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Design of Functionally Graded Structures Using Topology Optimization

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Abstract. The concept of functionally graded materials (FGMs) is closely related to the concept of topology optimization, which consists in a design method that seeks a continuum optimum material distribution in a design domain. Thus, in this work, topology optimization is applied to design FGM structures considering a minimum compliance criterion. The present approach applies the so-called "continuous topology optimization" formulation where a continuous change of material properties is considered inside the design domain by using the graded finite element concept. A new design is obtained where distribution of the graded material itself is considered in the design domain, and the material properties change in a certain direction according to a specified variation, leading to a structure with asymmetric stiffness properties.

Introduction

Functionally Graded Materials (FGMs) materials are materials that possess continuously graded properties with gradual change in microstructure [1,2]. The materials are made to take advantage of desirable features of its constituent phases. For instance, in a thermal protection system, FGMs take advantage of heat and corrosion resistance typical of ceramics, and mechanical strength and toughness typical of metals.

Topology optimization is a powerful structural optimization method that combines a numerical solution method, usually the Finite Element Method (FEM), with an optimization algorithm to find an optimal material distribution inside a given domain [3]. It is well-known that the optimum topology optimization result consists in a structure with intermediate (or composite) material -- thus its concept is strongly related to the concept of FGM materials, which essentially considers a continuous transition of material properties.

In traditional topology optimization formulations, the design variable is defined in a piecewise fashion in the discretized domain, which means that continuity of the material distribution is not realized between finite elements. However, considering the topology optimization results as an FGM-type material, a more natural way of representing the material distribution emerges by using the concept of the "graded finite elements," which leads to a continuous representation of material properties [4] that are interpolated inside the finite element using the FE shape functions. In fact, recent works [5,6] have suggested that this continuum material distribution alleviates the checkerboard problem [7], an old problem in topology optimization. By means of the continuum model approach, the design of FGM structures can be fully achieved by applying topology optimization because a continuous change of material properties is considered inside the design domain.

The design of FGM structures using topology optimization has been considered in previous works [8,9], which emphasized thermal and thermomechanical applications, including transient analysis, by defining the design variable in a piecewise fashion in the discretized domain. The objective of the present work is to design FGM structures by using the concept of continuum topology optimization [6].

By means of a continuum distribution of the design variable inside the finite element domain, a continuously graded material during the design problem is represented. A new structural design is obtained by considering the distribution of the FGM itself in the design domain, which leads to a structure with asymmetric stiffness properties. As objective function, the traditional formulation for a stiffness design problem is considered initially where the objective is to find the material distribution that minimizes the mean compliance. Examples are given to illustrate the method.

Theoretical Formulation

The objective of topology optimization is to determine holes and connectivities of the structure by adding and removing material in the extended domain which is a large fixed domain that must contain the whole structure to be determined [10]. A main question in topology optimization is how to change the material in a binary form (e.g. from zero to one). The use of a discrete approach would present difficulties in the numerical treatment of the problem due to multiple local minimum. Thus, a material model must be defined to allow the material to assume intermediate property values by defining a function of a continuous parameter (design variable) that determines a mixture of two materials throughout the domain. This provides enough relaxation for the design problem. In this work, the topology optimization implementation considers material models based on the so-called density methods or artificial power law approaches, which will be employed together with a filtering technique to control the mesh dependency

Material Model. The traditional SIMP (Solid Isotropic Material with Penalization) material model [3] states that in each point of the domain, the material property is given by $E^H = \rho^p E_0$ where E^H and E_0 are the Young modulus of the homogenized material and basic material that will be distributed in the domain, respectively, ρ is a pseudo-density describing the amount of material in each point of the domain which can assume values between 0 and 1, and p is a penalization factor to recover the discrete nature of the design. For ρ equal to 0 the material is equal to void and for ρ equal to 1 the material is equal to solid material.

