Module 10

Das

Module 10: CS-1319-1: Programming Language Design and Implementation (PLDI)

Global Register Allocation

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Module Objectives

- Issues in Global Register Allocation
- The Problem
- Register Allocation based on Usage Counts
- Chaitin's graph coloring based algorithm

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Module Outline

Some Issues in Register Allocation

- Which values in a program reside in registers? (Register Allocation)
- In which register? (Register Assignment)
  - The two together are usually loosely referred to as **Register Allocation (RA)**
- What is the unit at the level of which register allocation is done?
  - Typical units are *basic blocks*, *functions*, and *regions*
  - RA within *basic blocks* is called local RA
  - RA within *functions* and *regions* are known as global RA
  - Global RA requires lot more time than local RA

Some Issues in Register Allocation

- Phase ordering between *register allocation* and *instruction scheduling*
- In which register? (*register assignment*)
  - Performing RA first restricts movement of code during scheduling – *not recommended*
  - Scheduling instructions first cannot handle spill code introduced during RA
    - ▷ Requires another pass of scheduling
- Tradeoff between *speed* and *quality of allocation*
  - In some cases, for example, in Just-In-Time compilation, cannot afford to spend too much time in register allocation
  - Only local or both local and global allocation?

The Problem

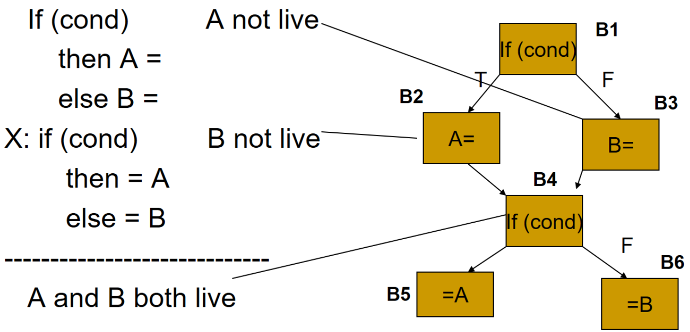
- Global Register Allocation assumes that allocation is done beyond basic blocks and **usually at function level**
- Decision problem related to register allocation
  - Given an intermediate language program represented as a control flow graph and a number k, is there an assignment of registers to program variables such that
    - *no conflicting variables* are assigned the same register,
    - *no extra loads or stores* are introduced, and
    - *at most k* registers are used
- This problem has been shown to be NP-hard (Sethi 1970)
- **Graph colouring** is the most popular heuristic used
- However, there are simpler algorithms as well

Conflicting Variables

- Two variables interfere or conflict if their **live ranges** intersect
  - A variable is **live** at a point p in the flow graph, if there is a **use** of that variable in the path from p to the end of the flow graph
  - The **live range** of a variable is the *smallest set of program points* at which it is live
  - The representation for a point is:
    - basic block number
    - instruction number in the basic block

Example

- **Live range of A: B2, B4, B5**
- **Live range of B: B3, B4, B6**



Global Register Allocation via Usage Counts (for Single Loops)

- Allocate registers for variables used within loops
- Requires information about liveness of variables at the entry and exit of each basic block (BB) of a loop
- Once a variable is computed into a register, it stays in that register until the end of the BB (subject to existence of next-uses)
- Load/Store instructions cost 2 units (because they occupy two words)

Global Register Allocation via Usage Counts (for Single Loops)

[1] For every usage of a variable v in a BB, until it is first defined, do:

- savings(v) = savings(v) + 1
- after v is defined, it stays in the register any way, and all further references are to that register

[2] For every variable v computed in a BB, if it is live on exit from the BB,

- count a savings of 2, since it is not necessary to store it at the end of the BB

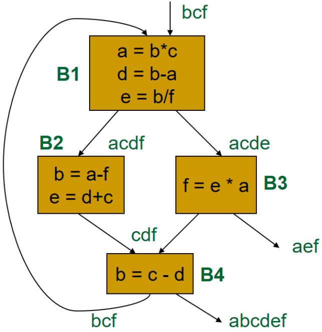
Global Register Allocation via Usage Counts (for Single Loops)

- Total savings per variable v are

$$\sum_{B \in Loop} (savings(v, B)) + 2 * liveandcomputed(v, B))$$

  - $liveandcomputed(v, B)$ in the second term is 1 or 0
- On entry to (exit from) the loop, we load (store) a variable live on entry (exit), and lose 2 units for each
  - But, these are *one time* costs and are neglected
- Variables, whose savings are the highest will reside in registers

Global Register Allocation via Usage Counts (for Single Loops)



