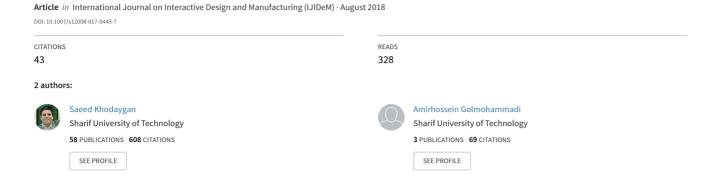
Multi-criteria optimization of the part build orientation (PBO) through a combined meta-modeling/NSGAII/TOPSIS method for additive manufacturing processes



ORIGINAL PAPER



Multi-criteria optimization of the part build orientation (PBO) through a combined meta-modeling/NSGAII/TOPSIS method for additive manufacturing processes

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Abstract

Additive manufacturing (AM), is a new technology for the manufacturing of the physical parts through an additive manner. In the AM process, the orientation pattern of the part is an important variable that significantly influences the product properties such as the build time, the surface roughness, the mechanical strength, the wrinkling, and the amount of support material. The build time and the surface roughness are the more important criteria than others that can be considered to find the optimum orientation of parts. The designers and manufacturing engineers usually attempt to find an optimum solution to reach the product with high quality at the minimum time. Determining the optimum build orientation of the virtual model in the design stage for the additive manufacturing to reach a real production with higher quality at the lower time can be an effective strategy to success in the competitive environment of manufacturing firms. In this paper, a new combined metamodeling/NSGA II/TOPSIS approach is introduced to search the accurate optimum PBO in the AM based on the multi-criteria optimization formulation. In order to reach this aim, first, a new formulation is proposed to model the build time with respect to the PBO in AM processes. Then, a proper formulation is developed to estimate the mean surface roughness based on the part orientations. By utilizing Kriging method as a powerful meta-modeling approach, the build time and the surface roughness as the objective functions are modeled in the explicit form in terms of the part orientation. Then, the non-dominated sorting genetic algorithm II (NSGA-II) is utilized to solve the multi-criteria optimization problem with the build time and the surface roughness as the objective functions. Consequently, Pareto-optimum solutions are obtained from the optimization problemsolving. The TOPSIS method is employed to rank all obtained optimum solutions for selecting the best solution. The proposed approach aims to precisely find the optimum PBO for the several AM processes under the low computational time. Finally, to illustrate and validate the efficiency and accuracy of the proposed approach two case studies are considered and the obtained results are compared and discussed.

Keywords Additive manufacturing · Multi-criteria optimization · Part build orientation · Build time · Surface roughness

1 Introduction

The AM as a revolutionary method of manufacturing refers to a new manufacturing process to build up the physical part in layers by depositing material based on the CAD model [1]. After establishing the first commercial device of the AM in 1986, the AM technique has been developed as a new

manufacturing technology to produce parts with the complex geometry in the low build time and high precision [2]. Unlike the machining process, the AM processes are an additive procedure in which the part is fabricated in layered form through overlaying of materials on the previous material layers [3]. For prototyping of a part based on the AM method, first, the designed part in CAD system should be saved in the STL format. The effective parameters of the AM process which are the laser power, layer thickness, the powder spray speed and the build orientation, should be adjusted to the proper level. Then, the part is built through the adding layered materials. Finally, the post-processing operations as the support structure removal are followed to finish the fabrication of the part by the AM [4]. The most well-known AM

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processes are including SLA,¹ SLS,² SLM,³ and so forth. These categories are usually based on the material used to make the part. Generally, in SLA, the resin and in both SLS and SLM processes, the metal or polymer powder materials are utilized [5,6].

The orientation pattern as the main factor can affect the properties of the manufactured part [7]. The part build orientation (PBO) influences the mechanical properties such as the ultimate tensile strength, the average plastic strain rate, and the microstructure properties [8,9]. Therefore, the PBO should be specified before beginning the manufacturing process.

Some researchers have conducted studies to obtain the optimum PBO in the AM processes for several goals, such as the surface roughness, the build time, and the support material amount. Frank and Fadel [10] proposed a method to select a suitable PBO through the AM. The build time and the surface roughness were assumed as the targets based on the experimental results. Cheng et al. [11] proposed a method to select the optimum orientation in the SLA process assuming the dimensional accuracy and the build times as targets. Lan et al. [12] presented a method to find the optimum PBO to maximize the surface quality and minimize the build time and the complexity of the support structures. In order to improve characteristics of the surface, the surface area with the worst quality was minimized. Padhye et al. [13] suggested a method to calculate the optimum PBO by minimizing the build time and the surface roughness together for SLS process based on the statistical measures. The optimization problem was solved using the evolutionary optimizers for the multi-objective problem. Alexander et al. [14] suggested a method to find the optimum PBO to reach the high quality of surface and the low required support materials. So, to achieve the desired surface quality, the stair stepping effect has been minimized. Moroni et al. [15] evaluated the optimum orientation of an assembled model with the cylindrical surfaces made by the AM technique. Phatak and Pande [16] presented a method to find the optimum PBO based on the genetic algorithm. The required height, the average value of roughness and the required material mass for the fabricating part were considered as a single objective optimization problem.

Some researchers have focused on estimating the build time for several AM processes. Giannatsis et al. [17] developed a method to compute the build time in terms of the surface area and volume of the part. Sanatinejad et al. [18] proposed a method to estimate the fabricating time for the constant and variable layer thicknesses. Pandy [19] presented a model to estimate the build time based on the build time of

³ Selective Laser Melting.



each layer and the number of layers. The fabricating time of each layer is the sum of times that are depended on the area of each surface and the amount of its support material. Sullivan et al. [20] empirically considered the effects of the scan algorithm and the laser movement speed on the fabricating time. Campbell et al. [21] proposed a method to approximate the fabricating time based on the converted volume and the free space under the part for using the support material. Zhang et al. [22] used the grey theory to present a method to estimate the build time in the AM processes. The inputs of the grey system are the fabrication machine factors such as the laser power, the part characteristics, the quality parameters, and the system output is the estimated build time. Choi [23] developed a model to calculate the build time in the SLS process. The model was established upon the speed of laser movement, the height of part and the temperature of the fabricating chamber. Also, some researchers have conducted the studies to find a proper relation between the build orientations and the surface roughness in the AM processes [24,25].

