Clearly the solution cannot be a function t alone. So $y' = y - t$ has to be the answer:

3.) Circle the differential equation whose direction field is given below:

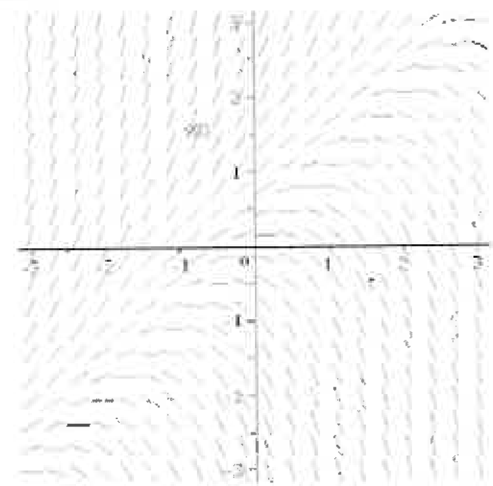
A) $y' = t^2$ B) $y' = \frac{1}{2}t + 1$

C) $y' = e^t$ D) $y' = t + 1$

E) $y' = -2e^t$ F) $y' = y - t$

G) $y' = ln(t)$ H) $y' = 0$

I) $y' = sin(t)$ J) $y' = cos(t)$



$y' - y = -t$; integrating factor is $e^{\int -1 dt} = e^{-t} = u$

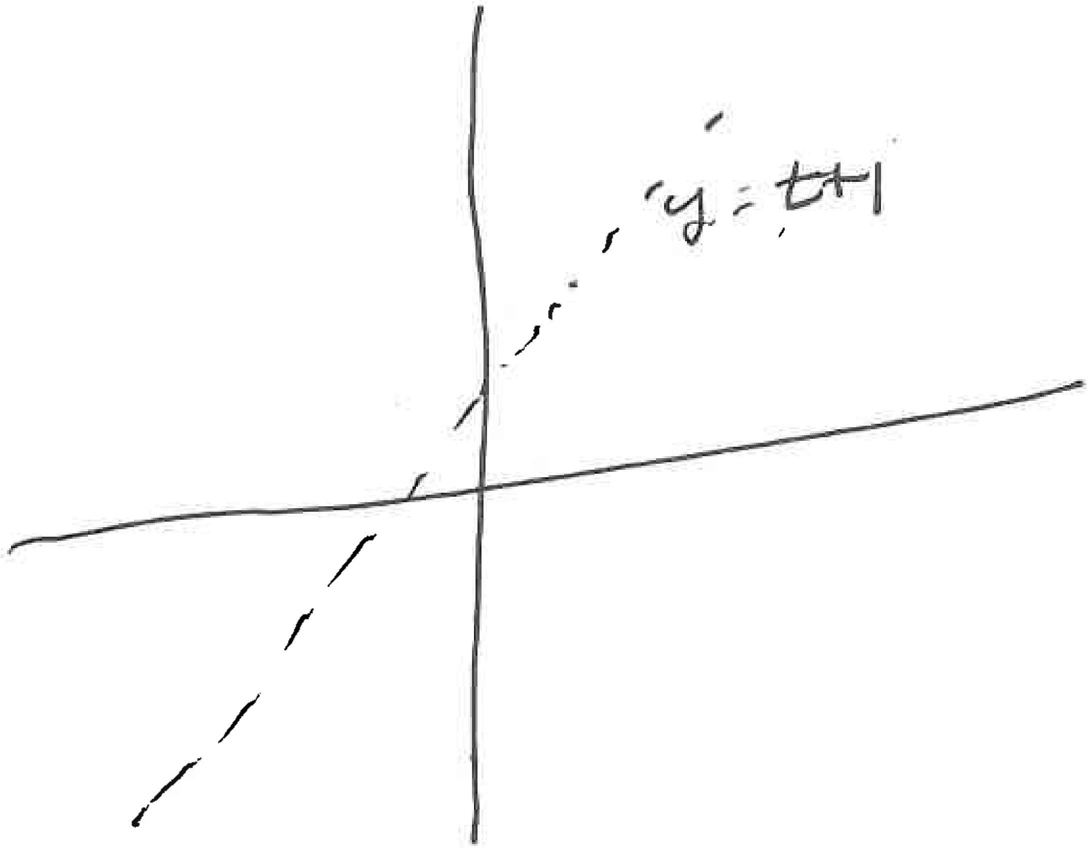
$\frac{1}{u}\left[\int -te^{-t} dt + c\right] = e^{+t}\left[te^{-t} + e^{-t} + c\right]$