Savings for the variables

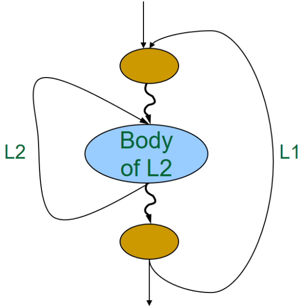
	B1	B2	B3	B4	
a:	**(0+2)**+	**(1+0)**+	**(1+0)**+	**(0+0)**=	**4**
b:	**(3+0)**+	**(0+0)**+	**(0+0)**+	**(0+2)**=	**5**
c:	(1+0)+	(1+0)+	(0+0)+	(1+0)=	3
d:	**(0+2)**+	**(1+0)**+	**(0+0)**+	**(1+0)**=	**4**
e:	(0+2)+	(0+0)+	(1+0)+	(0+0)=	3
f:	(1+0)+	(1+0)+	(0+2)+	(0+0)=	4

If there are 3 registers, they will be allocated to the variables, a, b, and d (or f)

Global Register Allocation via Usage Counts (for Nested Loops)

- We first assign registers for inner loops and then consider outer loops. Let L1 nest L2
- For variables assigned registers in L2, but not in L1
  - load these variables on entry to L2 and store them on exit from L2
- For variables assigned registers in L1, but not in L2
  - store these variables on entry to L2 and load them on exit from L2
- All costs are calculated keeping the above rules

Global Register Allocation via Usage Counts (for Nested Loops)

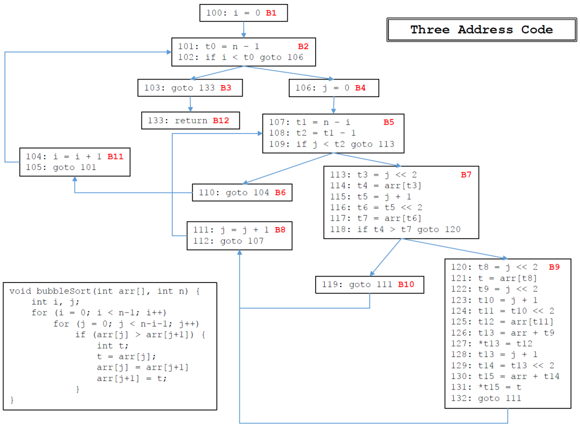


- **Case 1**: Variables x, y, z assigned registers in L2, but not in L1
  - Load x, y, z on entry to L2
  - Store x, y, z on exit from L2
- **Case 2**: Variables a, b, c assigned registers in L1, but not in L2
  - Store a, b, c on entry to L2
  - Load a, b, c on exit from L2
- **Case 3**: Variables p, q assigned registers in both L1 and L2
  - No special action

Sample Code Generation: Bubble Sort
Three Address Code

- Three Address Code for Bubble Sort as generated by syntax directed translation

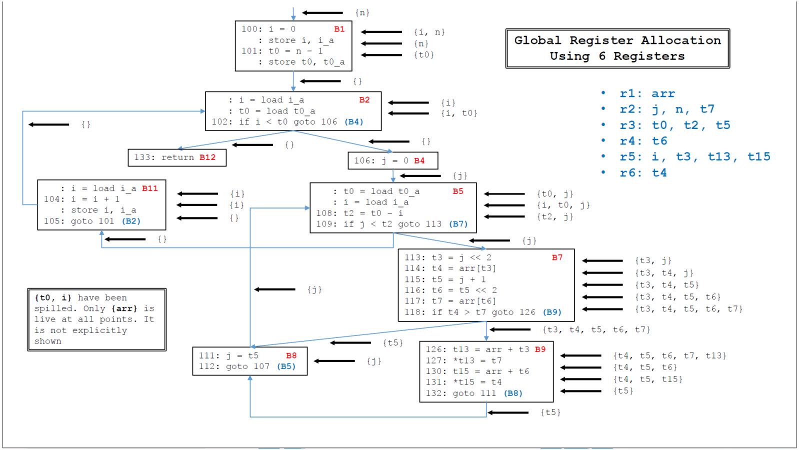
```
100*: i = 0
101*: t0 = n - 1
102 : if i < t0 goto 106
103*: goto 133
104*: i = i + 1
105 : goto 101
106*: j = 0
107*: t1 = n - i
108 : t2 = t1 - 1
109 : if j < t2 goto 113
110*: goto 104
111*: j = j + 1
112 : goto 107
113*: t3 = j << 2
114 : t4 = arr[t3]
115 : t5 = j + 1
116 : t6 = t5 << 2
117 : t7 = arr[t6]
118 : if t4 > t7 goto 120
119*: goto 111
120*: t8 = j << 2
121 : t = arr[t8]
122 : t9 = j << 2
123 : t10 = j + 1
124 : t11 = t10 << 2
125 : t12 = arr[t11]
126 : t13 = arr + t9
127 : *t13 = t12
128 : t13 = j + 1
129 : t14 = t13 << 2
130 : t15 = arr + t14
131 : *t15 = t
132 : goto 111
133*: return
```




```
void bubbleSort(int arr[], int n) {
    int i, j;
    for (i = 0; i < n-1; i++)
        for (j = 0; j < n-i-1; j++)
            if (arr[j] > arr[j+1]) {
                int t;
                t = arr[j];
                arr[j] = arr[j+1]
                arr[j+1] = t;
            }
}
```