Interactive design as a novel paradigm can integrate user expectations in the product development process, allowing the designer to interact with the virtual product and its environment [26]. Interactive product design is a major economic and strategic issue in innovative products generation. In interactive design, the creation of a product is considered by three factors: the experts' knowledge, the end-user satisfaction and the realization of functions.

The designers and manufacturing engineers usually attempt to find an optimum solution to reach the product with high quality at the minimum time. Today, with the advancement in additive manufacturing technologies, the need to produce the part with better quality at a shorter time is necessary. This leads to conquering of the target market and the satisfaction of customers [27]. In order to make progress in the manufactured parts, the interactive design aids engineers to implement virtual prototypes enabling the interaction between real and virtual elements [26]. In the case of additive manufacturing applications, the interaction of real and virtual objects has to be incorporated into the analysis and design of the virtual model. In order to reach this aim, the proposed method can be used as a useful and valuable tool. In the present study, a new efficient approach is proposed to determine the accurate optimum orientation of the build part in the AM processes through a combined meta-modeling/NSGA II/TOPSIS algorithm. The input of the proposed method is the virtual part in CAD format and its output is the optimum build orientation for construction by 3D printers Developing an efficient method for determining the optimum build orientation for minimization of the build time and the surface roughness in the additive manufacturing at the design stage, can improve the interactivity level of the design for manufacturing procedure. Since the

¹ Stereolithography.

² Selective Laser Sintering.

proposed algorithm can be simply automated for use within CAD/CAM software, it can be utilized as an interactive useful tool in the design for additive manufacturing of products. Based on the proposed method, determining the optimum build orientation of the virtual model in the design stage for the additive manufacturing to reach a real production with higher quality at the lower time can be an effective strategy to success in the competitive environment of manufacturing firms.

In order to reach this target, a new formulation to approximate the build time is proposed. Based on a proper model, the surface roughness of the part is described considering the build orientation. To formulate the optimum design problem, the main criteria are precisely modeled based on the Kriging method. To find the Pareto front of the optimum orientations, the multi-criteria optimization is solved by the NSGA II. Finally, the best orientation is obtained through the TOP-SIS method.

In the following; In Sect. 2, the basic steps of the proposed method are presented in details. In Sect. 3, to demonstrate the capability of the proposed method to find the accurate optimum PBO, two case studies are considered. Finally, the paper followed by the conclusions in Sect. 4.

2 The proposed method to find the multi-criteria optimum PBO

In this section, the proposed method to determine the optimum PBO are described in details. The proposed method can be presented in the following main steps:

- Step 1 Part modeling in STL format.
- Step 2 Estimating the build time in terms of the PBO based on a new formulation.
- Step 3 Estimating the mean roughness of the part in terms of the PBO.
- Step 4 Generating the sample orientations based on the Latin Hypercube Sampling (LHS) method and evaluating criteria at the sample PBOs.
- Step 5 Meta-modeling the objective functions based on the Kriging method.
- Step 6 Formulating and solving the multi-criteria optimization problem by the NSGAII method.
- Step 7 Selecting the optimum PBO from the Pareto front by the TOPSIS method.

2.1 Part modeling in STL format

"STL" format is commonly utilized for importing a CAD model into the 3D printing machine. The term "STL" usually is referred to "Stereolithography"; also it has been referred to "Standard Triangle Language" and "Standard Tessellation

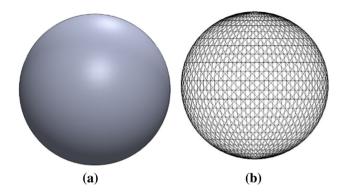


Fig. 1 The designed part in a CAD model, b STL format

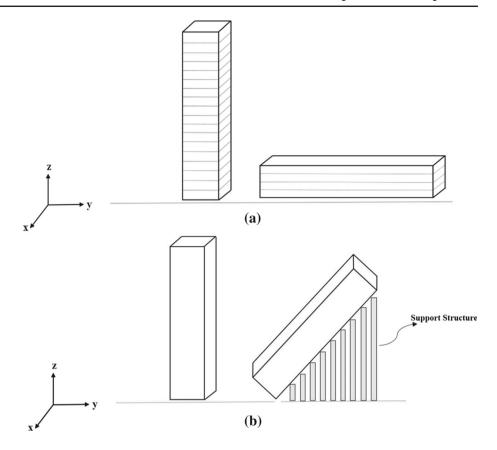
Language" in some other publications [28]. When a designed part is converted to the STL format, whole of the model is divided into a lot of the triangular elements in the 3D space. Figure 1a, b show the designed part in the CAD model and the STL formats, respectively. In the STL format, when the number of triangular elements increases, the dimensional accuracy, the file size, and also the creation time of the part sections are accordingly increased [29]. The coordinates of the triangular elements and the corresponding normal vectors can be extracted from the STL file. Therefore, in the first step of the proposed algorithm, the part is modeled in the STL format.

2.2 Estimating the build time in terms of the PBO based on a new formulation

The build time in the AM processes depends on many variables such as the power of the machine, the scan rate, the type of hatching layers, the hatch spacing, the velocity of powder spraying, the orientation of part, the number of constituent layers of the part and the amount of consumed support material. In the AM, the PBO directly affects the number of layers and the amount of support material. For example, in the building, a cube through the AM process, the number of layers in the vertical state is more than the horizontal state (see Fig. 2). Since in the AM process, the parts are made in layers form, the layers need to be supported by the material basis. If the support materials are not used, the part will be overturned. Although the support structures and materials are the essential components in many AM processes, the material supports are not required in all AM processes [30]. For example, the SLA process requires the support structure to build, while the SLS process does not need it. In AM processes that the support structure is needed, it is should be added to the part model and should finally be removed from the built part [31]. Accordingly, some studies have been conducted for minimizing the support-material that can lead to reducing the build time [32].



