attendance, quizzed are on Prof. Darcy's web page.

Some theory - homework.

Section 2.5 an autonomous equation is one like:

$$\frac{dy}{dt} = f(y) \mapsto \text{no } t \text{ here}$$

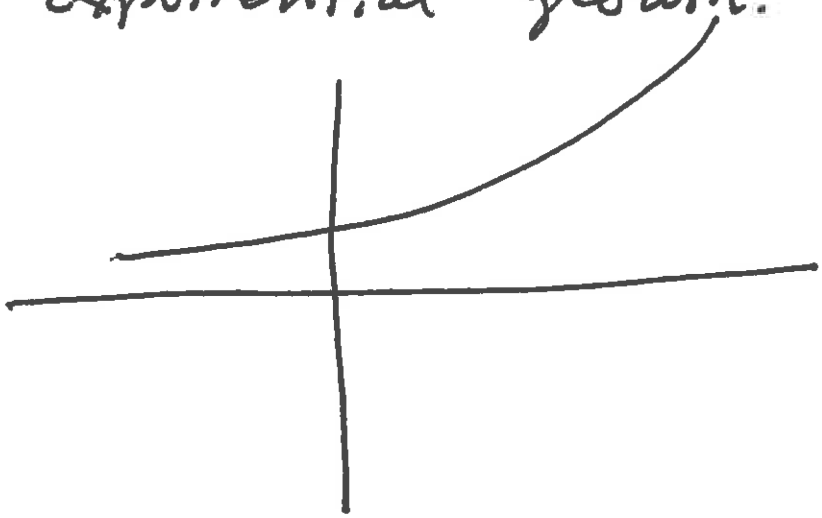
standard $\frac{dy}{dt} = ry$

so $\frac{dy}{y} = r\,dt$

$\ln y = rt + c$

$y = ce^{rt}$

Exponential growth.



better model is

$$\frac{dy}{dt} = h(y)\, y \quad \text{look like.}$$

ry sometimes.

chose an $h(y) = (r - ay)$

logustic equation.

but $r\left(1 - \frac{a}{r} y\right)$

real logustic equatim is:

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right) y. \qquad K = r/a$$

when $y = K$; everybody is

infected; $\frac{dy}{dt} = 0.$

Look for solutions with constant population:

$\frac{dy}{dt} = r\left(1 - y/K\right) y = 0.$

$y = 0$ and $y = K$.

