Negotiation Strategy for Self-Organizing Stable Matchings

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Introduction

Our objectives

- Stable Matching Problems with **no central orchestrator**
- Multi-agent framework to distributively solve assignment problems
- Assignments are viewed as emergent phenomena resulting from local agent negotiations: *stability*, *social optimal welfare*

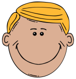
Casanova

- Distributed method to solve the stable marriage problem
- Agent behaviors leading negotiation processes to *social optimal assignments*
- *Minimal concession strategy*
- Preserves the privacy
- Improves the optimality of the solution and the equity amongst partners

Stable Marriage Problem

Alain

Barbara $\succ$ Alice $\succ$ Carla



Bob

Carla $\succ$ Barbara $\succ$ Alice

Charles

Alice $\succ$ Carla $\succ$ Barbara

Alice

Bob $\succ$ Alain $\succ$ Charles

Barbara

Charles $\succ$ Bob $\succ$ Alain

Carla

Alain $\succ$ Charles $\succ$ Bob

Stable Marriage Problem: Definition

- Subclass of stable matching problems [GS62]
- Two finite sets of participants: men and women (or *communities*)

Definition (Stable Marriage Problem)

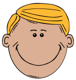
A **stable marriage problem** of size n (with $n \geqslant 1$) is a couple $\mathcal{SM} = \langle X, Y \rangle$ where:

- $X = \{x_1, \dots, x_n\}$ is a set n men ranking women in a strict and complete order forming his preference list.
  $\forall 1 \leqslant i \leqslant n,\ x_i = (y_i^0, \dots, y_i^{n-1})$
- $Y = \{y_1, \dots, y_n\}$ is a set n women ranking men in a strict and complete order forming her preference list.
  $\forall 1 \leqslant i \leqslant n,\ y_i = (x_i^0, \dots, x_i^{n-1})$

Matchings

Alain

Barbara$\succ$Alice$\succ$Carla



Bob

Carla$\succ$Barbara$\succ$Alice

Charles

Alice$\succ$Carla$\succ$Barbara

Alice

Bob$\succ$Alain$\succ$Charles

Barbara

Charles$\succ$Bob$\succ$Alain

Carla

Alain$\succ$Charles$\succ$Bob

Alain–Barbara

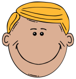
Bob–Alice

Charles–Carla

Unstable Matchings

Alain

Barbara $\succ$ Alice $\succ$ Carla



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Alice $\succ$ Carla $\succ$ Barbara

Alice

Bob $\succ$ Alain $\succ$ Charles

Barbara

Charles $\succ$ Bob $\succ$ Alain

Carla

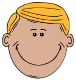
Alain $\succ$ Charles $\succ$ Bob

Alain–Barbara
Bob–Alice
Charles–Carla

Stable Matchings

Alain

Barbara$\succ$Alice$\succ$Carla



Bob

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Charles

Alice$\succ$Carla$\succ$Barbara

Alice

Bob$\succ$Alain$\succ$Charles

Barbara

Charles$\succ$Bob$\succ$Alain

Carla

Alain$\succ$Charles$\succ$Bob

Alain–Alice
Bob–Barbara
Charles–Carla

Charles–Alice
Alain–Barbara
Bob–Carla

Bob–Alice
Charles–Barbara
Alain–Carla

Stable Matchings: Definition

Definition (Matching)

Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geqslant 1$). A **matching** for SM is a n-uplet $M = \langle m_1, \ldots, m_n \rangle$ of n marriages where each m_i (with $1 \leqslant i \leqslant n$) is a couple $(x_i, y_i) \in X \times Y$ such that the matching is complete, i.e. each individual is married. Formally, $\forall x \in X\ \exists! y \in Y\ (x, y) \in M$.

Definition (Stable Matching)

Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $1 \geqslant n$). and M a matching for SM. M is **stable** iff:
$\forall (x_i, y_i) \in M,\ \nexists (x_j, y_j) \in M\ x_j \succ_{y_i} x_i$ and $y_j \succ_{x_i} y_i$

$\Rightarrow$ No pair of people like each other better than their current spouse
$\Rightarrow$ Find a stable assignment that is *optimal* with respect to a metric that depends on the preferences of the agents

Social Welfare

Social Welfare

- Aggregation of individual preferences can be modelled using the notion of social welfare as studied in Welfare Economics and Social Choice Theory [ASS02]
- individual agents evaluate their satisfaction using utility functions mapping assignments to numerical values
- The social welfare theory is used to evaluate the matching, considering the welfare of each person [ASS02]

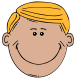
Definition (Utility function)

Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geqslant 1$), $z = (t_i^0, \ldots, t_i^k, \ldots, t_i^{n-1})$ an individual agent and T be the potential partners of z. The **utility function** of the agent z is a function $u_z : T \to \mathbb{R}$. If the matching assigns z with t_i^k, then $u_z(t_i^k) = \frac{(n-1)-k}{n-1}$.

Social Welfare in SM

Alain

Barbara≻Alice≻Carla



Bob

Carla≻Barbara≻Alice

Charles

Alice≻Carla≻Barbara

Alice

Bob≻Alain≻Charles

Barbara

Charles≻Bob≻Alain

Carla

Alain≻Charles≻Bob

Alain–Alice
Bob–Barbara
Charles–Carla
$u_M = 1.5$
$u_W = 1.5$

Charles–Alice
Alain–Barbara
Bob–Carla
$u_M = 3$
$u_W = 0$

Bob–Alice
Charles–Barbara
Alain–Carla
$u_M = 0$
$u_W = 3$

Social Welfare in SM: Definition

Definition (Social welfare)

Let $SM = \langle X, Y \rangle$ be a stable marriage problem of size n (with $n \geqslant 1$) and M a matching for SM.

- The **utilitarian welfare** considers the welfare of the whole society: $sw_u(X \cup Y) = \Sigma_{z \in X \cup Y} u_z(p_M(z))$.
- The **male welfare** considers the welfare of the men: $sw_u(X) = \Sigma_{x \in X} u_x(p_M(x))$.
- The **female welfare** considers the welfare of the women: $sw_u(Y) = \Sigma_{y \in Y} u_y(p_M(y))$.
- The **equity welfare** considers the fairness among partners' welfare in every marriage: $sw_e(X \cup Y) = 1 - \frac{|sw_u(X) - sw_u(Y)|}{n}$.

Social Welfare in SM: Definition (cont.)

Social welfare $\approx$ measure of the global quality of a matching

- e.g. the utilitarian social welfare a suitable indicator when we are considering the mean of agent's utilities whatever they are male or woman
- e.g. the male welfare (resp. female welfare) measures the quality of a matching from the viewpoint of the men (resp. women)
- e.g. the equity welfare may be a suitable indicator when we have to satisfy both the men and the women

Gale-Shapley Algorithm

First Algorithm for SMP [GS62]

- Centralized algorithm
- Always finds a stable matching for any instance of the SMP
- Produces a matching in which each man has the best partner he can have

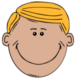
assign each person to be free
while *a man* x *is free* **do**
  $y \leftarrow$ first woman on x's list
  `//x proposes to y`
  if *a man* x_y *is already married to* y **then**
    assign x_y to be free
  assign x and y to be married to each other
  for *each successor* x_y *of* x *on* y*'s list* **do**
    delete x_y from y's list
    delete y from x_y's list

