Distributed Constraint Optimization for the Internet-of-Things

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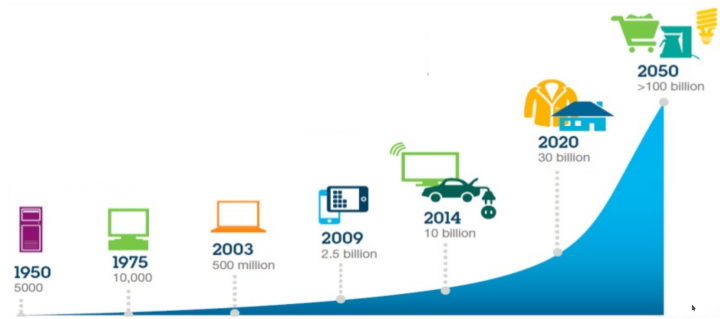
Internet-of-Things (IoT) and its Control

- Huge (marketing ?) trend today
- 25 billion of connected objects in 2020 ? (Gartner)
- Hardware and communication is cheaper and cheaper
- Contrained devices
  - limited cpu and memory resources
  - limited communication capabilities

- Connected things' actions should be **coordinated**
- Current approach: centralizing decisions
  - Communications
  - Resilience
  - Scalability
  - Cost



Distributed Coordination and Decision Making

Autonomous and spontaneous

Coordinating objects to achieve objectives

- Coordination
  - Decentralized
  - Spontaneous
  - Autonomous
- No central point
- Self-adaption to environmental changes
- Self-repair in case of one component failure

About decisions

$$x_i \ ?$$

About decisions

$$x_i \ ?$$

s.t. "I'm happy with x_i"

About decisions

x_i ?

s.t. "I'm happy with x_i"

x_j ?

s.t. "agent i is fine with x_j"

About decisions

x_i ?

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x_j ?

s.t. "agent i is fine with x_j"

How can agents autonomously make their decisions
in a coordinated way, without external control ?

About decisions

x_i ?

s.t. "I'm happy with x_i"

x_j ?

s.t. "agent i is fine with x_j"

How can agents autonomously make their decisions in a coordinated way, without external control ?

$\Rightarrow$ Decentralized decision making

About decisions

x_i ?

s.t. "I'm happy with x_i"

x_j ?

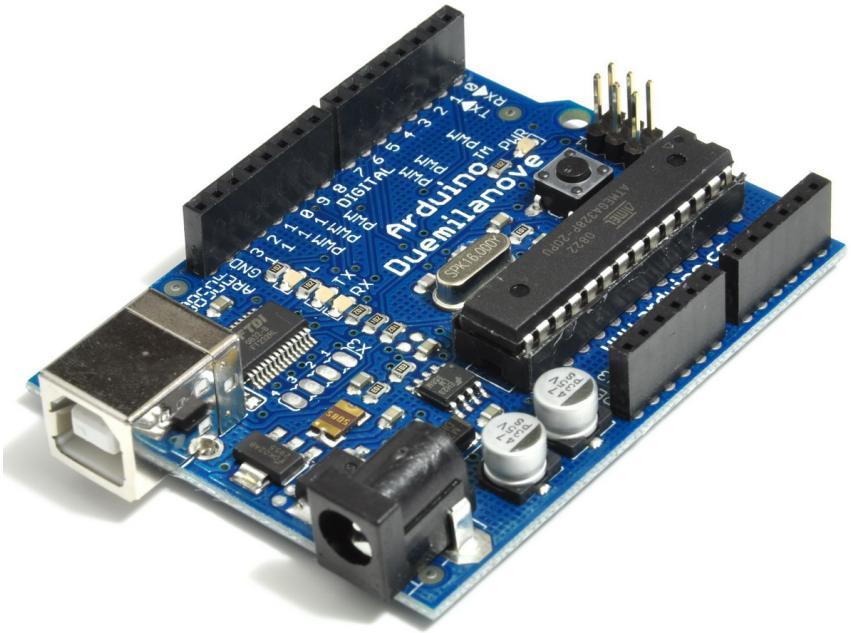
s.t. "agent i is fine with x_j"

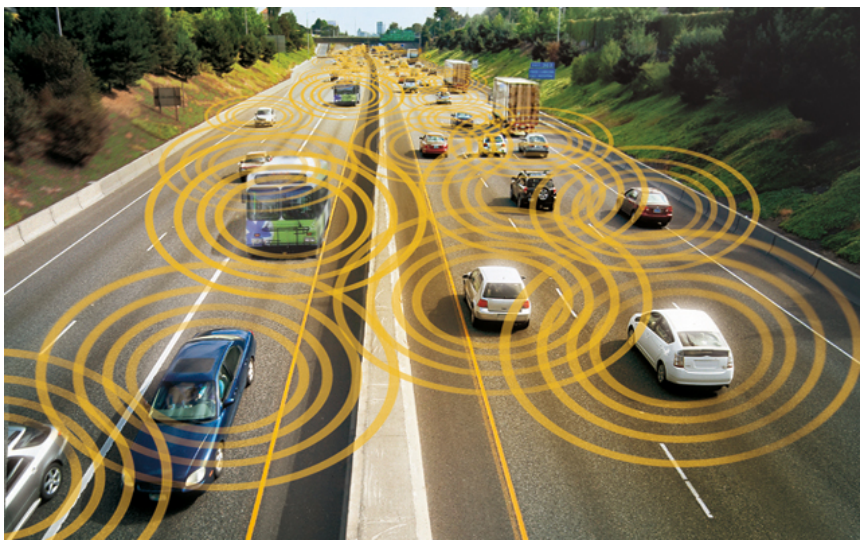
How can agents autonomously make their decisions in a coordinated way, without external control ?

$\Rightarrow$ Decentralized decision making

- Agents have to coordinate to perform best actions
- Agents form a team $\rightarrow$ best actions for the team

Application Domains





Menu

DCOP Framework

Focus on Some Solution Methods

Hands on PyDCOP I

Focus on Smart Environment Configuration Problems

Distributing Computations

Hands on PyDCOP II

Dynamic DCOPs

Conclusion

Menu

DCOP[1]

Distributed Constraints Optimization Problem

Definition (DCOP)

A DCOP is a tuple $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mu \rangle$, where:

- $\mathcal{A} = \{a_1, \ldots, a_{|A|}\}$ is a set of agents
- $\mathcal{X} = \{x_1, \ldots, x_n\}$ are variables
- $\mathcal{D} = \{\mathcal{D}_{x_1}, \ldots, \mathcal{D}_{x_n}\}$ is a set of finite domains, for the x_i variables
- $\mathcal{C} = \{f_1, \ldots, f_m\}$ is a set of soft constraints, where each c_i defines a cost $\in \mathbb{R} \cup \{\infty\}$ for each combination of assignments to a subset of variables
- μ is a function mapping variables to their associated agent

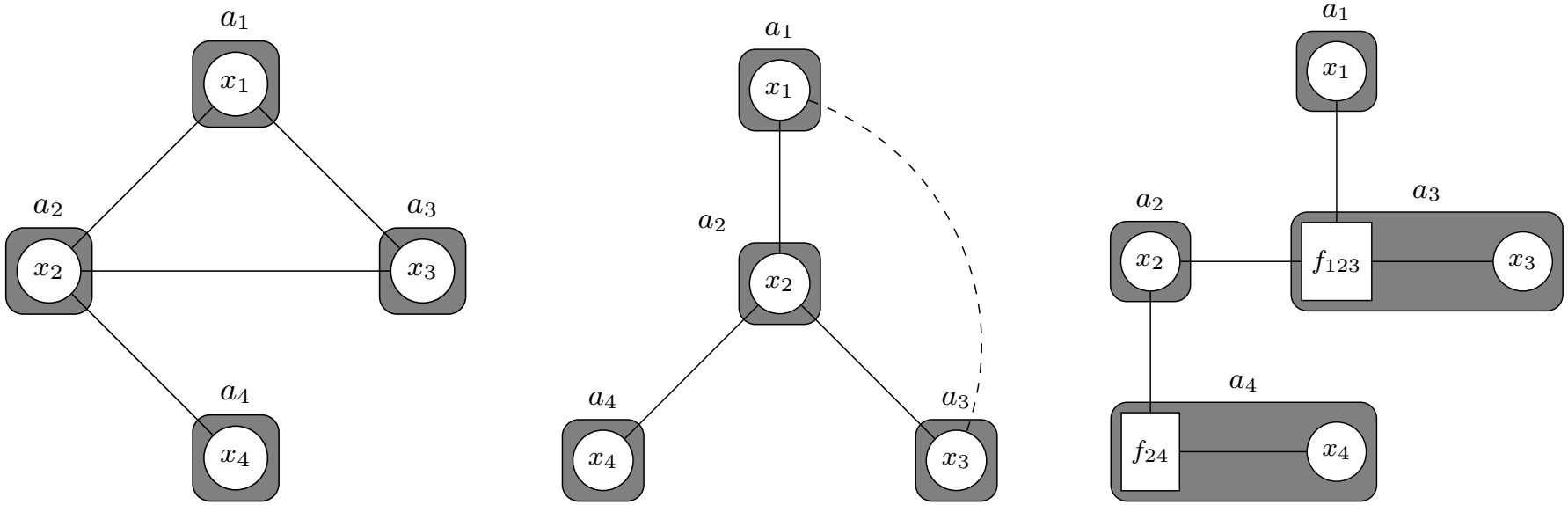
Definition (Solution)

A *solution* to the DCOP is an assignment $\mathcal{A}$ to all variables that minimizes $\sum_i f_i$

[1] Some contents taken from OPTMAS 2011 and OPTMAS-DCR 2014

DCOP

Example and Graphical Representation



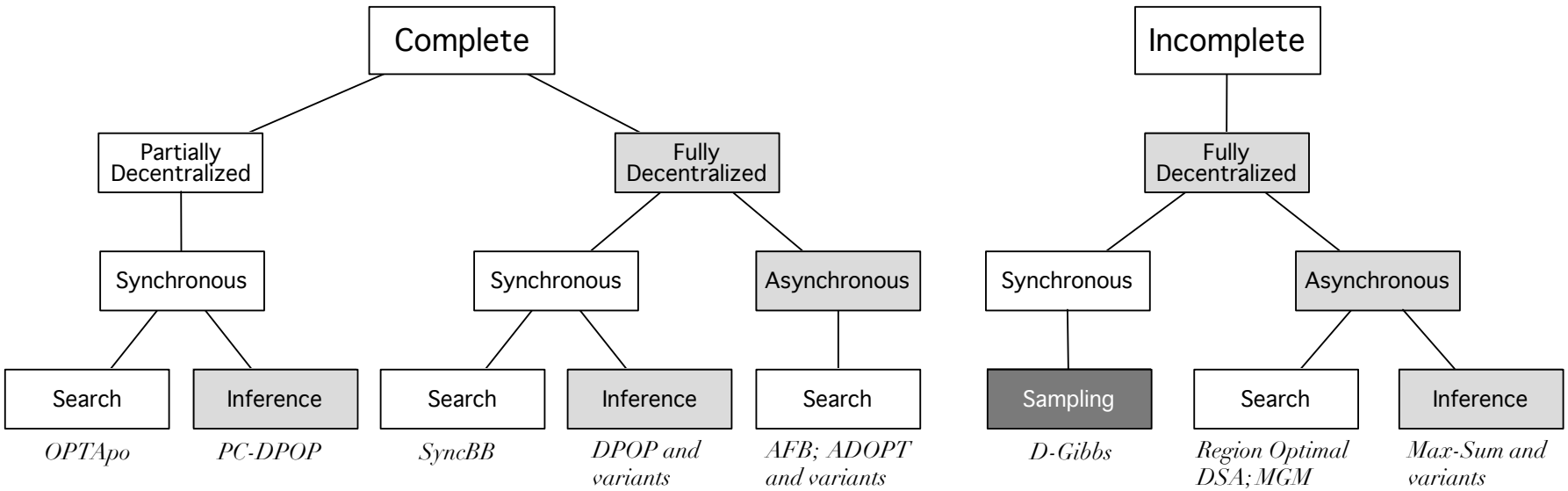
Objective Function

$$F(\mathcal{A}) = \sum_{x_i, x_j \in X} f_{ij} \quad \text{where } f_{ij} = (\mathbf{x}_i + \mathbf{x}_j + 1) mod 3$$

In figure *(a)*:

- $F(\{(x_1, 0), (x_2, 0), (x_3, 0), (x_4, 0)\}) = 4$
- $F(\{(x_1, 1), (x_2, 1), (x_3, 1), (x_4, 1)\}) = 0$

DCOP Algorithms



(FIORETTO et al., 2018)

Menu

DCOP Framework

Focus on Some Solution Methods
- DPOP
- Max-Sum
- DSA
- MGM

Hands on PyDCOP I

Focus on Smart Environment Configuration Problems

Distributing Computations

Hands on PyDCOP II

Dynamic DCOPs

Conclusion

Distributed Pseudotree Optimization Procedure (DPOP)

(PETCU and FALTINGS, 2005)

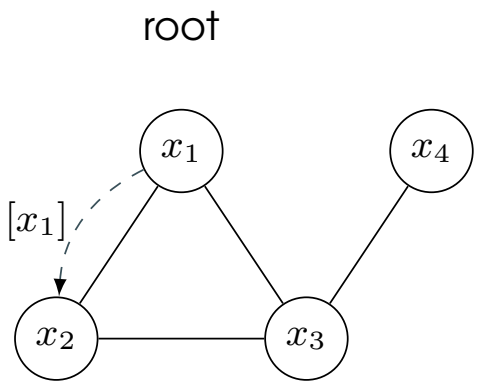
3-phase distributed algorithm

	PHASES	MESSAGES
1.	DFS Tree construction	token passing
2.	Utility phase: from leaves to root	**util** (child $\rightarrow$ parent, constraint table (-child))
3.	Value phase: from root to leaves	**value** (parent $\rightarrow$ children $\cup$ pseudochildren, parent value)

DFS Tree Phase

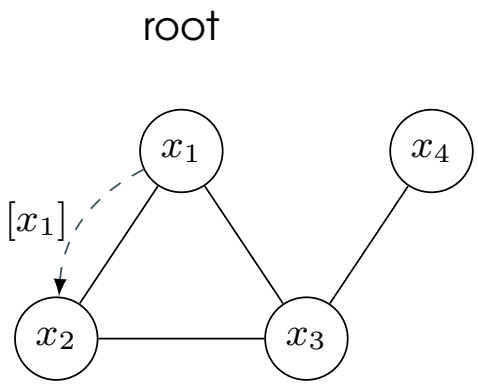
- **Distributed DFS graph traversal**: token, ID, $neighbors(X)$
  1. X owns the token: adds its own ID and sends it in turn to each of its neighbors, which become children
  2. Y receives the token from X: it marks X as visited. First time Y receives the token then $parent(Y) = X$. Other IDs in token which are also $neighbors(Y)$ are **pseudoparent**. If Y receives token from neighbor W to which it was never sent, W is pseudochild.
  3. When all $neighbors(X)$ visited, X removes its ID from token and sends it to $parent(X)$.
- A node is selected as root, which starts
- When all neighbors of root are visited, the DFS traversal ends

DFS Tree Phase: Example



x_1 parent of x_2

DFS Tree Phase: Example



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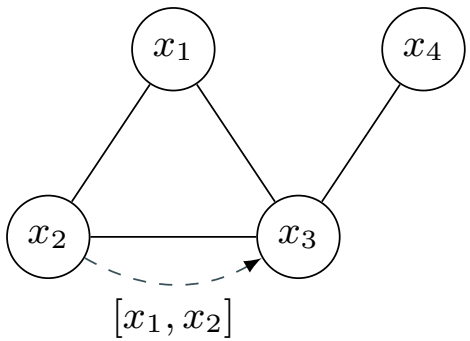
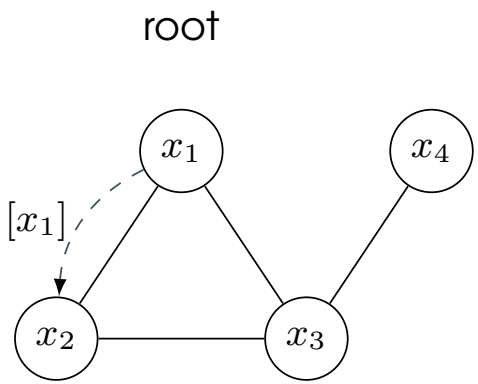
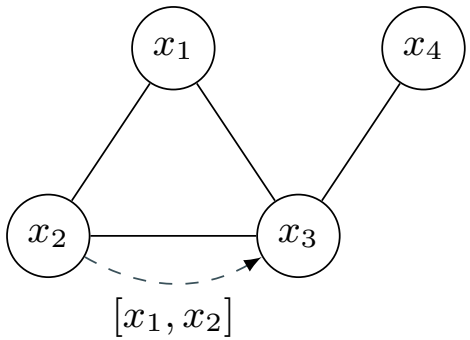


x_2 parent of x_3
x_1 pseudoparent of x_3

DFS Tree Phase: Example



x_1 parent of x_2



x_2 parent of x_3
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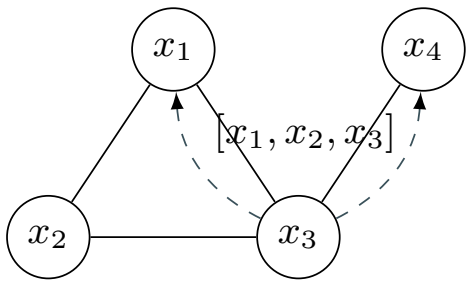
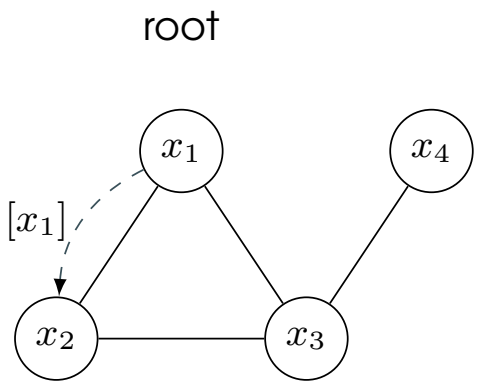
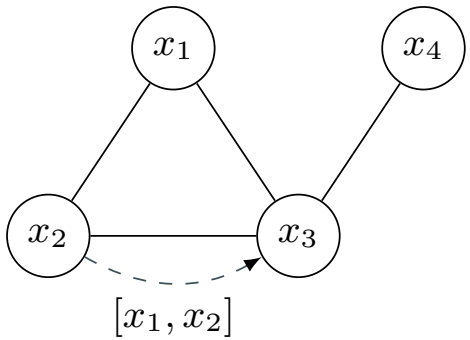


x_3 parent of x_4
x_3 pseudoparent of x_1

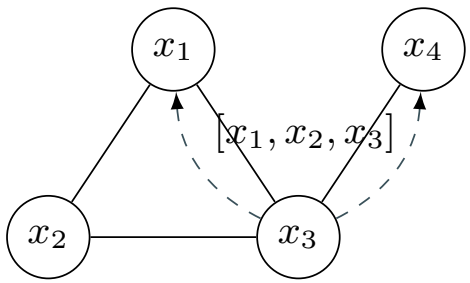
DFS Tree Phase: Example



x_1 parent of x_2



x_2 parent of x_3
x_1 pseudoparent of x_3



x_3 parent of x_4
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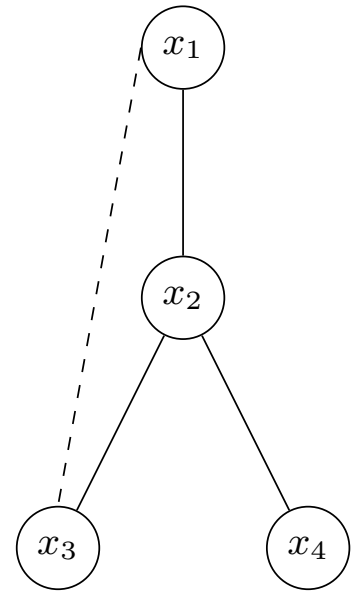

Util Phase

Agent X:

- receives from each child Y_i a cost function: $C(Y_i)$
- combines (adds, joins) all these cost functions with the cost functions with $parent(X)$ and $pseudoparents(X)$
- projects X out of the resulting cost function, and sends it to $parent(X)$

From the leaves to the root

Util Phase: Example

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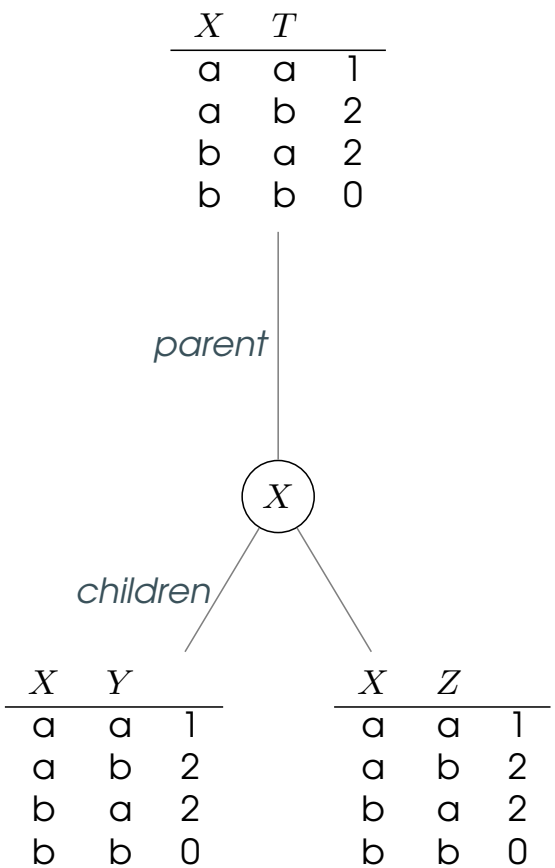
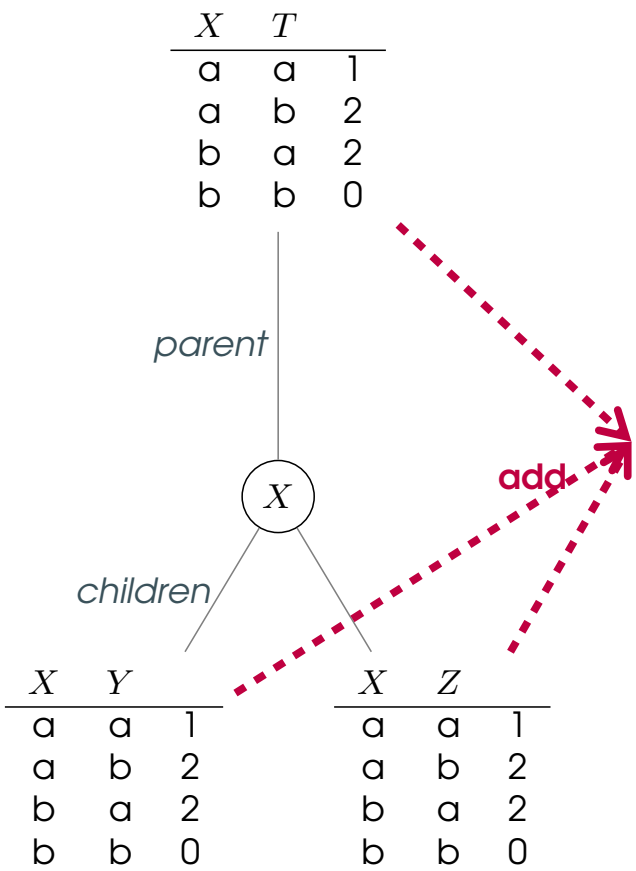


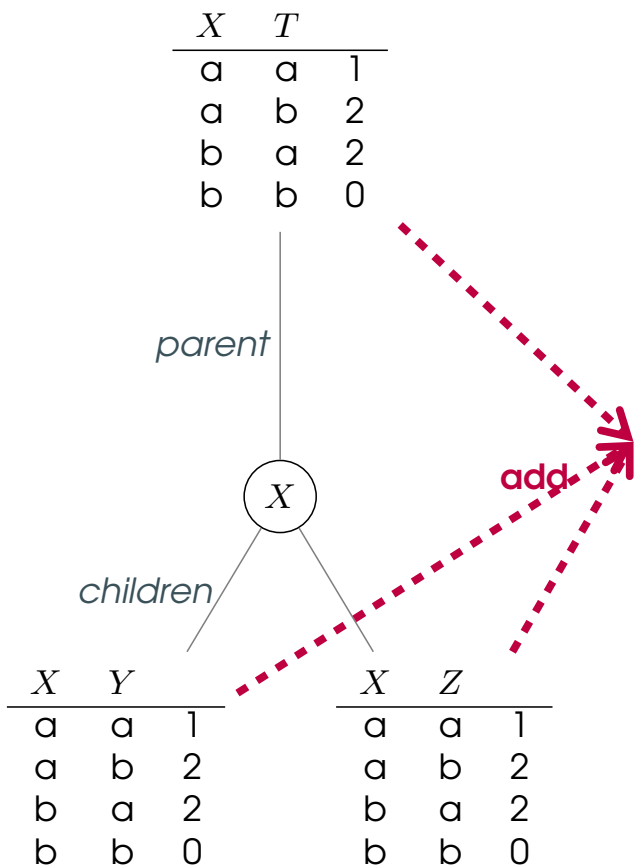
Util Phase: Example



X	Y	Z	T	
a	a	a	a	3
a	a	a	b	4
a	a	b	a	4
a	a	b	b	5
a	b	a	a	4
a	b	a	b	5
a	b	b	a	5
a	b	b	b	6
b	a	a	a	6
b	a	a	b	4
b	a	b	a	4
b	a	b	b	2
b	b	a	a	4
b	b	a	b	2
b	b	b	a	2
b	b	b	b	0

