4. a)

```
1   x = 0
2   for i = 1 to n + 12 do
3       x = x + 4
4       for j = 389 to 20100
5           for k = 2i to 3i
6               x = x + 77
```

Line 6 : 2 (# simple operations)
Lines 5-6: 2i
Lines 4-6: (20100 - 389) 2i
= 39422 i
Line 3 : 2 (# simple ops.)
Lines 2-6:

$$T(n) \sum_{i=1}^{n+12} [2 + 39422i]$$

$$= 2\sum_{i=1}^{n+12} 1 + 39422 \sum_{i=1}^{n+12} i$$

$$= 2(n+12) + 39422 \frac{(n+12)(n+13)}{2}$$

$$= 19711n^2 + 492777n + 180$$

or $T(n) \in \Theta(n^2)$.

b)

```
1   x = 0
2   for i = 1 to ceiling(log(n))
3       for j = 1 to i
4           for k = 1 to 10
5               x = x + 1
```

Line 5: 2 simple ops
Lines 4-5: 20 times (10 x 2)
Lines 3-5: 20i times
Lines 2-4:

$$T(n) = \sum_{i=1}^{\lceil \log(n) \rceil} 20i = 20 \sum_{i=1}^{\lceil \log n \rceil} i = 20 \frac{\lceil \log n \rceil (\lceil \log n \rceil + 1)}{2}$$

or $T(n) \in \Theta(\log^2(n) + \log n)$

c)
```
1   x = 0
2   for i = 1 to sqr(n)
3       for j = 1 to ceiling(log i)
4           x = x + 1
```

Line 4 : 2 simple ops

Lines 3-4 : $2(\lceil \log i \rceil)$ times

Lines 2-4 :

$$T(n) = \sum_{i=1}^{n^2} 2(\lceil \log i \rceil) = 2(\lceil \log(n^2!) \rceil)$$

or $T(n) \in \Theta(n^2 \log n)$ $\quad [\log n! \in \Theta(n \log n)]$