Algorithm 1: The men-propose Gale-Shapley algorithm for SM

GS Algorithm Sample Execution

Alice

Charles



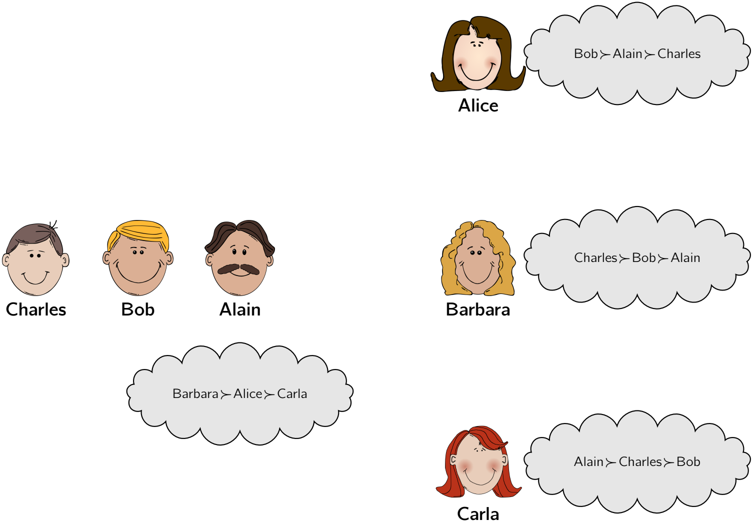
Bob

Alain

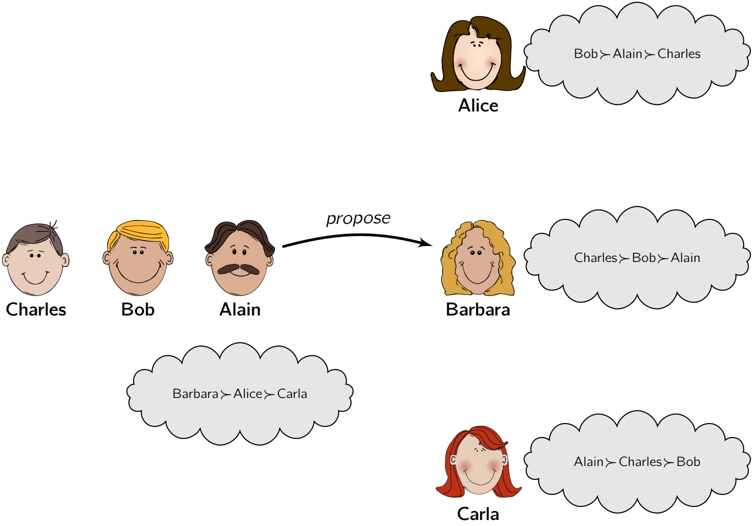
Barbara

Carla

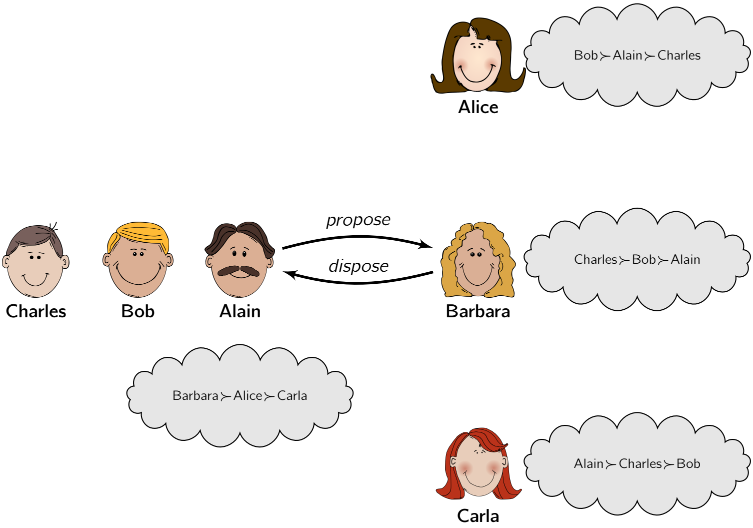
GS Algorithm Sample Execution



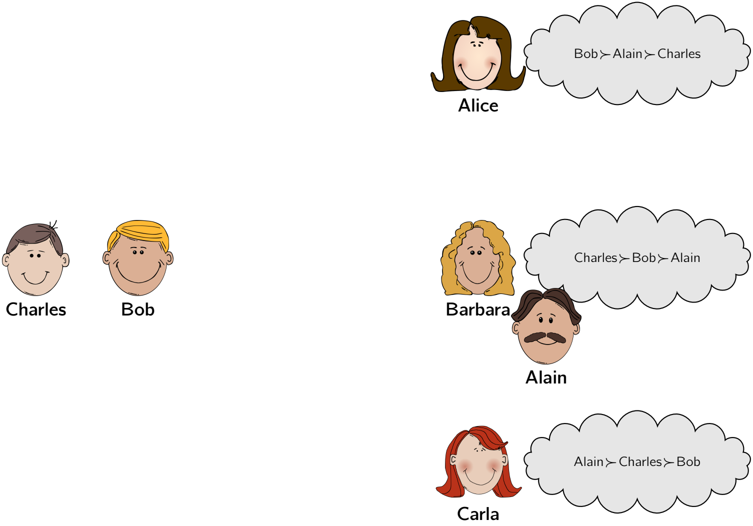
GS Algorithm Sample Execution



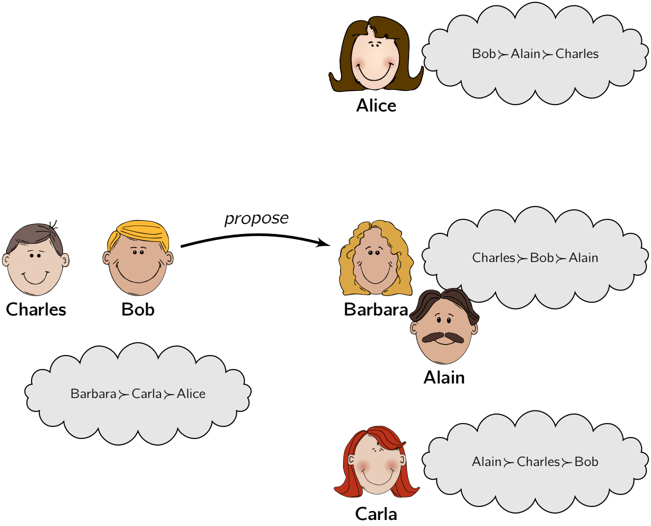
GS Algorithm Sample Execution



GS Algorithm Sample Execution



GS Algorithm Sample Execution



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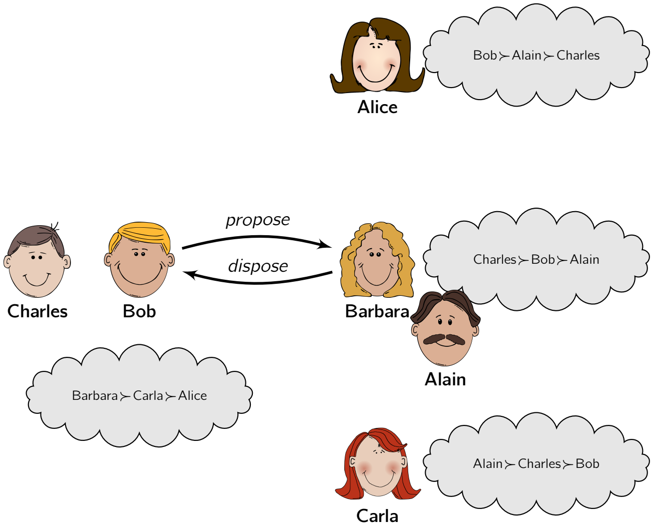

