2. a) $f(n) = n^2 + 27n\log n + 2016$ $\quad$ $g(n) = n^2 \log n + 2016$

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2 + 27n\log n + 2016}{n^2\log n + 2016}$$

$$= \lim_{n\to\infty} \frac{\frac{\cancel{n^2}}{n^2\log n} + \frac{27\cancel{n\log n}}{n^2\cancel{\log n}} + \frac{2016}{n^2\log n}}{\frac{\cancel{n^2\log n}}{\cancel{n^2\log n}} + \frac{2016}{n^2\log n}}$$

$$= \lim_{n\to\infty} \frac{0+0+0}{1+0}$$

$$= 0$$

As $L=0$, $f(n) \in o(g(n))$. (little - o)

b) $f(n) = 10^n + 99n^{10}$ $\quad$ $g(n) = 75^n + 25n^{27}$

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \underbrace{\lim_{n\to\infty} \frac{10^n}{75^n + 25n^{27}}} + \underbrace{\lim_{n\to\infty} \frac{99n^{10}}{75^n + 25n^{27}}}$$

$$= \lim_{n\to\infty} \frac{10^n}{75^n} + \lim_{n\to\infty} \frac{99\cancel{n^{10}}}{25n^{\cancel{27}\,17}}$$

$\left[\forall n \geq 0,\ 75^n \geq 0\ \&\ 25n^{27} \geq 0 \right.$
$\left. \Rightarrow \frac{1}{75^n + 25n^{27}} \leq \frac{1}{75^n}\ \&\ \frac{1}{75^n + 25n^{27}} \leq \frac{1}{25n^{27}} \right]$

$$= 0 + 0 = 0 \qquad [75^n >> 10^n]$$

As $L=0$, $f(n) \in o(g(n))$ (little - o)

c) $f(n) = \sqrt{n}$ $\qquad$ $g(n) = (\log n)^7$

$$L = \lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^7}$$

$$= \lim_{n\to\infty} \left(\frac{\sqrt[14]{n}}{\log n}\right)^7$$

$$= \lim_{n\to\infty} \left(\frac{n}{14n^{13/14}}\right)^7 \qquad \text{[L'Hopital rule]}$$

$$= \lim_{n\to\infty} \left(\frac{n^{1/4}}{14}\right)^7$$

$$= \infty$$

As $L = \infty$, $f(n) \in \omega(g(n))$.