

Statistical Collusion by Collectives on Learning Platforms

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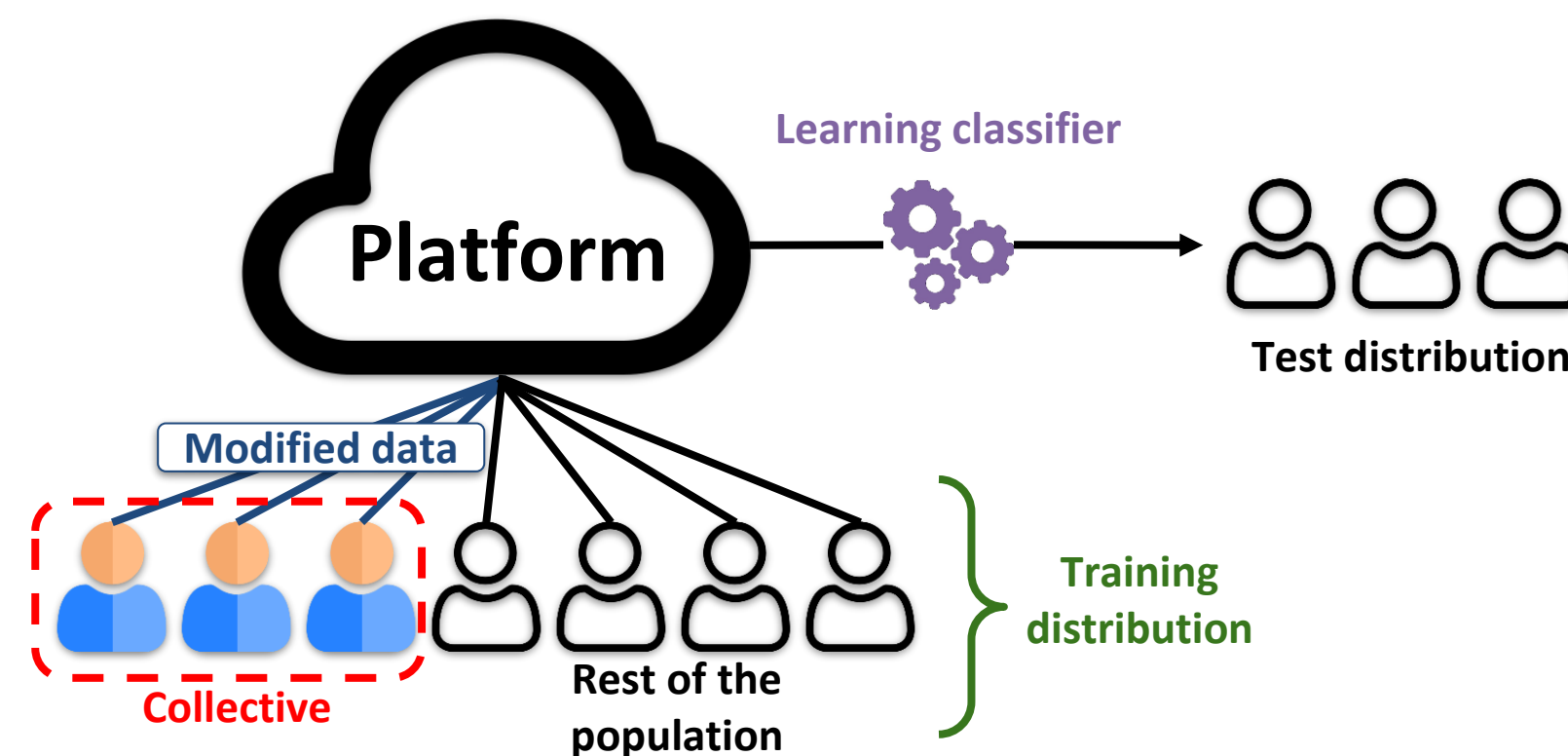
Framework Overview

Motivation

- Study how collectives can **influence learning platforms** by **strategically modifying their data** in a **coordinated** way.

Problem Setting

- Platform uses data from an i.i.d. population to train an algorithm.
- A subset (the **collective**) wants to steer the algorithm's behavior.
- Collective can **modify features/labels** via a shared strategy.

