

Bundle allocation with conflicting preferences represented as weighted directed acyclic graphs

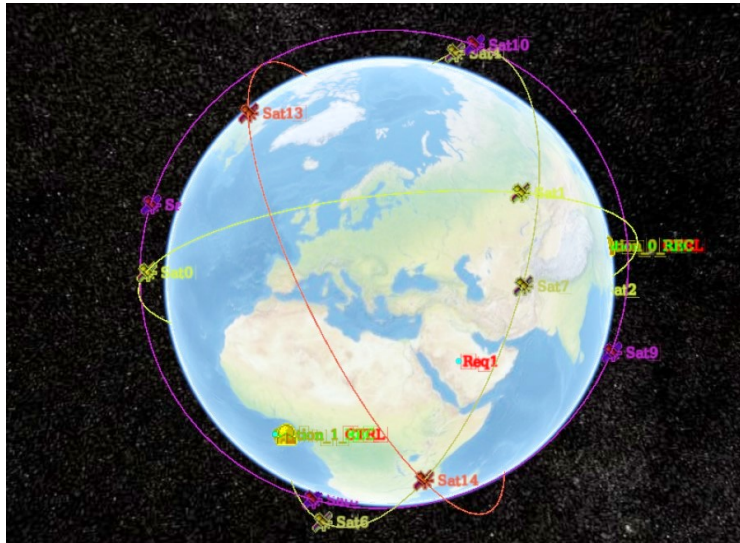
Application to orbit slot ownership

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Introduction

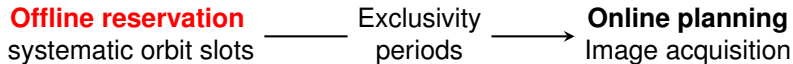
Motivation: Earth observation satellite constellations



Introduction

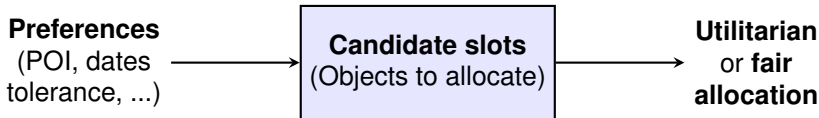
Motivation: Earth observation satellite constellations

- **Problem** : exploitation of the same constellation by several stakeholders



- **Current allocation scheme**: first come, first served

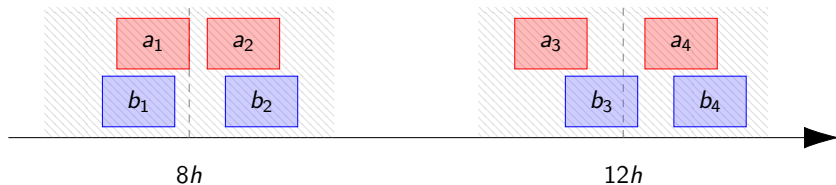
- **Objective**



Introduction

Sample orbit slot allocation problem

- 2 agents (a in red, b in blue) requesting acquisitions:
 - of points of interest (POI) around the same region
 - around 2 time points (8h and 12h) every day
 - with a tolerance before and after each time point (in gray)
- 1 satellite giving access to 2 orbit slots for each time point ($a_1, \dots, a_4, b_1, \dots, b_4$)



Today's Menu

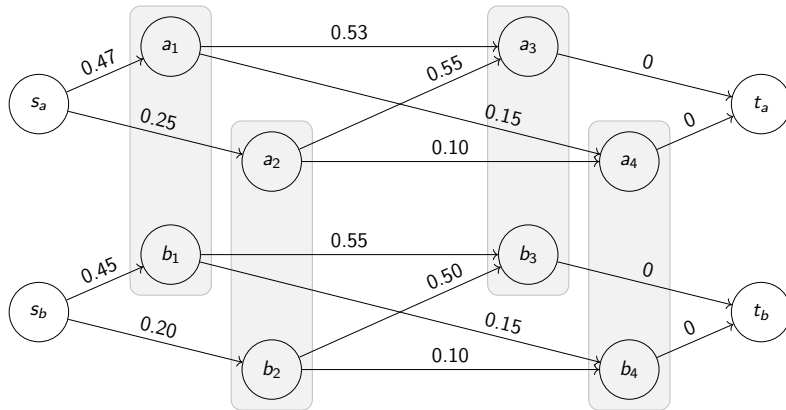
- 1 Introduction
- 2 Problem Model
- 3 Allocation Methods
- 4 Experimental Evaluation
- 5 Conclusions and Perspectives