$= (t+1) + ce^{+t}$

as $t \to -\infty$ the solutions approach $t+1$

because $\lim_{t \to -\infty} e^{t} = 0$.

attendence quiz 7



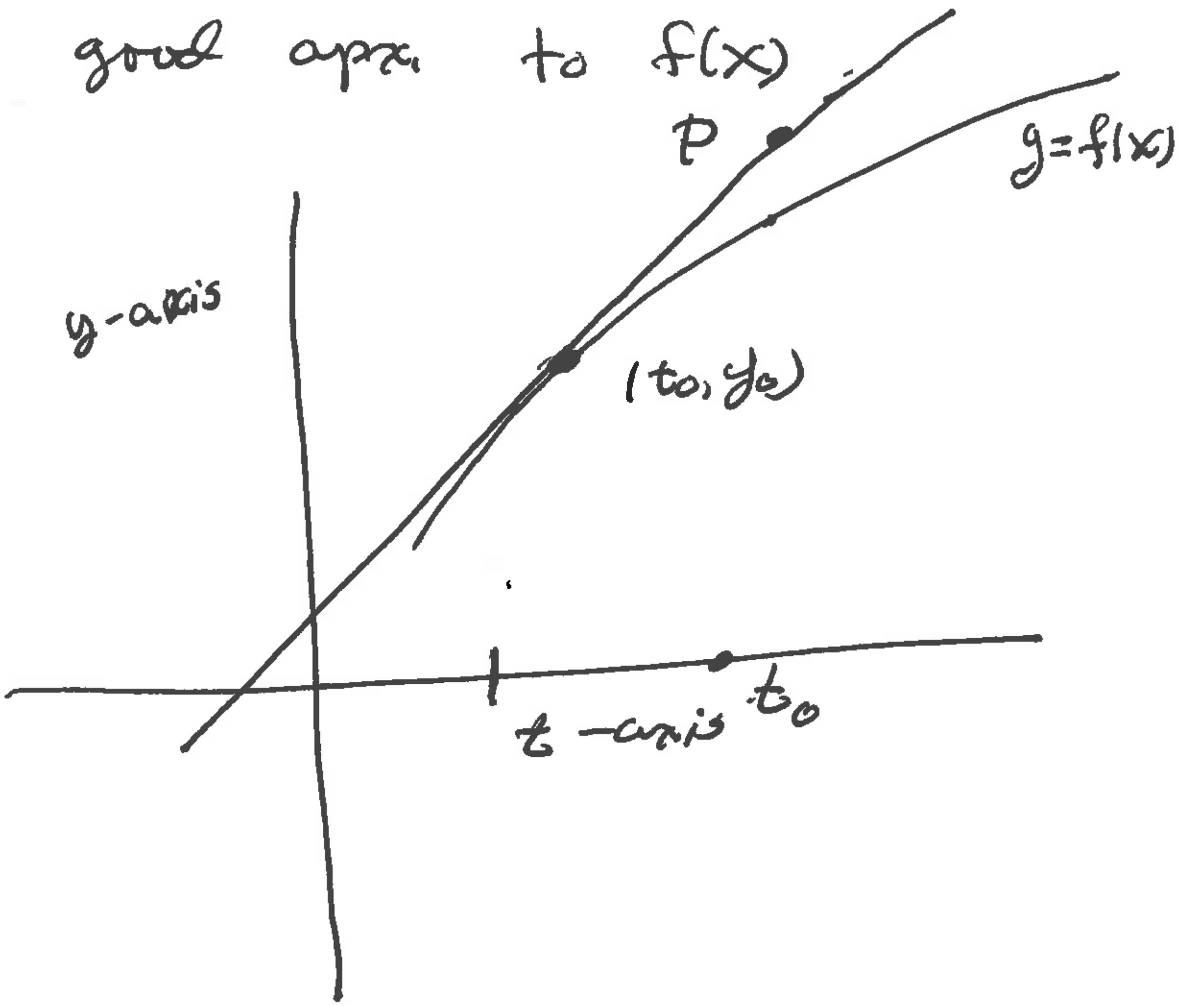
is a solution.