Fig. 2 The effect of the PBO on a the number of layers, b the amount of support material



For developing a proper method to approximate the build time with related to orientations of the part, the build time (T) can be considered as a function of the number of material layers (n) and the amount of the support material (S):

$$T = f(n, S) \tag{1}$$

For calculating the number of layers and the support material, the STL format can be used. For this purpose, a function is defined that its input is the STL format of the CAD model and its outputs are coordinates of the triangular elements and the corresponding normal vectors. Figure 3 shows the coordinates of a triangular element of the part and the corresponding normal vector in the STL format.

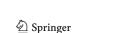
The number of layers (n) can be obtained as follow:

$$n = \frac{h}{d} \tag{2}$$

where d is the layer thickness and h is the effective height of the part in the fabrication.

The effective height of the part in the fabrication is the difference between the highest and the lowest values of the vertices coordinates along *z*-direction in the global coordinate system as (see Fig. 4);

$$h = z_{max} - z_{min} \tag{3}$$



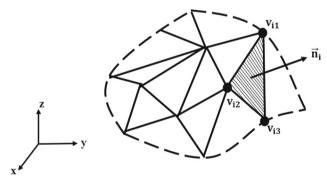


Fig. 3 The coordinates of a triangular element of the part and the corresponding normal vector in the STL format

 z_{min} and z_{max} are the lowest and highest z- components of vertices coordinates, respectively. Since the thickness of the layers is constant, the number of layers is only related to the effective height of the fabrication.

The amount of support material (S) can be determined based on the downward faces and the corresponding normal vectors. The normal vectors with third components in the opposite of z-direction (the z-component with a negative value) in the global coordinate system belong to the downward faces. In this work, the downward faces are named A_{down} . Also, the effective height of A_{down} from the bottom

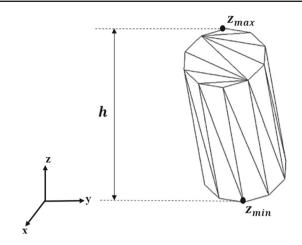


Fig. 4 The effective height of the part

of the build chamber (\bar{z}) is a key factor because the support material needed is directly related to it (see Fig. 5).

For computed the support material of a part, first the specifications of the triangular elements and the corresponding normal vectors are derived from the STL file (Fig. 6). For the specified triangular element, the corresponding normal vector and the coordinates of its vertices can be written as;

$$\vec{n} = (n_1, n_2, n_3)$$

$$v_1 = (x_1, y_1, z_1)$$

$$v_2 = (x_2, y_2, z_2)$$

$$v_3 = (x_3, y_3, z_3)$$
(4)

According to Fig. 6, since the third component of the normal vector of the specific triangular element is negative, the element is the downward face and it needs the support material.

For determining the effective height of A_{down} , the average of z-component of the coordinates can be calculated as below;

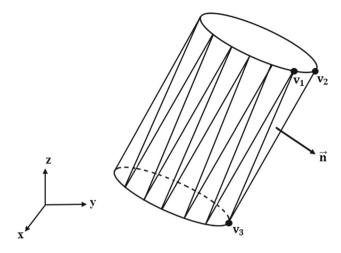


Fig. 6 The coordinates of the triangular elements and the corresponding normal vectors

$$Z_{dowm} = \frac{z_1 + z_2 + z_3}{3} \tag{5}$$

Also, the area of A_{down} can be obtained as follow;

$$A_{\text{down}} = 0.5 \left| (v_1 - v_3) \times (v_2 - v_3) \right| \tag{6}$$

Therefore, the amount of the support material (S) for building the part can be obtained from;

$$S = \frac{\sum_{i=1}^{m} Z_{\text{down}_i} A_{\text{down}_i}}{\sum_{i=1}^{m} A_{\text{down}_i}} \left(1 + \sum_{i=1}^{m} \left| n_{3_i} \right| \right)$$
 (7)

where m is the number of downward faces, n_3 is the normal vector z-component. The summation of the absolute z-component of the normal vectors, in Eq. 7, presents the deviation of downward faces from the z-axis (vertical direction).

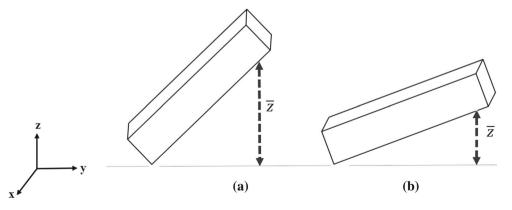


Fig. 5 The PBO effect on the effective height of surfaces and the needed support material. a The part with high effective height, b the part with the low effective height

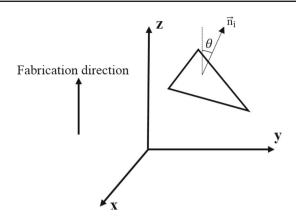


Fig. 7 The STL triangle orientation according to fabrication direction

Using Eq. 7, the build time (T) as the first objective can be estimated as follow:

$$T = K \left(z_{max} - z_{min} \right) (1 + S) \tag{8}$$

where K is the time calibration constant for expressing the built time in units of seconds, minutes or hours.

2.3 Estimating the mean roughness of the part in terms of the PBO

One of the common problems in the rapid prototyping processes is the stair stepping effect that is happened in the layered manufacturing. This effect can lead to the high surface roughness and the undesirable generated surface. The PBO can directly affect the surface roughness. The several studies have been conducted to find a mathematical relationship between the surface roughness and the PBO in the AM processes.

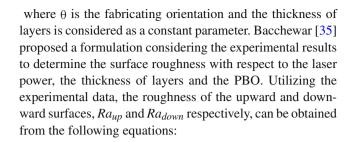
Campbell et al. [33] investigated the effect of the surface angle on the roughness of the experimental samples which have been produced by various rapid prototyping methods. The result of their research has been formulated as follow;

$$Ra = \alpha \times \sin\frac{\theta}{4} \times \tan\theta \tag{9}$$

where α layer thickness and θ is is the angle between fabricating direction and normal vector on each of the surfaces (see Fig. 7).