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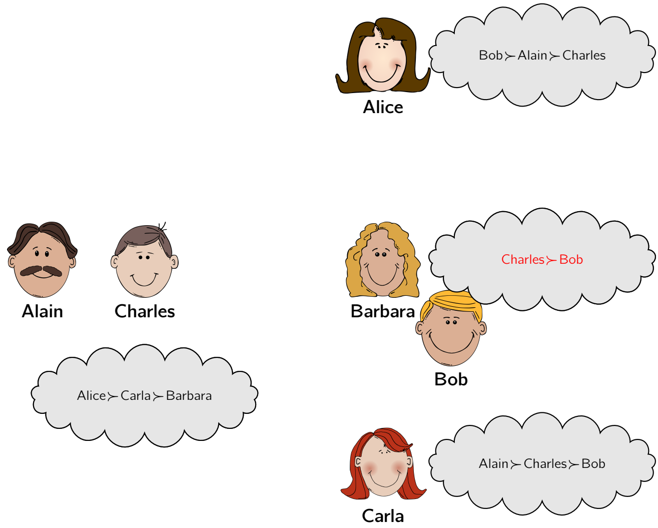

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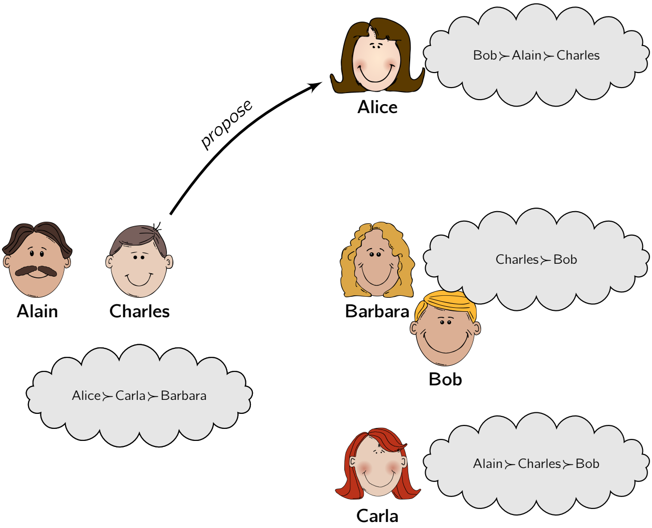

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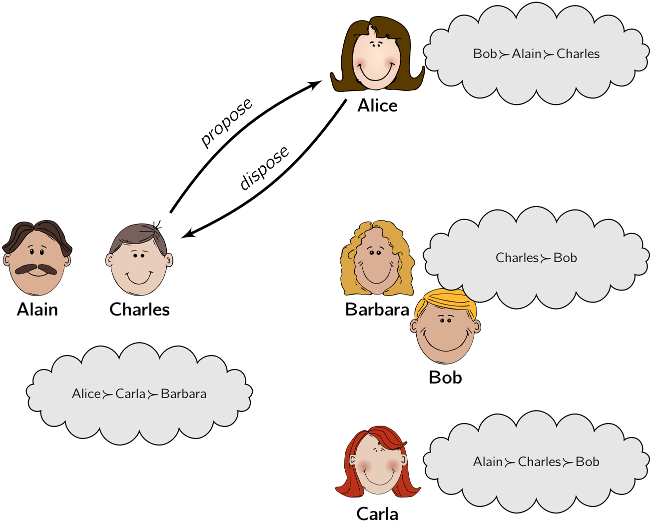

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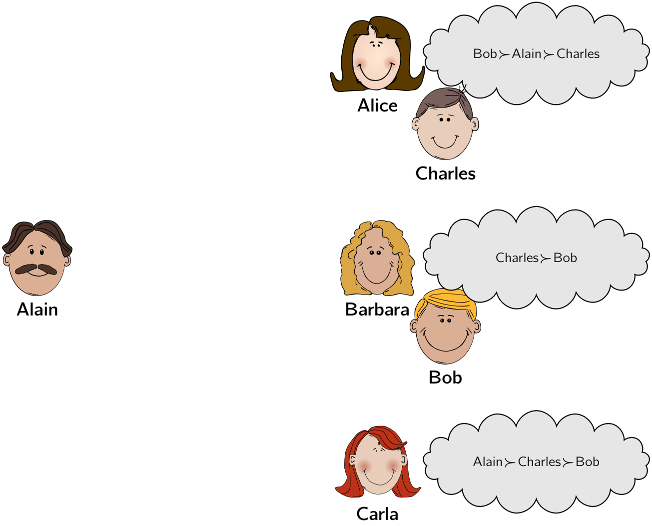

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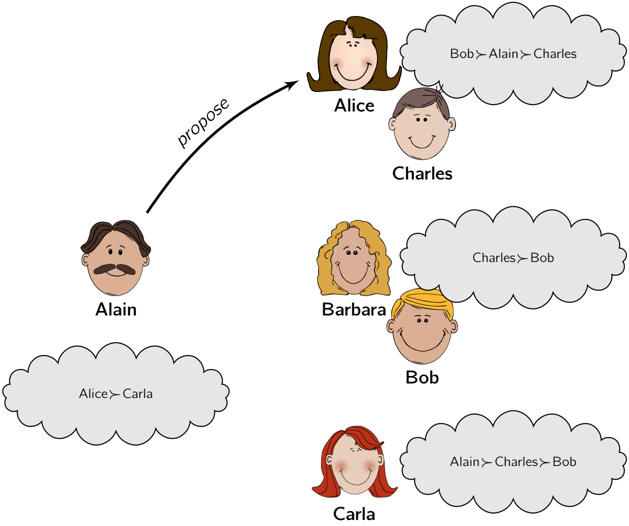

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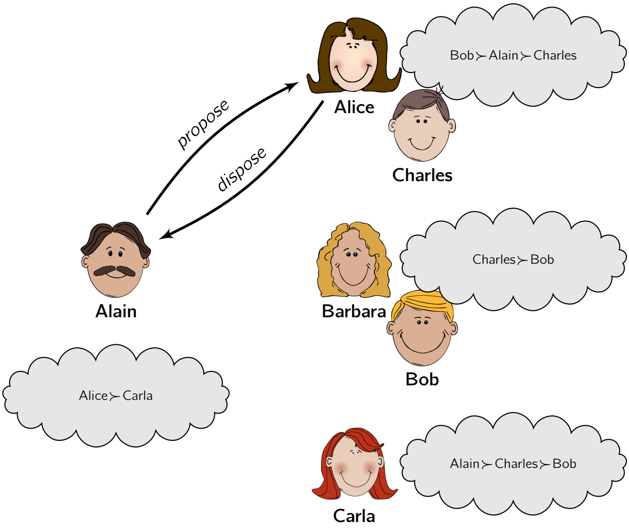

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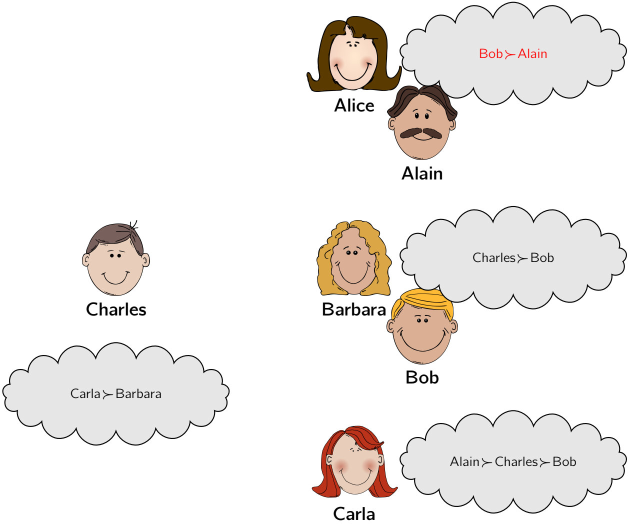