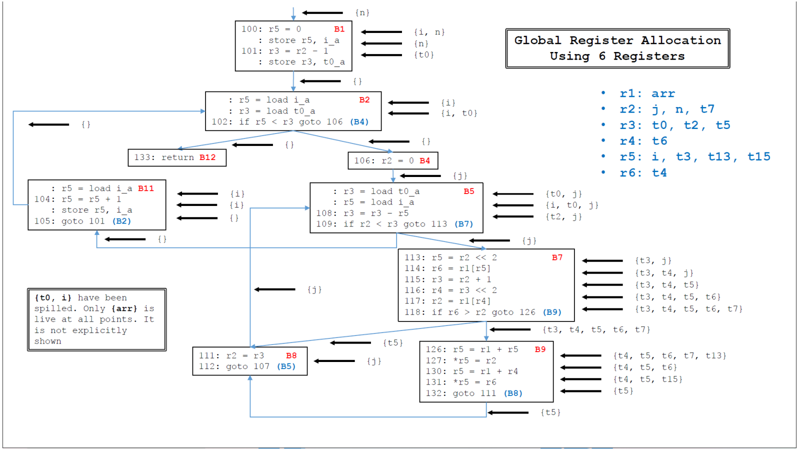
Sample Code Generation: Bubble Sort Liveness after LCSE, GCSE Optimization

- Three Address Code Optimized by peephole, LCSE by VN and GCSE by DFA. Finally, live variables are computed by DFA



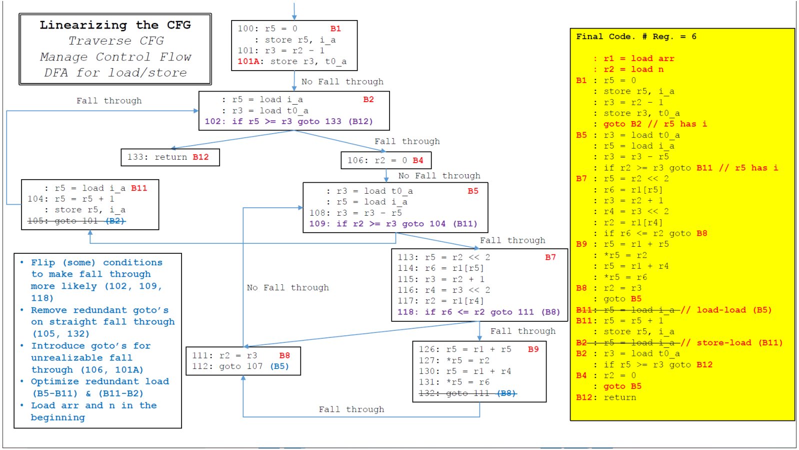
Sample Code Generation: Bubble Sort Global Register Allocation

- Registers are allocated globally using graph coloring based on the liveness information
- Variables are replaced by respective registers



Sample Code Generation: Bubble Sort Linearized and Optimized Target Code

- The CFG is linearized and further optimized to get the final target code

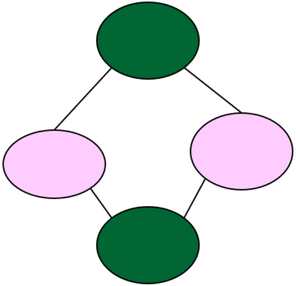


Chaitin's Formulation of the Register Allocation Problem

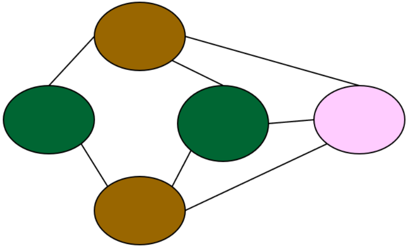
- A graph colouring formulation on the interference graph
- Nodes in the graph represent either live ranges of variables or entities called webs
- An edge connects two live ranges that interfere or conflict with one another
- Usually both adjacency matrix and adjacency lists are used to represent the graph.
- Assign colours to the nodes such that two nodes connected by an edge are not assigned the same colour
  - The number of colours available is the number of registers available on the machine
  - A k-colouring of the interference graph is mapped onto an allocation with k registers

Example

Two Colorable



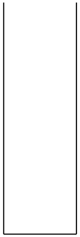
Three Colorable



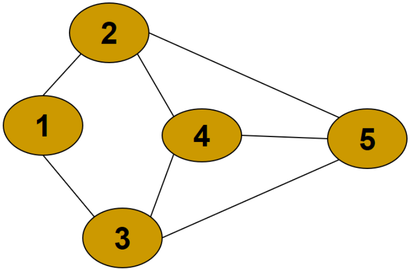
Idea behind Chaitin's Algorithm

- Choose an arbitrary node of degree less than k and put it on the stack
- Remove that vertex and all its edges from the graph
  - This may decrease the degree of some other nodes and cause some more nodes to have degree less than k
- At some point, if all vertices have degree greater than or equal to k, some node has to be spilled
- If no vertex needs to be spilled, successively pop vertices off stack and colour them in a colour not used by neighbours (reuse colours as far as possible)