pb → analyze this

draw $ry(1 - y/K)$

$= r(y - y^2/K)$

parabola, opens down; y^2 coeff. is negative

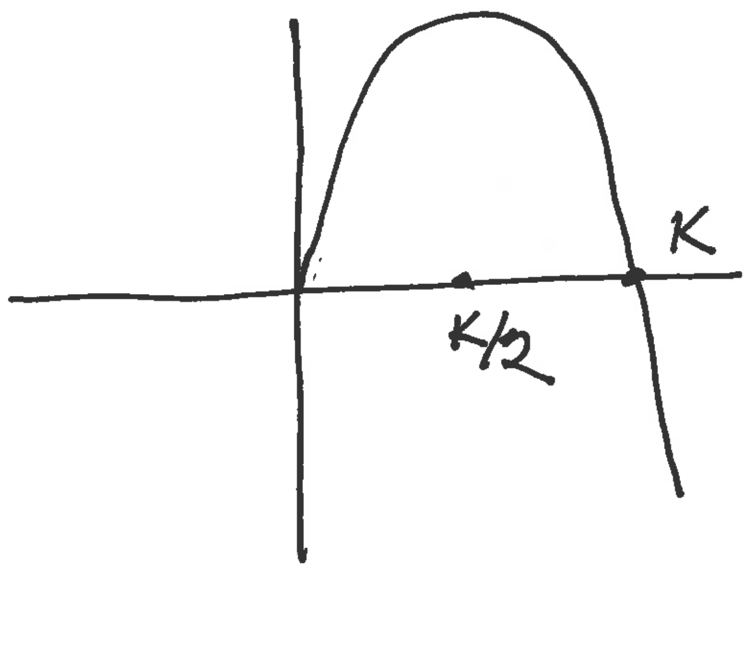
$= 0$ at $y = 0$; $y = K$

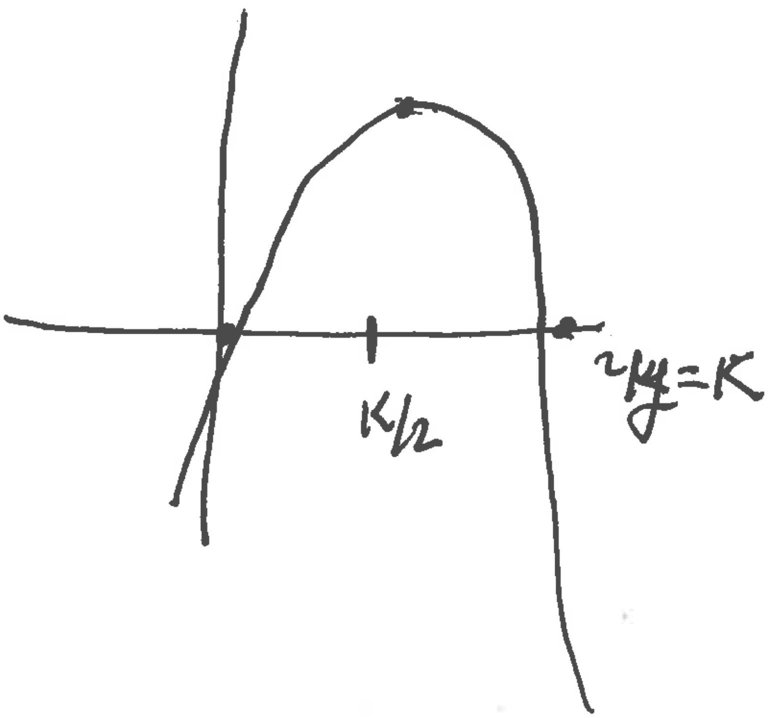
think of this as population.
find constant solutions.

$$r(1-y/K)y = 0$$

$y=0$ $\quad$ $y=K$

discuss this, $f(y)=(1-y/K)y$
parabola opens down
$f(y) = y - \frac{1}{K}y^2$ has zeros at
$y=0$; $y=K$





Since $\frac{dy}{dt} = f(y)$, $f(y)$ is

positive for $0 \le y \le K$

so $\frac{dy}{dt}$ is increasing

on $0 \le y \le K$

9/8 — 4

2.5 4

$$\frac{dy}{dt} = f(y) \quad \text{so}$$

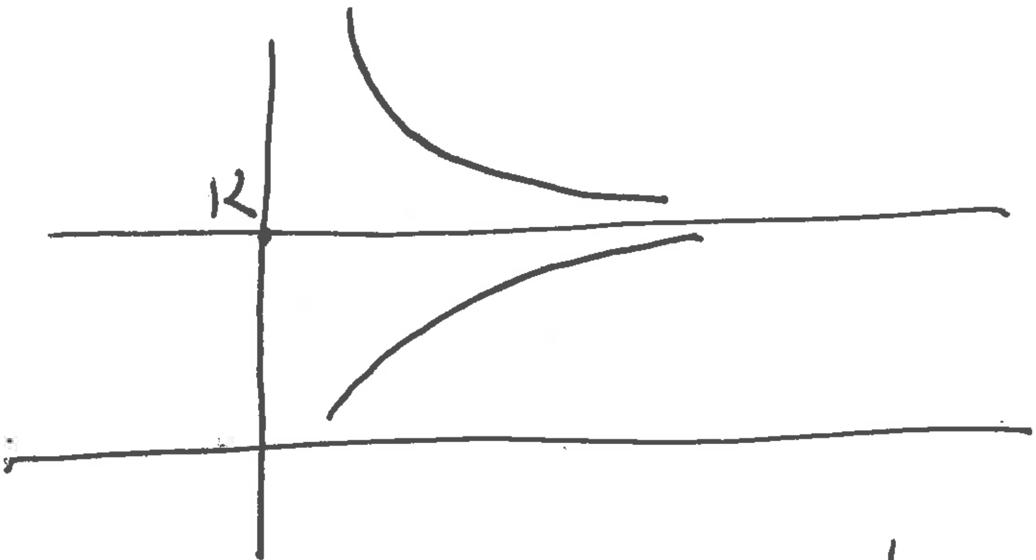
$\frac{dy}{dt}$ is positive when $f(y)$ is pos.

Suppose $\frac{dy}{dt} = f(y(t))$ ← solution.

if $y > K$ $\frac{dy}{dt}$ is negative

$f(y)$ is positive btw 0 and K, $\frac{dy}{dt}$ is pos. so the solution is increasing

9/8 5



If $y > K$

$\frac{dy}{dt} < 0$ so y is decreasing.

we say $y = K$ is a stable equilibrium solution.

y is an unstable equilibrium.

