

$$J(j,k) = \begin{pmatrix} \frac{\partial F_j}{\partial u_k^{n+1}} & \frac{\partial F_j}{\partial u_{k+1}^{n+1}} & \frac{\partial F_j}{\partial u_{k+1}^{n+1}} \\ \frac{\partial F_{j+1}}{\partial u_{k-1}^{n+1}} & \frac{\partial F_{j+1}}{\partial u_k^{n+1}} & \frac{\partial F_{j+1}}{\partial u_{k+1}^{n+1}} \\ \dots & \dots & \dots \end{pmatrix}$$

$$\frac{\partial F_j(u_k^{n+1})}{\partial u_k^{n+1}} = \frac{\partial}{\partial u_k^{n+1}} \left(u_k^{n+1} - u_k^n - \nu (u_{k+1}^{n+1} - 2u_k^{n+1} + u_{k-1}^{n+1}) \right)$$

$$= \cancel{\frac{\partial u_k^{n+1}}{\partial u_k^{n+1}}} \cdot 1 - 0 - \nu \frac{\Delta t}{\Delta x^2} (0 - 2 + 0) = 1 - 2\nu$$

$$\boxed{\frac{\partial F_j(u_k^{n+1})}{\partial u_k^{n+1}} = 1 - 2\nu \frac{\Delta t}{\Delta x^2}}$$

$$\boxed{\frac{\partial F_j(u_{k+1}^{n+1})}{\partial u_{k+1}^{n+1}} = \frac{\partial}{\partial u_{k+1}^{n+1}} (\dots) = 0 - 0 - \nu (1 - 0 + 0) \frac{\Delta t}{\Delta x^2} = -\nu \frac{\Delta t}{\Delta x^2}}$$

$$\frac{\partial F_j(u_{k-1}^{n+1})}{\partial u_{k-1}^{n+1}} = 0 - 0 - \nu (0 - 0 + 1) \frac{\Delta t}{\Delta x^2} = -\nu \frac{\Delta t}{\Delta x^2}$$

$$\boxed{\frac{\partial F_j(u_{k-1}^{n+1})}{\partial u_{k-1}^{n+1}} = -\nu \frac{\Delta t}{\Delta x^2}}$$

$$\Rightarrow J(j,k) = \begin{pmatrix} 1 - 2\nu \frac{\Delta t}{\Delta x^2} & -\nu \frac{\Delta t}{\Delta x^2} & 0 \\ -\nu \frac{\Delta t}{\Delta x^2} & 1 - 2\nu \frac{\Delta t}{\Delta x^2} & -\nu \frac{\Delta t}{\Delta x^2} \\ 0 & -\nu \frac{\Delta t}{\Delta x^2} & 1 - 2\nu \frac{\Delta t}{\Delta x^2} \end{pmatrix}$$