Simple example – Given Graph

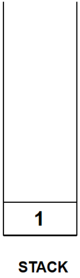


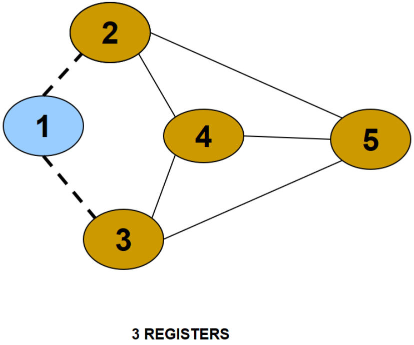
STACK



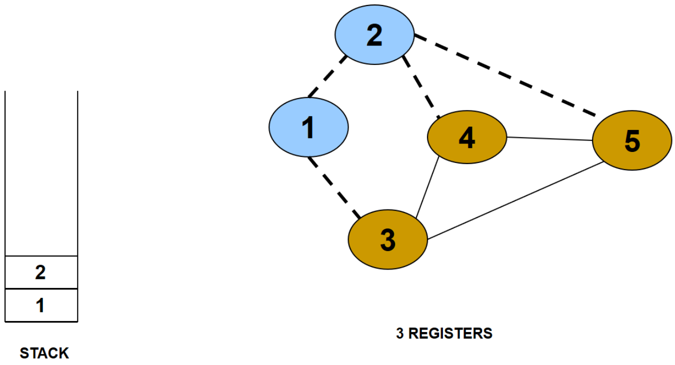
3 REGISTERS

Simple example – Delete Node 1

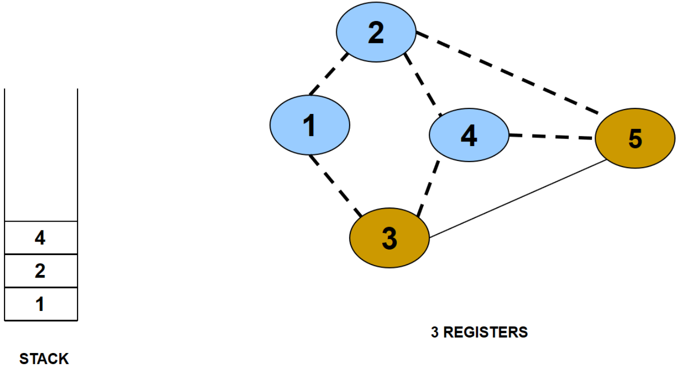




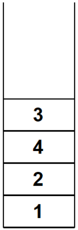
Simple example – Delete Node 2



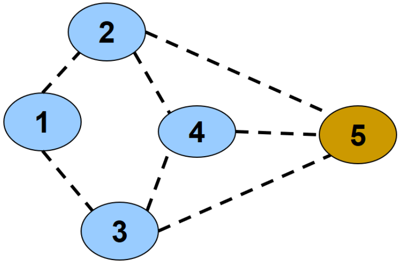
Simple example – Delete Node 4



Simple example – Delete Node 3



STACK



3 REGISTERS

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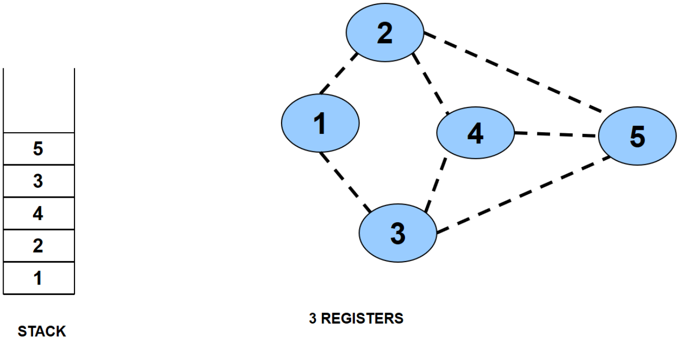
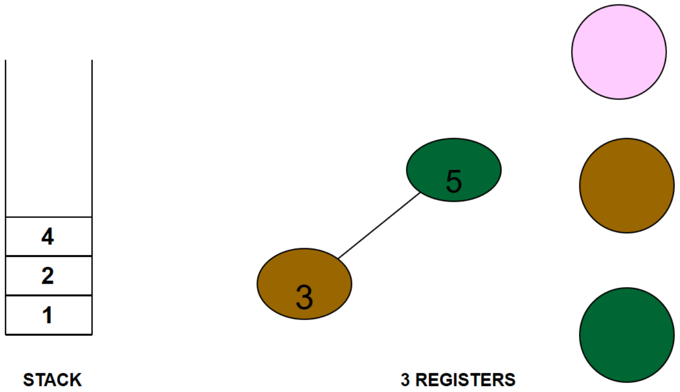
Simple example – Delete Node 5