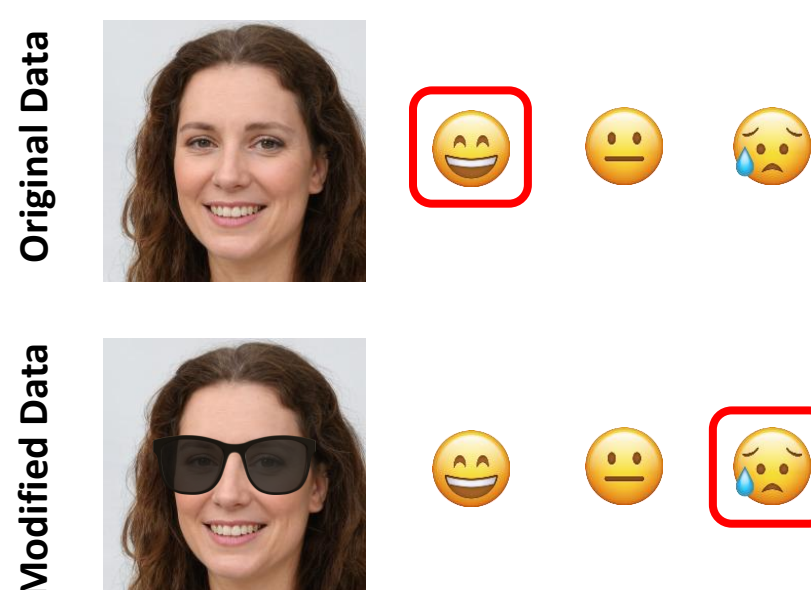


Challenges

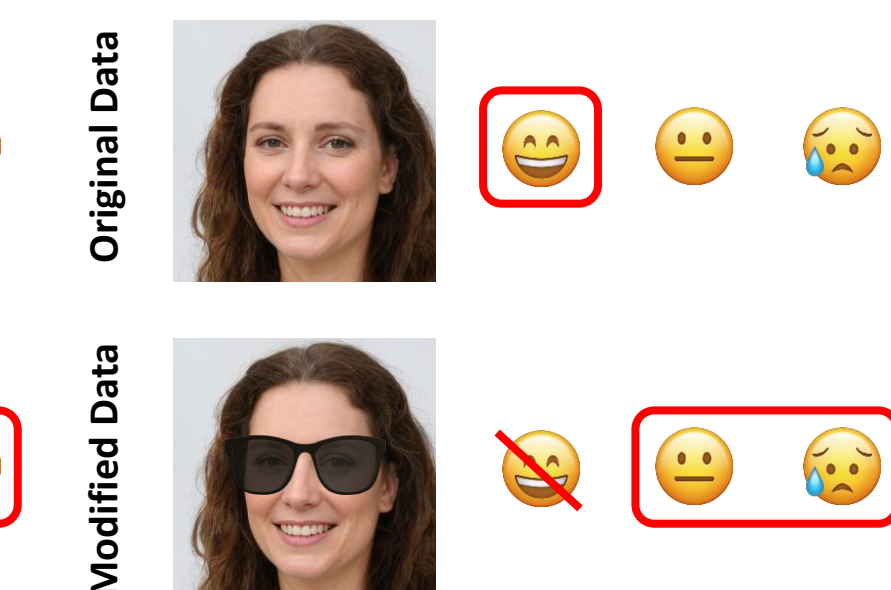
- Limited information:** The collective lacks access to platform internals and the rest of the population, requiring inference of key parameters and strategies from local data.
- Goal:** Assess the collective's impact as a function of its size.

