

University of Waterloo

CS240 Spring 2016

Assignment 1

Due Date: Wednesday, May 18, at 5:00pm

Problem 5 [3+5+10 marks]

- a) Show that an *arbitrary* item can be removed from a heap in $O(\log n)$ time, if the item's index into the heap is known. Specifically, give an algorithm *heapRemove*(*h*, *i*) that removes the element at index *i* from heap *h*. Justify your algorithm's running time.

Answer:

```
/**
 * h: Heap
 * i: Index of element to be removed
 */
heapRemove(h, i) {
    n = sizeof(h)

    swap(h[i], h[n])
    remove(h[n])
    bubbleDown(h[i])

    sizeof(h) = sizeof(h) - 1
}
```

We first swap the i^{th} and n^{th} elements where *n* is the number of elements in the heap. We then remove the specified element from its swapped position and call *bubbleDown*().

Analysis: *sizeof*(), *swap*(), *remove*(), should all be $O(1)$ operators. *bubbleDown*() goes down the height of the heap which is $\leq \log n$. Therefore the run-time of the algorithm is $O(\log n)$.