

Assignment 5

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Download all python codes from

<https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment5/Codes>

and latex-tikz codes from

<https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment5>

1 PROBLEM (GATE 2019 ST, Q43 STATISTICS SECTION)

Let X be a random variable with uniform distribution on the interval $(-1, 1)$ and $Y = (X + 1)^2$. Then the probability density function $f(y)$ of Y , over the interval $(0, 4)$, is

- 1) $\frac{3\sqrt{y}}{16}$
- 2) $\frac{1}{4\sqrt{y}}$
- 3) $\frac{1}{6\sqrt{y}}$
- 4) $\frac{1}{\sqrt{y}}$

2 SOLUTION

We know that, since $Y = (X + 1)^2$, for $y < 0$,

$$F_Y(y) = 0 \quad (2.0.1)$$

Therefore, for $y \geq 0$,

$$F_Y(y) = \Pr((x + 1)^2 \leq y) \quad (2.0.2)$$

$$= \Pr(-\sqrt{y} - 1 \leq x \leq \sqrt{y} - 1) \quad (2.0.3)$$

$$= \Pr(-\sqrt{y} - 1 \leq x \leq \sqrt{y} - 1) \quad (2.0.4)$$

$$= F_X(\sqrt{y} - 1) - F_X(-\sqrt{y} - 1) \quad (2.0.5)$$

Since X is a uniform random variable in $(-1, 1)$,

$$f_X(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.6)$$

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{x}{2} + \frac{1}{2} & -1 < x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2.0.7)$$

Using 2.0.7 in 2.0.5, and using the fact that

$$-\sqrt{y} - 1 \leq -1 \quad \forall y \geq 0, \quad (2.0.8)$$

we get

$$F_Y(y) = \begin{cases} F_X(\sqrt{y} - 1) - 0 & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (2.0.9)$$

$$= \begin{cases} 0 & y < 0 \\ \frac{\sqrt{y}}{2} & 0 \leq y \leq 4 \\ 1 & y > 4 \end{cases} \quad (2.0.10)$$

Therefore,

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}} & 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.11)$$

Therefore, **option 2** is correct. Fig. 4 shows a theoretical vs simulated plot of the PDF of random variable Y .

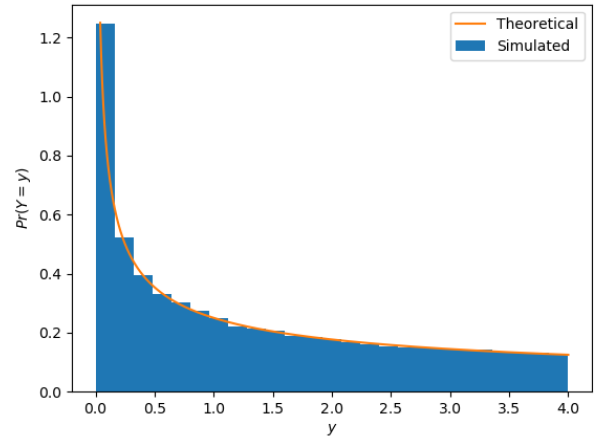


Fig. 4: The PDF of Y