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S. Nikbakht, S. Kamarian, M. Shakeri

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Review

A Review on Optimization of Composite Structures Part II: Functionally Graded Materials

S. Nikbakht[c], S. Kamarian[a,b,c*], M. Shakeri[c]

a. Department of Mechanical Engineering, Ilam Branch, Islamic Azad University, Ilam, Iran

b. Young Researchers Club, Ilam Branch, Islamic Azad University, Ilam, Iran

c. Department of Mechanical Engineering, Amirkabir University of Technology, Tehran, Iran

Abstract

Functionally Graded (FG) structures are a novel design through which the material properties vary smoothly and this feature leads these structures to have better mechanical or thermal performances. They are mostly constituted from two or more materials with gradually varying volume fraction distribution. Most of the publications on the optimization of laminated composite structures were investigated in the first part of the authors' review paper [1]. In this research which acts as the second part, the majority of publications on optimization of FG structures are reviewed. In addition to FG beams, plates and shells, various structures such as tubes, implants, rotating disks, sport instruments, etc. are investigated. Furthermore, the key outputs of each publication are represented to make this article an asset source for mechanical engineers since there has not been any comprehensive review article on optimal designs of FG structures in the literature.

Keywords: Optimization; Composite structures; Functionally graded materials

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1. Introduction

In 1980s, a group of materials scientists in Japan introduced a new class of materials called Functionally Graded Materials (FGMs) resistant to high temperatures and suitable for many engineering applications, such as thermal barrier coatings, engine components or rocket nozzles. FGMs are advanced composite materials whose equivalent properties vary gradually along one (or more) direction(s), usually in the thickness direction, to obtain modified response to external loadings. FG structures are used in a variety of engineering applications including aircraft, construction and transportation where strong, stiff and light structures are required. The high specific stiffness and strength of these structures along with their low weight can be mentioned as the advantages of these materials over their homogeneous counterparts. Furthermore, in contrast to conventional composites whose steep shifts in material properties cause the problem of interfacial stresses, the strategy of changing the geometry or the volume fraction distribution gradually through the FG structure avoids this problem. In the simplest form of FGMs, two different material components change gradually from one surface to the other according to a certain function. The most familiar FGM is compositionally graded from a refractory ceramic to a metal (Fig 1).

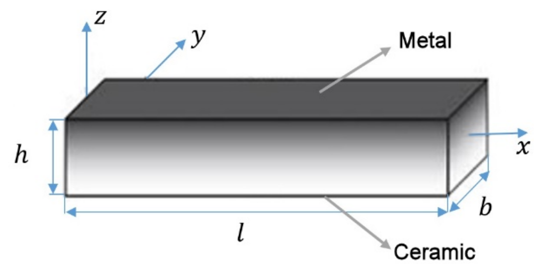

Fig.1. A FGM slab with volume fraction distribution through the thickness direction

The variation in material properties of FGM is examined via two types of functions. The first set of functions are related to the volume fraction distribution of these materials. The second group of functions are associated with the relationship between material gradation and the material properties variation through FGMs.

1.1. Volume fraction distribution functions

The majority of FG structures consist of two phases whose volume fractions vary gradually. Therefore, the functions employed for determining the volume fraction distribution of material constituents of these structures are of significant importance. The most applicable formulas regarding material gradation through FGMs are given below.

Power-law model:

$$V_1 = \left(\frac{1}{2} + \frac{z}{h}\right)^p \quad \text{Eq.1}$$

Where p is the power-law index which defines the level of material inhomogeneity through the FG structure.

Trigonometric model:

$$V_1 = Sin^2\left[\left(\frac{1}{2} + \frac{z}{h}\right)^p\right] \quad \text{Eq.2}$$

Viola-Tornabene three-parameter model:

$$V_1 = \left(\frac{1}{2} - \frac{z}{h} + b\left(\frac{1}{2} + \frac{z}{h}\right)^c\right)^p \quad \text{Eq.3}$$

Viola-Tornabene four-parameter model:

$$V_1 = \left(1 - a\left(\frac{1}{2} - \frac{z}{h}\right) + b\left(\frac{1}{2} + \frac{z}{h}\right)^c\right)^p \quad \text{Eq.4}$$

Where a, b and c are constant parameters which dictate the material propagation in the FG structure.

Four-parameter model:

$$V_1 = C\left[1 - \left(\frac{z}{h}\right) + \alpha\left(\frac{z}{h}\right)^{\beta}\right]^{\gamma} \quad \text{Eq.5}$$

Five-parameter trigonometric model:

$$V_1 = C\left[\frac{1}{2} - \frac{\alpha}{2}\sin\left(\frac{\eta\pi z}{h} + \varphi\right)\right]^{\gamma} \quad \text{Eq.6.a}$$

Five-parameter trigonometric model for 2D FGM:

$$V_1 = \left[1 - \left|\sin\left(\frac{\eta_x\pi x}{L_x} + \varphi_x\right)\sin\left(\frac{\eta_y\pi y}{L_y} + \varphi_y\right)\right|\right]^{\gamma} \quad \text{Eq.6.b}$$

Where $C, \alpha, \beta, \gamma, \eta$ and φ are controlling parameters which determine the material variation pattern through the FG structure. Two other functions for volume fraction distribution of 2D FG plates have been recently introduced by Lieu et al. [2], [3] as follows.

$$V_1 = \left(\frac{x}{a}\right)^{k_x}\left(\frac{y}{b}\right)^{k_y} \quad \text{Eq.7}$$

$$V_1 = (V_{1,x})(V_{1,y}) \quad \text{Eq.8.a}$$

$$V_{1,x} = \begin{cases} \left(\frac{2x}{a}\right)^{k_x}, 0 \le x \le \frac{a}{2} \\ \left(2 - \frac{2x}{a}\right)^{k_x}, \frac{a}{2} \le x \le a \end{cases} \quad \text{Eq.8.b}$$

$$V_{1,y} = \begin{cases} \left(\frac{2y}{b}\right)^{k_y}, 0 \le y \le \frac{b}{2} \\ \left(2 - \frac{2y}{b}\right)^{k_y}, \frac{b}{2} \le y \le b \end{cases} \quad \text{Eq.8.c}$$

Where a and b are the dimensions of FG plates in x and y directions, respectively. k_x and k_y are the constants which define the level of material inhomogeneity in the in x and y directions, respectively.

Sigmoid model:

$$V_1 = \begin{cases} 1 - \frac{1}{2}\left(1 - \frac{2z}{h}\right)^p ; 0 \leq z \leq \frac{h}{2} \\ \frac{1}{2}\left(1 + \frac{2z}{h}\right)^p ; -\frac{h}{2} \leq z \leq 0 \end{cases} \qquad \text{Eq.9}$$

There are a number of volume fraction approximating functions defined for FG structures considering a certain number of control points through them. In other words, according to these models, FGMs are assumed as multi-layered structures and the volume fraction distribution is determined in each layer individually according to the volume fraction and/or the slope of volume fraction gradation at the top and bottom of each layer. These functions are given as follows.

B-Spline Basis Function:

$$V_1(\xi) = \sum_{i=1}^{n} B_{i,p}(\xi) V_{1,i}(z) \qquad \text{Eq.10}$$

Where $V_{1,i}(z)$, p and n are the volume fraction of the first phase at the i^{th} control point, the number of basis function and the polynomial order, respectively. $B_{i,p}(\xi)$ are the B-Spline basis functions defined via different formula and can be found in the literature.

Piece-wise cubic interpolation function:

$$V_1(z) = V_i H_1(z) + S_i H_2(z) + V_{i+1} H_3(z) + S_{i+1} H_4(z); z_i \leq z \leq z_{i+1} \qquad \text{Eq.11}$$

Where V_i and S_i are the volume fraction and slope of volume fraction at the i^{th} control point, respectively. These values along with Hermite basis functions $H_k(z)$ can be found in a study represented by Vel and Pelletier [4].

In all of the above mentioned functions, V_1 is the volume fraction of the first phase and the volume fraction of the second phase can be obtained as

$$V_2(z) = 1 - V_1(z) \qquad \text{Eq.12}$$

It is to be noted that all the above equations are defined in Cartesian coordinate system and it is assumed that the volume fraction of each phase varies through the thickness direction (Fig. 1). These formulations can be calculated for axially FG beams in which the volume fraction of materials vary through longitudinal direction by applying a simple mathematical technique. For FG hollow cylinders, these equations should be defined in cylindrical coordinate system. In order to avoid any redundancy, the authors decided to only mention the functions related to plates in which the material constituents vary through their thickness.

1.2. Material properties variation functions

Regarding the aforementioned volume fraction distribution functions, equivalent material properties of FGMs could be determined by various functions. In this sub-section, a number of the most important formulas in terms of material properties variation in FGMs are introduced. In order to avoid any repetitive information, only the formulas related to the elastic modulus gradation are mentioned here.

Rule of Mixture (ROM):

$$E = E_1V_1 + E_2V_2 \quad \text{Eq.13}$$

Mori-Tanaka Scheme [5]:

$$\frac{K - K_1}{K_2 - K_1} = \frac{V_2}{1 + \frac{3V_1(K_2 - K_1)}{(3K_1 + 4G_1)}} \quad \text{Eq.14.a}$$

$$\frac{G - G_1}{G_2 - G_1} = \frac{V_2}{1 + \frac{V_1(G_2 - G_1)}{(G_1 + f_1)}} \quad \text{Eq.14.b}$$

$$f_1 = \frac{G_1(9K_1 - 8G_1)}{6(K_1 + 2G_1)} \quad \text{Eq.14.c}$$

$$E = \frac{9KG}{3K + G} \quad \text{Eq.14.d}$$

$$\nu = \frac{3K - 2G}{2(3K + G)} \quad \text{Eq.14.e}$$

Where K, G and ν are the local bulk modulus, local shear modulus and Poisson's ratio, respectively.

Exponential function [6]:

$$E(z) = Aexp(Bz) \quad \text{Eq.15}$$

TTO model: The name of this model which was firstly represented by Tamura et al. [7] indicates the initials of the names of the researchers who introduced it. The basic assumption of this model is that the magnitude of stresses and strains in each area of a two-phased material (σ, ε) depends on the stresses and strains each phase undergoes (σ_1, σ_2, ε_1, ε_2) and their volume fractions (V_1, V_2).

$$\sigma = \sigma_1V_1 + \sigma_2V_2 \quad \text{Eq.16.a}$$

$$\varepsilon = \varepsilon_1V_1 + \varepsilon_2V_2 \quad \text{Eq.16.b}$$

$$E = \frac{V_1E_1\left(\frac{q + E_2}{q + E_1}\right) + V_2E_2}{V_1\left(\frac{q + E_2}{q + E_1}\right) + V_2} \quad \text{Eq.16.c}$$

$$q = \frac{\sigma_1 - \sigma_2}{|\varepsilon_1 - \varepsilon_2|}; 0 \le q \le \infty \quad \text{Eq.16.d}$$

Where q is the stress to strain ratio. This parameter was proved to depend on the type of materials and the microstructural interactions between the two phases. TTO model is also capable of determining the yielding strength of FGMs as well as other material properties such as Young's modulus and thermal expansion coefficient.

$$\sigma_Y = \sigma_{Y_1}\left[V_1 + \left(\frac{q+E_1}{q+E_2}\right)\left(\frac{E_2}{E_1}\right)(1-V_1)\right] \qquad \text{Eq.17}$$

Where σ_{Y_1} is the yielding strength of the ductile phase. Fig. 2 clearly demonstrates the yielding strength variation of a FGM constituted of metal (ductile phase) and ceramic (brittle phase).

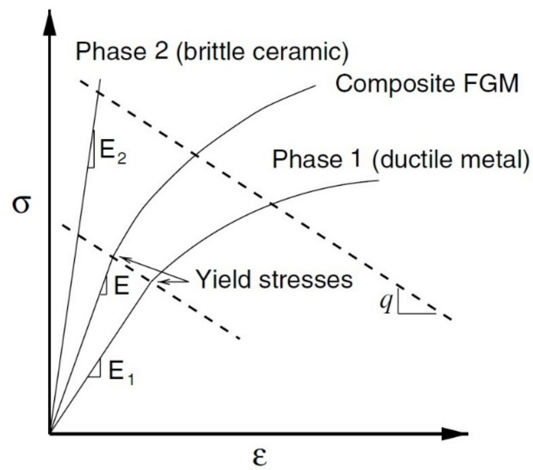

Fig. 2. The stress-strain curves for an arbitrary two phased composite [8]

Bhattacharyya et al. [9] conducted an experimental study and claimed that TTO model outperformed ROM in determining material properties of FGMs. Nikbakht et al. [10] and Komarsofla et al. [11] exploited TTO scheme to obtain the yielding initiation of FG plates and shells, respectively. More detailed information about this model could be found in the mentioned references.