All value combinations
Costs are the sum of applicable costs

Util Phase: Example



X	Y	Z	T	
a	a	a	a	3
a	a	a	b	4
a	a	b	a	4
a	a	b	b	5
a	b	a	a	4
a	b	a	b	5
a	b	b	a	5
a	b	b	b	6
b	a	a	a	6
b	a	a	b	4
b	a	b	a	4
b	a	b	b	2
b	b	a	a	4
b	b	a	b	2
b	b	b	a	2
b	b	b	b	0

All value combinations
Costs are the sum of applicable costs

Project out X

~~X~~	Y	Z	T	
~~a~~	a	a	a	3
~~a~~	a	a	b	4
~~a~~	a	b	a	4
~~a~~	a	b	b	5
~~a~~	b	a	a	4
~~a~~	b	a	b	5
~~a~~	b	b	a	5
~~a~~	b	b	b	6
~~b~~	a	a	a	6
b	a	a	b	4
~~b~~	a	b	a	4
b	a	b	b	2
~~b~~	b	a	a	4
b	b	a	b	2
b	b	b	a	2
b	b	b	b	0

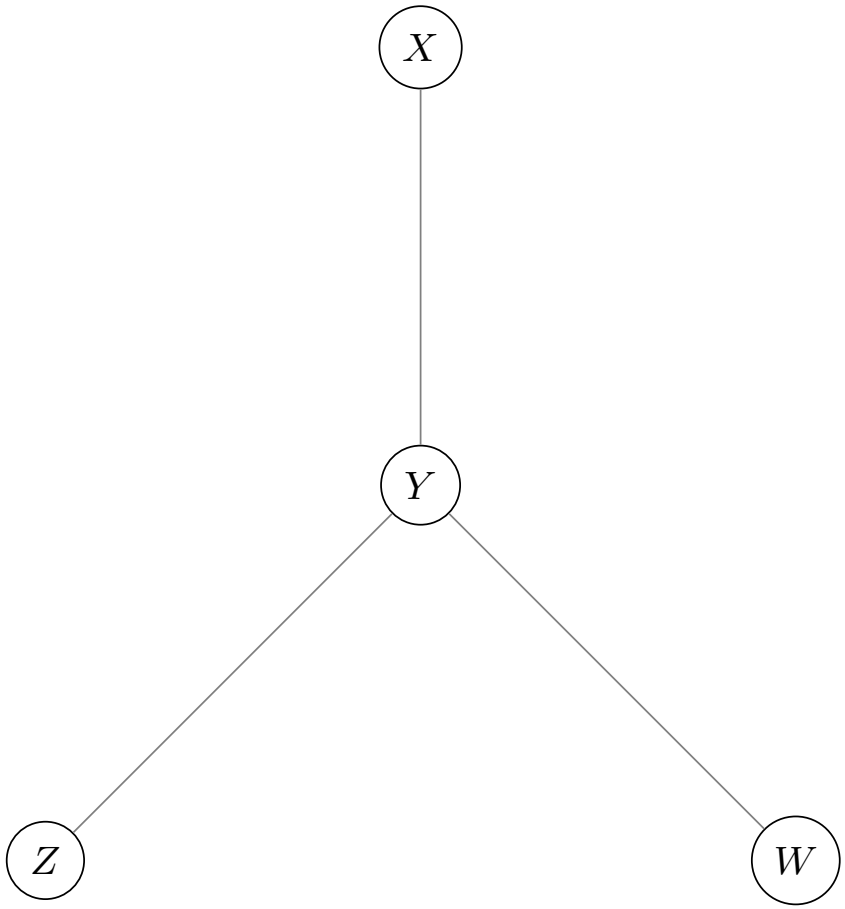
Remove X
Remove duplicates
Keep the min cost

Value Phase

1. The root finds the **value that minimizes the received cost function** in the util phase, and informs its descendants (children ∪ pseudochildren)
2. Each agent **waits to receive** the value of its parent / pseudoparents
3. Keeping fixed the value of parent/pseudoparents, finds **the value that minimizes the received cost function** in the Util phase
4. Informs of this value to its children/pseudochildren

This process starts at the root and ends at the leaves

DTREE : DPOP for DCOPs without backedges



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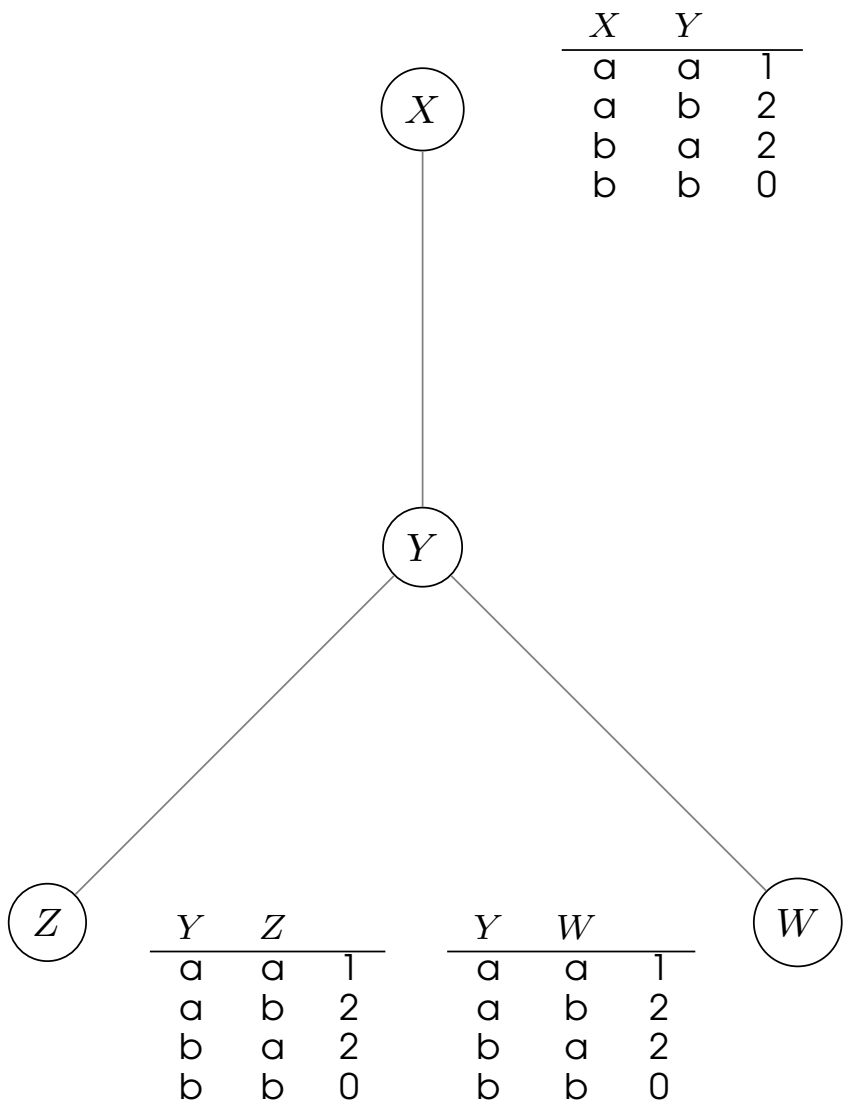

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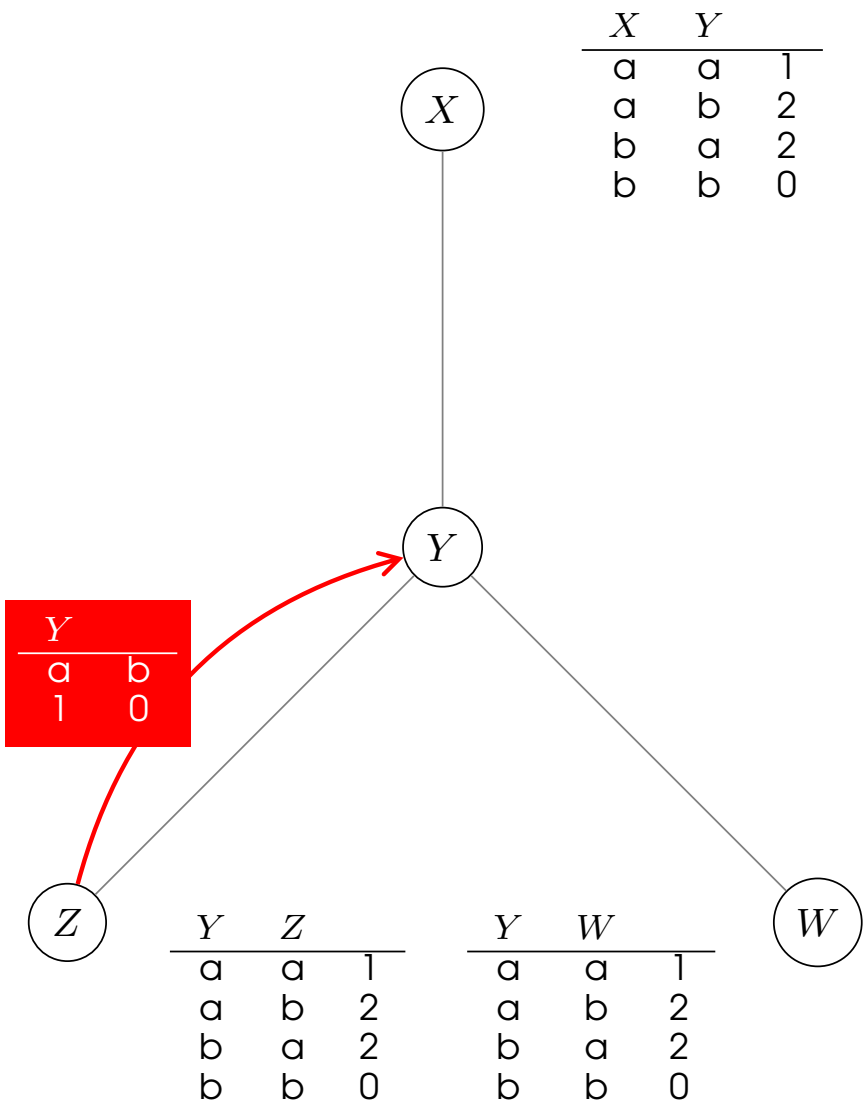

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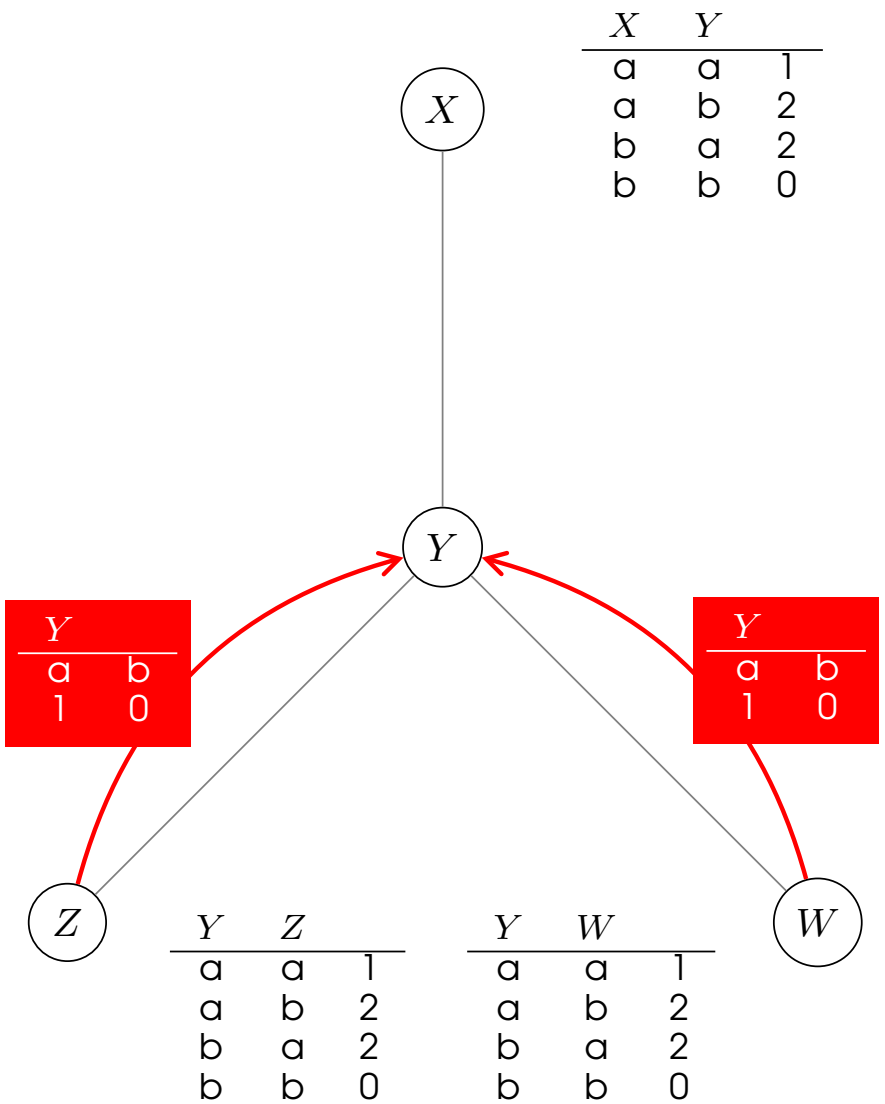

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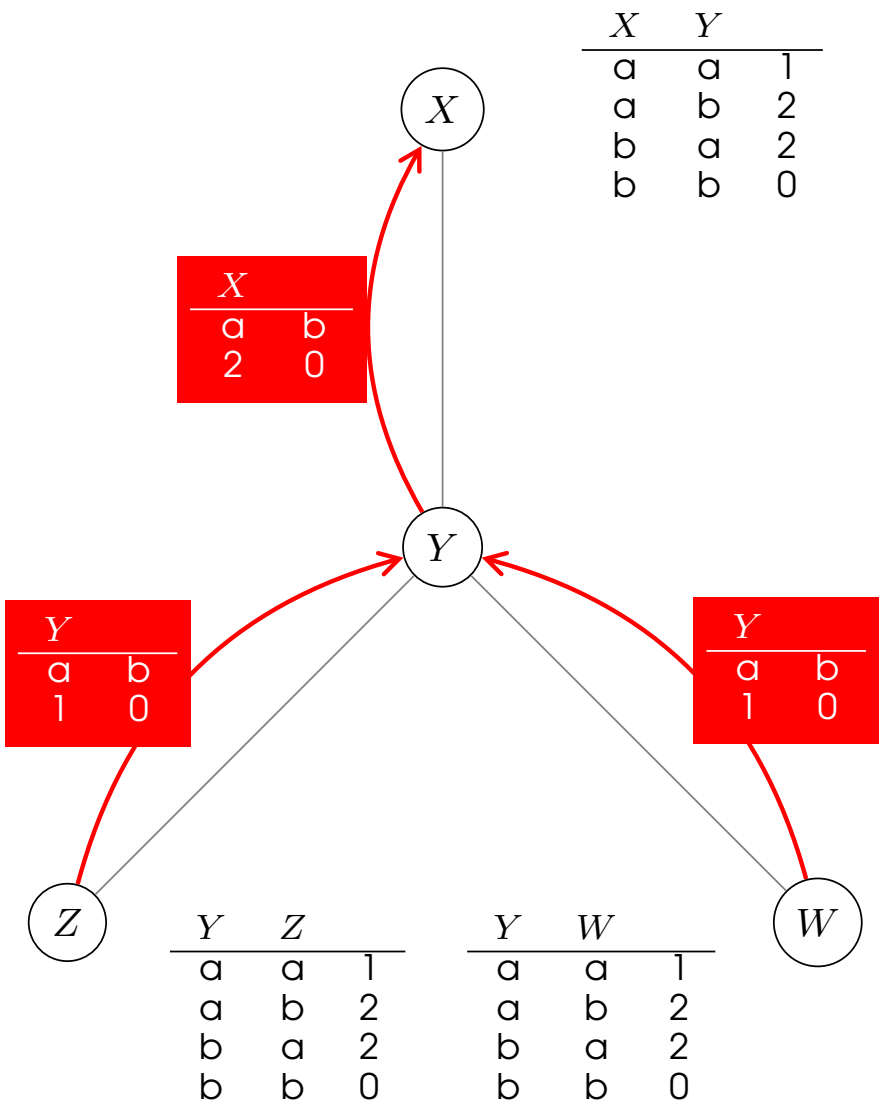

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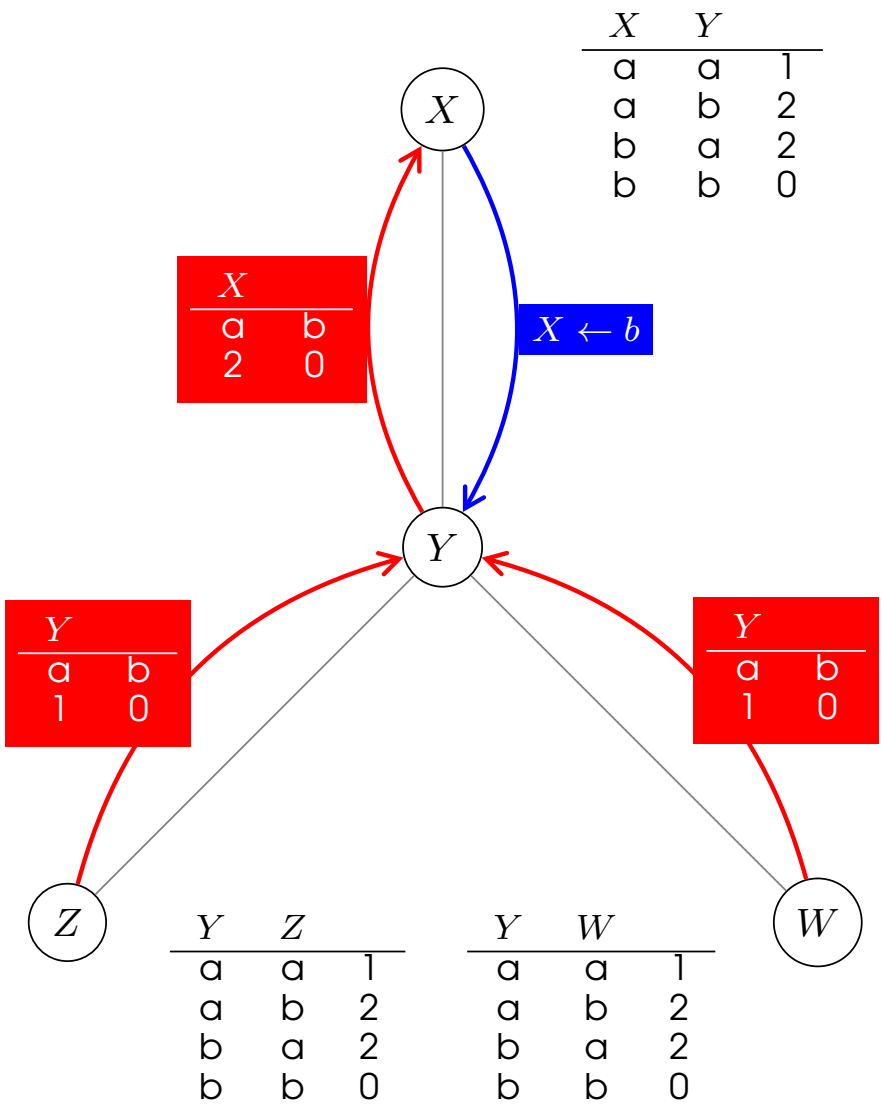

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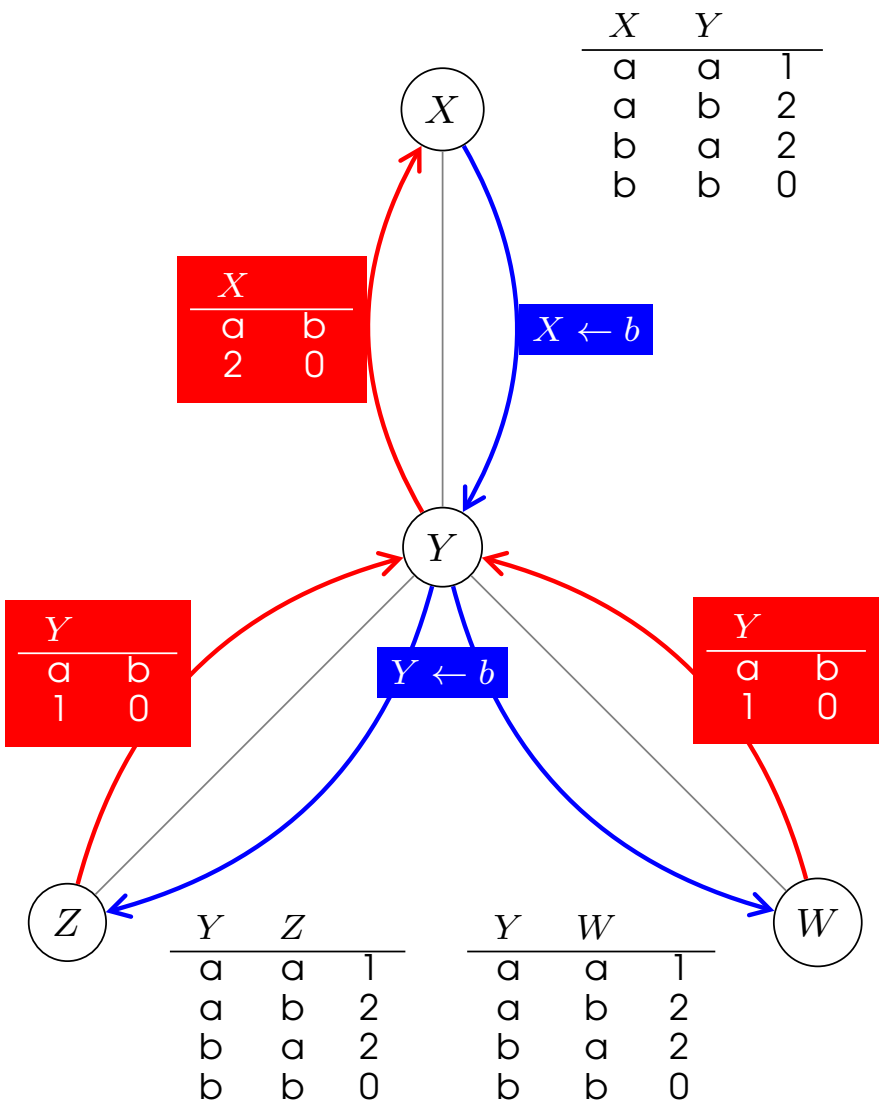

