Assignment 6

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Download all latex-tikz codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment6

1 Problem

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n (≥ 2) from a distribution having the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}\exp(-\frac{x}{\theta}) & x > 0, \\ 0, & \text{otherwise,} \end{cases} \tag{1.0.1}$$

where $\theta \in (0, \infty)$. Let $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $T = \sum_{i=1}^{n} X_i$. Then $E(X_{(1)}|T)$ equals ....

- $\dfrac{T}{n^2}$
- $\dfrac{T}{n}$
- $\dfrac{(n+1)T}{2n}$
- $\dfrac{(n+1)^2T}{4n^2}$

2 Solution

For $n = 2$,

$$E(X_{(1)}|T) = \int_{-\infty}^{\infty} x f_{X_{(1)}}(x|T)dx \tag{2.0.1}$$

$$= \int_{-\infty}^{\infty} x \frac{f_{X_1,X_2}(x, T-x) + f_{X_1,X_2}(T-x, x)}{2f_{X_1+X_2}(T)}dx \tag{2.0.2}$$

$$= \int_0^T x \frac{\exp(-\frac{x}{\theta})\exp(-\frac{T-x}{\theta})}{\theta^2(\frac{1}{\theta^2}T\exp(-\frac{T}{\theta}))}dx \tag{2.0.3}$$

$$= \int_0^T x\frac{1}{T}dx \tag{2.0.4}$$

$$= \frac{1}{T}\int_0^T xdx \tag{2.0.5}$$

$$= \frac{T}{2} \tag{2.0.6}$$

The given key is $\dfrac{T}{n^2} = \dfrac{T}{4}$