McClurkin and Rosen [34] developed a model using the experimental results to estimate the roughness of the triangle surfaces in the STL file. According to their research, the roughness of surfaces can be computed as below;

$$Ra_{i} = \begin{cases} 0.0254 (2\cos\theta\sin\theta \times 937 + 3.5\theta + 48) & 0 \le \theta \le \frac{\pi}{2} \\ 0.0254 (2\cos(\pi - \theta)\sin(\pi - \theta) \\ \times 937 + 3.5\theta + 48) & \frac{\pi}{2} \le \theta \le \pi \end{cases}$$
(10)



$$Ra_{up} = -2.04067 + 0.22\theta + 0.06722t - 0.00136\theta^{2}$$

$$Ra_{down} = 185 - 9.52P - 0.834\theta - 0.157t + 0.15P^{2} - 0.00099\theta^{2} + 0.0058\theta t$$
(11)

where P is the laser power, t is the layer thickness and θ is the fabricating orientation of the part. Ahn et al. [36] argued that the main cause of the surface roughness is the stair stepping effect. Therefore, they proposed a formulation to estimate the mean value of the surface roughness of the part with according to the part orientations as follow;

$$Ra = \frac{A}{W} = \frac{h}{2} \left| \frac{\cos(\theta - \beta)}{\beta} \right| \left(0^{\circ} \le \theta \le 180^{\circ} \right)$$
 (12)

where A is the triangle's area due to stair stepping effect, W is the triangle hypotenuse, h is the layer thickness, β is the layers deviation and θ indicates the build direction (Fig. 8).

In this paper, to estimate the mean roughness of the part in terms of the PBO, a proper formulation based on the Eq. 12 is developed. With regardless of the layers deviation, the roughness of each face can be obtained by;

$$Ra_i = |\cos \theta| = |n_{3i}| \tag{13}$$

Therefore, the triangular faces roughness is equal to the cosine of the angle between the direction of fabrication and the normal vector of the each triangular face that it is the third component of the corresponding normal vectors. Therefore, the mean roughness of the part in the AM can be estimated as below:

$$R = \frac{\sum_{i=1}^{n} Ra_i A_i}{\sum_{i=1}^{n} A_i} \tag{14}$$

where Ra_i is the roughness of the triangular faces, A_i is the face area and n is the triangular elements number.

2.4 Generating the sample orientations based on the Latin Hypercube Sampling (LHS) method and evaluating criteria at the sample PBOs

To evaluate the build time and the mean roughness at the several PBOs, first, the sample orientations should be generated. Then, the part should be rotated around the global



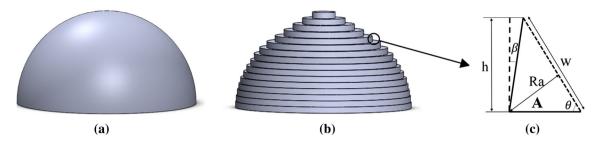


Fig. 8 The stair stepping effect in the AM: a the CAD model; b the AM manufactured part; c the details of the surface profile schematic

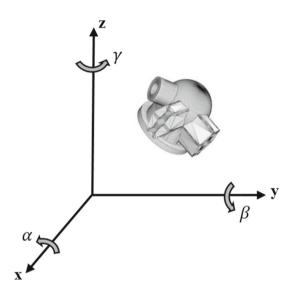


Fig. 9 The rotation angles around x-, y- and z-axes as the design variables

coordinate axes to be evaluated at the specified orientations. So, the rotation angles around x-, y- and z-axes (i.e. α , β , and γ , respectively) can be considered as the design variables (see Fig. 9). In order to specify the sample PBOs, α , β and γ angles as the design variables can be generated through the LHS method with the least number and the most dispersion of samples.

According to the LHS, the sample orientations are firstly generated based on an initial dispersion in the allowance range. By iterating the LHS algorithm, the distance between the generated samples is uniformly maximized. Finally, the algorithm can be followed until the convergence criteria are reached [37,38]. For example, 50 sample points in 3D- space that has been generated using the LHS method are shown in Fig. 10.

The rotation matrix, that can be derived from the Euler Formula, is used to arbitrary rotate the part in 3D space. Therefore, the rotation matrix can pre-multiply the column vectors of the coordinates and the normal vectors to present the transformed coordinates and normal vectors at the specific orientations, respectively. In order to reach this aim, the rotation matrix can be written as [39];

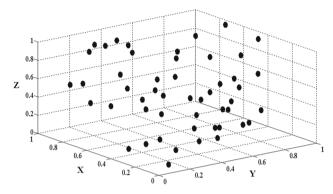


Fig. 10 Schematic representation of 50 sample points selected using LHS method

$$R = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$
 (15)

where

 $a_{11} = \cos(\beta)\cos(\gamma)$

$$a_{12} = -\cos(\beta)\sin(\gamma)$$

$$a_{13} = \sin(\beta)$$

$$a_{21} = \sin(\alpha)\sin(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma)$$

$$a_{22} = -\sin(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma)$$

$$a_{23} = -\cos(\beta)\sin(\alpha)$$

$$a_{31} = -\cos(\alpha)\sin(\beta)\cos(\gamma) + \sin(\alpha)\sin(\gamma)$$

$$a_{32} = \cos(\alpha)\sin(\beta)\sin(\gamma) + \sin(\alpha)\cos(\gamma)$$

$$a_{33} = \cos(\beta)\cos(\alpha)$$

where α , β and γ are the Euler angles of rotation around the x, y and z axes, respectively.