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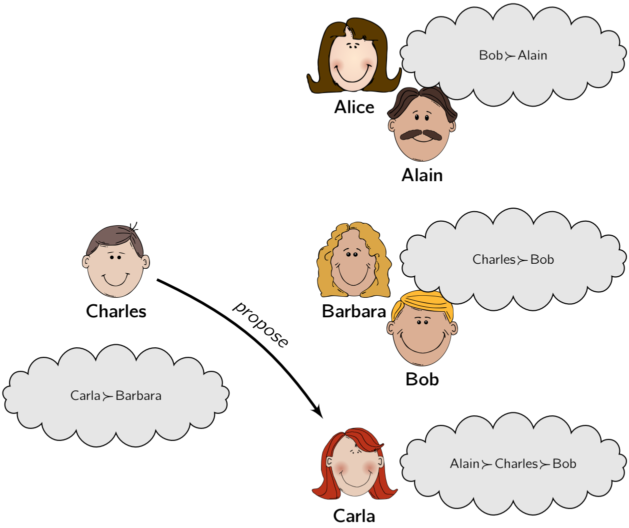

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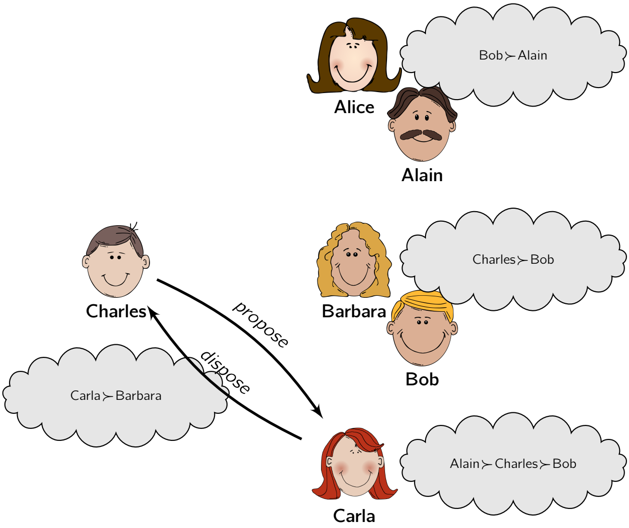

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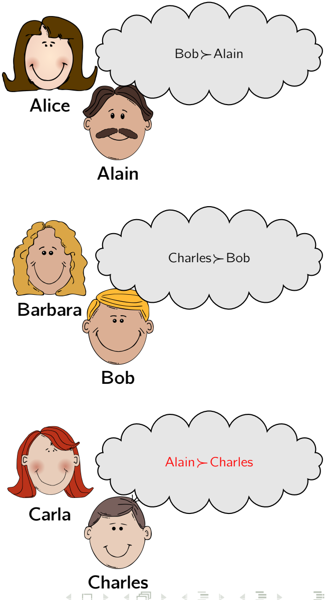
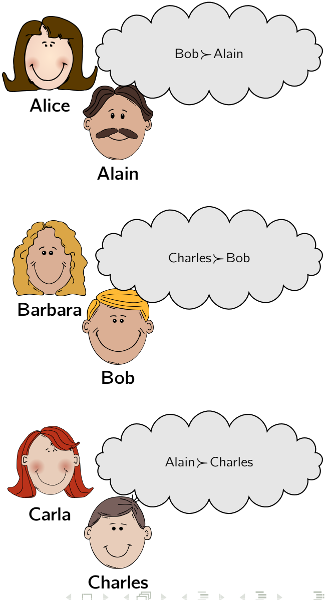


GS Algorithm Sample Execution



Gale-Shapley Algorithm (cont.)

Distributed Extended Gale-Shapley Algorithm (DisEGS) [BM05]

- Distributed version of the GS algorithm
- Each man (and woman) is represented by an agent
- Agents exchange messages (*propose*, *accept* and *delete*) as to reproduce the GS algorithm and find a stable assignment
- Agents do not follow the same procedure:
  - men propose
  - women accept or decline proposals
- each agent keeps its own preferences → privacy

Casanova Algorithm

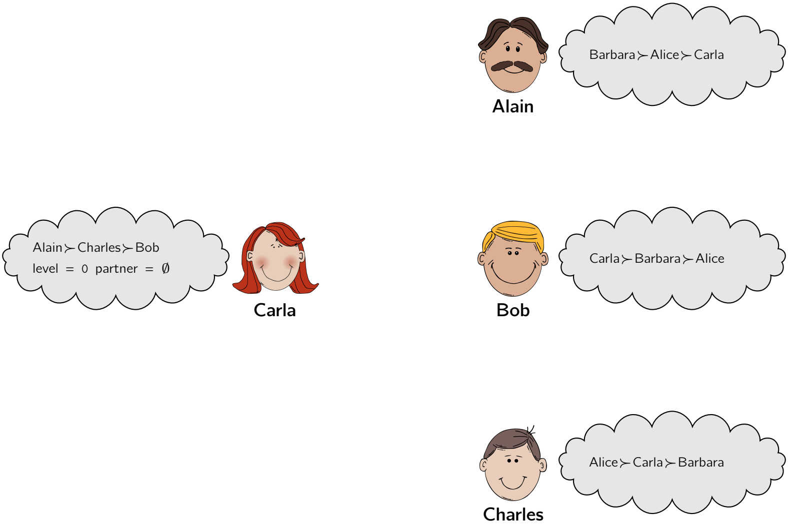
Casanova Characteristics

- Multi-agent algorithm: men and women are agents
- Privacy preserving: preferences are not shared
- Equity preserving: men and women follow the same protocol
- Optimal wrt social utilitarian welfare

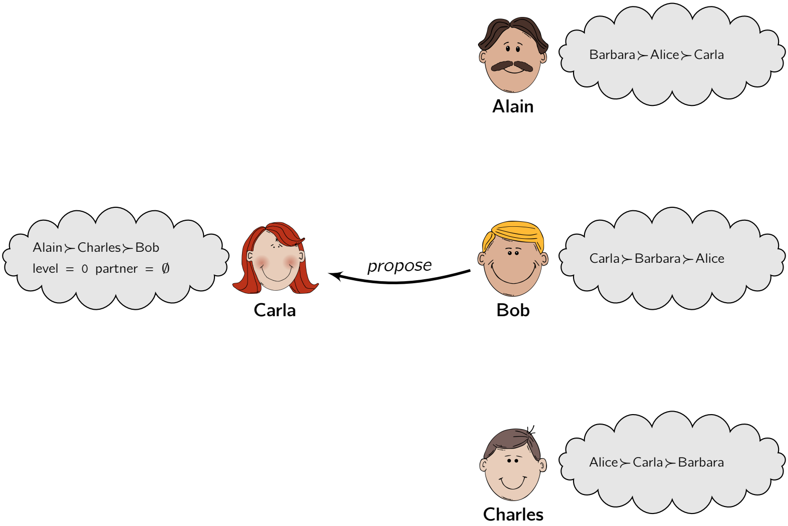
Casanova Principles

- Agents send concurrently proposals and reply with acceptance or rejections
- Agents use the minimal concession strategy [RZ94, MM10]
  - from the most preferred partners
  - to the less preferred ones
  - but still trying to attract the preferred ones