1.3. Motivation

Technological advances in engineering designs motivated many scientists to investigate the mechanical behavior of novel materials such as FGMs under static, dynamic and impact loadings. The huge number of studies in these fields convinced researchers to compile review articles regarding FGMs with different perspectives. Liew et al. [12] presented a review paper in the area of static, dynamic, free vibration and buckling analyses of FG structures with meshless methods. Their main concentration was on Element Free Galerkin (EFG) and reproducing kernel particle methods. The cons and pros of each study were represented and possible future works were suggested in their article as well. A critical review on FG plates was carried out by Jha et al. [13] in which the most publications in the field of vibration and thermo-elastic analyses of these structures were reviewed. As a conclusion, the researchers compared 2D and 3D methods in terms of their accuracy and computational costs. A bulk of publications associated with FG plates and shells subjected to thermal and mechanical loads were gathered in a comprehensive review article represented by Thai and Kim [14]. The main focus of their paper was on the implemented theories. Classical, firs order shear deformation theories, higher order shear deformation theories and 3D elasticity theory employed for analyzing FG structures were also detailed thoroughly in their research. Compiling a review article on FG plates and shells, Gupta and Talha [15] discussed different fabrication processes of FGMs, different adopted theories for static and dynamic analyses of FG structures and the recent application of these novel materials. They also represented a number of suggestions for future studies regarding FGMs. Swaminathan and Sangeetha [16] attempted to gather most of the published researches on thermal analysis of FG structures under various types of thermal loadings. After reviewing the studies regarding static, vibrational and buckling analyses of FG structures, these authors compared different solution methods and mentioned their positive and negative points.

Albeit numerous review studies on mechanical behavior of FG structures, lack of a comprehensive review article on optimization of these materials can be seen in the literature. To this end, in the present work, researches on the optimum design of various types of FG structures are reviewed and the most prominent results of each publication are highlighted. According to the structural classification in this research, firstly the publications related to FG beams are reviewed. Following this, two sections are devoted to publications on optimization of FG plates and shells. Subsequently, FG tubes which are advanced crash structures with vast application in industries are investigated in the next section. Finally, other structures such as implants, disks, sport instruments, etc. made of FGMs are detailed in the last section. For the convenience of the readers, all the abbreviations exploited in this review article are gathered in Table 1. in alphabetical order.

Table 1. The alphabetically ordered abbreviations

Abbreviation	**Stands for**
AEDEA	Adaptive Elicit Differential Evolution Algorithm
AHEFA	Adaptive Hybrid Evolutionary Firefly Algorithm
ANFIS	Adaptive Neuro-Fuzzi Interference System
ANN	Artificial Neural Network
APDL	ANSYS Parametric Design Language
ASA	Adaptive Simulated Annealing
BESO	Bi-directional Evolutionary Structural Optimization
CAMD	Continuous Approximation of Material Distribution
CG	Conjugate Gradient
CNT	Carbon Nanotube
CPSO	Co-evolutionary Particle Swarm Optimization
DAF	Dynamic Amplification Factor
DE	Differential Evolution
DMS	Direct Multi-Search
DMO	Discrete Material Optimization
DNN	Deep Neural Network
DOE	Design of Experiments
DQM	Differential Quadrature Method
EFG	Element Free Galerkin
FA	Firefly Algorithm
FAIPA	Feasible Arc Interior Point Algorithm

FEM	Finite Element Method
FG	Functionally Graded
FGCNT	Functionally Graded Carbon Nanotube
FGF	Functionally Graded Foam
FGFCS	Functionally Graded Foam-filled Cellular Structure
FGFGT	Functionally Graded Foam-filled Graded Thickness
FGFTT	Functionally Graded Foam-filled Tapered Tubes
FGH	Functionally Graded Honeycomb
FGM	Functionally Graded Materials
FGPT	Functionally Graded Piezoelectric Transducers
FGS	Functionally Graded Sandwich
FGT	Functionally Graded Thickness
FGTIT	Functionally Graded Thickness Inversion Tube
FLGFT	Functionally Lateral Graded Foam-filled Tubes
GA	Genetic Algorithm
GDO	Goal Driven Optimization
GLODS	Global and Local Optimization using Direct Search
GS	Golden Section
HAP	Hydroxyapatite
HFT	Heterogeneous Feature Tree
HSDT	Higher order Shear Deformation Theory
ICA	Imperialistic Competitive Algorithm
MBB	Messerschmitt-Bolkow-Blohm
MC	Monte-Carlo
MIGA	Multi-Island Genetic Algorithm
MOGA	Multi-Objective Genetic Algorithm
MOPSO	Multi-Objective Particle Swarm Optimization
mSOS	modified Symbiotic Organisms Search
MWCNT	Multi-Walled Carbon Nanotube
NN	Neural Network
NSGA	Non-dominated Sorting Genetic Algorithm
NURBS	Non-Uniform Rational Basis Spline
PCF	Peak Crushing Force
PRS	Polynomial Response Surface
PSACO	Particle Swarm with Ant Colony Optimization
PSO	Particle Swarm Optimization
PSOPC	Particle Swarm Optimization with Passive Congregation
PSPCACO	Particle Swarm with Passive Congregation and Ant Colony Optimization
PSOSM	Particle Swarm Optimization Sliding Mode
PVDF	Polyvinylidene Fluoride
PZT	Lead Zirconate Titanate
RBDO	Reliability Based Design Optimization
RBF	Radial Basis Function
RCGA	Real Coded Genetic Algorithm
ROM	Rule of Mixture
RSM	Response Surface Methodology
SA	Simulated Annealing
SCM	Simple Cell Mapping
SEA	Specific Energy Absorption
SGO	Social Group Optimization
SLP	Sequential Linear Programming
SMOSA	Suppapitnarm Multi-Objective Simulated Annealing
SOS	Symbiotic Organisms Search
SQP	Sequential Quadratic Programming
SVR	Super Vector Regression
TDFGFT	Two Dimensional Functionally Graded Foam-filled Tube

TSM	Traditional Sliding Mode
UF	Uniform Foam-filled
UFCS	Uniform Foam-filled Cellular Structure
UHF	Uniform Honeycomb Filled

2. Functionally graded beams

Beams are one of the main structures in mechanical engineering that primarily resist loads applied perpendicular to their longitudinal axis. They are traditionally descriptions of civil engineering structural elements. However, many primary and secondary structural elements such as helicopter rotor blades, turbine blades, robot arms and space erectable booms can be assumed as beams in accordance with their geometry. As beams are vastly implemented in industries, a wide range of materials, from homogeneous to composite and FGM, are utilized to construct them. FG beams are divided into two main categories with regard to the direction of material variation. These two categories are referred to as transversely and axially FG beams which can be seen in Fig. 3. Many researchers have investigated optimization of FG beams for different objectives which are mentioned in this section.

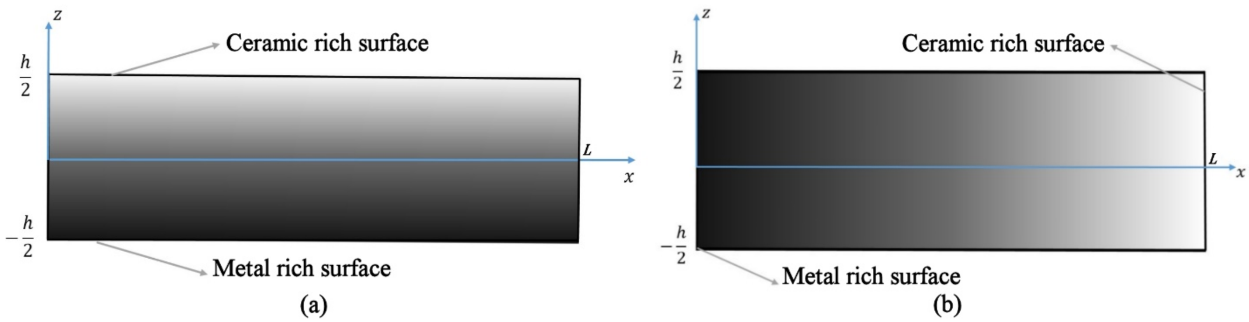

Fig.3. Schematic of FG beams. (a) Transversely FG beam. (b) Axially FG beam.

2.1. *Transversely functionally graded beams*

Goupee and Vel [17] represented an optimization approach combining EFG and Real Coded Genetic Algorithm (RCGA) to reveal the results associated with the fundamental frequency maximization of three different models of FG beams and to show the outperformance of these materials over their monotonic counterparts. They proved that significantly larger amounts of fundamental frequencies could be achieved for beams made up of copper/tungsten FGM compared to those homogeneous materials made of copper or tungsten. Moreover, adding geometrical parameters to the design variables of their study resulted in more light-weight structures. Almeida et al. [18] implemented Continuous Approximation of Material Distribution (CAMD) method for the topology optimization of FG beams subjected to thermal and mechanical loads. Their aim was to find the best material distribution for such materials which resulted in the minimum compliance of the beams taking into consideration symmetry and pattern repetition constraints. In order to consider the safety of mechanical structures, scientists input safety factor in the formulation of their optimization approaches. These methods however were found to be too conservative or too unsafe. To tackle this problem, Reliability Based Design Optimization (RBDO) method is suggested which restrains the design to a certain reliability level. This method in the framework of Sequential Quadratic Programming (SQP) was utilized by Noh et al. [19] to minimize the thermal stresses of FG beams subjected to thermal loads. These researchers managed to decrease the thermal stress of such structures significantly regardless of the reliability. Subsequently, adding RBDO method to their calculations led to a slightly higher amount of thermal stresses while the reliability of the beams increased noticeably. Different structures with various shapes constituted from FGM were optimized by Taheri and Hassani [20] in an attempt to modify the structural eigenfrequency. The mass of an FG beam with clamped-clamped boundary conditions was minimized with eigenfrequency as the constraint. Next, the natural frequency of FG cylinders with different cross-sections was maximized. Subsequently, the difference between the first and second eigenfrequencies of a square plate with different types of holes in the middle was maximized. Finally, a particular eigenfrequency of a complex geometry made of FGM was increased considering the mass as the constraint. In all cases above, both material distribution and shape of the structures were selected as the design variables, and the researchers revealed that inserting the shape as an additional design variable could lead to better optimized results. Combining Imperialistic Competitive Algorithm (ICA) with Artificial Neural Network (ANN), Yas et al. [21] improved the speed of an optimization approach in which the material variation profile of FG beams was optimized. They demonstrated the efficiency of this hybrid method comparing the optimal results with those of GA. Subsequently, Kamarian et al. [22] proved the outperformance of the integration of Firefly Algorithm (FA) and Adaptive Neuro-Fuzzi Interference System (ANFIS) over hybrid algorithms such as ICA integrated with ANN or Genetic Algorithm (GA) combined with ANN in optimizing the material distribution of FG beams resting on a two-parameter elastic foundation. The objective function of both studies was to maximize the fundamental frequency of the beams and a three-parameter model was employed to examine the material distribution through the thickness of beams. Roque and Martins [23] implemented Differential Evolution (DE) optimization method to obtain the optimum material distribution of FG beams which led to the maximum first fundamental frequency of these structures choosing the volume fraction of one of the material constituents as the constraint. For this purpose, they investigated different materials and the influence of the ratio of material constituents' Young's moduli on the optimum design. Using a three-parameter material model referred to as Viola-Tornabene formulation helped them to include a wider variety of material distribution in their study. In another work carried out by Roque et al. [24], the same material model and optimization methodology were employed along with a modified couple stress theory to minimize the free vibration frequency of the Timoshenko FG nano beams. The researchers concluded that the optimal parameters of the applied material model did not depend on the scale parameter and boundary conditions. Trying to increase accuracy and decrease the computational time of an optimization approach regarding the fundamental frequency of FG nano beams, Vosoughi and Darabi [25] combined GA and Conjugate Gradient (CG) method. These researchers took the volume fraction distribution along with the small scale parameter as the design variables of their study. They also revealed results associated with FG nano beams with different boundary conditions and proved the efficiency and accuracy of their methodology. Pham et al. [26] employed the novel Social Group Optimization (SGO) algorithm to maximize the first natural frequency of FG beams. Three different material models with 3, 4 and 5 variables were utilized in their study and the last case was demonstrated to be the most efficient in representing the best optimal material distribution leading to maximum fundamental frequency. Taati and Sina [27] employed NSGA-II to optimize the thickness and the level of inhomogeneity in FG-micro beams resting on elastic foundation. The objective functions of their study were to minimize the maximum stress and deflection and to maximize the fundamental frequency and critical buckling load. The coefficient of elastic foundation and internal length scale parameter were found to have noticeable influence on the optimal design. Selecting lateral and flexural-torsional buckling of I-shaped FG beams as the objective function, Nguyen and Lee [28] conducted a GA-based optimization approach in which the material distribution and geometrical parameters were the design variables. A piece-wise cubic interpolation function was utilized in their study to examine the material distribution through the thickness of the beams. The researchers claimed that although both geometrical and material parameters are influential on buckling of FG beams, the latter was more dominant for shorter beams. Composite box beams with variable volume fraction of fibers and ply thickness were optimized by Maalavi [29] using MATLAB optimization toolbox. The volume fraction of materials was assumed to vary according to a power-law function. Maximizing the natural frequency and harnessing them to certain target values were the key objective functions, and the fiber orientation angle and volume fraction along with the ply thickness distribution formed the design variables.

With the aim of minimizing the compliance of FG beams and bridge structures, Xia and Wang [30] developed a level set based optimization approach and found the optimal topology and volume fraction distribution of these structures. The material was assumed to vary through these structures from a soft material to a stiff material smoothly. According to their results, although the FG designs had more compliance than their homogeneous counterparts made of the stiff material, FG structures contained less volume fraction of stiff material and this could lead to a design with lower cost. Maleki Jebeli and Shariat Panahi [31] integrated their Bi-directional Evolutionary Structural Optimization (BESO) method with GA to prove the outperformance of their methodology over that of Xia and Wang [30] when the design variables, objective function and the type of structures in the two aforementioned researches were the same. Less computational time, less minimized compliance and design manufacturability after the optimization process were of virtues of their approach over Xia and Wang’s.