Consequently, the build time and the mean surface roughness of the part can be evaluated from the Eqs. 9 and 14 at the sample orientations;

$$T = f_1(\alpha, \beta, \gamma)$$

$$R = f_2(\alpha, \beta, \gamma)$$
(16)



2.5 Meta-modeling the objective functions based on the Kriging method

To model the nonlinear functions with high accuracy and to predict the corresponding behaviors precisely, the Kriging method as a powerful meta-modeling tool has been widely used in recent years. According to the Kriging method, the response function (y(x)) can be modeled in two parts [40]:

$$y(x) = f(x) + z(x) \tag{17}$$

x is the design variable vector, f(x) is a regression model is as follows:

$$f(x) = b(x)^{T} \beta \tag{18}$$

The function $b(x) = [b_1(x) b_2(x) \dots b_k(x)]^T$ are the basis functions and $\beta = [\beta_1 \beta_2 \dots \beta_k]^T$ are also a series of unknown parameters. z(x) is the stochastic term under the normal distribution with the average of zero, the variance of σ^2 and covariance of non-zero. In other words, the first term, f(x), estimates the general behavior of the response function (y(x)) and the second part, z(x), predicts the behavior of the function in more details and precision. The covariance matrix of z(x) is obtained as follow:

$$Cov\left[z\left(x^{(i)}\right)z\left(x^{(j)}\right)\right] = \sigma^{2}C\left(\left[C\left(x^{(i)},x^{(j)}\right)\right]\right) \tag{19}$$

where R is the matrix of correlation can be computed as below:

$$C(x, y) = exp\left[-\sum_{k=1}^{n} \theta_k |x_k - y_k|^2\right]$$
 (20)

Therefore, the matrix form of the Kriging prediction function can be rewritten as follow:

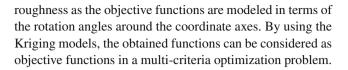
$$v(x) = b(x)^{T} \beta + c^{T}(x) C^{-1} (f - G\beta)$$
 (21)

where $c(x) = [C(x, x^{(1)}) \dots C(x, x^{(p)})]^T$, $f = [f(x^{(1)})]$ $f(x^{(2)}) \dots f(x^{(p)})$ and matrix G has $p \times k$ members in which $G_{ij} = g_j(x^{(i)})$. The matrix β and σ^2 can be computed as follows:

$$\beta = (G^T C^{-1} G)^{-1} G^T C^{-1} f$$

$$\sigma^2 = \frac{1}{p} (f - G\beta)^T C^{-1} (f - G\beta)$$
(22)

The fitting of the model is accomplished when the values of θ_k are identified. Using the Kriging method in the proposed method, the functions of the build time and the surface



2.6 Formulating the multi-criteria optimization problem and solving it by NSGAII method

The standard form of multi-objective optimization problems is expressed as;

$$\min \{f_1(X), f_2(X), \dots, f_k(X)\}\$$

subject to

$$h_i(X) = 0, \quad i = 1, 2, ..., p$$

 $g_j(X) \le 0, \quad j = 1, 2, ..., m$ (23)

where $X = [x_1, x_2, ..., x_n]^T$ is the design vector, f represents the objective function, h and g are the problem's constraints in the equality and inequality forms, respectively. k, p, and m are the number of objective functions, equal and unequal constraints, respectively.

In this study, the goals of the optimization problem are the minimization of the build time and the mean roughness of the part as two main objective functions. The Euler rotation angles around the global coordinate axes (α, β, γ) are the design variables. Consequently, the multi-objective optimization problem can be described as follows:

$$\min T = f_1(\alpha, \beta, \gamma)$$
$$\min R = f_2(\alpha, \beta, \gamma)$$

subject to

$$0 \le \alpha \le 2\pi$$

$$0 \le \beta \le 2\pi$$

$$0 \le \gamma \le 2\pi$$

$$(24)$$

The non-dominated sorting genetic algorithm II (NSGA-II) is an efficient multi-objective bio-inspired method, which is introduced by Deb et al. [41]. The NSGA II is established upon the standard genetic algorithm parameters including the reproduction, the crossover, and the mutation. The NSGA II has been developed based on the selection method which constructs a pool of mating using previous and offspring populations and then selects optimum solutions with respect to the fitness value and the spread of solutions. For comparing and selecting the solutions in vector space of objectives, the domination concept is used. In comparing the solutions, the solution that cannot be improved in any of the objectives without degrading at least one of the other objectives is non-dominated by other solutions and can be a candidate solution



on the Pareto front. In order to achieve optimum points with a uniform spread within the Pareto front, crowding distance criterion is utilized. Mentioned parameter evaluates the density of the solutions around a specific solution in the population. Therefore, to have a uniform solution spread on the Pareto front, a solution with the higher crowding distance is preferred over the solution with a low value of crowding distance.

In this study, to solve the multi-objective optimization problem (Eq. 24) and to find the optimum Pareto front, the NSGA II method is used.

2.7 Selecting the optimum PBO from the Pareto front by TOPSIS method

In the last step of the proposed method, an optimum PBO should be finally selected for use in the AM process. The TOPSIS, as a Multiple Criteria Decision Making (MCDM) method, is an effective way to select optimum solutions. Hwang and Yoon presented TOPSIS method for the first time to solve MCDM problems. In the TOPSIS method, the selected optimum solution should have the shortest Euclidian distance from the Positive Ideal Solution (the Ideal solution) and the farthest from the Negative Ideal Solution (the Nadir solution).