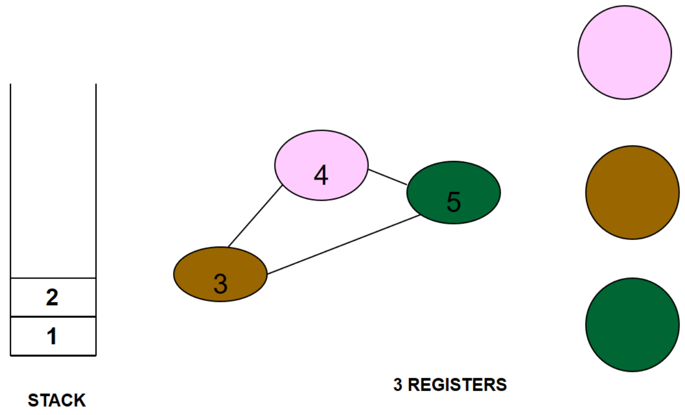


Simple example – Colour Node 5

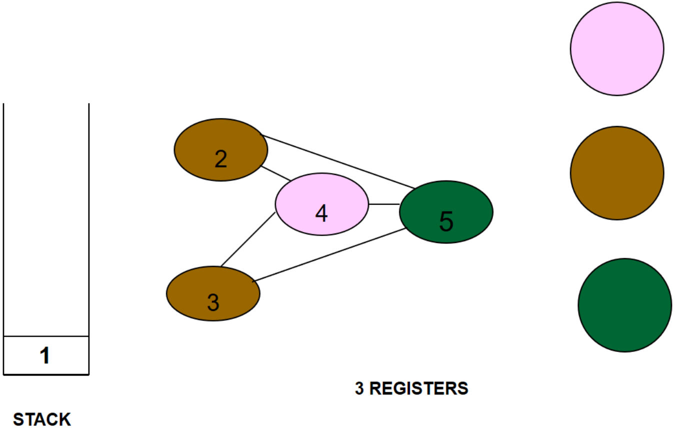
Simple example – Colour Node 3



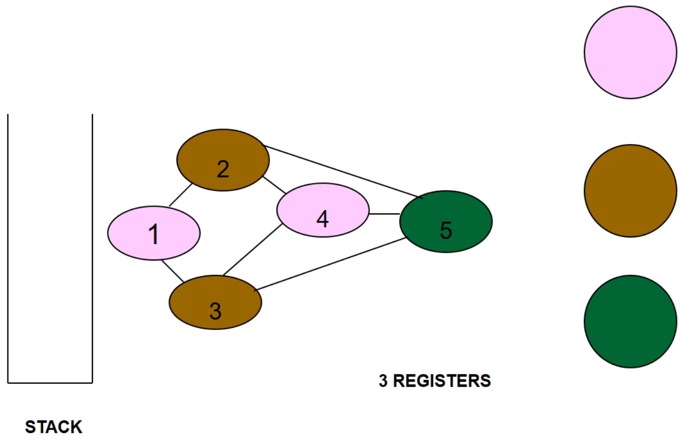
Simple example – Colour Node 4



Simple example – Colour Node 2



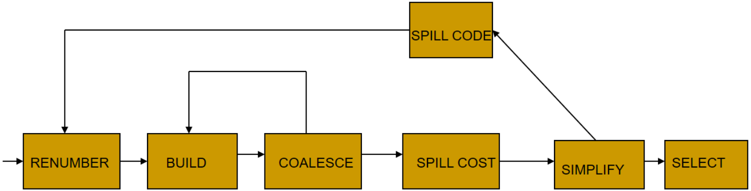
Simple example – Colour Node 1



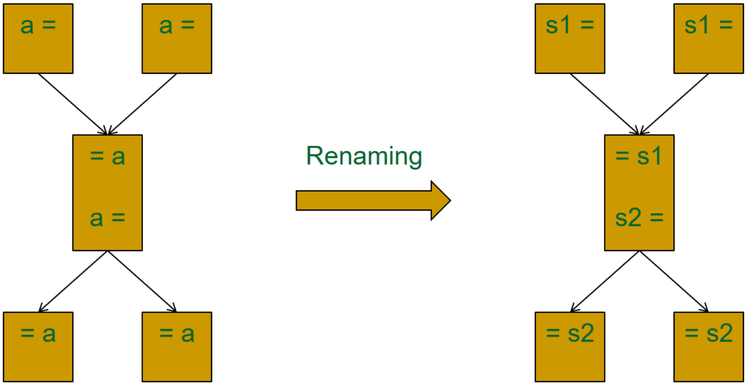
Steps in Chaitin's Algorithm

- Identify units for allocation
  - Renames variables/symbolic registers in the IR such that each live range has a unique name (number)
- Build the interference graph
- Coalesce by removing unnecessary move or copy instructions
- Colour the graph, thereby selecting registers
- Compute spill costs, simplify and add spill code till graph is colourable