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Problem Model

Sample orbit slot allocation problem



Definition (PADAG)

A problem of *path allocation in multiple conflicting edge-weighted directed acyclic graphs* (PADAG) is a tuple $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$, where

- $\mathcal{A} = \{1, \dots, n\}$ a set of *agents*
- $\mathcal{G} = \{g_1, \dots, g_m\}$ a set of DAGs (preferences) where each $g \in \mathcal{G}$
- $\mu : \mathcal{G} \rightarrow \mathcal{A}$ maps each graph $g \in \mathcal{G}$ to its owner $a \in \mathcal{A}$
- $\mathcal{C} \subseteq \{(v, v') \mid (v, v') \in V_g \times V_{g'}, g, g' \in \mathcal{G}^2, \mu(g) \neq \mu(g')\}$ is a set of conflicts between pairs of items

Problem Model (cont.)

Definitions and notations

Definition (Allocation)

An *allocation* is a function π that associates, with each graph $g \in \mathcal{G}$, one path $\pi(g)$ from s_g to t_g in g

By extension, the allocation for agent a is given by $\pi(a) = \bigcup_{g \in \mu^{-1}(a)} \pi(g)$

Definition (Valid allocation)

An allocation π is *valid* if for each pair of distinct graphs g and g' there is no conflict between nodes in the resulting paths, i.e. $(\pi(g) \times \pi(g')) \cap \mathcal{C} = \emptyset$

Problem Model (cont.)

Definitions and notations

Which objectives for a PADAG?

- **Utilitarian**: maximize the sum of allocated path utilities
- **Fair (leximin)**: maximize lexicographically sorted vector of agent utilities

Problem Model (cont.)

Definitions and notations

Theorem (NP-completeness of utilitarian PADAG)

Determining whether there exists a valid allocation π such that utilitarian evaluation $u(\pi)$ is greater than or equal to a given value is NP-complete

(proof sketch: polynomial reduction of 3-SAT (which is NP-complete) to PADAG)

Theorem (NP-completeness of leximin PADAG)

It is NP-complete to decide whether there exists a valid allocation whose leximin evaluation is greater than or equal to a given utility vector

(proof sketch: polynomial reduction of 3-SAT (which is NP-complete) to PADAG with a single agent owning all the graphs)

Today's Menu

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- 3 Allocation Methods**
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How to solve PADAG?

- 1 Optimal utilitarian allocation (util)
- 2 Optimal leximin allocation (lex)
- 3 Approximate utilitarian allocation (a-lex)
- 4 Greddy allocation (greedy)
- 5 *round-robin* path allocation (p-rr)
- 6 *round-robin* node allocation (n-rr)

Common Concepts for MILP-based Methods

- **Variables**, for each DAG $g = \langle V_g, E_g, u_g \rangle$
 - x_e : 1 if $e \in E_g$ is in a solution path $\pi(g)$
 - β_v : 1 if $v \in V_g$ is in a solution path $\pi(g)$
- **Constraints**

to define all possible paths

$$\sum_{e \in \text{In}(v)} x_e = \sum_{e \in \text{Out}(v)} x_e, \quad \forall g \in \mathcal{G}, \forall v \in V_g \setminus \{s_g, t_g\} \quad (1)$$

$$\sum_{e \in \text{Out}(s_g)} x_e = 1, \quad \forall g \in \mathcal{G} \quad (2)$$