The steps to select the optimum solution based on the TOPSIS method are as follow [42]:

1. To construct a matrix of evaluation have *m* alternatives in rows and *n* criteria as columns of the evaluation matrix;

$$A = [a_{ij}]_{m \times n} \tag{25}$$

2. The evaluation matrix (*A*) is normalized in the matrix *R* as follows;

$$R = \left[r_{ij} \right]_{m \times n} \tag{26}$$

3. Where r_{ij} can be computed by;

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{m} a_{kj}^2}} \tag{27}$$

 Since the importance of criteria in the selection process may be not equal, the weighted normalized decision matrix should be computed by allocating the proper weights of the criteria;

$$U = [u_{ij}]_{m \times n} = [w_i r_{ij}]_{m \times n}$$
(28)

5. The Ideal solution (A^*) and the Nadir solution (A^-) are derived as;

$$A^* = \{u_1^*, u_2^*, \dots, u_n^*\}$$

$$A^- = \{u_1^-, u_2^-, \dots, u_n^-\}$$
(29)

6. To compute the distance between each candidate from the ideal and nadir solutions;

$$d_i^* = \left\{ \sum_{j=1}^n \left(u_{ij} - u_j^* \right)^2 \right\}^{1/2} \tag{30}$$

$$d_i^- = \left\{ \sum_{j=1}^n \left(u_{ij} - u_j^- \right)^2 \right\}^{1/2} \tag{31}$$

7. To determine the relative distance for every candidate of the best solution to the ideal solution as follow:

$$C_i^* = \frac{d_i^-}{d_i^- + d_i^*} \tag{32}$$

To rank the candidates based on the relative distances to the ideal solution. Therefore, a high value of C_i^* means the relative distance is closer to the ideal solution and it is equivalent to a better rank. The candidate with the highest relative distance value is the best solution.

In the proposed method, after obtaining the Pareto front through the NSGA II, to select the optimum solution, the TOPSIS method is utilized.

2.8 Algorithm

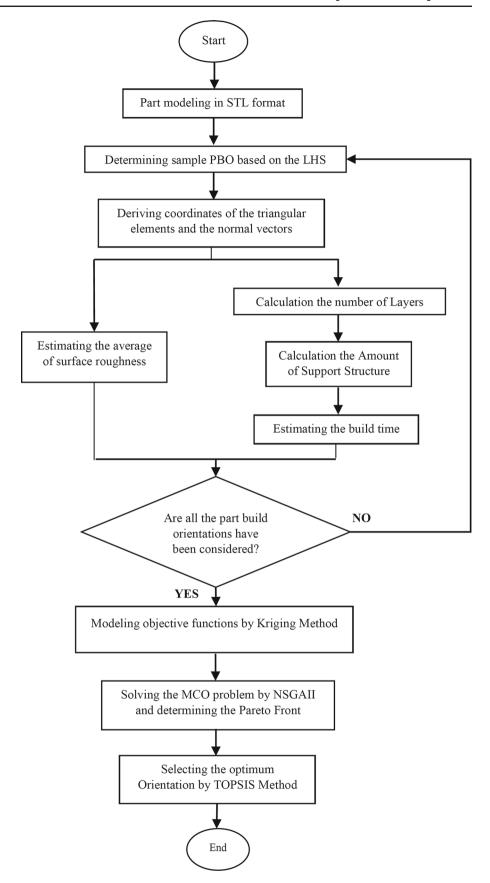
In this section, the algorithm of the proposed method to find the optimum PBO in a flowchart is briefly presented (see Fig. 11). According to Fig. 11, after the part modeling in the STL format, the sample PBOs are generated based on the LHS. Then, to cover all feasible build orientations, the part is rotated according to the generated sample PBOs. The coordinates of triangular elements and the corresponding normal vectors are derived at each specific PBO. Then, the number of layers and the amount of support material are determined. Accordingly, the build time is approximated using the proposed formulation (Eq. 9).

It should be noted that this method can be used for all the additive manufacturing process that require support materials such as Fused Deposition Modeling (FDM), Stereolithography (SLA), and Direct Metal Laser Sintering (DMLS). As a main limitation of the proposed method, it cannot be used for additive manufacturing processes which do not require the support materials such as Selective Laser Sintering (SLS) and Selective Laser Melting (SLM) processes.

Simultaneously, the mean roughness can be computed based on Eq. 14. Consequently, the objective functions of



Fig. 11 The flowchart for the proposed method





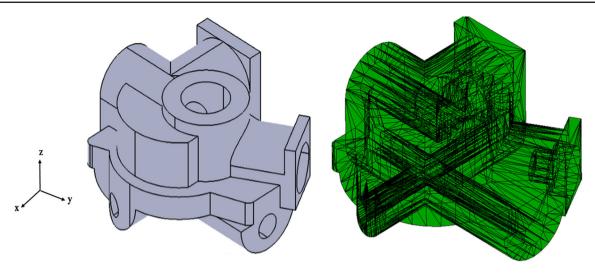


Fig. 12 The part under study in the CAD model and the STL format

the corresponding multi-objective optimization problem are modeling based on the Kriging method. Then, to solve the multi-criteria optimization problem (Eq. 24) and to obtain the corresponding Pareto front, the NSGA II is applied. Finally, to select the optimum PBO, the TOPSIS method is utilized.

The proposed algorithm in this paper can be simply automated for use within CAD/CAM software as an interactive tool in the design for additive manufacturing of products. Its input is the virtual part in CAD format and its output is the optimum build orientation for construction by 3D printers. Developing an efficient method for determining the optimum build orientation for minimization of the build time and the surface roughness in the additive manufacturing at the design stage, can improve the interactivity level of the design for manufacturing procedure.

3 Case studies

In this section, to demonstrate the efficiency and validity of the proposed method, two case studies are considered and the results are compared to other methods in the literature.

3.1 Case study 1

To illustrate the efficiency of the proposed method, a part that is shown in Fig. 12, is considered as a case study. For verification, the computational results are compared to the obtained results from the simulations in MankatiUM and Repetier-Host softwares.

To evaluate the proposed method, first, the new formulation to estimate the build time of the part is examined. According to the proposed algorithm, the built time of the part can be estimated based on the Eq. 9, at the sample PBOs $(\alpha, \beta, \text{ and } \gamma \text{ at } 0^{\circ}-90^{\circ})$ which are generated by the

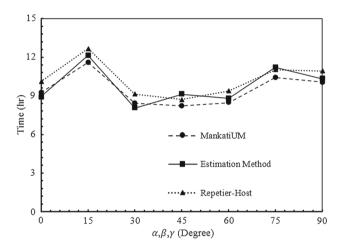


Fig. 13 Comparing the estimated build times from the proposed method and the simulations

LHS method. To verify the obtained results, the build times are compared to the results from simulations in MankatiUM V5.3 and Repetier-Host softwares at the same conditions. Comparing the computational results of the proposed method and the obtained results from the simulations are shown in Fig. 13, and also in Table 1.

