \frac{1}{12} \cdot 100000 \qquad 327 \approx \frac{1}{3} \cdot 1000$$

For approximations you will rarely ever see them equate to almost exactly to the correct fraction. For example you might use $\frac{2}{3} \cdot 1000$ for any value from $654 - 678$. Usually you can tell for the approximation problems what fraction the test writer is really going for.

### §1.1.1 Multiplying by Any Fraction of 100, 1000, etc. Problems

1. $125 \times 320 =$ ________
2. (*) $8333 \times 24 =$ ________
3. $138 \div 125 =$ ________
4. (*) $57381 \div 128 =$ ________
5. (*) $245632 \div 111 =$ ________
6. (*) $\mathbf{16667 \div 8333 \times 555} =$ ________
7. (*) $12.75 \times 28300 \div 102 =$ ________
8. $375 \times 24.8 =$ ________
9. (*) $857142 \times 427 =$ ________
10. $.0625 \times .32 =$ ________
11. (*) $16667 \times 369 =$ ________
12. (*) $375.1 \times 83.33 \times 1.595 =$ ________
13. (*) $\mathbf{8333 \div 6666 \times 4444} =$ ________
14. (*) $\mathbf{8333 \times 12\frac{1}{2}\% \times .12} =$ ________
15. (*) $639 \times 375 \div 28 =$ ________
16. (*) $6250 \div 8333 \times 8888 =$ ________

## §1.2 Calculator: Number Crunchers

**Number Crunchers** are problems involving just pushing buttons to get a specific answer. There are a few things hard about these problems:

1. They require little to no math skill, so you must be **fast**.
2. When you are fast, you make more mistakes. So you must be **careful**.
3. Even little mistakes like not converting to `rad` and using `deg` instead are important.

Let's try a few (these are real problems!):

**Example 1.** $193 \times \pi \times 155 =$ ______

*Solution.* Here are the steps:

1. Type 193
2. Press `enter`
3. Press $\leftarrow$ then `cos`
4. Press $\times$
5. Press 155
6. Press $\times$

This should display 93980.74423..., which gives us our answer of $\boxed{9.40 \times 10^4}$. ☐

**Example 2.** $\left[\frac{(131+137)}{134/130}\right]\left[\frac{0.00247}{157}\right] =$ ______

*Solution 1.* Here are the steps:

1. Type 131
2. Press `enter`
3. Type 137
4. Press $+$
5. Type 134
6. Press `enter`
7. Type 130
8. Press $\div$
9. Press $\div$
10. Type .00247
11. Press `enter`
12. Type 157
13. Press $\div$
14. Press $\times$

This should display 0.00409044586..., which gives us our answer of $\boxed{4.09 \times 10^{-3}}$. ☐

**Remark 1.** *This is actually not the fastest solution.*

Let's try again:

*Solution 2.* Here are the steps:

1. Type 131
2. Press `enter`
3. Type 137
4. Press $+$
5. Type 134
6. Press $\div$
7. Type 130
8. Press $\times$
9. Type .00247
10. Press $\times$
11. Type 157
12. Press $\div$

This should display 0.00409044586..., which gives us our answer of $\boxed{4.09 \times 10^{-3}}$. ☐

Notice how we ignored some of the `enter`s and still got the same answer. This may only save a few seconds, but in the end, a few seconds are a lot!

**Fact 1.** Remember that when the problem has (rad) in it, it means **convert to radians**! If it is (deg), **convert to degrees**!

### §1.2.1 Number Crunchers Problems

1. $\sqrt{\frac{1/(18.2-11.8)}{(43.8)(17.6+75)^2}} =$ ______

2. $\sqrt[3]{4.65-1190/998}+1/\sqrt{0.0205+0.0045} =$ ______

3. $\frac{26!+25!}{25!} =$ ______

4. (rad) $\frac{\sin(0.507)-\tan(0.507)}{\sin(0.507)}$ ______

5. $(161-132)^{0.304-0.142} =$ ______

6. $12.2^{1.2} =$ ______

7. (deg) $\frac{\sin(30)-\cos(30)}{\cos 45} =$ ______

8. $\sqrt[4]{\frac{1/3+2/3-4/5}{14.3-13.3+1}} =$ ______

## §1.3 General Math: Solving Quadratics

Two weeks ago, we covered **quadratics** and a few hard facts regarding them. Now we will focus a lot more on how to solve them.