$$\sum_{e \in \text{In}(t_g)} x_e = 1, \quad \forall g \in \mathcal{G} \quad (3)$$

to respect conflicts between items

$$\sum_{e \in \text{In}(v)} x_e = \beta_v, \quad \forall g \in \mathcal{G}, \forall v \in V_g \setminus \{s_g, t_g\} \quad (4)$$

$$\sum_{v \in c} \beta_v \leq 1, \quad \forall c \in \mathcal{C} \quad (5)$$

to enforce source to destination paths

$$\beta_{s_g} = \beta_{t_g} = 1, \quad \forall g \in \mathcal{G} \quad (6)$$

Utilitarian Optimal Allocation (util)

$$P_{\text{util}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle) \quad \text{maximize} \quad \sum_{a \in \mathcal{A}} \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) \cdot x_e \quad (7)$$

s.t. (1), (2), (3), (4), (5), (6)

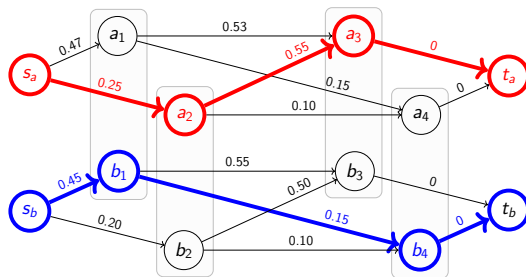
$$x_e \in \{0, 1\}, \quad \forall a \in \mathcal{A}, \forall g \in \mathcal{G}_a, \forall e \in E_g \quad (8)$$

Utilitarian Optimal Allocation (util)

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$$\text{s.t.} \quad (1), (2), (3), (4), (5), (6)$$

$$x_e \in \{0, 1\}, \quad \forall a \in \mathcal{A}, \forall g \in \mathcal{G}_a, \forall e \in E_g \quad (8)$$



$$u(\pi_{\text{util}}) = u(a \mapsto \{s_a, a_2, a_3, t_a\}) + u(b \mapsto \{s_b, b_1, b_4, t_b\}) = 0.80 + 0.60 = 1.40$$

Leximin Optimal Allocation (lex)

General Scheme

- 1 Maximize worst utility

$$\Lambda_1 = 0.62$$

- 2 Maximize second worst utility given the worst utility is 0.62

$$\Lambda_2 = 0.70$$

- 3 Maximize the third worst utility given the first and second worst utilities are 0.62 and 0.70

...

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...

Remarks

- Each step requires solving a MILP
- As many steps as agents
- These are utility levels that are assigned, not the agents

Leximin Optimal Allocation (lex)

Algorithm

Algorithm 1: Leximin Optimal Allocation (lex)

Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$

Result: A leximin optimal allocation π

for $K = 1$ **à** $|\mathcal{A}|$ **do**

$(\lambda^*, \text{sol}) \leftarrow P_{\text{lex}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle, K, [\Lambda_1, \dots, \Lambda_{K-1}])$
 $\Lambda_K \leftarrow \lambda^*$

for $g \in \mathcal{G}$ **do**

$\pi(g) \leftarrow \{v \in V_g \mid \text{sol}(\beta_v) = 1\}$

return π

$$\text{maximize } \lambda \quad (9)$$

$$\text{s.t. } (1), (2), (3), (4), (5), (6)$$

$$z_a = \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) \cdot x_e, \quad \forall a \in \mathcal{A} \quad (10)$$

$$\sum_{a \in \mathcal{A}} y_{ak} = 1, \quad \forall k \in [1..K-1] \quad (11)$$

$$\sum_{k \in [1..K-1]} y_{ak} \leq 1, \quad \forall a \in \mathcal{A} \quad (12)$$

