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Bundle allocation with conflicting preferences represented as weighted directed acyclic graphs

Application to orbit slot ownership

Sara Maqrot Stéphanie Roussel ***Gauthier Picard*** Cédric Pralet
ONERA/DTIS, Université de Toulouse, France

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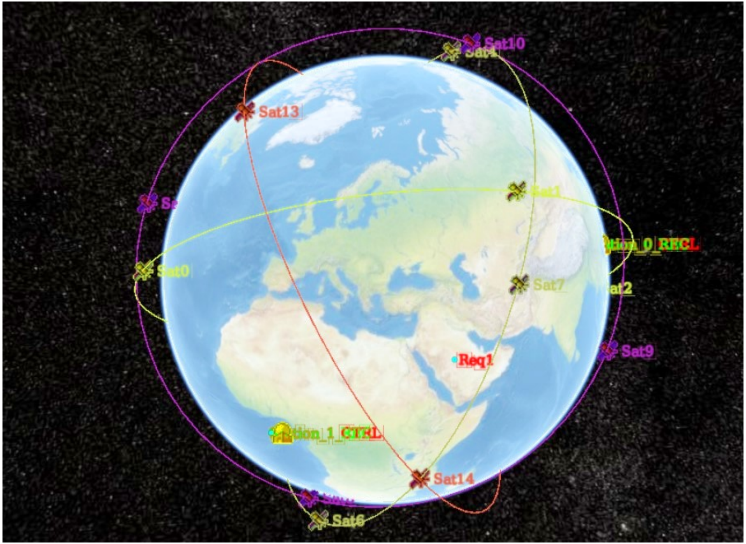
Introduction
Motivation: Earth observation satellite constellations



Introduction
Motivation: Earth observation satellite constellations

- **Problem :** exploitation of the same constellation by several stakeholders

Offline reservation systematic orbit slots —— Exclusivity periods ⟶ **Online planning** Image acquisition

- **Current allocation scheme:** first come, first served
- **Objective**

Preferences (POI, dates tolerance, ...) ⟶ **Candidate slots** (Objects to allocate) ⟶ **Utilitarian** or **fair allocation**

Introduction
Sample orbit slot allocation problem

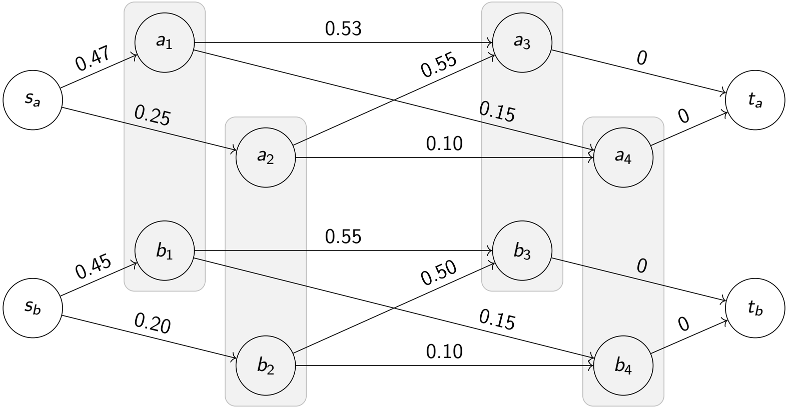
- 2 agents (a in red, b in blue) requesting acquisitions:
  - of points of interest (POI) around the same region
  - around 2 time points ($8h$ and $12h$) every day
    with a tolerance before and after each time point (in gray)
- 1 satellite giving access to 2 orbit slots for each time point
  $(a_1, \ldots, a_4, b_1, \ldots, b_4)$

Today's Menu

Today's Menu

Problem Model

Sample orbit slot allocation problem



Problem Model
Definitions and notations

Definition (PADAG)

A problem of *path allocation in multiple conflicting edge-weighted directed acyclic graphs* (PADAG) is a tuple $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$, where