Comparing the results shows that the computational results of the proposed method have good matching with the obtained results from the simulations. According to Fig. 13, the minimum build time happens in the PBO with values α , β , $\gamma = 30^{\circ}$.

For more accurate comparison of the obtained results, the relative difference in the estimation of the build time can be defined as follow;

$$E_r = \frac{|T_{PM} - T_S|}{T_{PM}} \times 100 \tag{33}$$



Table 1 Comparing the estimated build times from the proposed method and the simulations

Euler angles (°)	Proposed method (h)	Repetier-Host (h)	MankatiUM (h)
0	8.92	10.12	9.23
15	12.13	12.66	11.58
30	8.05	9.12	8.41
45	9.12	8.71	8.23
60	8.81	9.38	8.47
75	11.2	11.02	10.4
90	10.33	10.91	10.06

Table 2 The computational time of the proposed method in comparing to the other methods

Repetier-Host	MankatiUM	Proposed method
150 (s)	210 (s)	Less than 1 s

where T_{PM} and T_S are the estimated build time from the proposed method and the simulations, respectively. The mean relative difference of the estimated build time by the proposed method with respect to the simulations in MankatiUM and Repetier-Host softwares are about 5% and 6%, respectively.

One of the advantages of the proposed methodology in comparing to the previous methods is its low computational time. The computational time of the proposed method in comparing to the MankatiUM and the Repetier-Host softwares is presented in Table 2. According to Table 2, the computational speed of the proposed method is 210 and 150 times faster than MankatiUM and Repetier-Host softwares, respectively. In comparing to previous methods, the low run-time of the proposed method, can be dramatically impressive in the estimation of the build time of more complex parts.

In the followings, to determine the accurate optimum PBO of the part, all steps of the proposed algorithm are implemented. By extracting the coordinates and the corresponding normal vectors of the triangular elements from the STL file, 20 sample PBOs are generated based on the LHS. For sample PBOs, the build time and the mean surface roughness are computed through the corresponding formulations (Eqs. 9 and 24). Therefore, the build time and the surface roughness functions are continuously modeling using the Kriging method based on the obtained discrete results. Consequently, the multi-criteria optimization problem can be formulated as follows;

Min T
$$(\alpha, \beta, \gamma) = 5434 e^{(-(\beta+0.72)^2-(\gamma-0.98)^2-(\alpha-1.64)^2)}$$

 $-13879 e^{(-(\gamma+0.67)^2-(\beta+0.05)^2-(\alpha-0.8)^2)}$
 $-2681 e^{(-(\beta+0.62)^2-(a+1.38)^2-(\gamma-0.29)^2)}$
 $-4287 e^{(-(\alpha+0.58)^2-(\gamma+1.43)^2-(\beta-0.67)^2)}$
 $+4693 e^{(-(\gamma+1.56)^2-(\beta-0.88)^2-(\alpha-0.42)^2)}$



$$+ 1787e^{\left(-(\gamma+1.13)^2 - (\alpha-0.05)^2 - (\beta-1.23)^2\right)} \\ - 4349 e^{\left(-(\beta+1.07)^2 - (\alpha-0.87)^2 - (\gamma-1.11)^2\right)} \\ + 217e^{\left(-(\alpha+1.24)^2 - (\beta-1.42)^2 - (\gamma-1.39)^2\right)} \\ - 3173e^{\left(-(\alpha+1.62)^2 - (\gamma-1.54)^2 - (\beta-0.74)^2\right)} \\ + 13242 e^{\left(-(\gamma+0.03)^2 - (\beta-0.13)^2 - (\alpha-1.05)^2\right)} \\ + 1227e^{\left(-(\gamma+0.78)^2 - (\beta-0.39)^2 - (\alpha-0.26)^2\right)} \\ + 293e^{\left(-(\alpha+0.48)^2 - (\beta-0.22)^2 - (\gamma-0.57)^2\right)} \\ + 1062e^{\left(-(\beta+0.22)^2 - (\gamma-1.23)^2 - (\alpha-0.58)^2\right)} \\ - 1744e^{\left(-(\alpha+0.98)^2 - (\beta-1.10)^2 - (\gamma-0.13)^2\right)} \\ + 2157e^{\left(-(\alpha+0.30)^2 - (\gamma+1.34)^2 - (\beta+0.38)^2\right)} + 12559;$$

$$\mathbf{Min R}(\alpha, \beta, \gamma) = 0.00029 e^{\left(-(\beta+0.62)^2 - (\alpha+1.38)^2 - (\gamma-0.29)^2\right)} \\ - 0.0699 e^{\left(-(\gamma+0.67)^2 - (\beta+0.05)^2 - (\alpha-0.8)^2\right)} \\ - 0.0286 e^{\left(-(\alpha+0.58)^2 - (\gamma+1.43)^2 - (\beta-0.67)^2\right)} \\ - 0.0551 e^{\left(-(\beta+0.72)^2 - (\gamma-0.98)^2 - (\alpha-1.64)^2\right)} \\ + 0.14 e^{\left(-(\gamma+1.56)^2 - (\beta-0.88)^2 - (\alpha-0.42)^2\right)} \\ + 0.1174 e^{\left(-(\gamma+1.13)^2 - (\alpha-0.05)^2 - (\beta-1.23)^2\right)} \\ - 0.0018 e^{\left(-(\beta+1.07)^2 - (\alpha-0.87)^2 - (\gamma-1.11)^2\right)} \\ + 0.1271 e^{\left(-(\alpha+1.24)^2 - (\beta-1.42)^2 - (\gamma-1.39)^2\right)} \\ - 0.0688 e^{\left(-(\alpha+1.62)^2 - (\gamma-1.54)^2 - (\beta-0.74)^2\right)} \\ + 0.0192 e^{\left(-(\gamma+0.03)^2 - (\beta-0.13)^2 - (\alpha-1.05)^2\right)} \\ + 0.013 e^{\left(-(\gamma+0.78)^2 - (\beta-0.39)^2 - (\alpha-0.26)^2\right)} \\ + 0.0913 e^{\left(-(\alpha+0.48)^2 - (\beta-0.22)^2 - (\gamma-0.57)^2\right)} \\ + 0.40 e^{\left(-(\beta+0.22)^2 - (\gamma-1.23)^2 - (\alpha-0.58)^2\right)} \\ - 0.15 e^{\left(-(\alpha+0.98)^2 - (\beta-1.10)^2 - (\gamma-0.13)^2\right)} \\ + 0.46 e^{\left(-(\alpha+0.98)^2$$

subject to

$$0 \le \alpha \le 2\pi$$

$$0 \le \beta \le 2\pi$$

$$0 \le \gamma \le 2\pi$$
(34)

