Name:______________________________ Student ID:____________________

Quiz 1 SHOW ALL WORK

[4] 1a.) The equilibrium solutions for the differential equation $y' = (y+1)(y-4)$ are

$y = -1, y = 4$

[2] 1b.) If $y(t)$ is the solution to the initial value problem $y' = (y+1)(y-4)$, $y(0) = -2$, then

$$\lim_{t \to +\infty} y(t) = \underline{\ -1\ }$$

[5] 2.) Find the integrating factor needed to solve the differential equation $ty' - 3y = 4$

Do NOT solve, just find the integrating factor and use log rules to simplify your answer.

$y' - \frac{3}{t}y = \frac{4}{t}$

$u(t) = e^{\int p(t)dt} = e^{\int -\frac{3}{t}dt} = e^{-3ln(t)} = e^{ln(t^3)} = \cancel{t^3}$ t^3 $e^{ln\, t^{-3}}$

The integrating factor is $u(t) = \cancel{t^3}$ t^{-3}

[9] 3.) Solve by separating varables $(x+1)y' = \frac{5x+4}{xy}$

Hint: $y' = \frac{dy}{dx}$. Also after separating variables, you will need to use integration by partial fractions to integrate one of the sides.

$(x+1)\frac{dy}{dx} = \frac{5x+4}{xy}$

Separate variables: $ydy = \frac{(5x+4)dx}{x(x+1)}$

$\int ydy = \int \frac{(5x+4)dx}{x(x+1)}$

$\frac{(5x+4)}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$5x + 4 = A(x+1) + Bx = (A+B)x + A$

Thus $A = 4$ and $B = 1$

$\frac{(5x+4)}{x(x+1)} = \frac{4}{x} + \frac{1}{x+1}$

$\int ydy = \int \frac{4dx}{x} + \int \frac{dx}{x+1}$

$\frac{1}{2}y^2 = 4ln|x| + ln|x+1| + C$

$y^2 = 8ln|x| + 2ln|x+1| + C$

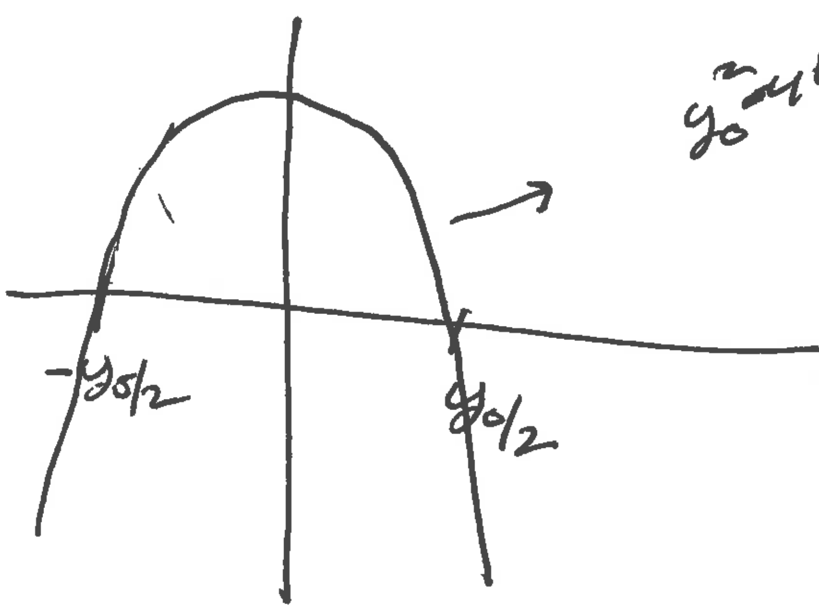
$y = \pm\sqrt{8ln|x| + 2ln|x+1| + C}$

I need $y_0^2 - 4t^2 \geq 0$

$z = y_0^2 - 4t^2$ we could factor. Easier to draw a piture $y_0^2 - 4t^2$ has negative leading coefficent opens down.

solve $y_0^2 - 4t^2 = 0$

$t = \pm \frac{y_0}{2}$



$y_0^2 - 4t^2$ is positive for

$-\frac{y_0}{2} < t < y_0/2$

or

$|t| \leq \frac{y_0}{2}$

⑤

9-6 $\frac{dy}{dt} = -\frac{4t}{y}$; $y(0) = y_0$

Separable:

so $y\,dy = -4t\,dt$

$$\frac{y^2}{2} = -\frac{4t^2}{2} + c$$

mult by 2 get

$$y^2 = -4t^2 + c$$

$y(0) = y_0$ so

$$y_0^2 = 0 + c$$

so $y^2 = y_0^2 - 4t^2$

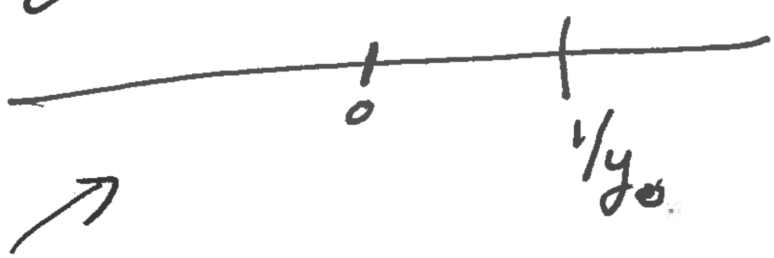
$$y = \sqrt{y_0^2 - 4t^2}$$

this solution has trouble

at $1-y_0t=0$; $\frac{1}{y_0}=t$

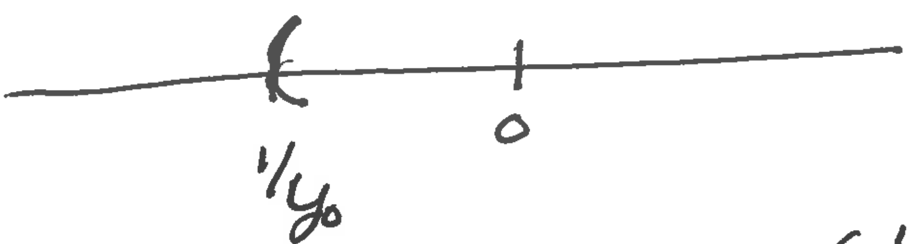
solution interval, check "0"

if $y_0>0$



chose this; solution is on $(-\infty, 1/y_0)$

if $y_0<0$



solution lives in $(1/y_0, \infty)$

9-6 3.4.

$$y = \frac{-1}{t-1}$$

$$y = \frac{1}{1-t}$$

homework? valid. Blows up a $t=1$

initial point. Chose an interval that contains the initial point.

Solution is valid in $(-\infty, 1)$.

Continue:

Suppose $y(0) = y_0$. discuss.

the solution $y = \dfrac{y_0}{1 - y_0 t}$

note $y(0) = y_0$.

9-6 3.4

Example: discuss $y' = y^2$; $y(0) = 1$

$\frac{df}{dy} = 2y$; f is continuous so
I have a solution some interval.
Solve to find a plausable interval

$$\frac{dy}{dt} = y^2;$$

$$\frac{dy}{y^2} = 1\,dt$$

$$\text{so} \int y^{-2}\,dy = t + c$$

$$-y^{-1} = t + c$$

$$-\frac{1}{y} = t + c$$

$$y = \frac{-1}{t+c}$$

when ~~y~~. $t = 0$ $y = 1$

$$71 = \frac{-1}{c} \quad \text{so}$$

$$c = -1$$

9/6 ~3.4

Section 3.4

Existance of solutions.

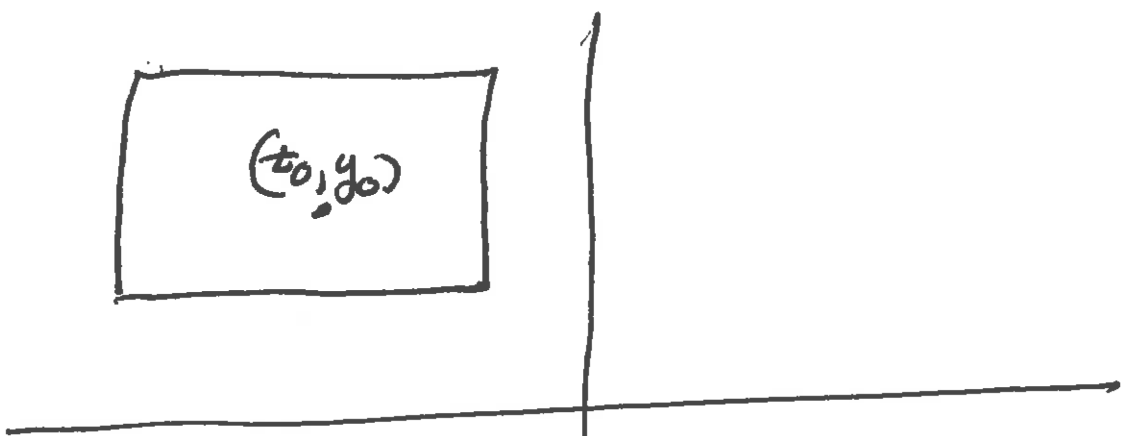
Have $y' = f(t, y)$ I

I want to solve this.

Can I solve it?

If $\frac{df}{dy}$ is continuous and:

$f(y)$ is continuous



then there is a unique solution to I on some interval $(t_0 - h, t_0 + h)$.