Casanova Sample Execution



Casanova Sample Execution



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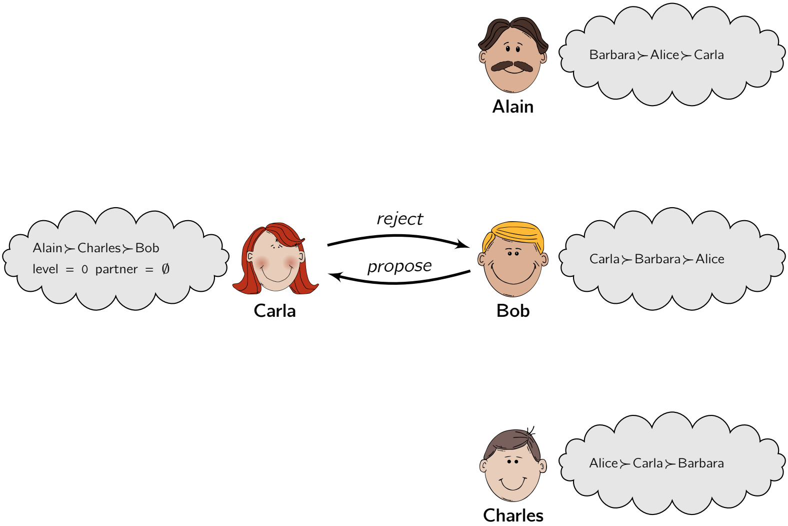

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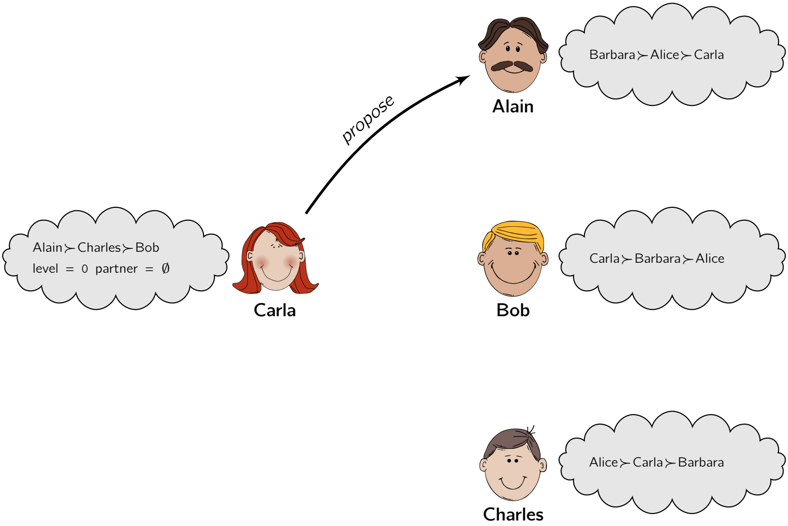

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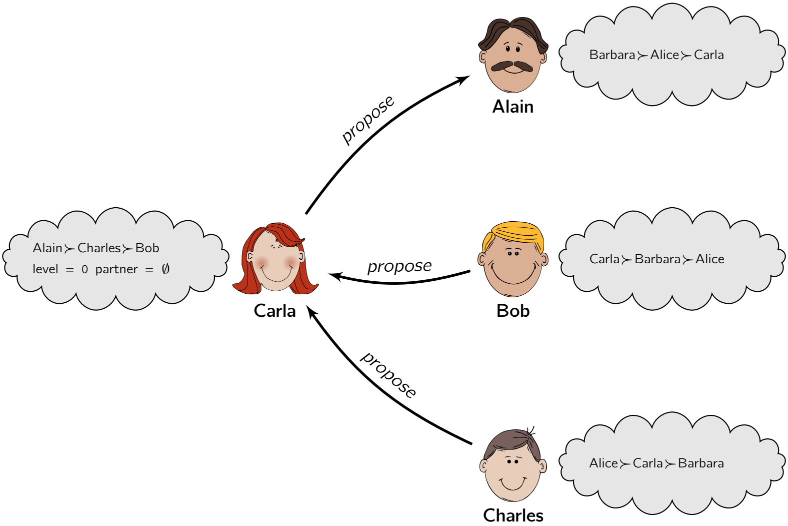

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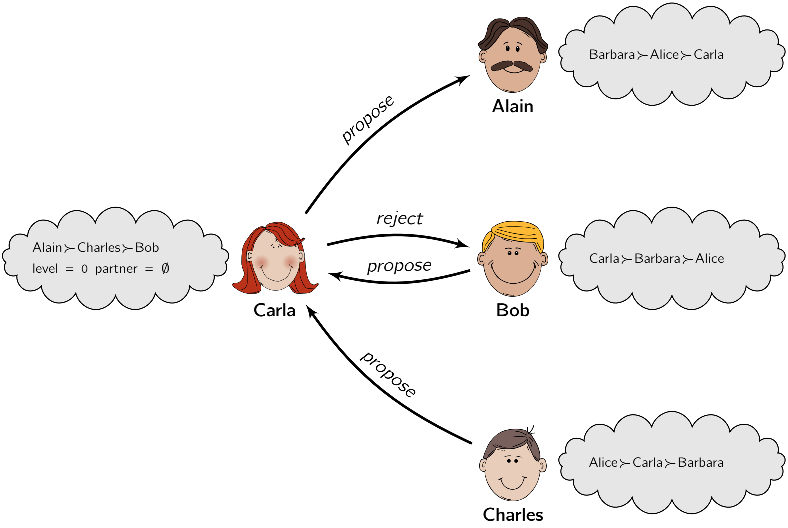

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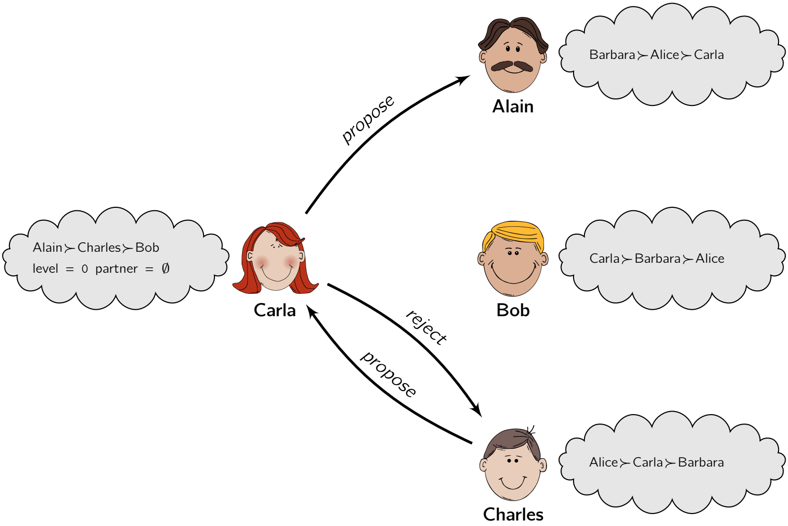

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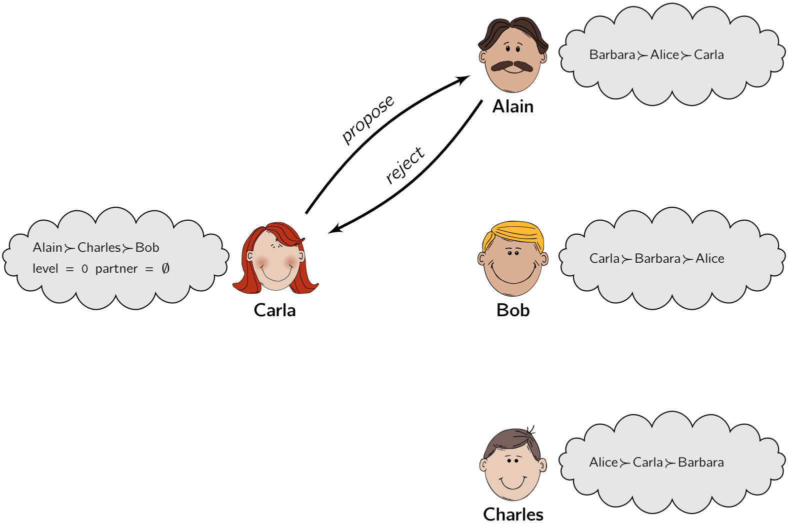