**Definition 1 (Quadratic Equation).** A **quadratic equation** is a polynomial with $n = 2$:

$$ax^2 + bx + c.$$

A common way to solve the equation $ax^2 + bx + c = 0$ is using the **Quadratic Formula**:

**Theorem 1 (Quadratic Formula).** For the equation $ax^2 + bx + c = 0$, the roots $x_1, x_2$ must be equal to

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a},$$

$$x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Some people may ask, what if $b^2 - 4ac$ is less than 0? What do we do then? This leads us to the **Discriminant Test**:

**Theorem 2 (Discriminant Test).** For the equation $ax^2 + bx + c = 0$, we have the following cases:

- If $b^2 - 4ac > 0$, we have **two real solutions.**
- If $b^2 - 4ac = 0$, we have **one real solutions.**
- If $b^2 - 4ac < 0$, we have **zero real solutions.**

Now let's discuss **completing the square**:

**Example 3.** Find the roots of $(x+4)^2-4=0$.

*Solution.* This is equal to $(x+4)^2=4 \implies x+4=\pm 2$, so $x+4=2$ or $-2$, so $x=\boxed{-2,-6}$. □

**Example 4.** Find the roots of $x^2+8x+12=0$.

*Solution.* Let's try to make the left hand side a **square**. The idea to do this is to take $x^2+8x$, which looks like $x^2+8x+16=(x+4)^2$, but it is missing the 16, so $x^2+8x=(x+4)^2-16$. Thus, $x^2+8x+12=(x+4)^2-16+12=0$, so $(x+4)^2-4=0$, but this is the same as the last problem! Thus, the roots are $x=\boxed{-2,-6}$. □

This is the motivation behind completing the square. This next theorem is the general form, but it is **not** useful. The idea behind it, the **method**, makes a lot more sense than using the formula.

**Theorem 3 (Completing the Square).** The quadratic $ax^2+bx+c$ can be rewritten as

$$a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right).$$

Now let's talk about **Rational Root Theorem**:

**Theorem 4 (Rational Root Theorem).** Let $P(x)$ be a polynomial such that $P(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$ with integral coefficients, and $a_n \neq 0$. If $P(x)$ has a rational root $r=\pm\frac{p}{q}$ with $p,q$ relatively prime positive integers, $p$ is a divisor of $a_0$ and $q$ is a divisor of $a_n$.

This is a little complicated, since it can be applied to **all polynomials**, but it is important when thinking of factoring. I am sure for a lot of you **distributing** is easy:

**Example 5.** Distribute $(x+3)(2x+1)$.

*Solution.* Using **FOIL**, we get $(x+3)(2x+1)=2x^2+x+6x+3=\boxed{2x^2+7x+3}$. □

But **factoring** is a lot harder:

**Example 6.** Factor $2020x^2+4040x+2020$.

*Solution.* If we factor out 2020, we get $2020(x^2+2x+1)$, and so we have $\boxed{2020(x+1)^2}$. □

Here are some more examples:

1. $(x-2)(x+3)=x^2+1x-6$
2. $(x-4)(x+7)=x^2+3x-28$
3. $(x-7)(x+2)=x^2-5x-14$
4. $(x-9)(x+5)=x^2-4x-45$
5. $(x-8)(x+10)=x^2+2x-80$
6. $(x-7)(x+10)=x^2+3x-70$
7. $(x-6)(x+2)=x^2-4x-12$
8. $(x-5)(x+8)=x^2+3x-40$
9. $(x-6)(x+10)=x^2+4x-60$
10. $(x-5)(x+5)=x^2+0x-25$

## §1.3.1 Solving Quadratics Problems

**Problem 1.** Find the roots of $3x^2 - 2x + 1$ using the Quadratic Formula.

**Problem 2.** Find the roots of $2x^2 - 4x - 6$ using completing the square.

**Problem 3.** Distribute these polynomials:

1. $(x-7)(x+4)$
2. $(x-8)(x+3)$
3. $(x-6)(x+2)$
4. $(x-3)(x+2)$
5. $(x-9)(x+5)$
6. $(x-8)(x+2)$
7. $(x-4)(x+5)$
8. $(x-8)(x+9)$
9. $(x-8)(x+9)$
10. $(x-9)(x+9)$
11. $(x-2)(x+4)$
12. $(x-3)(x+8)$
13. $(x-2)(x+8)$
14. $(x-5)(x+3)$
15. $(x-2)(x+2)$
16. $(x-10)(x+2)$
17. $(x-10)(x+4)$
18. $(x-4)(x+5)$
19. $(x-10)(x+2)$
20. $(x-10)(x+10)$

**Problem 4.** Factor these polynomials:

1. $x^2 - 4x - 32$
2. $x^2 + 5x - 36$
3. $x^2 + 2x - 48$
4. $x^2 - 3x - 10$
5. $x^2 - 4x - 5$
6. $x^2 + 8x - 9$
7. $x^2 + 4x - 45$
8. $x^2 + 2x - 3$
9. $x^2 + 3x - 18$
10. $x^2 - 5x - 36$
11. $x^2 + 6x - 7$
12. $x^2 + 0x - 1$
13. $x^2 + 2x - 3$
14. $x^2 - 1x - 30$
15. $x^2 - 7x - 30$
16. $x^2 + 6x - 7$
17. $x^2 - 4x - 32$
18. $x^2 + 2x - 80$
19. $x^2 - 5x - 36$
20. $x^2 + 1x - 6$