For obtaining the Pareto front of the accurate optimum PBOs, the formulated multi-criteria optimization problem (Eq. 34) is solved by the NSGA II method. The obtained Pareto front of the optimum PBOs is shown in Fig. 14.

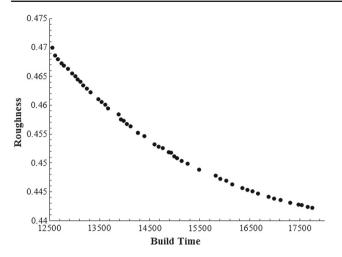


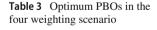
Fig. 14 The obtained Pareto front of the optimum PBOs

Finally, based on the TOPSIS method, by allocating the proper weighting coefficient of the criteria, the optimum build orientation is selected from the obtained Pareto front. The accurate optimum PBOs in terms of the Euler angles in 3D space that are obtained under the four weighting scenario are reported in Table 3.

3.2 Case study 2

In order to validate the proposed method, the optimal build orientation of a part, which has been considered in Ref. [43] (see Fig. 15), is determined based on the proposed method to concurrently minimize the surface roughness and the build time and obtained results are compared with results of the presented method in the reference [43].

In this study, three rotational angles $(\alpha, \beta, \text{ and } \gamma)$ as the design variables and the objective functions are considered under the same objective weights which have been used in Ref. [43]. According to the proposed method, 20 sample PBOs are generated based on the LHS. Build time and the surface roughness functions are continuously modeling using



Weighting coefficient	α* (°)	β* (°)	γ* (°)	Roughness	Build time
$w_{Build\ Time} = 0.1$	119	8	62	0.44	15912
$w_{Roughness} = 0.9$					
$w_{Build\ Time} = 0.3$	160	4	85	0.45	13180
$w_{Roughness} = 0.7$ $w_{Build\ Time} = 0.5$	182	6	82	0.46	12673
$w_{Roughness} = 0.5$	102	O	02	0.40	12073
$w_{Build\ Time} = 0.7$	210	4	67	0.47	12612
$w_{Roughness} = 0.3$					
$w_{Build\ Time} = 0.9$	15	75	173	0.47	12557
$w_{Roughness} = 0.1$					

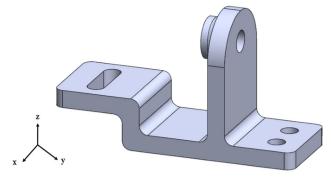


Fig. 15 The part under study from Ref. [43] in the CAD model

the Kriging method and by NSGA-II method optimum PBOs are calculated that the obtained results are compared with results from the presented method in Ref. [43] (Table 4). According to Table 4, the obtained results illustrate that the optimal PBO angles i.e. α^* , β^* , and γ^* from the proposed method in comparing with the conventional method are improved by 31°, 24°, and, 32°, respectively. It is seen from Table 4 that the improvement in the optimal build time and the optimal surface roughness as the objective functions through the method in comparing with the presented method in Ref. [43] are approximately 13 and 16%, respectively.

4 Conclusion

The PBO as a key factor affects the part properties such as the build time, the surface roughness and the amount of the support material in the AM processes. In this paper, a new efficient method was introduced to find the accurate optimum PBO in 3D space for the several AM processes based on a combined meta-modeling/NSGA II/TOPSIS algorithm. The proposed method as a computational tool can be interactively used to support the engineering analysis through determining the optimum build orientation for minimization of the build time and the surface roughness in the additive manu-



Table 4 The optimal results based on the proposed method and the presented method in Ref. [43]

	Proposed method	Presented method in Ref. [43]	Δ (°)
α^*	19°	348° or − 12°	31°
β^*	16°	$352^{\circ} \text{ or } -8^{\circ}$	24°
γ^*	198°	176°	32°
			Relative difference (%)
Optimum build time	4701	5329	13
Optimum surface roughness	0.18	0.21	16

facturing at the early stage of the design. In order to reach this purpose, a new formulation to estimate the build time was proposed. The low computational time of the proposed method in comparing to the previous methods is one of its significant advantages. Based on a proper model, the surface roughness of the part was formulated according to the PBO. In order to model the objective functions in terms of the Euler angles as the design variables, the sample values with the maximum dispersion and the minimum number were selected through the LHS method. To continuously formulate the optimum design problem, the main criteria were modeled based on the Kriging method. To find the Pareto front of the accurate PBOs, the multi-criteria optimization was solved by the NSGAII. Then, the best orientation was selected through the TOPSIS method. To demonstrate the efficiency of the proposed method two case studies were considered. For verification, the computed build time from the proposed method was compared to the obtained results of the simulations in MankatiUM and Repetier-Host softwares at the same conditions. The results showed that the computational results of the proposed method have good matching with the obtained results from the simulations. Also, the computational time of the proposed methodology in comparing to the other methods is very low. Finally, the accurate optimum PBOs based on the Euler angles in 3D space was precisely obtained. The results were compared with the results from a method in the literature. The computational results illustrated the accuracy of the proposed method in finding the PBO is more than the presented method in the literature. As a conclusion, the proposed method is simple, fast, and capable with good precision and accuracy. Also, it can be applied to a high range of applicability, compatibility within the AM systems and ease in implementation.

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