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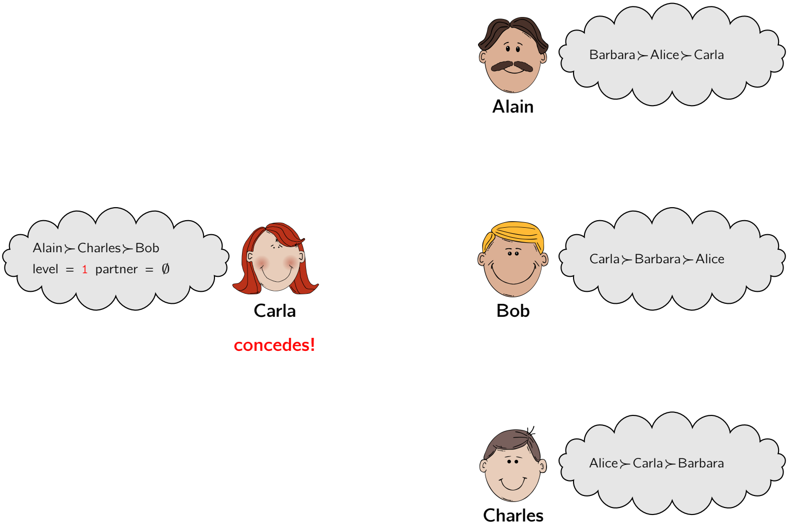

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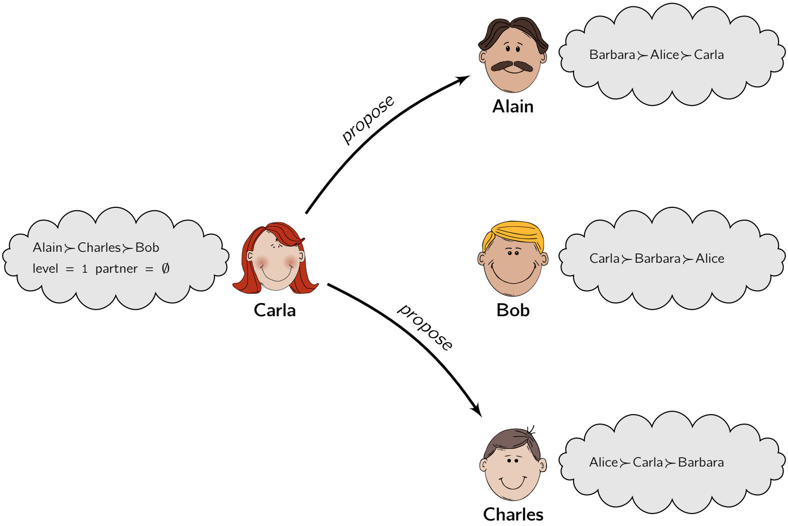

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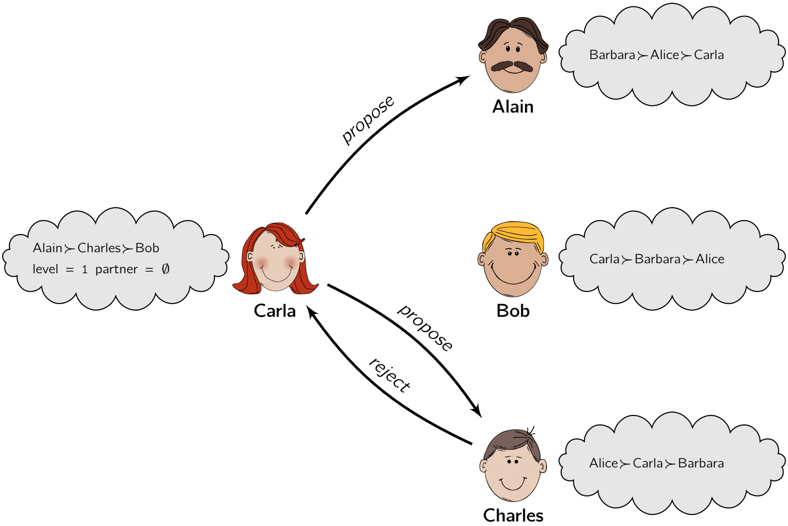

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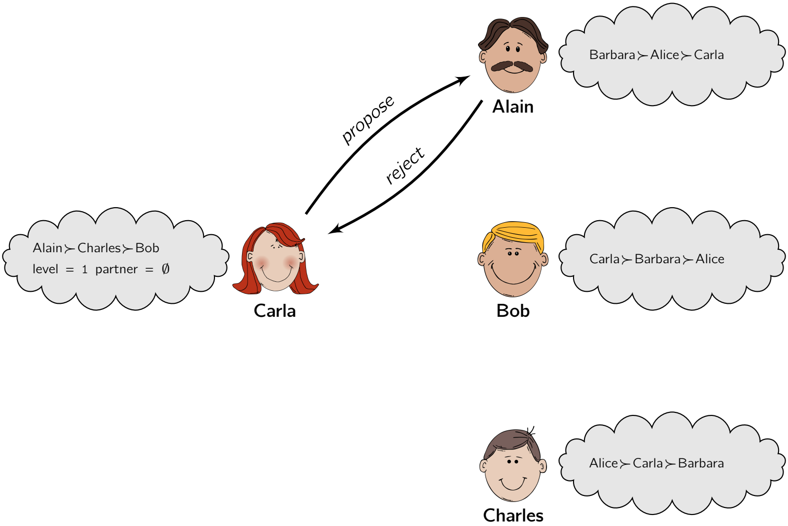

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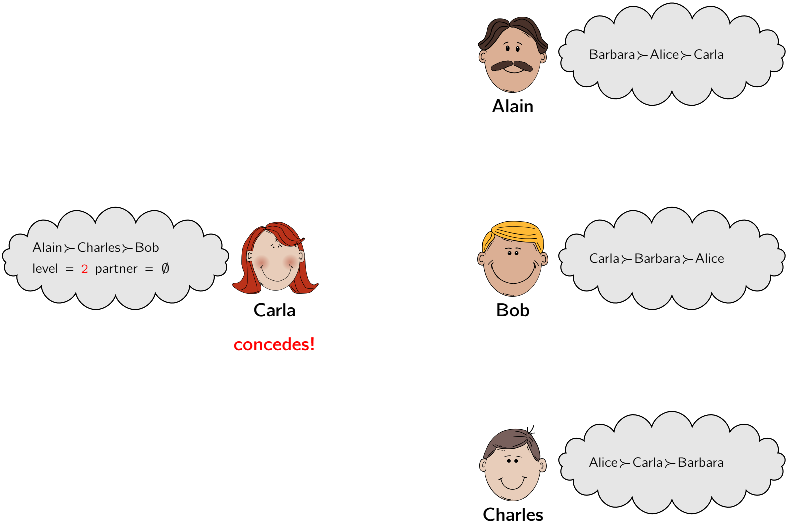

