Assignment 6

Gautham Bellamkonda - CS20BTECH11017

Download all python codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment6/Codes

and latex-tikz codes from

https://github.com/GauthamBellamkonda/AI1103/tree/main/Assignment6

1 Problem

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n (≥ 2) from a distribution having the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}\exp(-\frac{x}{\theta}) & x > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{1.0.1}$$

where $\theta \in (0, \infty)$. Let $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $T = \sum_{i=1}^{n} X_i$. Then $E(X_{(1)}|T)$ equals .....

2 Solution

For $n = 2$,

$$E(X_{(1)}|T) = \int_{-\infty}^{\infty} x f_{X_{(1)}}(x|T)dx \tag{2.0.1}$$

$$= \int_{-\infty}^{\infty} x\frac{f_{X_1,X_2}(x, T-x) + f_{X_1,X_2}(T-x, x)}{2f_{X_1+X_2}(T)}dx \tag{2.0.2}$$

$$= \int_{0}^{T} x\frac{\exp(-\frac{x}{\theta})\exp(-\frac{T-x}{\theta})}{\theta^2(\frac{1}{\theta^2}T\exp(-\frac{T}{\theta}))}dx \tag{2.0.3}$$

$$= \int_{0}^{T} x\frac{1}{T}dx \tag{2.0.4}$$

$$= \frac{1}{T}\int_{0}^{T} xdx \tag{2.0.5}$$

$$= \frac{T}{2} \tag{2.0.6}$$