9-11 #1

2.7 Approximating solutions:

Example get a good apx. to $y' = f(t, y)$ with initial conditions.

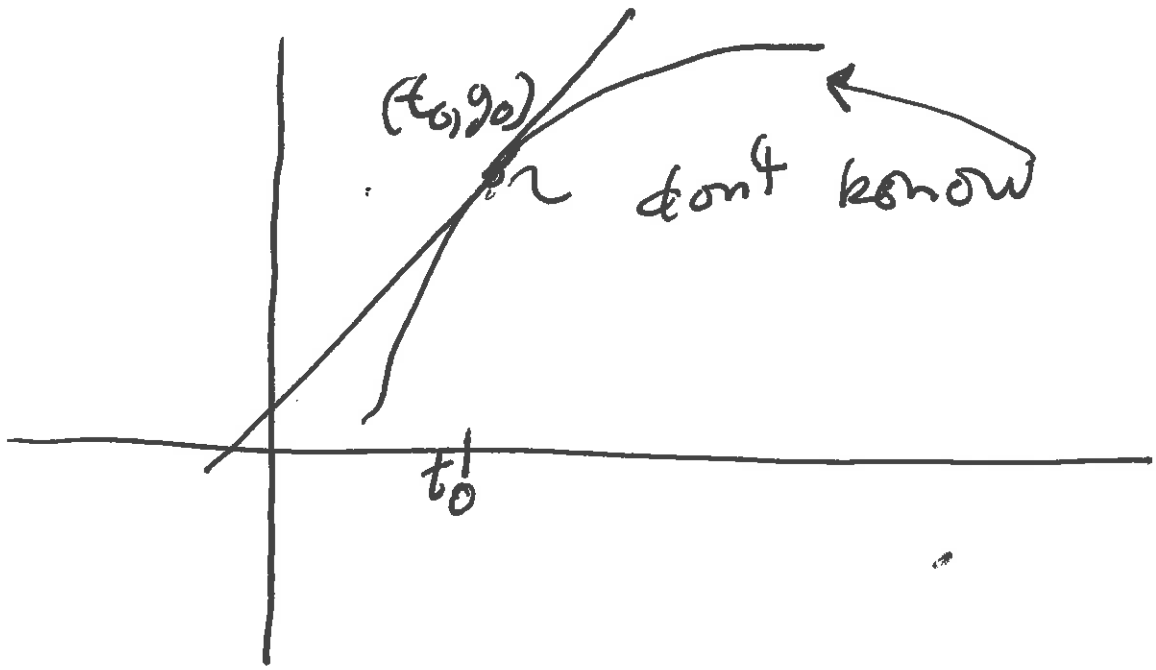
Basic idea use the tangent line: to $y = f(x)$ near (t_0, y_0) is a good apx to $f(x)$