Multi-material L-shaped brackets and Messerschmitt-Bolkow-Blohm (MBB) beams constituted from three material phases were optimized by Taheri and Suresh [32] utilizing perimeter penalization technique. As the volume fraction of material phases varied through these structures smoothly, they were considered as FG structures. However, there was no regular material gradation pattern in the optimized results. The total mass and volume fraction of particular phases were chosen as the design variables in this topology optimization research whose aim was to minimize the compliance of the mentioned beams and brackets. The authors noticed that this technique led to more efficient and manufacturable designs than the other conventional methods in the literature. A comprehensive multi-resolution topology optimization approach based on isogeometric analysis on different types of structures namely MBB beams, cantilever beams, bridge structures, curved beams and L-shaped beams was conducted by Lieu and Lee [33]. All of these structures were multi-material and the objective function was to minimize their compliance. The researchers changed each multi-phase optimization problem containing several volume fraction constraints into a number of 2-phase sub-problems with one volume fraction constraint. Every sub-problem was subsequently solved by optimality criteria method. The authors finally demonstrated the outperformance of their methodology over conventional topology approaches in terms of computational costs. The weight of FG lattice beams and L-shaped brackets were minimized in a topology optimization approach conducted by Cheng et al. [34] with the stress as the constraint. The researchers implemented a modified Hill's yield criterion and asymptotic homogenization method to obtain the yielding strength and effective elastic properties of these structures, respectively. After conducting experimental tests, they proved the accuracy of the utilized scheme. According to their results the optimized FG structures showed significantly better elastic properties than their uniform counterparts.

2.2. Axially functionally graded beams

Alshabatat and Naghshineh [35] utilized Finite Element Method (FEM), lumped parameter model and GA to represent the optimal design of clamped and cantilever axially FG beams with the aim of minimizing the sound power radiation and maximizing the fundamental frequency of these structures. Four and five-parameter trigonometric models were represented to determine the material distribution of the beams. These models were proved to be more flexible than conventional power-law material models. Axially FG cantilever polymer composite beams reinforced by Multi-Walled Carbon Nanotube (MWCNT) were optimized by Rokni et al. [36] to maximize the fundamental frequency. The proportion of Carbon Nanotube (CNT) through axial direction was approximated by an exponential function, and different indices were investigated to obtain the best CNT distribution regarding this objective function. According to the final results, the optimized FG distribution of these structures could lead to nearly 11% higher values of fundamental frequency than that of the uniformly distributed design while both contained 1 wt.% of MWCNT. The combination of Particle Swarm Optimization (PSO) method and Simple Cell Mapping (SCM) helped He and Sun [37] to optimize the material distribution of axially FG beams. The objective functions were to achieve a design with minimum total mass and maximum radiated sound power with upper and lower bounds of the desired frequencies as the constraints. The authors also observed that the hybrid SCM integrated with an evolutionary algorithm outperformed an individual evolutionary algorithm. The combination of isogeometric analysis based on 2D Non-Uniform Rational Basis Spline (NURBS) functions with DE method helped Kim et al. [38] to optimize the material distribution of 2D FG beams. The purpose of their study was to maximize the first three fundamental frequencies. Investigating the influence of different parameters on the optimal design of 2D FG beams, they reached two important conclusions. Firstly, the most significant influence of the optimization process can be seen in clamped-free boundary conditions while the increase in the fundamental frequency of simply supported beams were less than those with other boundary conditions such as clamped-clamped and clamped-simply supported. Secondly, the slenderness has negligible influence on the maximization of fundamental frequency of the beams.

2.3. *Functionally graded sandwich beams*

Sandwich structures are implemented in various engineering applications where high strength and lightweight structures are required. The main disadvantage of these structures is the mismatch of stiffness properties between their core and face sheets. This causes a certain type of damage referred to as debonding particularly when the sandwich beam or plate is subjected to impact loadings. In order to decline the vulnerability of sandwich structures to this type of failure, scientists have combined the concept of FGMs with sandwich structures and this has brought them a new type of structures named Functionally Graded Sandwich (FGS) structures. There are two main types of FGS structures. The first one is a FG core sandwiched between two homogeneous face sheets. In the second type, a homogeneous core is sandwiched between two FG face sheets. Both of these structures are depicted in Fig.4.

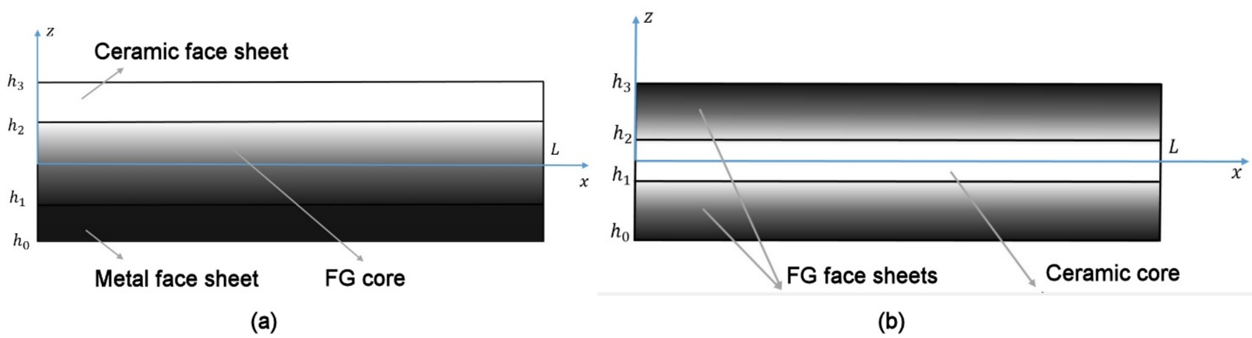

Fig.4. Schematic of FGS structure. (a) Type-I. (b) Type-II.

In recent years, many researchers have been motivated to optimize the FGS structures. In order to illustrate the applicability of PSO method in optimizing FGM and exploiting re-initialization strategy in this very algorithm, Loja [39] attempted to optimize FGS beams using this algorithm. The objective function of this research was to maximize the bending stiffness of these beams. To this end, two case studies with different sets of design variables were chosen. In the first case which was a symmetric sandwich beam, the thickness of ceramic core and the material distribution of FG face sheets were the design variables. The second case was an asymmetric FGS beam and the thickness and material distribution of face sheets were taken as the design variables. The author finally noted the importance of re-initialization in the process of PSO method. The material distribution of FGS beams and plates was optimized by Shi and Shimoda [40] to minimize their compliance using traction method. In these structures, face sheets and cores were constituted of metal and ceramic, and the shape of interfaces between the core and face sheets were assumed to vary according to a shape gradient function, considering the total volume of the structure as the constraint. This function was the design variable of their study. The authors realized that the larger the differences between the Young's moduli of the ceramic and metal phases, the more optimization procedure could lead to lower values of compliance. In a comprehensive optimization study provided by Correia et al. [41] FGS plates with FG cores were optimized considering several conflicting objective functions such as weight and cost minimization, natural frequency maximization and stress reduction. The thickness of core and face sheets along with the material inhomogeneity were chosen as the design variables while manufacturing limitations were also taken into account. GLODS and DMS were the implemented optimization methods in their study.

2.4. Functionally graded porous beams

Porous materials which bring engineers lightweight designs with advanced performances are widely utilized in different sections of industries these days. Engineers are also inspired by the primary natural porous bones and have succeeded in modifying the characteristics of porous materials even more by introducing the FG porosity idea and optimizing the porosity pattern in these materials. Implementing Non-dominated Sorting Genetic Algorithm (NSGA-II), Beluch and Hatlas [42] managed to optimize the porosity distribution of FG porous cantilever beams subjected to vertical loads. The objective functions of this multi-objective optimization procedure were minimizing the maximum deflection and maximizing the total porosity of the beams. A polynomial function was used for determining the porosity variation in the mentioned structure. The optimal results proved that the minimized maximum deflection at the end of the beam was achieved when the porosity was minimized. A multi-objective optimization procedure founded on NSGA-II was adopted in Jamshidi and Arghavani's work [43] to optimize the porosity distribution of 2-D FG porous beams. The objective functions were to maximize the buckling load and minimize the weight. The researchers presented several examples with different symmetrical clamped and hinged boundary conditions. They claimed that the concentration of porosity was on the middle of the beams rather than outer corners in most of the optimal designs. Moreover, boundary conditions were found to be influential on the final results and optimal porosity distribution.

3. Functionally Graded Plates

Various shapes of plates such as circular, square, rectangular, trapezoidal and skew plates have been investigated by the scientists since these structures are remarkably applicable in engineering designs. The splendid thermo-mechanical behavior of FGMs have inspired many researchers, over the last twenty years, to probe the optimal design of FG plates with regards to different objective functions. Fig. 5 depicts a schematic of an arbitrary rectangular FG plate and its coordinate system.

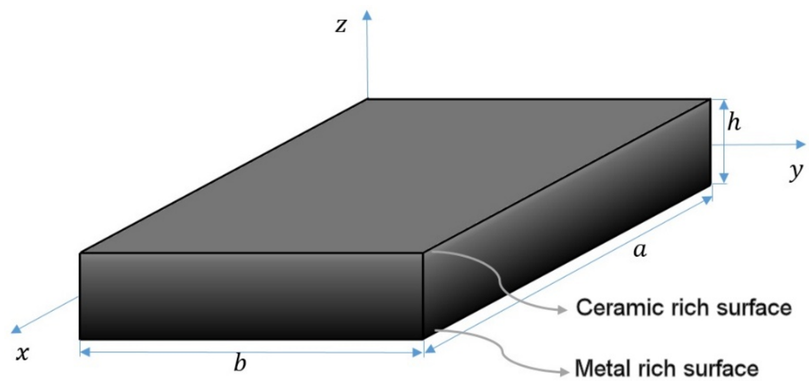

Fig.5. Schematic of transversely FG plate and the corresponding coordinate system

3.1. Transversely functionally graded plates

Fares et al. [44] attempted to optimize the structure and closed loop control function of FG plates. The objective function associated with the control problem was defined as minimizing the dynamic responses of these plates while the structural objective was to maximize the critical buckling load. These optimization problems were imposed to constraints such as control energy and total thickness. It was proved that ignoring the normal strain effects could lead to errors up to 22% when the power-law exponent by which the material distribution of the plates was determined was higher than 1. A vibrational optimization study was presented by Alshabatat et al. [5] which was associated with FG plates with various boundary conditions. The distinctive in-plane material distribution was taken into consideration by using a five-parameter trigonometric function (Eq.6.b). The coefficients of this function were chosen as the design variables in the two key problems of their work. The first one dealt with maximizing the fundamental frequency while the second one involved minimizing the kinetic energy of the mentioned structure vibrating under a particular excitation frequency. The effective material properties were calculated via Mori-Tanaka scheme and the researchers reported that this type of material variation model had great influence on the vibrational characteristics of these plates. Correia et al. [45] implemented Direct Multi-Search (DMS) method and GLODS method to optimize the thickness and material distribution of FG plates in three different examples. The objective functions were to maximize critical buckling load and natural frequency and to minimize the mass and cost of FG plates. Several optimum design domains were achieved and the best possible design for these structures was proposed depending on the priority of the four mentioned objective functions.

Ootao et al. [46] implemented ANN to optimize the material distribution of FG plates under uniform heating on one surface with the aim of minimizing the thermal stresses. They assumed the materials to be temperature-dependent and finally compared their results with those of an ordinary non-linear mathematical programming. The results were in line with each other. Goupee and Vel [6] introduced two optimum solutions for simply supported FG plates subjected to thermo-mechanical loads using RCGA. In the first model, they managed to decrease the peak effective stress of FG plates made of Ni/Al_2O_3 nearly 55% by optimizing the material distribution through the thickness of these plates. Using the same design variable in the second model, they managed to decrease the total volume fraction of ceramic from 0.5 to 0.44 and maximum temperature that the metal experienced from 500°C to 275°C while the amount of effective stress was considered as the constraint. Mori-Tanaka and self-consistent homogenization models were utilized in their study to obtain the material properties variation. The maximum Von Mises stress, shear stress and deflection of rectangular FG plates were minimized in a research performed by Helal and Shi [47] employing ANSYS Parametric Design Language (APDL). The volume fraction distribution through the thickness of these plates was chosen as the design variable. The researchers used different types of material models namely power-law, exponential and sigmoid models to investigate the influence of these models on the optimal design and finally managed to decrease the deflection, Von Mises stress and shear stress by 24%, 22% and 11%, respectively. Additionally, the best optimal results were related to those following sigmoid and power-law models. Ding and Wu [48] adopted a GA-based optimization approach to minimize the peak thermal induced stress of FG plates whose surfaces were imposed to particular temperatures and heat convention. The plates were artificially assumed as multi-layered plates and four different material distribution models namely power-law, sigmoid, linear layer-wise and step layer-wise models were taken into account. Regarding this specific objective function, the results proved that the best material model was the linear layer-wise scheme and the authors suggested that a higher order layer-wise model would lead to even better optimum results.