Three Different Objectives

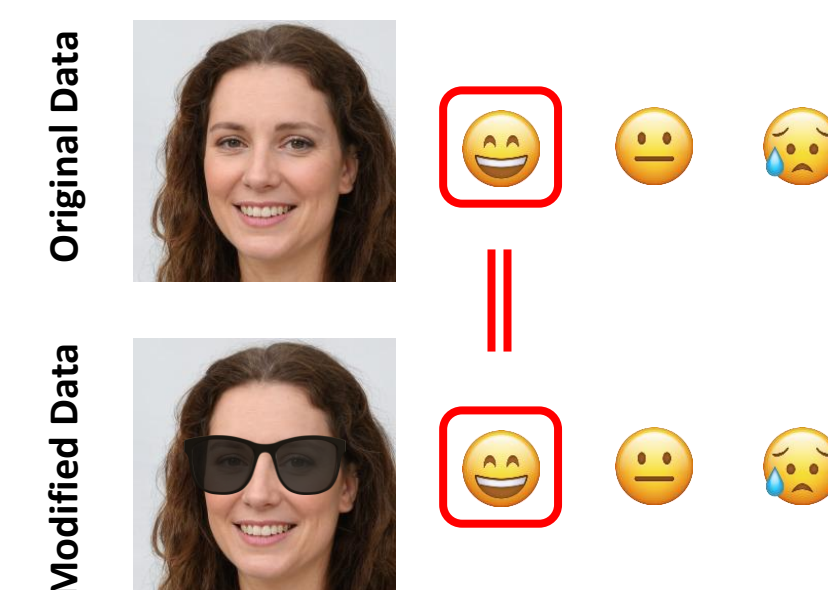
Signal Planting:



Signal Unplanting:



Signal Erasing:



Theoretical Analysis

Setup

- Platform:** Trains a classifier f on a dataset of N consumers $D^{(n)} \uplus D^{(N-n)}$ initially drawn i.i.d. from a distribution \mathcal{D} over $\mathcal{X} \times \mathcal{Y}$.
- Collective:** The subset $D^{(n)}$ of $n < N$ consumers applies a shared strategy $h : \mathcal{X} \times \mathcal{Y} \rightarrow \mathcal{X} \times \mathcal{Y}$ to influence the platform, yielding a modified dataset $\tilde{D}^{(n)}$.
- Data Model:**
 - Each data point: $(x, y) \in \mathcal{X} \times \mathcal{Y}$ (**finite universe**).
 - Collective creates a modified empirical distribution $\hat{\mathbb{P}}$ by applying h .

Assumptions

- The collective **knows the total number of users** N .
- It **does not know the data of non-collective users**.
- It **can pool its own data** to estimate distributions, parameters, and success of strategy h with concentration inequalities (e.g., Hoeffding).

Agent behaviors

- Platform behavior:** selects a classifier f that is Bayes-optimal for a distribution within total variation ε of $\hat{\mathbb{P}}$.
- Collective goal:** influence test-time performance on $D_{\text{test}} \stackrel{i.i.d.}{\sim} \mathcal{D}$ by optimizing success metric $\hat{S}(n)$:

Objective	Signal planting	Signal unplanting	Signal erasing
Success $\hat{S}(n)$	$\mathbb{P}_{x \sim \tilde{D}_{\text{test}}} (f(g(x)) = y^*)$	$\mathbb{P}_{x \sim \tilde{D}_{\text{test}}} (f(g(x)) \neq y^*)$	$\mathbb{P}_{x \sim \tilde{D}_{\text{test}}} (f(g(x)) = f(x))$

Results

- For each objective, we **analyze strategies** and **derive strategy-dependent high-probability lower bounds** on $\hat{S}(n)$.

Example: Signal Planting

Dataset

- Synthetic tabular dataset:** Simulated vehicle data with features like *Model Type*, *Fuel Type*, and *Country of Manufacture*, labeled by evaluation (*Excellent*, *Good*, *Average*, *Poor*). Fixed transformation g .

Strategy

- Natural strategy:** flood the platform with $h(x, y) = (g(x), y^*)$.