9/8.

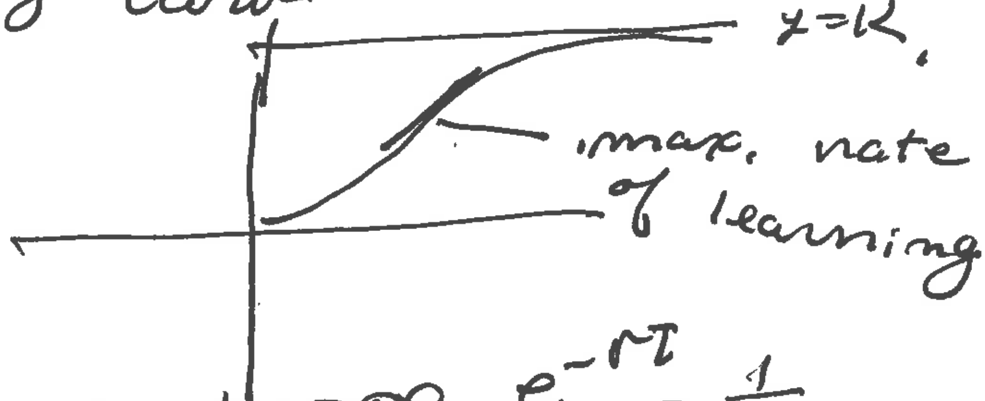
6

Solve $\frac{dy}{dt} = r\,y(1 - y/K)$

the solution is:

$$y = \frac{K y_0}{y_0 + (K - y_0)\,e^{-rt}}$$

learning curve



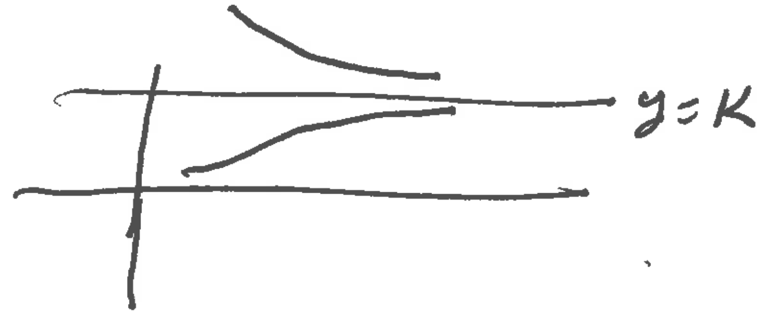
note as $t \to \infty$ $e^{-rt} = \frac{1}{e^{rt}}$

approaches 0.

so $\lim_{t \to \infty} y(t) = \frac{K y_0}{y_0} = K.$

so K is the limiting population.

9-8



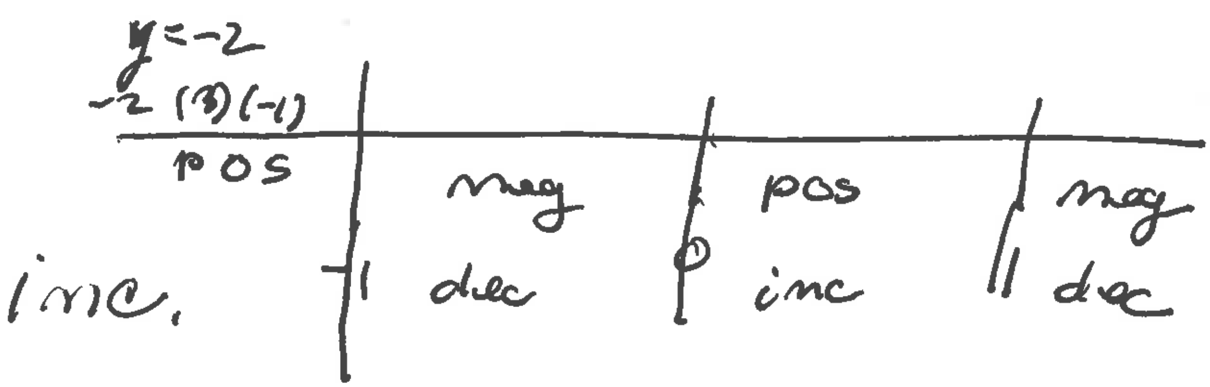
in this case we say $y=K$ is asymptotically stable.

