## INTRODUCTION TO MACHINE LEARNING CS-1390/PHY-1390 HOMEWORK ASSIGNMENT-1

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TOTAL MARKS - 78 marches BONUS MARKS - 28 marches

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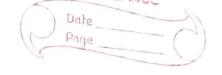
## IML - A1



2 7 1 1	Consider the 5X5 matrix
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× (5) /s	1 2 -1 -1 0
	$A = \begin{bmatrix} 0 & 0 & 1 & 4 & 0 \end{bmatrix}$
	2 4 1 10 1
	L 0 0 0 0 1
(.a)	Ind an invertible matrix P such
- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	that PA is a row-reduced
	echlon matrix R.
(b)	Find a basis for the row space w
	of R
(c)	say which rectors (b1, b2, b3, b4, b5)
	are in with the second
(d)	Find the coordinates natrin of each
	vector (b1, b2, b3, b4, b5) € W in the
<sup>4</sup> gr	ordered basia chosen in (b).
(e)	Write each vector (b, b2, b3, b4, b5) EW
	as a linear combination of the roles of A.
(4)	give an explicit description of the
F	mill space of A
(9)	Find a basis for the null space
(1)	For what solution matrices of does the
	equation $AX = 7$ have solutions $X?$
(i)	Enficitly find the range space of
	A and find a vasio.
(j)	verify the relations regarding the
	verify the relations regarding the nullify, row rank and solunin rank of to

consider the space of square - integrable real functions on the interval [-7, 1] 12 ([-x, x]), and the associated orthonormal basis given by  $\left\{ \frac{1}{\sqrt{2\pi}}, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin nx}{\sqrt{\pi}} \right\}, n=1,2,$ consider the following two subspaces: 5-space of symmetric functions, that is, f(x) = f(-x), on [-x, x], and A - spaceof antisymmetric functions, f(x) = -f(-x), on [-x, x] a) show how many function f(n) from  $L_2([-\sqrt{\Lambda}])$  can be weither as f(n) = $f_s(n) + f_a(n)$ , where  $f_s(n) \in S$  and  $fa(x) \in A$ . (b) give orthonormal vasus for s and A (c) Verify that  $L_2([-\pi, \pi]) = 5 \oplus A$ 3. We have 2 spagne vags, each containing 2 balls. One wag now 2 black balls and the other has a black ball and a white wall. you jick a wag at random and then pick one of the valls in that wag at random. When you look at The ball, it is black you now pick the second wall from that same wag what is the probability that this ball is also black?

(a) 1/4 (b) 1/3 (c) 1/2  $(d)^{2}/3$ (e) 3/4 4. Consider the least squares problem: Given mxn set of equations An=b, where m>n and rank (A) = n. Show that the following (a-d) together constitute a least square solution (a) Find the SVD A=UEV (b) Set b' = UTb (c) Find me vertor y defined by y = bi/oi, where 6i is the in diagonal entry of 5 (d) The solution is n= Vy Consider the least squares problem: Given mxn set of equations An=b, where m>n and siank (A) = r<n. Show that the following (a-e) together constitute a general least square solution (a) Find the SVD A=UZVT (b) Set b' = UTb (c) Find the vector y defined by y= bi/oi for i=1,...r ad y;=0 otherwise (d) The solution is of minimum norm I'm I is



(e) The general solution is  $x = Vy + \lambda x + \lambda x$ 

and you draw cards from the dick with replacement uniformly at rondom independently. Let X, X, , , X50 be the outcomes of the first 50 draws.

Thus, each random variable X; can take values 1, . . , 62 and the probability that it takes each of their values is  $\frac{1}{2}$ .

(a) what is E[X,J?

(b) Let z = x, -2x, +3x3. What is E[z]?

(c) Let  $Y = X_1 - X_2 + X_3 - X_4 + - - + X_{49} - X_{50}$ 

What is E[Y]?

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3 (m) 182		
2. Age X and Y independent? Tustify		
your answer.  3. What is the conditional distribution of X,		
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10. Assume a finite dimensional space  $|R^N|$  and an orthonormal basis  $\{n_1, n_2, \dots, n_N\}$ . Vector y can thus be written as  $y = \sum_i a_i n_i$  where  $a_i = \langle n_i, y \rangle$  consider the best approximation to y in the least squares sense and living on the subspace spanned by the first K vectors  $\{n_1, n_2, \dots, n_K\}$  or  $\hat{y} = \sum_{i=1}^K \beta_i n_i$ . Prove that  $\beta_i = a_i$  for  $i = 1, \dots, K$  by showing that it minimizes  $\|y - \hat{y}\|$ 

by showing that it minimizes  $\|y-\hat{y}\|$  (Kint: Use Parseval's equality)