Na and Kim [49] provided an optimization research related to thermo-mechanical buckling of FG plates. They calculated the best volume fraction distribution through the thickness of these plates which resulted in minimum temperature of the metal phase and the minimum total stress both phases experienced. Improving the thermo-mechanical loads was another objective of their study. Utilizing GA and PSO method, Ashjari and Khoshravan [50] optimized the material distribution of simply supported FG plates under sinusoidal and uniformly distributed transverse loads in order to minimize the total mass. Deflection and stress were considered as the constraints individually and simultaneously through three case studies. The best results were achieved when both constraints were imposed on the process of optimization. The researchers demonstrated that the best performance was associated with symmetrical sandwich plates with homogeneous cores and FG face sheets. They also claimed that PSO method was slightly more efficient than GA. Hossein and Mulani [51] chose to analyze the shear and uniaxial buckling of simply supported and clamped FG plates. These plates were actually made up of aluminum reinforced with silicon carbide nano-particles. The authors subsequently decided to minimize the required amount of reinforcing material which led to the minimum cost of these structures. They let material distribution be the design variable which was determined by an in-plane polynomial function. The researchers claimed that this function outperformed other schemes such as power-law since it was capable of controlling the material distribution more efficiently. Finally, they managed to minimize the amount of ceramic powder approximately 55% after the optimization process. A similar approach was taken by the same authors [52] to decrease the cost and total volume fraction of silicone carbide in these types of simply supported FG plates under uniformly distributed loading, non-uniform loading, central point loading and flutter speed constraint. After the optimization process, this value decreased by 16.3%, 22%, 82% and 18.6% for these four load cases, respectively. These authors [53] also tried another optimization approach towards minimization of the reinforcement material (silicon carbide) in three types of FG plates namely unstiffened, stiffened and plates with cutouts with critical buckling load as the constraint. The impact of several parameters such as boundary conditions, the type of structure, the direction of material variation and the type of imposed loads on the optimal design were demonstrated. It was proved when looking at the final optimal results that the most decrease in the amount of reinforcement material was related to clamped rather than simply supported boundary conditions, vertical (perpendicular to the direction of stiffeners) rather than horizontal material variation direction and shear rather uniaxial loadings. The largest amount of silicon carbide saving was found to be up to 200% compared to that of homogeneous panels.

In order to obtain a certain desired temperature distribution through FG plates, Golbahar Haghighi et al. [54] assumed the six surfaces of these structures to be imposed to heat fluxes. Accordingly, these authors employed CG method combined with Differential Quadrature Method (DQM) to minimize the discrepancy between the desired and actual temperatures in the presence of heat fluxes. They also provided different examples dealing with various shapes, material distributions and desired temperature domain distributions to show the efficiency of their method. Chiba and Sugano [55] utilized GA to optimize the material variation of FG plates under temperature loads with regard to the microstructural morphology and microstresses into consideration. The objective function was to maximize the safety index defined as the relationship between microstress domains and phase-specific failure criteria. The researchers claimed that exploiting the knowledge of microstresses in the optimization process had considerable influence on the final optimal volume fraction distribution of the material constituents. Chen et al. [56] performed an elastic analysis of FG plates via a meshless collocation method. This method was founded on generalized multi-quadrics Radial Basis Functions (RBF) which was defined by a function with two parameters referred to as the shape parameter and exponent. These authors then employed GA to minimize the discrepancies between the results of their study and those obtained by Zenkour's research [57]. Both of these studies were based on Higher order Shear Deformation Theory (HSDT). The optimal design variables in their study were the exponent and shape parameter which were found to be 0.73 and 1.3, respectively. Lieu and Lee [58] approximated the volume fraction of ceramic of a FG plate at specific grid points along the thickness using *Greville abscissae* and material distribution through the thickness via a B-spline basis function. The constituent materials were assumed to be temperature dependent. The plates were subjected to thermo-mechanical loads, and the optimum material distribution which led to the minimum compliance was determined using Adaptive Hybrid Evolutionary Firefly Algorithm (AHEFA). According to the optimal results, the middle plane was metal-rich and the surfaces were ceramic-rich. Additionally, the mentioned algorithm resulted in more accurate results with less computational costs.

3.2. 2D functionally graded plates

2D-FG plates are those in which the volume fraction of each constituent material varies through the plane in both directions (Fig. 6). The application of this kind of FGMs can be seen mostly in designs which contain cutouts and consequently the variation of materials in two dimensions results in better stress distribution or effective high temperature-resistance.

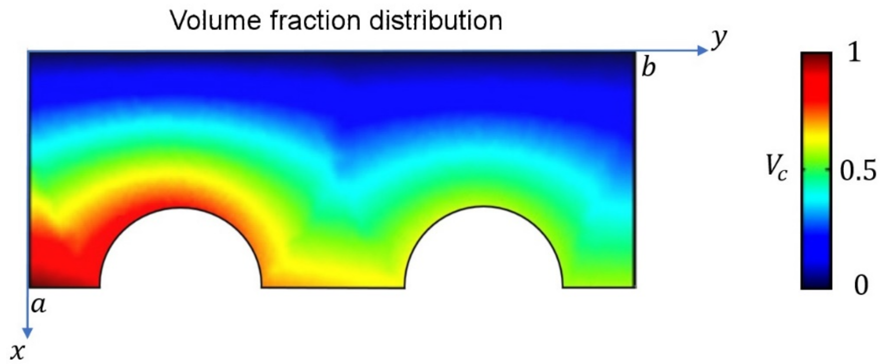

Fig.6. In-plane volume fraction distribution in a 2D-FG plate. [59]

Gradual variation of materials in two or more directions can be seen in many different natural phenomena. This can be seen nowhere clearer than the microstructure of bones. Although bones contain a lot of holes, they could carry huge loads while maintaining their low weight. These significant features are attributed to the multi-directional material distribution around these holes. Inspired by the structure of bones, Huang and Rapoff [60] analyzed plates with a circular hole in the middle when the material around the hole was assumed to vary according to a power-law function. They demonstrated that this design led to a higher ratio of strength to weight of the plates compared to the homogeneous plates. Another optimization study on FG plates with a hole in the middle was performed by Venkataraman and Sankar [61] employing GA. The hole was assumed to be encircled by a number of rings when the material type and properties of these rings varied gradually. Thus, this design was considered as a 2D-FG plate with a hole. The objective function in this study was to minimize the failure index by finding the best material distribution around the hole. The authors compared their design with perforated homogeneous plates and proved the significant influence of gradually varying materials around the hole on the strength of the plate. A 2D-FG plate, constituted from tungsten and copper, with a hole near to its southern edge was optimized by Vel and Goupee [62] implementing NSGA-II. The northern edge was the tungsten rich edge which was assumed to have 500°C while the cutout edge's temperature was 0°C. The researchers' goal was to maximize the dynamic and static safety factors of these plates under the mentioned temperature gradient. To this end, they tried to find various optimal results and represented a Pareto front regarding the objective functions. According to the final optimal design, as the volume fraction of tungsten increased, the static safety factor increased while the dynamic safety factor showed an opposite downward trend. Nemat-allah [63] conducted an optimization approach to minimize the thermal and residual stresses of 2D-FG plates composed of ZrO_2/*6061-T6/Ti-6Al-4V* subjected to several thermal loading cycles. The volume fraction of material constituents were assumed to vary via power-law functions in both directions. Thus, the power-law exponents were considered as the design variables and the optimum values for these parameters which led to minimized maximum temperature and minimized maximum equivalent stress were found. A plate with semicircular cutouts on one edge, made up of 2D-FGM, was optimized by kou et el. [59] to minimize the mass and maximum equivalent Von Mises stress. This plate was subjected to a linear temperature gradient. The main objective of this study was to illustrate the superiority of PSO method over mathematical programming methods such as attractive set and trust region algorithms. The material distribution of the mentioned FG plates was determined by a Heterogeneous Feature Tree (HFT) model which allowed the researchers to approximate a versatile material propagation via a number of explicit functions. The authors finally managed to decrease the mentioned objectives by nearly 4% and 24%, respectively. In order to demonstrate the outperformance of modified Symbiotic Organisms Search (mSOS) method over DE and Symbiotic Organisms Search (SOS) methods Do et al. [64] conducted two optimization studies on 2D FG plates. The fundamental frequency and buckling load were maximized by obtaining the optimum material gradation through these structures. These authors also proved that combining mSOS with Deep Neural Network (DNN) method decreased the computational time significantly and outperformed the mSOS in the framework of isogeometric analysis. Lieu and Lee [65] exploited AHEFA in the framework of isogeometric multi-mesh design to maximize the fundamental frequency of multi-directional FG plates. Different plates with constant and variable thickness were analyzed in their study and the material inhomogeneity as well as thickness variation profile were taken as the design variables. The aforementioned method enabled the researchers to generate two independent NURBS surfaces which led the optimization process to have remarkably lower computational costs.

3.3. Functionally graded sandwich plates

Utilizing DE method, Silva and Loja [66] could optimize FGS panels constituted from aluminum core and two FG face sheets aiming at minimizing the thermal residual stress distribution. The design variables were defined as the thicknesses of the core and face sheets and the volume fraction variation pattern in the face sheets which was calculated by a power-law function. The researchers restrained the range of power-law index between 0 and 10. The larger this parameter was, the less the thermal residual stresses were. Therefore, the optimal exponent was calculated to be 10. Ashjari and Khoshravan [67] represented an optimization solution based on NSGA-II to minimize the mass and deflection of FGS plates subjected to mechanical loads with the stress field as the constraint. The material volume fraction was defined in some control points and its distribution was calculated via a piecewise cubic interpolation function. These control points, the volume fraction distribution and the thickness of the face sheets were the design variables of this study. As transverse shear stress played an important role in the maximum equivalent Von Mises stress at the vertical edges, the researchers reported that the maximum shear stress occurred in designs with stiffer face sheets and steeper material variation. Utilizing PSO method, Torabi and Afshari [68] optimized trapezoidal FGS symmetric plates as a model of aircraft wings in order to maximize the critical aerodynamic pressure. These plates were made of two FG face sheets and a homogeneous core. Material variation through the thickness as well as several geometrical parameters such as the thickness of core and plate, aspect ratio and plate angles were taken as the design variables. The best optimal design regarding the mass and angles of each corner as the constraints was achieved for these symmetric FGS plates. However, the authors recommended that asymmetric designs might be more capable of increasing the mentioned objective function. Lieu et al. [69] developed AHEFA to optimize the material variation of FGS plates with regards to their vibrational behavior. These plates are considered as multi-layer plates. Therefore, a multi-patch

B-spline basis function was employed to determine the thickness of each layer and the volume fraction of ceramic at each control point. The set of control points was examined by *Greville abscissae.* Two different problems with different objective functions and constraints were explored. The objective function of the first problem was to maximize the natural frequency having total volume as the constraint. The second problem dealt with minimizing the mass imposing constraints on fundamental frequencies. The authors employed DE method and FA as well as AHEFA and demonstrated the outperformance of the last case in terms of accuracy and convergency speed. Moreover, they revealed that the B-Spline functions led to a totally different optimal design compared to their power-law counterparts. It is to be noted that utilizing B-spline functions in determining the material propagation of FG plates resulted in more feasible and convenient manufacturing process.

3.4. Step-formed functionally graded plates

Step-formed FG plates are those which are made of a number of homogeneous layers, and the volume fraction of materials varies gradually through the thickness from a layer to another. Detailed schemes of these types of FG plates and an arbitrary step-formed functionally graded volume fraction through the thickness of these plates can be observed in Fig. 7.

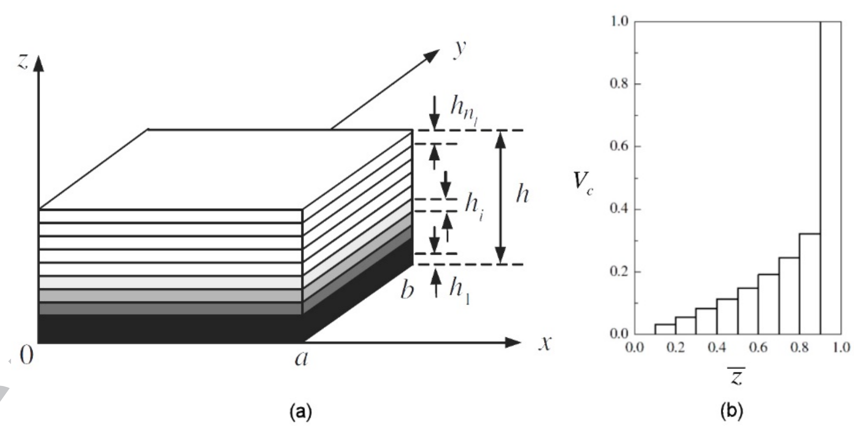

Fig.7. (a) Geometry of a step-formed FG plate. (b) An arbitrary volume fraction distribution of ceramic in the step-formed FG plate. [70]

Ootao et al. [71] applied GA to optimize the material composition of ten-layered step-formed FG plates whose ceramic rich surface was subjected to uniform heating. The objective function was to decrease the maximum thermal stress in the transient state. Three different cases were considered in this study. In the first one, eight design variables associated with the volume fraction of the eight middle layers were chosen while the volume fractions of metal in the first and tenth layers were 0 and 1 as the constraints. These constraints were omitted in the second case. In the third case, the material distribution of layers through the thickness was assumed to follow a power-law function and the number of design variables declined to only the power-law index. The researchers noted that the best stress reduction was related to the second design. The material distribution of step-formed FG plates was optimized by Na and Kim [70] to reduce the stress and to enhance their thermo-mechanical buckling. These types of structures were composed of several layers made up of homogeneous materials and the material properties of the layers were assumed to be temperature-dependent and vary gradually from layer to layer by controlling the volume fraction of their material constituents. A similar optimization approach was performed by these authors [72] on FGS plates consisting of an FG core and homogeneous face sheets considering core to face sheet thickness ratio and material variation pattern as the design variables. Comparing these plates with a two-layered composite plate made up of the same ceramic and metal, the best results were related to those of FGS plates. However, the optimum results of step-formed FG plates were found to be close to those of the aforementioned FGS plates.