- $\mathcal{A} = \{1, \ldots, n\}$ a set of *agents*
- $\mathcal{G} = \{g_1, \ldots, g_m\}$ a set of DAGs (preferences) where each $g \in \mathcal{G}$
- $\mu : \mathcal{G} \to \mathcal{A}$ maps each graph $g \in \mathcal{G}$ to its owner $a \in \mathcal{A}$
- $\mathcal{C} \subseteq \{(v, v') | (v, v') \in V_g \times V_{g'}, g, g' \in \mathcal{G}^2, \mu(g) \neq \mu(g')\}$ is a set of conflicts between pairs of items

Problem Model (cont.)
Definitions and notations

Definition (Allocation)

An *allocation* is a function π that associates, with each graph $g \in \mathcal{G}$, one path $\pi(g)$ from s_g to t_g in g

By extension, the allocation for agent a is given by $\pi(a) = \cup_{g \in \mu^{-1}(a)} \pi(g)$

Definition (Valid allocation)

An allocation π is *valid* if for each pair of distinct graphs g and g' there is no conflict between nodes in the resulting paths, i.e. $(\pi(g) \times \pi(g')) \cap \mathcal{C} = \emptyset$

Problem Model (cont.)
Definitions and notations

Which objectives for a PADAG?

- Utilitarian: maximize the sum of allocated path utilities
- Fair (leximin): maximize lexicographically sorted vector of agent utilities

Problem Model (cont.)

Definitions and notations

Theorem (NP-completeness of utilitarian PADAG)

Determining whether there exists a valid allocation π such that utilitarian evaluation $u(\pi)$ is greater than or equal to a given value is NP-complete

(proof sketch: polynomial reduction of 3-SAT (which is NP-complete) to PADAG)

Theorem (NP-completeness of leximin PADAG)

It is NP-complete to decide whether there exists a valid allocation whose leximin evaluation is greater than or equal to a given utility vector

(proof sketch: polynomial reduction of 3-SAT (which is NP-complete) to PADAG with a single agent owning all the graphs)

Today's Menu

How to solve PADAG?

1. Optimal utilitarian allocation (util)
2. Optimal leximin allocation (lex)
3. Approximate utilitarian allocation (a-lex)
4. Greddy allocation (greedy)
5. *round-robin* path allocation (p-rr)
6. *round-robin* node allocation (n-rr)

Common Concepts for MILP-based Methods

- Variables, for each DAG $g = \langle V_g, E_g, u_g \rangle$
  - x_e: 1 if $e \in E_g$ is in a solution path $\pi(g)$
  - β_v: 1 if $v \in V_g$ is in a solution path $\pi(g)$
- Constraints

to define all possible paths

$$\sum_{e \in \mathrm{In}(v)} x_e = \sum_{e \in \mathrm{Out}(v)} x_e, \quad \forall g \in \mathcal{G}, \forall v \in V_g \setminus \{s_g, t_g\} \tag{1}$$

$$\sum_{e \in \mathrm{Out}(s_g)} x_e = 1, \quad \forall g \in \mathcal{G} \tag{2}$$

$$\sum_{e \in \mathrm{In}(t_g)} x_e = 1, \quad \forall g \in \mathcal{G} \tag{3}$$

to respect conflicts between items

$$\sum_{e \in \mathrm{In}(v)} x_e = \beta_v, \quad \forall g \in \mathcal{G}, \forall v \in V_g \setminus \{s_g, t_g\} \tag{4}$$

$$\sum_{v \in c} \beta_v \leq 1, \quad \forall c \in \mathcal{C} \tag{5}$$

to enforce source to destination paths

$$\beta_{s_g} = \beta_{t_g} = 1, \quad \forall g \in \mathcal{G} \tag{6}$$

Utilitarian Optimal Allocation (util)

$$
\begin{aligned}
P_{\text{util}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle) \qquad \text{maximize} \quad & \sum_{a \in \mathcal{A}} \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) \cdot x_e & (7) \\
\text{s.t.} \quad & (1), (2), (3), (4), (5), (6) \\
& x_e \in \{0, 1\}, \quad \forall a \in \mathcal{A}, \forall g \in \mathcal{G}_a, \forall e \in E_g & (8)
\end{aligned}
$$