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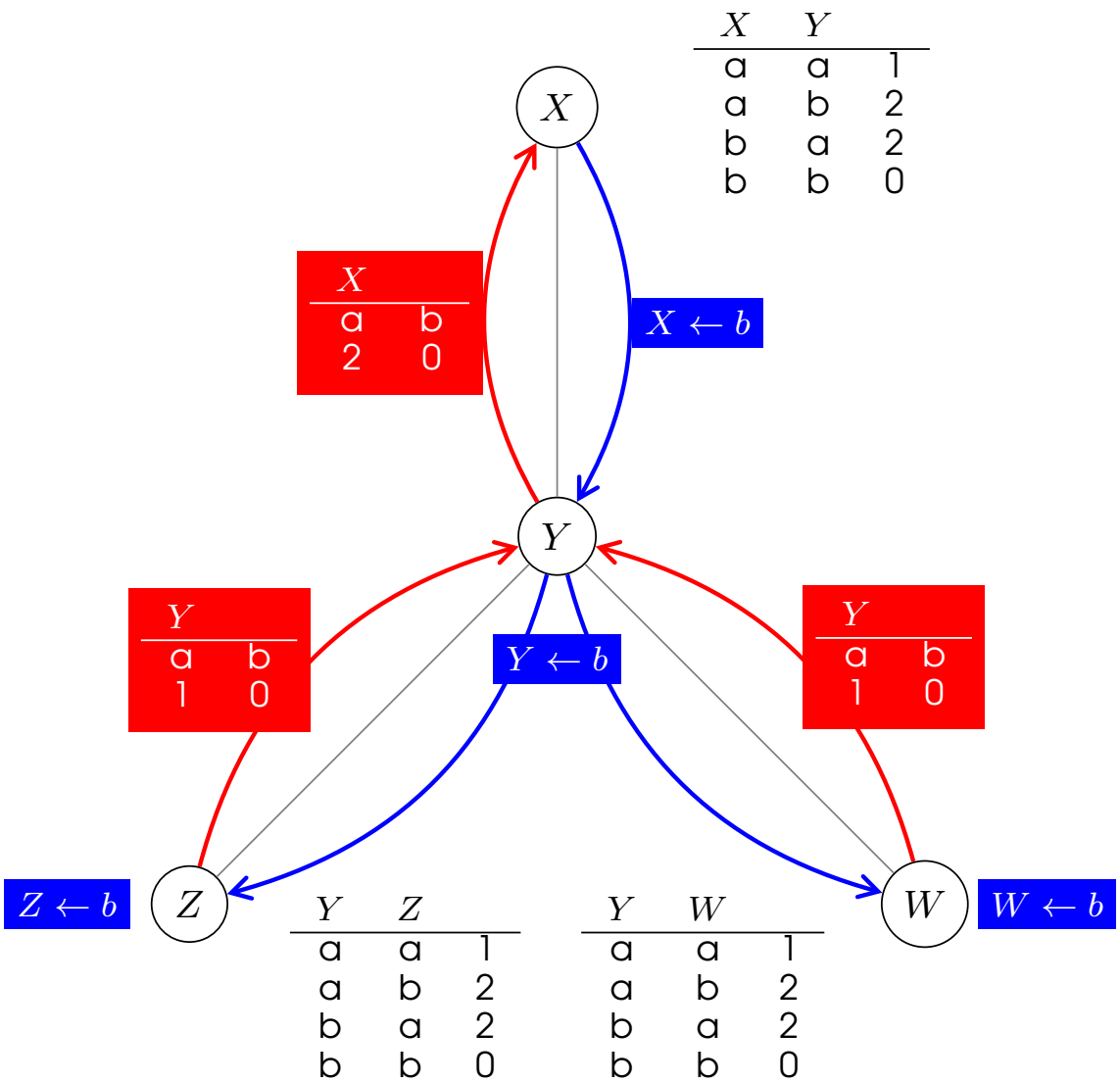

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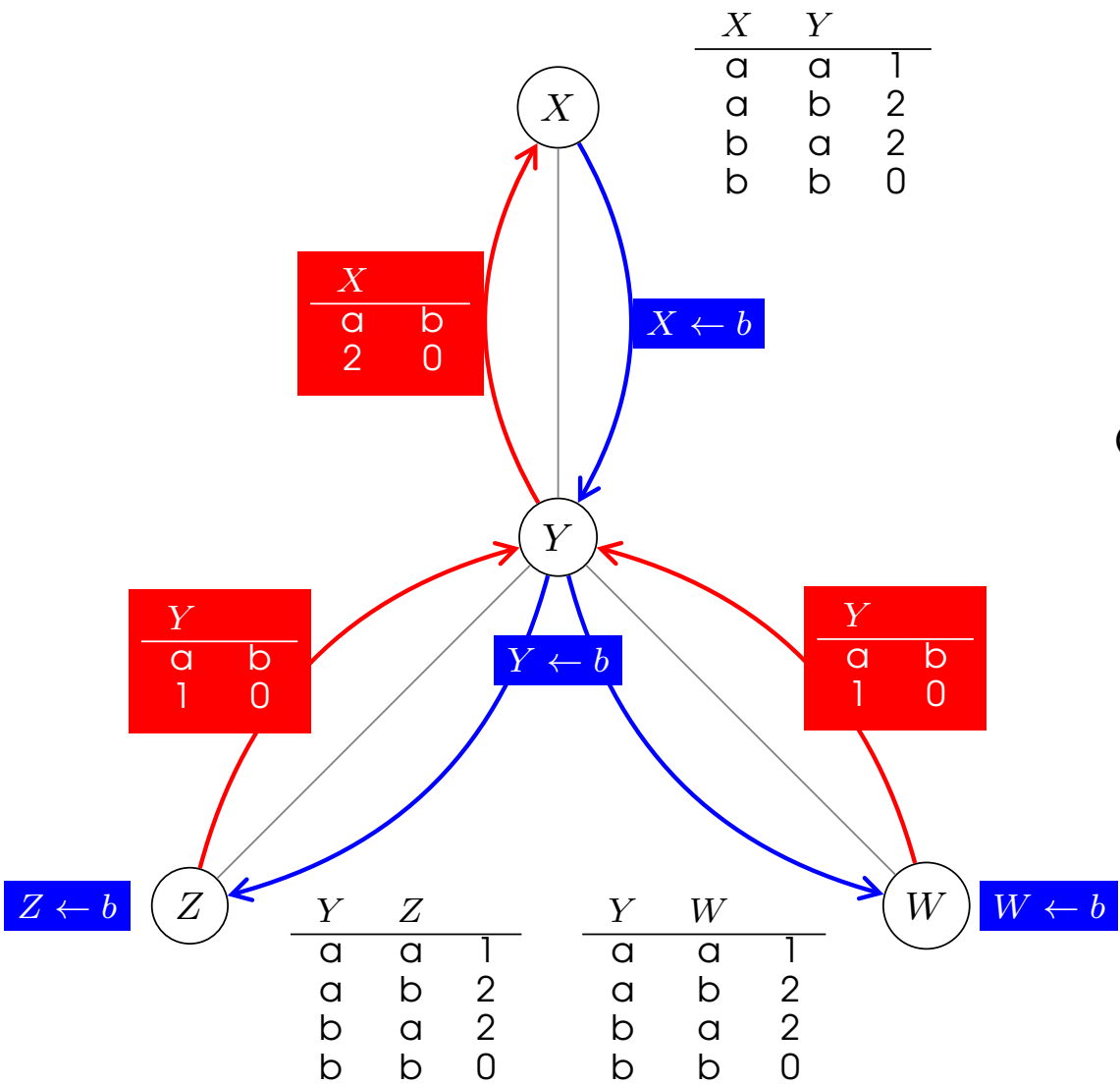
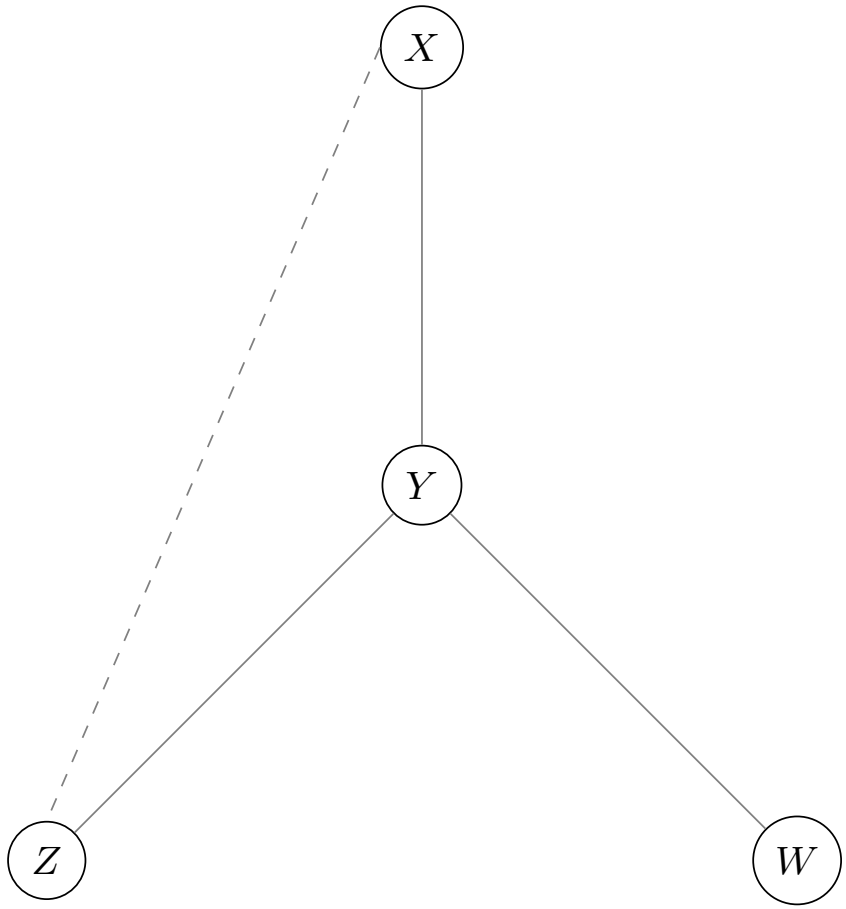


Optimal solution:

- linear number of messages
- message size: linear

DPOP for any DCOP



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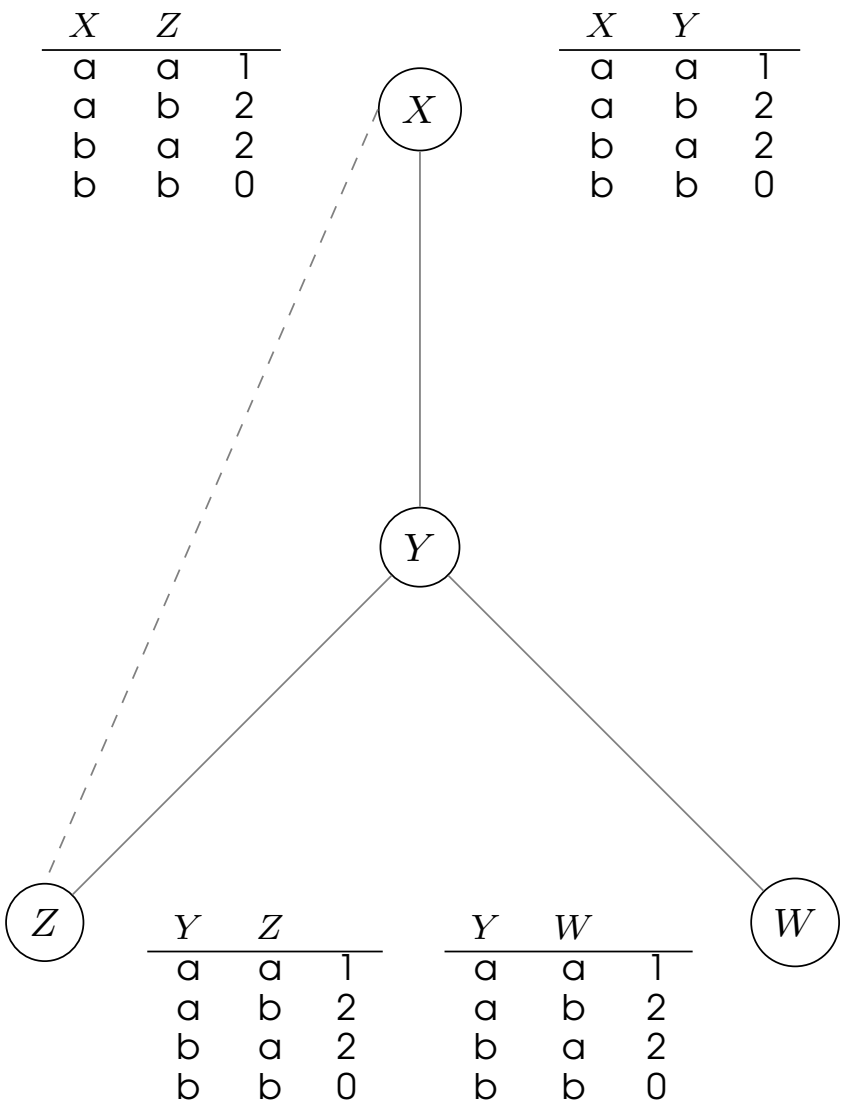

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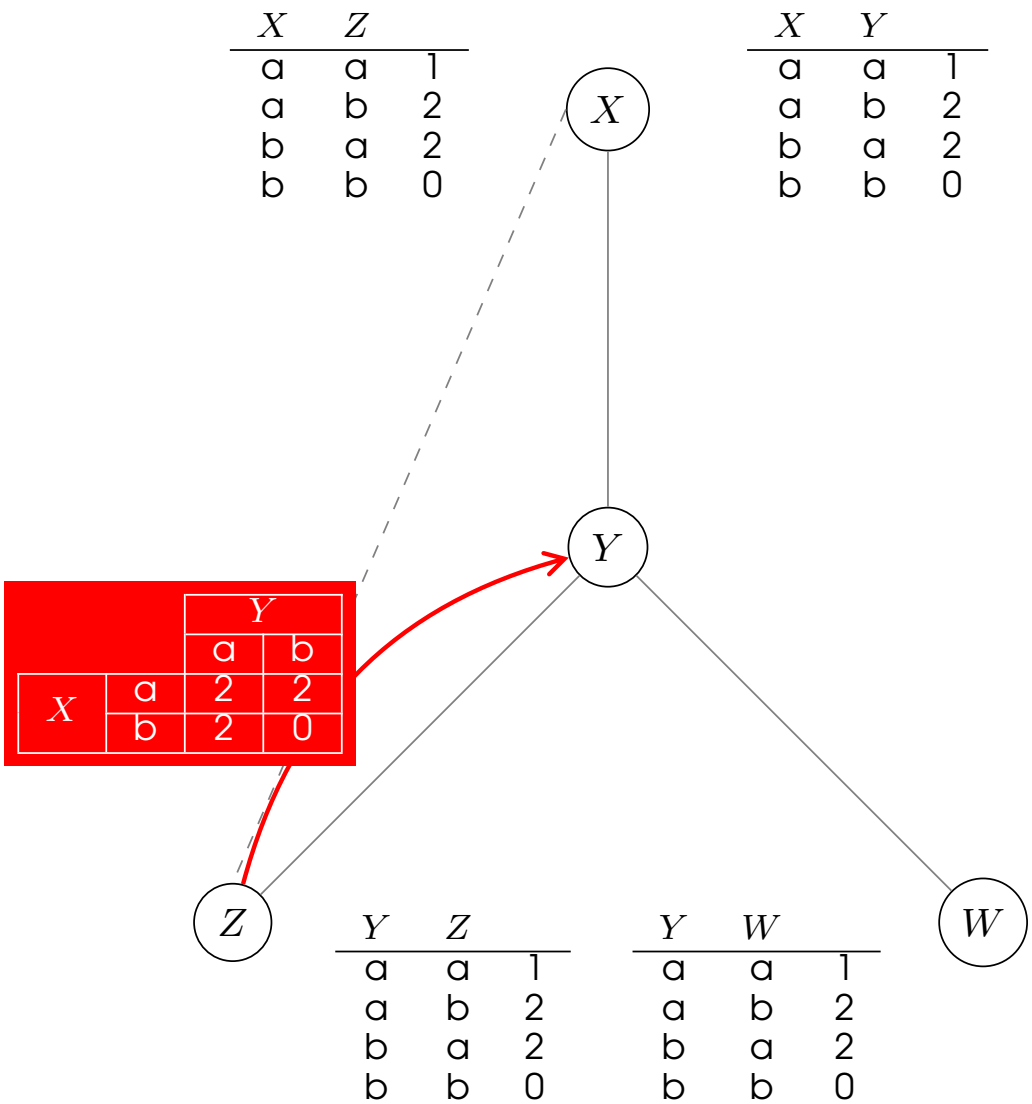

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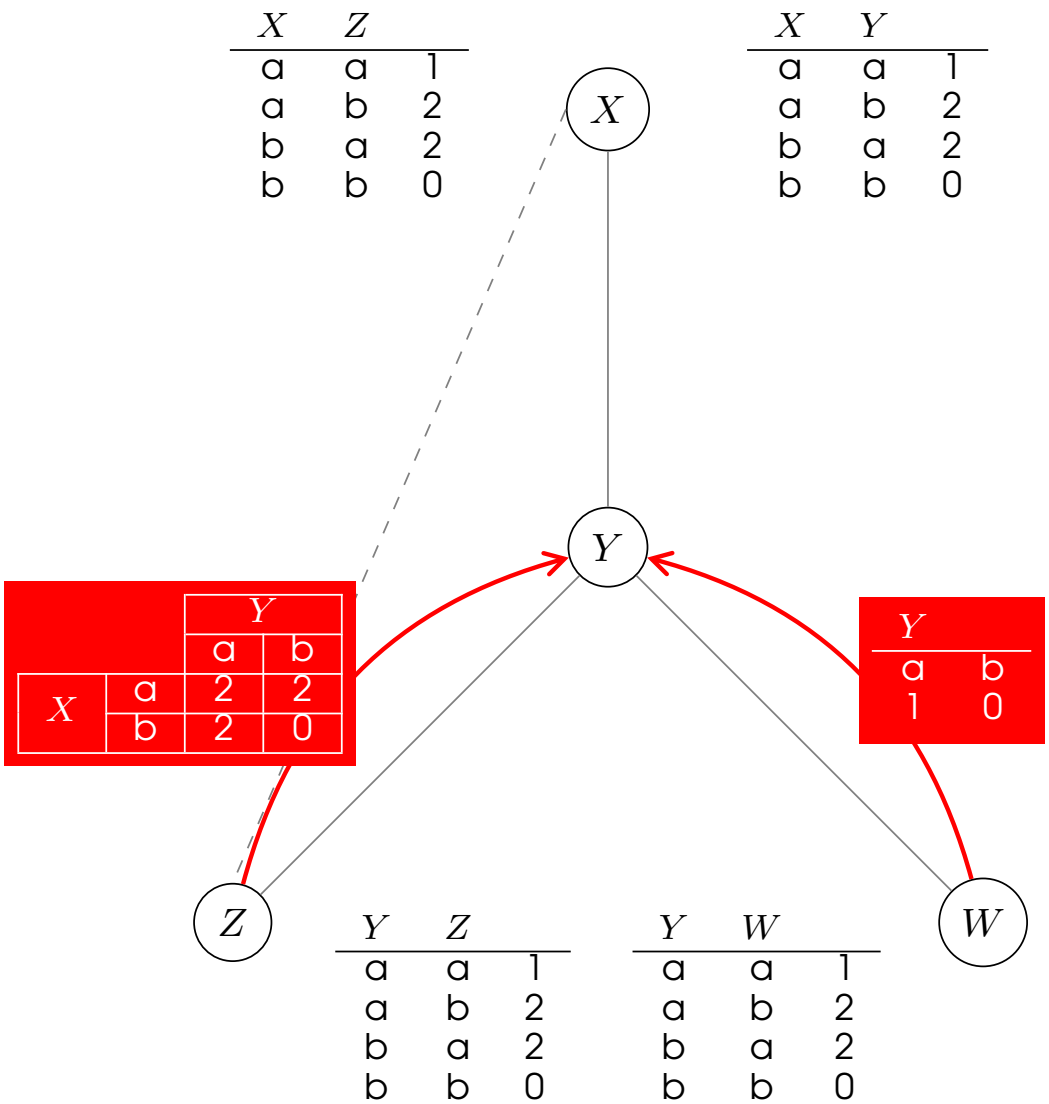

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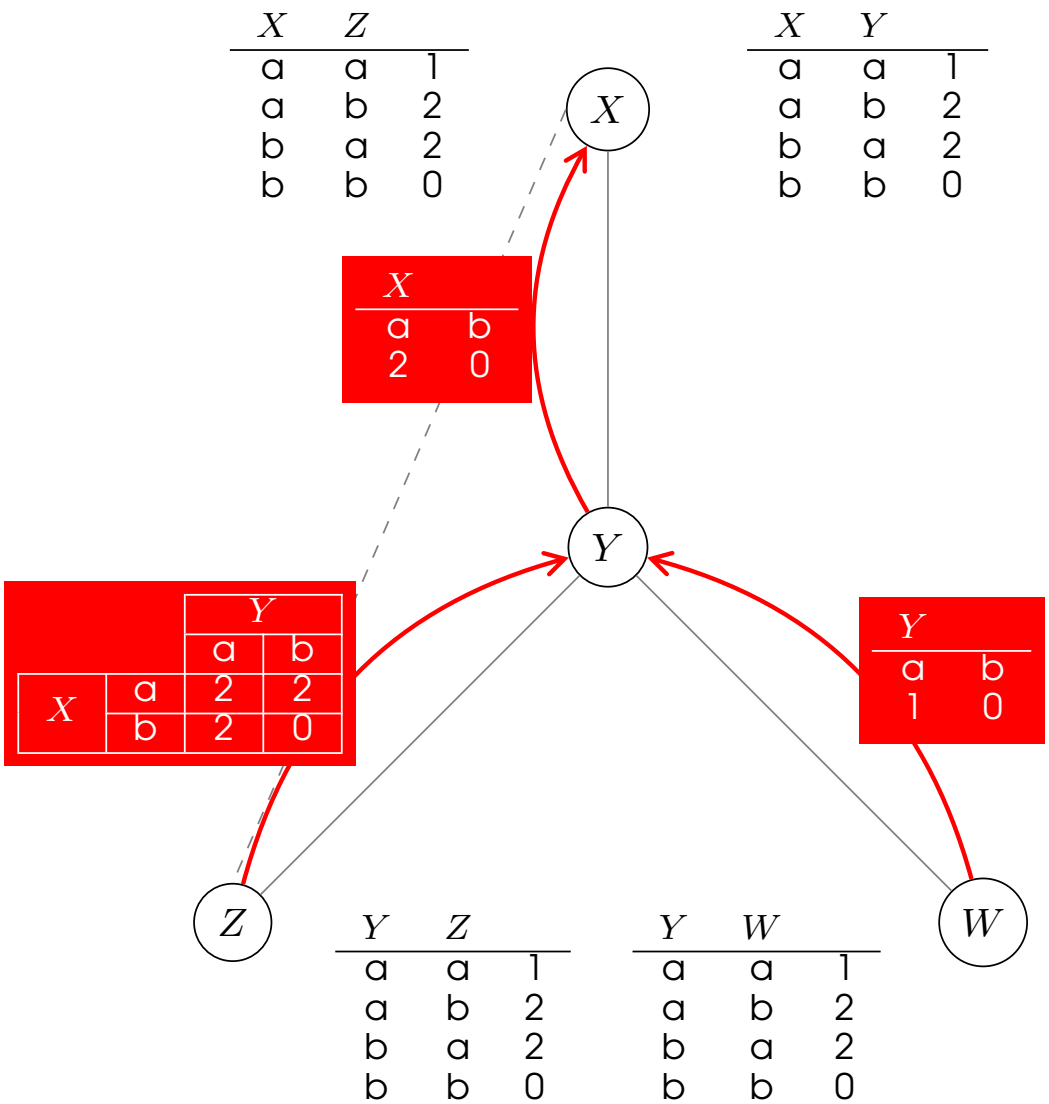