Now, consider the objective of designing a structure in an FGM domain, that is, a domain where the properties change in a certain direction according to a specified law. In this case, the property E_0 considered above is not constant along the domain but it depends on the position **x**. Thus, the previous SIMP model can be recast in the following functional form,

$$E^H = \rho^p E_0 e^{\alpha x + \beta y}, \tag{1}$$

where α and β are coefficients that define the change of material property in the domain, and x and y are the position Cartesian coordinates. This is a common situation when dealing with FGMs, which is expected to result in non-symmetric designs. This novel material model is called FGM-SIMP. Material functions other than the exponential variation of Eq. (11) can also be considered as discussed by Silva *et al.* [11].

Considering the discretized domain using graded finite elements [4], the concept of the continuum distribution of design variable based on the CAMD (continuous approximation of material distribution) method [6,12] is considered. In this method, the pseudo-density inside each finite element is given by

$$\rho(\mathbf{x}) = \sum_{I=1}^{n_d} \rho_I N_I(\mathbf{x}), \tag{2}$$

where ρ_I is the nodal design variable, N_I is the finite element shape function and n_d is the number of nodes in each element. Then, using Eq. (1), the material property (Young´s modulus) is calculated inside each finite element. This formulation allows a continuous distribution of material along the design domain, instead of the traditional piecewise material distribution typical of previous topology optimization formulations [3].

Design Problem Formulation. As objective function, the traditional formulation for stiffness design problem is considered where the objective is to find the material distribution that minimizes the mean compliance (C_{mean}) given by [3]. Considering the FEM formulation for the discretized domain, a general form of the topology optimization problem for stiffness design can be defined as

$$\begin{aligned} &\text{Min } C_{mean} = \boldsymbol{U}^t\boldsymbol{F} \\ &\rho_I \text{ (for each node)} \\ &\text{Such that } \ \mathbf{KU=F};\ \sum_{I=1}^{N}\rho_I \le V_{des};\quad 0 \le \rho_I \le 1 \end{aligned} \tag{3}$$

together with a filtering technique that allows us to control the mesh-dependency of the problem [3, 13].

Sensitivity Calculation. The sensitivity calculation for the mean compliance design problem defined above is well-known in the literature. Here, the formulation is extended considering the concept of the continuous approximation of material distribution.

Assuming that the external load is independent of design, the sensitivity of mean compliance objective function C_{mean} in relation to a design variable ρ_I is given by [10],

$$\frac{\partial C_{mean}}{\partial \rho_I} = -\int_{\Omega} \boldsymbol{\varepsilon}^T \frac{\partial \mathbf{E}^H}{\partial \rho_I} \boldsymbol{\varepsilon} dV\ . \tag{4}$$

Considering the continuous material distribution we can write $\frac{\partial \mathbf{E}^H}{\partial \rho_I} = \frac{\partial \mathbf{E}^H}{\partial \rho} N_j(\mathbf{x})$, and thus the derivative $\frac{\partial \mathbf{E}^H}{\partial \rho}$ can be easily calculated by considering the material model formulation described by Eq. (1). Therefore

$$\frac{\partial C_{mean}}{\partial \rho_I} = -\int_{\Omega} \boldsymbol{\varepsilon}^T \frac{\partial \mathbf{E}^H}{\partial \rho} N_I \boldsymbol{\varepsilon} dV\ . \tag{5}$$

By discretizing the domain into finite elements, the above integral will include all m_I elements associated with I-th node [6], thus,

$$\frac{\partial C_{mean}}{\partial \rho_I} = -\sum_{e=1}^{m_I} \int_{\Omega_e} \boldsymbol{\varepsilon}^T \frac{\partial \mathbf{E}^H}{\partial \rho} N_I(\mathbf{x}) \boldsymbol{\varepsilon} dV_e\ . \tag{6}$$

where e is the element number.

Numerical Results

A flow chart of the optimization algorithm is shown in Fig. 1. The algorithm was implemented using C language. Four node bilinear elements considering plane stress formulation are used in the finite element formulation.