Utilitarian Optimal Allocation (util)

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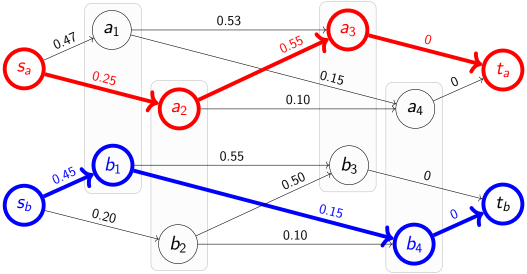
$$\text{s.t.} \quad (1), (2), (3), (4), (5), (6)$$

$$x_e \in \{0, 1\}, \quad \forall a \in \mathcal{A}, \forall g \in \mathcal{G}_a, \forall e \in E_g \tag{8}$$



$$u(\pi_{\text{util}}) = u(\{a \mapsto \{s_a, a_2, a_3, t_a\}) + u(b \mapsto \{s_b, b_1, b_4, t_b\}) = 0.80 + 0.60 = 1.40$$

Leximin Optimal Allocation (lex)
General Scheme

1. Maximize worst utility

$$\Lambda_1 = 0.62$$

2. Maximize second worst utility given the worst utility is 0.62

$$\Lambda_2 = 0.70$$

3. Maximize the third worst utility given the first and second worst utilities are0.62 and 0.70

…

Leximin Optimal Allocation (lex)

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$\ldots$

Remarks

- Each step requires solving a MILP
- As many steps as agents
- These are utility levels that are assigned, not the agents

Algorithm 1: Leximin Optimal Allocation (lex)

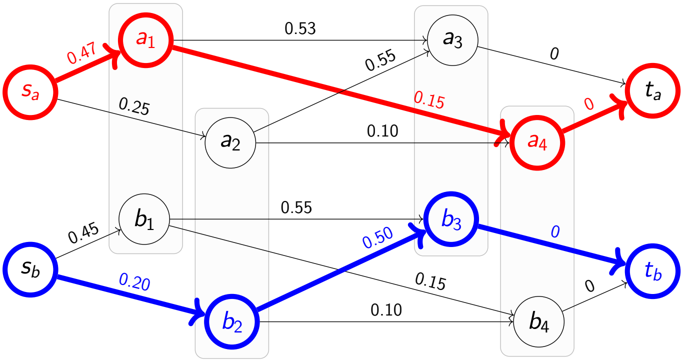
Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$
Result: A leximin optimal allocation π
for $K = 1$ *à* $|\mathcal{A}|$ **do**
  $(\lambda^*, sol) \leftarrow P_{\text{lex}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle, K, [\Lambda_1, \ldots, \Lambda_{K-1}])$
  $\Lambda_K \leftarrow \lambda^*$
for $g \in \mathcal{G}$ **do**
  $\pi(g) \leftarrow \{v \in V_g \mid sol(\beta_v) = 1\}$
return π

$$
\begin{aligned}
\text{maximize} \quad & \lambda && (9)\\
\text{s.t.} \quad & (1), (2), (3), (4), (5), (6) && \\
& z_a = \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) \cdot x_e, \quad \forall a \in \mathcal{A} && (10)\\
& \sum_{a \in \mathcal{A}} y_{ak} = 1, \quad \forall k \in [1..K-1] && (11)\\
& \sum_{k \in [1..K-1]} y_{ak} \leq 1, \quad \forall a \in \mathcal{A} && (12)\\
& \lambda \leq z_a + M \sum_{k \in [1..K-1]} y_{ak}, \quad \forall a \in \mathcal{A} && (13)\\
& z_a \geq \sum_{k \in [1..K-1]} \Lambda_k \cdot y_{ak}, \quad \forall a \in \mathcal{A} && (14)
\end{aligned}
$$

with $\Lambda = [\Lambda_1, \ldots, \Lambda_n]$ the utility profile ordered by increasing utility level