0 is unstable

Pb 7 $\frac{dy}{dt} = y(1-y^2)$ analyze

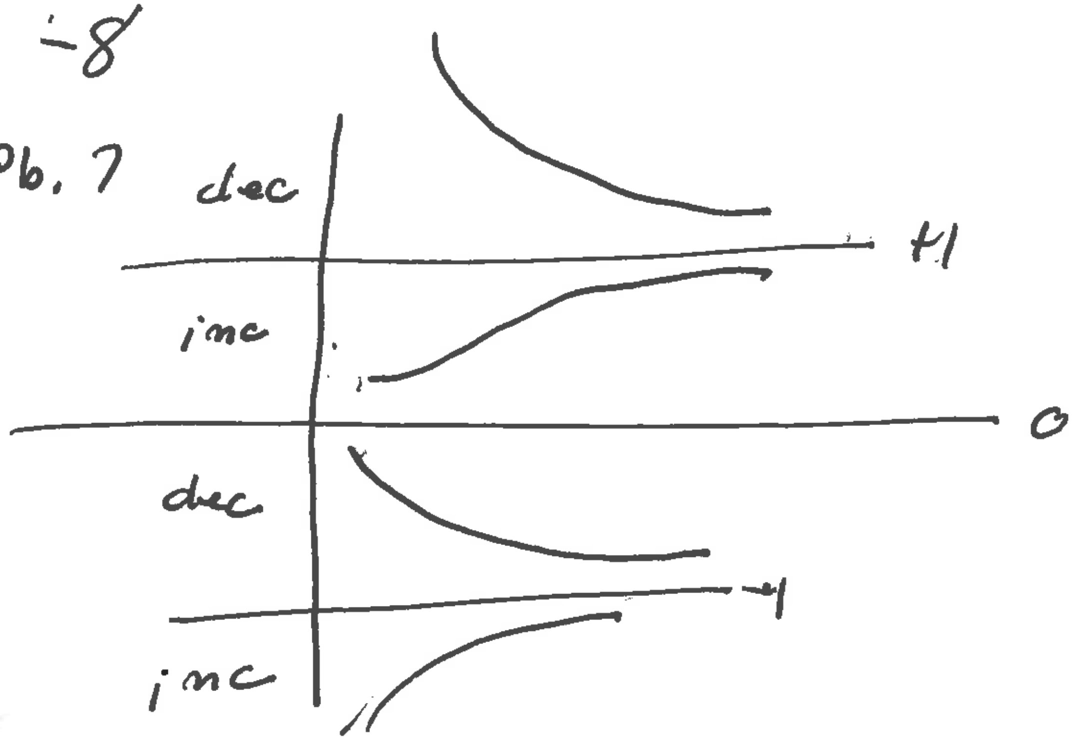
eq. solutions $y(1-y^2)=0$

$y(1-y)(1+y)$



9 - 8

Pb. 7



so $y=1$ and $y=-1$ are stable equil. solns.

$y=0$ is unstable.