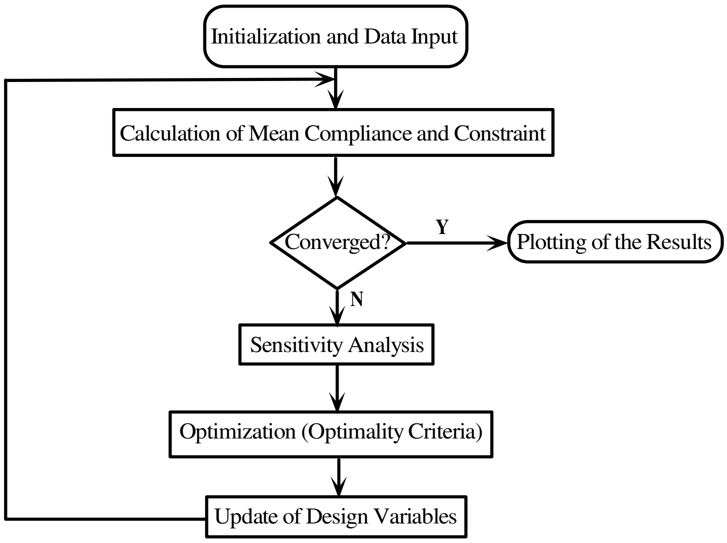

Fig. 1: Flow Chart of Optimization Procedure.

The traditional stiffness design problem is considered, and the *optimality criteria method* is applied to solve the optimization problem because it is very efficient. This method is well-described in [3]. The filtering technique applied is also presented in detail in [13].

The given example is related to the structural design in an FGM domain, that is, a domain where the properties change in a certain direction according to a specified variation. This design, called graded structural design, is obtained by distributing the FGM itself in the design domain. The idea is to obtain a structure with asymmetric stiffness properties. The distribution of the FGM is considered according to Eq. (2). The normalized Young`s modulus E_0 is equal to 10. The design domain considered, with corresponding applied distributed load, is shown in Fig. 2(a). A finite element mesh of 50 X 50 elements was considered. A volume material constraint of 30% and a filter with radius equal to 1.5 were applied to all the examples. The volume material constraint is the amount of material allowed for the final structural topology. It is a percentage of the amount of material in the actual design domain (considered full of material). The filter radius essentially defines which finite elements in the neighborhood of a finite element will be taken into account for the filter calculation for this specific finite element [13]. Figure 2(b) shows the resulting topology using the SIMP method and considering no material gradation.

Figs. 3(a) and 3(b) show the results obtained considering material variation in the x direction for both positive and negative values of α (see Eq. (1)). By comparing with the homogeneous result of Fig. 2(b) (where no material variation is considered), one notices that for a positive value of α, the method enlarges the thickness of the structure close to the supports where the material has a lower stiffness. For a negative value of α, the opposite trend is obtained, that is, the method tries to increase the material in the front part of the structure where the material has a lower stiffness. In both cases, the final topology is changed as a function of the material gradient. This feature indicates that the ability to control the material variation in the design process offers a convenient tool to change the resulting structural topology.

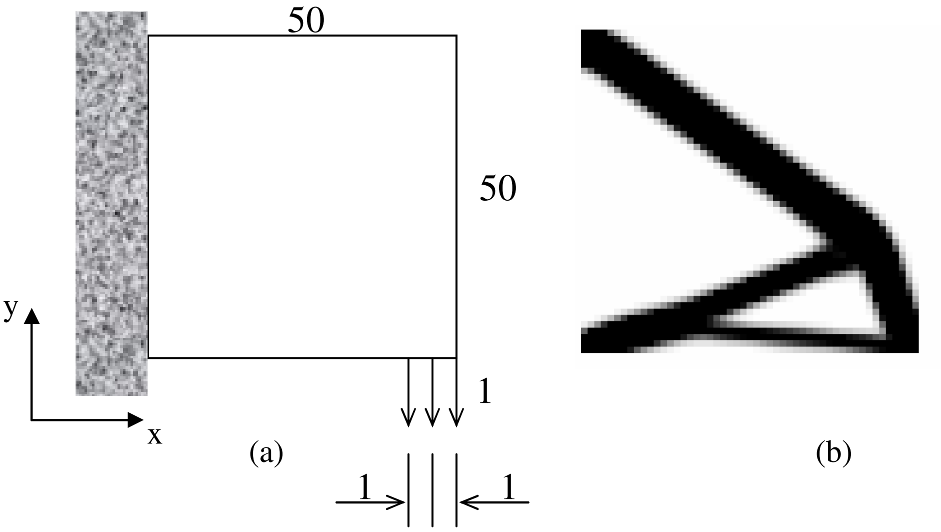

Fig. 2: (a) Initial domain discretized with a 50 x 50 mesh; (b) Topology for comparison obtained considering SIMP model for homogeneous material (α=β=0) with 30% volume constraint and filtering (radius equal to 1.5).