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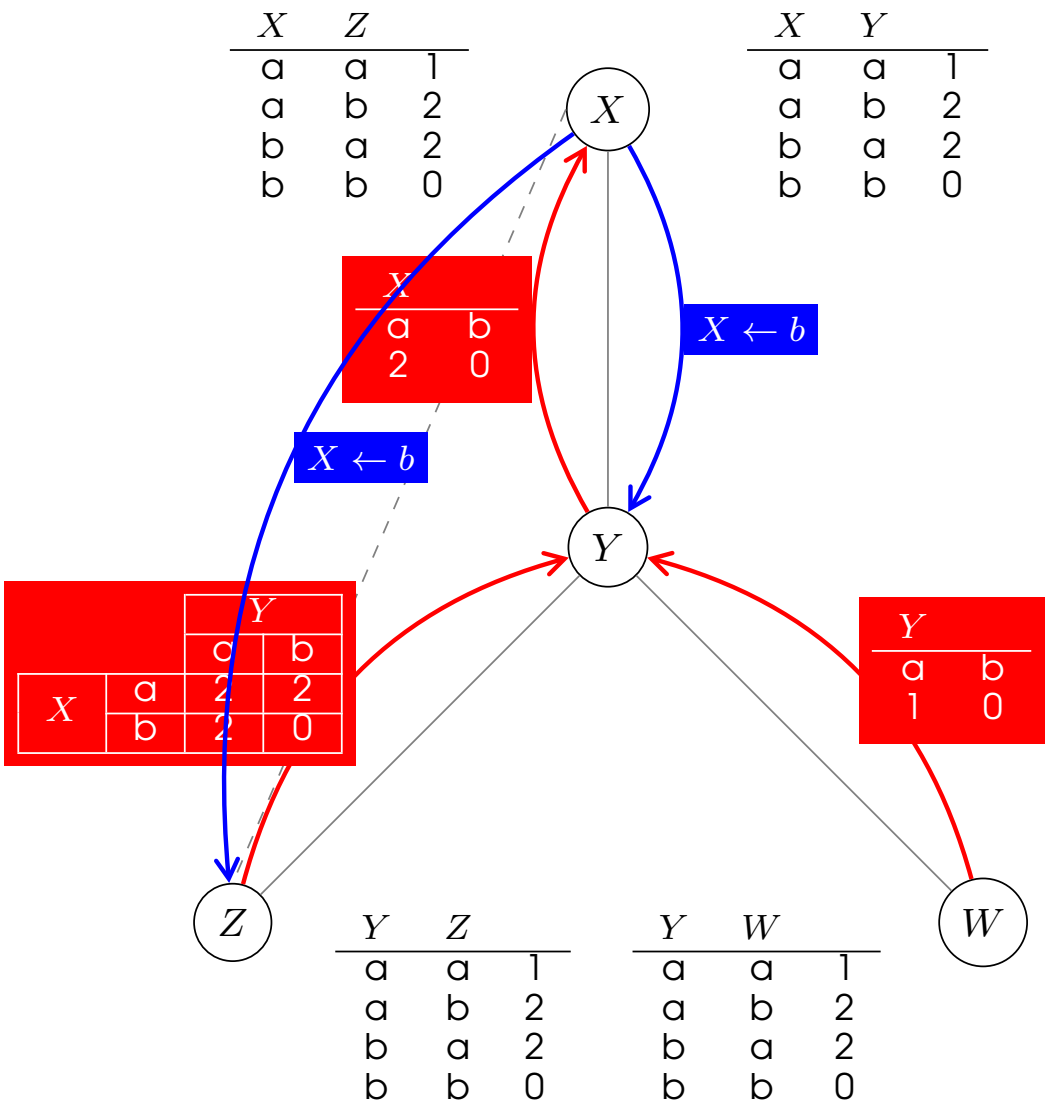

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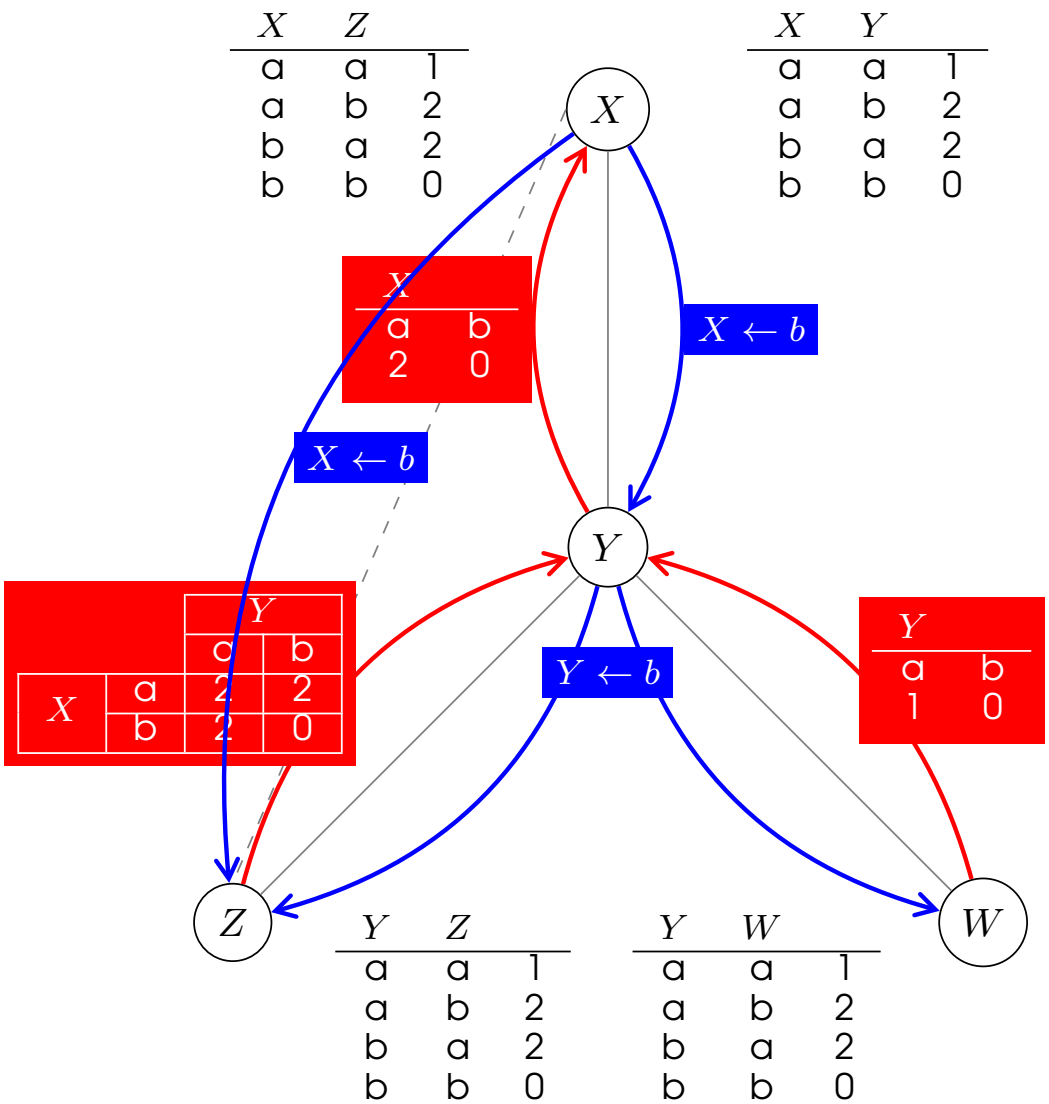

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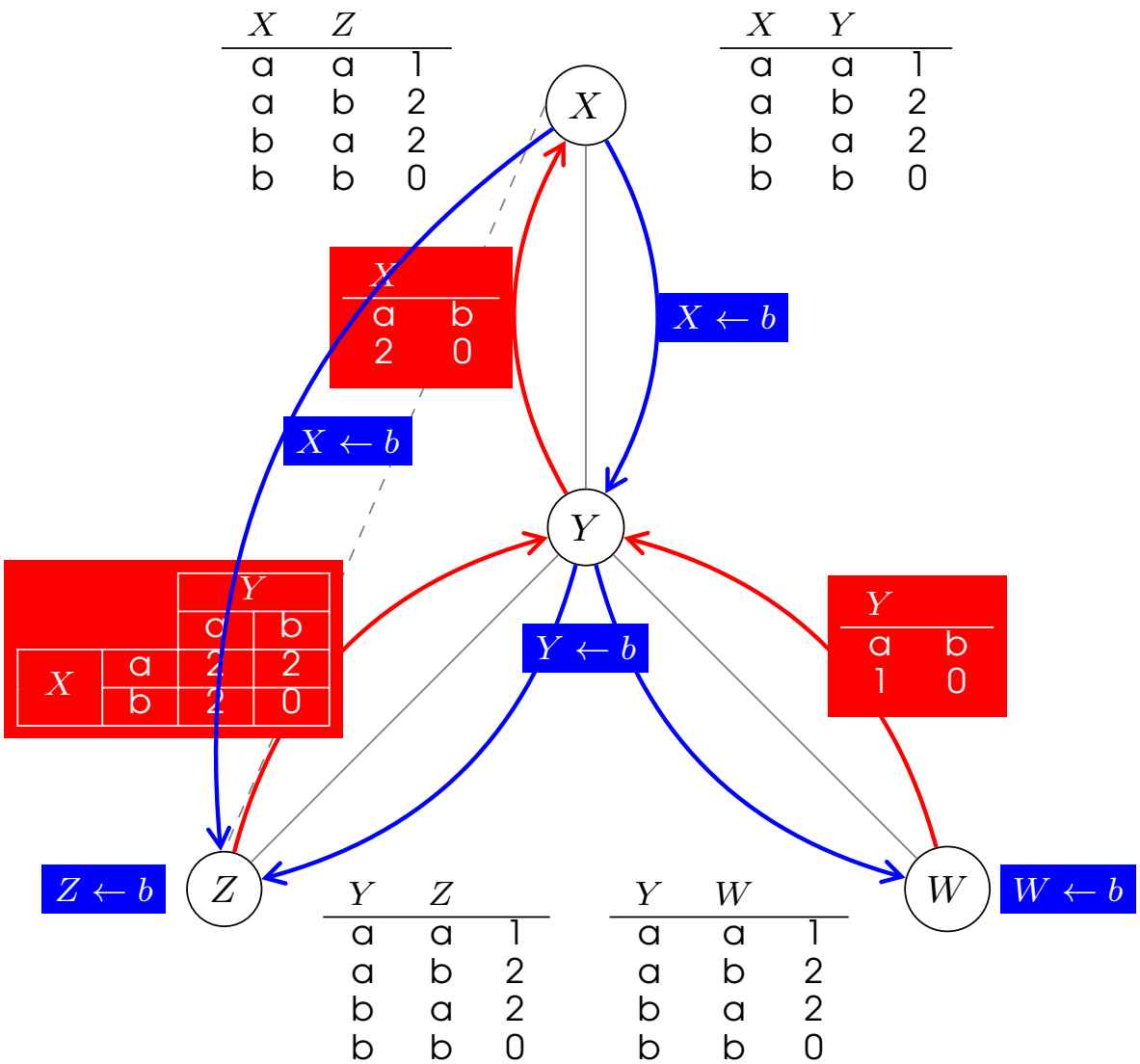

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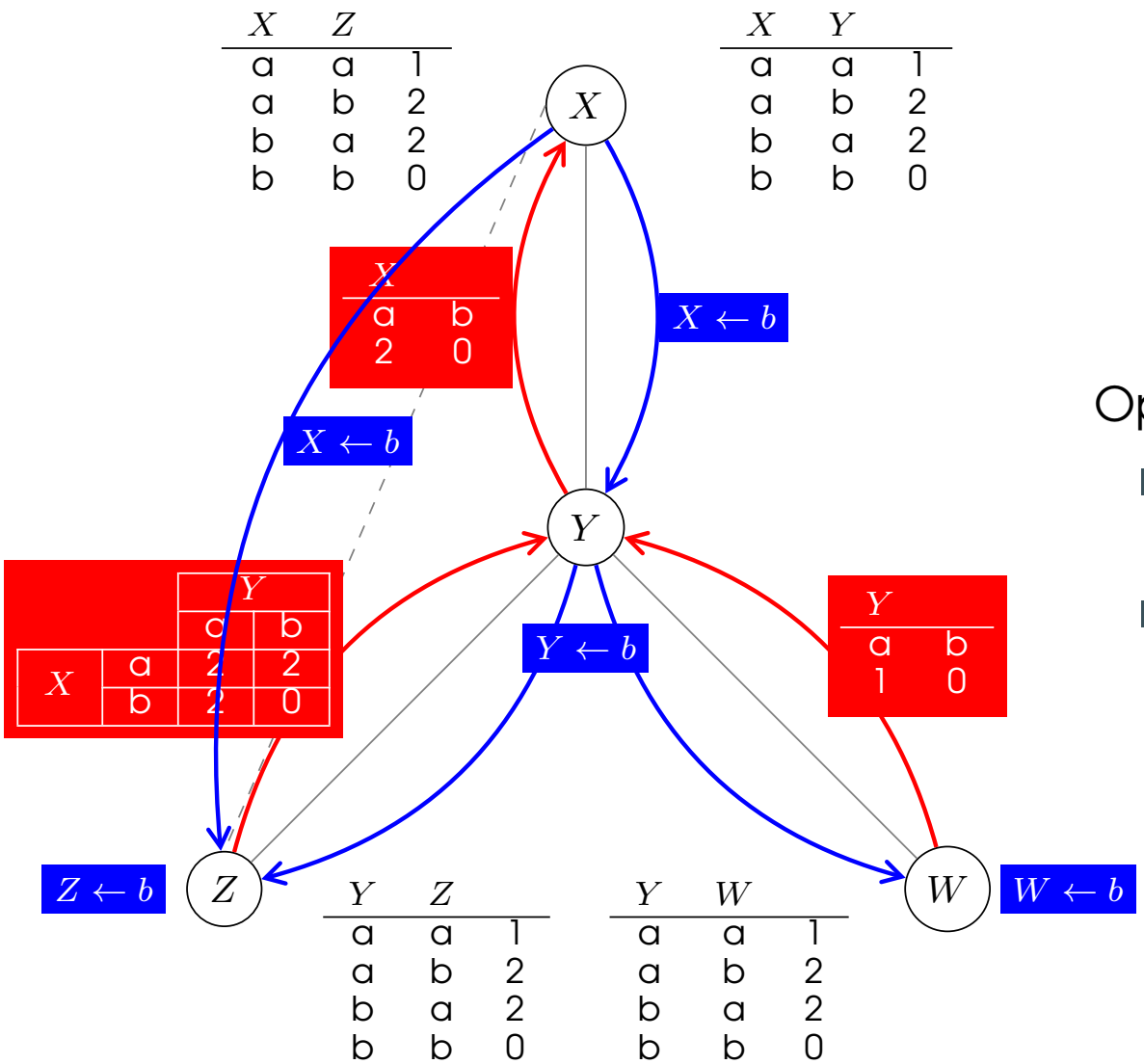


Optimal solution:

- linear number of messages
- message size: exponential

GDL-based approaches

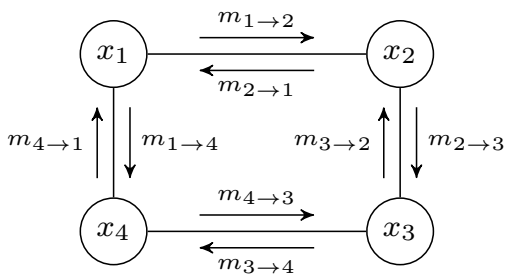
- Generalized Distributive Law (AJI and MCELIECE, 2000)
  - Unifying framework for inference in Graphical models
  - Builds on basic mathematical properties of semi-rings
  - Widely used in Info theory, Statistical physics, Probabilistic models
- Max-sum
  - DCOP settings: maximise social welfare

	K	"$(+,0)$"	"$(\cdot,1)$"	short name
1.	A	$(+,0)$	$(\cdot,1)$	
2.	$A[x]$	$(+,0)$	$(\cdot,1)$	
3.	$A[x,y,\ldots]$	$(+,0)$	$(\cdot,1)$	
4.	$[0,\infty)$	$(+,0)$	$(\cdot,1)$	sum-product
5.	$(0,\infty]$	$(\min,\infty)$	$(\cdot,1)$	min-product
6.	$[0,\infty)$	$(\max,0)$	$(\cdot,1)$	max-product
7.	$(-\infty,\infty]$	$(\min,\infty)$	$(+,0)$	min-sum
8.	$[-\infty,\infty)$	$(\max,-\infty)$	$(+,0)$	max-sum
9.	$\{0,1\}$	$(\mathrm{OR},0)$	$(\mathrm{AND},1)$	Boolean
10.	2^S	$(\cup,\emptyset)$	$(\cap,S)$	
11.	Λ	$(\vee,0)$	$(\wedge,1)$	
12.	Λ	$(\wedge,1)$	$(\vee,0)$.	

Max-Sum

(FARINELLI et al., 2008)

Agents iteratively computes local functions that depend only on the variable they control



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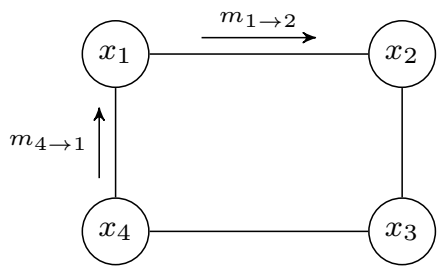

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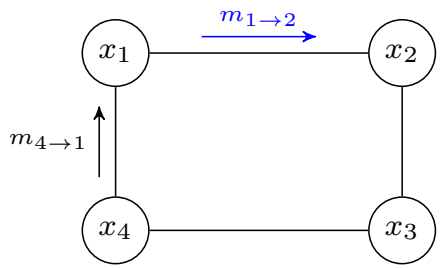

$$m_{1\to2}(x_2) = max_{x_1}(F_{12}(x_1, x_2) + m_{4\to1}(x_1))$$

Max-Sum

(FARINELLI et al., 2008)

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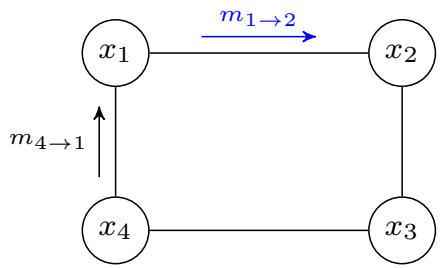


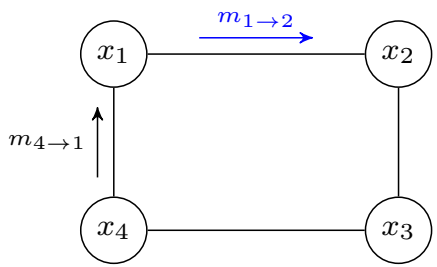
Shared constraint

$$m_{1\to2}(x_2) = max_{x_1}(\quad + m_{4\to1}(x_1))$$

Max-Sum

(FARINELLI et al., 2008)

Agents iteratively computes local functions that depend only on the variable they control



Shared constraint

$$m_{1\to2}(x_2) = max_{x_1}(\quad + \quad)$$

All incoming messages except x_2

Max-Sum

(FARINELLI et al., 2008)

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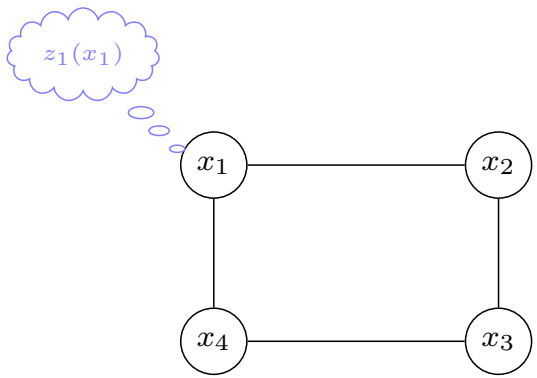

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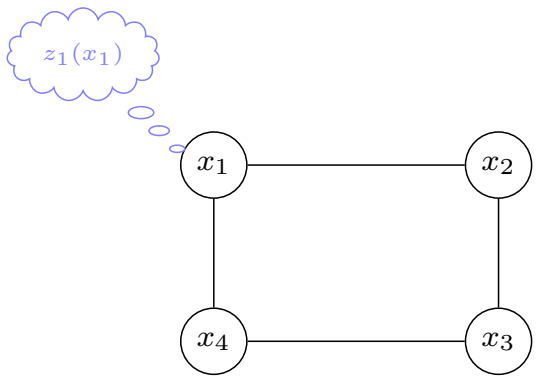

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All incoming messages except x_2

$$z_1(x_1) = m_{4\to1}(x_1) + m_{2\to1}(x_1)$$

Max-Sum

(FARINELLI et al., 2008)

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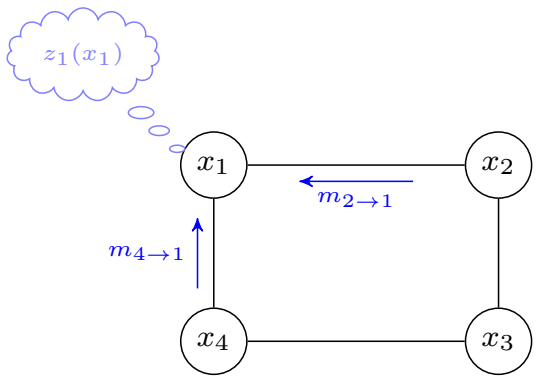

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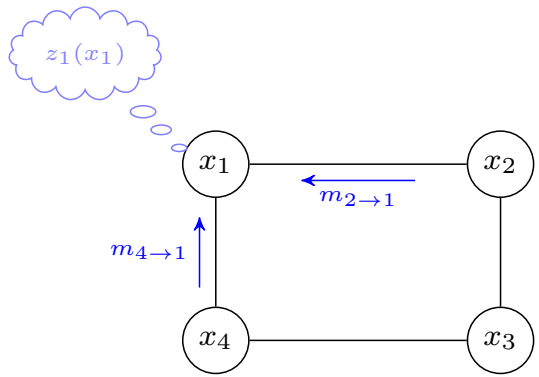

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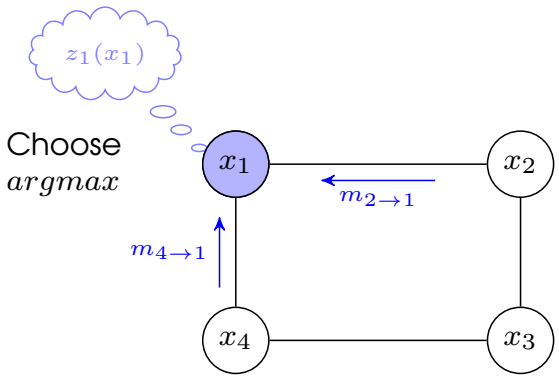

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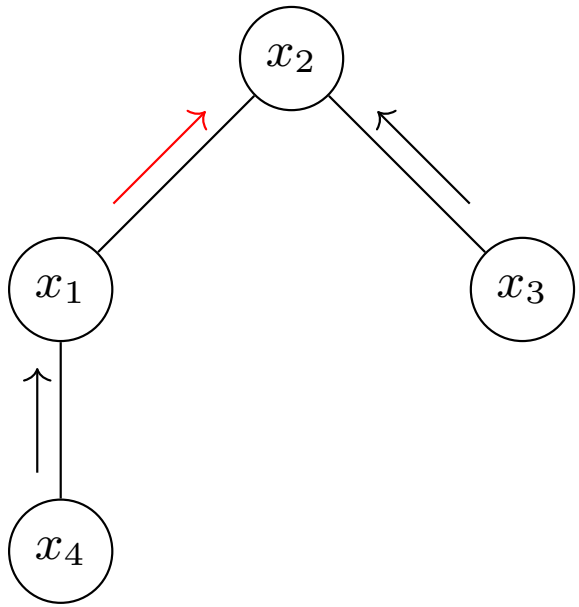
All incoming messages except x_2

$$z_1(x_1) =$$

All incoming messages

Max-Sum on acyclic graphs

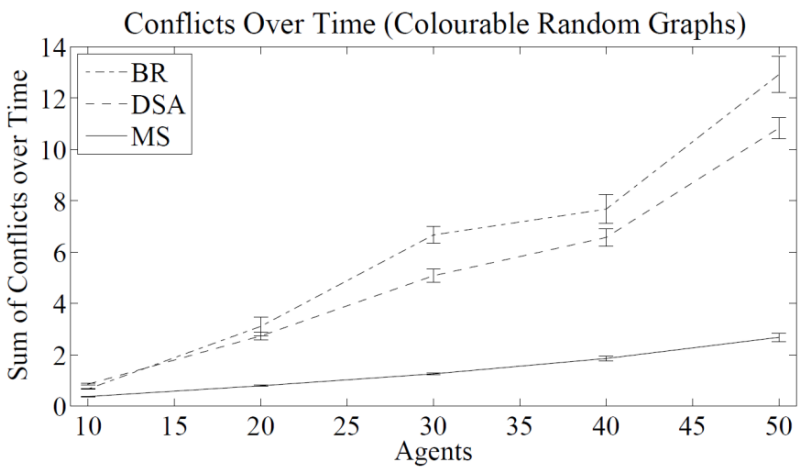
- Max-sum Optimal on acyclic graphs
  - Different branches are independent
  - Each agent can build a correct estimation of its contribution to the global problem (z functions)
- Message equations very similar to Util messages in DPOP
  - Sum messages from children and shared constraint
  - Maximize out agent variable
  - GDL generalizes DPOP (VINYALS et al., 2011)