3.5. Multi-layered plates containing functionally graded layers

Shape control optimization of FG beams and plates was conducted by Liew et al. [73] utilizing GA. Both of the mentioned structures were sandwiched between two layers containing piezo-electric patches. The top and bottom layers were playing the role of actuator and sensor, respectively. The researchers finally obtained the optimum voltage distribution through these layers which led to the desired shape of the beams and plates. They also investigated the influence of material distribution through the thickness of these FG structures restrained to clamped and simply supported boundary conditions on their optimal designs. FG plates sandwiched between two piezo-electric layers made of Lead Zirconate Titanate (PZT) and/or Polyvinylidene Fluoride (PVDF) were analyzed by Nourmohammadi and Behjat [74] in terms of their bending behavior. The material properties were assumed to vary according to a power-law function and the exponent was considered as the design variable by which the deflection of these plates was intended to be minimized. These plates were subjected to electrical, mechanical and thermal loads. The researchers proved that each type of loading led to a distinct optimum value of power-law index. Three different layer arrangements namely PVDF-PVDF, PVDF-PZT and PZT-PZT were investigated and the last case proved to result in the lowest minimized deflection.

Multilayered plates made up of ZrO_2 and Ti_6Al_4V alloy whose thermal properties significantly depend on temperature were optimized by Parashkevova et al. [75] when undergoing temperature gradient. The layers were made up of ceramic and metal decussately and the thicknesses of layers were assumed to vary through the thickness on the basis of an exponential function. Optimizing the distribution of thicknesses through the thickness, the researchers intended to minimize the stress jumps. Tsai-Wu, Mohr, maximal stress and deformation criteria were utilized in this research to determine the stress through the thickness of these plates. Moreover, various boundary conditions were imposed on these plates amongst which the simply supported boundary conditions were observed to lead to the highest stress jumps. Gunes et al. [76] made an attempt to optimize the adhesively bonded FG single lap joints to maximize their fundamental frequency and to minimize their modal strain energy. The material distribution of FG plates and the geometry of the joint formed the design variables of this research. The effects of several parameters such as the thickness of the plate, overlap length, material composition of plate and the thickness of the adhesive layer on the mentioned objectives were investigated, and the last parameter had negligible influence in contrast with the first two variables. According to the optimum design, when the overlap length increased, the thickness of the plates had to be decreased and when the overlap length decreased, the thickness of plates had to be increased. Additionally, increasing the volume fraction of ceramic in plates led to higher natural frequencies. In order to demonstrate the outperformance of plates constituted from two homogeneous face sheets and a FG core over a two-layered composite made up of the same metal and ceramic in terms of thermo-mechanical buckling behavior, Na and Kim [77] compared these two types of plates and carried out an optimization study on the former case to obtain the best volume fraction distribution of ceramic through the thickness. The constituent materials were assumed to be temperature dependent varying through the thickness via a power-law function. Protecting the battle vehicle bodies against radar surveillance and blast waves motivated Zhu et al. [78] to design an elaborate type of sandwich plates constituted from a glass composite face sheet at the top, a carbon composite face sheet at the bottom and an Functionally Graded Foam (FGF) core. Additionally, these authors conducted an optimization approach based on Response Surface Methodology (RSM) to find the best material variation in the core which led to the minimum vulnerability of this intrinsic structure against electro-magnetic or shock wave mitigation. The material distribution of the core was assumed to follow a power-law index and the optimum value of this design variable was found to be between 8 and 10. Moreover, the ascending gradient model (increasing the stiffness of the FGF from the bottom to the top of the core) led to the best performance of these plates. Utilizing Particle Swarm Optimization Sliding Mode (PSOSM) control system, Moghaddam and Bagheri [79] optimized a three-layer composite made up of an FG panel sandwiched between two piezo-electric layers at the top and bottom as the actuator and sensor, respectively. The aim of their study was to suppress the vibration of these structures which were imposed to three free and one clamped boundary conditions. The researchers also compared the results with those of Traditional Sliding Mode (TSM) control system and demonstrated the efficiency and outperformance of PSOSM over TSM in terms of convergency speed. Utilizing the combination of GA and ANN, Daynes et al. [80] conducted a topology optimization on flat sandwich panels containing an FG lattice core. They calculated the best orientation and geometrical properties of the lattice cells which led to the highest ratio of strength to weight. According to the final results of this research, the stiffness and strength of FG lattice panels could be increased significantly through the optimization process. The stiffness and strength of the optimum FG lattice panels were found to be 101% and 172% more than those of their uniform lattice counterparts, respectively, while their weights were equal.

3.6. *Functionally graded carbon nanotube plates*

Setoodeh and Shojaei [81] conducted an optimization study based on GA to maximize the buckling load of Functionally Graded Carbon Nanotube (FGCNT) reinforced composite quadrilateral plates. Three different volume fraction distributions of CNTs through the thickness of these plates were considered. The fiber orientation angles were chosen as the design variables in this study and the influence of other parameters such as volume fraction of CNTs, geometry and boundary conditions on the buckling of these plates were also demonstrated. The best results were related to the plates in which the volume fraction of the CNTs at the center and surfaces of the plate was maximum and minimum, respectively. Vo-Duy et al. [82] proposed an optimization method based on Adaptive Elicit Differential Evolution Algorithm (AEDEA) to optimize the fiber orientation angles of CNTs in FGCNT reinforced composite quadrilateral plates considering maximization of fundamental frequency as the objective function. The influences of other parameters such as volume fraction distribution of CNTs through the thickness, number of layers and boundary conditions on the optimal design were illustrated the last of which was found to be the most influential. Hussein and Mulani [83] employed RBDO method and a deterministic optimization approach based on SQP to determine the best possible material gradation of in-plane FGCNT-reinforced polymer composite plates aiming at improving the stiffness of these structures. Accordingly, these researchers calculated the optimum volume fraction distribution through plates which led to the minimum amount of required stiffening CNT considering deflection as the constraint. A comparison between the optimized FGCNT plates with homogeneous CNT composites made evident that the former case required 45% less amount of CNT for the same values of stiffness. Additionally, the optimal volume fraction of CNT in the FGCNT plates examined by RBDO method was 48% more than that of the deterministic approach.

3.7. Functionally graded foam-filled plates

Thermal protection Systems containing FG metal foam insulations were optimized in a study performed by Zhu et al. [84], implementing SQP. As these systems should keep the maximum temperature below a certain limit, the objective function was to minimize the maximum temperature by optimizing the solidity profile of the FGF. The total thickness and mass of the panel which was supposed to be insulated by FGF were chosen as the constraints. According to the outcomes related to the optimal design of these systems, the solidity of the inner layer (which was cooler) was high while the solidity of the outer layer (which was hotter) was low. Furthermore, the optimal results of transient heat conduction showed a huge discrepancy with those associated with steady state analysis. FGF insulations were optimized by Venkataraman et al. [85] in order to minimize the heat transmission. These foams were compared to their uniform counterparts and were proved to outperform them regarding the mentioned objective function. Closed and open cells were considered and the heat transmission of the former did not change significantly during the optimization process. In contrast, the open cell FGF’s heat transmission was proved to be 10% higher than that of the uniform foam. Moreover, a FGF could transmit the same amount of heat as the uniform foam while it is 12% thinner than its uniform density counterpart. RSM helped Zhang and Zhang [86] to optimize the density variation of FGF blocks so as to enhance the crashworthiness behavior of these structures. Accordingly, these blocks were assumed to be under middle and high velocity ball impact loads. Various density distributions for these foam blocks were studied and the linear decreasing density model was proved to outperform other designs in terms of crush force efficiency. This design was also the most optimal when the foam was sandwiched between two layers.

3.8. *Other functionally graded plates*

A commercial optimization program named DOT helped Aboudi et al. [87] to optimize FG composite plates in which the fiber volume fraction varied through the thickness. These plates were assumed to be under temperature gradient. The optimization problem was solved with and without constraint on fiber spacing and the objective function in both problems was to minimize the moment resultants attributed to thermal loading. Moreover, the researchers entered generalized plane strain and plane strain boundary conditions into the optimization formulations and compared the results. According to the final results, the boundary conditions and the mentioned constraint had great influence on the reduction of the objective function and the pattern by which the volume fraction of fibers varied in the optimal designs. Increasing the buckling load of rib stiffened FG plates, Birman and Byrd [88] decided to add some stringers to these plates. After investigating the positive influence of these stringers on the critical buckling load of FG plates, the researchers minimized the mass of the stringers letting critical buckling load be the constraint. Considering neither the local buckling nor the buckling of stringers is the flaw which can be seen in this research. Putting these two objectives aside, the best optimized results were achieved when the minimum number of stringers were widely situated. Hedia et al. [89] utilized the ANSYS package to optimize the material distribution of a perforated porous FG plate aiming at minimizing stress concentration near the hole when undergoing thermal and pressure loads individually and simultaneously. A non-liner function was employed to determine the effective material properties of these plates. This non-linearity is attributed to the porosity distribution through these materials. After obtaining the optimized results, the authors claimed that while one of the loads (thermal or pressure) was imposed, a ceramic-rich design was optimum. On the other hand, a metal-rich plate was found to be the best optimized design when both loads were applied on the plates.

4. Functionally Graded Shells

Shell structures are vastly employed in different engineering designs such as fluid containers or conveyers, aerospace and naval construction industries. Their curvature helps them to carry out huge loadings effectively. Therefore, they are often used as load bearing structures for aircrafts, rockets, submarines and missile bodies. As FGMs have extraordinary features, many researchers have been encouraged to optimize FG shells with different purposes recently. There are several types of shells namely cylindrical, conical and spherical shells. In this section, these types of shells along with cylinders, vessels and pipes made of FGM are investigated.

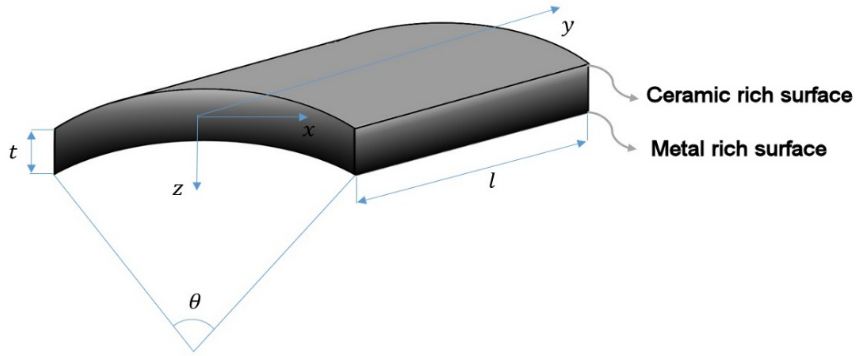

Fig.8. Geometry of an FG cylindrical shell and the corresponding coordinate system

4.1. Functionally graded cylindrical and conical shells

The material distribution of FG cylindrical shells was optimized by Ootao et al. [90] using mathematical programming and Neural Network (NN) method. These structures were assumed to be under various thermal conditions and the aim was to minimize the thermal stresses. The volume fraction of material constituents through the thickness of these shells was considered to vary according to the power-law function and the exponent was taken as the design variable. The optimum outcomes of the aforementioned methods were in line with each other. The researchers also noted that although the first training of the neural network approach required a long time of CPU processing, once completed, learned and unlearned thermal conditions would take a very short amount of time for CPU processing. This feature was highlighted as the advantage of NN over the mathematical programing method. In order to show how thermo-mechanical behavior of FGM depends on the volume fraction distribution of materials through the thickness of these structures, Cho and Ha [91] were motivated to adopt an optimization approach to this type of materials. They incroporated a combined interior penalty function and GS method to optimize their material propagation aiming at minimizing the steady-state thermal stresses. The material properties were assumed to vary according to TTO model. The researchers proved that varying the material distribution gradually has a significant effect on decreasing the thermal stresses by presenting examples of a FG beam and cylinder. A four-layered cylinder made up of a steel layer, a metal layer, a FG layer and a ceramic layer subjected to temperature gradient and inner and outer pressure was optimized by Huang et al. [92] to reduce the tensile stresses at the outer ceramic rich surface. Material distribution through the thickness of FG layer was estimated by a power-law function and the exponent as well as the geometry of the cylinder were chosen as the design variables of this optimization study which was based on a quadratic programming method. The optimal outcomes demonstrated that increasing the thickness of the FG layer resulted in lower tensile stress of the ceramic outer surface. Axially FG cylinders made up of nickel and alumina were modeled by Choi and Cho [93] as a ten-layer structure. With the purpose of minimizing the maximum hoop stress at the metal layer, they found the optimum material distribution of the multi-layered cylinders and subsequently with the aid of an interpolation function, they managed to obtain the best volume fraction distribution through the longitudinal direction of these FG cylinders. The peak stress which the metal rich surface experienced was declined from 430Mpa to

231Mpa after the optimization process. Batra [94] analyzed the FG incompressible linear elastic spherical and cylindrical shells subjected to inner and outer hydrostatic pressure. The aim of this research was to find the best shear modulus variation through the thickness of these shells which led to optimal circumferential and hoop stresses. Linear distribution of shear modulus through the thickness was finally found to be the best design according to the aforementioned objectives for both types of shells. Vel and Pelletier [2] applied an elastic NSGA to optimize the material distribution of plates and shells made of FGM introducing two models. In the first model, the objective function was to minimize the peak hoop stress and mass simultaneously under thermal loads considering the maximum temperature of the metal as the constraint. The researchers reported that the optimized design led to a slight increase in mass but a remarkable decrease in hoop stress. In the second model, they took minimizing the mass and maximizing the safety factor of shell under heat flux as the objective functions. Asgari [95] implemented Multi-Objective Genetic Algorithm (MOGA) to optimize the volume fraction distribution of 2D-FG hollow cylinders. The objective functions were considered to be minimizing the mass, stress wave propagation and the displacement of these structures under impulsive internal pressure. Cubic spline interpolation functions were chosen for determining the material distribution variation in both radial and axial directions. Moreover, the outperformance of these functions over regular power-law functions was noticed. In order to reduce the vibration and suppress the undesirable vibrations of shear deformable FG cylindrical shells, Biglar and Mirdamadi [96] tried GA to optimize the orientation angle and location of piezo-electric sensors and actuators installed on the surface of these shells. After analyzing three different examples, the researchers claimed that attaching sensors and actuators at the optimal angle and position could lead to higher transmission of mechanical energy to the shells and this resulted in a very quick elimination of vibrations. The influences of the material distribution of FG shells and control gain on the suppression of vibrations were determined as well.