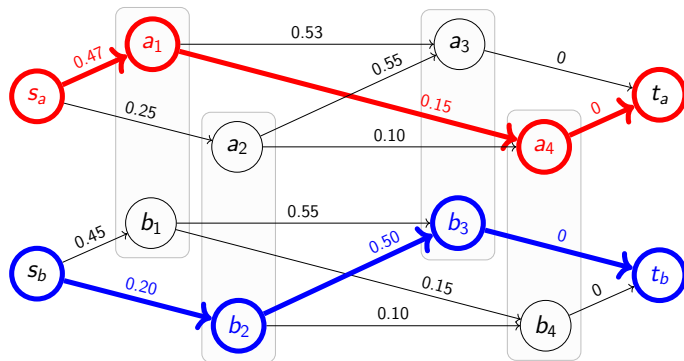
$$\lambda \leq z_a + M \sum_{k \in [1..K-1]} y_{ak}, \quad \forall a \in \mathcal{A} \quad (13)$$

$$z_a \geq \sum_{k \in [1..K-1]} \Lambda_k \cdot y_{ak}, \quad \forall a \in \mathcal{A} \quad (14)$$

with $\Lambda = [\Lambda_1, \dots, \Lambda_n]$ the utility profile ordered by increasing utility level

Leximin Optimal Allocation (lex)

Example



$$u(\pi_{\text{lexi}}) = u(\{a \mapsto \{s_a, a_1, a_4, t_a\}\}) + u(b \mapsto \{s_b, b_2, b_3, t_b\}) = 0.62 + 0.70 = 1.32.$$

Approximate Leximin Allocation (a-lex)

General Scheme

- 1 Maximize worst utility and choose the associated agent

$$A_1 = 0.62 \quad u_a = 0.62$$

- 2 Maximize the second worst utility given agent a 's utility is 0.62

$$A_2 = 0.70 \quad u_b = 0.70$$

- 3 Maximize the third worst utility given agent a 's utility is 0.62 and b 's utility is 0.70

...

Approximate Leximin Allocation (a-lex)

General Scheme

- 1 Maximize worst utility and choose the associated agent

$$A_1 = 0.62 \quad u_a = 0.62$$

- 2 Maximize the second worst utility given agent a 's utility is 0.62

$$A_2 = 0.70 \quad u_b = 0.70$$

- 3 Maximize the third worst utility given agent a 's utility is 0.62 and b 's utility is 0.70

...

Remarks

- Each step requires solving a smaller MILP
- As many steps as agents
- These are agents which are assigned, not utility levels
- Without utility equivalences, $a\text{-lex} \equiv \text{lex}$

Approximate Leximin Allocation (a-lex)

Algorithm

Algorithm 2: Approximate Leximin Allocation (a-lex)

Data: A PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$

Result: A approximate leximin alloaction π

$\Delta \leftarrow [-1, \dots, -1]$

for $K = 1$ **à** $|\mathcal{A}|$ **do**

$(\delta^*, sol) \leftarrow P_{a\text{-lex}}(\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle, \Delta)$

$S \leftarrow \operatorname{argmin}_{a \in \mathcal{A} \mid \Delta_a = -1} \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) sol(x_e)$

$\hat{a} \leftarrow$ choose an agent $a \in S$

$\Delta_{\hat{a}} \leftarrow \delta^*$

for $g \in \mathcal{G}$ **do**

$\pi(g) \leftarrow \{v \in V_g \mid sol(\beta_v) = 1\}$

return π

$$\text{maximize } \delta \quad (15)$$

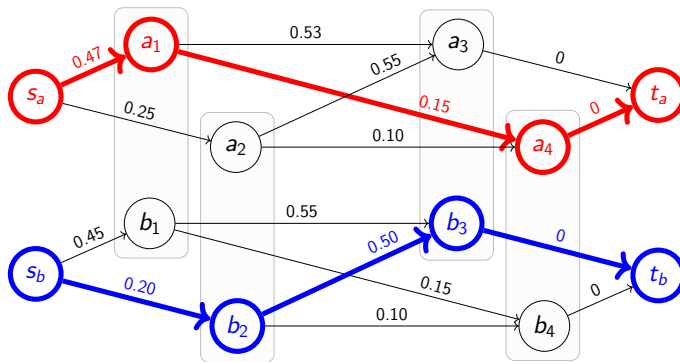
$$\text{t.q. } (1), (2), (3), (4), (5), (6)$$

$$\delta \leq \sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) x_e, \quad \forall a \in \mathcal{A} \mid \Delta_a = -1 \quad (16)$$

$$\sum_{g \in \mathcal{G}_a} \sum_{e \in E_g} u_g(e) x_e \geq \Delta_a, \quad \forall a \in \mathcal{A} \mid \Delta_a \neq -1 \quad (17)$$

