ECE345 HW1

Homework Group #85
Yanni Alan Alevras | yanni.alevras@mail.utoronto.ca | 1009330706
Evan Banerjee | evan.banerjee@mail.utoronto.ca | 1009682309
Gautam | |

Total Pages: 7

Thursday 19th September, 2024

1 Question 1 Examples of proof

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when $\square$

b. Time complexity: $\mathcal{O}(n)$

Proof. $(\Leftarrow)$ Assume by Contradiction,

$(\Rightarrow)$ $\square$

2 Question 2 Example Pseudocode

```
function GALE-SHAPLEY(E, S)
    initialize all employers in E and students in S to unmatched
    while an unmatched employer with at least one student on its preference list remains do
        choose such an employer e ∈ E
        make offer to next student s ∈ S on e's preference list
        if s is unmatched then
            Match e with s                                  ▷ s accepts e's offer
        else if s prefers e to their current employer e′ then
            Unmatch s and e′                                ▷ s rejects e′
            Match e with s                                  ▷ s accepts e's offer
        end if
        cross s off e's preference list
    end while
    report the set of matched pairs as the final matching
end function
```

3 Question 3

Figure 1: Example image

This figure shows xxx

4 Question 4 Induction

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when $\square$

b. Time complexity: $\mathcal{O}(n)$

Proof. ($\Leftarrow$) Assume by Contradiction,

($\Rightarrow$) $\square$

5 Question 5 Probability

a.

Theorem. *Some theorem here.*

Proof. Proof by induction

Base Step: When $n = 1$, is true.

Induction Hypothesis: Suppose is true

Induction Step: Consider when $\square$

b. Time complexity: $\mathcal{O}(n)$

Proof. ($\Leftarrow$) Assume by Contradiction,

($\Rightarrow$) $\square$

6 Question 6 Graphs, Proof by Contradiction

Proof. Assume by Contradiction,

Assume $d(u, w) + d(w, v) < d(u, v)$.

This implies that there is a path from u to v via w whose total distance is less than the direct distance between u and v.

However, by the definition of $d(u, v)$, it is the shortest distance between u and v. Therefore, no other path, including one through w, can have a smaller distance.

This contradiction arises from our assumption, so the assumption must be false. Hence, we conclude that $d(u, w) + d(w, v) \geq d(u, v)$. $\square$

7 Question 7 Trees, Proof by induction

Definition: A *stable parent* is defined as a node with two leaves. Let:

- n be the number of nodes,
- l be the number of leaves,
- p be the number of stable parents.

Proof. Proof by induction.

Assume the inductive hypothesis:

$$l_n = p_n + 1 \quad \Rightarrow \quad l_{n+1} = p_{n+1} + 1$$

We will now consider two cases:

Case 1: Add a child to a leaf.

- Since the leaf gains a child, while a new leaf is added, we have $l_{n+1} = l_n$.
- Since no node with 1 child gains another child, $p_{n+1} = p_n$.
- Therefore, $l_{n+1} = l_n = p_n + 1 = p_{n+1} + 1$.

Case 2: Add a child to a node with 1 child already.

- Since no leaf stops being a leaf, you have simply added a leaf, so $l_{n+1} = l_n + 1$.
- Since the node with 1 child gains another child, a new stable parent is formed, so $p_{n+1} = p_n + 1$.
- Therefore, $l_{n+1} = l_n + 1 = p_{n+1} + 1 = p_n + 1 + 1$.

Base case: For $n = 1$, we have:

- 1 node implies 1 leaf and 0 stable parents.
- Thus, $l_1 = 1$ and $p_1 = 0$.
- Therefore, $1 = 0 + 1$, which satisfies $l_1 = p_1 + 1$.

Q.E.D.