In a GA-based optimization approach, Nabian and Ahmadian [97] maximized the natural frequency and minimized the weight of simply supported FG hollow cylinders by finding the best material gradation pattern through the thickness of these structures. Piece-wise cubic interpolation function was implemented in this research to determine the volume fraction of material constituents at specific control points and to examine the volume fraction distribution through the thickness of cylinders. The authors finally extracted a Pareto front for these two contradictory objectives as a benchmark for other engineers. Tornabene and Ceruti [98] employed PSO method, GA and Monte-Carlo (MC) algorithm to optimize material variation in FG panels and doubly curved shells to minimize the deflection and maximize the fundamental frequency of these structures. The analyzed shells were assumed to consist of a number of laminas and the material distribution of each lamina was determined by a four-parameter power-law function. A comparison between the mentioned algorithms depicted that PSO method and GA represented more exact results than MC did; particularly when both of the objective functions were taken into consideration. Looking from the lens of a designer, the main disadvantage of this approach was attributed to the lack of any relationship between the four parameters of the power-law function and the dynamic or static performance of the mentioned shells. Maximizing the sound transmission loss of FG cylindrical shells which stems from the optimum first resonant frequenct encouraged Nouri and Astaraki [99] to optimize the volume fraction distribution of materials and geometry of these shells via GA. The thickness, frequency range and weight of these structures acted as the constraints. Different sets of materials were analyzed in this research and finally, the combination of nickel-aluminum and steel-aluminum were found to present the maximum sound transmission loss and minimum weight. In order to spend the minimum amount of control energy for suppressing the vibration of FG truncated conical shells, Fares et al. [100] attempted to optimize the control forces and deflection of these shells when they were imposed to simply supported and clamped boundary conditions. The integration of total energy of the shell and control energy was taken as the objective function which was intended to be minimized using Lyapunov-Bellman theory. The most important quality of this investigation was related to incorporating the stretching effects in optimization formulations which was claimed by the authors to be significantly influential on the optimal design; especially in case of moderately thick and short conical shells. Feasible Arc Interior Point Algorithm (FAIPA) helped Moita et al. [101] to optimize FG plates and cylindrical panels choosing fundamental frequency, mass, central deflection and critical load as the objective functions or the constraints. Material variation through the thickness was assumed to follow a power-law function and the power-law index plus the thickness were taken as the design variables in all case studies. Finally, after representing five different examples, the researchers introduced the utilized algorithm as an efficient and accurate method of optimization in case of FGM.

4.2. Functionally graded vessels and pipes

Pressure vessels and pipes which are mechanical elements for containing and conveying fluids used to be made of steel. During the past decades, engineers have been attracted to manufacturing these structures from composite materials. As the safety of these elements are of significant importance, researchers tried to design FG pressure vessels and pipes and optimize their material propagation with the aim of achieving designs with the best thermal and mechanical performances. Maalavi and El-Sayed [102] exploited a mathematical programming technique via MATLAB toolbox to optimize the distribution of materials in the longitudinal direction of FG pipes. The aim of their research was to maximize the ratio of stiffness to weight and the stability of the pipes without increasing the mass even slightly when the fluid current reached the divergence velocity. They considered three different boundary conditions namely hinged-hinged, clamped-hinged and clamped-clamped for these pipes. In a comparison between the results of the FG pipes with the existing baseline designs, the former showed greater divergence velocity and higher bending stability. It is to be noted that the most important parameter in this optimization approach was the length of pipe elements. Utilizing APDL, Wang et al. [103] optimized the material composition of FG vessels with two closed ends subjected to thermal loads and internal pressure. They aimed to find the best stress propagation through these structures. The material properties such as Young's modulus and coefficient of thermal expansion were determined by power-law functions with different indices. Finally, these optimal indices which led to the minimum equivalent Von Mises stresses average variance were obtained and presented by the researchers. In order to minimize the failure probability of step-formed FG pressure vessels, Shabana et al. [104] optimized the number of layers and material distribution through the thickness of these shells using PSO method. To this end, the researchers decided to choose the maximum hoop stress jump at the interfaces and the maximum hoop stress through the thickness as the objective functions. Their results documented that changing the priority of the objective functions led to different structural stiffness. Moreover, they managed to decrease these stresses by nearly 18% and 5%, respectively.

4.3. Functionally graded spheres

A gradient based method was implemented by Boussaa [105] on temperature-dependent FG thick hollow spheres under thermal gradient to minimize the maximum Von Mises equivalent stress. Material distribution through the thickness was chosen as the design variable. The final optimized design was compared to homogeneous spheres with the same material constituents, geometry and loading and the optimized FG spheres was found to be capable of bearing nearly 100% higher magnitude of thermal loads than their homogeneous counterparts. The applicability of various types of PSO namely Particle Swarm Optimization with Passive Congregation (PSOPC), Particle Swarm with Ant Colony Optimization (PSACO) and Particle Swarm with Passive Congregation and Ant Colony Optimization (PSPCACO) methods was explained by Fereidoon et al. [106] in an optimization approach with two main examples. The first one dealt with an FG plate cooled down from 300K to 100K while minimizing its peak residual stress. The second example was related to temperature-dependent FG spheres subjected to gradient temperature which were intended to have the maximum magnitude of safety factor against yielding. The material variation profile of these structures was the design variable in both cases. The safety factor of the optimal design of the spheres was found to be 28% higher than that of their non-optimized counterparts.

5. Functionally Graded Tubes

In order to reduce injuries in accidents, crash structures are vastly utilized in automobiles. The quality of making passengers safe during any accident is referred to as crashworthiness characteristics. This factor is defined by two parameters namely Specific Energy Absorption (SEA) and Peak Crushing Force (PCF). An optimum design of any structure with high crashworthiness quality has the maximum SEA and minimum PCF. Empty tubes and tubes filled with foam are of the most suitable crash structures which have attracted many scientists' attention to optimize them so as to improve their crash behavior. To this end, during the past decade, many scientists have attempted to design FG tubes which are divided into two types. The first case are FGF tubes which contain foam with variable density. The other type is called Functionally Graded Thickness (FGT) tubes which are those with gradually varying thickness along the longitudinal direction. These two kinds of FG tubes are shown in Fig. 9 and Fig. 10.

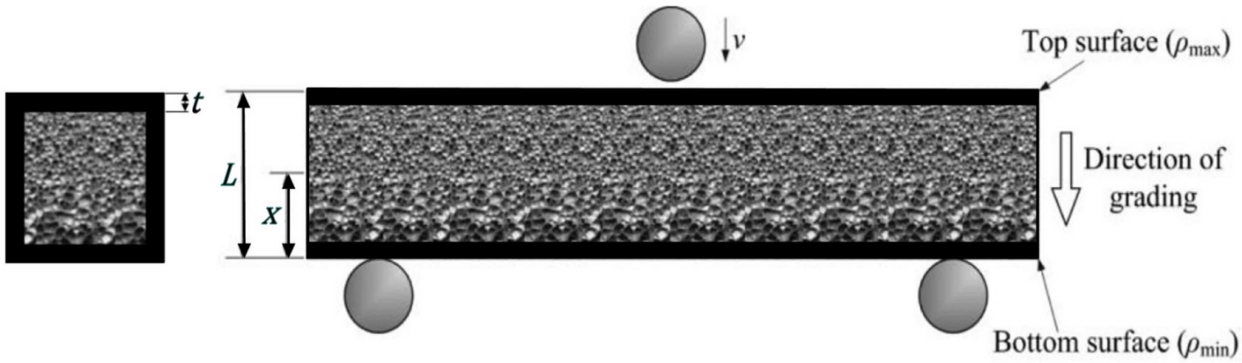

Fig.9. Schematic of an FGF tube with square cross section under lateral impact loading.

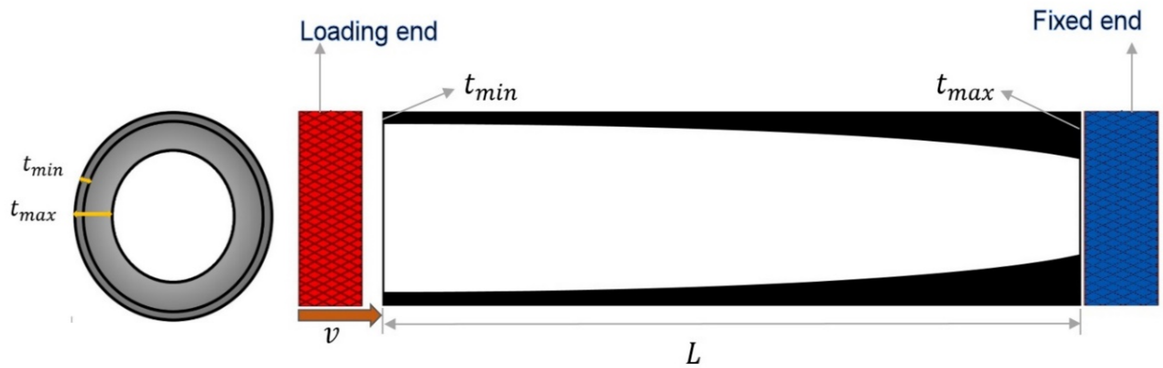

Fig.10. Schematic of an FGT tube with circular cross section under axial impact loading.

5.1. Functionally graded foam-filled tubes

Two types of Functionally Lateral Graded Foam-filled Tubes (FLGFT) were analyzed by Yin et al. [107] to boost the crashworthiness characteristics of these structures when undergoing axial impact loads. After comparing these tubes with their Uniform Foam-filled (UF) counterparts, the researchers realized the outperformance of FLGFTs over UF tubes. Subsequently, Multi-Objective Particle Swarm Optimization (MOPSO) method helped these authors to optimize the material gradation of foam evaluated by a power-law function to reduce the PCF and to increase the SEA even more. The Pareto front related to these two conflicting objectives was provided as a benchmark for engineers. A comprehensive comparative optimization study was performed by Yin et al. [108] in terms of crashworthiness behavior of FG tubes. These researchers optimized the density distribution pattern through Functionally Graded Foam-filled Tapered Tubes (FGFTT) utilizing the combination of dynamic ensemble metamodeling and MOPSO methods. Afterwards, they compared the optimized FGFTT with the conventional UF tubes and demonstrated that the ascending pattern of foam in FGFTTs had worse performance than UF tubes. However, the Pareto front extracted for the descending pattern of FGFTT proved to have better crashworthiness characteristics than UF tubes. Another comparison of this research was related to the efficiency of different methods. A number of traditional optimization methods such as RBF, Design of

Experiment (DOE), Super Vector Regression (SVR) and Polynomial Response Surface (PRS) were compared with the employed hybrid technique. The MOPSO method combined with the metamodeling method was observed to be the most efficient approach. Fang et al. [109] investigated the impact behavior of foam-filled thin walled tubes subjected to lateral impact loadings. These tubes were assumed to be filled with FGF and UF and the crash behavior of these two cases were compared with each other. This comparison demonstrated that FGF tubes had higher SEA but lower PCF compared to their UF counterparts. Subsequently, these researchers utilized PSO method and Kriging model to optimize both types of these tubes aiming to improving their crashworthiness behavior. In FGF cases, density gradation was considered as the design variable with the wall thickness, range of foam density and yielding strength as the constraints while in the UF case, the foam density was taken as the design parameter. According to the optimal designs, FGF showed better Pareto fronts. Moreover, axially graded FGF tubes outperformed their transversely distributed foam-filled counterparts in terms of SEA. In another study, Li et al. [110] applied NSGA-II to optimize the material gradation of FGF tubes subjected to oblique impact loads which led to enhanced crash behavior. Four impact angles of 0°, 10°, 20° and 30° were considered. Each impact load led to a particular set of optimal designs, PCF and SEA. Therefore, the authors decided to take all these load angles into consideration by adding weighting factors attributed to each angle in the optimization formulation. They concluded that incorporating all the impact angles resulted in a different optimal material variation through the longitudinal direction of the tubes. A comparison between conical tubes filled with FGFs and UFs subjected to axial impact loads was also represented by Mohammadiha and Beheshti [111]. Similar to cylindrical tubes, FGF tubes outperformed UF tubes regarding SEA and PCF. After this comparison, the researchers employed RSM to optimize the material distribution of foam determined by a power-law function in the tubes. The thickness of the wall was the other design variable in their study. The results showed that the optimal design, maximized SEA and minimized PCF were significantly influenced by whether the variation of materials from the top to the bottom of conical tubes were ascending or descending. The idea of filling crash boxes with honeycomb instead of foam inspired Mohammadiha et al. [112] to design and analyze these structures under oblique impact loads. These authors firstly compared Uniform Honeycomb Filled (UHF) crash boxes with their Functionally Graded Honeycomb (FGH) counterparts and noticed that the PCF of FGH tubes was always higher than UHF tubes. Moreover, they attempted to optimize FGH tubes considering the material density propagation through the box as the design variable. With this intention, four different material models namely ascending, descending, double ascending and double descending models were studied. In double patterns, density variation was calculated by two different power-law functions for each half of the crash box and these models were found to outperform simple ascending and descending models. Another factor significantly influencing the SEA was the number of honeycomb grading layers. As this parameter increased, the SEA saw a remarkable rise. Mohammadiha and Ghariblu [113] attempted to develop optimum results related to crash behavior of FG tubes and conducted a RSM-based optimization study on multi-tubes filled with FGF. In their study, sinusoidal density patterns with various numbers of extremum points were considered as well as ascending and descending patterns determined by a power-law function. According to the optimal results, ascending pattern led to higher SEA compared to its descending counterpart. Moreover, considering 11 extremum points resulted in better crashworthiness characteristics than