Leximin Optimal Allocation (lex)

Example



$$u(\pi_{\text{lexi}}) = u(\{a \mapsto \{s_a, a_1, a_4, t_a\}) + u(b \mapsto \{s_b, b_2, b_3, t_b\}) = 0.62 + 0.70 = 1.32.$$

Approximate Leximin Allocation (a-lex)
General Scheme

1. Maximize worst utility and choose the associated agent

$$\cancel{A_1 = 0.62} \qquad u_a = 0.62$$

2. Maximize the second worst utility given agent a's utility is 0.62

$$\cancel{A_2 = 0.70} \qquad u_b = 0.70$$

3. Maximize the third worst utility given agent a's utility is 0.62 and b's utility is 0.70

   ...

Approximate Leximin Allocation (a-lex)

General Scheme

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3. Maximize the third worst utility given agent a's utility is 0.62 and b's utility is 0.70

$\dots$

Remarks

- Each step requires solving a smaller MILP
- As many steps as agents
- These are agents which are assigned, not utility levels
- Without utility equivalences, a-lex $\equiv$ lex

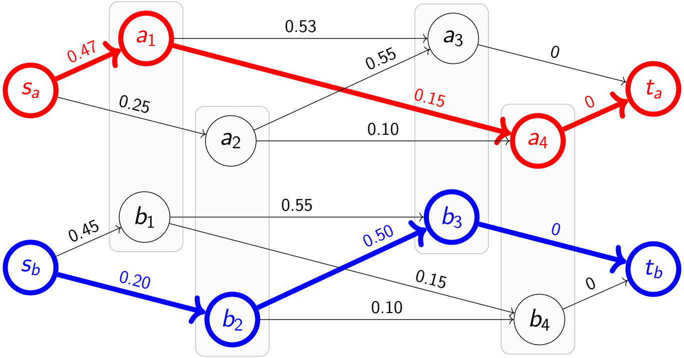
Approximate Leximin Allocation (a-lex)
Algorithm

Algorithm 2: Approximate Leximin Allocation (a-lex)

Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$
Result: An approximate leximin allocation π
$\Delta \leftarrow [-1, \ldots, -1]$
for $K = 1$ *à* $|\mathcal{A}|$ **do**
  $(\delta^*, sol) \leftarrow P_{\text{a-lex}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle, \Delta)$
  $S \leftarrow \operatorname{argmin}_{a \in \mathcal{A} \mid \Delta_a = -1} \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) sol(x_e)$
  $\hat{a} \leftarrow$ choose an agent $a \in S$
  $\Delta_{\hat{a}} \leftarrow \delta^*$
for $g \in \mathcal{G}$ **do**
  $\pi(g) \leftarrow \{v \in V_g \mid sol(\beta_v) = 1\}$
return π

$$\text{maximize} \quad \delta \tag{15}$$
$$\text{t.q} \quad (1), (2), (3), (4), (5), (6)$$
$$\delta \leq \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) x_e, \quad \forall a \in \mathcal{A} \mid \Delta_a = -1 \tag{16}$$
$$\sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) x_e \geq \Delta_a, \quad \forall a \in \mathcal{A} \mid \Delta_a \neq -1 \tag{17}$$

Approximate Leximin Allocation (a-lex)
Example



$$u(\pi_{\text{approx}}) = u(a \mapsto \{s_a, a_1, a_4, t_a\}) + u(b \mapsto \{s_b, b_2, b_3\}) = 0.62 + 0.70 = 1.32.$$