The Chaitin Framework



Example of Renaming

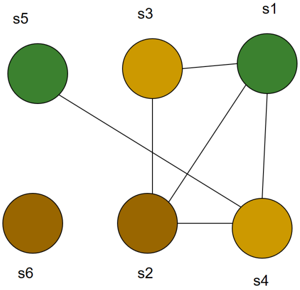


An Example

Original code		Code with symbolic registers		
1.	x = 2	1.	s1 = 2;	(lv of s1: 1-5)
2.	y = 4	2.	s2 = 4;	(lv of s2: 2-5)
3.	w = x + y	3.	s3 = s1 + s2;	(lv of s3: 3-3)
4.	z = x + 1	4.	s4 = s1 + 1;	(lv of s4: 4-6)
5.	u = x * y	5.	s5 = s1 * s2;	(lv of s5: 5-5)
6.	x = z * 2	6.	s6 = s4 * 2;	(lv of s6: 6-...)

An Example: Interference Graph

Interference Graph



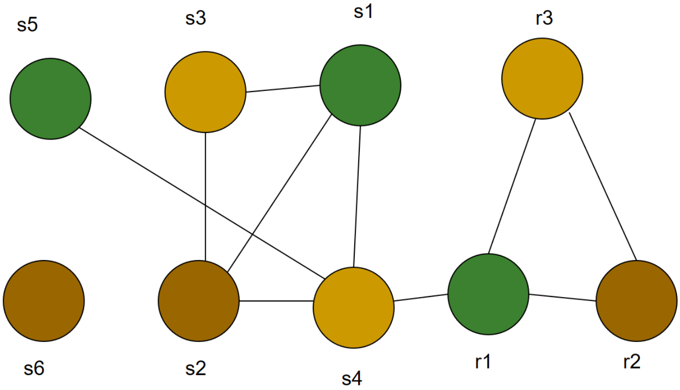
Stack Order for Colouring & Register Allocation (Number of Registers = 3)

s1	$\rightarrow$	r1
s2	$\rightarrow$	r2
s3	$\rightarrow$	r3
s4	$\rightarrow$	r3
s5	$\rightarrow$	r1
s6	$\rightarrow$	r2

1.	x = 2
2.	y = 4
3.	w = x + y
4.	z = x + 1
5.	u = x * y
6.	x = z * 2

1.	s1 = 2;	(lv of s1: 1-5)
2.	s2 = 4;	(lv of s2: 2-5)
3.	s3 = s1 + s2;	(lv of s3: 3-3)
4.	s4 = s1 + 1;	(lv of s4: 4-6)
5.	s5 = s1 * s2;	(lv of s5: 5-5)
6.	s6 = s4 * 2;	(lv of s6: 6- ...)

An Example: Interference Graph



Interference Graph

Here assume variable Z (s4) cannot occupy r1

An Example: Interference Graph

s1 → r1	1.	x = 2
s2 → r2	2.	y = 4
s3 → r3	3.	w = x + y
s4 → r3	4.	z = x + 1
s5 → r1	5.	u = x * y
s6 → r2	6.	x = z * 2

1.	s1 = 2;	(lv of s1: 1-5)
2.	s2 = 4;	(lv of s2: 2-5)
3.	s3 = s1 + s2;	(lv of s3: 3-3)
4.	s4 = s1 + 1;	(lv of s4: 4-6)
5.	s5 = s1 * s2;	(lv of s5: 5-5)
6.	s6 = s4 * 2;	(lv of s6: 6- ...)

Final Code:

3 reg. are sufficient for no spills

r1 = 2
r2 = 4
r3 = r1 + r2
r3 = r1 + 1
r1 = r1 * r2
r2 = r3 * 2

Another Example

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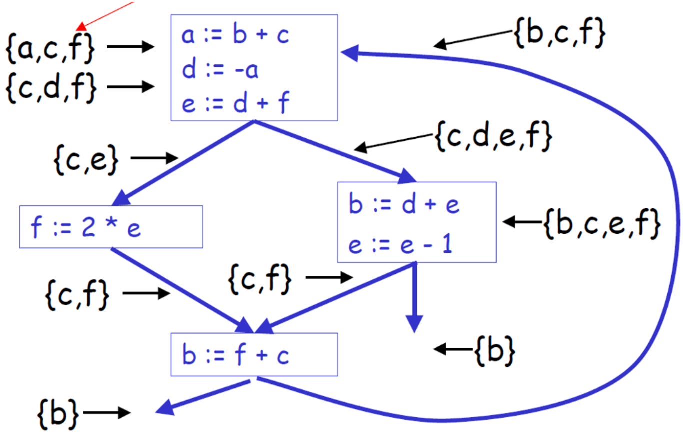
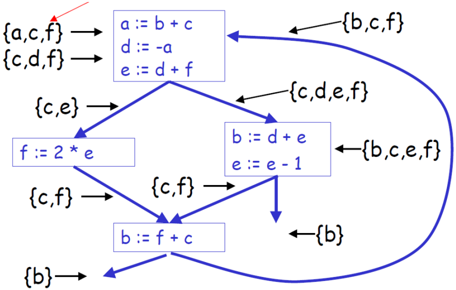
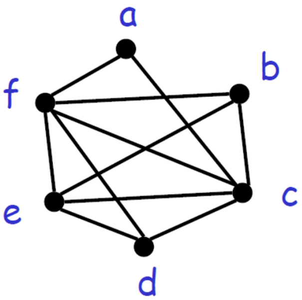
Compute live variables at each point