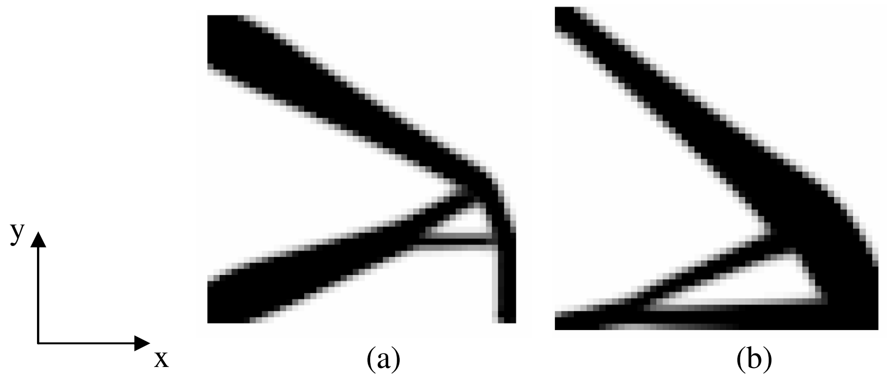

Fig. 3: (a) Topology obtained using the FGM-SIMP material model in the x direction with α α=0.06, and filter (radius equal to 1.5); (b) same, with α=-0.06.

Conclusions

This work shows that continuum topology optimization can be successfully applied to design FGM structures. The material distribution is based on graded finite elements [4] that allow change of material properties inside the design domain in a continuous manner (which is close related to the FGM concept). A new design is obtained where the distribution the FGM material itself is considered in the design domain leading to a structure with asymmetric stiffness properties. Thus, novel types of structures can be obtained by exploring the FGM idea. The length-scale of material gradation (e.g. $1/\alpha$) competes with the geometric length-scale of the design problem and leads to novel structural topologies. As a future work, the design of composite unit cells made of FGMs will be considered using the continuum topology optimization concept.

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References

[1] T. Hirai: *Materials Science and Technology: Processing of Ceramics* (Part 2, R.J.Brook, ed., VCH Verlagsgesell-schaft mbH, Weinheim, Germany, 17B, 1996), p. 292.
[2] S. Suresh and A. Mortensen: *Fundamentals of Functionally Graded Materials*, (IOM Communications, London 1998).
[3] M. P. Bendsøe, O. Sigmund: *Topology Optimization: Theory, Methods and Applications* (Springer-Verlag, Berlin, 2003).
[4] J. H. Kim and G. H. Paulino: ASME Journal of Applied Mechanics Vol. 69 (4) (2002), p. 502.
[5] C. S. Jog, R. B. Haber: Computer Methods in Applied Mechanics and Engineering, Vol. 130 (1996), p. 203.
[6] K. Matsui, K. Terada: International Journal for Numerical Methods in Engineering Vol. 59 (2004), p. 1925.
[7] R. Diaz, O. Sigmund: Structural Optimization, Vol. 10 (1) (1995), p. 40.
[8] S. Turteltaub: Structural and Multidisciplinary Optimization Vol.22 (2001), p. 157.
[9] S. Turteltaub: Computer Methods in Applied Mechanics and Engineering, Vol.191 (2002), p.2283.
[10] M. P. Bendsoe, N. Kikuchi: Computer Methods in Applied Mechanics and Engineering, Vol. 71(2) (1988), p.197.
[11] E.C.N. Silva, G.H. Paulino and M.F.T. Silva: (submitted for journal publication).
[12] S. F. Rahmatalla, C. C. Swan: Structural and Multidisciplinary Optimization, (2004), (in press).
[13] O. Sigmund: Structural and Multidisciplinary Optimization, Vol. 21(2) (2001), p. 120.