W2

March 27, 2020

Worksheet 2 Gradient and Hessian by Prof J.Morlier February 2020

#import Base.print_matrix #import Pkg; Pkg.add("SymPy")

[191]:
```
using LinearAlgebra, SparseArrays, SuiteSparse, Random
using SymPy
import Base.print_matrix
```

[192]:
```
versioninfo()
```

```
Julia Version 1.3.1
Commit 2d5741174c (2019-12-30 21:36 UTC)
Platform Info:
  OS: macOS (x86_64-apple-darwin18.6.0)
  CPU: Intel(R) Core(TM) i5-8259U CPU @ 2.30GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.1 (ORCJIT, skylake)
```

#1

A 2x2 Recap

[193]:
```
entries = @syms x1 x2  real=true
```

[193]: (x1, x2)

[194]:
```
J=2*x1^3 +3*x2^2 +3*x2*x1^2 -24*x2
```

[194]:

$$2x_1^3 + 3x_1^2x_2 + 3x_2^2 - 24x_2$$

```
Gradient computing
```

[195]:
```
dJdx1=diff(J,x1)  ;

dJdx2=diff(J,x2) ;

dJ=[dJdx1; dJdx2]
```

[195]:

$$\begin{bmatrix} 6x_1^2 + 6x_1x_2 \\ 3x_1^2 + 6x_2 - 24 \end{bmatrix}$$

Hessian computing

[196]:
```
Hdx11=diff(dJdx1,x1);
```

[197]:
```
Hdx12=diff(dJdx2,x1);
```

[198]:
```
Hdx21=diff(dJdx2,x1);
```

[199]:
```
Hdx22=diff(dJdx2,x2);
```

[200]:
```
H=[Hdx11 Hdx12; Hdx21 Hdx22]
```

[200]:

$$\begin{bmatrix} 12x_1 + 6x_2 & 6x_1 \\ 6x_1 & 6 \end{bmatrix}$$

#2

[202]:
```
J=(x1^2 +x2^2 -1)^2 + (x2^2 -1)^2
```

[202]:

$$\left(x_2^2 - 1\right)^2 + \left(x_1^2 + x_2^2 - 1\right)^2$$

[238]:
```
function gradient2(J,x1,x2)

dJdx1=diff(J,x1)  ;

dJdx2=diff(J,x2) ;
dJ=[dJdx1; dJdx2]
return dJ
end
```

[238]: gradient2 (generic function with 1 method)

[239]:
```
function hessian2(dJ,x1,x2)

Hdx11=diff(dJ[1],x1);
Hdx12=diff(dJ[1],x2);

Hdx21=diff(dJ[2],x1);

Hdx22=diff(dJ[2],x2);
H=[Hdx11 Hdx12; Hdx21 Hdx22]
return H
end
```

[239]: hessian2 (generic function with 1 method)

[240]:
```
dJ=gradient2(J,x1,x2)
```

[240]:

$$\left[\begin{array}{c} -\frac{2x_1(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} + \frac{1}{x_1^2+x_2^2+1} \\ -\frac{2x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} + \frac{1}{x_1^2+x_2^2+1} \end{array}\right]$$

[241]:
```
H=hessian2(dJ,x1,x2)
```

[241]:

$$\left[\begin{array}{cc} \frac{8x_1^2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{4x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} & \frac{8x_1x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{2x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2x_2}{\left(x_1^2+x_2^2+1\right)^2} \\ \frac{8x_1x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{2x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2x_2}{\left(x_1^2+x_2^2+1\right)^2} & \frac{8x_2^2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{4x_2}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} \end{array}\right]$$

[242]:
```
#3
```

[243]:
```
J=-2*x1^2 +x1*x2^2 +4*x1^4
```

[243]:

$$4x_1^4 - 2x_1^2 + x_1x_2^2$$

[244]:
```
dJ=gradient2(J,x1,x2)
```

[244]:

$$\left[\begin{array}{c} 16x_1^3 - 4x_1 + x_2^2 \\ 2x_1x_2 \end{array}\right]$$

[245]:
```
H=hessian2(dJ,x1,x2)
```

[245]:

$$\left[\begin{array}{cc} 48x_1^2 - 4 & 2x_2 \\ 2x_2 & 2x_1 \end{array}\right]$$

[246]:
```
#4
```

[247]:
```
J=(x1+x2)/(x1^2 +x2^2 +1)
```

[247]:

$$\frac{x_1 + x_2}{x_1^2 + x_2^2 + 1}$$

[248]:
```
dJ=gradient2(J,x1,x2)
```

[248]:

$$\begin{bmatrix} -\frac{2x_1(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} + \frac{1}{x_1^2+x_2^2+1} \\ -\frac{2x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} + \frac{1}{x_1^2+x_2^2+1} \end{bmatrix}$$

[249]:
```
H=hessian2(dJ,x1,x2)
```

[249]:

$$\begin{bmatrix} \frac{8x_1^2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{4x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} & \frac{8x_1x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{2x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2x_2}{\left(x_1^2+x_2^2+1\right)^2} \\ \frac{8x_1x_2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{2x_1}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2x_2}{\left(x_1^2+x_2^2+1\right)^2} & \frac{8x_2^2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^3} - \frac{4x_2}{\left(x_1^2+x_2^2+1\right)^2} - \frac{2(x_1+x_2)}{\left(x_1^2+x_2^2+1\right)^2} \end{bmatrix}$$