Register Interference Graph

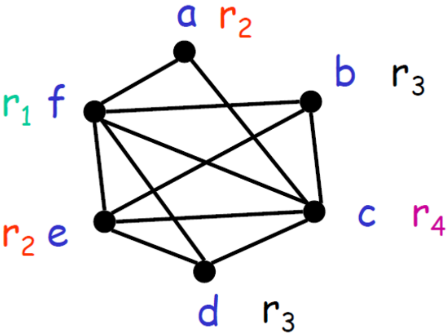
- b and c cannot be in the same register
- b and d can be in the same register





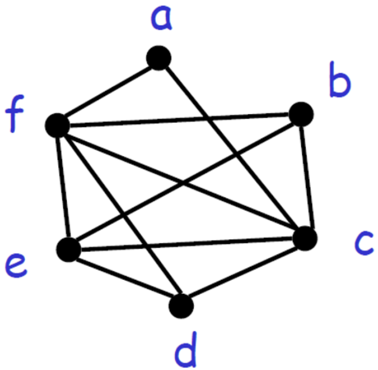
Graph Coloring: Example

- There is no coloring with less than 4 colors (has two 4-cliques)
- There are 4 colorings of the graph



Graph Coloring: Example

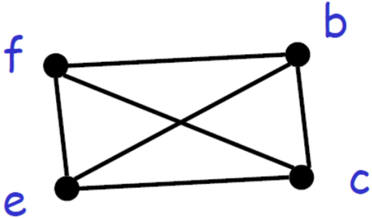
- Start with the RIG and with k = 4. Stack = {}



- Remove a and then d: Stack = {d, a}

Graph Coloring: Example

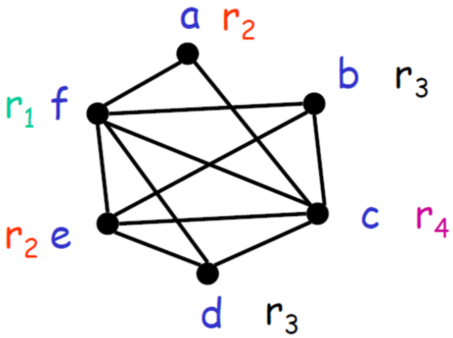
- Now all nodes have less than 4 neighbors and can be removed. Say, as: c, b, e, f



- Stack = {f, e, b, c, d, a}

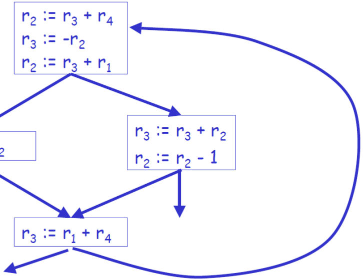
Graph Coloring: Example

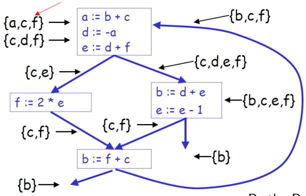
- Start assigning colors to: f, e, b, c, d, a

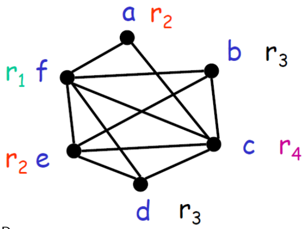


Code with Registers Allocated

- With the coloring the code becomes

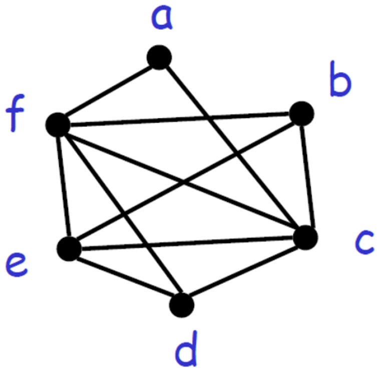






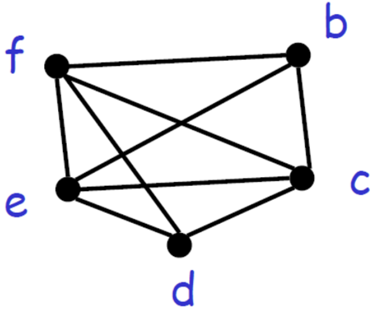
What if the Heuristic Fails?