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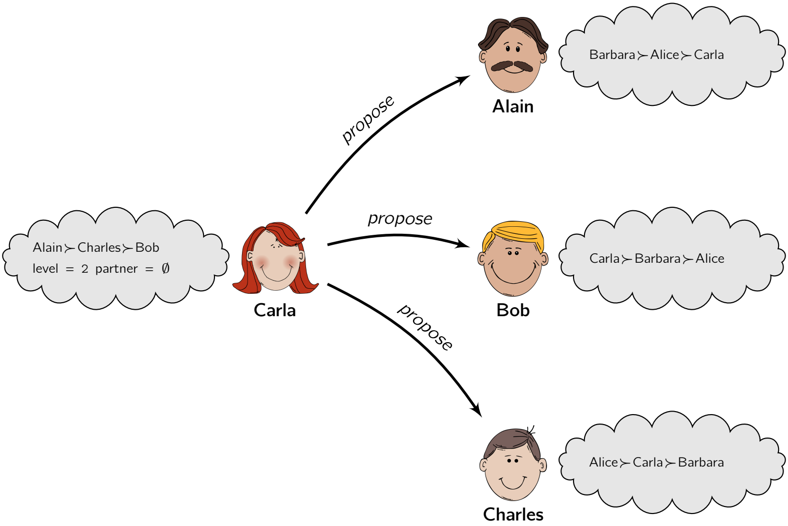

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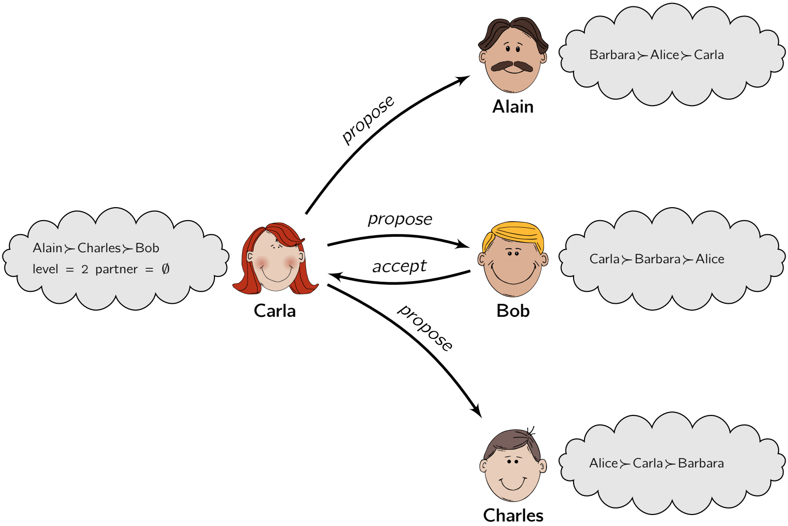

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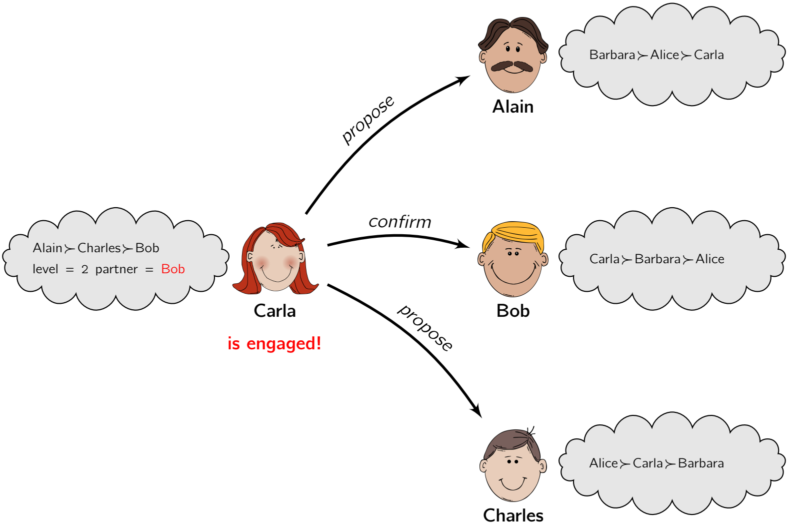

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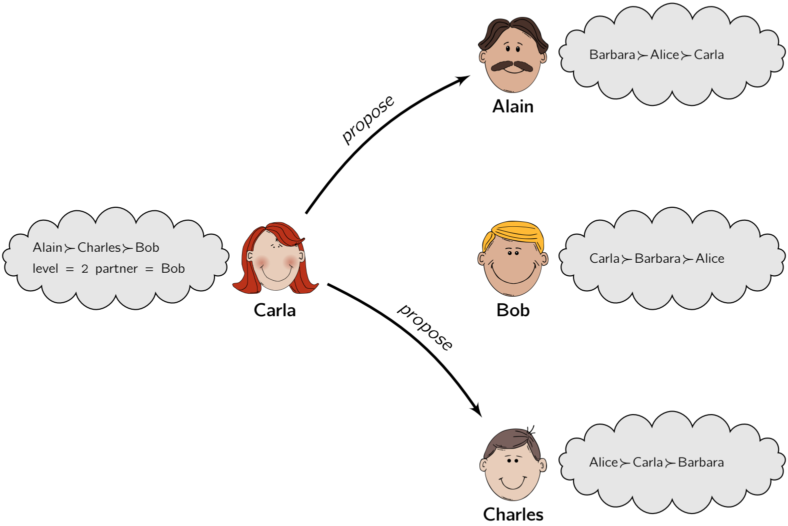

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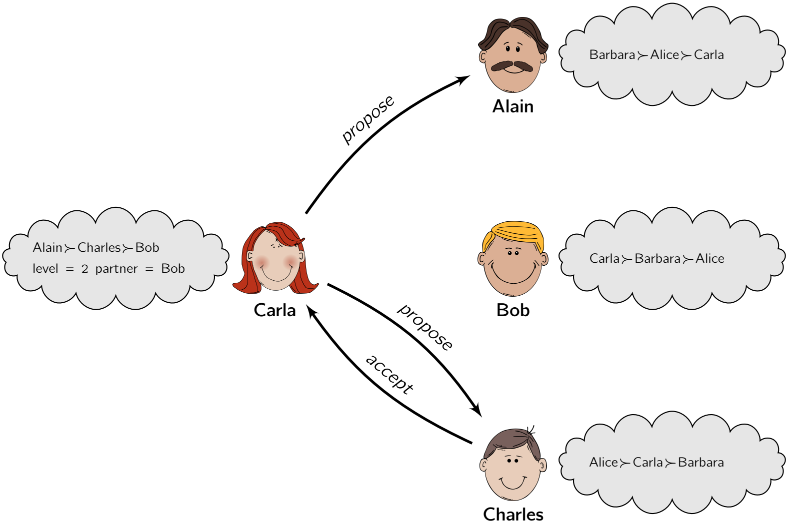

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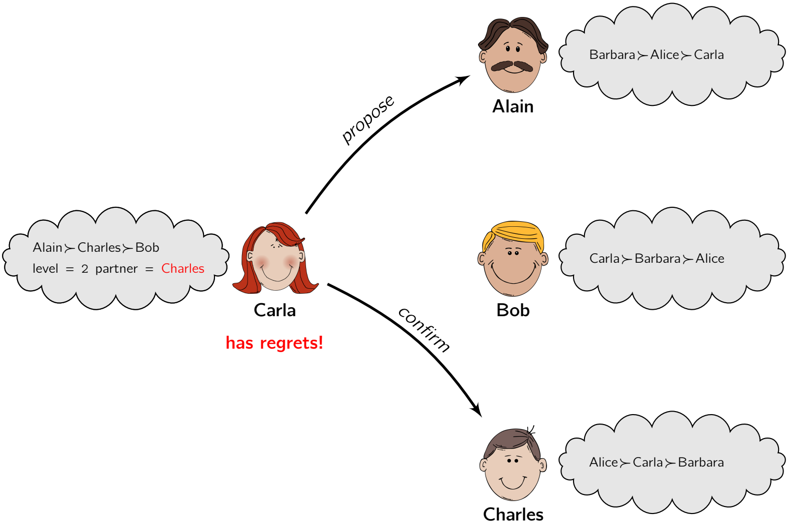