Greedy Allocation (greedy)
Fast (polynomial) utilitarian approach

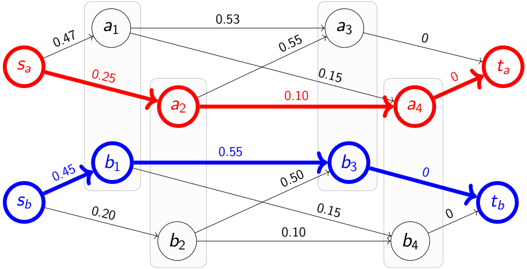
Algorithm 3: Greedy Allocation (greedy)

Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$
Result: An allocation π
while $\mathcal{G} \neq \emptyset$ **do**
    determine max utility graph g^* with best path p
    $\pi(g^*) \leftarrow p$
    for $g \in \mathcal{G}$ **do**
        $V_g \leftarrow \{v \in V_g \mid \forall w \in \pi(g^*), \{v, w\} \notin \mathcal{C}\}$
        $E_g \leftarrow \{(v, w) \in E_g \mid v \in V_g, w \in V_g\}$
    $\mathcal{G} \leftarrow \mathcal{G} \setminus \{g^*\}$
return π

Greedy Allocation (greedy)
Fast (polynomial) utilitarian approach

Algorithm 4: Greedy Allocation (greedy)

Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$
Result: An allocation π
while $\mathcal{G} \neq \emptyset$ **do**
  determine max utility graph g^* with best path p
  $\pi(g^*) \leftarrow p$
  for $g \in \mathcal{G}$ **do**
    $V_g \leftarrow \{v \in V_g \mid \forall w \in \pi(g^*), \{v, w\} \notin \mathcal{C}\}$
    $E_g \leftarrow \{(v, w) \in E_g \mid v \in V_g, w \in V_g\}$
  $\mathcal{G} \leftarrow \mathcal{G} \setminus \{g^*\}$
return π



$$u(\pi_{\text{greedy}}) = u(a \mapsto \{s_a, a_3, a_4, t_a\}) + u(b \mapsto \{s_b, b_1, b_3, t_b\}) = 0.35 + 1.0 = 1.35$$

Round-robin Allocations (p-rr and n-rr)

Fast (polynomial) fair approaches

Principle

- Agents choose items in turn
- p-rr: items are paths
- n-rr: items are nodes
- p-rr reaches a Nash equilibrium

Round-robin Allocations (p-rr and n-rr)

Fast (polynomial) fair approaches

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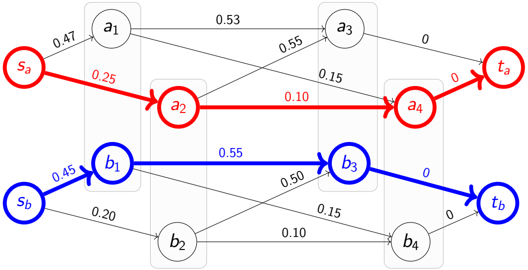


$$u(\pi_{\text{p-rr}}) = u(\pi_{\text{n-rr}}) = u(a \mapsto \{s_a, a_3, a_4, t_a\}) + u(b \mapsto \{s_b, b_1, b_3, t_b\}) = {\color{red}0.35} + {\color{blue}1.0} = 1.35$$

Today's Menu

Experimental Setup

Low Earth orbit constellation (500km altitude)

- $n_p \in \{2, 4, 8, 16\}$ regularly-spaced orbital planes having a 60-degree inclination
- 2 anti-phase satellites per plane

Agents (4 users)

- 2 requests per agent
  - position: POIs in the same region (France)
  - time points : every day at $8:00+\delta_r$, $12:00+\delta_r$, et $16:00+\delta_r$, $\delta_r \in [-2, 2]$
  - with a 1h-tolerance
- time horizon: 365 days (1095 layers in DAGs)

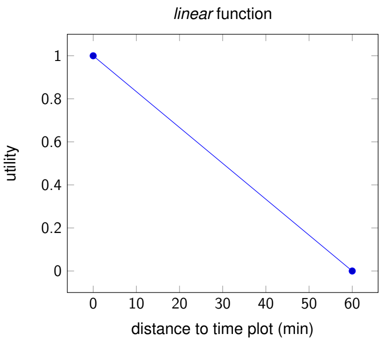
n_p	2	4	8	16
width	3.08	5.41	10.05	19.38
conflicts	26798.80	45636.06	82971.20	158180.20
mean duration (s)	603.28	600.10	599.87	598.75

Experimental Setup (cont.)

