

ECE345 HW1

Homework Group #85

Yanni Alan Alevras | yanni.alevras@mail.utoronto.ca | 1009330706

Evan Banerjee | evan.banerjee@mail.utoronto.ca | 1009682309

Gautam | |

Total Pages: 7

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1 Question 1 Examples of proof

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when

□

b. Time complexity: $\mathcal{O}(n)$

Proof. (\Leftarrow) Assume by Contradiction,

(\Rightarrow)

□

2 Question 2 Example Pseudocode

```
1: function GALE-SHAPLEY( $E, S$ )
2:   initialize all employers in  $E$  and students in  $S$  to unmatched
3:   while an unmatched employer with at least one student on its preference list remains do
4:     choose such an employer  $e \in E$ 
5:     make offer to next student  $s \in S$  on  $e$ 's preference list
6:     if  $s$  is unmatched then
7:       Match  $e$  with  $s$                                      ▷  $s$  accepts  $e$ 's offer
8:     else if  $s$  prefers  $e$  to their current employer  $e'$  then
9:       Unmatch  $s$  and  $e'$                                      ▷  $s$  rejects  $e'$ 
10:      Match  $e$  with  $s$                                        ▷  $s$  accepts  $e$ 's offer
11:     end if
12:     cross  $s$  off  $e$ 's preference list
13:   end while
14:   report the set of matched pairs as the final matching
15: end function
```

3 Question 3

This figure shows xxx

Figure 1: Example image

4 Question 4 Induction

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when

□

b. Time complexity: $\mathcal{O}(n)$

Proof. (\Leftarrow) Assume by Contradiction,

(\Rightarrow)

□

5 Question 5 Probability

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when

□

b. Time complexity: $\mathcal{O}(n)$

Proof. (\Leftarrow) Assume by Contradiction,

(\Rightarrow)

□

6 Question 6 Graphs, Proof by Contradiction

Proof. Assume by Contradiction,

Assume $d(u, w) + d(w, v) < d(u, v)$.

This implies that there is a path from u to v via w whose total distance is less than the direct distance between u and v .

However, by the definition of $d(u, v)$, it is the shortest distance between u and v . Therefore, no other path, including one through w , can have a smaller distance.

This contradiction arises from our assumption, so the assumption must be false. Hence, we conclude that $d(u, w) + d(w, v) \geq d(u, v)$. \square

7 Question 7 Trees, Proof by induction

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when

□

b. Time complexity: $\mathcal{O}(n)$

Proof. (\Leftarrow) Assume by Contradiction,

(\Rightarrow)

□