Assignment 5

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Download all python codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment5/Codes

and latex-tikz codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment5

1 Problem (Gate 2019 ST, Q43 Statistics Section)

Let X be a random variable with uniform distribution on the interval $(-1, 1)$ and $Y = (X+1)^2$. Then the probability density function $f(y)$ of Y, over the interval (0,4), is

1) $\frac{3\sqrt{y}}{16}$
2) $\frac{1}{4\sqrt{y}}$
3) $\frac{1}{6\sqrt{y}}$
4) $\frac{1}{\sqrt{y}}$

2 Solution

We know that, since $Y = (X+1)^2$,

$$F_Y(y) = 0 \ \forall \ y < 0 \tag{2.0.1}$$

Therefore, for $y \geq 0$,

$$\begin{aligned} F_Y(y) &= \Pr\left((x+1)^2 \leq y\right) && (2.0.2)\\ &= \Pr\left(-\sqrt{y} - 1 \leq x \leq \sqrt{y} - 1\right) && (2.0.3)\\ &= \Pr\left(-\sqrt{y} - 1 \leq x \leq \sqrt{y} - 1\right) && (2.0.4)\\ &= F_X(\sqrt{y} - 1) - F_X(-\sqrt{y} - 1) && (2.0.5) \end{aligned}$$

Since X is a uniform random variable in $(-1, 1)$,

$$f_X(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{x}{2} + \frac{1}{2} & -1 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.7}$$

Using (2.0.7) in (2.0.5), and using the fact that

$$-\sqrt{y} - 1 \leq -1 \ \forall \ y \geq 0, \tag{2.0.8}$$

we get

$$F_Y(y) = \begin{cases} F_X(\sqrt{y} - 1) - 0 & y \geq 0 \\ 0 & y < 0 \end{cases} \tag{2.0.9}$$

$$= \begin{cases} 0 & y < 0 \\ \frac{\sqrt{y}}{2} & 0 \leq y \leq 4 \\ 1 & y > 4 \end{cases} \tag{2.0.10}$$

Therefore,

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}} & 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.11}$$

Therefore, **option 2** is correct. Fig. 4 shows a theoretical vs simulated plot of the PDF of random variable Y.

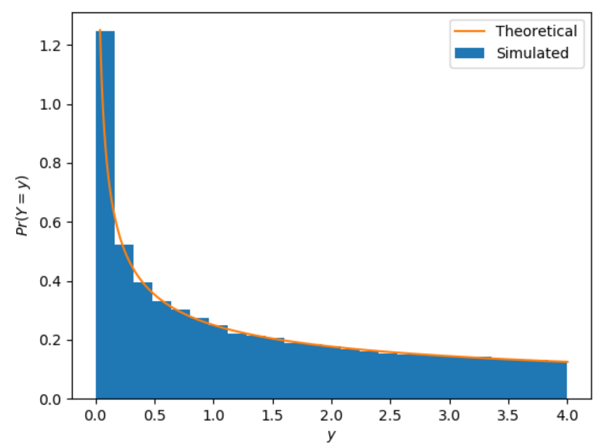

Fig. 4: The PDF of Y