5 extremum pointed sinusoidal pattern. Two Dimensional Functionally Graded Foam-filled Tubes (TDFGFT) in which the foam density was assumed to vary along the longitudinal and transversal directions were optimized by Ebrahimi et al. [114] utilizing PSO method. Two power-law functions with different exponents namely n and m were used in the two aforementioned directions, respectively. Several models with respect to the range of these exponents were introduced as the optimal designs. Among all these optimized tubes, the ascending pattern with $0.2 \leq m,n \leq 1$ and the descending pattern with $2 \leq m,n \leq 10$ were proved to have the best outcomes in SEA and PCF, respectively. The researchers also pointed the noticeable superiority of TDFGFTs over UF tubes. Uniform Foam-filled Cellular Structures (UFCS) and Functionally Graded Foam-filled Cellular Structures (FGFCS) were optimized by Yin et al. [115] to enhance their crashworthiness characteristics under lateral impact loadings. NSGA-II helped these scientists to optimize the density variation of foam in the FGFCSs which led to maximum SEA and minimum PCF. Nine different cross sections which were defined by the number of cells (1, 2, 3, 4, 6 and 9) and their configurations were studied. Among all these cross sections, that with 9 cells represented the highest SEA regardless of PCF as the constraint. Taking PCF into account, several Pareto fronts were presented according to the final optimal designs of this research which are an excellent benchmark for engineers in terms of manufacturing car bodies. Ying et al. [116] utilized RBF combined with NSGA-II to minimize PCF and maximize the SEA of thin walled tubes made up of steels with FG strength in which the strength was assumed to vary via a power-law function along the tube. The gradient exponent was considered as the design variable. Critical impact load angle was another objective which rose as the gradient exponent approached 1. The results were validated by experimental and theoretical studies. The researchers also compared FG strength tubes with their uniform strength counterparts and showed the outperformance of the former over later regarding their crashworthiness characteristics.

5.2. Functionally graded thickness tubes

FGT tubes were analyzed by Sun et al. [117] in terms of their crashworthiness characteristics. The thickness tended to vary following a power-law function. The authors compared the SEA and PCF of these tubes with those of uniform thickness tubes and observed that the former outperformed the latter. Following this, they implemented RSM and NSGA-II to optimize the power-law index indicating the variation pattern of thickness aiming to maximize the SEA and minimize the PCF. They adopted several response surface models to obtain the optimal exponent and demonstrated that the higher order response surface models did not necessarily represent better results. To illustrate, in some cases, a 2nd order polynomial function led to better results than its third order counterparts. The crash behavior of FGT tubes was investigated by Sun et al. [118] when undergoing lateral impact loads. After a comparison between these tubes and the uniform thickness tubes, it was claimed that SEA and PCF of FG tubes were higher than those of uniform thickness tubes. Subsequently, these authors employed PSO method to improve the crashworthiness behavior of FG tubes under lateral impact by finding the best thickness gradation pattern. Although the results of some FGT cases could not outperform the uniform tubes in terms of PCF, the researchers managed to improve the overall crash behavior of FGT tubes by increasing SEA and decreasing PCF to a great extent. RBF models combined with NSGA-II helped Yin et al. [119] to optimize the thickness distribution pattern of FGT multi-cell tubes whose thicknesses were assumed to vary based on power-law functions. Improving their crashworthiness behavior defined by maximizing SEA and minimizing PCF acted as the objective function. Two types of RBF namely adaptive RBF and conventional static RBF were employed in this research and the former proved to be more accurate and efficient. FGT tubes made up of aluminum were optimized by Baykasoglu and Baykasoglu [120] regarding their crash behavior under impact loads. The cross section of these tubes was circular and the thickness distribution through the longitudinal direction was examined by a power-law function. The aspect ratio (length to diameter ratio) and the power-law index formed the design variables of this research. The researchers utilized GA combined with ANN acting as the objective function evaluator to carry out their optimization approach. Finally, the Pareto fronts with respect to PCF and SEA as the objective functions were represented. Depending on the priority of the mentioned objectives, the optimal design could be obtained from these Pareto fronts. Implementing an integrated NSGA-II and RSM, the same optimization problem was solved by Yao et al. [121], and a number of interesting results were extracted. Increasing the aspect ratio and decreasing the power-law exponent led to higher SEA while increasing the two mentioned design variables resulted in lower PCF. Oblique impact loads were also investigated in this study by considering 10°, 20°, 30° and 40° of load angles. Increasing the angle loads caused decreasing trend in the PCF and SEA of the mentioned structures. In order to make the design of crash tubes more flexible and improve their crashworthiness characteristics, Yin et al. [122] modeled a new type of these structures in which the foam density varied along the transverse direction and the thickness of the tube varied along the longitudinal direction. These structures were referred to as Functionally Graded Foam-filled Graded Thickness (FGFGT) tubes. According to the final results of FGFGT tubes, the best optimal design was associated with tubes whose middle was the thickest part. In this case, when the weight of the optimized FGFGT tubes were equal to that of their UF tubes, the former showed better crashworthiness behavior. On the other hand, the design with minimum thickness the middle of FGFGT tubes showed lower SEA in comparison to their UF counterparts with the same weight. The grading pattern of the thickness and foam density were the design variables of this study.

5.3. Functionally graded thickness inversion tubes

An Inversion tube is another crash structure which consists of a die and a tube. When this structure undergoes impact loads, the solid die fills the tube and collapses it. Therefore, the energy of impact load transforms to distortion of the tube. Functionally Graded Thickness Inversion Tubes (FGTIT) are an innovative design of crash structures in which the thickness of the tube varies gradually along the longitudinal direction. These structures can be seen in Fig. 11.

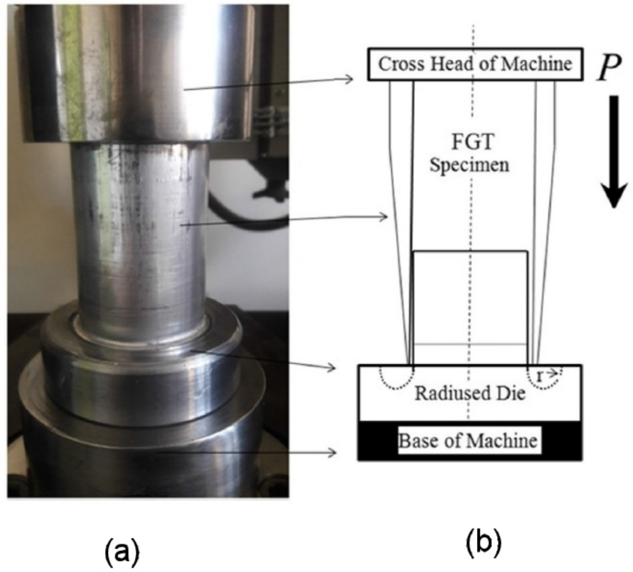

Fig.11. (a) An FGTIT specimen under impact loading. (b) A schematic of the die and FGT in FGTIT [123]

Mohammadiha and ghariblou [123] performed a numerical optimization study on the crashworthiness characteristics of FGTIT which contained a FGT tube and a circular die. Dynamic Amplification Factor (DAF) with which dynamic deformation performance of the tubes are assessed was considered as extraordinary objective function along with SEA and PCF. Die radius, coefficient of friction between die and tube and the gradient index by which the thickness varied were taken as the design variables in this research. The results demonstrated that concave function along the tube could significantly increase SEA with no remarkable increase in PCF. The optimum friction coefficient values were found to be under 0.04 which noticeably enhanced the crashworthiness characteristics of FGTITs. In another similar study regarding FGTITs subjected to 5° and 10° oblique impact loads, these authors [124] considered the same objective functions and design variables. The best friction coefficient was calculated to be under 0.033, and the optimum die diameter was 3.5mm. Moreover, the researchers proved that the non-linear thickness variation of the tubes resulted in the best crashworthiness behavior of FGTITs under oblique loads. The results of this numerical study was verified by experimental tests and in both cases of oblique and axial impact loads, the authors countered that FGTITs were superior to uniform thickness tubes.

6. Other Functionally Graded Structures

Numerous advantages of FGMs have attracted many scientists to implement these materials in different areas such as orthopedic and dental implants, rotating disks, turbine disks, sports equipment, structures composed of piezoelectric materials, etc. These types of structures which cannot be classified in three main mechanical structures namely beams, plates and shells as well as tubes, are compiled in this section. Moreover, there are a few studies describing the behavior of FGMs regardless of the structural shape which are reviewed and presented at the end of this section.

There are two key parameters in designing a dental implant namely Osseo-integration and bone remodeling. The mechanical properties of implants' synthetic materials mismatch the native biomaterials and this fact deteriorates the two aforementioned essential parameters of dental implants. Manufacturing these implants from FGM varying from biomaterials to synthetic materials would be a wise idea to overcome this problem. Accordingly, Lin et al. [125] designed and constructed a model containing a FG dental implant and four other teeth. They subsequently calculated the bone remodeling stemming from implementing different FG implants over a four-year period. The results helped these scientists to optimize the material gradation of FG implants and enhance the process of bone remodeling which was determined by three specialized functions. The implant material distribution was assumed to follow a power-law function and the results showed that reducing the power-law exponent led to higher performance of the implant. However, reduction in the power-law index resulted in lower stiffness which made the whole design vulnerable over the healing period. Therefore, Pareto fronts were represented by the researchers as a benchmark to tailor these types of structures in the most appropriate manner. Following this research, Sadollah and Bahreininejad [126] utilized Suppapitnarm Multi-Objective Simulated Annealing (SMOSA) and MOGA to improve the results of RSM taking the same objective function and design variables into consideration. Their results were improved up to 33% compared to those of RSM. They also reported that the two meta-heuristic algorithms showed close optimal results. They finally represented a wider range of optimal power-law index varying from 0.1 to 0.65. A similar study conducted by Sadollah et al. [127] employing MOPSO method sought the same objectives and led to quite the same results as the previous research. Utilizing the optimization toolbox of Solidworks software founded on the DOE Rechtschafner design, Ichim et al. [128] represented a new design of dental implants enveloped by two coatings made of porous zirconia to minimize the stresses in their surrounding bones. These implants were analyzed when undergoing loads with angles of 0°, 15°, 30° and 45°. The elastic moduli of the coatings were the design parameters of this study and the researchers demonstrated that covering the stiff core of the implant (E_{core} = 110Mpa) with softer porous coatings ($E_{coating}$ = 45Mpa) was sufficient. In other words, a grading pattern of changing the young modulus from the stiff core to the soft coatings had negligible impact on stress minimization. According to the final optimum results, these resilient coatings led the average stresses in peri-implant bone to decrease by 15%. The equivalent value for compressive and tensile stresses was 50%.

After hip implant surgeries, in order to reduce all the adverse consequences which are attributed to two main factors namely bone resorption and bone-implant interface instability, Khonaki and Pasini [129] made an attempt to design a novel graded cellular hip implant. Subsequently, they determined the bone-implant interface instability and bone resorption by two specific functions and carried out an NSGA-II-based optimization experiment to minimize both parameters. Material gradation was chosen as the design variable of their study. In order to illustrate how efficient and superior this novel design was, the researchers compared their optimal design with hip implants made up of titanium and those constituted from foam. The interface stresses between the implant and bone as well as bone resorption of the optimized design were found to be 50% and 70% less than those of Titanium design, respectively. These values were 65% and 53% while comparing the foam design with optimized graded hip implant in favor of the latter case. Hedia and Fouda [130] attempted to design artificial hip stems which outperformed the former designs in terms of strength and life of the joint implant. The rivals of this novel FG design consisting of collagen and Hydroxyapatite (HAP) were titanium stems with and without HAP coatings. Choosing the elimination of stress shielding at the medial proximal region of the femur as the objective function, the authors conducted an optimization approach in which the material distribution and thickness of their novel design played the role of the design variables. The volume fraction distribution of material constituents was calculated by a power-law function, and the optimized power-law index was found to be 0.1. This design demonstrated noticeable declines in the maximum lateral and medial shear stress while the Von Mises equivalent stress at the medial proximal region increased significantly. Bahraminasab et al. [131] employed FEM combined with RSM to optimize the femoral component of knee implants made up of FGM. This optimization study was carried out to eliminate the loosening problem of knee implants. Therefore, the authors defined maximizing stresses in the distal femur at the interface of the bone/implant, minimizing micro-motions at the interface between the bone and the implant and minimizing wear index of polyethylene insert as the objective functions. The volume fraction of material constituents and their porosity variation pattern were taken as the design variables in this multi-objective study. The researchers compared their optimal result with a benchmark alloy made of cobalt and chromium and achieved averages of 3.8%, 13.6% and 0.6% enhancement in the aforementioned objectives, respectively. Mehboob and Chang [132] took advantage of ABAQUS software to simulate fractured tibia and the bone fragments which were assembled by a FG biodegradable bone plate and a number of screws. Subsequently, mechano-regulation theory was adopted along with deviatoric strain and FEM analyses to determine the healing performance of the fractured bone. Finally, Taguchi method and DOE were exploited to maximize the healing performance of the mentioned structures. The average elasticity modulus, material distribution and thickness of these FG structures were the design variables of this research and the optimum values of these parameters were obtained after the optimization process.

