3.1 #20

$ay'' + by' + cy = d$

we want a number e

that is a constant solution.

so $a \underset{0}{e''} + b\overset{0}{e'} + ce = d$

so $e = d/c$

21. Consider $ay'' + by' + c = 0$, $a > 0$

a) when are the solns,

real, different and negative

we have $ar^2 + br + c = 0$.

so $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ If $c < 0$

then $b^2 - 4ac > 0$ since $a > 0$

and $\sqrt{b^2 - 4ac} > b$ so

$\dfrac{-b + \sqrt{b^2 - 4ac}}{2a} > 0$

but we want < 0

3.1 p21 continued.

so $c > 0$

real means $b^2 - 4ac > 0$

if b is negative

$-b \pm \sqrt{b^2 - 4ac} > 0$, both roots positive

so $\boxed{b > 0}$ then

$b^2 - 4ac > 0$

so $\boxed{\frac{b^2}{4a} > c > 0}$

7. $y'' + y' - 2y = 0$ $\quad y(0) = 1$, $y'(0) = 1$

$r^2 + r - 2 = 0$ factor

$(r+2)(r-1) = 0$

basic solutions e^{-2t}; e^{t}

$y = c_1 e^{-2t} + c_2 e^{t}$, $y' = -2c_1 e^{-2t} + c_2 e^{t}$

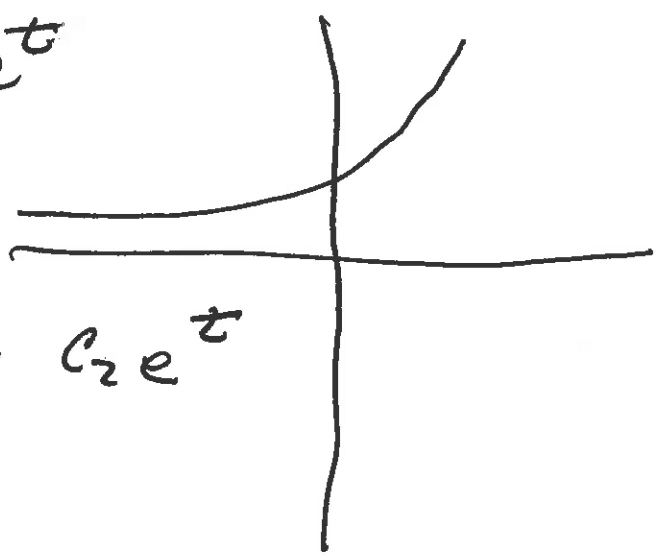
$y(0) = 1 \qquad c_1 + c_2 = 1 \qquad$ i.e. $e^0 = 1$

$y'(0) = 1 \qquad -2c_1 + c_2 = 1$

$-3c_1 + 0 = 0$

$c_1 = 0 \qquad c_2 = 1$

$y = e^t$



$y = 0 + c_2 e^t$

4

3.1 #8

$y'' + 4y' + 3y = 0 \qquad y(0) = 2; \; y'(0) = -1$

$r^2 + 4r + 3 = 0$

$(r+1)(r+3) = 0$

$r = -1 \; ; \; r = -3$

$y = c_1 e^{-t} + c_2 e^{-3t}$

$y(0) = 2 = c_1 + c_2$

$y'(0) = -1 = -c_1 - 3c_2$

add $\quad 1 = -2c_2$

$c_2 = -1/2 \; ; \; c_1 = 5/2$

$5/2\, e^{-t} - 1/2\, e^{-3t} = y(t)$

$\lim_{t \to \infty} = 0.$

$\lim_{t \to \infty} \{ e^{-t}, e^{-3t} \} = 0$ because

the exponents are negative

#17. Find α so that $\lim\limits_{t\to\infty} y(t) = 0$

or $\lim\limits_{t\to\infty} y(t) = \infty$ also $\lim\limits_{t\to\infty} y(t) = \infty$

$y'' - (2\alpha-1)y' + \alpha(\alpha-1)y = 0,$

$r^2 - (2\alpha-1)r + \alpha(\alpha-1) = 0,$

factors $(r-\alpha)(r-(\alpha-1))$

so solutions are

$y(t) = c_1 e^{\alpha t} + c_2 e^{(\alpha-1)t}$

to go to 0 $\quad \alpha;\ \alpha-1 < 0$

so $\alpha < 0$.

to go to ∞

$\alpha,\ \alpha-1 > 0$, so $\alpha > 1$.

