CS 240 Assignment 1

1. a) $27n^7 + 17n^3 \log n + 2016$ is $O(n^9)$

$$27n^7 + 17n^3 \log n + 2016 \le 27n^9 + 17n^9 + 2016n^9 \quad (n \ge 1)$$
$$[\log n < n \quad \forall n \ge 1]$$
$$\le 2060n^9$$

Let $c = 2060$, $n_0 = 1$

Since $\exists\, c > 0, n_0 > 0$ st $0 \le 27n^7 + 17n^3 \log n + 2016 \le cn^9$ $\forall n \ge n_0$, the statement is proved.

b) $f(n) = n^2 (\log n)^{1.0001}$ $\qquad g(n) = n^2 \qquad f(n) \in \Omega(g(n))$

$$n^2 \le n^2 (\log n)^{1.0001}$$
$$\Rightarrow cn^2 \le n^2 (\log n)^{1.0001} \qquad [(\log n)^{1.0001} \ge 1 \quad \forall n \ge 2]$$

Let $c = 1$, $n_0 = 2$.
Since $\exists\, c > 0, n_0 > 0$ st $0 < cg(n) \le f(n)$ $\forall n \ge n_0$, $f(n) \in \Omega(g(n))$.

c) $f(n) = \dfrac{n^2}{n + \log n}$ $\qquad g(n) = n \qquad f(n) \in \Theta(g(n))$

$$\frac{n^2}{n + \log n} \ge \frac{n^{\cancel{2}}}{2\cancel{n}} = c_1 n \quad (c_1 = \tfrac{1}{2})$$
$$[\log n < n \quad \forall n \ge 1]$$

Also, $\dfrac{n^2}{n + \log n} \le \dfrac{n^{\cancel{2}}}{\cancel{n}} = c_2 n \quad (c_2 = 1)$

$$[\log n \ge 0 \quad \forall n \ge 1]$$

$c_1 = \frac{1}{2}$, $c_2 = 1$, $n_0 = 1$
Since $\exists\, c_1, c_2 > 0, n_0 > 0$ st $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ $\forall n \ge n_0$, $f(n) \in \Theta(g(n))$.

d) $f(n) = n^n$ $\quad g(n) = n^{20}$ $\quad f(n) \in \omega(g(n))$

$c n^{20} < n^n$ $\quad \forall n > 21, \ 0 < c < n$

$\Rightarrow c < \dfrac{n^n}{n^{20}}$

$\Rightarrow c < n^{n-20}$

$\Rightarrow \log_{n-20} c < n$

$\therefore$ for all $c > 0$, $\exists n_0 = \log_{n-20} c > 0$ st $0 \leq c g(n) < f(n)$ $\forall n \geq n_0$

$\Rightarrow f(n) \ \omega(g(n))$.