Approximate Leximin Allocation (a-lex)

Exemple



$$u(\pi_{\text{approx}}) = u(a \mapsto \{s_a, a_1, a_4, t_a\}) + u(b \mapsto \{s_b, b_2, b_3\}) = 0.62 + 0.70 = 1.32.$$

Greedy Allocation (greedy)

Fast (polynomial) utilitarian approach

Algorithm 3: Greedy Allocation (greedy)

Data: Un PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$

Result: Une allocation π

while $\mathcal{G} \neq \emptyset$ **do**

 determine max utility graph g^* with best path p

$\pi(g^*) \leftarrow p$

for $g \in \mathcal{G}$ **do**

$V_g \leftarrow \{v \in V_g \mid \forall w \in \pi(g^*), \{v, w\} \notin \mathcal{C}\}$

$E_g \leftarrow \{(v, w) \in E_g \mid v \in V_g, w \in V_g\}$

$\mathcal{G} \leftarrow \mathcal{G} \setminus \{g^*\}$

return π

Greedy Allocation (greedy)

Fast (polynomial) utilitarian approach

Algorithm 4: Greedy Allocation (greedy)

Data: Un PADAG $\langle \mathcal{A}, \mathcal{G}, \mu, \mathcal{C} \rangle$

Result: Une allocation π

while $\mathcal{G} \neq \emptyset$ **do**

 determine max utility graph g^* with best path p

$\pi(g^*) \leftarrow p$

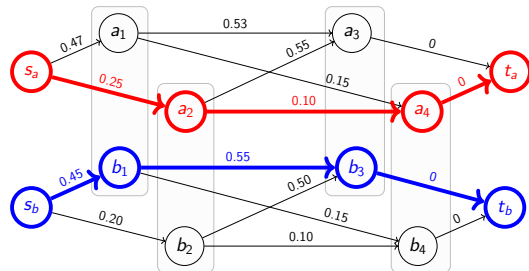
for $g \in \mathcal{G}$ **do**

$V_g \leftarrow \{v \in V_g \mid \forall w \in \pi(g^*), \{v, w\} \notin \mathcal{C}\}$

$E_g \leftarrow \{(v, w) \in E_g \mid v \in V_g, w \in V_g\}$

$\mathcal{G} \leftarrow \mathcal{G} \setminus \{g^*\}$

return π



$$u(\pi_{\text{greedy}}) = u(a \mapsto \{s_a, a_3, a_4, t_a\}) + u(b \mapsto \{s_b, b_1, b_3, t_b\}) = 0.35 + 1.0 = 1.35$$

Round-robin Allocations (p-rr and n-rr)

Fast (polynomial) fair approaches

Principle

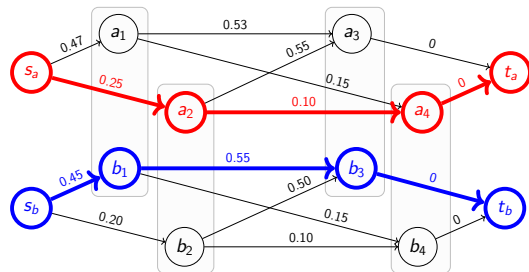
- Agents choose items in turn
- p-rr: items are paths
- n-rr: items are nodes
- p-rr reaches a Nash equilibrium

Round-robin Allocations (p-rr and n-rr)