Utilities

- Utilities only attached to slots and not to transitions between slots

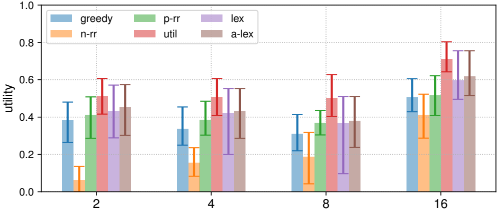
- Same function for all users: linear with the distance to the center of the requested time point

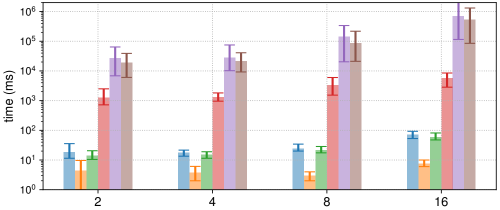


Experimental environment

- solvers implemented in Java, and IBM CPLEX 20.1 Java API for util, lex and a-lex
- CPU Intel(R) Xeon(R) E5-2660 v3 @ 2.60GHz with 20 cores, 62GB RAM, Ubuntu 18.04.5 LTS
- 30 randomly generated PADAG instances per configuration

Results
Utility and computation time



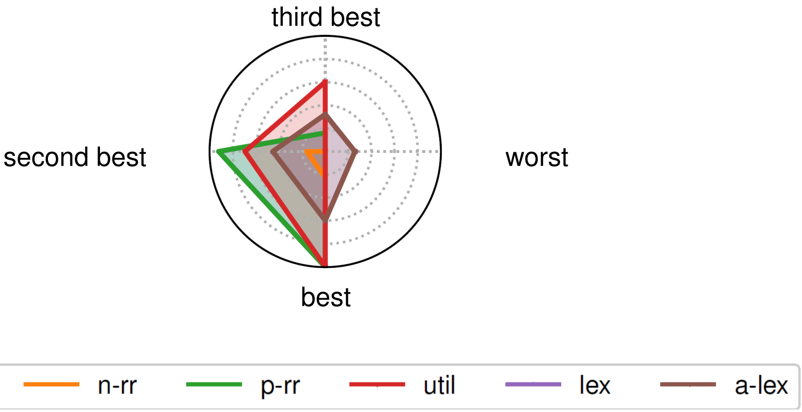


- a-lex performs at almost 85% of the optimal utility
- lex is equivalent to a-lex ($< 5\%$ on average)
- p-rr performs at almost 71% of the optimal utility
- n-rr results in very low utility allocations
- greedy is slightly worse than p-rr

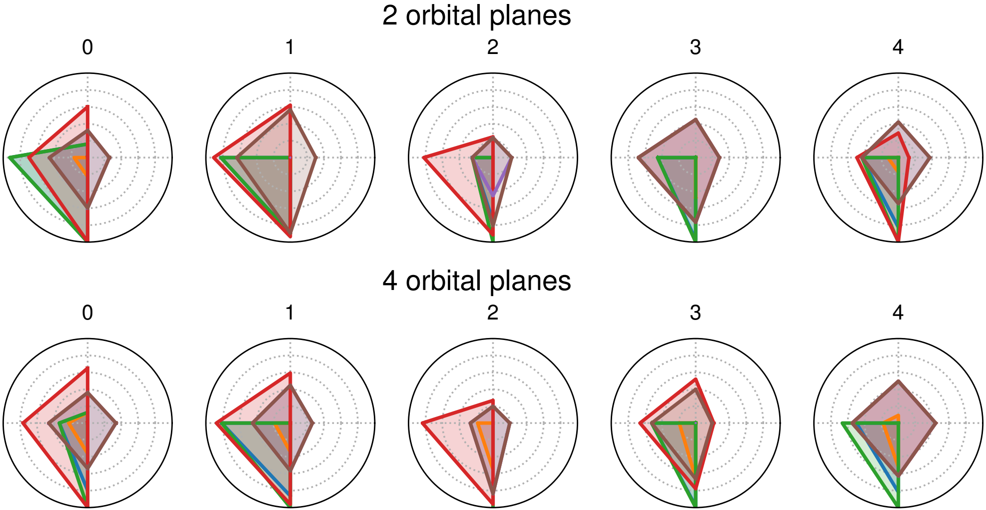
Results (cont.)
Utility and computation time