I can use P as an approximation to $y_0 = f(t_0)$

9/11 ②

Let say I have $y' = f(t, y)$



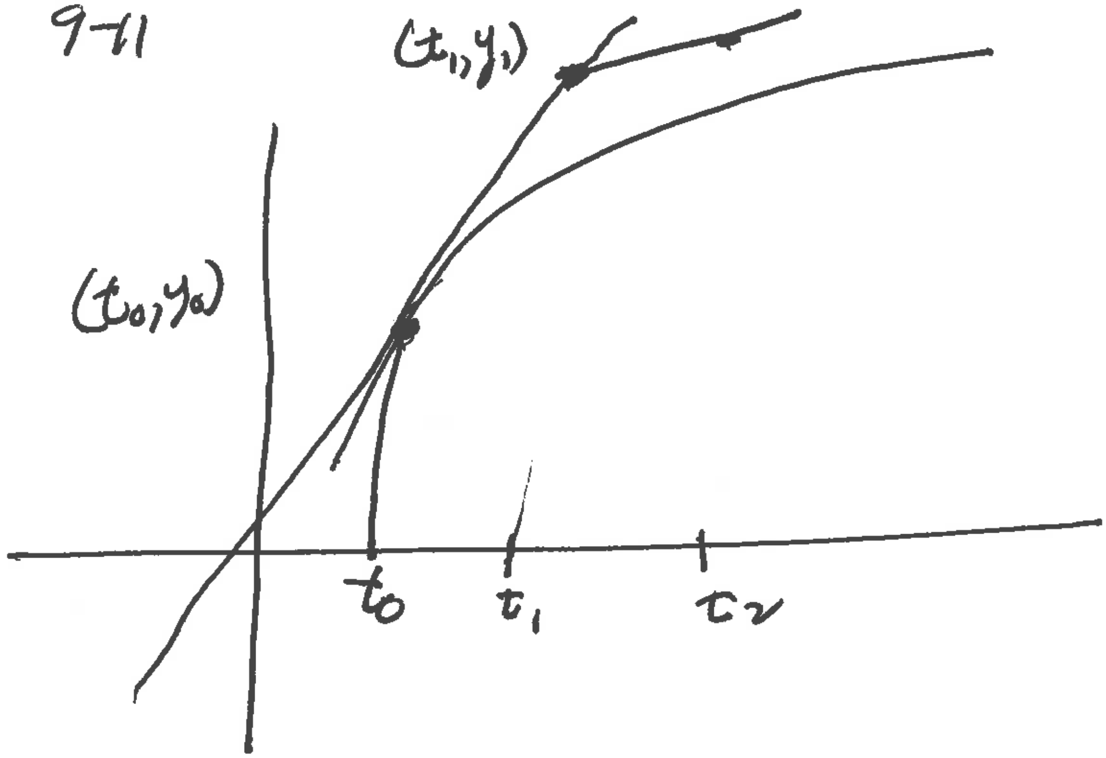
by definition the tangent line has slope $f(t_0, y_0)$

So the equation of the tan line is

$$y = y_0 + f(t_0, y_0)(t - t$$

use the tan line as an app solution.

9-11



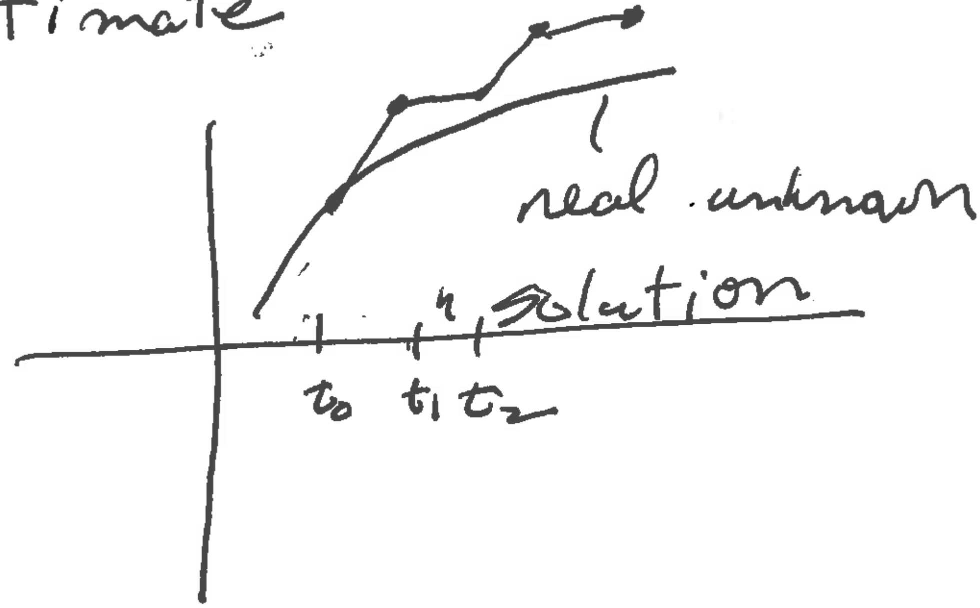
I use the tan line to compute y_1. not exact. the second line has slope $f(t_1, y_1)$