Fast (polynomial) fair approaches

Principle

- Agents choose items in turn
- p-rr: items are paths
- n-rr: items are nodes
- p-rr reaches a Nash equilibrium



$$u(\pi_{p-rr}) = u(\pi_{n-rr}) = u(a \mapsto \{s_a, a_3, a_4, t_a\}) + u(b \mapsto \{s_b, b_1, b_3, t_b\}) = 0.35 + 1.0 = 1.35$$

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Experimental Setup

Low Earth orbit constellation (500km altitude)

- $n_p \in \{2, 4, 8, 16\}$ regularly-spaced orbital planes having a 60-degree inclination
- 2 anti-phase satellites per plane

Agents (4 users)

- 2 requests per agent
 - position: POIs in the same région (France)
 - time points : every day at $8 : 00 + \delta_r$, $12 : 00 + \delta_r$, et $16 : 00 + \delta_r$, $\delta_r \in [-2, 2]$
 - with a 1h-tolerance
- time horizon: 365 days (1095 layers in DAGs)

n_p	2	4	8	16
width	3.08	5.41	10.05	19.38
conflicts	26798.80	45636.06	82971.20	158180.20
mean duration (s)	603.28	600.10	599.87	598.75

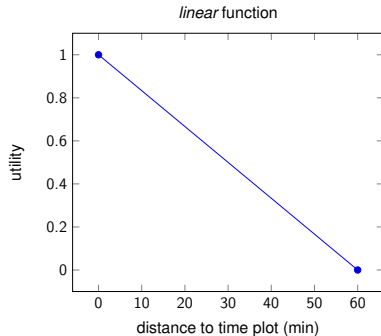
Experimental Setup (cont.)

Utilities

- Utilities only attached to slots and not to transitions between slots



- Same function for all users: linear with the distance to the center of the requested time point

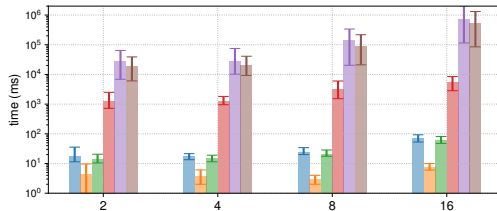
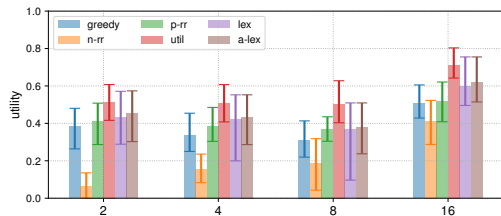


Experimental environment

- solvers implemented in Java, and IBM CPLEX 20.1 Java API for util, lex and a-lex
- CPU Intel(R) Xeon(R) E5-2660 v3 @ 2.60GHz with 20 cores, 62GB RAM, Ubuntu 18.04.5 LTS
- 30 randomly generated PADAG instances per configuration

Results

Utility and computation time



- a-lex performs at almost 85% of the optimal utility
- lex is equivalent to a-lex (< 5% on average)
- p-rr performs at almost 71% of the optimal utility
- n-rr results in very low utility allocations
- greedy is slightly worse than p-rr

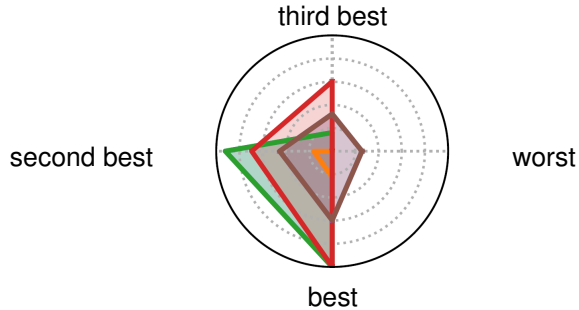
Results (cont.)

