Control Systems Problem 11

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Problem

A system is described by the following differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = 1 \tag{2.1}$$

with the initial conditions x(0) = 1, x'(0) = -1. Show a block diagram of the system, giving its transfer function and all pertinent inputs and outputs. (Hint: the initial conditions will show up as added inputs to an effective system with zero initial conditions.)

Solution

Consider the following equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = r(t) \tag{3.1}$$

where r(t)=1.

On applying laplace transform on both sides we get

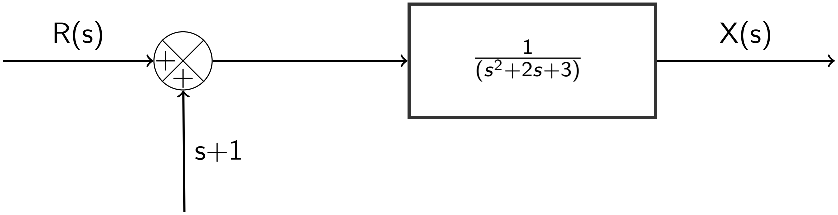
$$\begin{aligned} s^2X(s) - s + 1 + 2sX(s) - 2 + 3X(s) &= R(s) \\ (s^2 + 2s + 3)X(s) - s - 1 &= R(s) \\ X(s) &= \frac{R(s)}{(s^2 + 2s + 3)} + \frac{s+1}{(s^2 + 2s + 3)} \end{aligned} \tag{3.2}$$

where R(s) $= \frac{1}{s}$

Block Diagram

$$X(s) = \frac{R(s)}{(s^2 + 2s + 3)} + \frac{s + 1}{(s^2 + 2s + 3)} \tag{4.1}$$

Here, s+1 is due to the initial conditions and are getting added to the input R(s). So, the block diagram is



Plots

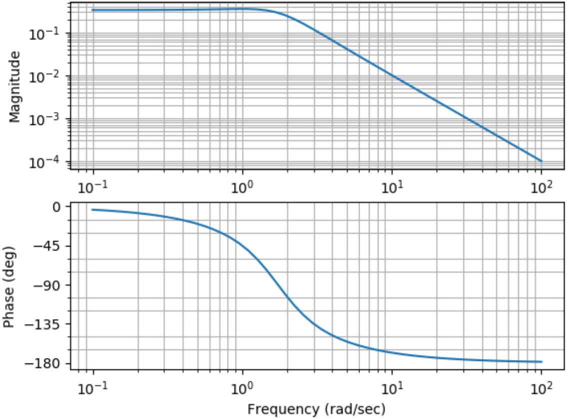


Figure: Bode Plot of Transfer Function

Plots

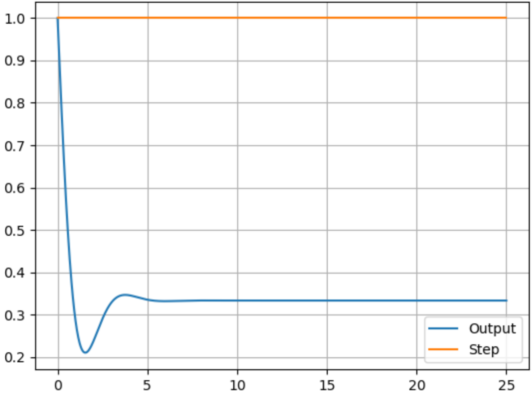


Figure: Output Response