It's equation is

$$y - y_1 = f(t_1, y_1)(t - t_1)$$

9/11

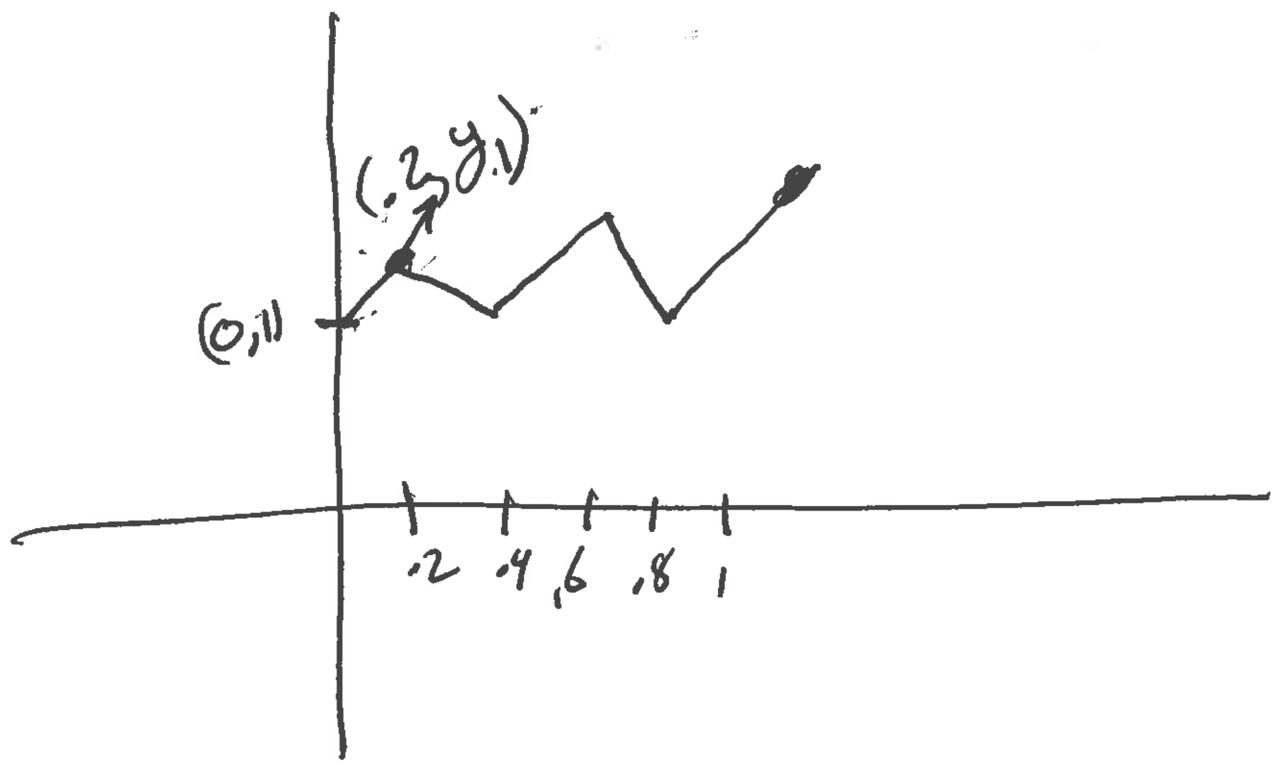
Estimate



make the increments very small and "hope" you drift too far from the solution.

step size $= h$

Example 2.7.1



Example $n=2$

1.87 is the y_2 value

1.756 is the actual value

these are close.
compare Col. 3 with the last column

2.7

3

If we introduce the notation $f_n = f(t_n, y_n)$, then we can rewrite equation (8) as

$$y_{n+1} = y_n + f_n \cdot (t_{n+1} - t_n), \qquad n = 0, 1, 2, \ldots. \tag{9}$$

Finally, if we assume that there is a uniform step size h between the points $t_0, t_1, t_2, \ldots$, then $t_{n+1} = t_n + h$ for each n, and we obtain Euler's formula in the form

$$y_{n+1} = y_n + f_n h, \qquad n = 0, 1, 2, \ldots. \tag{10}$$

To use Euler's method, you repeatedly evaluate equation (9) or equation (10), depending on whether or not the step size is constant, using the result of each step to execute the next step. In this way you generate a sequence of values $y_1, y_2, y_3, \ldots$ that approximate the values of the solution $\phi(t)$ at the points $t_1, t_2, t_3, \ldots$. If, instead of a sequence of points, you need a function to approximate the solution $\phi(t)$, then you can use the piecewise linear function constructed from the collection of tangent line segments. That is, let y be given in $[t_0, t_1]$ by equation (7) with $n = 0$, in $[t_1, t_2]$ by equation (7) with $n = 1$, and so on.