Utility and computation time

- ⇒ Larger constellation problems are easier to solve for the utilitarian perspective for non optimal algorithms, because there are more options to build conflict-free paths

Results

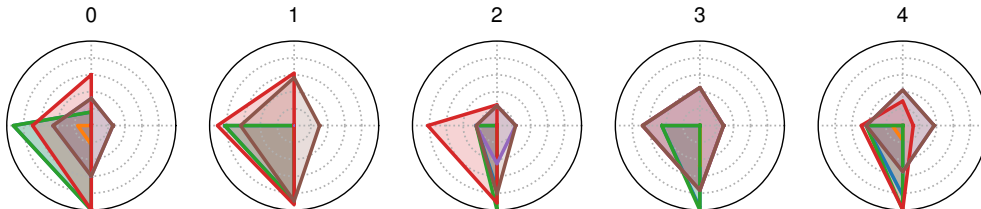
Utility profiles (leximin ordering)



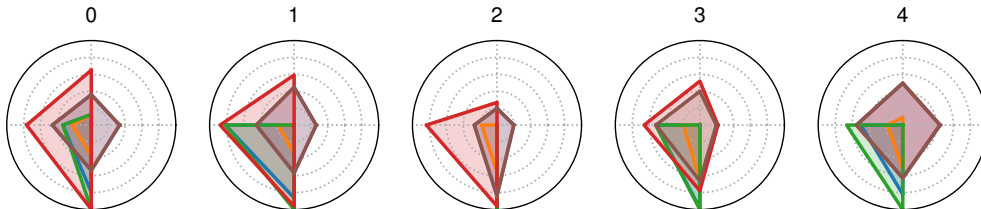
Results

Fairness (10 first instances over 30)

2 orbital planes



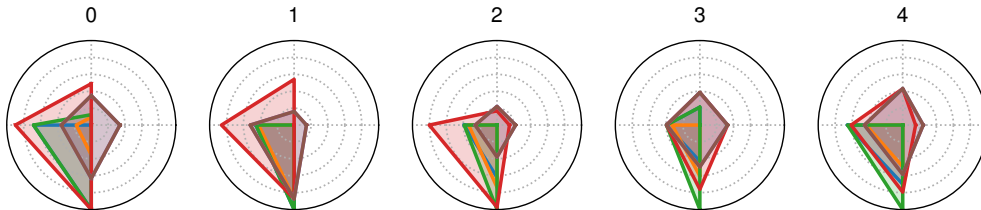
4 orbital planes



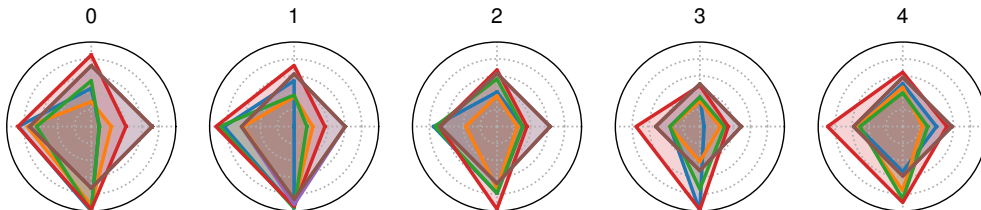
Results (cont.)

Fairness (10 first instances over 30)

8 orbital planes



16 orbital planes



— greedy — n-rr — p-rr — util — lex — a-lex

Results (cont.)

Fairness (10 first instances over 30)

- greedy unfairly allocate to the first agents
- *round-robin* methods are more fair, but struggle on larger constellations
- util neglects the last user
- lex and a-lex are the most fair and are equivalent

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Conclusions

- First approach to allocate bundles with conflicting preferences represented as DAGs
- Several investigated methods
 - util, lex, a-lex, greedy, p-rr, n-rr
- Experiments on orbit slot allocation problems
- a-lex is the best compromise between utilitarianism, fairness and computation time

- Work in progress
 - Consider other types of requests (and graphs)
 - systematic requests
 - stereoscopic requests
 - Consider other methods for these graphs
 - Constraint programming (PAIS'22)
 - Local search and Min-conflict
- Future work
 - Consider areas (AOI instead of POI)
 - Consider other applications (e.g. NFV)

Acknowledgements

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Thank you for your attention!

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