9/8

Pb. 9 $\quad \dfrac{dy}{dt} = y^2 (1-y)^2$

find the sign of $\dfrac{dy}{dt}$

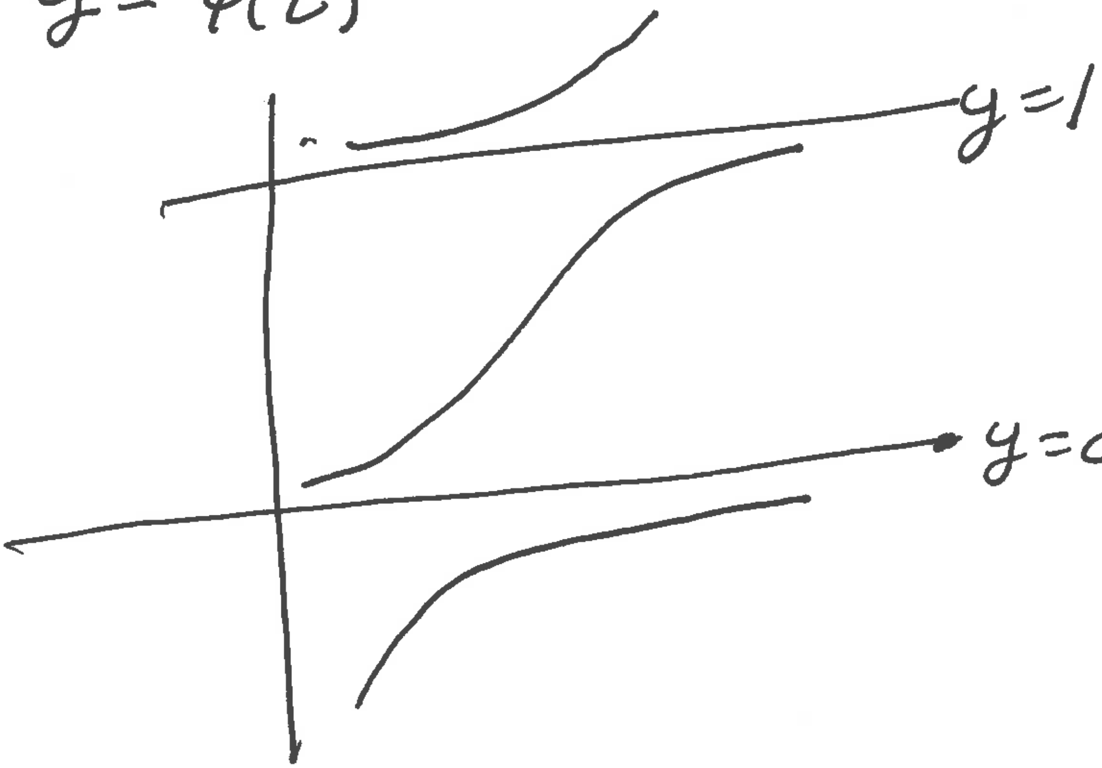
as a function of y

but $y^2 \geq 0$; $(1-y)^2 \geq 0$

so $\dfrac{dy}{dt}$ is positive or

zero everywhere.

$y = \phi(t) \rightarrow$ always increasing



new word in Pb 9

$y=0$ and $y=1$ are said to be semi-stable.