EXAMPLE 2.7.1

Consider the initial value problem

$$\frac{dy}{dt} = 3 - 2t - \frac{y}{2}, \qquad y(0) = 1. \tag{11}$$

Use Euler's method with step size $h = 0.2$ to find approximate values of the solution of initial value problem (9) at $t = 0.2, 0.4, 0.6, 0.8$, and 1. Compare them with the corresponding values of the actual solution of the initial value problem.

Solution Note that the differential equation in the given initial value problem is the same as in equation (2); its direction field is shown in Figure 2.7.1. Before applying Euler's method, observe that this differential equation is linear, so it can be solved as in Section 2.1, using the integrating factor $e^{t/2}$. The resulting solution of the initial value problem (9) is

$$y = \phi(t) = 14 - 4t - 13e^{-t/2}. \tag{12}$$

We will use this information to assess how the approximate solution obtained by Euler's method compares with the exact solution.

To approximate this solution by Euler's method, note that $f(t, y) = 3 - 2t - y/2$. Using the initial values $t_0 = 0$ and $y_0 = 1$, we find that

$$f_0 = f(t_0, y_0) = f(0, 1) = 3 - 0 - 0.5 = 2.5$$

and then, from equation (3), the tangent line approximation near $t = 0$ is

$$y = 1 + 2.5(t - 0) = 1 + 2.5t. \tag{13}$$

Setting $t = 0.2$ in equation (13), we find the approximate value y_1 of the solution at $t = 0.2$, namely,

$$y_1 = 1 + (2.5)(0.2) = 1.5.$$

At the next step we have

$$f_1 = f(t_1, y_1) = f(0.2, 1.5) = 3 - 2(0.2) - (1.5)/2 = 3 - 0.4 - 0.75 = 1.85.$$

Then the tangent line approximation near $t = 0.2$ is

$$y = 1.5 + 1.85(t - 0.2) = 1.13 + 1.85t. \tag{14}$$

Evaluating the expression in equation (14) for $t = 0.4$, we obtain

$$y_2 = 1.13 + 1.85(0.4) = 1.87.$$

Repeating this computational procedure three more times, we obtain the results shown in Table 2.7.1.

2.4
4

TABLE 2.7.1 **Results of Euler's Method with $h = 0.2$ for $y' = 3 - 2t - y/2,\ y(0) = 1$**

n	t_n	y_n	$f_n = f(t_n, y_n)$	Tangent Line	Exact $y(t_n)$
0	0.0	1.00000	2.5	$y = 1 + 2.5(t - 0)$	1.00000
1	0.2	1.50000	1.85	$y = 1.5 + 1.85(t - 0.2)$	1.43711
2	0.4	1.87000	1.265	$y = 1.87 + 1.265(t - 0.4)$	1.75650
3	0.6	2.12300	0.7385	$y = 2.123 + 0.7385(t - 0.6)$	1.96936
4	0.8	2.27070	0.26465	$y = 2.2707 + 0.26465(t - 0.8)$	2.08584
5	1.0	2.32363			2.11510

The second column contains the t-values separated by the step size $h = 0.2$. The third column shows the corresponding y-values computed from Euler's formula (10). Column four contains the slopes f_n of the tangent line at the current point, (t_n, y_n). In the fifth column are the tangent line approximations found from equation (7). The sixth column contains values of the solution (12) of the initial value problem (9), correct to five decimal places. The solution (12) and the tangent line approximation are also plotted in **Figure 2.7.3**.