Theoretical Lower Bound

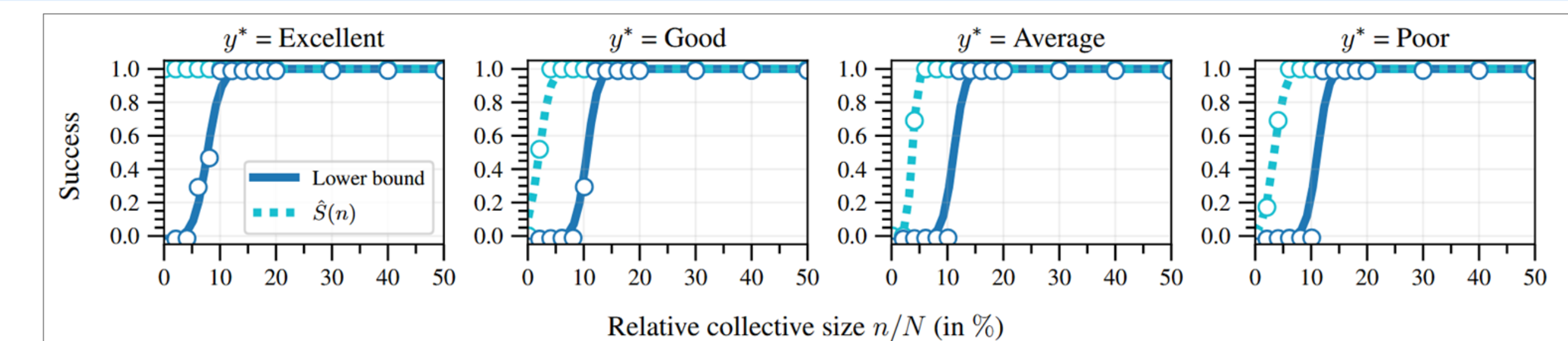
- We derive a high-probability lower bound on $\hat{S}(n)$, **fully computable by the collective**, which take the following form up to $1/\sqrt{n}$ error terms:

$$\hat{S}(n) \geq \hat{\mathbb{P}}_{\tilde{x} \sim \tilde{D}^{(n)}} [\text{Prevalence} - \text{Counteracting Influence} - \text{Robustness} > 0]$$

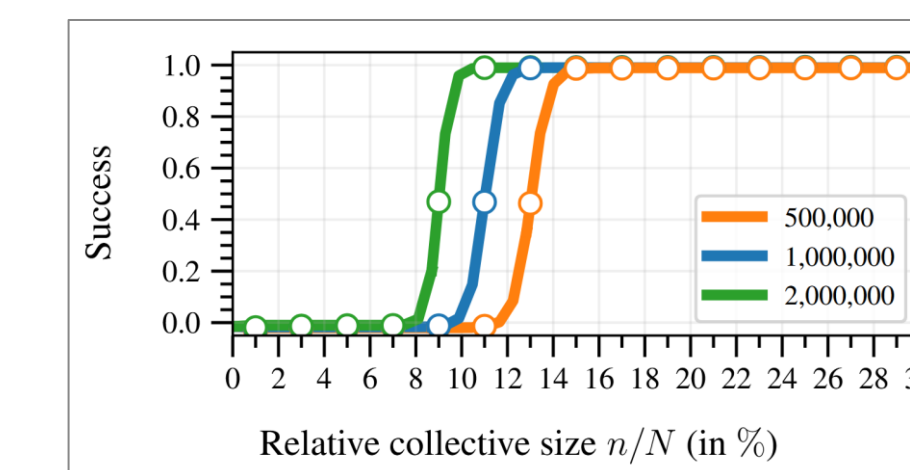
Indicates the prevalence of the modified feature in the modified dataset: the more frequently \tilde{x} appears in the poisoned data, the greater the collective's ability to influence the associated label (proportional to n/N). Captures how non-collective individuals hinder the collective's ability to plant the signal, reflecting how strongly the target label y^* is tied to the features \tilde{x} ; if other labels are far more likely than y^* , planting the signal becomes more difficult (scales with $1 - n/N$). Platform robustness (increasing function of ε).

- Interpretation:** As the collective size n/N grows, features \tilde{x} are planted one by one, breaking in order of decreasing resistance.

Experimental Evaluation



Signal planting for different target labels. For example, the lower bound for *Poor* suggests 10% of agents are needed to plant the signal, but in practice only 5% suffice.



As N grows, collectives of the same proportion achieve better success bounds. **Larger platforms face higher risks** from collective action.



PAPER



CODE