Assignment 4

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Download all python codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment4/Codes

and latex-tikz codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment4

1 Problem

GATE 2014 (CS-SET 3), Q.48 (CS/IT section)

Let S be a sample space and two mutually exclusive events A and B be such that $A + B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is

2 Solution

$$\Pr(A + B) = 1 \tag{2.0.1}$$

$$\Pr(A) + \Pr(B) = 1 \tag{2.0.2}$$

$$\Pr(A)\Pr(B) = \Pr(A)(1 - \Pr(A)) \tag{2.0.3}$$

$$= \Pr(A) - (\Pr(A))^2 \tag{2.0.4}$$

$$= \frac{1}{4} - \left(\Pr(A) - \frac{1}{2}\right)^2 \tag{2.0.5}$$

$$\leq \frac{1}{4} \tag{2.0.6}$$

$$\Pr(A) = \Pr(B) = \frac{1}{2} \Rightarrow \Pr(A)\Pr(B) = \frac{1}{4} \tag{2.0.7}$$

$$\therefore \max(\Pr(A)\Pr(B)) = \frac{1}{4} \tag{2.0.8}$$