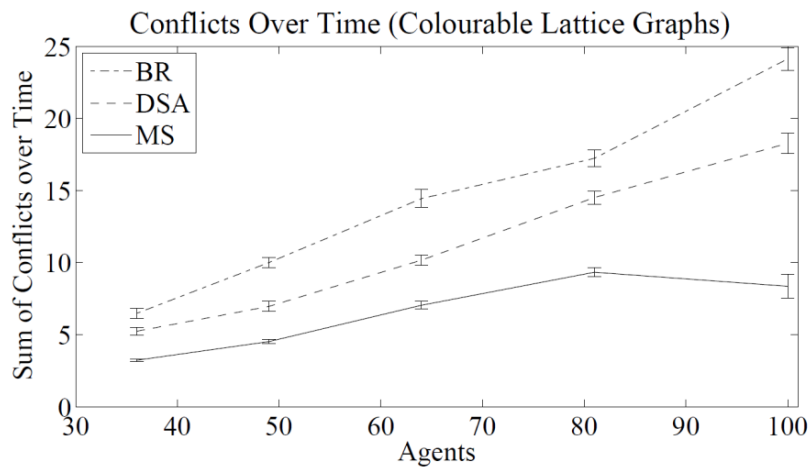


$$m_{1\to 2}(x_2) = max_{x_1}(F_{12}(x_1, x_2) + m_{4\to 1}(x_1))$$

Max-Sum Performance

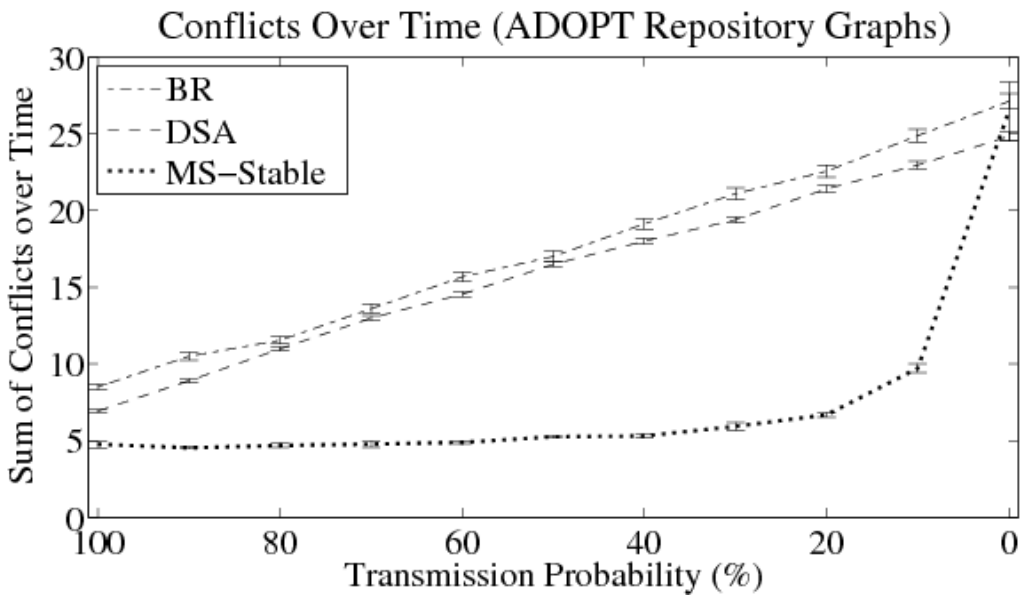
- Good performance on loopy networks (FARINELLI et al., 2008)
  - When it converges very good results
    - Interesting results when only one cycle (WEISS, 2000)
  - We could remove cycle but pay an exponential price (see DPOP)





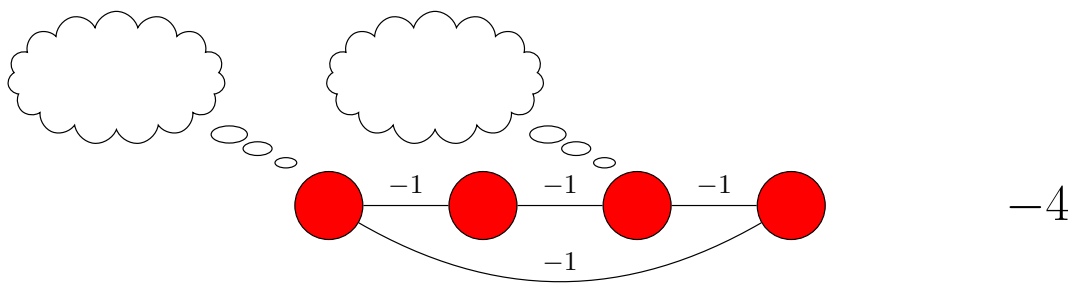
Max-Sum for low power devices

- Low overhead
  - Msgs number/size
- Asynchronous computation
  - Agents take decisions whenever new messages arrive
- Robust to message loss



Local Greedy Approaches

- Greedy local search
  - Start from random solution
  - Do local changes if global solution improves
  - Local: change the value of a subset of variables, usually one



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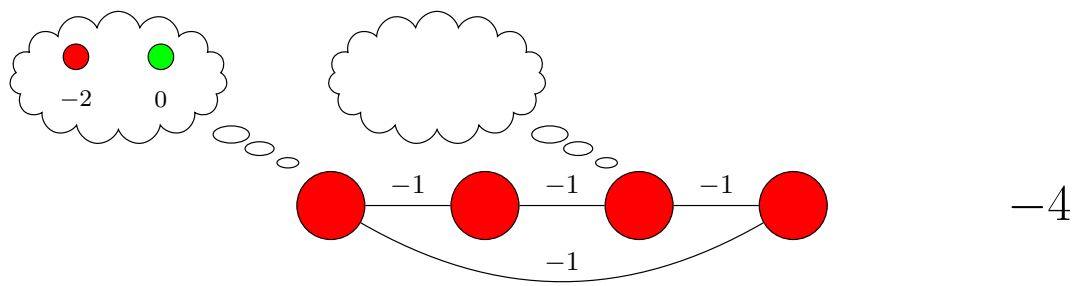

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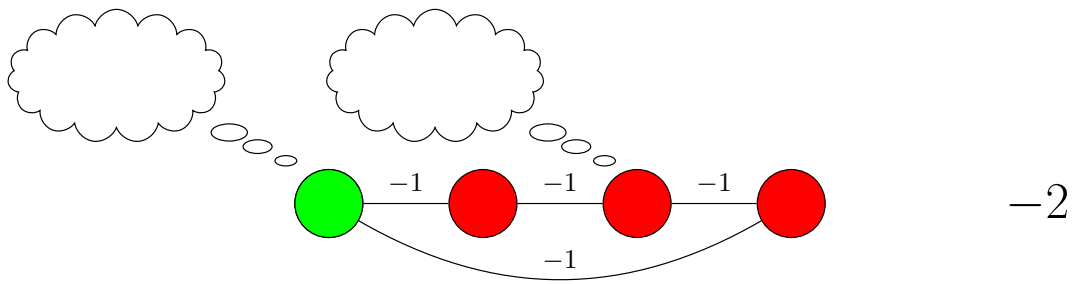

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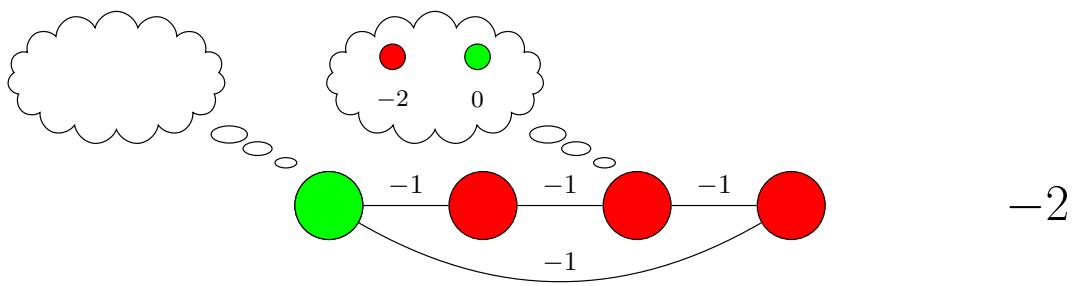

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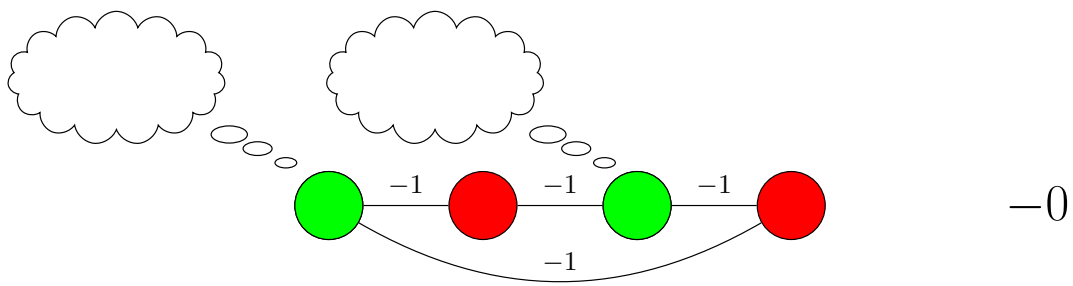
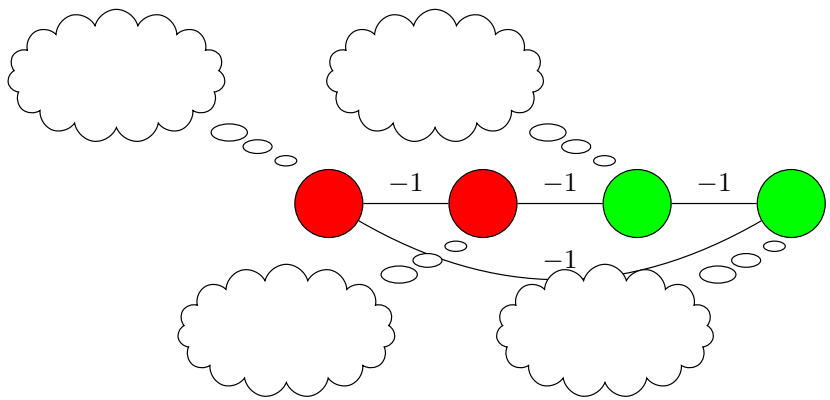


Local Greedy Approaches

- Problems
  - Local minima
  - Standard solutions: Random Walk, Simulated Annealing



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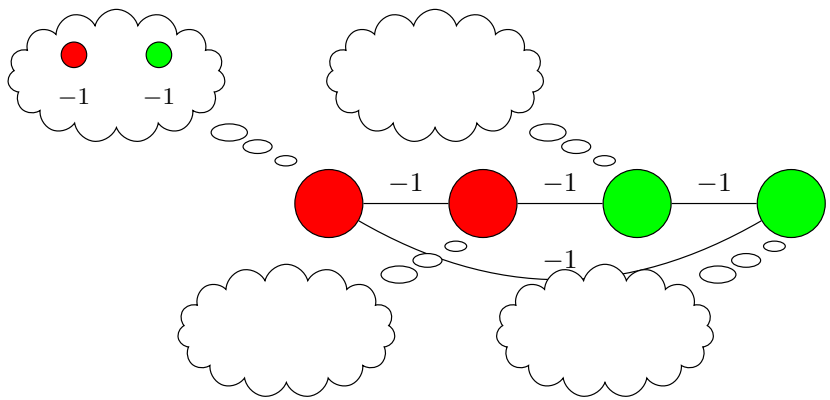

-2

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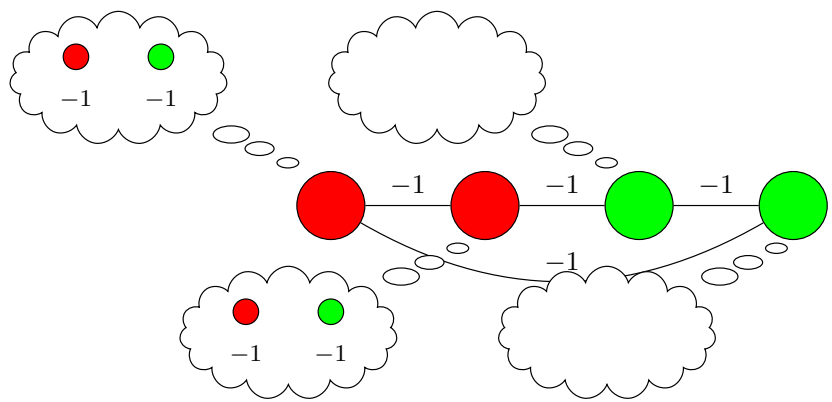

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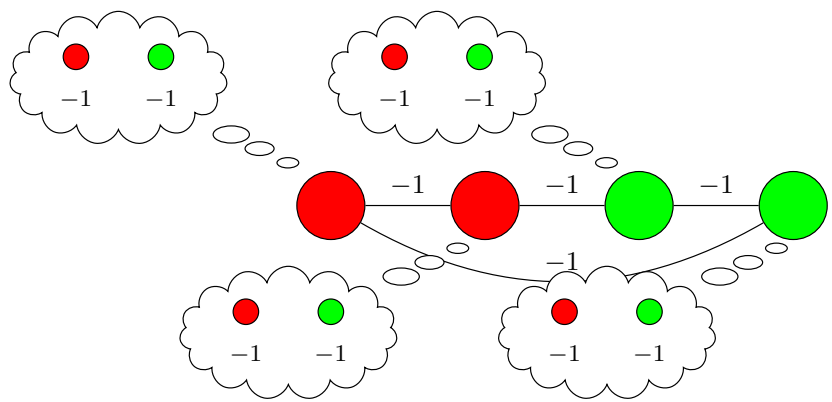
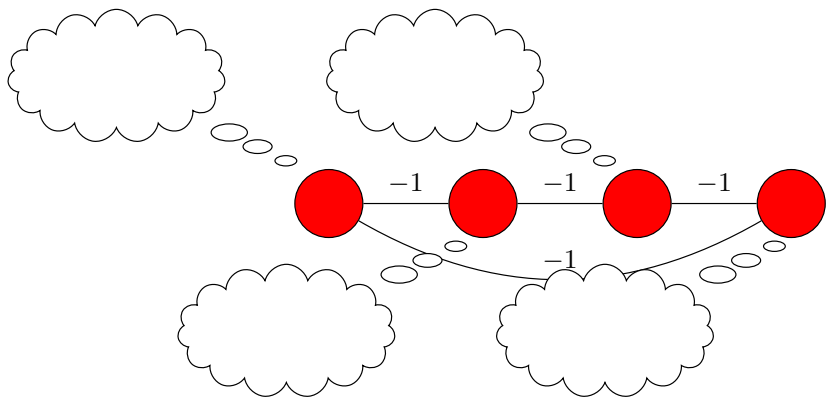


Distributed Local Greedy approaches

- Local knowledge
- Parallel execution
  - A greedy local move might be harmful/useless
  - Need coordination



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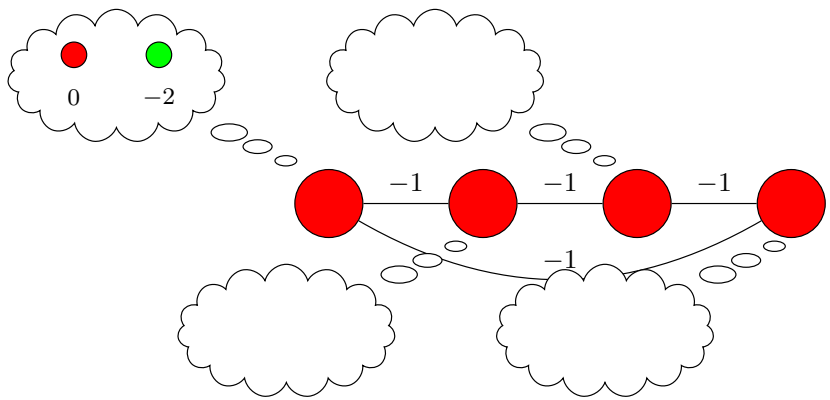

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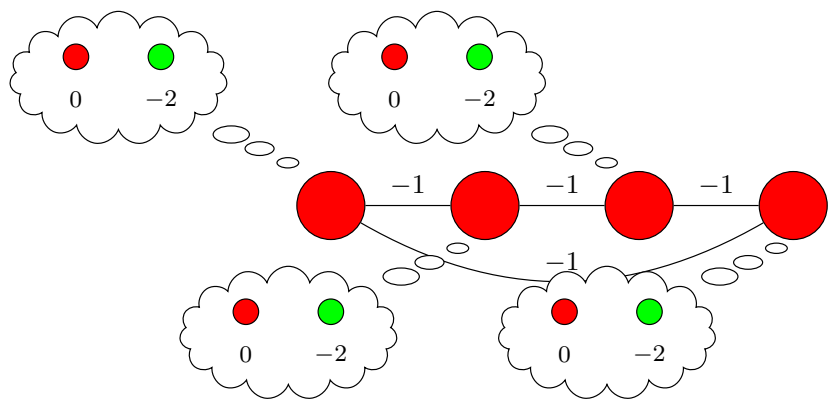

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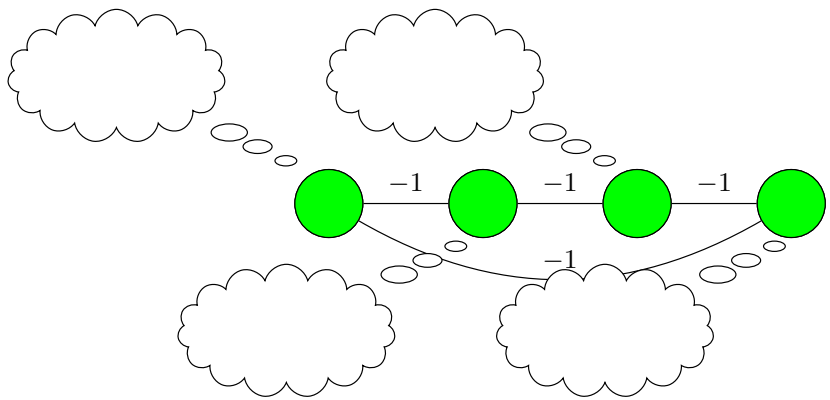

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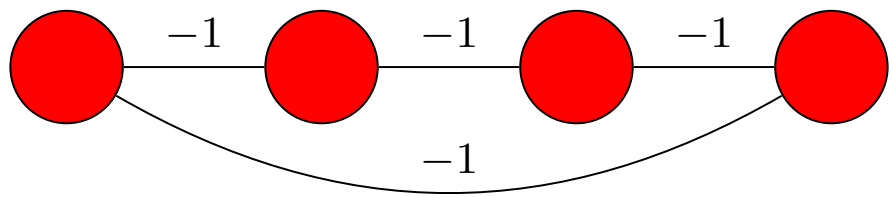
Distributed Stochastic Search Algorithm (DSA)

(ZHANG et al., 2005)

- Greedy local search with activation probability to mitigate issues with parallel executions
- DSA-1: change value of one variable at time
- Initialize agents with a random assignment and communicate values to neighbors
- Each agent:
  - Generates a random number and execute only if rnd less than activation probability
  - When executing changes value maximizing local gain
  - Communicate possible variable change to neighbors

DSA-1: Execution Example



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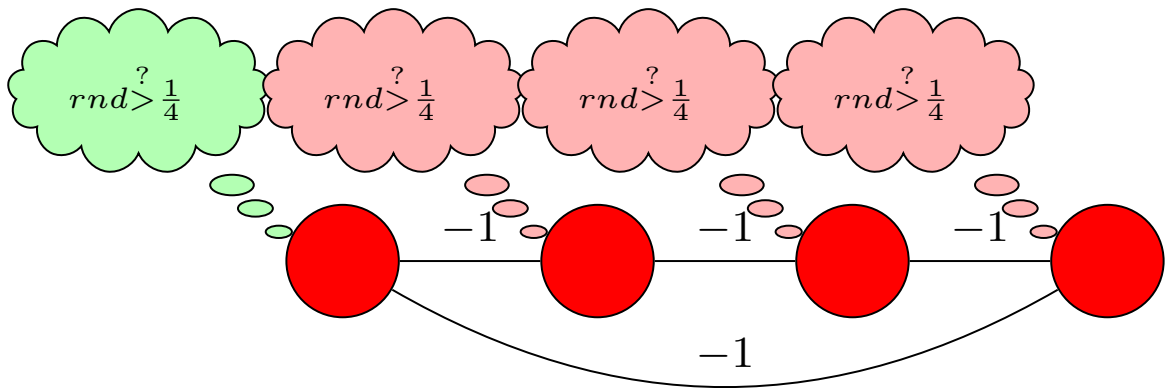

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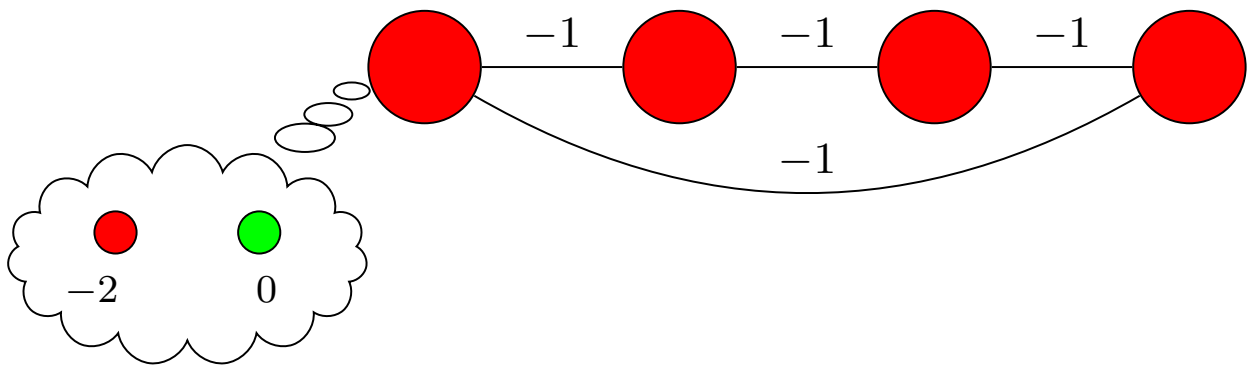

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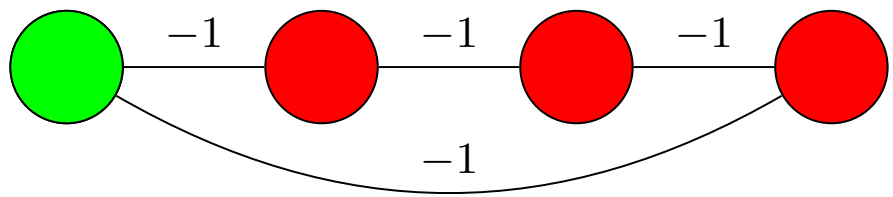

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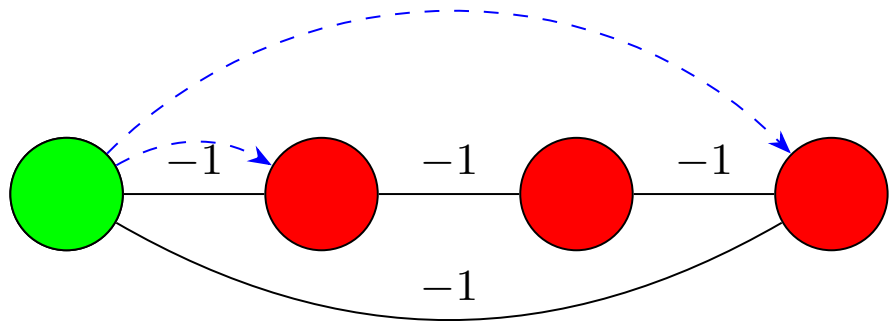


DSA-1: Discussion

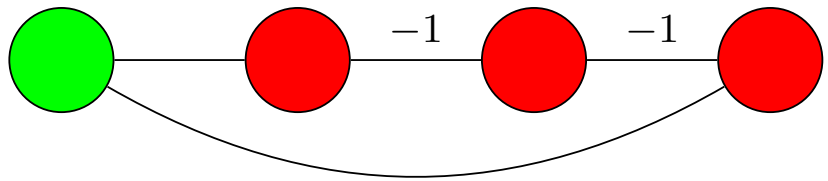
- Extremely "cheap" (computation/communication)
- Good performance in various domains
  - e.g. target tracking (Fitzpatrick and Meertens, 2003; Zhang et al., 2003)
  - Shows an anytime property (not guaranteed)
  - Benchmarking technique for coordination
- Problems
  - Activation probablity must be tuned (Zhang et al., 2003)
  - No general rule, hard to characterise results across domains

Maximum Gain Message (MGM-1)

(MAHESWARAN et al., 2004)

- Coordinate to decide who is going to move
  - Compute and exchange possible gains
  - Agent with maximum (positive) gain executes
- Analysis
  - Empirically, similar to DSA
  - More communication (but still linear)
  - No Threshold to set
  - Guaranteed to be monotonic (Anytime behavior)