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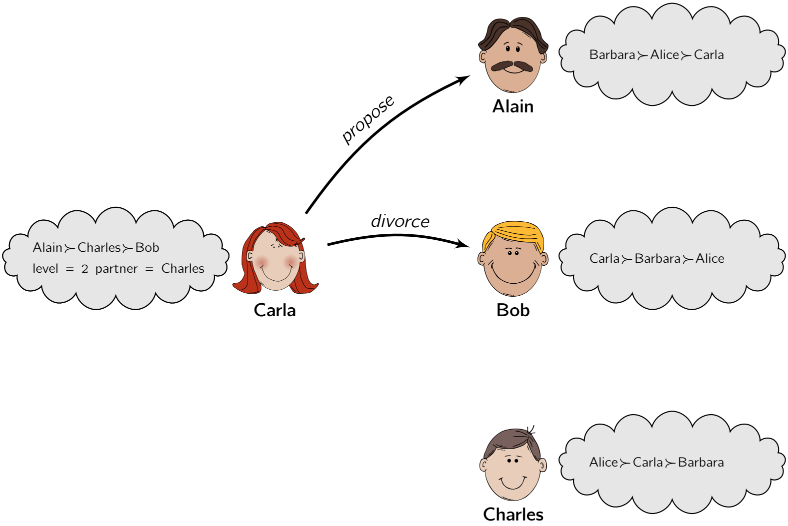

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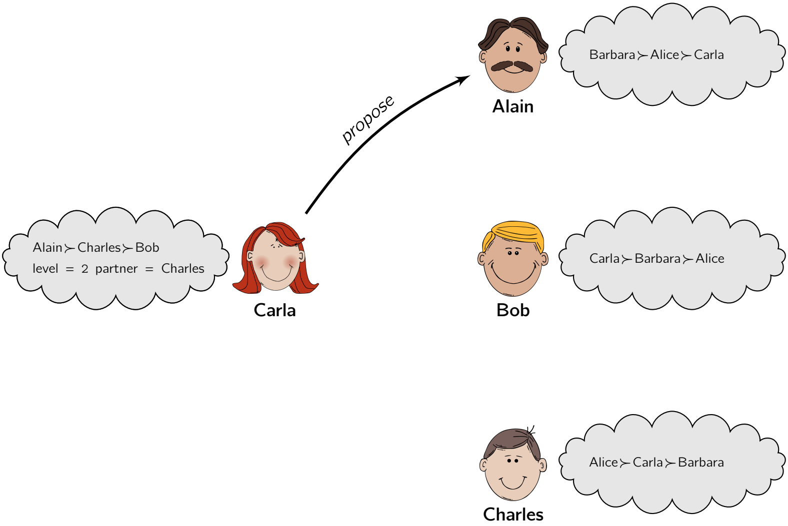

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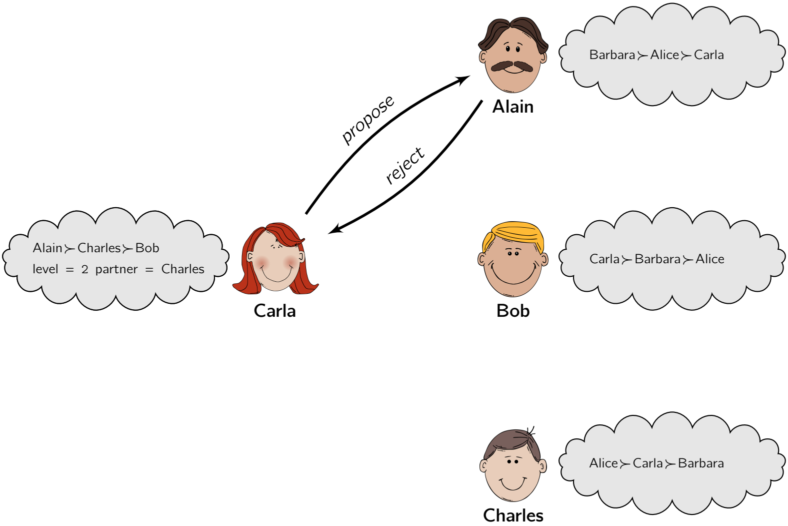

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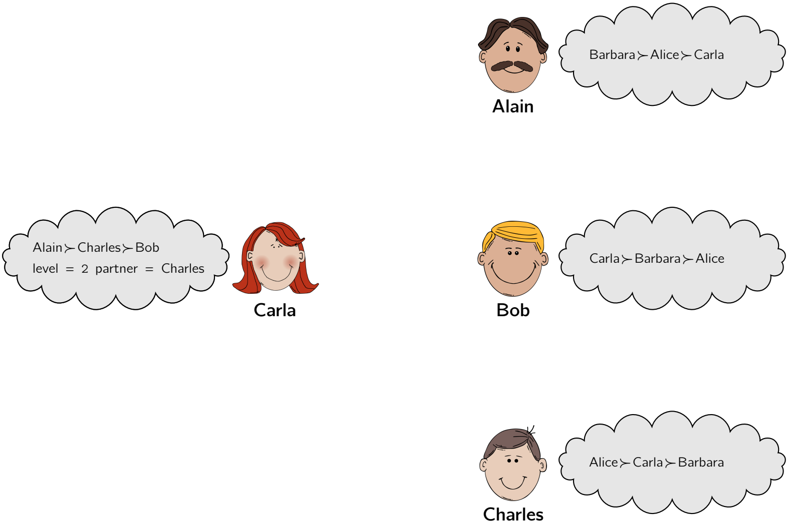
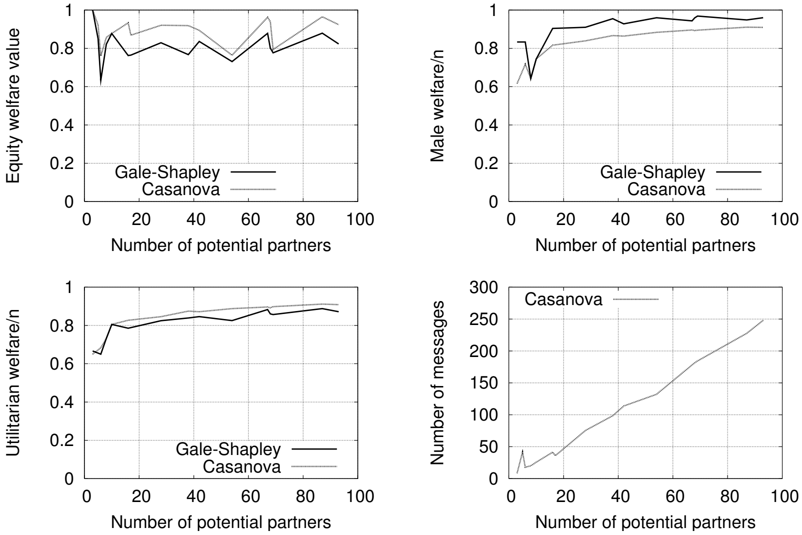


Evaluation


Equity welfare value
Gale-Shapley
Casanova
Number of potential partners
Male welfare/n
Gale-Shapley
Casanova
Number of potential partners
Utilitarian welfare/n
Gale-Shapley
Casanova
Number of potential partners
Number of messages
Casanova
Number of potential partners

Conclusion

Results

- Optimal matching = **emergent result** from the local interactions
- Realisation of the minimal concession strategy
- Preserves privacy
- Improves the optimality of the matching and the equity amongst the partners

Future Works

- Realize more experiments for evaluating other metrics of social welfare
- The assignment problem we consider here should be generalized in two directions:
  - SM with incomplete list and total order (e.g. home swapping)
  - SM with incomplete list and indifference (e.g. carpool)

Questions?

References

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