

Assignment 4

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Download all python codes from

<https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment4/Codes>

and latex-tikz codes from

<https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment4>

1 PROBLEM

GATE 2014 (CS-SET 3), Q.48 (CS/IT section)

Let S be a sample space and two mutually exclusive events A and B be such that $A + B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is

2 SOLUTION

Given that A and B are mutually exclusive events.

$$\Pr(A + B) = 1 \quad (2.0.1)$$

$$\Pr(A) + \Pr(B) = 1 \quad (2.0.2)$$

$$\Pr(A) \Pr(B) = \Pr(A) - (\Pr(A))^2 \quad (2.0.3)$$

$$= \frac{1}{4} - \left(\Pr(A) - \frac{1}{2} \right)^2 \quad (2.0.4)$$

$$\leq \frac{1}{4} \quad (2.0.5)$$

When $\Pr(A) = \Pr(B) = \frac{1}{2}$, $\Pr(A) \Pr(B) = \frac{1}{4}$.

Therefore, maximum value of $\Pr(A) \Pr(B)$ is $\frac{1}{4}$.