MGM-1: Example



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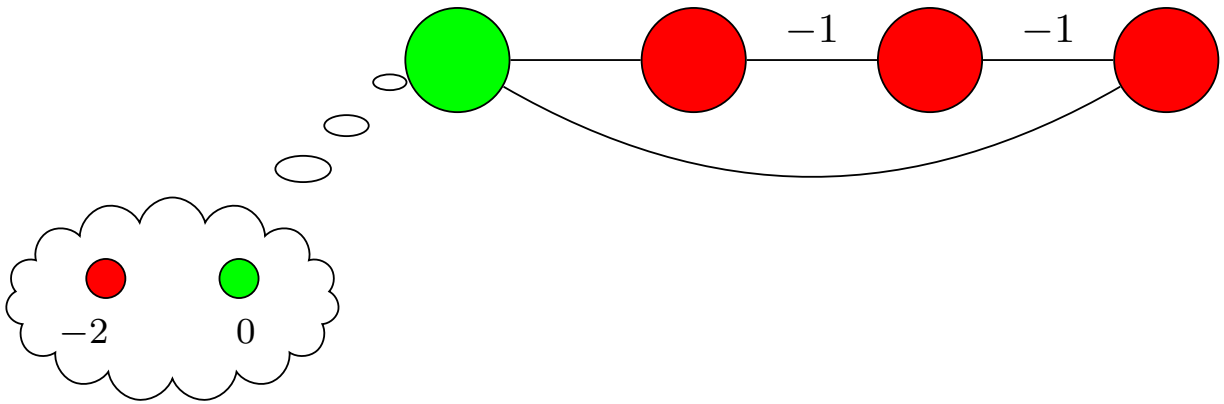

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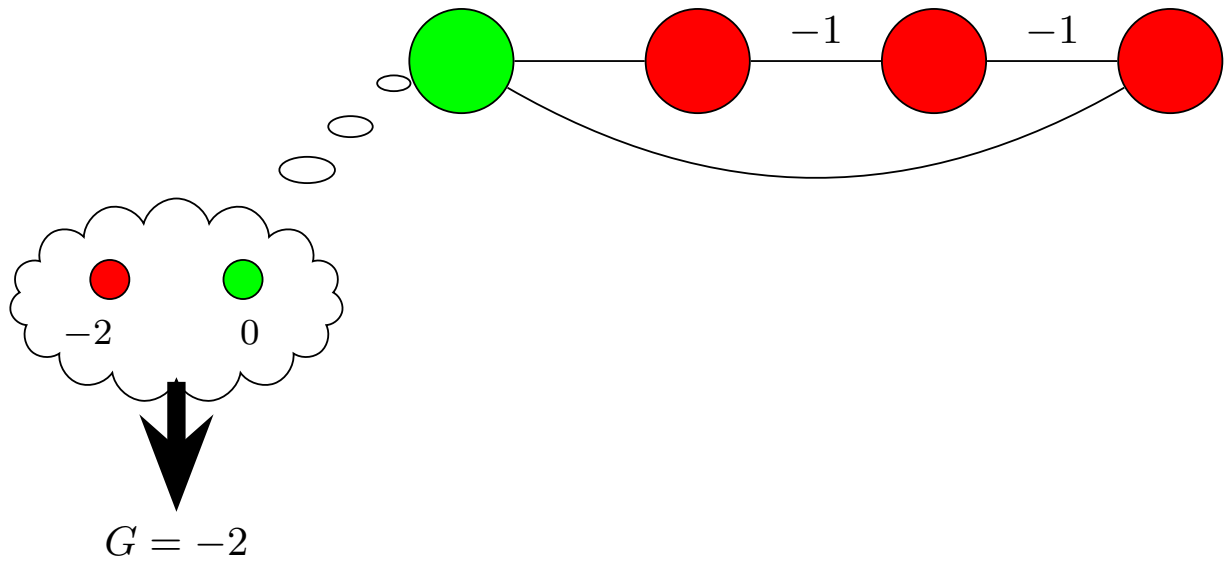

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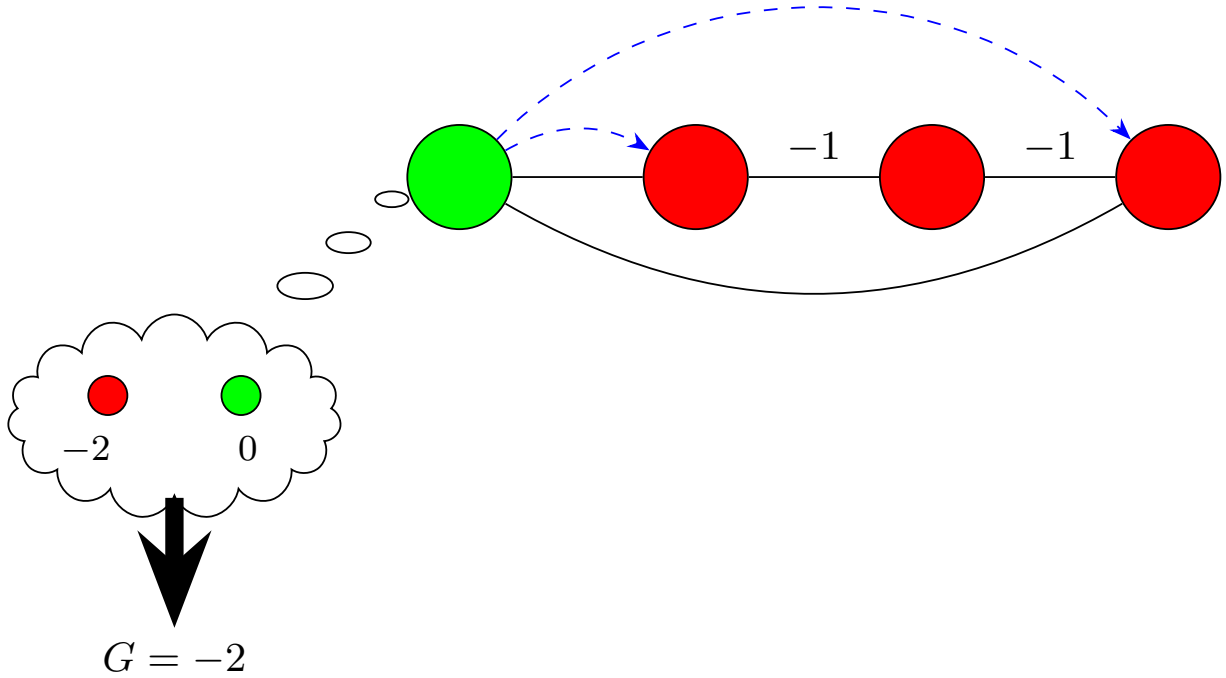

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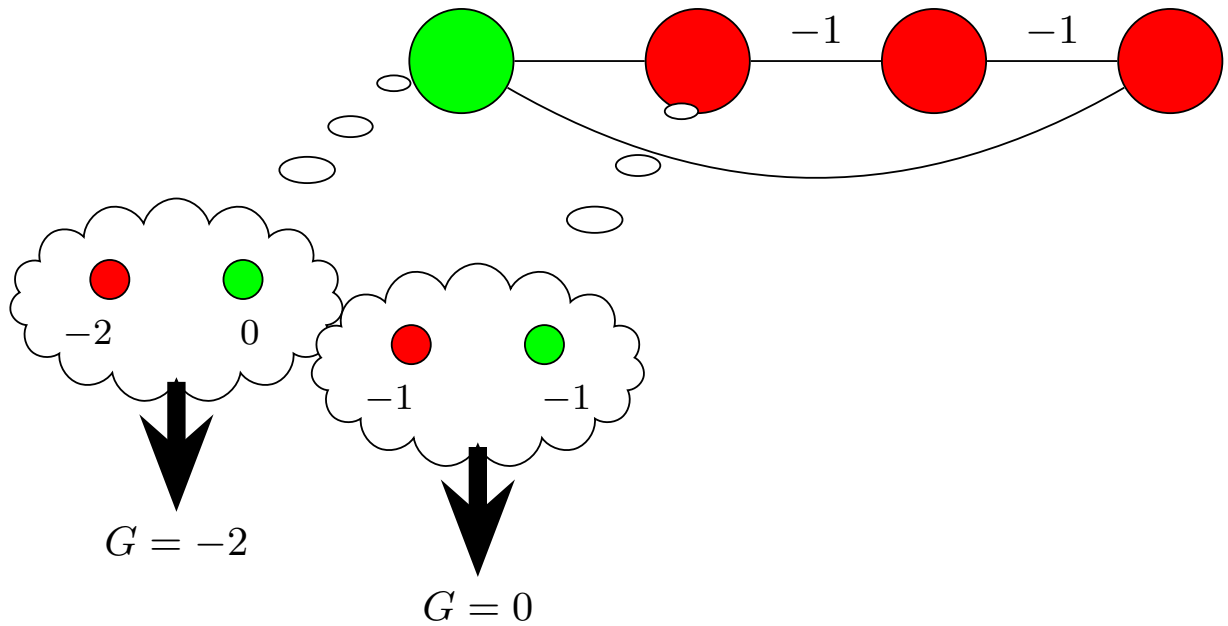

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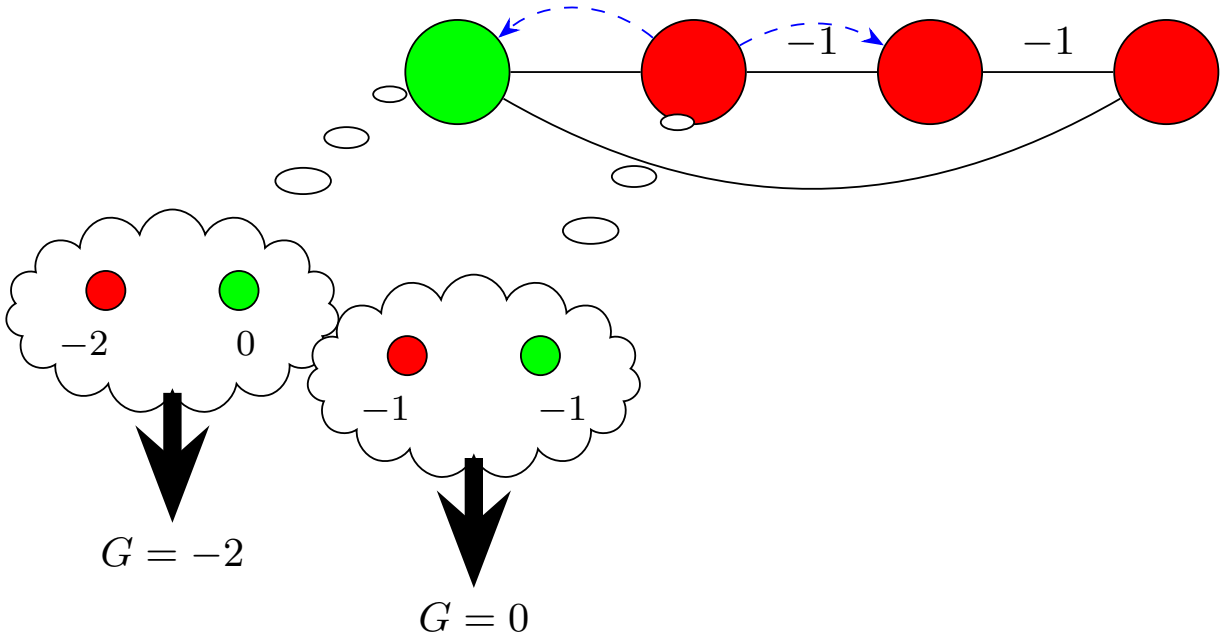
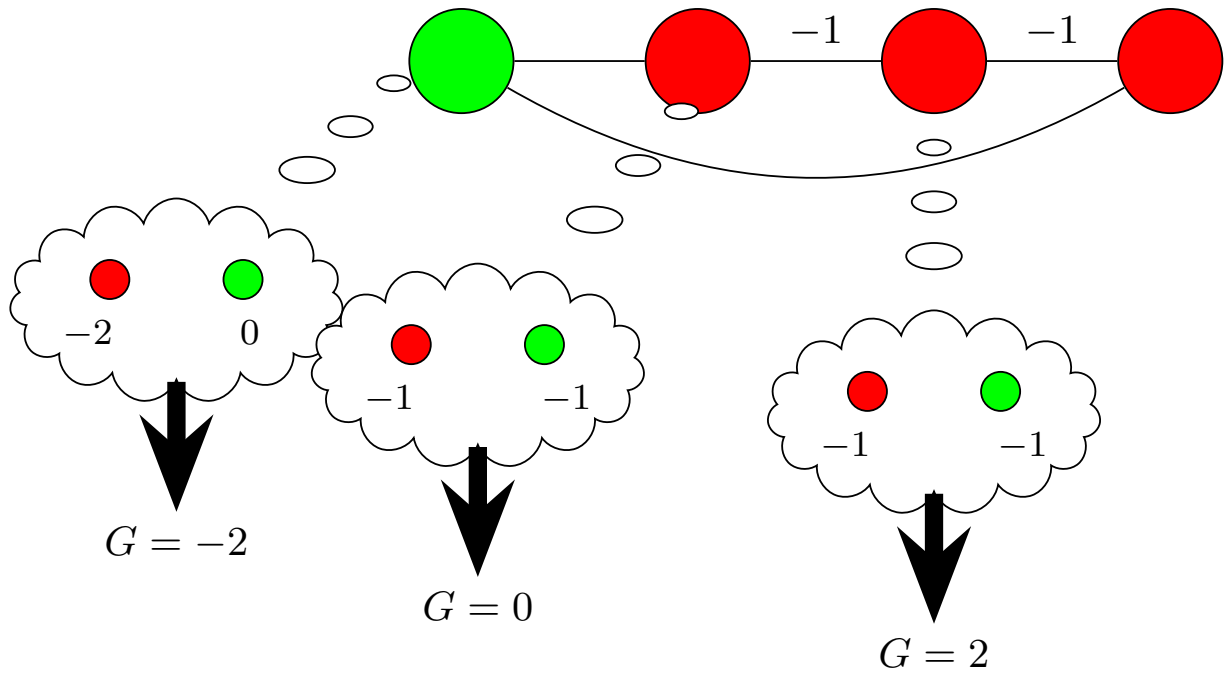
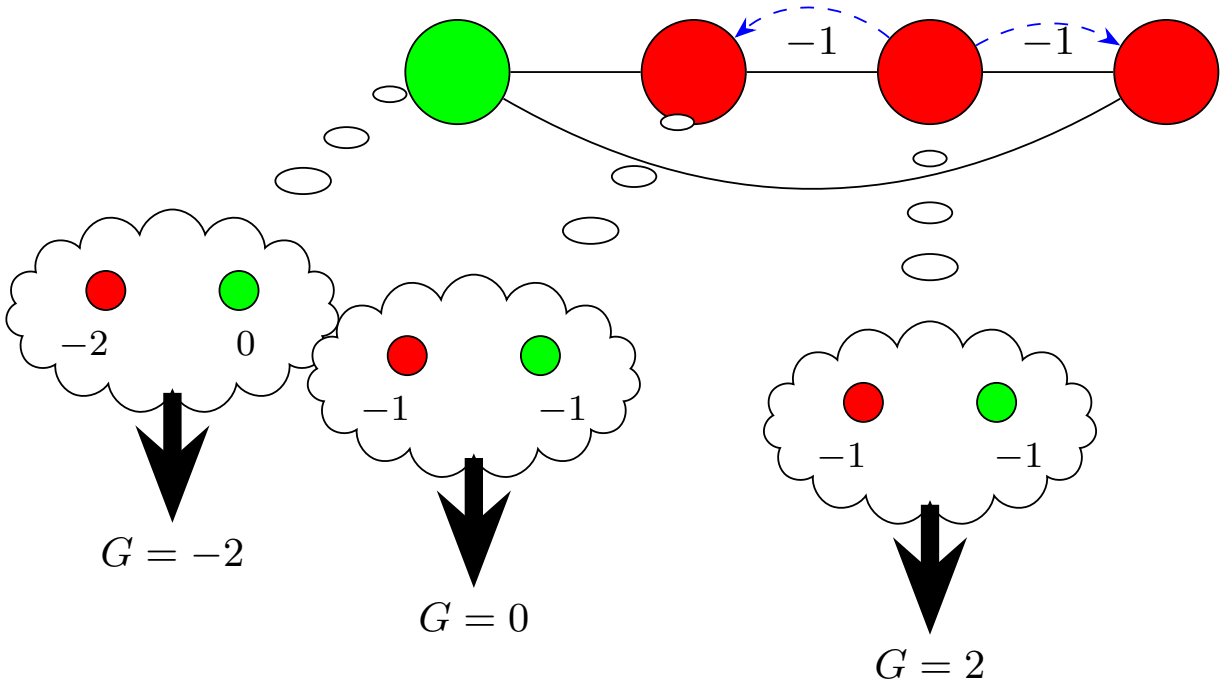


