Assignment-2

1) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

(a)

$x(n) = x(n-1)+5 \rightarrow ①$

sub $x(n-1)$ in ①

$x(n) = (x(n-2)+1)+5$

$= x(n-2)+6 \rightarrow ②$

sub $x(n-2)$ in ②

$x(n) = (x(n-3)+1)+6$

$= x(n-3)+7 \rightarrow ③$

$x(k) = x(n-k)+(n+4)$

$n-k = 0$

$n = k$

$x(n-n)+(n+4)$

$= x(0)+n+4$

$= 1+n+4 = n+5$

$\Rightarrow O(n) \rightarrow$ linear

$x(n) = x(n-1)+1$

$x(n-1) = x(n-1-1)+1$

$x(n-2)+1$

$x(n-2) \cdot x(n-2-1)+1$

$= x(n-3)+1$

(b) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

$x(n) = 3x(n-1) \rightarrow (1)$

Sub $x(n-1)$ in (1)

$x(n) = 3[3x(n-2)]$

$= 9x(n-2) \rightarrow (2)$

$x(n) = 9 \cdot 3x(n-3)$

$= 27x(n-3) \rightarrow (3)$

$x(k) = 1k \cdot 3x(n-k)$

$n - k = 0$

$n = 0$

$n \cdot 3x(n-n)$

$n3 \cdot 0 = n$

$= O(n) \rightarrow$ linear

$x(n) = 3x(n-1)$

$x(n-1) = 3x(n-1-1)$

$= 3x(n-2)$

$x(n-2) \cdot 3x(n-2-1)$

$x(n-2) \cdot 3x(n-2-1)$

$\cdot 3x(n-3)$

(2) (a) $T(n) = T(n/2) + 1 \rightarrow n \cdot 2^k$

$T(n) = T(n/2) + 1$

$T(n) = (T(n/4) + 1) + 1$

$= T(n/4) + 2$

$T(n) = T(n/8) + 1 + 2$

$= T(n/8) + 3$

$T(n) = T(n/2^k) + k$

$n/2^k = 1$

$2^k = n$

$k = \log_2 n$

$T(n) = \log_2 n + 1$

(b) $T(n) = T(n/3) + T(2n/3) + cn$

$T(n) = (\log_2 (n/3) + 1) + [\log (2n/3) + 1] + cn$

$= \log_2 (n/3) + \log_2 (2n/3) + 2 + cn$

$= \log_2 n + \log_2 3 + \log_2 2 + \log_2 (n/3) + 2 + cn$

$= \log_2 n - \log_2 3 + \log_2 n - \log_2 3 + 2 + cn$

$T(n) = 2 \log_2 n - 2 \log_2 3 + 3 + cn$

$T(n) = O(n)$

3) $F(n) = 2n^2 + 5$

$g(n) = 7n$

$= (2+5)n^2$

$= 7n^2$

$\Rightarrow g(n) = 7n$

$f(n) \geq g(n)$

$\Rightarrow F(n) = \Omega\, g(n)$

Hence proved

4) Fun Towers of Hanoi (n, source, destination, aux);

if n == 1:
move disk from source to destination

else:
Towers of Hanoi (n-1, source, aux, destination)
move disk from source to destination
Towers of Hanoi (n-1, aux, destination, source)

$\Rightarrow T(n) = 2T(n-1) + 1$

$T(n-1) \cdot 2T(n-2) + 1$

$T(n) = 2[2T(n-2) + 1] + 1$

$= 4T(n-2) + 3$

$T(n) = 2^2 T(n-2) - (2^2 - 1)$

$T(n) = 2^{n-1} + [2^n - 1 - 1]$

$= 2^n - 1$

$T(n) = O(2^n)$

5) (a) $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2$

$f(n) = n^2$

$\Rightarrow n^{\log_2 3}$

$f(n) \geq n^{\log_2 3}$

$T(n) = \theta(f(n))$

$= \theta(n^2)$.

(b) $T(n) = T(n/2)/2^n$

master Theorem can't be applied.