they are approached from one side only

9/8 11

$y = ay + by^2 \quad a \geq 0 \; ; \; b \geq 0$

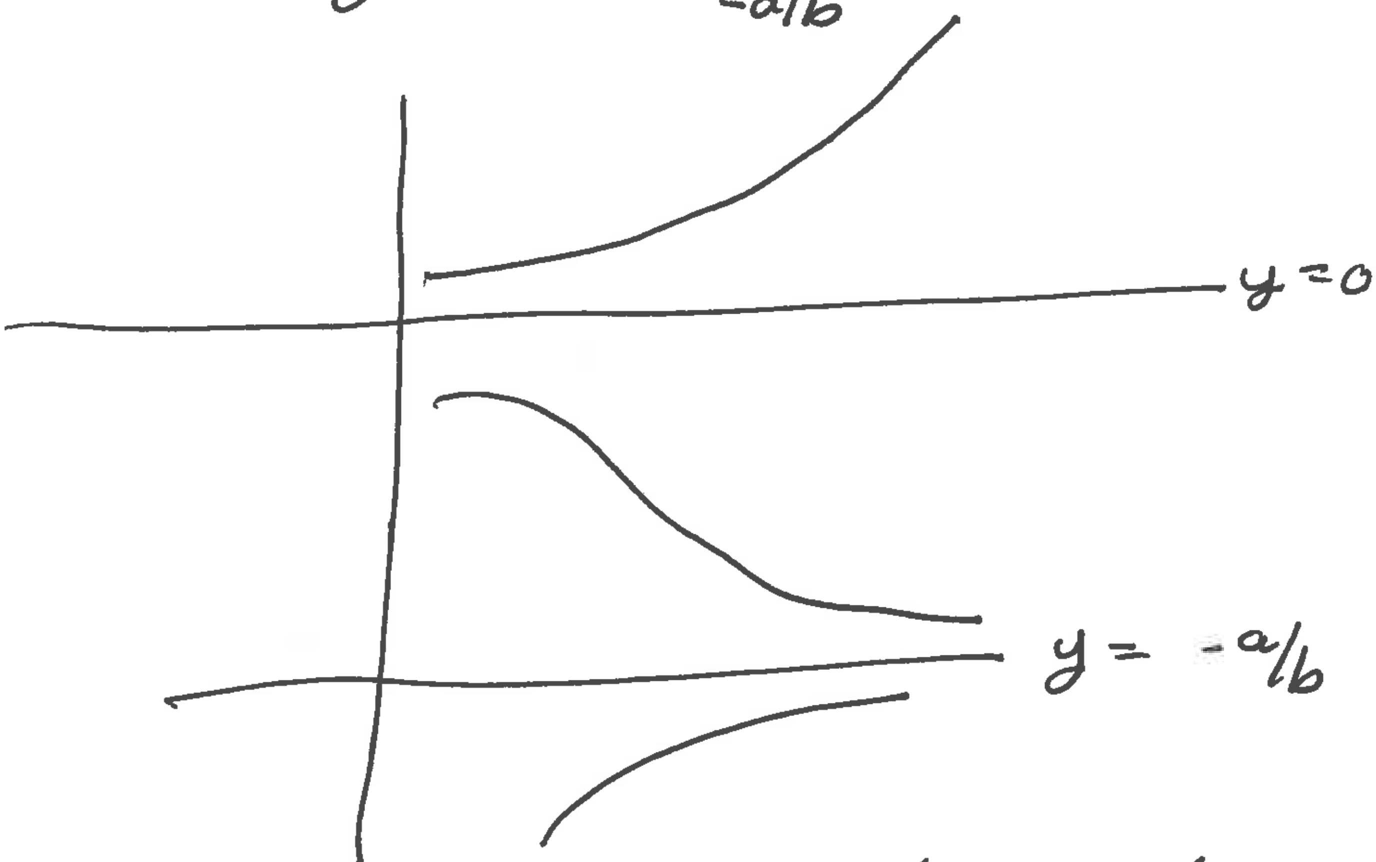
solve $\quad ay + by^2 = 0$

$y(a + by) = 0 \; ; \; y = 0 \; ; \; y = -\frac{a}{b}$

$y(a + by)$

pos | neg | pos

0     $-a/b$



$-\frac{a}{b}$ is asymptotically stable

0 is unstable

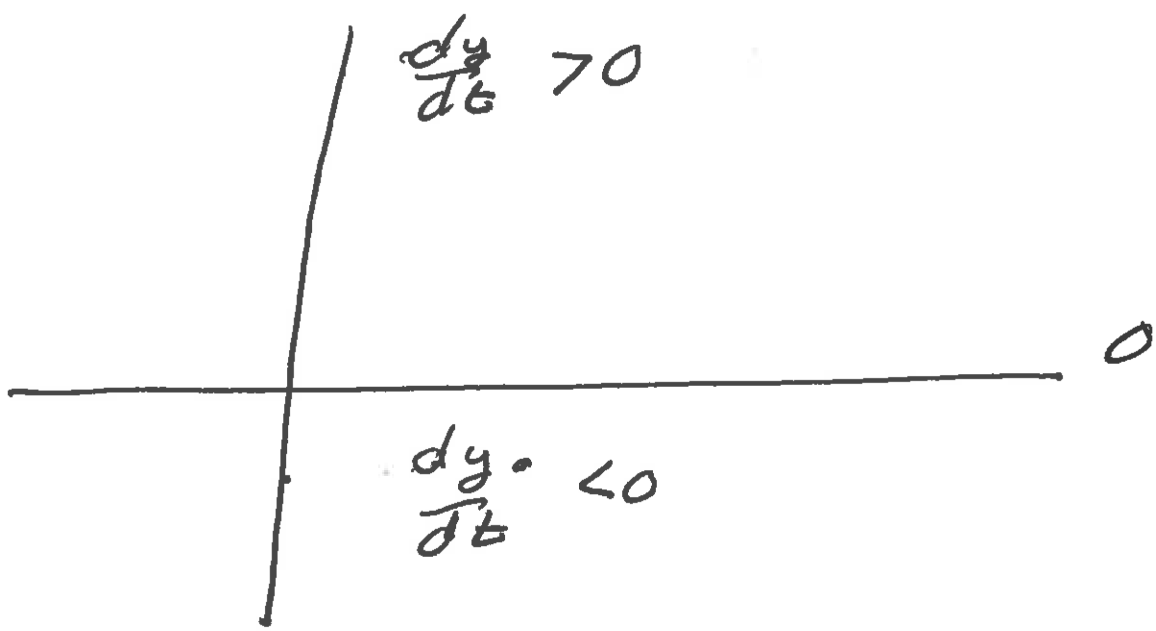
9/8

#3 $\frac{dy}{dt} = e^y - 1$

Set $e^y - 1 = 0$; $e^y = 1$

$e^y = 1$

$y = 0$



so 0 is <u>unstable</u>.