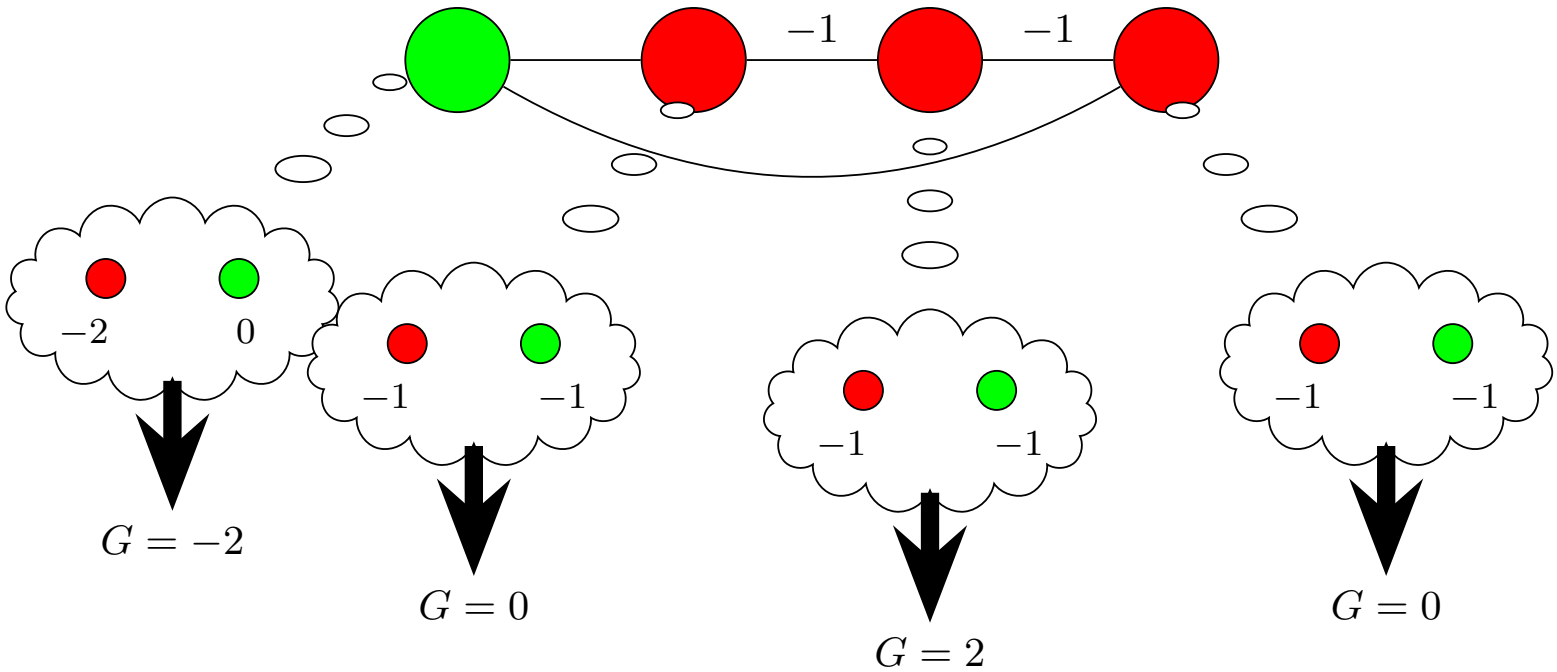
MGM-1: Example



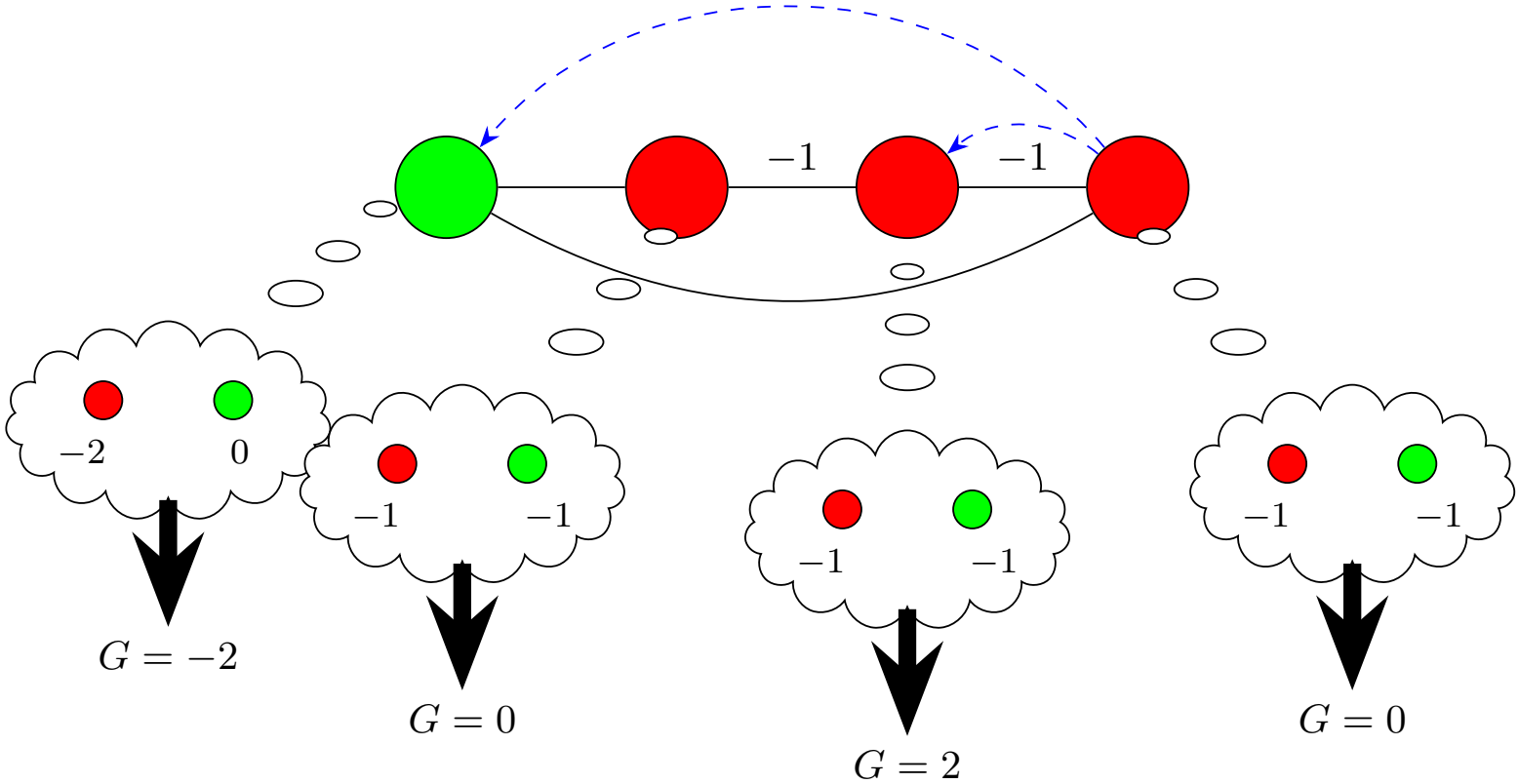
MGM-1: Example



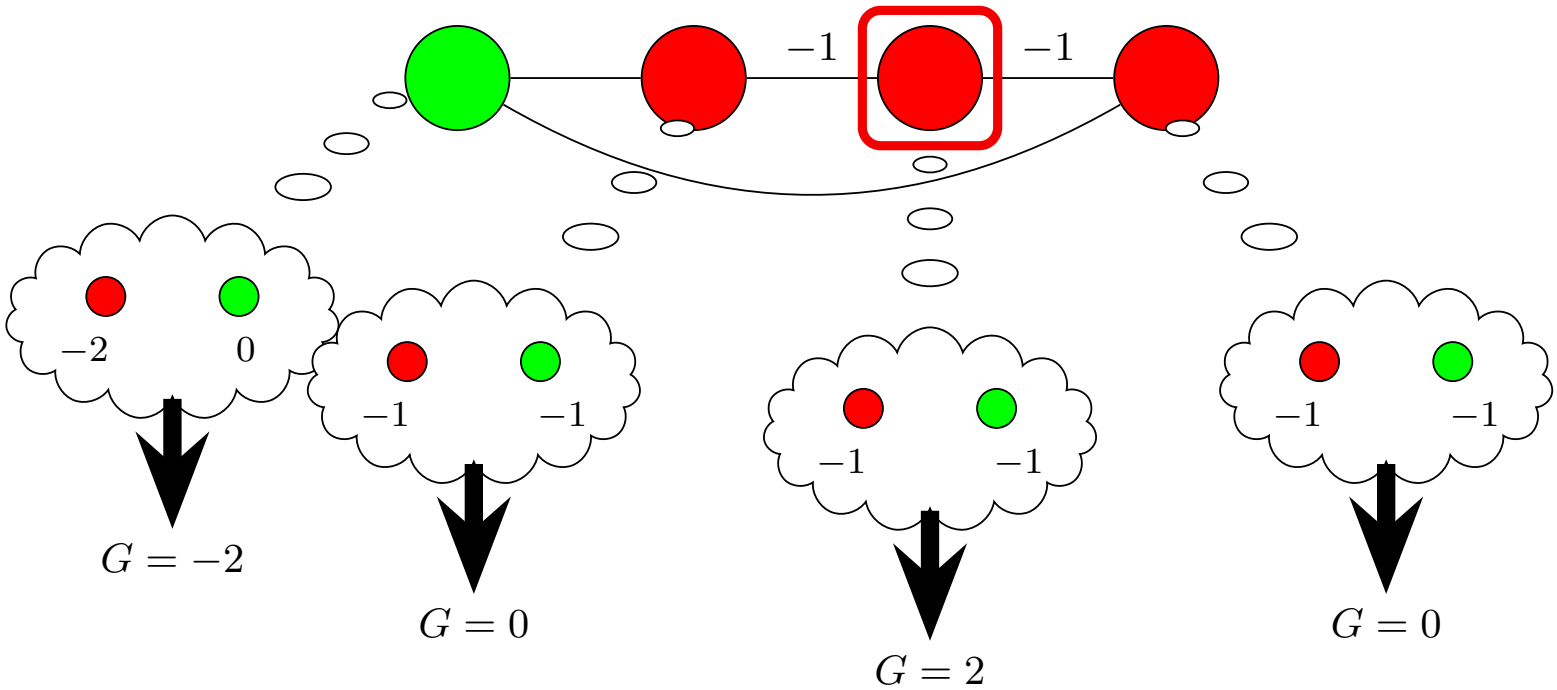
MGM-1: Example



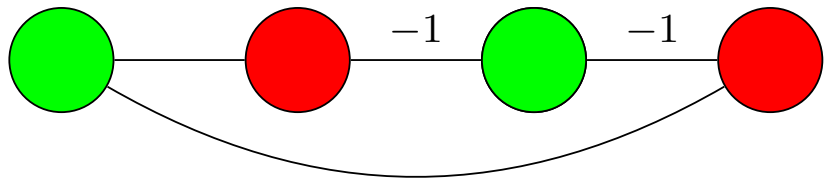
MGM-1: Example



MGM-1: Example



MGM-1: Example



Menu

Hands on PyDCOP I

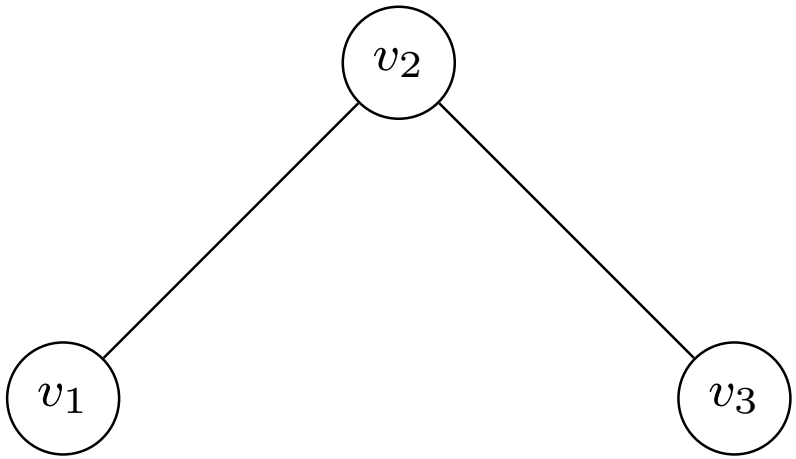
- Install VirtualBox
- Import the pyDCOP Virtual Machine (`https://bit.ly/2JUrrKP`)
  - It's a Debian image with everything preinstalled:
  - python3, pyDCOP, matplotlib, glpk, etc.
- Alternatively, follow

  `https://pydcop.readthedocs.io/en/latest/installation.html`

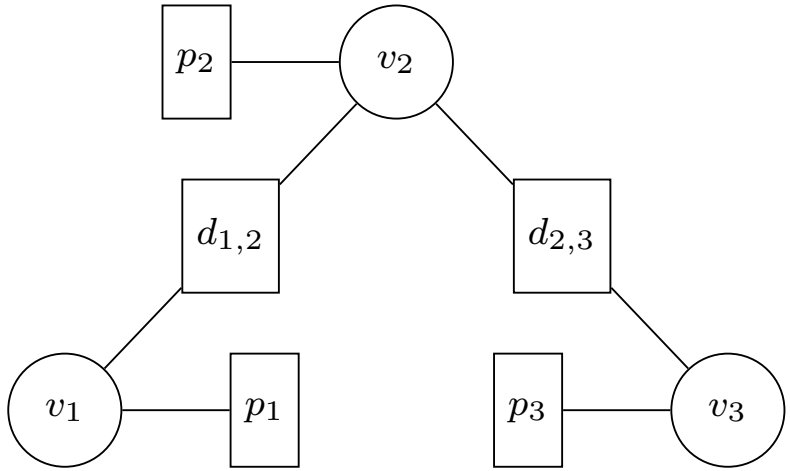
1. `https://pydcop.readthedocs.io/en/latest/tutorials/getting_started.html`
2. `https://pydcop.readthedocs.io/en/latest/tutorials/analysing_results.html`

Hands on PyDCOP I

DCOP - Graph Coloring



(a) constraints graph



(b) factor graph

- **Objective:** minimize
- **Domain:** 2 colors R and B
- **Variables:** V_1, V_2, V_3
- **Constraints:** neighbors must have different colors + preferences
- **Agents:** 3 agents

Yaml representation

Hands on PyDCOP I

Solving the Graph Coloring DCOP

Command:

```
$ pydcop solve --algo dpop graph_coloring.yaml
```

Output:

```
...
"assignment": {
  "v1": "R",
  "v2": "G",
  "v3": "R"
},
"cost": -0.1,
...
```

With other algorithms:

```
$ pydcop --timeout 2 solve --algo dsa graph_coloring.yaml
$ pydcop solve --algo mgm --algo_params stop_cycle:20 \
       graph_coloring.yaml
```

Hands on PyDCOP I

Results

Full results :


```
{
  "agt_metrics": {
    ...
  },
  "assignment": {
    "v1": "R",
    "v2": "G",
    "v3": "R"
  },
  "cost": -0.1,
  "cycle": 20,
  "msg_count": 158,
  "msg_size": 158,
  "status": "FINISHED",
  "time": 0.032010296999942 51,
  "violation": 0
}
```

Hands on PyDCOP I

Logs

Simple:

use `-v 0..3`

```
$ pydcop -v 2 solve --algo dpop graph_coloring.yaml
```

Precise :

use `-log <log.conf>`

```
$ pydcop --log log.conf -t 1 solve --algo dsa graph_coloring.yaml
```

Now, look at `algo.log`

Hands on PyDCOP I

Run-time metrics

periodic: "--collect_on period --period <p>"

```
$ pydcop --log log.conf -t 10 solve \
   --collect_on period --period <p>
   --algo dsa graph_coloring.yaml
```

cycle: "--collect_on cycle_change"

Only supported with synchronous algorithms !

```
$ pydcop solve --algo mgm --algo_params stop_cycle:20 \
    --collect_on cycle_change --run_metric ./metrics.csv \
    graph_coloring_50.yaml
```

value: "--collect_on value_change"

```
$ pydcop -t 5 solve --algo mgm --collect_on value_change \
    --run_metric ./metrics_on_value.csv \
    graph_coloring_50.yaml
```

Hands on PyDCOP I

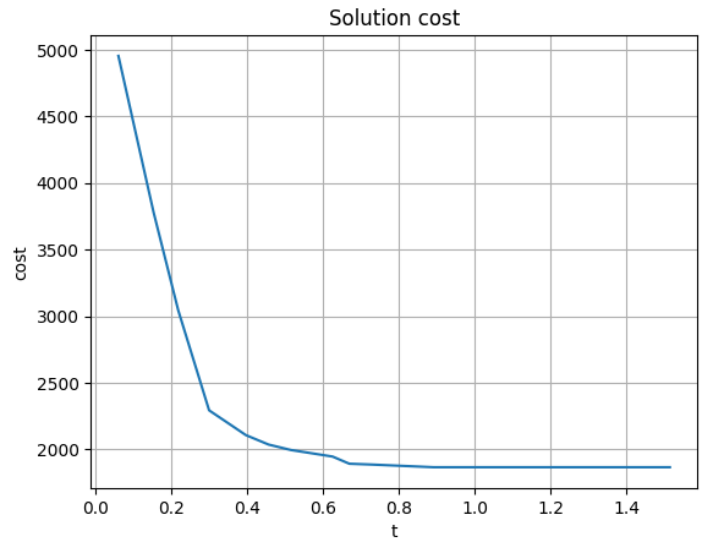
Run-time metrics

With a bigger graph coloring problem

```
$ pydcop solve --algo mgm --algo_params stop_cycle:20 \
           --collect_on cycle_change \
           --run_metric ./metrics.csv \
           graph_coloring_50.yaml
```

Plotting with matplotlib

```
$ python3 plot_cost.py ./metrics.csv
```



Menu

DCOP Framework

Focus on Some Solution Methods

Hands on PyDCOP I

Focus on Smart Environment Configuration Problems

Distributing Computations

Hands on PyDCOP II

Dynamic DCOPs

Conclusion

SECP model

Smart Environment Configuration Problem **(RUST et al., 2016)**

- Example of applying DCOPs to a "real" problem
- Coordinate objects in the building
- Model
  - objects
  - relations between objects and environment
  - user objectives and requirements
- Formulate the problem as an optimization problem

SECP model

Smart Environment Configuration Problem **(RUST et al., 2016)**

Focus on smart lighting use cases

- **Objects:** anything that can produce light: light bulbs, windows with rolling shutter, etc.
- **User preferences:** having a predefined luminosity level in a room, under some conditions
- **Energy efficiency**

Linking objects and user preferences:

- How to model the luminosity in a room ? **variable**
- How to model the dependency between the light sources and the luminosity ? **function / constraint**

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - Connected light bulbs, TV, Rolling shutters, ...
- **Sensors**
  - Presence detector, Luminosity Sensor, etc.
- **Physical Dependency Models**
  - E.g. Living-room light model
- **User Preferences**
  - Expressed as rules :

IF	presence_living_room	$=$	1
AND	light_sensor_living_room	$<$	60
THEN	light_level_living_room	$\leftarrow$	60
AND	shutter_living_room	$\leftarrow$	0

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - *Decision* variable x_i, domain $\mathcal{D}_{x_i}$
  - Cost function $c_i : \mathcal{D}_{x_i} \to \mathbb{R}$
- **Sensors**
  - *Read-only* variable s_l, domain $\mathcal{D}_{s_l}$
- **Physical Dependency Models** $\langle y_j, \phi_j \rangle$
  - Give the expected state of the environment from a set of actuator-variables influencing this model
  - Variable y_j representing the *expected* state of the environment
  - Function $\phi_j : \prod_{\varsigma \in \sigma(\phi_j)} \mathcal{D}_\varsigma \to \mathcal{D}_{y_j}$
- **User Preferences**
  - Utility function u_k
  - Distance from the current expected state to the target state of the environment

Formulating SECP as a DCOP

Multi-objective optimization problem

$$\min_{x_i \in \nu(\mathfrak{A})} \sum_{i \in \mathfrak{A}} c_i \quad \text{and} \quad \max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \sum_{k \in \mathfrak{R}} u_k$$

$$\textbf{s.t.} \;\; \phi_j(x_j^1, \ldots, x_j^{\overline{\phi_j}}) = y_j \quad \forall y_j \in \nu(\Phi)$$

Then mono-objective DCOP formulation

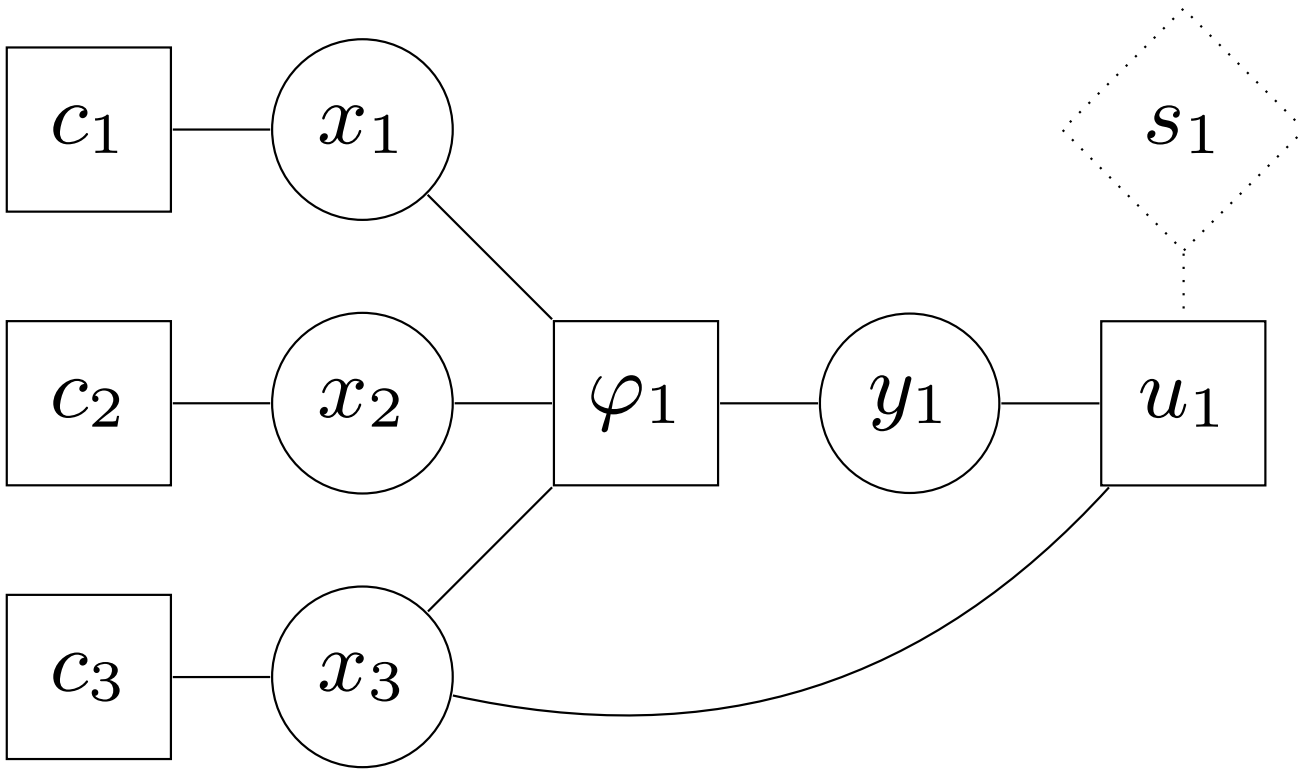
$$\max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \omega_u \sum_{k \in \mathfrak{R}} u_k - \omega_c \sum_{i \in \mathfrak{A}} c_i + \sum_{\varphi_j \in \mathbf{\Phi}} \varphi_j$$

with reformulation of hard constraints ϕ_j into soft ones:

$$\varphi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}, y_j) = \begin{cases} 0 & \text{if } \phi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}) = y_j \\ -\infty & \text{otherwise} \end{cases}$$

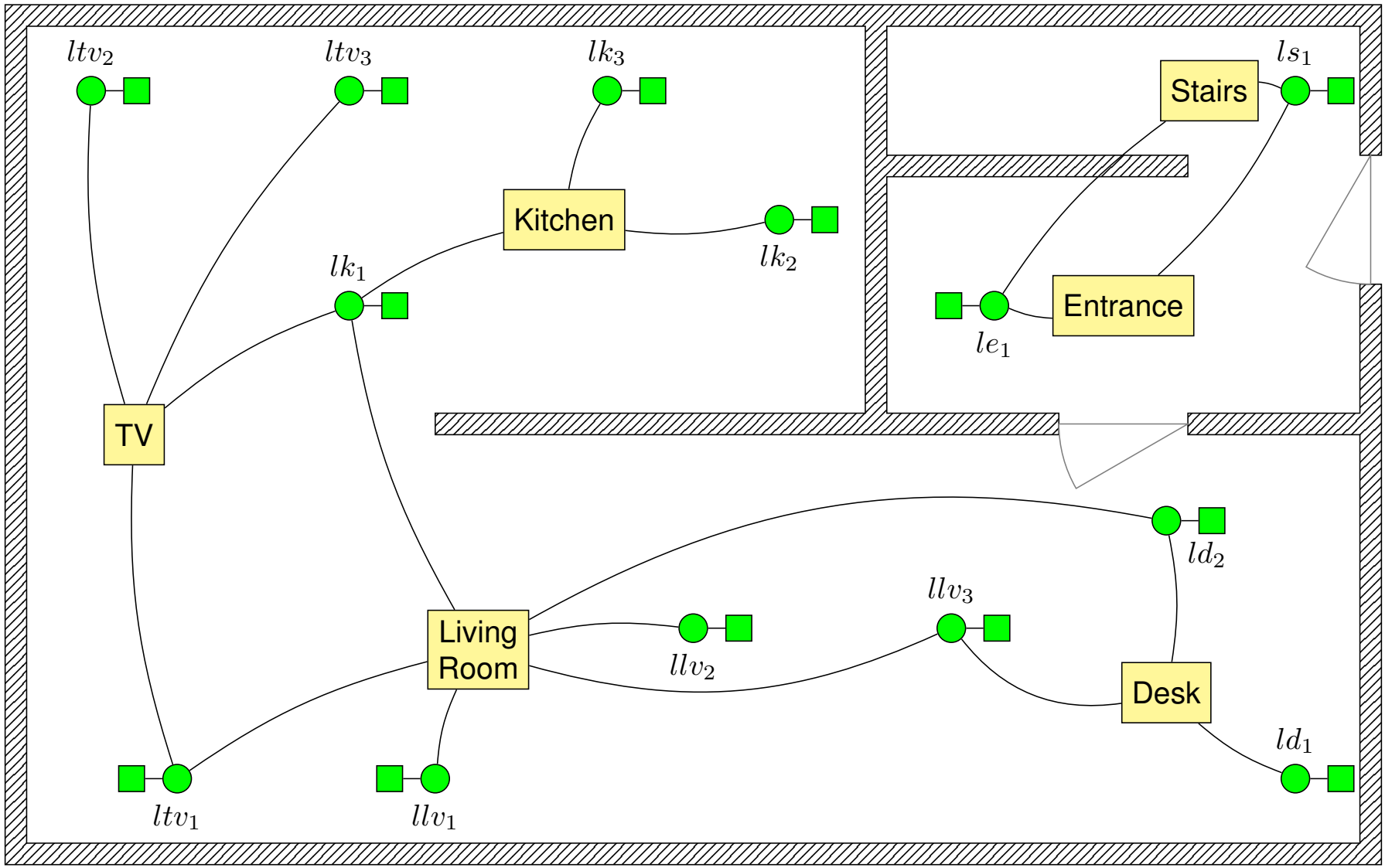
Formulating SECP as a DCOP

Representing a DCOP as a factor graph



SECP Factor Graph

in a house (without rules)



Menu

Distribution of computations

Allocating computations to agents

- DCOP: $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mu \rangle$
- μ : function mapping variables to their associated agent

Why is distribution needed ?

Common assumptions:

- computation $\equiv$ variable
- each agent controls exactly one variable (bijection)
- binary constraints

Real distributed problems:

- agents must be hosted on real devices
- the set of devices might be given by the problem
- for some variables the relation with an agent is obvious, but not always

Distribution of computations

Allocating computations to agents

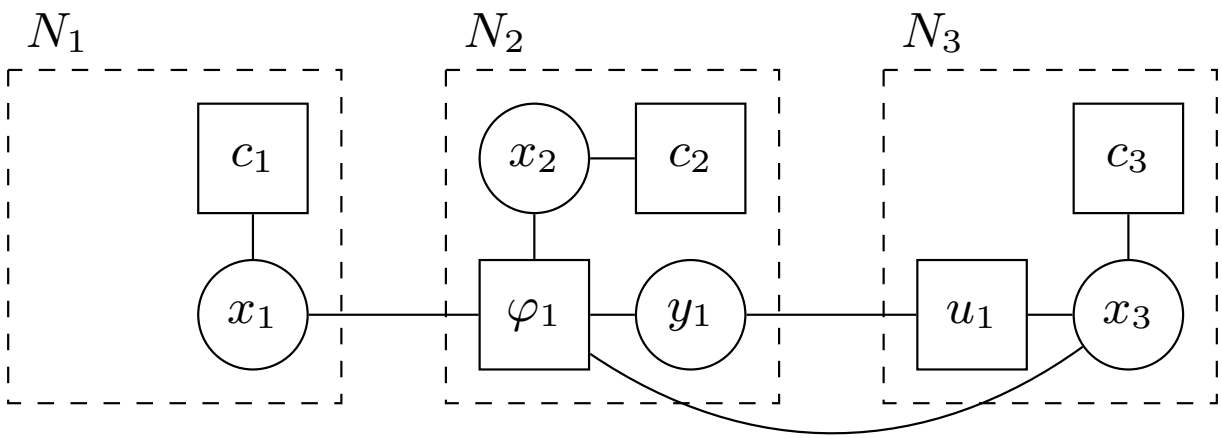
- Distributing computations
  - computations depends on the graph model
  - variables and / or factors

- Distribution impacts the system characteristics :
  - speed,
  - communication load,
  - hosting costs, etc.

Distribution of computations

Allocating computations to agents

- Simple heuristic
  - No computation on sleepy devices (sensors)
  - Computation should be close the the impacted variables
  - Spread the computation load amongst agents
- How good is it ?



Distribution of computations

Optimal definition

Optimal distribution ?

- Problem dependent
- Optimization problem :
  find the best distribution, for your problem's criteria
- Optimal distribution $\equiv$ graph partitioning,
  NP-hard in general (BOULLE, 2004)

Distribution of computations

Better definition

SECP distribution problem

- Devices have limited memory
- Communication is expensive and has limited bandwidth
- Variable related to an actuator are hosted by it
- Objective : **minimize overall communication between agents**

Optimization problem : define an ILP for it !

