2.3 #1

pb. 1

A tank contains 200 L. also dye at a concentration of 1 gr/L. Water is poured in and allowed out at 2 liters/min. How long before the concentration of the dye is 1%.

Let $a(t)$ be the amount of dye, then $\frac{a}{200}$ is concentration / liter.

so $\frac{da}{dt} = -2\left(\frac{a}{200}\right)$; notice the "$-$".

$\frac{da}{a} = -\frac{dt}{100}$, so $\ln(a) = -\frac{t}{100} + c$

when $t=0$, $a=200$, so $\ln(200) = -\frac{0}{100} + c$

so $c = \ln 200$

we have $\ln a - \ln(200) = -\frac{t}{100}$.

or $\ln\frac{a}{200} = -\frac{t}{100}$ so $t = -100 \ln\frac{a}{200}$.

we want $a=2$ so $t = -100 \ln\frac{1}{100}$

$t = -100 \ln 100^{-1}$ or

$t = 100 \ln 100$

(#2) 2.3 ~~ex~~ $120\gamma(1 - \exp^{-t/60})$ 120γ
problem 2

Tank has 120 mg. of pure water. Mixture has γL,~~gal~~/liter entering and leaving, at 2L/min.
Find a formula in terms of γ for $Q(t)$.

$$\frac{dQ}{dt} = \text{In} - \text{out}$$

$$= 2\gamma - \frac{Q}{120}\cdot 2$$

$$\frac{dQ}{2\gamma - \frac{Q}{60}} = dt \to$$

$$-60\ln(2\gamma - Q/60) = t + c$$

$$\ln(2\gamma - Q/60) = \frac{t}{-60} + c$$

$$2\gamma - Q/60 = ce^{-t/60};$$ $Q(0) = 0$

$c = 2\gamma$

$$2\gamma - Q/60 = 2\gamma e^{-t/60}$$

$$Q/60 = 2\gamma - 2\gamma e^{-t/60}$$

$$Q = 2\cdot 60\gamma(1 - e^{-t/60})$$

$$Q = 120\gamma(1 - e^{-t/60})$$

In the long run the tank contains 120γ gr.

(#3)

2.3 P. 3 Problem 5

#5 S_0 is invested at interest rate r

cp. continuously. This actually

means $S(t) = S_0 e^{rt}$

i.e. $\lim_{m \to \infty} S_0 \left(1 + \frac{r}{m}\right)^{mt} = e^{rt}$

a) What is the doubling time?

$$2 S_0 = S_0 e^{rt}$$

$$2 = e^{rt}, \quad t = \frac{\ln 2}{r}$$

b) T = doubling time = $\frac{\ln 2}{.07}$ at 7%

c) What r makes the investment double in 8 years.

$$8 = \frac{\ln 2}{r}$$

$$r = \frac{\ln 2}{8}$$

2.3 P4

#6 Invest k dollars/year.

Intrest rate r compounded continuously

Investments cpd. continuously

find $S(t)$ money after time t.

$$\frac{dS}{dt} = rS + k$$

$$\frac{dS}{rS+k} = dt$$

$$\frac{1}{r}\ln|rS+k| = t + c$$

$S(0)=0$

$$\frac{1}{r}\ln k = c$$

$$\frac{1}{r}\ln|rS+k| = t + \frac{\ln k}{r}$$

$$\ln|rS+k| = rt+\ln k$$

$$rS+k = e^{rt+\ln k}$$

$$= k e^{rt}$$

$$S = \frac{ke^{rt}-k}{r}$$

$$= \frac{k}{r}(e^{rt}-1)$$

p2 Pb 2 Jan 27, 2023

P5

12. newtons law of cooling

$$\frac{dT}{dt} = k(T - T_0)$$

A) Coffee has temp of 200° when freshly poured

B) 1 min later it is 190°

c) Room temp = 70°

d) Find ~~temp.~~ time when temp. = 150°

p3

2.3 #~~10~~ p#3

Jan 27, 2023

P6

Newton's law says

$$\frac{dT}{dt} = k(T - T_0) \quad \text{in this}$$

case $T_0 = 70$

so $\frac{dT}{dt} = k(T-70)$

then $\frac{dT}{T-70} = k\,dt$ integrate.

so $\ln(T-70) = kt + C$ ln(130)

when $t=0$; $T=200$

so $\ln(130) = C$

then $\ln(T-70) - \ln(130) = kt$

~~then~~ use $\ln A - \ln B = \ln A/B$.

2.3. ~~#12~~ Find k:

Jan 27, 2023

P7

or $\ln\left(\frac{T-70}{130}\right) = kt$

when $t=1$, $T = 190$

so $\ln \frac{120}{130} = k$

then $\ln\left(\frac{T-70}{130}\right) = \ln\left(\frac{120}{130}\right) t$

cool to $150°$?

when is $T = 150$

ans $\ln \frac{150-70}{130} = \ln\left(\frac{120}{130}\right) t$

$$t = \frac{\ln\left(\frac{80}{130}\right)}{\ln\left(\frac{120}{130}\right)}$$

not the answer in the book

$$t = \frac{-\ln(8/13)}{-\ln\left(\frac{12}{13}\right)} = \frac{\ln 13/8}{\ln(13/12)}$$

use $-\ln r = \ln r^{-1}$

2.3 Pb. 15 P8

1. A lake has volume V
2. At time t it hold $q(t)$ pollutant with concentration $c(t) = \frac{q(t)}{V}$
3. Water containing concentration R enters and leaves at rate r
4. Pollutants are added at constant rate P
5. At time 0 the concentration is c_0

Find an expression for the concentration.

$$\frac{dq}{dt} = Rr - \frac{q(t)}{V} \cdot r + P$$

$$\frac{dq}{Rr - \frac{q(t)}{V} r + P} = dt$$

pg

to integrate let

$$u = kr - \frac{\varphi(t)}{V} r + p$$

$$du = -\frac{r}{V} d\varphi$$

so $\int = -\frac{V}{r} \ln\left|kr - \frac{\varphi(t)}{V} r + p\right| = t + C$

~~$\varphi(0) = C_0$~~ mult by $\frac{r}{-V}$

so ~~$\frac{-V}{r}$~~

$$\ln\left|kr - \frac{\varphi(t)}{V} r + p\right| = -\frac{r}{V} t + C$$

Let $t = 0$ then

$$\ln\left|kr - \frac{\varphi(0) r}{V} + p\right| = C$$

$$= \ln\left|kr - \frac{C_0 r}{V} + p\right| = C$$

3

P10

so $\ln\left|kr - \frac{Q(t)}{V} r + P\right| = -\frac{r}{V}t + c$

at t_0; $\frac{Q(t)}{V} = C_0$

so $c = \ln\left|kr - \frac{Q(t)}{V} r + P\right|$

$c = \ln\left|kr - \frac{C_0}{\ } r + P\right|$

then: $\ln\left|kr - \frac{Q(t)}{V} r + P\right| = -\frac{r}{V}t + \ln\left|kr - \frac{C_0 r}{\ } + P\right|$

raise to the e power

$kr - \frac{Q(t)}{V} r + P = \left(kr - \frac{C_0 r}{\ } + P\right) e^{-\frac{rt}{V}}$

then $\frac{Q(t)}{V} r = kr + P + \left(-kr + \frac{C_0 r}{\ } - P\right) e^{-\frac{rt}{V}}$

$= k + P/r + \left(-k + C_0 - P/r\right) e^{-\frac{rt}{V}}$

$= C(t)$