- What if during simplification we get to a state where all nodes have k or more neighbors?
- Let us try a 3-coloring



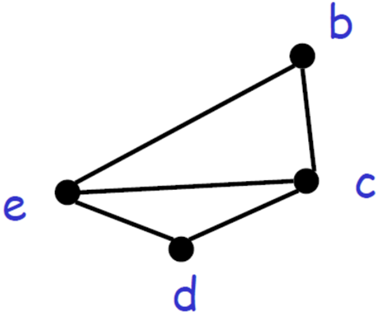
What if the Heuristic Fails?

- Remove a and get stuck
- Pick a node as a candidate for spilling
  - A spilled temporary “lives” in memory
- Assume that f is picked as a candidate



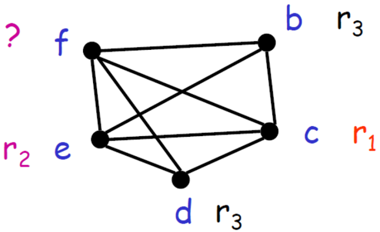
What if the Heuristic Fails?

- Remove f and continue the simplification
  - Simplification now succeeds: b, d, e, c



What if the Heuristic Fails?

- On the assignment phase we get to the point when we have to assign a color to f
- We hope that among the 4 neighbors of f we use less than 3 colors ⇒ **optimistic coloring**

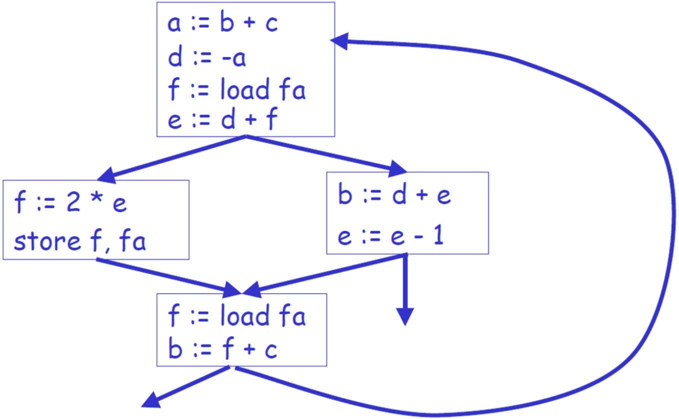


Spilling

- We fail and we must spill temporary f
- We must allocate a memory location as the home of f
  - Typically this is in the current stack frame
  - Call this address fa
- Before each operation that uses f, insert
  - f := load fa
- After each operation that defines f, insert
  - store f, fa

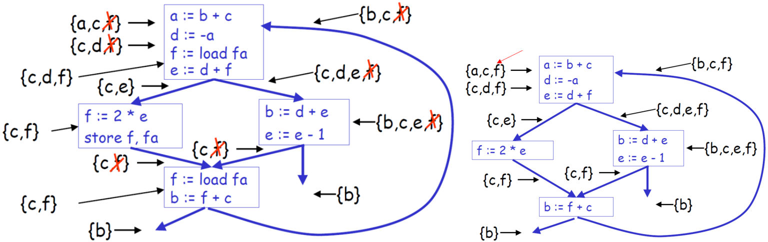
Code with Spilling

- The new code after spilling f



Recomputing Liveness Information

- The new liveness information after spilling

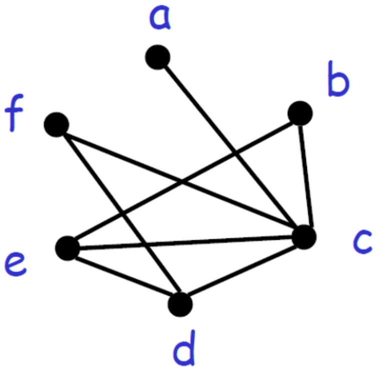


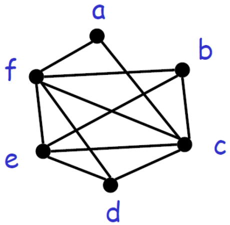
Recomputing Liveness Information

- The new liveness information is almost as before
- f is live only
  - Between a f := load fa and the next instruction
  - Between a store f, fa and the preceding instruction
- Spilling reduces the live range of f
- And thus reduces its interferences
- Which results in fewer neighbors in RIG for f

Recompute RIG after Spilling

- The only changes are in removing some of the edges of the spilled node
- In our case f still interferes only with c and d
- And the resulting RIG is 3-colorable





Spilling

- Additional spills might be required before a coloring is found
- The tricky part is deciding what to spill
- Possible heuristics:
  - Spill temporaries with most conflicts
  - Spill temporaries with few definitions and uses
  - Avoid spilling in inner loops
- Any heuristic is correct