Binary ILP for computation distribution

- x_i^k , binary variables that map computations to agents and $\alpha_{ij}^{mn} = x_i^m \cdot f_j^n$

$$\forall x_i \in X, \quad \sum_{a_m \in \mathbf{A}} x_i^m = 1 \tag{1}$$

- Message's size between variable x_i and factor f_j: $msg(i,j)$

$$\underset{x_i^m}{\textbf{minimize}} \quad \sum_{(i,j)\in D} \sum_{(m,n)\in \mathbf{A^2}} \mathbf{msg}(i,j) \cdot \alpha_{ij}^{mn} \tag{2}$$

- Memory footprint of a computation: $\mathbf{weight}(e)$, and memory capacity for a device: $\mathbf{cap}(a_k)$

$$\forall a_m \in \mathbf{A}, \quad \sum_{x_i \in D} \mathbf{weight}(x_i) \cdot x_i^m \leq \mathbf{cap}(a_m) \tag{3}$$

- and a few linearization constraints

Binary ILP for computation distribution

More generic case:

- Add route cost: $\mathbf{com}(i, j, m, n)$

$$\forall x_i, x_j \in \mathbf{X}, \forall a_m, a_n \in \mathbf{A},$$

$$\mathbf{com}(i, j, m, n) = \begin{cases} \mathbf{msg}(i, j) \cdot \mathbf{route}(m, n) & \text{if } (i, j) \in D, m \neq n \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\underset{x_i^m}{\textbf{minimize}} \sum_{(i,j)\in D} \sum_{(m,n)\in \mathbf{A}^2} \mathbf{com}(i, j, m, n) \cdot \alpha_{ij}^{mn} \quad (5)$$

- Add hosting costs : $\mathbf{host}(a_m, x_i)$

$$\underset{x_i^m}{\textbf{minimize}} \sum_{(x_i,a_m)\in X\times \mathbf{A}} x_i^m \cdot \mathbf{host}(a_m, x_i) \quad (6)$$

Binary ILP for computation distribution

$$\underset{x_i^m}{\textbf{minimize}} \quad \omega_{\mathbf{com}} \cdot \sum_{(i,j)\in D} \sum_{(m,n)\in \mathbf{A}^2} \mathbf{com}(i,j,m,n) \cdot \alpha_{ij}^{mn}$$

$$+ \quad \omega_{\mathbf{host}} \cdot \sum_{(x_i,a_m)\in X\times \mathbf{A}} x_i^m \cdot \mathbf{host}(a_m, x_i) \tag{7}$$

subject to

$$\forall a_m \in \mathbf{A}, \quad \sum_{x_i \in D} \mathbf{weight}(x_i) \cdot x_i^m \leq \mathbf{cap}(a_m) \tag{8}$$

$$\forall x_i \in X, \quad \sum_{a_m \in \mathbf{A}} x_i^m = 1 \tag{9}$$

$$\forall x_i \in X, \quad \alpha_{ij}^{mn} \leq x_i^m \tag{10}$$

$$\forall x_j \in X, \quad \alpha_{ij}^{mn} \leq x_j^m \tag{11}$$

$$\forall x_i, x_j \in X, a_m \in A, \quad \alpha_{ij}^{mn} \geq x_i^m + x_j^n - 1 \tag{12}$$

Solving the ILP for computation deployment

- NP-hard, but can be solved with branch-and-cut
  LP solvers are very good at this
- Yet, only possible for small instances
- Gives us a reference for optimality: benchmarking
- When not solvable, still gives us a metrics to compare heuristics

Menu

Hands on PyDCOP II

1. https://pydcop.readthedocs.io/en/latest/tutorials/deploying_on_machines.html
2. https://pydcop.readthedocs.io/en/latest/usage/cli/run.html
3. https://pydcop.readthedocs.io/en/latest/usage/cli/distribute.html

Hands on PyDCOP II

SECP

A Very simple SECP: single room

- 3 light bulbs, 1 model and 3 rules
- `/tutorials/hands-on_2/single_room.yaml`
- Solve with

```
pydcop --log log.conf -t 10 solve
  --algo maxsum --algo_params damping:0.8
  --dist adhoc single_room.yaml
```

- Result : `"cost": 702.3000000000004, ...`
  - not that good ...
  - Look at the yaml definition
  - the rules contradict each other !
- Change the yaml definition
  - comment out rules to keep only one active
  - could be done with *'read-only'* variables
  - solve it again

Hands on PyDCOP II

SECP - Running on several machines

- We used `solve`
  - great for testing
  - everything run locally, in the same process
- Launching several agents:
  - One agent for each light bulb `a1`, `a2` and `a3` (change port for each agent)

```
pydcop -v3 agent -n a1 -p 9001
   --orchestrator 127.0.0.1:9000
```

  - an orchestrator

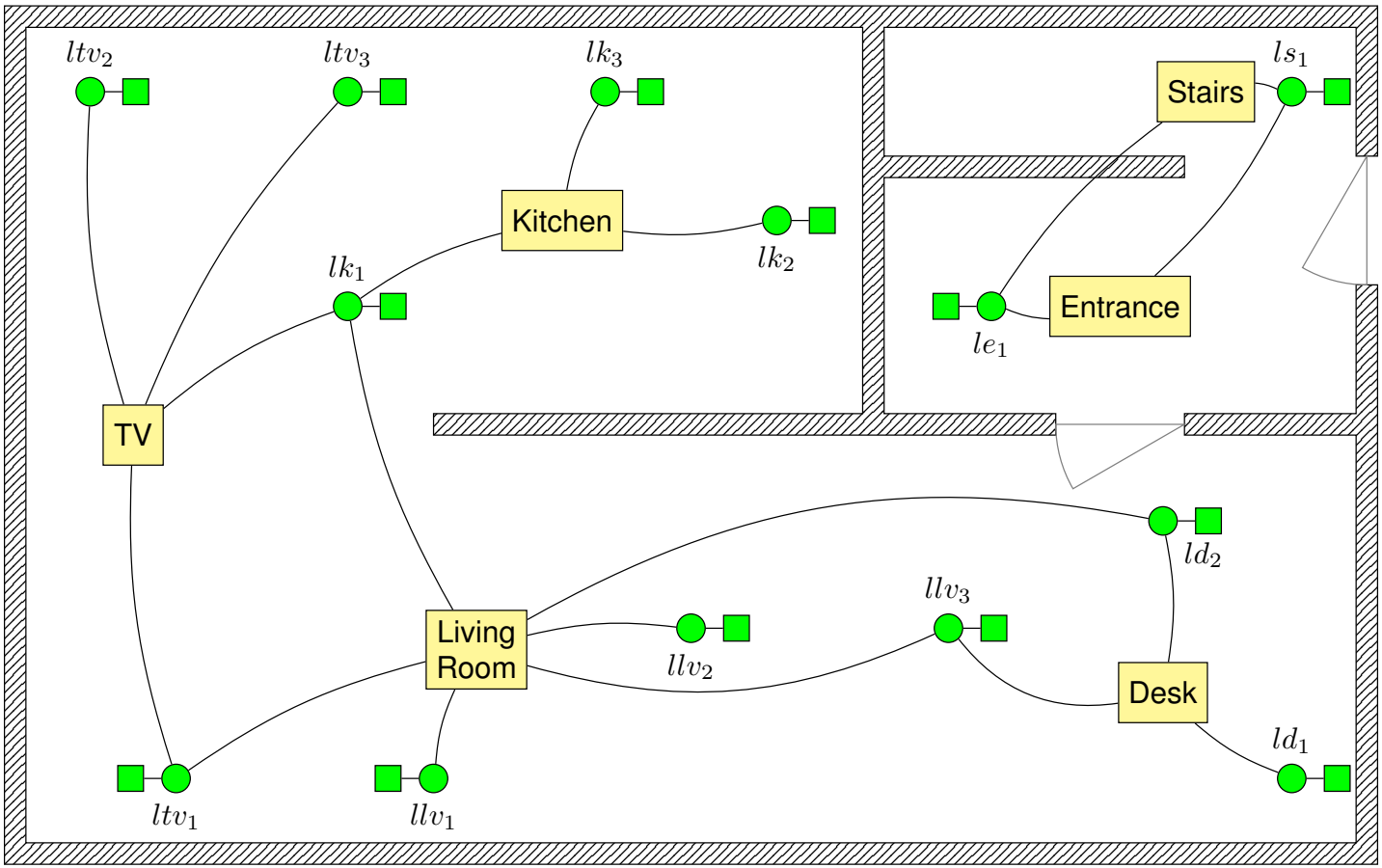
```
pydcop --log log.conf -t 10 orchestrator \
  --algo maxsum --algo_params damping:0.8 \
  --dist adhoc single_room.yaml
```

  - run the agents on different Virtual machines, different computers

Hands on PyDCOP II

A bigger SECP

in `/tutorials/hands-on_2/SimpleHouse.yml`
13 light bulbs, 6 models



Hands on PyDCOP II

Distributing a SECP

```
$ pydcop --output dist_house_fg_ilp.yaml distribute -d ilp_compref \
        -a maxsum SimpleHouse.yml
```

Need to specify the algorithm

- computation graph
- weight of computation's
- size of computations' messages

On such a small system, we can compute the optimal distribution !

Hands on PyDCOP II

Distributing a SECP

```
cost: 8725.0
distribution:
  a_d1: [mv_desk, mc_desk, l_d1, r_work, mc_livingroom, mv_livingroom]
  a_d2: [l_d2]
  a_e1: [mv_entry, r_entry, mv_stairs, l_e1, mc_entry, mc_stairs]
  a_e2: [l_e2]
  a_k1: [l_k1]
  a_k2: [l_k2]
  a_k3: [l_k3]
  a_lv1: [l_lv1]
  a_lv2: [mc_kitchen, l_lv2]
  a_lv3: [l_lv3]
  a_tv1: [l_tv1]
  a_tv2: [l_tv2]
  a_tv3: [r_lunch, l_tv3, mv_tv, r_cooking, r_homecinema, mc_tv, mv_kitchen]
inputs:
  algo: maxsum
  dcop: [SimpleHouse.yml]
  dist_algo: ilp_compref
  graph: factor_graph
```

Menu

SECP and DCOP

So far we have:

- Designed a model for SECP
- Formulated this model as a DCOP
- Distributed the computation of the DCOP on devices / agents (bootstrap)
- Run our system to get an auto-configuration of the devices

But what happens
in dynamic environments ?
if objects appear and disappear?

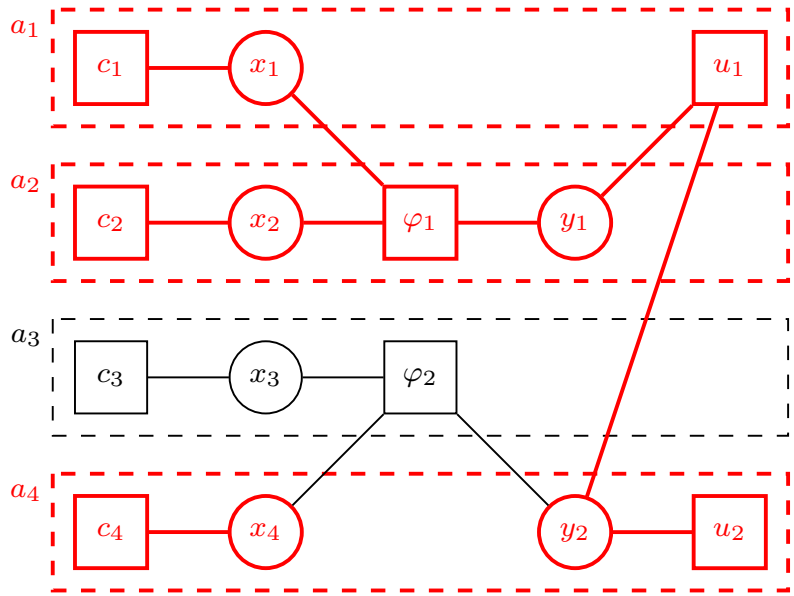
SECP is a dynamic problem

Dynamics in the infrastructure

- Devices can disappear
- New devices can be added to the system

At runtime..

- No powerful device available to solve the ILP
- The deployment must be repaired: self-adaptation
- Only consider a portion of the factor graph: the neighborhood



k-resilience

Dynamics in the infrastruture

Definition (k-resiliency)

k-resiliency is the capacity for a system to repair itself and operate correctly even in the case of the disappearance of up to k agents

- Two parts:
  - Do not loose the definition of the computations: **replication**
  - Migrate the orphaned computations to another agent: **selection** / activation
- Apply to any graph of computations, not only DCOP

Replication of computations

Replica distribution

- For each computation, place k replica on k other agents
  replica $equiv$ definition of the computation
- Must be distributed !
- Optimal replication ? impact the set of available agents when repairing
  which criteria ? too hard (quadratic multiple knapsack problem)...

Distributed Replica Placement Method (DRPM)

- Heuristic : place replica on agents close (network) the active computation, while respecting capacity
- Distributed version of iterative lengthening (aka uniform cost search based on path costs)

Replication of computations

iterative lengthening on route and hosting costs

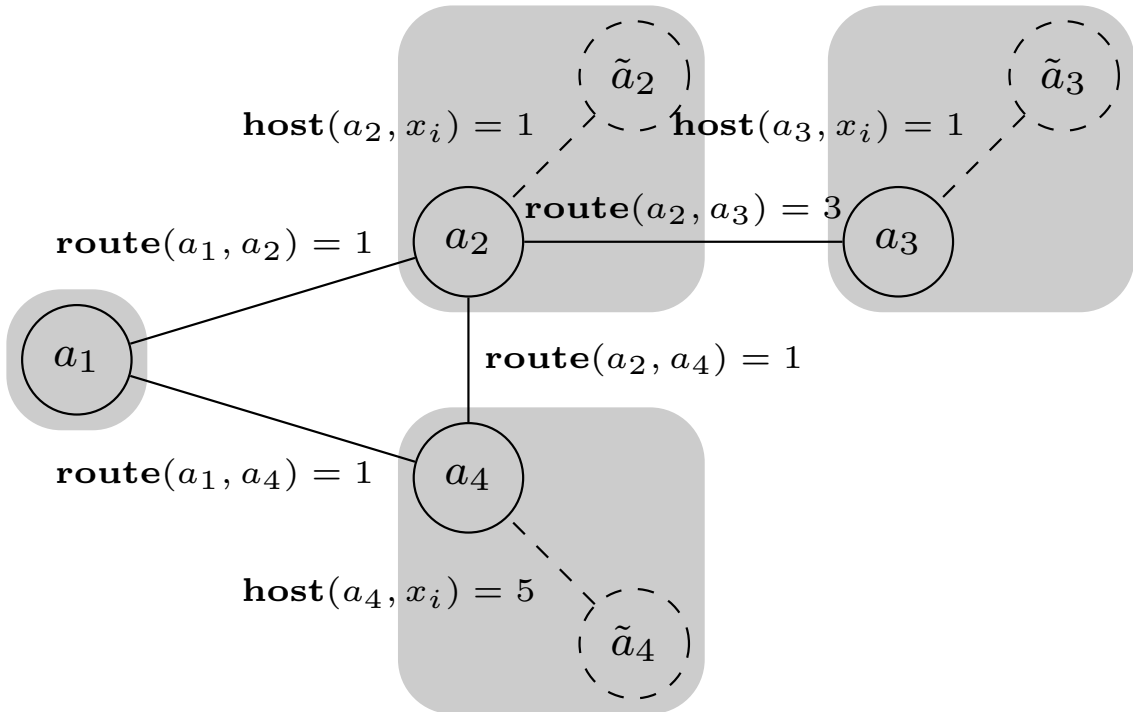


Figure: A sample **route+host**–graph with 4 agents (in gray), where a_1 search for hosting computation x_i. For $k = 2$, DRMP places a replica on a_2 (cost of $1 + 1 = 2$) and another on a_3 (cost of $1 + 3 + 1 = 5$) if enough capacity on these two agents, since the minimum cost path to host on a_4 is higher ($1 + 5 = 6$).

Migrating computations

Selecting an agent

Migrating a set of x_i computations X_c

- set of candidate agents A_c
- migrating the computation must not exceed agent's capacity
- for each computation, select the agent that minimize hosting and communication cost

Same optimization problem than for initial distribution, but on a subset of the of the graph

Distributed process !

Migrating computations

Selecting an agent

Distributed optimization problem $\Rightarrow$ let's use a DCOP!

- $\mathcal{A}$ is the set of candidate agents A_c
- $\mathcal{X}$ are the binary decision variables x_i^m
- $\mathcal{C}$ are the constraints ensuring that all computations are hosted, agent's capacities are respected and hosting and communication costs are minimized

Migrating computations

Selecting an agent

$$\sum_{a_m \in A_c^i} x_i^m = 1 \tag{13}$$

$$\sum_{x_i \in X_c^m} \mathbf{weight}(x_i) \cdot x_i^m + \sum_{x_j \in \mu^{-1}(a_m) \setminus X_c} \mathbf{weight}(x_j) \leq \mathbf{cap}(a_m) \tag{14}$$

$$\sum_{x_i \in X_c^m} \mathbf{host}(a_m, x_i) \cdot x_i^m \tag{15}$$

$$\begin{aligned} &\sum_{(x_i, x_j) \in X_c^m \times N_i \setminus X_c} x_i^m \cdot \mathbf{com}(i, j, m, \mu^{-1}(x_j)) \\ + &\sum_{(x_i, x_j) \in X_c^m \times N_i \cap X_c} x_i^m \cdot \sum_{a_n \in A_c^j} x_j^n \cdot \mathbf{com}(i, j, m, n)) \end{aligned} \tag{16}$$

Decentralized reparation

When agents are removed:

- computation to migrate = computation that where hosted on these agents
- candidate agents = remaining agents that posses a replica of these orphaned computation

Solving the migration DCOP

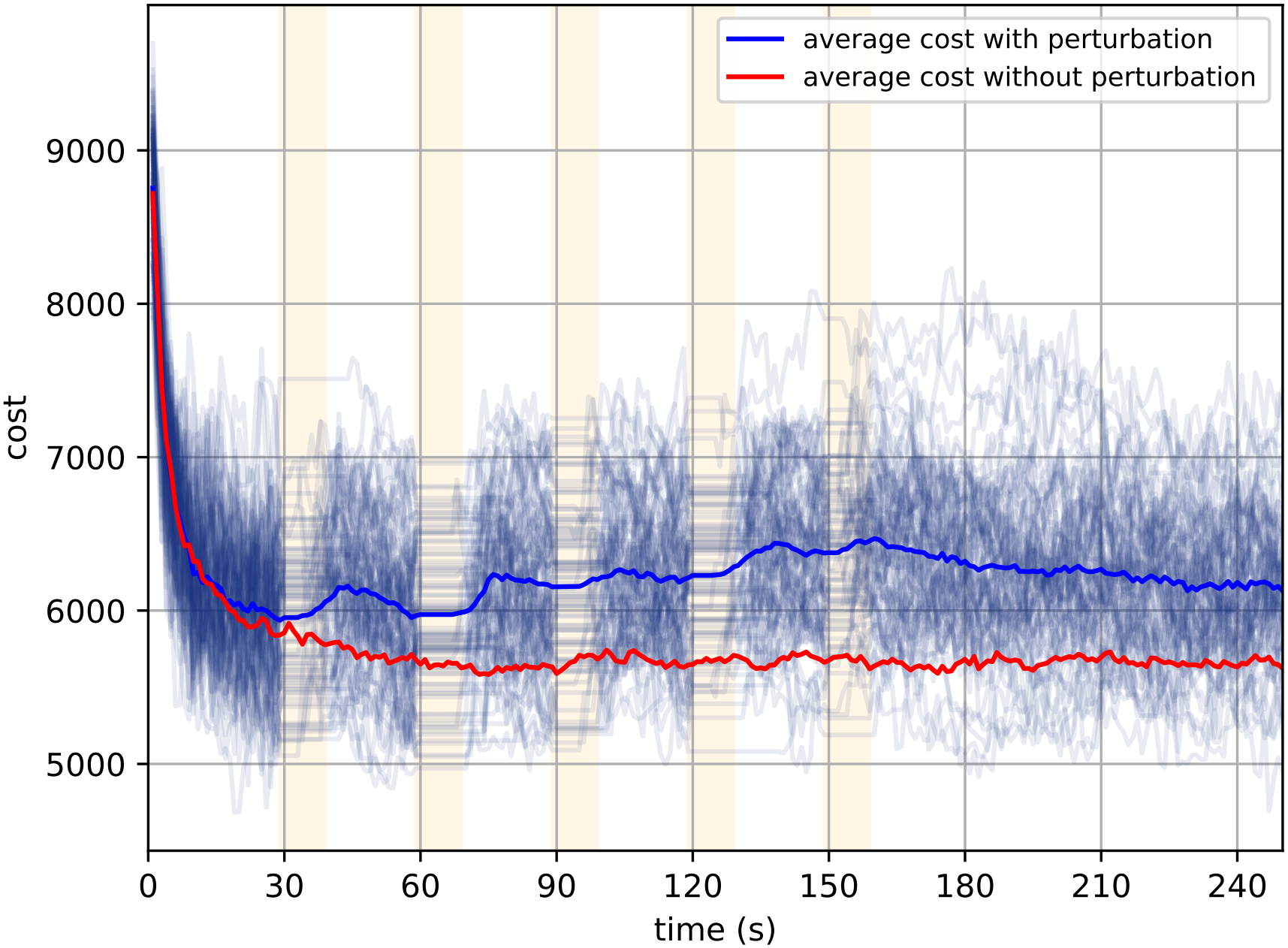
Which algorithm should we use ?

Criteria:

- lightweight
- fast (even if not optimal !)
- monotonic : mix of hard and soft constraints

MGM-2 : like MGM, with 2-coordination

Experimental results



Menu

To sum up

What we've seen today:

- Some generic concepts
  - How to model coordination problems using DCOP formalism
  - Some solution methods (complete and incomplete) to solve DCOP
- Some specificities of IoT-based apps
  - How to model a specific smart environment configuration problem as a DCOP
  - How to use PyDCOP to model, run, solve, and distribute DCOP
  - How to equip a system with resilience using replication and DCOP-based reparation

- Want to go deeper into DCOPs $\rightarrow$ OPTMAS-DCR workshop series (AAMAS/IJCAI), other tutorials at AAMAS/IJCAI

The End

References

AJI, S.M. and R.J. MCELIECE (2000). "The generalized distributive law". In: *Information Theory, IEEE Transactions on* 46.2, pp. 325–343. ISSN: 0018-9448. DOI: 10.1109/18.825794.

BOULLE, M. (2004). "Compact Mathematical Formulation for Graph Partitioning". In: *Optimization and Engineering* 5.3, pp. 315–333. ISSN: 1573-2924. DOI: 10.1023/B:OPTE.0000038889.84284.c7. URL: http://dx.doi.org/10.1023/B:OPTE.0000038889.84284.c7.

BÜRGER, M., G. NOTARSTEFANO, F. BULLO, and F. ALLGÖWER (2012). "A distributed simplex algorithm for degenerate linear programs and multi-agent assignments". In: *Automatica* 48.9, pp. 2298 –2304. ISSN: 0005-1098. DOI: http://dx.doi.org/10.1016/j.automatica.2012.06.040. URL: //www.sciencedirect.com/science/article/pii/S0005109812002956.

FARINELLI, A., A. ROGERS, A. PETCU, and N. R. JENNINGS (2008). "Decentralised Coordination of Low-power Embedded Devices Using the Max-sum Algorithm". In: *Proceedings of the 7th International Joint Conference on Autonomous Agents and Multiagent Systems - Volume 2*. AAMAS '08. International Foundation for Autonomous Agents and Multiagent Systems, pp. 639–646. ISBN: 978-0-9817381-1-6.

FIORETTO, F., E. PONTELLI, and W. YEOH (2018). "Distributed Constraint Optimization Problems and Applications: A Survey". In: *Journal of Artificial Intelligence Research* 61, pp. 623–698.

FITZPATRICK, Stephen and Lambert MEERTENS (2003). "Distributed Coordination through Anarchic Optimization". In: *Distributed Sensor Networks: A Multiagent Perspective*. Ed. by Victor LESSER, Charles L. ORTIZ, and Milind TAMBE. Boston, MA: Springer US, pp. 257–295. ISBN: 978-1-4615-0363-7.

MAHESWARAN, R.T., J.P. PEARCE, and M. TAMBE (2004). "Distributed Algorithms for DCOP: A Graphical-Game-Based Approach". In: *Proceedings of the 17th International Conference on Parallel and Distributed Computing Systems (PDCS), San Francisco, CA*, pp. 432–439.

PETCU, Adrian and Boi FALTINGS (2005). "A scalable method for multiagent constraint optimization". In: *IJCAI International Joint Conference on Artificial Intelligence*, pp. 266–271. ISBN: 1045-0823.

References (cont.)

RUST, P., G. PICARD, and F. RAMPARANY (2016). "Using Message-passing DCOP Algorithms to Solve Energy-efficient Smart Environment Configuration Problems". In: *International Joint Conference on Artificial Intelligence (IJCAI)*. AAAI Press.

— (2017). "On the Deployment of Factor Graph Elements to Operate Max-Sum in Dynamic Ambient Environments". In: *Autonomous Agents and Multiagent Systems – AAMAS 2017 Workshops, Best Papers, Sao Paulo, Brazil, May 8-12, 2017, Revised Selected Papers*. Ed. by G. SUKTHANKAR and J.A. RODRIGUEZ-AGUILAR. Vol. 10642. Lecture Notes in Artificial Intelligence (LNAI). Extended Version. Springer, pp. 116–137. DOI: 10.1007/978-3-319-71682-4_8.

VINYALS, Meritxell, Juan A. RODRÍGUEZ-AGUILAR, and Jesus CERQUIDES (2011). "Constructing a unifying theory of dynamic programming DCOP algorithms via the generalized distributive law". In: *Autonomous Agents and Multi-Agent Systems* 3.22, pp. 439–464. ISSN: 1387-2532. DOI: 10.1007/s10458-010-9132-7.

WEISS, Yair (2000). "Correctness of Local Probability Propagation in Graphical Models with Loops". In: *Neural Comput.* 12.1, pp. 1–41. ISSN: 0899-7667. DOI: 10.1162/089976600300015880. URL: http://dx.doi.org/10.1162/089976600300015880.

ZHANG, W., G. WANG, Z. XING, and L. WITTENBURG (2005). "Distributed stochastic search and distributed breakout: properties, comparison and applications to constraint optimization problems in sensor networks.". In: *Journal of Artificial Intelligence Research (JAIR)* 161.1-2, pp. 55–87.

ZHANG, Weixiong, Guandong WANG, Zhao XING, and Lars WITTENBURG (2003). "A Comparative Study of Distributed Constraint Algorithms". In: *Distributed Sensor Networks: A Multiagent Perspective*. Ed. by Victor LESSER, Charles L. ORTIZ, and Milind TAMBE. Boston, MA: Springer US, pp. 319–338.