Improving the performance of safety helmets motivated Rueda et al. [133] to optimize the foam liner part without increasing the size or weight of these helmets. Increasing the weight of these structures leads to higher inertia of the helmets. Therefore, the possibility of neck injuries are increased. The performance of helmets depends on reduction in peak acceleration which is related to three key features namely contact areas, distribution of material stresses and dissipated plastic energy density. The researchers claimed that varying the foam material through the thickness and increasing the contact area had great influence on the energy absorption of these helmets. In another study, the same authors [134] decided to use FGF liners to avoid crack propagation and delamination of liners. After modelling these new designed helmets, they attempted to find the optimum foam configuration through the thickness of liners which led to the best energy absorbing behavior. They finally compared FGF liner helmets with UF liner helmets and demonstrated the outperformance of the former when both types were assumed to be subjected to various impact velocities and positions.

Corrugated rods which contain a number of curvatures with specific steps and amplitude are implemented as reinforcement in many engineering structures. FG corrugated rods were introduced and optimized by Andrianov et al. [135] to increase their stiffness. The study succeeded in optimizing the variation of amplitude and steps of corrugation in these structures and concluded that these parameters are mostly effective on longitudinal stiffness and compression-extension stiffness, respectively. In order to demonstrate the outperformance of FG strength structures consisting a gradual range of low strength to high strength materials over their uniform strength counterparts in terms of crashworthiness characteristics, Ying et al. [136] analyzed these two types of materials using nonlinear finite element program LS_DYNA. For this purpose, columns made of these materials were considered to be under crashing and oblique impact loads. Subsequently, an NSGA-II-based multi-objective optimization study was conducted to find the best strength propagation through FG strength columns which led to minimum PCF and maximum SEA. According to the results, the best crashworthiness behavior was related to columns with the lowest strength gradation index and designs with the highest values of top strength.

In order to increase the safety of roller compacted concrete gravity dams in terms of seepage and cracking, Zhang et al. [137] designed dams in which the material properties along the water flow varied functionally. These dams are called FG partition structures . Implementing optimization approaches based on GA, Multi-Island Genetic Algorithm (MIGA) and Adaptive Simulated Annealing (ASA), they managed to minimize the cost of these structure considering their hydraulic fracture as the constraint. ASA was the fastest algorithm among the mentioned utilized methods according to the final results.

As disks are vastly implemented in different industrial applications and undergo large amounts of thermal and centrifugal loads, there is a desire to boost their mechanical characteristics. An optimization approach founded on Sequential Linear Programming (SLP) was conducted by Chen and Tong [138] to minimize the total weight of FGM subjected to heat flux and temperature loads. A long slab, a circular disk and a turbine disk made up of these materials were investigated as examples. In the last example, the researchers took deflection and maximum equivalent stress as the constraints and managed to minimize the weight of FG turbine disk by 10%. Utilizing CAMD method, Stump et al. [139] attempted to minimize the volumetric density of FG rotating and turbine disks. The material variation profile was the design parameter of their study with the Von Mises yield criterion as the constraint. The same turbine disks subjected to centrifugal force was optimized by these authors [140] with the aim of minimizing the volume fraction of one of the material constituents considering Von Mises equivalent stress as the constraint. SLP was the optimization method they utilized to optimize the material distribution of FG turbine disks. Khorsand and Tang [141] employed a combined Co-evolutionary Particle Swarm Optimization (CPSO) method and DQM to optimize the weight of FGT rotating hollow disks and to minimize the deflections and centrifugal and Von Mises equivalent stresses of these structures. These disks were assumed to be under thermo-mechanical loads. The weight reduction results associated with this approach were found to be greater than those of other methods. The influences of angular velocity along with the thickness variation pattern on the mentioned stresses, deflection and weight optimization were also evaluated in this research. The material propagation and geometry of FG rotating hollow disks were optimized by Tharun et al. [142] in order to minimize the mass, maximum deflection and stresses of these structure when they undergo thermo-mechanical loads. Mechanical loading was attributed to rotating with certain angular speed and thermal load stem from temperature gradient along the radial direction. The researchers also considered constant and variable thickness disks in their study. Finally, they demonstrated that the most significant factors on the optimality of these structures were the ratio of inner to outer radii and angular speed while the temperature had the minimum influence on the optimal design of these disks.

The tribological application of coatings encouraged many surface engineers to optimize their characteristics. The reason lies in the significant influence of coatings on enhancing the environment sustainability and declining the life-cycle costs of many structures made of sensitive materials. Nadeau and Ferrari [143] implemented Downhill simplex method to optimize the material properties distribution of FG composite coatings made up of silicon/carbide and aluminum. The properties and propagation pattern of the reinforcing fibers embedded in the matrix of these composites acted as the design variables of this research. The curvature and strain energy density of these layers were taken as the objective functions. The authors compared the results of FG coatings with the equivalent optimized bi-layered coatings and claimed that the former could represent nearly 86% better performance taking both objective functions into account. Xu et al. [144] employed a modified PSO method to minimize the thermal residual stresses of step formed five-layered FG carbon/silicon carbide coatings as face sheets of sandwich structures with carbon/carbon composite core after cooling down. They considered the thicknesses and material distributions as the design variables. The volume fraction distribution through the thickness of these coatings followed a power-law function with the exponent varying from 0.02 to 2. With respect to the optimized results, the best optimized power-law index was nearly 0.08 which led to the minimum amount of residual stresses. In addition, two completely different optimum designs were achieved depending on whether $p \geq 0.08$ or $p < 0.08$.

The application of gears in industries is remarkable, and the most prominent objective of these structures is to transmit energy. Minimizing the weight of gears has always been important to engineers since this leads to less energy consumption and higher efficiency of these structures. Jing et al. [145] utilized Goal Driven Optimization (GDO) method to optimize gear teeth constituted of FGM. The aim of their research was defined as minimizing the maximum deflection of the teeth and the total weight of these gears taking the maximum allowable stress as the constraint. After optimizing the teeth material, the researchers compared the gears with FG teeth with their totally homogeneous counterparts and reported that the former design had 32% less weight and 6% less maximum deflection in the teeth. They also recommended an optimization analysis on gears whose all parts are made of FGM. Considering the printability constraint was another suggestion the authors mentioned which could result in a more practical design.

Injection molding is the process of producing specific parts in large scale. As the mechanical stability of injection molds deters during molding cycles, it is important to improve their design by increasing their load bearing capacity and their resistance against thermal loads. Using FGMs for manufacturing injection molds and optimizing them are of viable options to reach these objectives. FG lattice injection molds were optimized by Wu et al. [146] to reduce their weight with their thermal conduction and stiffness being the constraints. Accordingly, a number of lattice cells were distributed through the core of the injection mold so that the mass of these structures reduced significantly with a negligible penalty on the thermal and mechanical constraints. The final optimum design demonstrated nearly 9% and 3.5% reduction in the stiffness and heat conduction, respectively while the structural weight reduced up to 30%. Although huge amount of weight increase was saw in this optimization process, enhancements in stiffness and heat conduction of the optimized injection molds were perceived as compensation.

Piston rings which are located between the cylinder and piston of engine have four principal functions namely compression gas sealing, lubricating oil film control, heat transfer and supporting piston in the cylinder. Due to the significant role these structures play in the engine, engineers are driven to improve their performance. Carvalho et al. [147] optimized the material gradation of AlSi-CNT FG composites to make a compromise between the wear, fatigue, tensile strength, hardness and cost of these materials which were intended to be implemented in producing piston rings. The best results related to fatigue and wear characteristics of these materials was obtained when the composite was reinforced with 6% wt. CNT. In this case however, the other two mechanical properties degraded. 2% wt. CNT designs were found to have the best behavior regarding all the mentioned characteristics while the cost increased significantly. Therefore, the researchers finally introduced a linearly graded pattern for CNT as the optimal design so that the surfaces contained 2% wt. CNT and the middle of these structures contained 0% wt. CNT.

There is a long history with regards to materials of golf club heads from wood to steel, titanium and recently FG porous materials. The main reason for producing different designs of golf club heads was related to their beauty and functionality. Over the last decade, the dominance of the latter is getting more and more obvious. In order to improve the performance of golf clubs, Ray et al. [148] optimized the porous propagation of club heads made up of porous titanium. Three different objective functions namely reducing the weight, reducing the coefficient of restitution and increasing the contact time were taken into account, and all of these parameters were found to be influential on the shot performance of clubs to a great extent. It is noteworthy that the numerical results of the impact behavior of club heads against the steel balls were extracted from LS DYNA commercial software.

Piezoelectric materials are novel designed materials which are capable of converting electrical energy to mechanical energy and vice versa. Since these materials are vastly implemented in various applications such as computer, military, medical and automotive industries, researchers are trying to improve their characteristics. Recently, employing the concept of FGM in piezoelectric materials has enabled scientists to promote the performance of these novel materials. Energy harvesting of FG piezocomposites which is mostly attributed to the coefficient of electromechanical coupling and microscopic stresses was maximized in an approach based on Discrete Material Optimization (DMO) method presented by Vatanabe et al. [149]. Accordingly, the polarization direction of piezoelectric material and material gradation were selected as the two design parameters. The researchers compared the FG piezocomposites with their non-FG counterparts. Although the latter case showed lower electromechanical coupling coefficient, the microscopic stresses in the FG piezocomposites were claimed to be significantly lower than non-FG designs. Therefore, the FG pattern in piezocomposites resulted in harvesting more electrical energy. Material gradation in Functionally Graded Piezoelectric Transducers (FGPT) was optimized through a topology optimization approach represented by Rubio et al. [150]. They intended to find the maximum eigenfrequency of a selective eigenmode. The optimization process was conducted through the framework of SLP and CAMD. Looking at the optimal results, the gradation in the thickness direction was significantly more effective on the eigenfrequencies than the gradation through the longitudinal direction.

Schaller and Yang [151] implemented swarm search algorithm and GA in an optimization study with the goal of stress minimization in FGMs. They incorporated a three-layer FGS structure whose core was constituted from FGM and the face sheets were homogeneous metal and ceramic. The objective was to minimize the stress of the ceramic surface when this structure was assumed to be under thermal loads. They finally managed to decrease maximum stress at the ceramic surface from 45Mpa to 20Mpa by optimizing material gradation and changing the pattern from a linear to an optimal function. Goupee and Vel [152] implemented NSGA combined with EFG method to optimize the material distribution of FGM subjected to thermal loadings in two problems. The objective functions in the first problem were defined as simultaneously minimizing mass and peak effective stress under temperature gradient. The second problem involved minimizing the mass and maximizing the safety factor of the structure simultaneously under intense heat flux. In this case, the maximum temperature experienced by the metal phase as well as the factor of safety played the role of constraints. The researchers finally prepared Pareto fronts offering a wide range of minimized mass and maximum equivalent Von Mises stresses as a benchmark to designers. In order to enhance the heat treatability and minimize the thermal stress distribution of FG slabs in thermal environments, Molaei Najafabadi et al. [153] investigated the influences of Fourier number in the thermo-elastic solution and the impact of material distribution of these structures on their deflection, stresses and temperature. This optimization problem which was solved via quasi-Newton method was restrained by the critical temperature. The researchers realized that exploiting materials (whether the ceramic or metal phase) with higher temperature diffusivity led to better heat treatability. Furthermore, it was claimed that for all values of Fourier number, as the critical temperature increased, the optimized power-law exponent was seen to be higher.

7. Conclusion

The majority of publications on optimization of FG structures were reviewed and compiled in this paper. These researches were categorized in accordance with the type of structures. Beams, plates, shells, tubes and other structures constituted from FGMs were sequentially investigated in this research. As the key feature of FGMs is their gradual and smooth variation of material constituents, the most common design variable in the reviewed researches was the material distribution pattern. Stress distribution, critical buckling load, fundamental frequency and the weight of FG structures were the most popular objectives researchers sought to improve through optimization studies. Methodologies based on GA and PSO were the most frequently utilized algorithms in the discussed field. There were a number of studies carried out with the aim of reducing the computational time and increasing the accuracy. According to these researches, combining stimulating techniques such as ANN or ANFIS with a metaheuristic algorithm could lead researchers towards these objectives. To conclude, the following optimization approaches are suggested by the authors to conduct in the future.

- There were a few optimization studies in which the manufacturability of the FG structure was taken into consideration. Adding manufacturability to the optimization studies is recommended since it leads to more practical designs with prospects of being produced in large scales.
- There were a few studies on FG structures with piezo electric patches or layers. As piezo-electric materials demonstrated to be influential on the thermo-mechanical behavior of various designs, it is suggested to investigate the optimum FG structures embedded between two piezo-electric layers or FG structures with attached piezo patches to their surfaces.
- The cost of a design is one of the most important issues which should be considered by designers. However, there were a few studies selecting this prominent feature as the objective function. Therefore, adding the cost of optimal design of a FG structure to the optimization procedure is highly recommended.
- The impact performance of FG beams, plates or shells has attracted little attention in the literature. Therefore, the authors suggest more contribution to the development of optimal designs for these structures when undergoing impact loads.

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