#19 $y'' + 5y' + 6y = 0$; $y(0) = 2$; $y'(0) = \beta$

a) Solve in terms of β.

Char. poly: $r^2 + 5r + 6 = 0$

so $(r+2)(r+3) = 0$.

$r = -2$; $r = -3$

$y = c_1 e^{-2t} + c_2 e^{-3t}$

find c_1 and c_2 with initial conditions

$c_1 + c_2 = 2$

$y' = -2c_1 e^{-2t} - 3c_2 e^{-3t}$

$y'(0) = -2c_1 - 3c_2 = \beta$.

$2c_1 + 2c_2 = 4$

solve

then $-c_2 = \beta + 4$

so $c_2 = -(\beta + 4)$

#19 (continued)

then $c_1 = 2 - c_2$

$$c_1 = 2 - (-(B+4))$$

$$= 6 + B$$

solution is

$$y = (6+B)e^{-2t} - (4+B)e^{-3t}$$

as $t \to \infty$ the solution goes to 0.

#19 continued

Find the max in terms of β

$$y = (6+\beta)e^{-2t} - (4+\beta)e^{-3t}$$

$$y' = -2(6+\beta)e^{-2t} + 3(4+\beta)e^{-3t} = 0$$

so

$$e^{-2t}\left[-12 - 2\beta + (12+3\beta)e^{-t}\right] = 0$$

so $(12 + 3\beta)e^{-t} = 12+2\beta$

so $e^{+t} = \dfrac{12+3\beta}{12+2\beta}$

and $t_m = \ln\left|\dfrac{12+3\beta}{12+2\beta}\right|$

#9 solve , $y(1) = 1$; $y'(1) = 0$

$$y'' + 8y' - 9y = 0$$

$$(r+9)(r-1) = 0$$

$$r = -9, \quad r = 1$$

$$y = c_1 e^{-9t} + c_2 e^{t}$$

$$y' = -9c_1 e^{-9t} + c_2 e^{t}$$

use initial conditions

$$\begin{cases} c_1 e^{-9} + c_2 e = 1 \\ -9c_1 e^{-9} + c_2 e = 0. \\ 9c_1 e^{-9} + 9c_2 e = 9 \end{cases}$$

so add

$$10 c_2 e = 9$$

so $c_2 = \frac{9}{10} e^{-1}$

then:

$$-9 c_1 e^{-9} + \frac{9}{10} e^{-1} \cdot e = 0.$$

$$\text{so } -9 c_1 e^{-9} = -\frac{9}{10}$$

$$c_1 = \frac{1}{10} e^{9}$$

$$\text{so } y = \frac{1}{10} e^{9} e^{-9t} + \frac{9}{10} e^{-1} e^{t}$$

$$= \frac{1}{10} e^{-9t+9} + \frac{9}{10} e^{t-1}$$

so

$$y = \frac{1}{10} e^{-9(t-1)} + \frac{9}{10} e^{t-1}$$

$$\lim_{t \to \infty} y(t) = \infty = \frac{1}{10} \cdot 0 + \frac{9}{10} \lim_{t \to \infty} e^{t-1}$$

#14. $y'' - y = 0 \qquad y(0) = 5/4 \quad , \; y(0) = \; -3/4$

$r^2 - 1 = 0$

$$y = c_1 e^{t} + c_2 e^{-t}$$

$$y' = c_1 e^{t} - c_2 e^{-t}$$

so

$$c_1 + c_2 = 5/4$$

$$c_1 - c_2 = -3/4$$

$$2c_1 = 2/4$$

$$c_1 = 1/4$$

$$c_2 = 1$$

$$y = \frac{1}{4} e^{t} + e^{-t}$$

Find the min. value

for $y = \frac{1}{4}e^{t} + e^{-t}$

$y' = \frac{1}{4}e^{t} - e^{-t} = 0$ mult. by e^{t}

$\frac{1}{4}e^{2t} - 1 = 0$

so $e^{2t} = 4$

$2t = \ln 4$

$t = \frac{1}{2}\ln 4 = \ln 4^{1/2}$

$= \ln\sqrt{4}$.

$\boxed{t = \ln 2}$

$y = \frac{1}{4}e^{\ln 2} + e^{-\ln 2}$ $e^{\ln x} = x$

$= \frac{1}{4}\cdot 2 + e^{\ln 1/2}$

$= \frac{1}{2} + \frac{1}{2} = 1$