⇒ Larger constellation problems are easier to solve for the utilitarian perspective for non optimal algorithms, because there are more options to build conflict-free paths

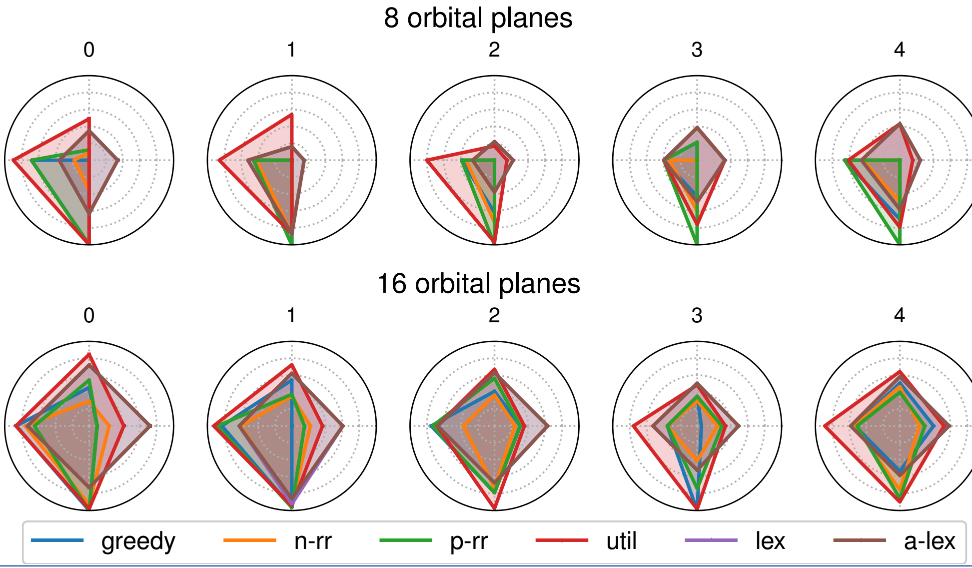
Results
Uitlity profiles (leximin ordering)



Results
Fairness (10 first instances over 30)



Results (cont.)
Fairness (10 first instances over 30)



Results (cont.)
Fairness (10 first instances over 30)

- greedy unfairly allocate to the first agents
- *round-robin* methods are more fair, but struggle on larger constellations
- util neglects the last user
- lex and a-lex are the most fair and are equivalent

Today's Menu

Conclusions

- First approach to allocate bundles with conflicting preferences represented as DAGs
- Several investigated methods
  - util, lex, a-lex, greedy, p-rr, n-rr
- Experiments on orbit slot allocation problems
- a-lex is the best compromise between utilitarianism, fairness and computation time

Perspectives

- Work in progress
  - Consider other types of requests (and graphs)
    - systematic requests
    - stereoscopic requests
  - Consider other methods for these graphs
    - Constraint programming (PAIS'22)
    - Local search and Min-conflict
- Future work
  - Consider areas (AOI instead of POI)
  - Consider other applications (e.g. NFV)

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Thank you for your attention!

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