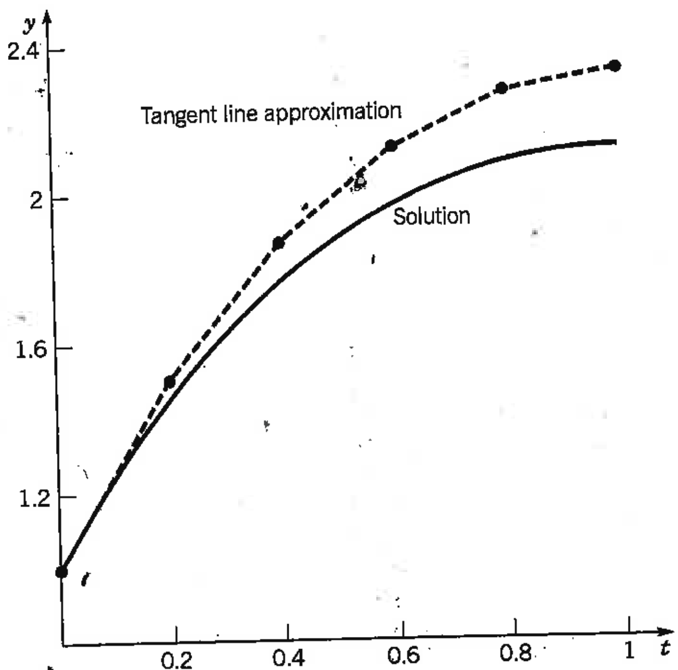


FIGURE 2.7.3 Plots of the solution and a tangent line approximation with $h = 0.2$ for the initial value problem (9): $dy/dt = 3 - 2t - y/2,\ y(0) = 1$.

From Table 2.7.1 and Figure 2.7.3 we see that the approximations given by Euler's method for this problem are greater than the corresponding values of the actual solution. This is because the graph of the solution is concave down and therefore the tangent line approximations lie above the graph.

The accuracy of the approximations in this example is not good enough to be satisfactory in a typical scientific or engineering application. For example, at $t = 1$ the error in the approximation is $2.32363 - 2.11510 = 0.20853$, which is a percentage error of about 9.86% relative to the exact solution. One way to achieve more accurate results is to use a smaller step size, with a corresponding increase in the number of computational steps. We explore this possibility in the next example.

Problem 2a ① 5

approximate solutions to $y' = 2y - 1$ $\quad y' = 2y - 1$

0	.1	.2	.3	.4	
0	1.1	1.22	1.364	1.736	$f(t, y)$

assume $y(0) = 1$

actual solution:

$$\frac{dy}{2y-1} = dt$$

$$\frac{1}{2} \ln|2y - 1| = t + c$$

so $\ln|2y-1| = 2t + c$

$$2y - 1 = ce^{2t}$$

$$y = \frac{1}{2}e^{2t} + \frac{1}{2}$$

2s

start with $y(0) = 1$ value initial condition

slope $\boxed{y'(1)} = 2 \cdot 1 - 1 = 1 = f(t, y)$

Equation $y = y_0 + 1 \cdot (t - 0)$ ✓

$= 1 + t$

Compute y_1

$\boxed{y_1 = 1 + .1 = 1.1}$ → put in the first value

$y'(1.1) = 2(1.1) - 1 = \boxed{1.2}$ - slope tan line

what is the equation of the second line:

$y = 1.1 + 1.2(t - .1)$ eq. 2nd tan line

now

$y_2 = 1.1 + 1.2(.2 - .1)$ ↗ 2nd point

$= 1.1 + 1.2(.1)$

$= 1.1 + .12 = \boxed{1.22 = y_2}$

③ s

to evaluate at $t = .3$.

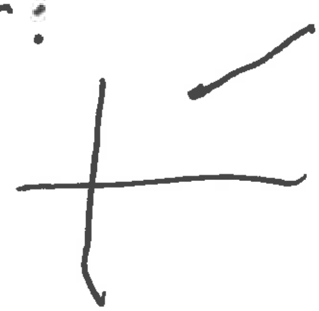
find the tangent line at $.2$

and then evaluate at $.3$

we are starting at:

$(.2, \boxed{1.22})$

y value



use $y' = 2y - 1$

to find the solpe of the next time $2y - 1$

$y' = 2(1.22) - 1$

$= 2.44 - 1$

$= \boxed{1.44 = \text{slope}}$

#5

$y = y_2 + m(t - .2)$

$y = 1.22 + 1.44(t - .2)$

to find y_3 at .3

Let $t = .3$

$y = 1.22 + 1.44(.3 - .2)$

$= 1.22 + (1.44)(.1)$

$= 1.220 + .144 = 1.364$