Solitons in Nonlinear Optics

S. Gautameshwar

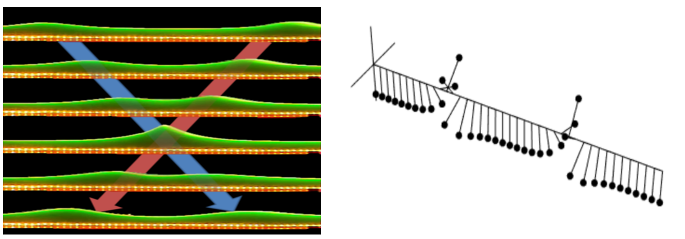
National Institute of Science Education and Research

1. Introduction

Solitons are wave packets that travel in a medium by retaining their shape. As simple as it sounds, this outcome of wave equations in both mechanical and optical systems has very interesting properties that make physicists consider it as a kind of particle by itself:

1. They maintain their shape throughout their motion in the medium. Thus, the energy is localized on the soliton and remains preserved since the shape does not change. This property is similar to a particle having finite energy while moving.
2. When two solitons interact by colliding, it emerges from the "collision" unchanged. A very intriguing property that is different from how conventional particles interact elastically/ inelastically during collisions. This also makes it very robust against any external perturbations.

There are many physical systems where solitons exist. For example, in liquids, solitons can be generated and made to move along the surface, as shown in Fig. 1(a). Another such mechanical system is in a weakly coupled set of pendulums when we flip one pendulum quickly. Solitons can also be formed in optical systems using EM waves due to nonlinear effects in crystals.



(a) Timewise snapshots of two solitons (shown in red and blue arrows) on water surface crossing each other

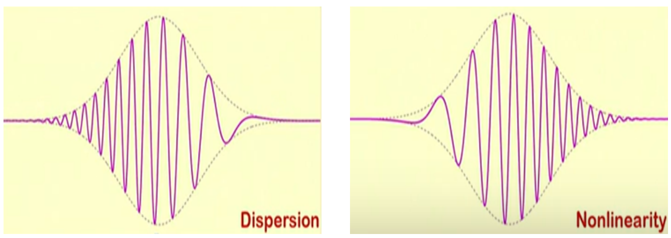
(b) Two solitons formed by quickly flipping two pendulums in a weakly coupled pendulums system

Fig. 1: Mechanical systems where solitons exist

2. Physics of solitons in a nonlinear optical medium

Consider an EM wave packet that is travelling inside an optical material. This single wave packet is a superposition of several frequencies if we decompose the wave packet in to frequencies in Fourier space. It is a fact that not all frequencies are treated the same way by the material. If we take optical dispersion for instance, it results in higher frequencies getting extinguished easily compared to the lower frequency components of the wave packet. This leads to broadening of our cute lil wave packet as it travels through the medium. However, if our crystal has strong nonlinear properties (i,e) $\chi^{(3)}$ coefficient, it leads to something called Kerr effect which in turn causes a nonlinear phenomenon called "Self-focusing" of light.

The process of self focusing creates more higher frequency in our wave packet and broadens the frequency spectrum. This in turn leads to sharpening of our wave packet in time domain and causes it to squeeze itself into a smaller wave packet. And if these two effects balance each other perfectly, then we will have a stable soliton moving in our nonlinear medium!



(a) Dispersion leading to broadening of the wavepacket in time domain

(b) Nonlinearity leading to narrowing of the wavepacket in time domain

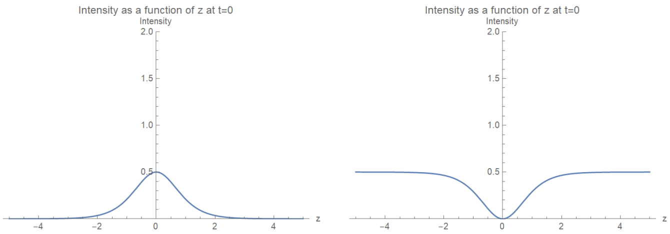
Fig. 2: Two optical effects on a wavepacket of light moving in a nonlinear medium

The existence of solitons in nonlinear medium can be shown analytically by considering nonlinear RI in the Maxwell's equations and deriving the dimensionless PDE of E-field amplitude u as:

$$i\frac{\partial u}{\partial z'} - \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + N^2|u|^2 u = 0$$

where z' is the dimensionless distance (chosen by appropriate normalisation) along direction of propagation, $\tau = \frac{t}{T_0} - \frac{z}{T_0 v_g}$, T_0 is another scaling factor with dimension of t, v_g being the group velocity of the wave packet and N is the order of solitons.

The double derivative term encapsulates the effect of normal dispersion and the $|u|^2u$ term, the nonlinear Kerr effect. If the dispersion in medium is anomalous, the double derivative term will have positive sign. It can be seen that for first order soliton equation (N=1), we have $sech(\tau)e^{iz'/2}$ as a feasible solution for anomalous, and $tanh(\tau)e^{iz'/2}$ for normal dispersion. This corresponds to a bright, and dark soliton respectively whose intensity plot has been shown in Fig.3. So the wave packets, if have the form of a sech or tanh distribution, will propagate in the medium immune to effects of decay or dying out due to perturbations.



(a) Spacial distribution of intensity of a bright soliton (b) Spacial distribution of intensity of a dark soliton

Fig. 3: Two types of solitons existing in our nonlinear medium depending on the type of dispersion

Thus, we also get to know from the analytical solutions that a bright soliton can only exist in a medium if it has anomalous dispersion while a dark soliton can exist only when the medium has normal dispersion. Nevertheless, due to nonlinear effects, optical solitons can exist in our materials and they are extremely stable since they exhibit "order" in the nonlinear equations from which they arise. That also explains why it cant be regarded exactly as a wave despite looking like one.

3. Where are optical solitons today?

Optical solitons were first experimentally produced by Mollenauer in 1980. Using silica glass fiber having characteristic negative group velocity dispersion, they made solitons and used it for sending information and extending its application in telecommunication due to its stable transfer of energy. Today, they are realised in many other ways like the ones below.

1. **Solitons using laser setup and reflecting mirrors:** This setup uses a nonlinear material such as Titanium Saffire that has a positive dispersion. To bring in anomalous dispersion, they add two reflecting mirrors that are coated with thin films that cause negative dispersion at each round trip, and thus lead to bright solitons. These laser setups can compress solitons by increasing the laser light intensity as long as the sample doesn't get damaged with the sudden high power generated by such compression.
2. **Solitons in semiconductors:** Recent advancement in nanotechnology has made it possible to realise solitons in photonic crystals . They have compressed the soliton pulse to orders of 1ps by cleverly engineering the photonic crystals to match with the desired dispersion relation of light and using sensitive frequency dependent electrical gates to read these short pulses.

References

[1] Cundiff, Steven Soto-Crespo, Jose Akhmediev, Nail. (2002). *Experimental Evidence for Soliton Explosions.* Physical review letters. 88. 073903. 10.1103/PhysRevLett.88.073903.

[2] Blanco-Redondo.A, Husko.C, Eades.D, Zhang.Y, Li.J. *Observation of soliton compression in silicon photonic crystals.* Nature Communication (2014). https://doi.org/10.1038/ncomms4160

[3] Mollenauer, L. F. and Stolen, R. H. and Gordon, J. P. *Experimental Observation of Picosecond Pulse Narrowing and Solitons in Optical Fibers.* American Physical Society(1980). https://link.aps.org/doi/10.1103/PhysRevLett.45.1095